1. Suppose that we wish to approximate the first derivative $u'(x)$ of a very smooth function with an error of only $O(h)^4$, where h is the step size. Which difference approximation could we use? (Hint: you may consider to use more than two points in the neighborhood)

2. Let $f : \mathbb{R} \mapsto \mathbb{R}$ be a smooth even function satisfying $f(0) = 0$. Our objective is to approximate the second order derivative $f''(0)$.

   - Prove that $f'(0) = 0$.
   - Gwan proposes the following estimator for $f''(0)$: for a step size h

     $$a_h = \frac{2f(h)}{h^2}.$$

     Please justify that Gwan's estimation has its convergence $O(h^2)$.
   - Is there anyway to improve the above convergence to $O(h^4)$ in the form of

     $$b_h = \frac{c_1 f(h) + c_2 f(2h)}{h^2}$$

     for some constants c_1 and c_2?
   - If the above function f is odd and other properties remain the same, how do you want to find the $f''(0)$ efficiently?