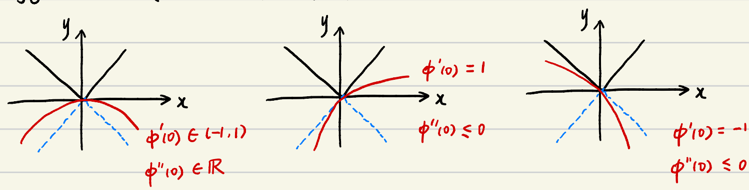
Question:

Let $O = [-1,1]$ and $f(x) = |x|$. Find superjets and subjets of f at all $x \in O$.

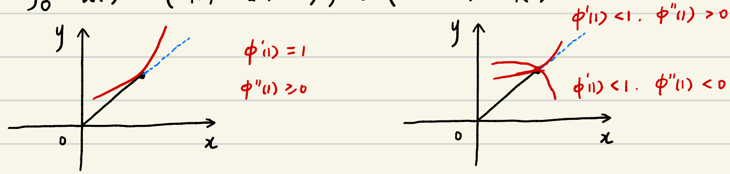
Solution:

- For $x = 0$, $J_O^{2,+} u(0) = \phi$

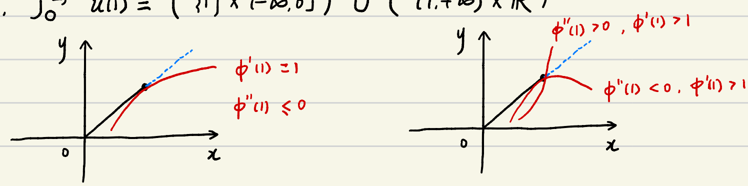
  $J_O^{2,-} u(0) = \big((-1,1) \times \mathbb{R} \big) \cup \big(\{1\} \times (-\infty, 0] \big) \cup \big(\{-1\} \times (-\infty, 0] \big)$



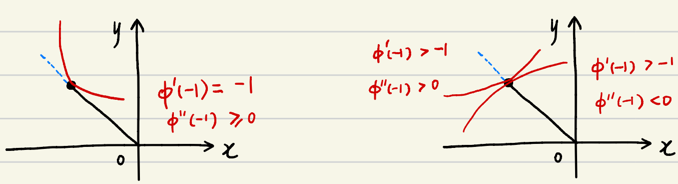
- For $x = 1$, $J_O^{2,+} u(1) = \big(\{1\} \times [0, +\infty) \big) \cup \big((-\infty, 1) \times \mathbb{R} \big)$



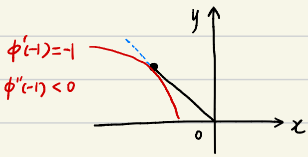
- For $x = 1$, $J_O^{2,-} u(1) = \big(\{1\} \times (-\infty, 0] \big) \cup \big((1, +\infty) \times \mathbb{R} \big)$

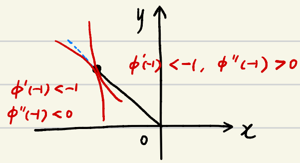


- For $x = -1$, $J_O^{2,+} u(-1) = \big(\{-1\} \times [0, +\infty) \big) \cup \big((-1, +\infty) \times \mathbb{R} \big)$

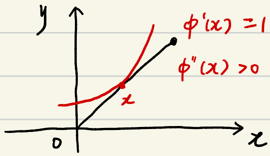


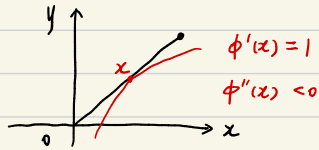
- For $x=-1$, $J_0^{2,-}u(-1) = (\{-1\}\times(-\infty,0]) \cup ((-\infty,-1)\times\mathbb{R})$



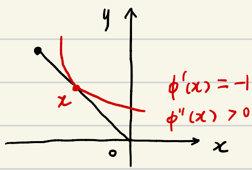


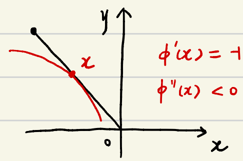
- For $x\in(0,1)$, $J_0^{2,+}u(x) = \{1\}\times[0,+\infty)$, $J_0^{2,-}u(x) = \{1\}\times(-\infty,0]$





- For $x\in(-1,0)$, $J_0^{2,+}u(x) = \{-1\}\times[0,+\infty)$, $J_0^{2,-}u(x) = \{-1\}\times(-\infty,0]$





Conclusion:

- For $x=0$, $J_0^{2,+}u(0) = \phi$

  $J_0^{2,-}u(0) = ((-1,1)\times\mathbb{R}) \cup (\{1\}\times(-\infty,0]) \cup (\{-1\}\times(-\infty,0])$
- For $x=1$, $J_0^{2,+}u(1) = (\{1\}\times[0,+\infty)) \cup ((-\infty,1)\times\mathbb{R})$

  $J_0^{2,-}u(1) = (\{1\}\times(-\infty,0]) \cup ((1,+\infty)\times\mathbb{R})$
- For $x=-1$, $J_0^{2,+}u(-1) = (\{-1\}\times[0,+\infty)) \cup ((-1,+\infty)\times\mathbb{R})$

  $J_0^{2,-}u(-1) = (\{-1\}\times(-\infty,0]) \cup ((-\infty,-1)\times\mathbb{R})$
- For $x\in(0,1)$, $J_0^{2,+}u(x) = \{1\}\times[0,+\infty)$, $J_0^{2,-}u(x) = \{1\}\times(-\infty,0]$
- For $x\in(-1,0)$, $J_0^{2,+}u(x) = \{-1\}\times[0,+\infty)$, $J_0^{2,-}u(x) = \{-1\}\times(-\infty,0]$