Midterm

Name:
ID:
All answers shall be justified properly to get full credits. Each question counts 10 points.

Throughout this exam, let $A = \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$, and $v = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$.

(1) Consider the matrix equation $Ax = v$.
   (a) Write its augmented matrix.

   (b) Find its reduced row echelon form.

   (c) Write its solution set into a parametric vector form, if there is any.

(2) Is the transformation defined by $Tx = Ax$ a one-to-one or onto mapping?

(3) We denote by $A = [a_1, a_2, a_3]$, where a_1, a_2, a_3 are column vectors of A.
(a) Does a_3 belong to the span generated by a_1 and a_2?

(b) Are vectors $\{a_1, a_2, a_3\}$ independent?

(c) Write a parametric vector form for the line going through a_1 and a_2.

(4) (a) Compute $v^T Av$, $v^T Bv$, respectively, whenever they are well defined.

(b) Compute BA, BAB, respectively, whenever they are well defined.

(c) Compute $B^7 A$ whenever they are well defined.