

Midterm (sample)

Name:

ID:

All answers shall be justified properly to get full credits. Each question counts 10 points.

Throughout this exam, let $A = \begin{bmatrix} 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, and $v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(1) Consider the matrix equation $Ax = v$.

(a) Write its augmented matrix, and find its reduced row echelon form.

(b) Write its solution set into a parametric vector form, if there is any.

(2) Is the transformation defined by $Tx = Bx$ a one-to-one or onto mapping? Explain its geometric meaning.

- (3) We denote by $A = [a_1, a_2, a_3, a_4]$, where a_1, a_2, a_3, a_4 are column vectors of A .
- (a) Does v belong to the span generated by $\{a_1, a_2, a_3, a_4\}$? If yes, find a linear combination.

(b) Are vectors $\{a_1, a_2, a_3, a_4\}$ independent?

- (4) (a) Compute $v^T Av, v^T Bv$, respectively, whenever they are well defined.

(b) Compute BA, BAB , respectively, whenever they are well defined.