CMS 380: Sprint 2 - Deliverables

Wizard People, Dear Reader?

P(She's a witch) = .75
P(Not receiving a letter | She's a witch) = .03 (because owls are actually pretty dumb)
P(Not receiving a letter | She's not a witch) = .99

$$P(A \mid B) = \frac{P(A) * P(B \mid A)}{P(B)}$$

A = She's a witch
B = Not receiving a letter

P(B) is therefore the total probability of not receiving a letter.

SOLUTION:

P(B) = P(B | A) * P(A) + P(B | !A) * P(!A)
P(B) = (0.03 * 0.75) + (0.99 * 0.25) = 0.27

$$P(A \mid B) = \frac{0.75 * 0.03}{0.27} = \mathbf{0.0833}$$

Chocolate Frogs

Mean of Geometric Distribution $= \frac{1}{p} => Trial(1) = \frac{30}{30} => Trial(2) = \frac{29}{30} * \frac{1}{p} = \frac{30}{29}$

SOLUTION:

$$i = 1; n = 29 : \sum_{i}^{n} = \frac{1}{p(x)} => \frac{1}{\frac{n-i}{n}} => \frac{n}{n-i} = 118.85 + 1(FirstTrial) = 119.85$$

Hat Problem

P(Not Evil) = 0.90 P (Evil) = 0.10 P(Slytherin | Evil) = 1 P(Slytherin | Not Evil) = 0.20

SOLUTION:

$$P(Evil|Slytherin) = \frac{P(Slytherin|Evil) * P(Evil)}{P(Slytherin)} = \frac{1 * 0.10}{P(Slytherin)} = \frac{0.10}{0.28} = 0.3571$$

$$P(Slytherin) = P(Slytherin|NotEvil) * P(NotEvil) + P(Slytherin|Evil) * P(Evil)$$
$$0.20 * 0.90 + 1 * 0.1 = 0.28$$

Dragon's Dice

3 Trials = P(Success) = 1 / 6 each die n = 3 p = 1/6

SOLUTION:

$$E[X] = -1P(0) + 1P(1) + 2P(2) + 3P(3)$$

$$P(0) = (\frac{5}{6})^3 = \frac{125}{216} \qquad P(1) = \frac{1}{6} * (\frac{5}{6})^2 = \frac{25}{216} \qquad P(2) = (\frac{1}{6})^2 * \frac{5}{6} = \frac{5}{216} \qquad P(3) = (\frac{1}{6})^3 = \frac{1}{216}$$

$$E[X] = -1 * \frac{125}{216} + 1 * \frac{25}{216} + 2 * \frac{5}{216} + 3 * \frac{1}{216} = \frac{-125 + 25 + 10 + 3}{216} = \frac{-87}{216} = -0.40$$

Urn While You Learn

10 Black, 5 Red

SOLUTION:

$$E[X] = xP(x)$$

$$E[X] = 0 * \frac{1}{15} + 1 * \frac{2}{5} + 2 * \frac{2}{15} = 0.666$$

$$P(2ndRed) = 1 - P = 1 - 0.666 = 0.333$$

Arithmancy

Prove that the expected value of a discrete uniform distribution with a = 1 and b = n is

$$E[X] = \frac{n+1}{2}$$

SOLUTION:

$$E[X] = \sum_{x} xP(x)$$

$$E[X] = a = 1, \sum_{a}^{n} a\frac{1}{n}$$

$$E[X] = \frac{1}{n}\sum_{a}^{n} a => \frac{1}{n} * \frac{n(n+1)}{2} => \frac{n+1}{2}\square$$

Birthday Attack

Probability of no two Students share the same birthday. Total of 40 Students.

SOLUTION:

$$P(X = 2) = \frac{365}{365} * \frac{364}{365} = 1 * 0.9972 = 0.9972$$

$$P(Sameday) = (1 - P) = 1 - 0.9972 = 0.002739$$

$$E[X] = xP(x) \ = 40 * 0.002739 = 0.1096$$