What are Systems of Linear Equations?

WPI

March 17, 2020

- Linear algebra is to study system of linear equations via matrix and vector
- They have many applications
  - Machine learning
  - Economics and finance
  - Statistics
  - Image processing
  - and more ...

If you google “Matrix”, you probably get ...

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Figure: From movie “The Matrix”

Definitions

- A matrix is a two-dimensional array of numbers,
  - A 2 by 3 matrix

$$A = \begin{bmatrix} 2 & 1 & 15 \\ 1 & 1 & 5 \end{bmatrix}$$

- If a matrix has only one row or only one column it is called a vector,
  - A 2 dimensional column vector

$$b_1 = \begin{bmatrix} 15 \\ 5 \end{bmatrix},$$

  - A 2 dimensional row vector

$$b_2 = \begin{bmatrix} 15 & 5 \end{bmatrix},$$

Before and after a transformation

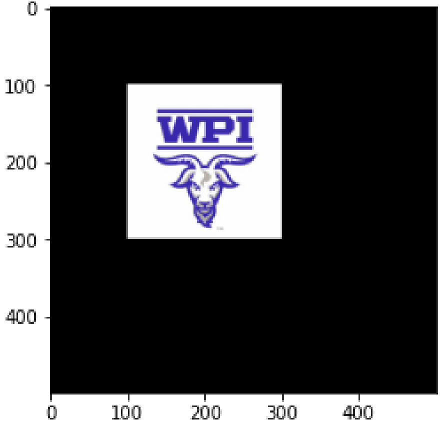

Figure: Before Transormation

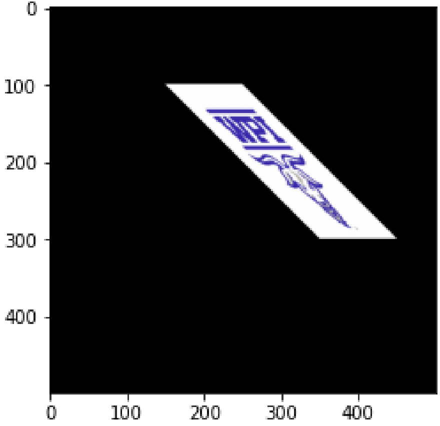

Figure: After Transormation

Transformation as a mapping

- Transformation takes each coordinate vector to another coordinate vector by mapping

$$T : \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} \frac{1}{2}x_1 + x_2 \\ x_2 \end{bmatrix}$$

- (Q.) A corner of the new picture is $\begin{bmatrix} 350 \\ 300 \end{bmatrix}$. What is its original coordinate?

As a linear system

- Set the original corner as $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$
- Solve

$$\begin{cases} \frac{1}{2}x_1 + & x_2 = 350 \\ & x_2 = 300 \end{cases}$$

- The solution is ...

General form - 1

- $m \times n$ linear system is

$$\begin{cases} a_{11}x_1 + \cdots + a_{1n}x_n & = b_1 \\ a_{21}x_1 + \cdots + a_{2n}x_n & = b_2 \\ \vdots & \\ a_{m1}x_1 + \cdots + a_{mn}x_n & = b_m. \end{cases}$$

- Coefficients a_{ij} and b_i are given numbers
- The goal is to find the solution

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

General form - 2

For $m \times n$ linear system,

- coefficient matrix

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

- augmented matrix $[A \mid b]$, i.e.

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} & | & b_1 \\ a_{21} & \cdots & a_{2n} & | & b_2 \\ \vdots & & & & \\ a_{m1} & \cdots & a_{mn} & | & b_m \end{bmatrix}$$