

Final (4 pages)

Name:

ID:

All answers shall be justified to get full credits. Each question counts 10 points.

Throughout this exam, let $A = [a_1, a_2, a_3] = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ -1 & -1 & 1 \end{bmatrix}$

(1) Find A^2 .

(2) Is a_2 orthogonal to a_3 ?

(3) Find the inverse of A , if it exists.

Throughout this exam, let $A = [a_1, a_2, a_3] = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ -1 & -1 & 1 \end{bmatrix}$

- (4) Find determinant of A . What is the rank of A ?
- (5) Write the characteristic polynomial and compute all eigenvalues.
- (6) Find an orthogonal basis for each eigenspace.

Throughout this exam, let $A = [a_1, a_2, a_3] = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ -1 & -1 & 1 \end{bmatrix}$

- (7) Diagonalize A , i.e. Find the invertible matrix P and diagonal matrix D satisfying $A = PDP^{-1}$. (You do not have to find P^{-1} .)

- (8) Compute the projection of a_2 onto $\text{span}\{a_3\}$.

- (9) Find a point y in $\text{span}\{a_2, a_3\}$ having the closest distance to a_1 . What is the distance from y to a_1 ?

Throughout this exam, let $A = [a_1, a_2, a_3] = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ -1 & -1 & 1 \end{bmatrix}$

(10) Solve $A^{2019}x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

(11) Using two sentences, write how "linear algebra" can help you in your career?