where

$$G(z) = \begin{cases} -z/|z| & \text{if } z \in \mathbf{R}^d \setminus \{0\}, \\ 0 & \text{if } z = 0. \end{cases}$$

Lemma 7.1. *For any $n \in \mathbf{N}_+$, there exists $C > 0$ such that*

$$E[|x^*(t)|^{2n}] \leq C(1+t).$$

Proof. The proof follows from the relation:

$$E[|x^*(t)|^{2n}] = |x|^{2n} + E[\int_0^t \{-2n|x^*(s)|^{2n-1} + n(2n+d-2)|x^*(s)|^{2n-2}\}dt].$$

Theorem 7.2. *We make the assumptions of Theorem 6.2. Then the optimal control $p^*(t)$ is given by*

$$p^*(t) = G(D\psi(x^*(t))) = G(x^*(t)),$$

and the value by

$$J(p^*) = \lambda.$$

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OPEN PROBLEMS ON BACKWARD STOCHASTIC DIFFERENTIAL EQUATIONS *

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1 BASIC RESULTS ON BACKWARD STOCHASTIC DIFFERENTIAL EQUATIONS

Recent years, many interesting problems in the theory of backward stochastic differential equations (in short, BSDE) have been solved. Others still remain open. In this paper, we will discuss those related to the stochastic control theory.

Let $(\Omega, \mathcal{F}, P)$ be a probability space, let $W_t, t \geq 0$ be a standard Brownian motion. For the sake of simplification, W_t is of 1-dimension. We are also limited to the interval $[0, T]$, with a finite constant $T > 0$. (see Peng [1991], Duffie, Ma and Yong [1994], Hu [1995], Chen [1997], Peng and Shi [1998] for infinite horizon cases) . All processes are assumed to be $\mathcal{F}_t$-adapted. Here the filtration $\{\mathcal{F}_t\}_{t\geq 0}$ is the natural one generated by the Brownian motion W. It is the information we have possessed up to time t. We will denote by $M^2(0, T; \mathbf{R}^n)$ the set of all $\mathbf{R}^n$-valued square-integrable $\mathcal{F}_t$-adapted processes.

We are given ξ, an $\mathcal{F}_T$-measurable $\mathbf{R}^m$-valued square-integrable random vector, and a function $f(y, z, t, \omega) : \mathbf{R}^m \times \mathbf{R}^m \times [0, T] \times \Omega \longrightarrow \mathbf{R}^m$ such that, for each $(y, z) \in \mathbf{R}^m \times \mathbf{R}^m$, $f(x, y, \cdot) \in M^2(0, T; \mathbf{R}^m)$. The problem of backward stochastic differential equation is to look for a pair of processes $(y, z) \in M^2(0, T; \mathbf{R}^m)^2$, that solves

$$-dy_t = f(y_t, z_t, t)dt - z_t dW_t, \quad t \in [0, T],$$
$$y_T = \xi.$$
(1.1)

Bismut [1973] studied the case when f is linear in (y, z). Pardoux and Peng [1990] obtained a general result: if f is Lipschitz in (y, z), then the solution $(y_t, z_t)_{0\leq t\leq T}$ of BSDE (1.1) exists and is unique.

Peng [1991a] observed a mutual relation between the solution $u(x,t)$: $\mathbb{R}^n \times [0, T] \longrightarrow \mathbb{R}^m$ of the the system of parabolic PDE

$$-\frac{\partial u^k}{\partial t} = \frac{1}{2}\sum_{i,j=1}^{n}[\sigma\sigma^T]_{ij}\frac{\partial^2 u^k}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b_i \frac{\partial u^k}{\partial x_i} + f^k(u, \nabla u\sigma, x),$$
$$u^k(x,T) = \Phi^k(x) \qquad k = 1, \cdots, m.$$
(1.2)

and the solution of $(y_s^{x,t}, z_s^{x,t})$, $s \in [t, T]$ of BSDE (1) in the time interval $[t, T]$, with the following special form of f and ξ: $f(y, z, \cdot) = g(y, z, x^{x,t}(\cdot))$, $\xi = \Phi(x_T^{x,t})$, where $x^{x,t}(s)$, $s \in [t, T]$ is an n-dimensional diffusion process with the initial condition (x, t). It evolves according to the following SDE

$$dx_s = b(x, s)ds + \sigma(x, s)dW_s, \quad s \in [t, T], \quad x_t = x.$$
(1.3)

The mutual relation is: $u(x^{x,t}(s), s) = y^{x,t}(s)$, $\sigma^T \nabla u(x^{x,t}(s), s) = z^{x,t}(s)$. Particularly

$$u(x,t) = y^{x,t}(t), \quad \sigma^T \nabla u(x,t) = z^{x,t}(t).$$
(1.4)

This can be formally checked by simply applying Itô's formula to $u(x,t)$. For $m = 1$, Peng [1992b] and then Pardoux and Peng [1992] showed that the function u defined by (1.4) is the unique viscosity solution of PDE (1.2) (see Crandall, Ishii and Lions [1992] for the theory of viscosity solution) Pardoux and Peng [1992] showed that if coefficients are smooth, say C^3, then $u(x,t)$ defined in (1.4) is the $C^{2,1}$ solution of (1.2) (m is allowed to be bigger than 1).

An interesting problem is, for $m \geq 2$, to find a suitable notion of the solution of PDE of "viscosity type" (1.2) that characterizes u in (1.4). Barles and Lesigne [1997] studied L^2-space PDE method. El Karoui, Peng and Quenez [1997] introduced a notion of system of quasi linear PDE (1.2). Pardoux and Rao [1997] discussed the viscosity solution for the system of PDE (1.2) with some additional assumptions such as f^i does not depends on z^j for $i \neq j$. But the general problem still remains an open.

We give two interesting special cases of formula (1.4) (for $m = 1$). The first one is $f(y, s) = c(x_s^{x,t})y + g(x_s^{x,t})$.

The function $u(x,t)$ defined by (1.4) is just the well-known Feynman-Kac formula. It is for this reason that we call (1.4) Nonlinear Feynman-Kac Formula.

The second case is the well-known Black & Scholes formula of option pricing. In this case $f = ay + bz$. [EPQ,1997] gives many further applications of BSDE in finance.

A useful tool in BSDE is Comparison Theorem introduced in Peng [1992b] and improved in Pardoux, Peng [1992], El Karoui, Peng and Quenez [1997].

Briefly speaking, if f and ξ are dominated respectively by f' and ξ', then the corresponding solution (y, z) is also dominated by (y', z'): $y_t \leq y'_t$. Many difficulties concerning the existence theorem in multi-dimensional case ($m \geq 2$) are due to the lack of this important property.

2 STOCHASTIC HAMILTON-JACOBI-BELLMAN EQUATIONS

We consider the following standard control system driven by a control process $\alpha. \in M^2(0, T; U)$ (called an admissible control), its value must be chosen in a given subset $U \subset \mathbb{R}^k$. The state variable evolves according to

$$dX_s^{x,t} = b(X_s^{x,t}, \alpha_s, s)ds + \sigma(X_s^{x,t}, \alpha_s, s)dW_s, \quad s \in [t, T],$$
$$X_t^{x,t} = x.$$
(2.1)

Some assumptions are needed to ensure the existence and uniqueness of the above stochastic differential equation. We are given the following utility function:

$$J_{x,t}(\alpha(\cdot)) = \mathbb{E}^{\mathcal{F}_t}\{\int_t^T f(X_s^{x,t}, \alpha_s)ds + \Phi(X_T^{x,t})\},$$
(2.2)

where $f(x, \alpha, t)$ and $\Phi(x)$ are real-valued functions. Our objective is to maximize the utility function over the set of admissible control. The maximum value is called the value function:

$$u(x,t) = ess\sup\{J_{x,t}(\alpha(\cdot)); \alpha(\cdot)) \in M^2(0,T;U)\}$$
(2.3)

A classic situation is when all coefficients b, σ, f and Φ are deterministic functions of x, α and t. In this case u is a deterministic function of (x, t). It is the unique viscosity solution of the following Hamilton-Jacobi-Bellman (HJB in short) equation:

$$-\frac{\partial u}{\partial t} = \sup_{v\in U}\left\{\frac{1}{2}(D^2u\sigma(x,\alpha), \sigma(x,\alpha)) + \langle Du, b(x,\alpha,t) \rangle + f(x,\alpha,t)\right\},$$
$$u(x,T) = \Phi(x).$$
(2.4)

But if one of the coefficients are random, e.g., for each x, $\Phi = \Phi(x, \omega)$ is $\mathcal{F}_T$-measurable), the value function $u(x, \cdot)$, for each x, becomes an $\mathcal{F}_t$-adapted process. It is no longer a solution of HJB equation (2.4). Peng [1992a] pointed out that, provided every thing is "smooth enough", the value function $u(x, \cdot)$ together with $u(x, \cdot)$, another function-valued $\mathcal{F}_t$-adapted process, consists the solution of the following backward stochastic PDE (Verification Theorem):

$$-du = \sup_{v\in U}\left\{\frac{1}{2}(D^2u\sigma(x,\alpha,t), \sigma(x,\alpha,t)) + \langle Du, b(x,\alpha,t) \rangle + \langle Dv, b(x,\alpha,t) + f(x,\alpha,t)\right\}dt - v(x,t)dW,$$
$$u(x,T) = \Phi(x).$$
(2.5)

This equation is called stochastic HJB equation. The problem is, in most circumstances, one can not expect that the pair $(u,v)(x,\cdot)$ is 'smooth enough'. Peng [1992a] studied the case where the diffusion term is independent of the control variable: $\sigma = \sigma(x,t)$. An existence and uniqueness theorem has been obtained. A difficult problem is when $\sigma = \sigma(x,\alpha,t)$. Two problems are:
(i) To introduce a generalized notion of viscosity solutions of (2.5). (ii) To develop a $C^{2+\alpha}$-theory.

3 STOCHASTIC MATRIX-VALUED RICCATI EQUATION

A typical situation of the above optimization is the so-called linear quadratic case, i.e., $U = \mathbf{R}^k$, b and σ are linear functions, f and Φ are quadratic functions:

$$b_t(x,\alpha) = A_t x + B_t \alpha, \quad \sigma_t(x,\alpha) = C_t x + D_t \alpha \qquad (3.1)$$

$$f_t(x,\alpha) = -\frac{1}{2}[\langle R_t x, x\rangle + \langle N_t\alpha, \alpha\rangle], \quad \Phi(x) = -\frac{1}{2}\langle Q x, x\rangle = \dots \qquad (3.2)$$

where $A_t, B_t, C_t, D_t, R_t, N_t$ and Q are bounded and matrix-valued with suitable dimensions. R_t and Q are symmetric and nonnegative. N_t is strictly positive. It is easy to check that the value function u is also quadratic in x: $u(x,t) = \frac{1}{2}\langle Y_t x, x\rangle$. Consequently, v is also quadratic: $v(x,t) = \frac{1}{2}\langle Z_t x, x\rangle$. Here Y_t and Z_t are $n\times n$-symmetric matrix-valued processes. Y_t is nonnegative. From (14) one can formally write:

$$-dY_t = F(Y_t, Z_t, t)dt - Z_t dW_t, \quad Y_T = Q, \qquad (3.3)$$

where $F(Y, Z, t)$ is an $n\times n$-valued function:

$$F(Y,Z,t) = A_t^T Y + Y A_t + C_t^T Z + Z C_t + C_t^T Y C_t + R_t$$
$$-(Y B_t + Z D_t + D_t Y C_t^T)(N_t + D_t^T Y D_t)^{-1}(Y B_t + Z D_t + D_t Y C_t^T)^T \qquad (3.4)$$

The problem is to prove the existence and uniqueness of (3.3). When $D = 0$, the problem is solved, see Bismut [1976], Peng [1992a,1993]. The case $n = 1$, $D \neq 0$ is solved ([OT,1997]).

4 GENERAL MAXIMUM PRINCIPLE UNDER RECURSIVE UTILITIES

The above stochastic optimization problem can also be treated by the method of stochastic maximum principle of Pontryagin's type. Let H be the "stochastic Hamiltonian" introduced in Bismut [1973] (see also Bensoussan [1981]) defined by $H = \langle p, b(x,\alpha,t)\rangle + \langle q, \sigma(x,\alpha,t)\rangle + f(x,\alpha,t)$. Based on the Bismut's 'local maximum principle', Peng [1990] obtained the following 'general stochastic

maximum principle':

$$\sup_{\alpha\in U}\left\{ H(x_t, \alpha, p_t, q_t - P_t\sigma(x_t,\alpha_t), t) - \frac{1}{2}\langle P(t)\sigma(x_t,\alpha), \sigma(x_t,\alpha)\rangle\right\}$$
$$\leq H(x_t, \alpha_t, p_t, q_t - P_t\sigma(x_t,\alpha_t), t) - \frac{1}{2}\langle P(t)\sigma(x_t,\alpha), \sigma(x_t,\alpha_t)\rangle \qquad (4.1)$$

where the pair $(p,q) \in M^2(0,T;\mathbf{R})^2$ is the solution of the following linear BSDE

$$-dp_t = \partial_x H(x_t, \alpha_t, p_t, q_t, t)dt - q_t dW_t, \quad p_T = \Phi(x_T). \qquad (4.2)$$

and (P_t, Q_t) solves the following $n\times n$-matrix valued BSDE:

$$-dP_t = [b_x^T P + P b_x + \sigma_x^T Q + Q\sigma_x + \sigma_x^T P\sigma_x$$
$$+ H_{xx}(x_t, \alpha_t, p_t, q_t, t)]dt - Q_t dW_t,$$
$$P_T = \Phi_{xx}(x_T). \qquad (4.3)$$

Recently, Duffie, Epstein [1992a,b] and other economists are introduced the notion of recursive utilities in continuous time (see also Duffie, Lions [1992]). El Karoui, Peng and Quenez [1997] give the following general form of recursive utility:

$$-dy_t = f(x_t, \alpha_t, y_t, z_t, t)dt - z_t dW_t, \quad y_T = \Phi(x_T), \qquad (4.4)$$

where y_0 is solved via the following BSDE

$$J_{x_0,0}(\alpha(\cdot)) = y_0 \qquad (4.5)$$

It is easy to check that, when f only depends on (x,α); i.e., $f = f(x,\alpha,t)$, then $J(\alpha(\cdot))$ is the classical utility function of type (2.2). Local maximum principle under the recursive utility is given in Peng [P,1993] (see also Xu [1994], Wu and Xu [1997]. The corresponding 'global maximum principle' for the case where f depends nonlinearly on z is open, except for some special case (see Peng and Xu [1998]).

5 EXISTENCE OF BSDE WITH NON LIPSCHITZ COEFFICIENTS

We return to a general BSDE of form (1.1). Since [PP,1990] many efforts have been made to relax the assumption of the uniform Lipschitz condition imposed on f. For 1-dimensional case, i.e., $m = 1$, see Lepeltier, San Martin [1996]; Kobylansky [1997] shows that the linear growth condition with respect to (y,z) can be relaxed to the quadratic growth condition provided the prescribed terminal value $y_T = \xi$ is bounded (see also [HLS,1998] for more general situation). This is a very interesting result. But the boundness of ξ excludes a very interesting application, i.e., the problem of risk sensitive control (see

Bensoussan and Nagai[1996], Bielecki and Pliska [1998]). This problem can be naturally formulated as a BSDE with f quadratic in z. In this domain one often interested in the situations where $\xi \in L^2$. Risk sensitive control problems also indicate a totally open topic in BSDE: the problem of explosions.

Another challenge in BSDE is for multi-dimensional case: $m \geq 2$. The problem of the existence with coefficient f only continuous in (y, z) becomes very hard (a Markov case was treated in Hamadane Lepeltier and Peng [1997]). Similar problem appears in situation of the quadratic growth condition. The techniques that successfully applied in 1-dimensional case fails since the lack of the comparison theorem. Multi-dimensional case has many interesting applications. For example: an important open questions in fining the equilibrium strategies in non-zero-sum differential games is formulated as a BSDE where f is only continuous (see [EPQ,1997], [HLP,1997]). The quadratic growth condition relates to harmonic mappings in differential geometry (see Darling [1995]).

6 FORWARD-BACKWARD STOCHASTIC DIFFERENTIAL EQUATIONS

Forward-Backward Stochastic Differential Equations (in short, FBSDE) is derived from many stochastic optimization problems. For fully coupled situation, a local existence and uniqueness result was obtained in Antonelli [1992] (see also [P.1992c]). The first global existence and uniqueness results, i.e., in an arbitrarily large interval $[0, T]$, was given in Ma and Yong [MY,1995], Ma, Protter and Yong [1994], which is now well known as "four-step-approach". Recently, Pardoux and Tang [1997] also give a new approach.

Here we discuss a different global existence and uniqueness approach given in Hu and Peng [1994] with a continuation method (see also Peng [1991a]). This approach is generalized in Peng and Wu [1996], Yong [1997] (Bridge Method).

Let b, σ and f be three $\mathbf{R}^n$-valued function of $\zeta = (x, y, z) \in (\mathbf{R}^n)^3$. We assume that the mapping A defined by $A(\zeta) = (-f, b, \sigma)(\zeta) : (\mathbf{R}^n)^3 \longrightarrow (\mathbf{R}^n)^3$ is Lipschitz in ζ and is monotonic:

$$\langle A(\zeta) - A(\zeta'), \zeta - \zeta' \rangle \leq -\beta \| \zeta - \zeta' \|^2 \tag{6.1}$$

We also assume that Φ is Lipschitz in x and is monotonic: $(\Phi(x) - \Phi(x'), x - x') \geq 0$. For each fixed $(t, x) \in \mathbf{R}^n \times [0, T]$, we consider the following problem of FBSDE: to find a triple of processes $(X_s^{x,t}, Y_s^{x,t}, Z_s^{x,t})$, $s \in [t, T]$, that solves

$$
\begin{aligned}
dX_s^{x,t} &= b(X_s^{x,t}, Y_s^{x,t}, Z_s^{x,t})ds + \sigma(X_s^{x,t}, Y_s^{x,t}, Z_s^{x,t})dW_s, \quad X_t^{x,t} = x. \\
-dY_s^{x,t} &= f(X_s^{x,t}, Y_s^{x,t}, Z_s^{x,t})ds - Z_s^{x,t}dW_s, \quad Y_T^{x,t} = \Phi(X_T^{x,t}).
\end{aligned}
$$

By Hu and Peng [1994], this equation has a unique solution. We then can define the following two (deterministic) function of (x, t),

$$u(x, t) = Y_t^{x,t}, \quad v(x, t) = Z_t^{x,t} \tag{6.2}$$

If the pair (u, v) is sufficiently smooth, then it is the solution of the following PDE

$$
\begin{aligned}
-\frac{\partial u}{\partial t} &= \frac{1}{2}[\sigma\sigma^T(x, u, v)]_{ij}\frac{\partial^2 u}{\partial x_i \partial x_j} + b_i(x, u, v)\frac{\partial u}{\partial x_i} + f(x, u, v), \\
v(x, t) &= \sigma^T(x, u, v)\nabla u, \quad (x, t) \in \mathbf{R}^n \times [0, T].
\end{aligned} \tag{6.3}
$$

with a Cauchy-type of terminal condition: $u(x, T) = \Phi(x)$. An interesting problem is how to find a suitable notion of the solution (u, v) of this PDE that coincides with (6.2). A more general case is when b, σ, f and Φ are random, e.g., for each $x \in \mathbf{R}^n$, $\Phi(x, \omega)$ is $\mathcal{F}_T$ measurable. In this situation the pair $(u(x, \cdot), v(x, \cdot))$ defined in (6.2) is an $(\mathbf{R}^n)^2$-valued $\mathcal{F}_T$-adapted process. The following backward stochastic PDE is then in order:

$$
\begin{aligned}
-du &= \{\frac{1}{2}[\sigma\sigma^T(x, u, v)]_{ij}\frac{\partial^2 u}{\partial x_i \partial x_j} + b_i(x, u, v)\frac{\partial u}{\partial x_i} \\
&\quad + \sigma^T(x, u, v)\nabla v + f(x, u, v)\}dt - vdW_t, \\
v(x, t) &= \sigma^T(x, u, v)\nabla u, \quad (x, t) \in \mathbf{R}^n \times [0, T].
\end{aligned} \tag{6.4}
$$

with the terminal condition: $u(x, T) = \Phi(x)$. The question is to prove the existence and uniqueness of (6.4).

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