

# Theoretical Analysis for Dynamic Service Placement Algorithm

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## 1 Theoretical Analysis

This section we will prove Theorem 2.

### 1.1 Introduction of the problem $P_3$

We define problem  $P_3$  as:

$$\begin{aligned}
 P_3 : \quad & \min_{\forall i, \mathbf{I}^i(t), \alpha_i(t)} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^N \mathbb{E}(G(\mathbf{I}^i(t))) \\
 & \text{s.t.} \quad (1), (9), (13) \\
 & \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\tilde{\zeta}_i(t) - \alpha_i(t)] = 0
 \end{aligned} \tag{1}$$

where  $\tilde{\zeta}_i(t) \triangleq \epsilon_i^l + \sum_{k=1}^{B_i} \epsilon_{i,k}^{tx}(t)$ , we can find that  $P_3$  is a relaxed version of  $P_1$ , Then  $P_3^* \leq P_1^*$ , where  $P^*$  is the optimal objective functions of  $P$ .

### 1.2 Optimal $(\mathbf{A}(t), \mathbf{E}^h(t))$ -only policy

**Lemma 1** For an arbitrary  $\delta > 0$ , there exists a stationary and randomized policy  $\pi^*$  for  $P_3$ , which satisfies

$$\begin{aligned}
 \mathbb{E}[\sum_i^N G^*(I^i(t))] &\leq P_3^* + \delta, \\
 |\mathbb{E}[\tilde{\zeta}_i^*(t) - \alpha_i^*(t)]| &\leq \delta, \\
 \mathbb{E}[E(t)] &\leq E_{avg} + \delta
 \end{aligned} \tag{2}$$

**Proof 1.1** The proof can be obtained by Theorem 4.5 in [1].

### 1.3 The specific proof

By utilizing Lemma 1 and Theorem 4.8 in [1], it is easy to obtain that

$$\begin{aligned}
 \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{i=1}^N \mathbb{E}[G(\mathbf{I}^i(t))] &\leq P_3^* + \frac{C}{V} \\
 \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} (\mathbb{E}[Q(t)]) &\leq \frac{VP_3^* + C}{\epsilon}
 \end{aligned} \tag{3}$$

Considering  $P_3^* \leq P_1^*$ , it can obtain that

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{i=1}^N \mathbb{E}[G(\mathbf{I}^i(t))] \leq P_1^* + \frac{C}{V} \quad (4)$$

Then Theorem 2 in our paper could be proved.

## References

- [1] Neely and J. Michael, "Stochastic network optimization with application to communication and queueing systems," *Synthesis Lectures on Communication Networks*, vol. 3, no. 1, p. 211, 2010.