- 1. Suppose that we wish to approximate the first derivative u'(x) of a very smooth function with an error of only $O(h)^4$, where h is the step size. Which difference approximation could we use? (Hint: you may consider to use more than two points in the neighborhood)
- 2. Let $f: \mathbb{R} \to \mathbb{R}$ be a smooth even function satisfying f(0) = 0. Our objective is to approximate the second order derivative f''(0).
 - Prove that f'(0) = 0.
 - Gwan proposes the following estimator for f''(0): for a step size h

$$a_h = \frac{2f(h)}{h^2}.$$

Please justify that Gwan's estimation has its convergence $O(h^2)$.

• Is there anyway to improve the above convergence to $O(h^4)$ in the form of

$$b_h = \frac{c_1 f(h) + c_2 f(2h)}{h^2}$$

for some constants c_1 and c_2 ?

• If the above function f is odd and other properties remain the same, how do you want to find the f''(0) efficiently?