CGMY Process On Option Pricing

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Abstract

In this project, the CGMY model introduced by *Carr, Geman, Madan and Yor* is investigated. The process is an infinite activity pure jump process that has these heavier peak and higher tails. In this project, we summarize theoretical searcher work; this provides a CGMY-FT closed form solution algorithm for pricing option. And we use the fast fourier transform to improve efficiency.

1. Introduction

Fast Fourier Transform Method reduced the complexity from Fourier Transform's N^2 to $NlogN$ (N represents the sample size). The purpose of this project is to investigate performance of CGMY model under the fast fourier transform method. our technique assumes that the characteristic function of the risk-neutral density is known analytically. For given characteristic function(under risk-neutral case), according to the work done by *Carr and Madan*, we can get the analytical solution of option price by using fast fourier transform to do the inversion.

The CGMY process is a generalization of a VG process. It has the L´evy density with four parameters, C, G, M and Y where C > 0, G 0, M 0 and Y < 2. When Y = 0, the CGMY process is exactly the VG process. The four parameters in CGMY process are different roles to capture the characters of the model. The parameter C, appeared both in x>0 and x<0, can be seen as a overall level of activity. The parameter G and M, appeared both in x < 0 and x > 0 separately, are exponential decay coefficient on both sides of the L´evy density. When G < M, the left tail of the distribution is heavier than the right tail, which can better represent the real world option market. The parameter Y in different range will lead the process have different properties. In this project, 1 < Y < 2, the process is completely monotone with infinite activity and infinite variation.

The outline of this paper is as follow. We introduce the Fourier Transform method of option pricing in section 2. In section 3, we show the detailed Fast Fourier Transform method on solving option price. In section 4, we show the Simpson's Rule method we use in the FFT to get the discrete segments. Section 5 contains mathematical properties of CGMY Process. In section 6 we illustrate our approach in the CGMY Model. Section 7 concludes.

2. Fourier Transform

There are two Fourier Transform applications on option pricing. One is fourier transforming the PDF of log stock price and integrate it to get the CDF of the log stock price. The other one is doing Fourier transform on the stock price and inverse the Fourier Transform to get characteristic function of option price.

a. Characteristic Function Characteristic function is the Fourier Transform of probability density function.

$$\Phi_X(u) = E[e^{iuX}] = \int_{-\infty}^{\infty} e^{iuX} f_x(X)dx \tag{1}$$

$$e^{iuX} = cos(uX) + isin(uX) \tag{2}$$

From equation(1) we know the characteristic function is symmetric.($\overline{\Phi_X}(u) = \Phi_X(-u)$) This property is important when we introduce the factorization method of Fast Fourier Transform.
Fourier Transform can be seen as a vector on complex plane, which defined as: $z = a+ib$. The conjugate transpose of fourier transform is $\bar{z} = a - ib$. The real part is $R = (z + \bar{z})/2$, and imaginary part is $I = (z - \bar{z})/2$. Then we implement the characteristic function.

$$\begin{aligned} R[\Phi_X(u)] &= \frac{1}{2}[\Phi_X(u) + \Phi_X(-u)] \\ I[\Phi_X(u)] &= \frac{1}{2i}[\Phi_X(u) - \Phi_X(-u)] \end{aligned} \tag{3}$$

Here we find the real part of characteristic function is always even, and imaginary part is odd.

b. Inverse Fourier Transform on Characteristic Function
There are 4 assumptions when we use Inverse Fourier Transform to price options.

I The market is risk neutral

II No tax or other trading fee exist

III Expected return rate is risk-free rate

IV No dividened in options

Call option price under risk neutral measure:

$$\begin{aligned} C &= e^{-rT} E^Q[(S_T - K)^+] \\ &= e^{-rT} \int_K^{\infty} (S_T - K) q_T(S) dS_T \end{aligned} \tag{4}$$

Here we let $s_t = lnS_t, k = lnK$

$$C_T(k) = \int_k^\infty e^{-rT}(e^s - e^k)q_T(s)ds \tag{5}$$

As $K \to -\infty$

$$\begin{aligned}
\lim_{k\to-\infty} C_T(k) &= \lim_{k\to-\infty} e^{-rT}\int_k^\infty (e^s - e^k)q_T(s)ds \\
&= \lim_{k\to-\infty} e^{-rT}\int_{-\infty}^\infty e^s q_T(s)ds \\
&= E^Q[e^{rT}e^{s_T}] \qquad (6)\\
&= E^Q[e^{-rT}S_T] \\
&= S_0
\end{aligned}$$

Thus $lim_{K\to-\infty}(C_T(K))^2 = S_0^2 > 0$

Now we assume $lim_{x\to-\infty}f^2(x) = a$ For $\forall\varepsilon < \frac{|a|}{2}$, $\exists M < 0$, s.t for all $x < M$, we have $|f^2(x) - a| < \varepsilon$. Then we have $0 < |a|-\varepsilon \le |f^2(x)-a+a| = f^2(x) \le \varepsilon+|a|$ Recall Caucky criteria, the improper integral converges at negative infinity iff $\forall\varepsilon > 0, \exists N \le 0$. s.t $\forall A.B \le N$ we have $|\int_A^B f(x)ds| < \varepsilon$. $|a|-\varepsilon \le |f^2(a)-a+a| \le |f^2(x)-a|+|a|$ Now we can prove disconvergence by proving that $\exists\varepsilon > 0, \forall N \le 0$ s.t $\forall A.B \le N$ we have $|\int_A^B f(x)ds| \ge \varepsilon$. We know $\exists\varepsilon \le \frac{|a|}{2}$ $\forall N < M < 0$ $\exists A.B \le N$ s.t $B - A = 1$ Then $\int_A^B f^2(x)dx \ge \int_A^B(|a|-\varepsilon)dx = (B-A)(|a|-\varepsilon) = |a|-\varepsilon > \frac{|a|}{2} > \varepsilon$

Then we prove that: if $f(x)$ satisfied $lim_{x\to-\infty}f^2(x) > 0$ Then $f \notin L_2$.

Since $E(S_T) = S_0e^{rT}$, we know the cumulative distribution function $C_T(k)$is not integrable. We add a variable α here and we get:

$$c_T(k) = e^{\alpha k}C_T(k) \tag{7}$$

Now we get the Fourier Transform of $c_T(k)$

$$
\begin{aligned}
\Psi_T(u) &= \int_{-\infty}^{\infty} e^{iuk} c_T(k) dk \\
&= \int_{-\infty}^{\infty} e^{\alpha k} e^{-rT} (e^s - e^k) q_T(s) ds dk \\
&= \int_{-\infty}^{\infty} e^{-rT} q_T(s) (\int_{-\infty}^{\infty} e^{\alpha k} e^{iuk} (e^s - e^k) dk) ds \\
&= \int_{-\infty}^{\infty} e^{-rT} q_T(s) (e^s \int_{-\infty}^{s} e^{(\alpha+iu)k} dk - \int_{-\infty}^{s} e^{(\alpha+iu+1)k} dk) ds \\
&= \int_{-\infty}^{\infty} e^{-rT} q_T(s) (\frac{e^s}{\alpha+iu} [e^{(\alpha+iu)k}]_{-\infty}^{s} - \frac{1}{\alpha+iu+1} [e^{(\alpha+iu+1)k}]_{-\infty}^{s}) ds \\
&= \int_{-\infty}^{\infty} e^{-rT} q_T(s) (\frac{e^{(\alpha+iu+1)s}}{\alpha+iu} - \frac{e^{(\alpha+iu+1)s}}{\alpha+iu+1}) ds \\
&= e^{-rT} \int_{-\infty}^{\infty} q_T(s) \frac{e^{(\alpha+iu+1)s}}{(\alpha+iu)(\alpha+iu+1)} ds \\
&= e^{-rT} \frac{\Phi_T(u - i(\alpha+1))}{(\alpha+iu)(\alpha+iu+1)} \qquad (8)
\end{aligned}
$$

$$(\int_{-\infty}^{\infty} q_T(s) e^{(\alpha+iu+1)s} ds = \int_{-\infty}^{\infty} q_T(s) e^{i(u-i(\alpha+1))s} ds = \Phi_T(u-i(\alpha+1)))$$

Now we want to do inverse Fourier Transform on equation(8), based on the definition of Fourier Transform inversion:

$$F^{-1}(\hat{f}) = f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\hat{u}) e^{iux} du \qquad (9)$$

Now we obtain call option price:

$$C_T(k) = \frac{e^{(-\alpha k)}}{2\pi} \int_{-\infty}^{\infty} e^{-iuk} \Psi_T(u) du \qquad (10)$$

Recall function(3), $\Psi_T(u) = \int_{-\infty}^{\infty} e^{iuk} c_T(k) dk$, c_T is a real number. Then $R[\Psi_T(u)] = \int_{-\infty}^{\infty} cos(iuk) c_T(k) dk$ and $I[\Psi_T(u)] = \int_{-\infty}^{\infty} isin(iuk) c_T(k) dk$. We can find that $R[\Psi_T(-u)] = R[\Psi_T(u)]$ and $I[\Psi_(-u)] = -[\Psi_(u)]$.
So we have:

$$C_T(k) = \frac{e^{(-\alpha k)}}{\pi} \int_{0}^{\infty} R[e^{-iuk} \Psi_T(u)] du \qquad (11)$$

The value of α is determined by $E^Q[S_T^{1+\alpha}] < \infty$.

3. Fast Fourier Transform

The Fast Fourier Transform method is based on the Fourier Transform method we introduced in the previous section, but here we will use Discrete Fourier Transform.

a. Standard Discrete Fourier Transform:

$$w_k = \sum_{j=1}^{N} e^{-i\frac{2\pi}{N}(j-1)(k-1)} x(j) \quad for\ k = 1, \ldots, N \tag{12}$$

b. Discrete Fourier Transform pricing call options:

$$C_T(k) \approx \frac{e^{-\alpha k}}{\pi} \sum_{j=1}^{N} e^{-iu_j k} \Phi_T(u_j)\eta \tag{13}$$

The equation (13) is from equation (10) when using Discrete Fourier Transform, with an even segment number N.
$N\eta$ is the upper limit of the integration, N is the number of segments of strike price, η is the interval of integration and λ is the regular spacing size of strike price. Then we have:

$$\begin{aligned} k_u &= -b + \lambda(u-1) \\ v_j &= (j-1)\eta \\ b &= \frac{N\lambda}{2} \end{aligned} \tag{14}$$

Now we take equation (13) back to equation (12), we can obtain the Call option price function under the Discrete Fourier Transform inversion:

$$\begin{aligned} C_T(k) &\approx \frac{e^{(-\alpha k_u)}}{\pi} \sum_{j=1}^{N} e^{-iv_j[-b+\lambda(u-1)} \Phi_T(v_j)\eta \\ &\approx \frac{e^{(-\alpha k_u)}}{\pi} \sum_{j=1}^{N} e^{-i\lambda\eta(j-1)(u-1)e^{ibv_j}\Phi_T(v_j)\eta} \\ &\approx \frac{e^{(-\alpha k_u)}}{\pi} \sum_{j=1}^{N} e^{-i\frac{2\pi}{N}(j-1)(u-1)e^{ibv_j}\Phi_T(v_j)\eta} \end{aligned} \tag{15}$$

with $\frac{2\pi}{N} = \lambda\eta$

c. Fast Fourier Transform Method

First we introduce the equation $z^n = 1$, the solutions z are the "nth roots of unity", which means there are n evenly spaced points around the unit circle in the complex plane. (Introduction to Linear Algebra, Gilbert Strang) Then we consider the complex number $\omega = e^{i\theta}$ and its polar form $e^{i\theta} = cos(\theta) + isin(\theta)$. Now, for a n by n fourier matrix, it contains powers of $\omega = e^{\frac{2\pi i}{n}}$.

$$F_n c = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \omega & \omega^2 & \dots & \omega^{n-1} \\ 1 & \omega^2 & \omega^4 & \dots & \omega^{2(n-1)} \\ \dots & \dots & \dots & \ddots & \dots \\ 1 & \omega^{n-1} & \omega^{2(n-1)} & \dots & \omega^{(n-1)^2} \end{bmatrix}_{n*n} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \dots \\ c_{n-1} \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \dots \\ y_{n-1} \end{bmatrix} = y \tag{16}$$

From function(16), we know the fourier matrix times a vector c will take n^2 multiplications, since the Fourier matrix has n^2 entries. To reduce the multiplication, we will do factorization on the Fourier matrix because the symmetric property on ω. (Factorization will generate zero entries in a matrix, which can be skipped during multiplication) Here we will directly introduce the factorization method in FFT.

First we want to prove the symmetric property of ω on complex plane.

$$\begin{aligned} \omega_n^k &= e^{\frac{2\pi k i}{n}} \\ \omega_{2n}^{2k} &= cos(2\pi \frac{2k}{2n}) + isin(2\pi \frac{2k}{2n}) \\ &= cos(2\pi \frac{k}{n}) + isin(2\pi \frac{k}{n}) = \omega_n^k \end{aligned} \tag{17}$$

And its Elimination Lemma:

$$\begin{aligned} \omega_n^{k+\frac{n}{2}} &= cos(2\pi \frac{k+\frac{n}{2}}{n}) + isin(2\pi \frac{k+\frac{n}{2}}{n}) \\ &= cos(2\pi \frac{k}{n} + \pi) + isin(2\pi \frac{k}{n} + \pi) \\ &= -cos(2\pi \frac{k}{n}) - isin(2\pi \frac{k}{n}) \\ &= -\omega_n^k \end{aligned} \tag{18}$$

Now, set we have a n terms polynomial function $A(x)$, and $n = 2^l$

$$A(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_{n-1} x^{n-1} \tag{19}$$

Then we serperate its even and odd parts

$$\begin{aligned} A(x) &= (a_0 + a_2 x^2 + a_4 x^4 + \cdots + a_{n-2} x^{n-2}) \\ &+ (a_1 x + a_3 x^3 + a_5 x^5 + \cdots + a_{n-1} x^{n-1}) \\ &= (a_0 + a_2 x^2 + a_4 x^4 + \cdots + a_{n-2} x^{n-2}) \\ &+ x(a_1 + a_3 x^2 + a_5 x^4 + \cdots + a_{n-1} x^{n-2}) \end{aligned} \tag{20}$$

$$
\begin{aligned}
A_1(x) &= (a_0 + a_2x + a_4x^2 + \cdots + a_{n-2}x^{\frac{n-2}{2}} \\
A_2(x) &= (a_1 + a_3x + a_5x^2 + \cdots + a_{n-2}x^{\frac{n-2}{2}} \\
A(x) &= A_1(x^2) + xA_2(x^2) \qquad (21)
\end{aligned}
$$

For $0 \leq k \leq \frac{n}{2} - 1$

$$
\begin{aligned}
A(\omega_n^k) &= A_1(\omega_n^{2k}) + \omega_n^k A_2(\omega_n^{2k}) \\
A(\omega_n^k) &= A_1(\omega_{\frac{n}{2}}^k) + \omega_n^k A_2(\omega_{\frac{n}{2}}^k) \qquad (22)
\end{aligned}
$$

For $\frac{n}{2} \leq k + \frac{n}{2} \leq n - 1$
$(\omega_n^{2k+n} = \omega_n^{2k}\omega_n^n = \omega_n^{2k} = \omega_{\frac{n}{2}}^k, \quad \omega_n^{k+\frac{n}{2}} = -\omega_n^k)$

$$
\begin{aligned}
A(\omega_n^{k+\frac{n}{2}}) &= A_1(\omega_n^{2k+n}) + \omega_n^{k+\frac{n}{2}} A_2(\omega_n^{2k+n}) \\
A(\omega_n^{k+\frac{n}{2}}) &= A_1(\omega_{\frac{n}{2}}^k) - \omega_n^k A_2(\omega_{\frac{n}{2}}^k) \qquad (23)
\end{aligned}
$$

Based on function (22) and (23), we find if we know the solution of $A_1(x)$ and $A_2(x)$ on $\omega_{\frac{n}{2}}^0, \omega_{\frac{n}{2}}^1, \omega_{\frac{n}{2}}^2, \ldots, \omega_{\frac{n}{2}}^{\frac{n}{2}-1}$, we can get the value of $A(x)$ in O(n) time. $A_1(x)$ and $A_2(x)$ here are half-size of orginial $A(x)$, so we can replicate this process multiple times to reduce the complexity of fourier matrix multiplication. For $n = 2^l$, we can reduce the time complexity from $O[N^2]$ to $O[N * log_2 N]$

4. Simpson's Rule

Definition:
$\int_a^b f(x)dx \approx \frac{b-a}{3}(f(x_0) + 4[f(x_1) + f(x_3) + f(x_5) + \ldots] + 2[f(x_2) + f(x_4) + f(x_6) + \ldots] + f(x_n))$ (*n must be even)

The simpson's rule approximates the curve by parabola, which requires 3 points to define. We need even number of strips to make it work. This prerequisite is automatically fulfilled under FFT circumstances since $n = 2^l$.

The reason we use Simpson's Rule here insteand of other two numeriacl integration methods is the error term of Simpson is smallest.

Proof:
Suppose f(x) is defined on interval [a,b], we evenly divided this interval into n parts. ($a < x_0 < x_1 < x_2 < \cdots < b$ and $x_i = a + i\Delta x$ for $i = 0, 1, 2, \ldots, n$, $\Delta x = \frac{b-a}{n}$) Since Simpson's rule approximate curve using parabola, here we define the parabola by three points $((x_0, f(x_0)), (x_1, f(x_1)), (x_2, f(x_2)))$ and we set $g(x) = ax^2 + bx + c$ to replace $f(x)$ on interval $[x_0, x_2]$ so we can approximate its inte-

gral in this interval. Now we can express:

$$\begin{aligned}
f(x_0) &= g(x_0) = \alpha x_0^2 + \beta x_0 + c \\
f(x_1) &= g(x_1) = \alpha(\frac{x_0+x_2}{2})^2 + \beta(\frac{x_0+x_2}{x}) + c \\
f(x_2) &= g(x_2) = \alpha x_2^2 + \beta x_2 + c
\end{aligned} \tag{24}$$

We have:

$$\begin{aligned}
\int_{x_0}^{x_2} f(x)dx &\approx \int_{x_0}^{x_2} g(x)dx \\
&= \int_{x_0}^{x_2} (\alpha x^2 + \beta x + c)dx \\
&= (\frac{\alpha}{3}x^3 + \frac{\beta}{2}x^2 + cx)|_{x_0}^{x_2} \\
&= \frac{\alpha}{3}(x_2^3 - x_0^3) + \frac{\beta}{2}(x_2^2 - x_0^2) + c(x_2 - x_0) \\
&= \frac{\Delta x}{3}[(\alpha x_0^2 + \beta x_0 + c) + 4(\alpha(\frac{x_0+x_2}{2})^2 + \beta(\frac{x_0+x_2}{2}) + c) + (\alpha x_2^2 + \beta
\\
&= \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + f(x_2)]
\end{aligned}$$

The coeffcient α, β and c are determined by three chosen points
Then we redo process above in the interval [a,b]

$$\begin{aligned}
\int_a^b f(x)dx &= \int_{x_0}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx + \cdots + \int_{x_{n-2}}^{x_n} f(x)dx \\
&\approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + f(x_2)] + \frac{\Delta x}{3}[f(x_2) + 4f(x_3) + f(x_4)] + \cdots + \\
&\quad \frac{\Delta x}{3}[f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)] \\
&= \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \ldots \\
&\quad +2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))
\end{aligned} \tag{26}$$

Then we get back to our FFT process

$$C_T(k_u) \approx \frac{e^(- \alpha k_u)}{\pi} \sum_{j=1}^{N} e^{-iv_j[-b+\lambda(u-1)]} \psi_T(v_j)\eta \tag{27}$$

We take $f(x_n)$ aside

$$\omega_j = \frac{\eta}{3}(1, 4, 2, 4, 2, 4, \ldots, 1) \approx \frac{\eta}{3}(3 + (-1)^j - \delta_{j-1}) \tag{28}$$

To transform into function (28), we ignore the last coefficient, and set $\delta_0 = 1$, $j = 1, 2, 3, \ldots$,n. This ignorant is reasonable since the last term is small.

$$C_T(k_u) \approx \frac{e^{(-\alpha k_u)}}{\pi} \sum_{j=1}^{N} e^{-iv_j[-b+\lambda(u-1)]} \psi_T(v_j) \frac{\eta}{3}(3 + (-1)^j - \delta_{j-1}) \tag{29}$$

5. CGMY Process

The VG process and classical representations for the Le´vy measures of gamma processes, Madan et al. (1998) show that the Le´vy density for the VG process is

$$\mathrm{k}_{vg} = \begin{cases} \frac{\mu_n^2}{v_n}\frac{exp(-\frac{\mu_n}{v_n}|x|)}{|x|} & for \quad x < 0 \\ \frac{\mu_p^2}{v_p}\frac{exp(-\frac{\mu_p}{v_p}|x|)}{|x|} & for \quad x > 0 \end{cases} \tag{30}$$

Then we generalize the VG Le´vy density to the CGMY Le´vy density with parameters C, G, M, and Y. Specifically, the Le´vy density of the CGMY process is given by

$$\mathrm{k}_{vg} = \begin{cases} C\frac{exp(-G|x|)}{|x|^{Y+1}} & for \quad x < 0 \\ C\frac{exp(-M|x|)}{|x|^{Y+1}} & for \quad x > 0 \end{cases} \tag{31}$$

It has the L´evy density with four parameters, C, G, M and Y where C $>$ 0, G $\geq$ 0, M $\geq$ 0 and Y $<$ 2. When Y $=$ 0, the CGMY process is exactly the VG process with the parameter identification

$$C = \frac{1}{V}$$
$$G = \frac{1}{\mu_n}$$
$$M = \frac{1}{\mu_p}$$

The four parameters in CGMY process are different roles to capture the characters of the model. The parameter C, appeared both in x>0 and x<0, can be seen as a overall level of activity. The parameter G and M, appeared both in x $<$ 0 and x $>$ 0 separately, are exponential decay coefficient on both sides of the L´evy density. When G $<$ M, the left tail of the distribution is heavier than the right tail, which can better represent the real world option market. The parameter Y in different range will lead the process have different properties. In this project, 1 $<$ Y $<$ 2, the process is completely monotone with infinite activity and infinite variation.//

From *Carr, Geman, Madan and Yor's* research, they displays the required characteristic function

$$\phi_{CGMY}(u,t;C,G,M,Y) = exp\{\ \mathrm{t}C\Gamma(-Y)[(M-iu)^Y - M^Y + (G+iu)^Y - G^Y]\ \} \tag{32}$$

Carr, Geman, Madan and Yor also derived some property of CGMY process

i) has a completely monotone Le´vy density for Y>-1 ;

ii) is a process of infinite activity for Y>0

iii) is a process of infinite variation for .

6. The market under the CGMY process

Asset price dynamics $S_{t\in[0,t]}$ is an exponential Levy process $L_{t\in[0,t]}$ of the form

$$S_t = S_0 exp L_t \quad (33)$$

The choice of the Levy process is the CGMY process plus a drift

$$L_t = \{\ (r-\beta)t + X_{CGMY}(t; C_Q, G_Q, M_Q, Y_Q)\} \quad (34)$$

Where $r \geq 0$ is the mean rate of return on the asset and β is the convexity correction in CGMY model takes the next form

$$\beta = \phi_{CGMY}(-i, t; C, G, M, Y),$$

$$= C_Q G_Q^{Y_Q} \Gamma(-Y_Q)\{\ (1+\frac{1}{G_Q})^{Y_Q} - 1 - \frac{Y_Q}{G_Q}\}$$

$$+ C_Q M_Q^{Y_Q} \Gamma(-Y_Q)\{\ (1-\frac{1}{M_Q})^{Y_Q} - 1 + \frac{Y_Q}{M_Q}\} \quad (35)$$

The risk-neutral log asset price dynamics can be obtained from the equation (34) as

$$lnS_t = lnS_0 + (r-\beta)t + X_{CGMY}(t; C_Q, G_Q, M_Q, Y_Q) \quad (36)$$

The characteristic function of log asset price lnS_t is obtained by substituting formula of the equation(31) and added the drift term $(r-\beta)t$, β is as defined in the equation(35). Without lost of generality, let t=T and $lnS_T = s_T$, then, drop Q we have

$$\phi_T(\omega) = exp\{\ i\omega s_0 + (r-\beta)T\}\ exp\{\ TCG^Y\Gamma(-Y)[(1+\frac{i\omega}{G})^Y - 1 - \frac{i\omega Y}{G}]$$

$$+ TCM^Y\Gamma(-Y)[(1-\frac{i\omega}{M})^Y - 1 + \frac{i\omega Y}{M}]\}$$

7. Numerical Results

We implement the CGMY Fourier transform formula with decay rates parameters G=10, M=5,overall arrival rate C=2 and Y=0.5. We consider the common parameters S_0 =100,r=0.03,T=0.5.Then we use fourier transform with simpson's rule and fast fourier transform to calculate option price. The fast fourier transform's parameter N=1024,η=0.15$and\alpha$=0.5

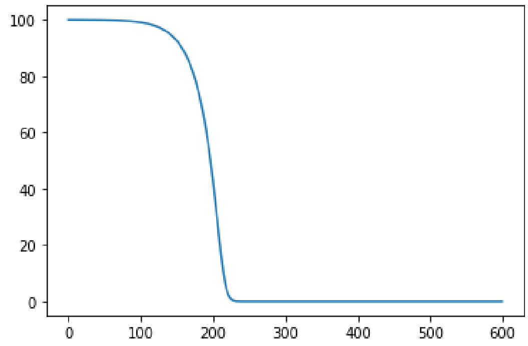

Figure 1: CGMY-FT

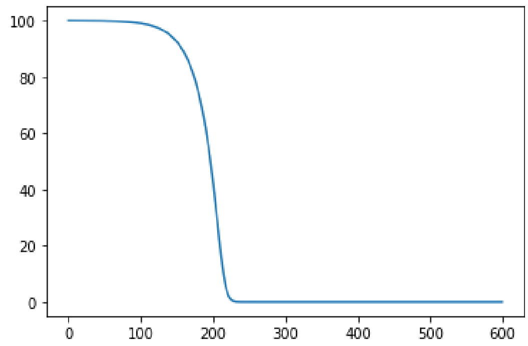

Figure 2: CGMY-FFT

And we use SPX's one month call option to calibrate CGMY parameters.

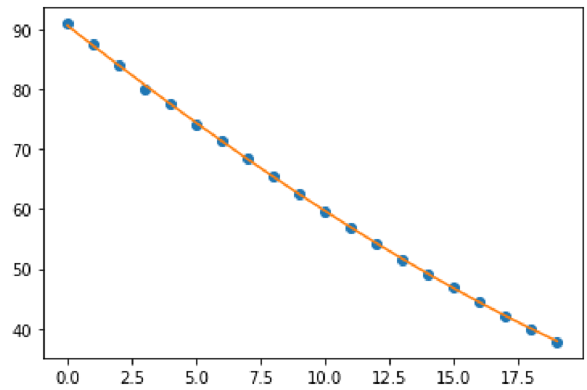

Figure 3: calibrated price and market price

8. Conclusion

Firstly, The fast fourier transform is more efficient than fourier transform with simpson's rule. However if we want one option price with specific strike price, the fourier transform with simpson's rule perform better.

Secondly, In the tail cases, the fast fourier transform have huge error. We need to choose these parameters carefully and use correct data range.

In addition, if we use market data to calibrate CGMY parameters, it will consume a lot of time. For example, it will take 10mins to train 20 option data. But the CGMY model can better predict the market than BSM model does. It will eliminate the volatility smile.

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