FORMAL LANGUAGES AND AUTOMATA, 2025 FALL SEMESTER

Lec 01. Intro & DFA

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Fundamental questions for CS

- What do we mean by "computation"?
  $\rightsquigarrow$ "Computation is to solve a problem by an effective manner."
  Vague.
- What is a computer? Why 'computers' are all called computers?
- What can it do and cannot?
- Computable vs not computable problems? 'Efficiently' computable problems and those which are not?
- Can anyone on earth devise a fundamentally more powerful computer? (An alien? In another universe?)

Two key features of Computation

What IS computed?
&
What COMPUTES it?

Throughout this course, we shall learn that these two features are intrinsically related.

How to express the object for computation: alphabet

Alphabet

- Alphabet, usually denoted as Σ, is a finite and nonempty set of symbols.
- Examples of alphabet: $\Sigma = \{0, 1\}$, $\{a, b, \ldots, z\}$, the set of all ASCII characters, etc.

How to express the object for computation: string

String

- String (a.k.a. word) is a finite sequence of symbols over Σ.
- Length of a string: number of symbols.
- Length-0 is a string itself, often denoted as ϵ.
- Σ^i: the set of all strings of length i.

How to express the object for computation: concatenation

Concatenation

- Operation on two strings.
- x, y are strings $\rightsquigarrow$ their concatenation xy is a string.
- $\epsilon x = x\epsilon = ?$

How to express the object for computation: language

Language

- Language (over alphabet Σ) is a set of strings over Σ.
- Simply put, $L \subseteq \Sigma^*$.
- Here, Σ^* is a set of all strings of finite length, $\Sigma^* := \bigcup_{i \geq 0} \Sigma^i$.
- Examples of languages: ...
- Both $\emptyset$ and $\{\epsilon\}(= \Sigma^0)$ are languages.

Two key features of Computation

- Any well-formulated information can be represented as a string of 0 and 1, or any finite alphabet Σ.
- The object for computation can be stated as a function.

Compute WHAT

computational problem $\Leftrightarrow$ compute a function $f : \Sigma^* \to \Sigma^*$.

Decision problem and Language

Compute WHAT

a decision problem $\Leftrightarrow$ compute a function $f : \Sigma^* \rightarrow \{0, 1\}$.

$\Leftrightarrow$ given $x \in \Sigma^*$, decide if $x \in L$

where $L = \{s : f(s) = 1\} \subseteq \Sigma^*$.

- Decision problem, or equivalently "membership test for a language", is easier to handle.
- ...while capable of capturing the essence of important computational problems.

Two key features of Computation

WHAT computes a function / language?

- Let us agree: "computing a function f" means "there is an ~~effective method~~ algorithm which outputs $f(x)$ for each input x".
- The concept of "algorithm" is still vague.

Two key features of Computation

What do we expect for an algorithm, intuitively?

- a finite number of finitely describable instructions.
- each instruction and what to do next are unambiguous.
- all the basic operation should be executable by the concerned executor.
- terminates at some point (i.e. in finite number of steps)

Toward a rigorous notion of algorithm

A mathematically rigorous description of an executor (computing device/machine...) and instructions is needed.

(Our) Model of Computation

Exercutor(machine) constituents:

- an alphabet Σ it recognizes,
- a gadget to read an input $x \in \Sigma^*$,
- a finite set of states to recognize its status ("where am I?"),
- memory to write and read later.

Basic operation:

- read one alphabet from input tape (or from memory),
- update its internal state,
- move the header (only in one fixed direction, or both direction, or neither) on input tape or memory,
- write/change on memory tape.

Set-up

- Concatenation xy of x and y.
- Cartesian product $A \times B$
- Notations: Σ, Σ^i, ϵ, Σ^*.
- Computing a function $f : \Sigma^* \to \Gamma^*$ means ...
- Special function $f : \Sigma^* \to \{0, 1\} \rightsquigarrow$ language.
- Language: a subset A of Σ^*, indicator function f_A.
- Computing $f_A \Leftrightarrow$ membership test for A

Finite (state) automata

Example: automatic door

Model of computation mimicking a simple computing device

- no/limited memory,
- basic operations: read one symbol from the input, update the state, and move to the next position in input.

(State) Transition Diagram

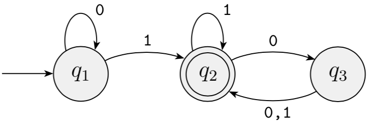


Figure 1.4, Sipser 2012.

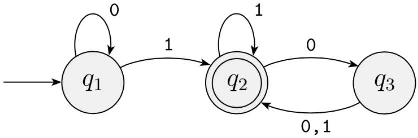
Strings accepted by M

The set of all $w \in \{0, 1\}^*$ such that...

Formal definition

A finite automata is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$

- Q a finite set called the states,
- Σ a finite set called the alphabet,
- δ a function from $Q \times \Sigma$ to Q called the (state) transition function,
- $q_0 \in Q$ the start state (a.k.a. initial states),
- $F \subseteq Q$ the set of accept states (a.k.a. final states).



Transition Diagram, Transition Table

(Other than listing the transition function) two common ways to express transition function.

Language recognized by FA

Definition

- Let M be a finite automaton.
- A string $w \in \Sigma^*$ is <u>accepted</u> by a finite automaton M if M ends in an accept state upon reading the entire w.
- $L(M)$ denotes the set of all strings accepted by M.
- A language A is said to be <u>recognized</u> by M if $A = L(M)$.

EXAMPLES OF FINITE AUTOMATA

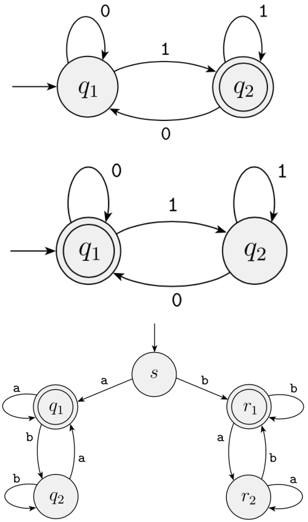


Figure 1.7, 9, 12 from Sipser 2012.