Theoretical Analysis

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1 Theoretical Analysis

This section we will prove Theorem 2.

1.1 Introduction of the problem P_3

We define problem P_3 as:

$$\begin{aligned} P_3 : \min_{\forall i, \mathbf{I^i(t)}, \alpha_i(t)} & \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} \mathbb{E}(G(\mathbf{I^i}(t))) \\ s.t. \qquad & (1), (9), (13) \\ \lim_{T\to\infty} \quad & \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\xi_i(t) - \alpha_i(t)] = 0 \end{aligned} \tag{1}$$

where $\xi_i(t) \triangleq \epsilon_i^l + \sum_{k=1}^{B_i} \epsilon_{i,k}^{tx}(t)$, we can find that P_3 is a relaxed version of P_1, Then $P_3^* \leq P_1^*$, where P^* is the optimal objective functions of P.

1.2 Optimal $(\mathbf{A}(t), \mathbf{E}^h(t))$-only policy

Lemma 1 *For an arbitrary $\delta > 0$, there exists a stationary and randomized policy π^* for P_3, which satisfies*

$$\begin{aligned} \mathbb{E}[\sum_i^N G^*(I^i(t))] &\leq P_3^* + \delta, \\ |\mathbb{E}[\xi_i^*(t) - \alpha_i^*(t)]| &\leq \delta, \\ \mathbb{E}[E(t)] &\leq E_{avg} + \delta \end{aligned} \tag{2}$$

Proof 1.1 *The proof can be obtained by Theorem 4.5 in [1].*

1.3 The specific proof

By utilizing Lemma 1 and Theorem 4.8 in [1], it is easy to obtain that

$$\begin{aligned} \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{i=1}^{N} \mathbb{E}[G(\mathbf{I}^i(t))] &\leq P_3^* + \frac{C}{V} \\ \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} (\mathbb{E}[Q(t)]) &\leq \frac{VP_3^* + C}{\epsilon} \end{aligned} \tag{3}$$

Considering $P_3^* \leq P_1^*$, it can obtain that

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{i=1}^{N} \mathbb{E}[G(\mathbf{I}^i(t))] \leq P_1^* + \frac{C}{V} \tag{4}$$

Then Theorem 2 in our paper could be proved.

References

[1] Neely and J. Michael, "Stochastic network optimization with application to communication and queueing systems," *Synthesis Lectures on Communication Networks*, vol. 3, no. 1, p. 211, 2010.