1 Abstract

We have learned OMC by exact sampling to evaluate a definite integral. We are going to use the same techniques to evaluate BSM option prices.

2 Project

BSM model assumes the distribution of stock as lognormal. In particular, with the parameters denoted by

- $S(0)$: The initial stock price
- $S(T)$: The stock price at T
- r: interest rate
- σ: volatility

the exact value of call and put prices with maturity T and K, denoted by C_0 and P_0, are given by

$$C_0 = \mathbb{E}[e^{-rT}(S(T) - K)^+] = S_0\Phi(d_1) - Ke^{-rT}\Phi(d_2),$$

and

$$P_0 = \mathbb{E}[e^{-rT}(S(T) - K)^-] = Ke^{-rT}\Phi(-d_2) - S_0\Phi(-d_1),$$

where d_i are given as

$$d_1 = \frac{(r + \frac{1}{2}\sigma^2)T - \ln\frac{K}{S_0}}{\sigma\sqrt{T}}, \quad d_2 = \frac{(r - \frac{1}{2}\sigma^2)T - \ln\frac{K}{S_0}}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

With parameters

$$S(0) = 100, K = 110, r = 4.75\%, \sigma = 20\%, T = 1$$

- Find BS call and put price using BS formula above.
- Approximate BS call and put price using exact sampling omc.
- Compute theoretical convergence rate and demonstrate its convergence rate numerically.