ex Asset

$$S_t = S_0 \exp\{\hat{\mu} t + \sigma W_t\} \qquad Q$$

Option payoff (digital put)

$$h(S_T) = I\left(S_T < S_0 e^{-b}\right).$$

→ Barrier price

for some constant b.

Fwd price.

$$v = \mathbb{E}^Q[h(S_T)].$$

para $r = 0.03$, $\sigma = 0.2$

$$\hat{\mu} = r - \tfrac{1}{2}\sigma^2 = 0.01$$

$T = 1$

$b = 0.39$

OMC

$$v \sim \hat{v}_{10} = \frac{1}{10}\sum_{i=1}^{10} I(Z_i < -2), \quad \text{where } Z_i \overset{iid}{\sim} N(0,1)$$

Algo ($n = 10$, $b = 2$)

① Take n samples from $N(-b, 1)$.

$\{X_1 \cdots X_{10}\}$

② Compute

$$\hat{v}_{10} = e^{\frac{1}{2}b^2} \frac{1}{10}\sum_{i=1}^{10} e^{-X_i \cdot b} I(X_i < -2)$$

(Hw)

① Find $\mathbb{E}[\hat{v}_{10}^2]$ for OMC

② — — — — — IS ($b=2$)

③ what b makes IS most efficient?

(in terms minimizing MSE?)