How to Value Employee Stock Options

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One of the arguments often used against expensing employee stock options is that calculating their fair value at the time they are granted is very difficult. This article presents an approach to calculating the value of employee stock options that is practical, easy to implement, and theoretically sound. It explicitly considers the vesting period, the possibility that employees will leave the company during the life of the option, the inability of employees to trade their options, and the relevant dilution issues. This approach is an enhancement of the approach suggested by the Financial Accounting Standards Board's Statement of Financial Accounting Standards No. 123 because it does not require an arbitrary reduction in the life of the option to allow for early exercise caused by the inability of employees to trade their options.

Shareholders are increasingly agitating for companies to recognize as an expense on the income statement the compensation their employees receive in the form of stock options. One of the major difficulties in accounting for employee stock options in this way is the determination of the fair value of the options.

Previous researchers have developed standard methods for valuing options that trade on an exchange and in the OTC market. But several features of a typical employee stock option make these standard methods difficult to apply to these options. In particular:

1. The options usually have a vesting period during which they cannot be exercised. This vesting period can be as long as four years.
2. When employees leave their jobs (voluntarily or involuntarily) during the vesting period, they forfeit unvested options.
3. When employees leave (voluntarily or involuntarily) after the vesting period, they forfeit out-of-the-money options and have to exercise in-the-money options immediately.
4. Employees are not permitted to sell their employee stock options. To realize a cash benefit or diversify their portfolios, they must exercise the options and sell the underlying shares. This requirement tends to lead to employee stock options being exercised earlier than similar regular options.
5. Some dilution arises from issuing employee stock options because if they are exercised, new treasury stock will be issued.

In Statement of Financial Accounting Standards No. 123, the Financial Accounting Standards Board (FASB 1995) makes some suggestions as to how employee stock options should be valued from the viewpoint of the company granting them. Rubinstein (1995) was one of the first researchers to critically examine the FAS 123 proposals. In this article, we extend Rubinstein's ideas to create a practical approach for valuing employee stock options that is theoretically sound, easy to implement, and in the spirit of FAS 123.

FAS 123

The traditional way of accounting for stock options in the United States is the intrinsic-value-based method, which is based on Accounting Principles Board Opinion No. 25, issued in 1972. Under this method, the compensation cost of an employee stock option is assumed to be the excess, if any, of the market price of the stock over the exercise price on the date the option is granted. In the most common situation, in which options are granted with an exercise price equal to the current market price, the intrinsic-value-based method calculates the compensation cost as zero.

In October 1995, the FASB published FAS 123, *Accounting for Stock-Based Compensation*. This statement encourages companies to adopt a fair-value-based method of accounting for stock options

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instead of the intrinsic-value-based method, but it does not require them to do so.[1] Following the publication of FAS 123, most companies continued to use the intrinsic-value-based method. Recently, however, a number of companies, such as the Coca-Cola Company and General Electric Company, have adopted the fair-value-based method. And in the near future, the International Accounting Standards Board and a number of national accounting boards will likely require the use of a fair-value-based method similar to that proposed by FAS 123.

Appendix B of FAS 123 discusses the value of employee stock options in some detail. It proposes a three-step valuation procedure:

1. Estimate the expected life of the option.
2. Use either the Black and Scholes (1973) model or the Cox, Ross, and Rubinstein (1979) binomial tree to value the option, with the expected life as the time to maturity.
3. Adjust the value to allow for the possibility of the employee leaving the company during the vesting period.

The procedure is best illustrated with an example. We will consider a company that grants the option shown in **Table 1** to its employees.[2]

Table 1. Sample Option

Parameter	Value
Life of option	10 years
Vesting period	3 years
Stock price	$50
Exercise price	$50
Risk-free rate	7.5%
Expected volatility	30%
Expected dividend yield	2.5%

A normal 10-year European option with the parameters in Table 1 would be valued at $20.47 in the Black–Scholes model. A normal 10-year American option would be valued at $21.03 in the Cox–Ross–Rubinstein binomial tree.

The FAS 123 three-step procedure requires that the analyst estimate two additional parameters:

- the employee exit rate during the vesting period, which is the probability that an employee will leave the company each year during the vesting period and is referred to as the "forfeiture rate" in FAS 123 and[3]
- the expected life of the option—that is, the average time the option stays in existence (under the assumption that the employee does not leave during the vesting period).

Suppose that the employee exit rate is 3 percent a year and the expected life of the option is estimated as six years. We reduce the option life parameter from 10 years to 6 years. If the Black–Scholes model is used, this change reduces the value of the option from $20.47 to $17.15. The employee exit rate of 3 percent means that, on average, 3 percent of this value is lost each year during the vesting period because employees leave the company. The FAS 123 value of the option is, therefore,

$$0.97 \times 0.97 \times 0.97 \times \$17.15 = \$15.65.$$

If the binomial tree model is used, the value of a six-year option is calculated as $17.25 and the FAS 123 value of the option is

$$0.97 \times 0.97 \times 0.97 \times \$17.25 = \$15.75.[4]$$

To determine the total value, the value of an individual option must be multiplied by the number of options of that type that are outstanding. Suppose the company has granted 100,000 options with the parameters shown in Table 1. Suppose further that the employee exit rate and expected life we have been using (3 percent and 6 years, respectively) are appropriate for all the options. Then, the fair value of the options will be calculated as $100{,}000 \times \$15.65 = \$1{,}565{,}000$ in the Black–Scholes model and $100{,}000 \times \$15.75 = \$1{,}575{,}000$ in the binomial tree model.[5]

How well does the basic FAS 123 approach deal with the five features of employee stock options listed at the beginning of this article? The first feature (that options cannot be exercised during the vesting period) is not incorporated. The Black–Scholes model assumes no exercise until the end of the expected life; the binomial tree model assumes exercise at any time until the end of the expected life.

The second feature concerns the possibility of employees leaving during the vesting period. FAS 123 handles this situation by estimating an employee exit rate during the vesting period. One can argue that this rate is negatively correlated with the stock price because (1) employees are more likely to be fired when the company is doing badly and (2) employees are less likely to choose to leave a company voluntarily if their employee stock options are in the money. Therefore, FAS 123's procedure for handling this feature of employee stock options probably understates the value of the options. Estimating the negative correlation between employee exit rates and the stock price is difficult. Luckily, however, the impact of the correlation appears to be low and can probably be ignored.[6]

The third and fourth features listed at the beginning of this article lead to options being exercised earlier than they would if they were regular

exchange-traded or OTC options. FAS 123 handles these features by reducing the life of the option. The problem is that estimating the appropriate reduction in life is difficult. For example, setting the life of the option equal to an estimate of the average time the options will remain unexercised may yield reasonable results in many situations, but it does not have a solid theoretical basis. The "true" value of an option once it has vested is determined by the exercise strategy of the employee and the possibility that the employee may be forced to exercise the option early or abandon it if he or she leaves the company. There is no reason for the FAS 123 estimation procedure to produce a value close to the "true" value.

The final feature of employee stock options concerns dilution. The basic FAS 123 method does not deal with this issue.

Enhanced FAS 123

Our "enhanced FAS 123" model deals better than the FAS 123 model with the features listed at the beginning of this article, and it overcomes FAS 123's theoretical weakness.[7] The enhanced FAS 123 model differs from the basic FAS 123 model in the following ways:

- It explicitly considers the possibility that the employee will leave the company after the vesting period. It requires the company to use employee turnover rates to estimate an employee exit rate that applies to both the prevesting period and the postvesting period.
- It explicitly incorporates the employee's early exercise policy by assuming that early exercise happens when the stock price is a certain multiple of the exercise price.

The employee exit rate, e, can be directly estimated from historical data on employee turnover rates for the category of option holders being considered.[8] The early exercise multiple, M, is likely to be more difficult to estimate in many situations. Whenever sufficient data are available, M should be set equal to the average ratio of the stock price to the strike price when employees have made voluntarily early exercise decisions in the past and these decisions were not made immediately after the end of the vesting period.[9]

The binomial model can be extended to value employee stock options in the enhanced FAS 123 model. The tree is constructed in the usual way, and the rules for calculating the value of the option at each node of the tree are as follows:

- Options can be exercised only after the vesting period.
- A vested option is exercised prior to maturity if the stock price is at least M times the exercise price.
- A probability $e\delta t$ exists that the option will be forfeited in each short period of time during the vesting period.
- The same probability $e\delta t$ exists that the option will terminate in each short period of time δt after the end of the vesting period. When this termination happens, the option is forfeited if it is out of the money and exercised immediately if it is in the money.

Suppose the tree has N time steps of length δt. Suppose further that $S_{i,j}$ is the stock price at the jth node of the tree at time $i\delta t$ and that $f_{i,j}$ is the value of the option at this node. Define K as the strike price of the option and v as the time when the vesting period ends. The equations describing the backward induction through the tree are as follows:

$$f_{N,j} = \max(S_{N,j} - K, 0).$$

When $0 \le i \le N - 1$,

if $i\delta t > v$ and $S_{i,j} \ge KM$, then

$$f_{i,j} = S_{i,j} - K;$$

if $i\delta t > v$ and $S_{i,j} < KM$, then

$$f_{i,j} = (1 - e\delta t)e^{-r\delta t}[pf_{i+1,j+1} + (1 - p)f_{i+1,j}] + e\delta t \max(S_{i,j} - K, 0);$$

if $i\delta t < v$, then

$$f_{i,j} = (1 - e\delta t)e^{-r\delta t}[pf_{i+1,j+1} + (1 - p)f_{i+1,j}],$$

where r is the risk-free rate and p is the probability of an up movement in the binomial tree. The value of the option is f_{00}.[10]

The early exercise strategy we are assuming leads to the employee stock option being a type of barrier option. As explained in Hull (2002), using a trinomial, rather than binomial, tree is computationally more efficient when valuing a barrier option. In a trinomial tree, three branches emanate from each node and the spacing between the stock prices considered is adjusted so that the tree has nodes where the stock price equals KM. The probabilities on the tree are chosen so that the expected change and standard deviation of change in the stock price in a short period of time are correct in a risk-neutral world.[11]

Table 2 shows the price of the option in Table 1 for different assumptions about M and e. As might be expected, the value of the option increases as M increases and e decreases. In practice, an analyst will typically have to exercise some discretion in the choice of parameters that determine the tree, such as the volatility and dividend yield.

Table 2. Impact on Valuation of Option in Table 1 from Alternative Values of *M* and *e*

M	$e = 3\%$	$e = 5\%$	$e = 7\%$	$e = 10\%$
1.2	$13.13	$12.28	$11.47	$10.33
1.5	15.13	14.06	13.07	11.69
2.0	17.09	15.80	14.61	12.97
2.5	17.97	16.57	15.28	13.53
3.0	18.34	16.89	15.56	13.75

Notes: The parameter M is the ratio of the stock price to the exercise price necessary to trigger voluntary early exercise. The parameter e is the employee exit rate (assumed to be the same prevesting and postvesting).

Empirical Evidence on Exercise Behavior

Relatively few statistics are available on the actual exercise behavior of employees in different types of companies that would assist in choosing the early exercise multiple, *M*. Carpenter (1998) and Huddart and Lang (1996) provided some results. Carpenter looked at a sample of option exercises by top executives at 40 companies between 1979 and 1994. All the options had 10-year lives. The average vesting period was 1.96 years; the average time of exercise was 5.83 years; and the stock price at the time of exercise was 2.8 times the exercise price. Huddart and Lang looked at eight companies issuing options to 58,316 employees during the 1980s and early 1990s. They considered all employees, not only top executives, and found that the average time of exercise was 3.4 years and that the average ratio of the stock price to the exercise price at the time of exercise was 2.2.

Unfortunately, the average ratio of the stock price to the exercise price at the time of exercise is only an approximate estimate of an employee's exercise policy, because at the end of the vesting period, the stock price might be well above the minimum necessary to trigger exercise. Also, at the end of the life of an option, exercise will take place for all stock prices above the exercise price.

The average time to exercise and the ratio of the stock price to the strike price at the time of exercise are lower for the Huddart and Lang sample than for the Carpenter sample. This difference suggests that top executives may wait longer than junior employees before exercising. One can conjecture that top executives wait longer because they have less need to exercise options for personal liquidity reasons.

Dilution

In many situations, the amount of equity underlying employee stock option plans is tiny in relation to the total amount of equity issued by a company, so the impact of dilution can be safely ignored. We argue that the impact of dilution can be ignored also in other situations.

To understand the nature of dilution, consider a company where 100,000 shares are outstanding and the current share price is $50. The company decides to grant 100,000 stock options to its employees with a strike price of $50 and a vesting period of three years. If the market anticipates this decision, the stock price is not affected. The adverse effect (if any) of this action is already reflected in the current $50 stock price. If the action is unanticipated and the market sees little benefit to the shareholders from the employee stock options in the form of reduced salaries and more highly motivated managers, the stock price will decline immediately after the announcement of the employee stock options. Suppose that the stock price declines to $45. The dilution cost to the current shareholders is $5 a share, or $500,000 in total.

Suppose that the company does well during the vesting period so that by the end of the vesting period the share price is $100. Suppose further that all the options are exercised at this point. The payoff to the employees is $50 an option. One might be tempted to argue that further dilution will take place because 100,000 shares worth $100 a share are now merged with 100,000 shares for which only $50 was paid, so (1) the share price falls to $75 and (2) the payoff to the option holders is only $25 an option. This argument, however, is flawed. The exercise of the options is anticipated by the market and already reflected in the share price.

Galai and Schneller (1978) provided a model for estimating the potential cost of a large issue of employee stock options that is not anticipated by the market. They modeled the situation in which a company makes a single warrant issue, and they assumed that the total of all equity (shares plus warrants) follows geometric Brownian motion. This model is useful to senior executives who wish to estimate the potential cost of a large issue of employee stock options. Our focus in this article is a little different. We are interested in estimating the cost of stock options once they have been granted. We can estimate this cost by observing the stock price immediately after the grant of the stock options is announced and then assuming a stochastic process for the stock price. The stock price immediately after the grant of the stock options is announced fully incorporates any dilution.

The natural assumption is that the stock-price process follows geometric Brownian motion. When this assumption is made, we are effectively in the same position as when we ignore dilution, providing we base our value of the option on the post-grant, rather than pre-grant, stock price. An alternative, more theoretically correct, assumption is that the value of the stock plus all outstanding employee stock options (as well as warrants and convertibles, if any) follows geometric Brownian motion. This assumption is consistent with Galai and Schneller's approach. Unfortunately, it is extremely difficult to extend their work to the situation in which more than one option issue is outstanding.

To test the assumption that the stock price follows geometric Brownian motion, we considered the case in which there is only one stock option issue and compared the assumption with the assumption in the Galai–Schneller model that the stock (plus options) follows geometric Brownian motion. Except in cases of extreme dilution, we found the price difference between the two models to be very small, which is reassuring.

An instructive exercise is to consider a situation in which two types of employee stock options have been granted to the same employees. The options have identical terms except that one is settled by issuing more treasury stock and the other by buying existing stock. As we have explained, the stock price already incorporates the impact of expected dilution. The payoffs from the two options are, therefore, the same, and they should be valued similarly. The standard practice when valuing exchange-traded and OTC options is to use a volatility skew. The impact of outstanding employee stock options, warrants, convertibles, and so on, is not explicitly considered. We should do the same when valuing employee stock options. Generalizing from this conclusion, analysts should always value employee stock options in the same way they value corresponding OTC or exchange-traded options and ignore dilution. The only proviso is that we base our valuations on the post-grant announcement stock price, not the pre-grant announcement price.[12]

Conclusions

We have discussed practical issues in valuing employee stock options. The basic FAS 123 model uses the expected life of the option as an input, which is a theoretical weakness because option-pricing models are constructed so that the correct input is the total potential life of the option (i.e., the maximum time that can elapse until the option is exercised). We have proposed a modification to the basic FAS 123 model that overcomes this weakness. This enhancement involves parameterizing the holder's early exercise strategy by estimating the amount by which the stock price must exceed the strike price to trigger early exercise.

The main focus of this article has been on the most common type of employee stock option plan, in which (1) the exercise price of an option remains constant during the option's life, (2) the option can be exercised at any time during its life after an initial vesting period, and (3) the employee cannot continue to hold the option after he or she has left the company. The approach we suggest can be extended to value other options—for example, those for which the exercise price changes through time and those for which the exercise price is linked to the value of a stock index.

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Notes

1. Companies electing not to use the fair-value-based method must make *pro forma* disclosures of the effect of using the fair-value-based method.
2. This example is based on one that appears in Appendix B of FAS 123.
3. We prefer the term "employee exit rate" because, as we will discuss later, employees may leave the company after the vesting period. In this case, they forfeit the time value but not the intrinsic value of their options.
4. An alternative to adjusting for the employee exit rate during the vesting period is to assume that all options will vest and then later to reverse the charge to income for those that do not vest.
5. In practice, under FAS 123, options are divided into groups in such a way that the employee exit rate and expected life for the options in each group are approximately the same.
6. In an example considered by Rubinstein, the use of a correlated forfeiture rate increases the option value from $30.75 to $31.63.
7. Using the enhanced FAS 123 method should not be a problem. To quote from paragraph 154 of FAS 123: "The Board's intent in this Statement is for the . . . illustrations in Appendix B to be sufficiently broad that employers may adopt future refinements in the models that improve their application to employee stock options without requiring the Board to amend this statement."

8. Some adjustments to the estimates obtained from historical employee turnover rates may be appropriate if the stock price has increased or decreased rapidly in the past.
9. Past exercise experience may depend on how optimistic employees were about future prospects for the company, but making adjustments for this factor is difficult.
10. These equations assume that the employee exit rate is expressed with continuous compounding. If u is the annual employee turnover rate, $e = \ln(1 + u)$. If the company wants to assume that all options vest and then later reverse the charge for those that do not, it should set $e = 0$ when $i\delta t < v$.
11. Software for carrying out the calculations described here can be downloaded from www.rotman.utoronto.ca/~hull/.
12. Cox and Rubinstein (1985) made a similar argument with respect to warrants.

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