MA573 - Capstone Project

Please address the following tasks step by step on the CEV model

$$dS_t = rS_t\,dt + \sigma S_t^{\beta} S_t\,dW_t$$

for a Brownian motion W_t, $\sigma > 0$ and $\beta \in [-1, 0]$.

- Design price engines for European call and put options using any of your favorite methods.
- Using at least two different methods, design price engine for your favorite discretely monitored barrier option pricing using both crude Monte Carlo method and importance sampling and compare their performances.
- Calibrate CEV model to the market data of call/put option prices underlying some of your carefully chosen stocks. The choice of the error function to be minimized is totally up to your taste. For simplicity, we treat the market call/put prices are European style.
- To get a sense of validity of the result, use your price engine on your calibrated model for the barrier option, and compare the results with its corresponding market price.
- Note that the stock and ETF put/call options (except Index options) are American style. Design price engines for American call and put options and repeat all above calibration procedure. Compare the outcome with the one with European pricing.

Some general suggestions are given in this below:

- This project should be done independently and shall be uploaded to your github at last.
- You can (and should) consult the (on- and off-line) literature and cite it correctly.
- All data sources of your market data shall be explicitly specified, for example, Bloomberg, quandl, etc.
- This should be a professional report, so the writing (English) should be up to professional standards. WPI's writing center (https://www.wpi.edu/student-experience/resources/writing-center) is a great resource to help you with this, you might reserve time in advance.
- Not only code and numerical results, but also the analytical part is essential. Literature review, theoretical background, mathematical derivation, and pseudo-codes must be included.
- *Performance comparison* among different pricing methods shall be up to you, but it's crucial. This may include analysis on convergence, convergence rate, stability, etc. In case the mathematical rigorous proof is not available, one shall demonstrate your point heuristically with some experimental evidence, for instance, using computer running time, statistical estimate of variance, etc.
- Although specified in the above explicitly, exotic option instruments and their methodologies on price engines are not limited to the above recommendations, if you have strong tendency to do so.
- You are encouraged to recover or outperform (bonus points) some numerical results from the existing literature;
- To get better performance in calibration, you may want to try different types of error functions, or different ticks of stocks.

- Last but not least, if you have new findings, you shall highlight on it. Some hypothetical observations are given here:
  - "More liquid stocks, better fit to CEV model",
  - "Financial sector stocks have better fit to CEV than energy sector stocks",
  - "CEV is not effective model whenever stock market is about big shock",
  - "Finite difference method outperforms Monte Carlo method",
  - "No boundary condition is needed if we revise the finite difference scheme",
  - "My choice on importance sampling is better than the paper XXX",
  - "I provided the convergence rate of order 1/2 of my importance sampling",
  - "I used deep learning for importance sampling",
  - ...

  That means, your "new findings" could be some empirical conclusion or theoretical conclusion, as long as it has not been discussed in our class. But if you could replicate an idea or observation from some existing literature with proper citation, we still count this as your "new findings" .

Look forward to your new findings!