Midterm (sample)

Name:
ID:
All answers shall be justified properly to get full credits.

Throughout this exam, let $A = \begin{bmatrix} 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, and $v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(1) Find a matrix E such that EA yields reduced row echelon form of A.

(2) Write its solution set of the matrix equation $Ax = v$ into a parametric vector form, if there is any.

(3) Is the transformation defined by $Tx = Bx$ a one-to-one or onto mapping? Explain its geometric meaning.

(4) We denote by $A = [a_1, a_2, a_3, a_4]$, where a_1, a_2, a_3, a_4 are column vectors of A.
   (a) Does v belong to the span generated by $\{a_1, a_2, a_3, a_4\}$? If yes, find a linear combination.

   (b) Are vectors $\{a_1, a_2, a_3, a_4\}$ independent?

(5) (a) Compute $v^T A^{100} v$, $v^T B^{100} v$, respectively, whenever they are well defined.

   (b) Compute BA, BAB, respectively, whenever they are well defined.