

All following questions are equivalent:
Given $A \in IR^{m \times n}$, $A = [a_1, \dots, a_m]$
(Does each row of A have pivot?
(2) Does Ax= f always have som tyelkm?
(3) Does span (a, and generats IRM?
(4) Is the Tx = Ax onto?
(5) Are there in columns of A linearly independent?
linearly in dependent?
Given A=[a, an] EIRMXN, all fallowings
Given A=[a, an] EIR ^{m×n} , all fallowings one equivalent questions.
Given A= [a, an] E RMXN, all fallowings are equivalent questions. U Is Tx = Ax 1-1 mapping?
Given A= [a, an] EIRMXN, all fallowings one equivalent questions. Doses Tx = Ax 1-1 mapping? Doses Ax = 0 have only trivial soln? Chomo system)
Given A=[a, an] EIRMXN, all followings one equivalent questions. () Is Tx = Ax 1-1 mapping? (2) Does Ax = 0 have only trivial soln? (homo system) A3 Does each column of A have privot? (4) Is there no free variable for Ax=0?

Midterm (sample)

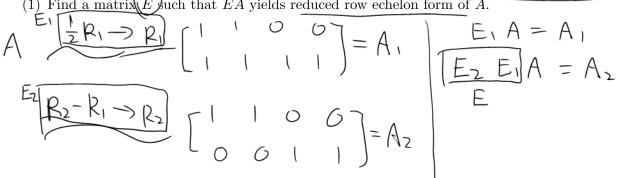
Name:

ID:

All answers shall be justified properly to get full credits.

Throughout this exam, let $A = \begin{bmatrix} 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, and $v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(1) Find a matrix E such that EA yields reduced row echelon form of A.



(2) Write its solution set of the matrix equation Ax = v into a parametric vector form, if there is any.

(3) Is the transformation defined by Tx = Bx a one-to-one or onto mapping? Explain its geometric meaning.

- (4) We denote by $A = [a_1, a_2, a_3, a_4]$, where a_1, a_2, a_3, a_4 are column vectors of A.
 - (a) Does v belong to the span generated by $\{a_1, a_2, a_3, a_4\}$? If yes, find a linear combination.
 - (b) Are vectors $\{a_1, a_2, a_3, a_4\}$ independent?

(5) (a) Compute $v^TA^{100}v$, $v^TB^{100}v$, respectively, whenever they are well defined.

(b) Compute BA, BAB, respectively, whenever they are well defined.