Theoretical Analysis for Dynamic Service Placement Algorithm

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1 Theoretical Analysis

This section we will prove Theorem 2 and Theorem 3.

1.1 Introduction of the problem P_3

We define problem P_3 as:

$$P_3 : \min_{\forall i, \mathbf{I^i}(\mathbf{t}), \alpha_i(t)} \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} \mathbb{E}(G(\mathbf{I^i}(t)))$$

$$s.t. \quad (1), (9), (13) \tag{1}$$

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\xi_i(t) - \alpha_i(t)] = 0$$

where $\xi_i(t) \triangleq \epsilon_i^l + \sum_{k=1}^{B_i} \epsilon_{i,k}^{tx}(t)$, we can find that P_3 is a relaxed version of P_1, Then $P_3^* \leq P_1^*$, where P^* is the optimal objective functions of P.

1.2 Optimal $(\mathbf{A}(t), \mathbf{E}^h(t))$-only policy

Lemma 1 For an arbitrary $\delta > 0$, there exists a stationary and randomized policy π^* for P_3, which satisfies

$$\mathbb{E}[\sum_i^N G^*(I^i(t))] \leq P_3^* + \delta,$$

$$|\mathbb{E}[\xi_i^*(t) - \alpha_i^*(t)]| \leq \delta, \tag{2}$$

$$\mathbb{E}[E(t)] \leq E_{avg} + \delta$$

Proof 1.1 The proof can be obtained by Theorem 4.5 in [?].

1.3 The specific proof

By utilizing Lemma 1 and Theorem 4.8 in [?], it is easy to obtain that

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{i=1}^{N} \mathbb{E}[G(\mathbf{I}^i(t))] \leq P_3^* + \frac{C}{V}$$

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} (\mathbb{E}[Q(t)]) \leq \frac{V P_3^* + C}{\epsilon} \tag{3}$$

Considering $P_3^* \leq P_1^*$, it can obtain that

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{i=1}^{N} \mathbb{E}_{COSPA}[G(\mathbf{I}^i(t))] \leq P_1^* + \frac{C}{V}$$

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} (\mathbb{E}[Q(t)]) \leq \frac{VP_1^* + C}{\epsilon}$$

(4)

Then Theorem 2 and Theorem 3 in our paper could be proved.