

Consider

$$\alpha = \int_0^1 h(x) dx$$

where

$$h(x) = 100 \cdot I_{(0,1/100]}(x) + 1 \cdot I_{(1/100,1)}(x).$$

- Implement Pseudocode for `omc_integral(n)`:

- Generate  $n$  iid samples

$$\{iid Y_i \sim U(0, 1) : i = 1, 2, \dots, n\};$$

- Compute  $n$   $X$  samples by

$$\{X_i = h(Y_i) : i = 1, 2, \dots, n\};$$

- Take average of  $X_i$ 's

- Demonstrate convergence rate numerically by doing the following:

- Fix a batch number  $m = 100$ ;

- For  $i$  in `range(5, 10)`:

- \* run  $m$  times of `omc_integral(n = 2i)`, store it into  $\{\alpha_{ij} : j = 1, \dots, m\}$ .

- \* compute standard deviation (`numpy.std`) of  $\{\alpha_{ij} : j = 1, \dots, m\}$ , save it to  $\sigma_i$ .

- plot and find slope (`scipy.stats.linregress`) for the data

$$\{(i, -\log_2 \sigma_i) : i = 5, \dots, 10\}.$$

- Find its convergence rate by the following procedure:

Compute RMSE (root MSE) for  $\hat{\alpha}_n$  in terms of  $Cn^{-\alpha}$ . We say  $\alpha$  as the convergence rate.