Final (4 pages)

Name:
ID:
All answers shall be justified to get full credits. Each question counts 10 points.

Let $A = [a_1, a_2, a_3] = \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ be a given matrix.

(1) Write the transpose of A.

(2) Is a_2 orthogonal to a_3?

(3) Find the inverse of A, if it exists.

$$A = [a_1, a_2, a_3] = \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

(4) Find determinant of A. What is the rank of A?

(5) Write the characteristic polynomial and compute all eigenvalues.

(6) Find an orthogonal basis for each eigenspace.

$$A = [a_1, a_2, a_3] = \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

(7) Diagonalize A, i.e. Find the invertible matrix P and diagonal matrix D satisfying $A = PDP^{-1}$. (You do not have to find P^{-1}.)

(8) Compute the projection of a_2 onto the space $span\{a_3\}$.

(9) Find a point y in $span\{a_2, a_3\}$ having the closest distance to a_1. What is the distance from y to a_1?

$$A = [a_1, a_2, a_3] = \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

(10) Solve $A^{2019}x = e_1$, where $e_1 = [1, 0, 0]^T$.

(11) Using two sentences, write how "linear algebra" can help you in your career?