NOTATIONS ON LINEAR SYSTEM

1. Goal

Our goal is to

- Understand the definitions: Linear system, Coefficient matrix, Augmented matrix
- Solve a linear system by Equivalent Row Operations (EROs)

2. Analysis

2.1. Notations.

- General form of $m \times n$ linear system (system of linear equation) is given by

$$\begin{cases} a_{11}x_1 + \cdots + a_{1n}x_n & = b_1 \\ a_{21}x_1 + \cdots + a_{2n}x_n & = b_2 \\ \vdots & \\ a_{m1}x_1 + \cdots + a_{mn}x_n & = b_m. \end{cases}$$

- Here, a_{ij} and b_i are given numbers(coefficients).
- A concise way to write the linear system is $Ax = b$, where
  - coefficient matrix

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

  - b-vector

$$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

- and we want to solve for the solution vector

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

- A more concise way to write linear system is by augmented matrix $[A \mid b]$, i.e.

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} & \mid & b_1 \\ a_{21} & \cdots & a_{2n} & \mid & b_2 \\ \vdots & & & & \\ a_{m1} & \cdots & a_{mn} & \mid & b_m \end{bmatrix}$$

2.2. **Example.**