Exercises

Let $A = [a_1, a_2, a_3] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ be a given matrix.

(1) Find the inverse of A, if it exists.
(2) Find determinant of A.
(3) Write the characteristic polynomial and compute all eigenvalues.
(4) Find an orthogonal basis for each eigenspace.
(5) Diagonalize A in terms of $A = PDP^{-1}$.
(6) Find A^{10}.
(7) Find a point y in $span\{a_2, a_3\}$ having the closest distance to a_1. What is the distance?

Sol 7

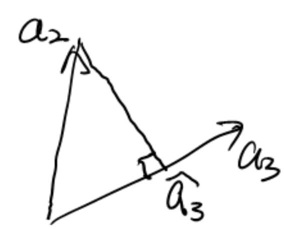
ⓐ check orthogonality

$$a_2 \circ a_3 = 1 \neq 0$$

a_2 is not orthognal to a_3

So we shall find orthogonal basis for span(a_2, a_3).

ⓑ Find $\hat{a}_3 = P_{a_3}(a_2) =$ Proj of a_2 on a_3.



$$\hat{a}_3 = \left(\frac{a_3 \circ a_2}{a_3 \circ a_3}\right) a_3 = \frac{1}{1} a_3 = a_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

ⓒ Find $\hat{a}_2 = a_2 - \hat{a}_3 = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$

ⓓ Now $\{\hat{a}_2, \hat{a}_3\}$ is orthogonal basis, so use proj form,

$$y = P_{\{\hat{a}_2, \hat{a}_3\}}(a_1) = \left(\frac{\hat{a}_2 \circ a_1}{\hat{a}_2 \circ \hat{a}_2}\right) \hat{a}_2 + \left(\frac{\hat{a}_3 \circ a_1}{\hat{a}_3 \circ \hat{a}_3}\right) \hat{a}_3$$

$$= \begin{pmatrix} 0.2 \\ -0.4 \\ 0 \end{pmatrix} \leftarrow \text{ans.}$$

ⓔ $$\|a_1 - y\| = \sqrt{0.8^2 + 0.4^2 + 0^2} = \frac{2}{\sqrt{5}} . \leftarrow \text{ans.}$$