Computer Vision HomeWork 02

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October 2017

1 Estimation of the Intrinsic and Extrinsic Parameters

The value of m_1^T, m_2^T, m_3^T and m_{14}, m_{24} have known, please give the procedures to computer the intrinsic and extrinsic parameters.

$$m_{34}\begin{bmatrix} \boldsymbol{m}_1^T & m_{14} \\ \boldsymbol{m}_2^T & m_{24} \\ \boldsymbol{m}_3^T & 1 \end{bmatrix} = \begin{bmatrix} \alpha\boldsymbol{r}_1^T + u_0\boldsymbol{r}_3^T & \alpha t_x + u_0 t_z \\ \beta\boldsymbol{r}_2^t + \upsilon_0\boldsymbol{r}_3^T & \beta t_y + \upsilon_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{bmatrix} \tag{1}$$

we know that m_{34} is a unknown scale factor, introduced here to account for the fact that we assumed $M_{34} = 1$ to get unique solution.

From eq. (1) we can get follow equations:

$$\begin{cases} m_{34}\boldsymbol{m}_1 = \alpha\boldsymbol{r}_1 + u_0\boldsymbol{r}_3 & (2) \\ m_{34}\boldsymbol{m}_2 = \beta\boldsymbol{r}_2 + \upsilon_0\boldsymbol{r}_3 & (3) \\ m_{34}\boldsymbol{m}_3 = \boldsymbol{r}_3 & (4) \end{cases}$$

we will use the fact that the rows of a rotation matrix have unit length and are perpendicular to each other yields some equations.

from eq. (4), apply norm in both side

$$m_{34} = 1/\|\boldsymbol{m}_3\| \tag{5}$$

first use eq. (2) dot multiply eq. (4), due to $\boldsymbol{r}_1 \cdot \boldsymbol{r}_3 = 0$ and $\boldsymbol{r}_3 \cdot \boldsymbol{r}_3 = 1$

$$u_0 = m_{34}^2(\boldsymbol{m}_1 \cdot \boldsymbol{m}_3) = \frac{\boldsymbol{m}_1 \cdot \boldsymbol{m}_3}{\|\boldsymbol{m}_3\|^2} \tag{6}$$

similarly, use eq. (3) dot multiply eq. (4), we can obtain

$$\upsilon_0 = m_{34}^2(\boldsymbol{m}_2 \cdot \boldsymbol{m}_3) = \frac{\boldsymbol{m}_2 \cdot \boldsymbol{m}_3}{\|\boldsymbol{m}_3\|^2} \tag{7}$$

then,use use eq. (2) cross multiply eq. (4), due to $\boldsymbol{r}_1 \times \boldsymbol{r}_3 = -\boldsymbol{r}_2$ and $\boldsymbol{r}_3 \times \boldsymbol{r}_3 = 0$

$$m_{34}^2(\boldsymbol{m}_1 \times \boldsymbol{m}_3) = -\alpha\boldsymbol{r}_2 \tag{8}$$

similarly, use eq. (3) cross multiply eq. (4)

$$m_{34}^2(\boldsymbol{m}_2 \times \boldsymbol{m}_3) = \beta \boldsymbol{r}_1 \tag{9}$$

apply norm in both side of eq. (8) and eq. (9), and the sign of parameters α and β can be taken to positive, thus:

$$\alpha = m_{34}^2 \|\boldsymbol{m}_1 \times \boldsymbol{m}_3\| = \frac{\|\boldsymbol{m}_1 \times \boldsymbol{m}_3\|}{\|\boldsymbol{m}_3\|^2} \tag{10}$$

$$\beta = m_{34}^2 \|\boldsymbol{m}_2 \times \boldsymbol{m}_3\| = \frac{\|\boldsymbol{m}_2 \times \boldsymbol{m}_3\|}{\|\boldsymbol{m}_3\|^2} \tag{11}$$

Combining eq. (8) with eq. (10) and eq. (9) with eq. (11)

$$\boldsymbol{r}_1 = \frac{\boldsymbol{m}_2 \times \boldsymbol{m}_3}{\|\boldsymbol{m}_2 \times \boldsymbol{m}_3\|} \tag{12}$$

$$\boldsymbol{r}_2 = \frac{\boldsymbol{m}_1 \times \boldsymbol{m}_3}{\|\boldsymbol{m}_1 \times \boldsymbol{m}_3\|} \tag{13}$$

$$\boldsymbol{r}_3 = \frac{\boldsymbol{m}_3}{\|\boldsymbol{m}_3\|} \tag{14}$$

Due to $m_{34}\boldsymbol{b} = \boldsymbol{K}\boldsymbol{t}$ so $\boldsymbol{t} = m_{34}\boldsymbol{K}^{-1}\boldsymbol{b}$, where t is transform matrix, M is intrinsic matrix and $b = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}^T$,so

$$\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = m_{34}\boldsymbol{K}^{-1}\boldsymbol{b} \tag{15}$$

$$= m_{34} \begin{bmatrix} \frac{1}{\alpha} & 0 & -\frac{u_0}{\alpha} \\ 0 & \frac{1}{\beta} & -\frac{\upsilon_0}{\beta} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_{14} \\ m_{24} \\ 1 \end{bmatrix} \tag{16}$$

So, we can get

$$t_x = m_{34}(\frac{m_{14}}{\alpha} - \frac{u_0}{\alpha}) = \frac{\|\boldsymbol{m}_3\|^2 (m_{14} - \boldsymbol{m}_1 \cdot \boldsymbol{m}_3)}{\|\boldsymbol{m}_3\| \cdot \|\boldsymbol{m}_1 \times \boldsymbol{m}_3\|} \tag{17}$$

$$t_y = m_{34}(\frac{m_{24}}{\beta} - \frac{\upsilon_0}{\beta}) = \frac{\|\boldsymbol{m}_3\|^2 (m_{24} - \boldsymbol{m}_2 \cdot \boldsymbol{m}_3)}{\|\boldsymbol{m}_3\| \cdot \|\boldsymbol{m}_2 \times \boldsymbol{m}_3\|} \tag{18}$$

$$t_z = m_{34} = \frac{1}{\|\boldsymbol{m}_3\|} \tag{19}$$