Final (4 pages)

Name:
ID:
All answers shall be justified to get full credits. Each question counts 10 points.

Throughout this exam, let $A = [a_1, a_2, a_3] = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ -1 & -1 & 1 \end{bmatrix}$

(1) Find A^2.

(2) Is a_2 orthogonal to a_3?

(3) Find the inverse of A, if it exists.

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(4) Find determinant of A. What is the rank of A?

(5) Write the characteristic polynomial and compute all eigenvalues.

(6) Find an orthogonal basis for each eigenspace.

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(7) Diagonalize A, i.e. Find the invertible matrix P and diagonal matrix D satisfying $A = PDP^{-1}$. (You do not have to find P^{-1}.)

(8) Compute the projection of a_2 onto $span\{a_3\}$.

(9) Find a point y in $span\{a_2, a_3\}$ having the closest distance to a_1. What is the distance from y to a_1?

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(10) Solve $A^{2019}x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

(11) Using two sentences, write how "linear algebra" can help you in your career?