Title: The Inverse First Passage Time Problem for killed Brownian motion

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Abstract: The inverse first passage time problem asks whether, for a Brownian motion B and a nonnegative random variable ζ, there exists a time-varying barrier b such that $\mathbb{P}\{B_s > b(s), 0 \leq s \leq t\} = \mathbb{P}\{\zeta > t\}$. We study a "smoothed" version of this problem and ask whether there is a "barrier" b such that $\mathbb{E}[\exp(-\lambda \int_0^t \psi(B_s - b(s))\, ds)] = \mathbb{P}\{\zeta > t\}$, where λ is a killing rate parameter and $\psi : \mathbb{R} \to [0, 1]$ is a non-increasing function. We prove that if ψ is suitably smooth, the function $t \mapsto \mathbb{P}\{\zeta > t\}$ is twice continuously differentiable, and the condition $0 < -\frac{d \log \mathbb{P}\{\zeta > t\}}{dt} < \lambda$ holds for the hazard rate of ζ, then there exists a unique continuously differentiable function b solving the smoothed problem. We show how this result leads to flexible models of default for which it is possible to compute expected values of contingent claims.