Variance Gamma Process On Option Pricing Application

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Abstract

The Variance Gamma Process is obtained by evaluating Brownian motion process with constant drift and volatility at a random time change given by gamma process. We are using characteristic function of the variance gamma process in the Fast Fourier Transform method to price options. This paper is focusing on investigating the accuracy and efficiency of the Variance Gamma process under Fast Fourier transform method when doing option pricing with Python.

1. Introduction

The Fast Fourier Transform Method is an algorithm computes the discrete Fourier Transform. It reduced the complexity of computational process like Fourier Transform from N^2 to $NlogN$ (N represents the sample size). The purpose of this paper is to investigate performance of variance gamma process under the fast fourier transform method. In this case, here we assume the characteristic function of variance gamma process is known. For given characteristic function(under risk-neutral case), according to the work done by *Carr and Madan*, we use the analytic expression for the Fourier transform of the option price and then use FFT do the inversion and get the real time price.

The Variance Gamma process is a Levy Process that can be written as a Brownian motion evaluated at a random time with a gamma process. It has economical meaning since the option price in real market jumps to different level instantaneously. Compared with Black-Scholes model, it has two additional parameters that control the kurtosis and skewness which create asymmetry of the left and right tails of the return density. The purpose of this paper is investigating the performance of the variance gamma model when pricing American options under different circumstances, and we will test its accuracy and efficiency.

The outline of this paper is as follow. We introduce the Fourier Transform method of option pricing in section 2. In section 3, we show the detailed Fast Fourier Transform method on solving option price. In section 4, we show the Simpson's Rule method we use in the FFT to get the discrete segments. Section 5 contains mathematical properties of Variance Gamma Process. In section 6 we illustrate our approach in the Variance Gamma Model. Section 7 concludes.

2. Fourier Transform

There are two Fourier Transform applications on option pricing. One is fourier transforming the PDF of log stock price and integrate it to get the CDF of the log stock price. The other one is doing Fourier transform on the stock price and inverse the Fourier Transform to get characteristic function of option price.

a. Characteristic Function Characteristic function is the Fourier Transform of probability density function.

$$\Phi_X(u) = E[e^{iuX}] = \int_{-\infty}^{\infty} e^{iuX} f_x(X)dx \tag{1}$$

$$e^{iuX} = cos(uX) + isin(uX) \tag{2}$$

From equation(1) we know the characteristic function is symmetric.($\overline{\Phi_X(u)} = \Phi_X(-u)$) This property is important when we introduce the factorization method of Fast Fourier Transform.
Fourier Transform can be seen as a vector on complex plane, which defined as: $z = a+ib$. The conjugate transpose of fourier transform is $\bar{z} = a - ib$. The real part is $R = (z + \bar{z})/2$, and imaginary part is $I = (z - \bar{z})/2$. Then we implement the characteristic function.

$$\begin{aligned} R[\Phi_X(u)] &= \frac{1}{2}[\Phi_X(u) + \Phi_X(-u)] \\ I[\Phi_X(u)] &= \frac{1}{2i}[\Phi_X(u) - \Phi_X(-u)] \end{aligned} \tag{3}$$

Here we find the real part of characteristic function is always even, and imaginary part is odd.

b. Inverse Fourier Transform on Characteristic Function
There are 4 assumptions when we use Inverse Fourier Transform to price options.

I The market is risk neutral

II No tax or other trading fee exist

III Expected return rate is risk-free rate

IV No dividened in options

Call option price under risk neutral measure:

$$\begin{aligned} C &= e^{-rT}E^Q[(S_T - K)^+] \\ &= e^{-rT}\int_K^{\infty}(S_T - K)q_T(S)dS_T \end{aligned} \tag{4}$$

Here we let $s_t = lnS_t, k = lnK$

$$C_T(k) = \int_k^{\infty} e^{-rT}(e^s - e^k)q_T(s)ds \tag{5}$$

As $K \to -\infty$

$$
\begin{aligned}
\lim_{k\to-\infty} C_T(k) &= \lim_{k\to-\infty} e^{-rT}\int_k^{\infty}(e^s-e^k)q_T(s)ds \\
&= \lim_{k\to-\infty} e^{-rT}\int_{-\infty}^{\infty} e^s q_T(s)ds \\
&= E^Q[e^{rT}e^{s_T}] \\
&= E^Q[e^{-rT}S_T] \\
&= S_0
\end{aligned} \tag{6}
$$

Thus $lim_{K\to-\infty}(C_T(K))^2 = S_0^2 > 0$
Now we assume $lim_{x\to-\infty}f^2(x) = a$ For $\forall\varepsilon < \frac{|a|}{2}$, $\exists M < 0$, s.t for all $x < M$, we have $|f^2(x) - a| < \varepsilon$. Then we have $0 < |a| - \varepsilon \le |f^2(x) - a + a| = f^2(x) \le \varepsilon + |a|$ Recall Caucky criteria, the improper integral converges at negative infinity iff $\forall\varepsilon > 0$, $\exists N \le 0$. s.t $\forall A.B \le N$ we have $|\int_A^B f(x)ds| < \varepsilon$. $|a| - \varepsilon \le |f^2(a) - a + a| \le |f^2(x) - a| + |a|$ Now we can prove disconvergence by proving that $\exists\varepsilon > 0$, $\forall N \le 0$ s.t $\forall A.B \le N$ we have $|\int_A^B f(x)ds| \ge \varepsilon$. We know $\exists\varepsilon \le \frac{|a|}{2}$ $\forall N < M < 0$ $\exists A.B \le N$ s.t $B - A = 1$ Then $\int_A^B f^2(x)dx \ge \int_A^B(|a| - \varepsilon)dx = (B - A)(|a| - \varepsilon) = |a| - \varepsilon > \frac{|a|}{2} > \varepsilon$ Then we prove that: if $f(x)$ satisfied $lim_{x\to-\infty}f^2(x) > 0$ Then $f \notin L_2$.
Since $E(S_T) = S_0e^{rT}$, we know the cumulative distribution function $C_T(k)$is not integrable. We add a variable α here and we get:

$$
c_T(k) = e^{\alpha k}C_T(k) \tag{7}
$$

Now we get the Fourier Transform of $c_T(k)$

$$\begin{aligned}
\Psi_T(u) &= \int_{-\infty}^{\infty} e^{iuk} c_T(k) dk \\
&= \int_{-\infty}^{\infty} e^{\alpha k} e^{-rT}(e^s - e^k) q_T(s) ds dk \\
&= \int_{-\infty}^{\infty} e^{-rT} q_T(s) (\int_{-\infty}^{\infty} e^{\alpha k} e^{iuk}(e^s - e^k) dk) ds \\
&= \int_{-\infty}^{\infty} e^{-rT} q_T(s) (e^s \int_{-\infty}^{s} e^{(\alpha+iu)k} dk - \int_{-\infty}^{s} e^{(\alpha+iu+1)k} dk) ds \\
&= \int_{-\infty}^{\infty} e^{-rT} q_T(s) (\frac{e^s}{\alpha+iu}[e^{(\alpha+iu)k}]_{-\infty}^{s} - \frac{1}{\alpha+iu+1}[e^{(\alpha+iu+1)k}]_{-\infty}^{s}) ds \\
&= \int_{-\infty}^{\infty} e^{-rT} q_T(s) (\frac{e^{(\alpha+iu+1)s}}{\alpha+iu} - \frac{e^{(\alpha+iu+1)s}}{\alpha+iu+1}) ds \\
&= e^{-rT} \int_{-\infty}^{\infty} q_T(s) \frac{e^{(\alpha+iu+1)s}}{(\alpha+iu)(\alpha+iu+1)} ds \\
&= e^{-rT} \frac{\Phi_T(u - i(\alpha+1))}{(\alpha+iu)(\alpha+iu+1)} \qquad (8)
\end{aligned}$$

$$(\int_{-\infty}^{\infty} q_T(s) e^{(\alpha+iu+1)s} ds = \int_{-\infty}^{\infty} q_T(s) e^{i(u-i(\alpha+1))s} ds = \Phi_T(u - i(\alpha+1)))$$

Now we want to do inverse Fourier Transform on equation(8), based on the definition of Fourier Transform inversion:

$$F^{-1}(\hat{f}) = f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\hat{u}) e^{iux} du \qquad (9)$$

Now we obtain call option price:

$$C_T(k) = \frac{e^{(-\alpha k)}}{2\pi} \int_{-\infty}^{\infty} e^{-iuk} \Psi_T(u) du \qquad (10)$$

Recall function(3), $\Psi_T(u) = \int_{-\infty}^{\infty} e^{iuk} c_T(k) dk$, c_T is a real number.
Then $R[\Psi_T(u)] = \int_{-\infty}^{\infty} cos(iuk) c_T(k) dk$ and $I[\Psi_T(u)] = \int_{-\infty}^{\infty} isin(iuk) c_T(k) dk$.
We can find that $R[\Psi_T(-u)] = R[\Psi_T(u)]$ and $I[\Psi_(-u)] = -[\Psi_(u)]$.
So we have:

$$C_T(k) = \frac{e^{(-\alpha k)}}{\pi} \int_{0}^{\infty} R[e^{-iuk} \Psi_T(u)] du \qquad (11)$$

The value of α is determined by $E^Q[S_T^{1+\alpha}] < \infty$.

3. Fast Fourier Transform

The Fast Fourier Transform method is based on the Fourier Transform method we introduced in the previous section, but here we will use Discrete Fourier Transform.

a. Standard Discrete Fourier Transform:

$$w_k = \sum_{j=1}^{N} e^{-i\frac{2\pi}{N}(j-1)(k-1)}x(j) \quad for\ k = 1, \ldots, N \tag{12}$$

b. Discrete Fourier Transform pricing call options:

$$C_T(k) \approx \frac{e^{-\alpha k}}{\pi} \sum_{j=1}^{N} e^{-iu_j k}\Phi_T(u_j)\eta \tag{13}$$

The equation (13) is from equation (10) when using Discrete Fourier Transform, with an even segment number N.
$N\eta$ is the upper limit of the integration, N is the number of segments of strike price, η is the interval of integration and λ is the regular spacing size of strike price. Then we have:

$$\begin{aligned} k_u &= -b + \lambda(u-1) \\ v_j &= (j-1)\eta \\ b &= \frac{N\lambda}{2} \end{aligned} \tag{14}$$

Now we take equation (13) back to equation (12), we can obtain the Call option price function under the Discrete Fourier Transform inversion:

$$\begin{aligned} C_T(k) &\approx \frac{e^{(-\alpha k_u)}}{\pi} \sum_{j=1}^{N} e^{-iv_j[-b+\lambda(u-1)}\Phi_T(v_j)\eta \\ &\approx \frac{e^{(-\alpha k_u)}}{\pi} \sum_{j=1}^{N} e^{-i\lambda\eta(j-1)(u-1)e^{ibv_j}\Phi_T(v_j)\eta} \\ &\approx \frac{e^{(-\alpha k_u)}}{\pi} \sum_{j=1}^{N} e^{-i\frac{2\pi}{N}(j-1)(u-1)e^{ibv_j}\Phi_T(v_j)\eta} \end{aligned} \tag{15}$$

with $\frac{2\pi}{N} = \lambda\eta$

c. Fast Fourier Transform Method
First we introduce the equation $z^n = 1$, the solutions z are the "nth roots of unity", which means there are n evenly spaced points around the unit circle in the complex plane. (Introduction to Linear Algebra, Gilbert Strang) Then we consider the complex number $\omega = e^{i\theta}$ and its polar form $e^{i\theta} = cos(\theta) + isin(\theta)$. Now, for a

n by n fourier matrix, it contains powers of $\omega = e^{\frac{2\pi i}{n}}$.

$$F_n c = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \omega & \omega^2 & \dots & \omega^{n-1} \\ 1 & \omega^2 & \omega^4 & \dots & \omega^{2(n-1)} \\ \dots & \dots & \dots & \ddots & \dots \\ 1 & \omega^{n-1} & \omega^{2(n-1)} & \dots & \omega^{(n-1)^2} \end{bmatrix}_{n*n} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \dots \\ c_{n-1} \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \dots \\ y_{n-1} \end{bmatrix} = y \tag{16}$$

From function(16), we know the fourier matrix times a vector c will take n^2 multiplications, since the Fourier matrix has n^2 entries. To reduce the multiplication, we will do factorization on the Fourier matrix because the symmetric property on ω. (Factorization will generate zero entries in a matrix, which can be skipped during multiplication) Here we will directly introduce the factorization method in FFT.

First we want to prove the symmetric property of ω on complex plane.

$$\begin{aligned} \omega_n^k &= e^{\frac{2\pi k i}{n}} \\ \omega_{2n}^{2k} &= cos(2\pi \frac{2k}{2n}) + isin(2\pi \frac{2k}{2n}) \\ &= cos(2\pi \frac{k}{n}) + isin(2\pi \frac{k}{n}) = \omega_n^k \end{aligned} \tag{17}$$

And its Elimination Lemma:

$$\begin{aligned} \omega_n^{k+\frac{n}{2}} &= cos(2\pi \frac{k+\frac{n}{2}}{n}) + isin(2\pi \frac{k+\frac{n}{2}}{n}) \\ &= cos(2\pi \frac{k}{n} + \pi) + isin(2\pi \frac{k}{n} + \pi) \\ &= -cos(2\pi \frac{k}{n}) - isin(2\pi \frac{k}{n}) \\ &= -\omega_n^k \end{aligned} \tag{18}$$

Now, set we have a n terms polynomial function $A(x)$, and $n = 2^l$

$$A(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_{n-1} x^{n-1} \tag{19}$$

Then we serperate its even and odd parts

$$\begin{aligned} A(x) &= (a_0 + a_2 x^2 + a_4 x^4 + \dots + a_{n-2} x^{n-2}) \\ &+ (a_1 x + a_3 x^3 + a_5 x^5 + \dots + a_{n-1} x^{n-1}) \\ &= (a_0 + a_2 x^2 + a_4 x^4 + \dots + a_{n-2} x^{n-2}) \\ &+ x(a_1 + a_3 x^2 + a_5 x^4 + \dots + a_{n-1} x^{n-2}) \end{aligned} \tag{20}$$

$$\begin{aligned} A_1(x) &= (a_0 + a_2 x + a_4 x^2 + \dots + a_{n-2} x^{\frac{n-2}{2}} \\ A_2(x) &= (a_1 + a_3 x + a_5 x^2 + \dots + a_{n-2} x^{\frac{n-2}{2}} \\ A(x) &= A_1(x^2) + x A_2(x^2) \end{aligned} \tag{21}$$

For $0 \leq k \leq \frac{n}{2} - 1$

$$\begin{aligned} A(\omega_n^k) &= A_1(\omega_n^{2k}) + \omega_n^k A_2(\omega_n^{2k}) \\ A(\omega_n^k) &= A_1(\omega_{\frac{n}{2}}^k) + \omega_n^k A_2(\omega_{\frac{n}{2}}^k) \end{aligned} \tag{22}$$

For $\frac{n}{2} \leq k + \frac{n}{2} \leq n - 1$
$(\omega_n^{2k+n} = \omega_n^{2k}\omega_n^n = \omega_n^{2k} = \omega_{\frac{n}{2}}^k, \quad \omega_n^{k+\frac{n}{2}} = -\omega_n^k)$

$$\begin{aligned} A(\omega_n^{k+\frac{n}{2}}) &= A_1(\omega_n^{2k+n}) + \omega_n^{k+\frac{n}{2}} A_2(\omega_n^{2k+n}) \\ A(\omega_n^{k+\frac{n}{2}}) &= A_1(\omega_{\frac{n}{2}}^k) - \omega_n^k A_2(\omega_{\frac{n}{2}}^k) \end{aligned} \tag{23}$$

Based on function (22) and (23), we find if we know the solution of $A_1(x)$ and $A_2(x)$ on $\omega_{\frac{n}{2}}^0, \omega_{\frac{n}{2}}^1, \omega_{\frac{n}{2}}^2, \ldots, \omega_{\frac{n}{2}}^{\frac{n}{2}-1}$, we can get the value of $A(x)$ in O(n) time. $A_1(x)$ and $A_2(x)$ here are half-size of orginial $A(x)$, so we can replicate this process multiple times to reduce the complexity of fourier matrix multiplication. For $n = 2^l$, we can reduce the time complexity from $O[N^2]$ to $O[N * log_2 N]$

4. Simpson's Rule

Definition:
$\int_a^b f(x)dx \approx \frac{b-a}{3}(f(x_0) + 4[f(x_1) + f(x_3) + f(x_5) + \ldots] + 2[f(x_2) + f(x_4) + f(x_6) + \ldots] + f(x_n))$ (*n must be even)
The simpson's rule approximates the curve by parabola, which requires 3 points to define. We need even number of strips to make it work. This prerequisite is automatically fulfilled under FFT circumstances since $n = 2^l$.
The reason we use Simpson's Rule here insteand of other two numeriacl integration methods is the error term of Simpson is smallest.
Proof:
Suppose f(x) is defined on interval [a,b], we evenly divided this interval into n parts. ($a < x_0 < x_1 < x_2 < \cdots < b$ and $x_i = a + i\Delta x$ for $i = 0, 1, 2, \ldots, n$, $\Delta x = \frac{b-a}{n}$) Since Simpson's rule approximate curve using parabola, here we define the parabola by three points $((x_0, f(x_0)), (x_1, f(x_1)), (x_2, f(x_2)))$ and we set $g(x) = ax^2 + bx + c$ to replace $f(x)$ on interval $[x_0, x_2]$ so we can approximate its integral in this interval. Now we can express:

$$\begin{aligned} f(x_0) &= g(x_0) = \alpha x_0^2 + \beta x_0 + c \\ f(x_1) &= g(x_1) = \alpha(\frac{x_0 + x_2}{2})^2 + \beta(\frac{x_0 + x_2}{x}) + c \\ f(x_2) &= g(x_2) = \alpha x_2^2 + \beta x_2 + c \end{aligned} \tag{24}$$

We have:

$$\begin{aligned}
\int_{x_0}^{x_2} f(x)dx &\approx \int_{x_0}^{x_2} g(x)dx \\
&= \int_{x_0}^{x_2} (\alpha x^2 + \beta x + c)dx \\
&= (\frac{\alpha}{3}x^3 + \frac{\beta}{2}x^2 + cx)|_{x_0}^{x_2} \\
&= \frac{\alpha}{3}(x_2^3 - x_0^3) + \frac{\beta}{2}(x_2^2 - x_0^2) + c(x_2 - x_0) \\
&= \frac{\Delta x}{3}[(\alpha x_0^2 + \beta x_0 + c) + 4(\alpha(\frac{x_0 + x_2}{2})^2 + \beta(\frac{x_0 + x_2}{2}) + c) + (\alpha x_2^2 + \beta \\
&= \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + f(x_2)]
\end{aligned}$$

The coeffcient α, β and c are determined by three chosen points
Then we redo process above in the interval [a,b]

$$\begin{aligned}
\int_{a}^{b} f(x)dx &= \int_{x_0}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx + \cdots + \int_{x_{n-2}}^{x_n} f(x)dx \\
&\approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + f(x_2)] + \frac{\Delta x}{3}[f(x_2) + 4f(x_3) + f(x_4)] + \cdots + \\
&\quad \frac{\Delta x}{3}[f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)] \\
&= \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots \\
&\quad +2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))
\end{aligned} \tag{26}$$

Then we get back to our FFT process

$$C_T(k_u) \approx \frac{e^(- \alpha k_u)}{\pi} \sum_{j=1}^{N} e^{-iv_j[-b+\lambda(u-1)]} \psi_T(v_j)\eta \tag{27}$$

We take $f(x_n)$ aside

$$\omega_j = \frac{\eta}{3}(1, 4, 2, 4, 2, 4, \dots, 1) \approx \frac{\eta}{3}(3 + (-1)^j - \delta_{j-1}) \tag{28}$$

To transform into function (28), we ignore the last coefficient, and set $\delta_0 = 1$, $j = 1, 2, 3, \dots$,n. This ignorant is reasonable since the last term is small.

$$C_T(k_u) \approx \frac{e^{(-\alpha k_u)}}{\pi} \sum_{j=1}^{N} e^{-iv_j[-b+\lambda(u-1)]} \psi_T(v_j) \frac{\eta}{3}(3 + (-1)^j - \delta_{j-1}) \tag{29}$$

5. Variance Gamma Process

This section introduce the Variance Gamma Process. The Variance Gamma Process was defined in the work done by Carr, Madan and Chang in 1998, by evaluating Brownian motion (with constant drift and volatility) at a random time change given by a gamma process.

$$b(t;\theta,\sigma) = \theta t + \sigma W(t) \tag{30}$$

$b(t;\theta,\sigma)$ here is a Brownian Motion with drift θ and volatility σ
The Variance Gamma Process has three parameters, the volatility σ of Brownian Motion, the θ controls the kurtosis and the ν controls the skewness. Then we introduce the gamma process $\gamma(t;\mu,v)$ with mean rate μ and variance rate v. The Variance Gamma Process can be expressed as:

$$X(t;\sigma,v,\theta) = b(\gamma(t;1,v);\theta,\sigma) \tag{31}$$

which is defined in terms of the Brownian motion with drift $b(t;\theta,\sigma)$ and the gamma process with unit mean rate, $\gamma(t;1,v)$
The characteristic function of the Variance Gamma process is:

$$\Phi_{X(t)}(u) = (\frac{1}{1-i\theta vu+(\sigma^2 v/2)u^2})^{(\frac{t}{v})} \tag{32}$$

The characteristic function of Variance Gamma process can first be developed from conditioning on the gamma time g

$$\begin{aligned} E(e^{iuX_t|g}) &= E(e^{iu(\theta g+\sigma W_g)}) \\ &= e^{iu\theta g}E(e^{iu\sigma W_g}) \\ &= e^{iu\theta g}E(e^{iu\sigma\sqrt{g}Z}) \\ &= e^{iu\theta g}e^{-\frac{(iu\sigma\sqrt{g})^2}{2}} \\ &= e^{iu\theta g}e^{\frac{-u^2\sigma^2 g}{2}} \\ &= e^{i(u\theta+i\frac{u^2\sigma^2}{2})g} \end{aligned} \tag{33}$$

Then we integrate over g

$$\begin{aligned} E(e^{iuX_t}) &= E_g(e^{i(u\theta g+i\frac{u^2\sigma^2}{2})g}) \\ &= \int_0^\infty e^{iu\theta g}e^{\frac{-u^2\sigma^2 g}{2}}\frac{g^{\frac{t}{v}-1}e^{\frac{-g}{v}}}{v^{\frac{t}{v}}\Gamma(\frac{t}{v})}dg \end{aligned} \tag{34}$$

Based on the characteristic function of gamma distribution:

$$\phi(u) = (\frac{\beta}{\beta-iu})^\alpha \tag{35}$$

This is the characteristic function of gamma process with shape parameter $\frac{t}{v}$ and scale parameter v evaluated at $u\theta+i\frac{u^2\sigma^2}{2}$

We transform the equation (34) into:

$$E_g(e^{i(u\theta+i\frac{u^2\sigma^2}{2})g}) = (\frac{\frac{1}{v}}{\frac{1}{v}-i(u\theta+i\frac{u^2\sigma^2}{2})})^{\frac{t}{v}} \tag{36}$$

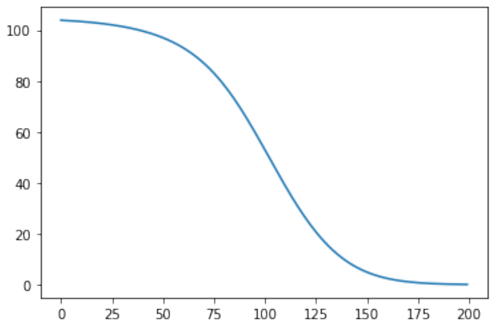
$$= (\frac{1}{1-iu\theta v+\frac{u^2\sigma^2 v}{2}})^{\frac{t}{v}} \tag{37}$$

Now we get the characteristic function of the Variance Gamma model.
Based on the Black-Scholes model, we have the European call option price:

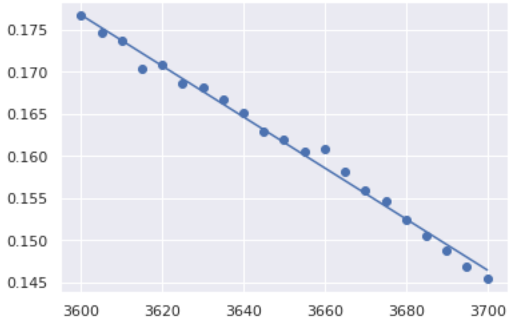
$$C(S_0,K,T) = \int_0^\infty Black-Scholes(S_0,K,T)\frac{g^{\frac{t}{v}-1}e^{\frac{-g}{v}}}{v^{\frac{t}{v}}\Gamma(\frac{t}{v})}dg \tag{38}$$

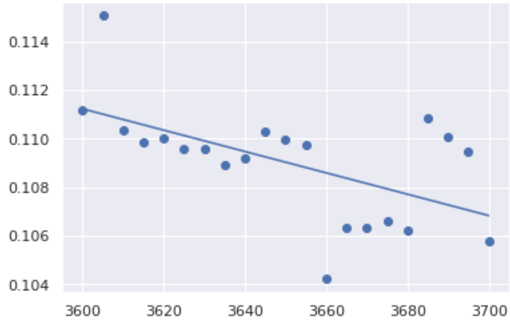
6. Variance Gamma Model Approach

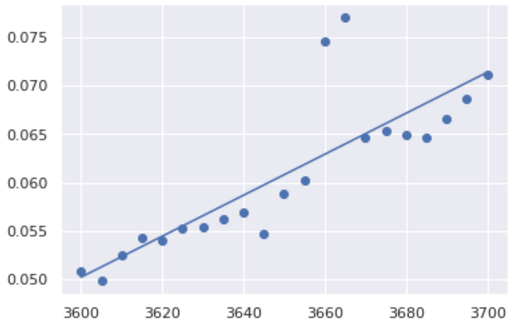
In this section, we will introduce the method we used to construct and calibrate our model. Since our model is based on the European option type, we choose SPX as our underlying asset. In our FFT method. we set the $\eta = 0.15$, $\alpha = 1.5$ and $N = 2^{10}$. We evaluating option price with a strike price = 110, initial stock price = 100 and maturity = 1. The performance of our model shown as below.



The CPU time of our model with parameter above is 31.6 ms, which is slower compared with the Black-Schols model under FFT method.
Then we test the accuracy based on the parameters given in *Computational Method in Finance* It contains 9 combinations of parameters and option prices, the average error of our model is 0.00467%. The calibration of our model based on real market data. The method we use on calibration is Nelder-Mead simplex algorithm implemented in python.
Here we do linear regressions for these three calibrated parameters:







The coefficient for implied sigma is -0.0003017, and intercept is 1.26819529. For implied theta, the coefficient is -4.402324e-05 and the intercept is 0.26971. The coefficient for implied v is 0.00021135 and intercept is -0.71065802.
The calibrated option price with linear regression parameters has a average error rate 0.368%.

7. Conclusion

The calibration result suggests the variance gamma model is accurate when pricing call options with size less than or equal to 20. The computation speed using VG model is slower than BSM model, but it removes the implied volatility bias. The Fast Fourier Transform method improve the computation performance of every model we have tested.

Citations

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2. Option Valuation using the fast Fourier Transform by Peter Carr and Dilip B. Madan.
3. The Variance Gamma Process and Option Pricing by Dilip B. Madan, Peter P. Carr and Eric C. Chang.
4. Option Pricing Under the Variance Gamma Process by Fiorani, Filo.
5. Pricing Options with VG model using FFT by Andrey Itkin