Assignment 01

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1 Condensation of the solar nebula

1.1 Question (a)

Suppose the number of Atoms condensed as metallic Fe on the fully condensed nebula is N, therefore

$$\begin{cases} N_{Fe} = N_{FeO} = N \\ N_{MgO} = N_{SiO_2} = 2N \\ N_{H_2O} = (24 - 1 - 2 - 4)N = 17N \end{cases} \tag{1}$$

So, we calculate ratio mass seperately as:

$$\begin{cases} \frac{M_{ice}}{M_{rock}} = \frac{N_{H_2O} \cdot 18}{N_{MgO} \cdot 40 + N_{SiO_2} \cdot 60 + N_{FeO} \cdot 72} = \frac{17N \cdot 18}{80N + 120N + 72N} = \frac{9}{8} \\ \frac{M_{ice}}{M_{Fe}} = \frac{N_{H_2O} \cdot 18}{N_{Fe} \cdot 56} = \frac{17N \cdot 18}{56 \cdot N} = \frac{153}{28} \end{cases} \tag{2}$$

The mass ratio of them are:

$$M_{ice} : M_{rock} : M_{metal} = 153 : 136 : 28 \tag{3}$$

1.2 Question (b)

$$\bar{\rho} = \frac{M_{total}}{V_{total}} = \frac{M_{ice} + M_{rock} + M_{metal}}{\frac{M_{ice}}{\rho_{ice}} + \frac{M_{rock}}{\rho_{rock}} + \frac{M_{metal}}{\rho_{metal}}} = \frac{153 + 136 + 28}{\frac{153}{1} + \frac{136}{3} + \frac{28}{8}} = 1.5706 g/cm^3 \tag{4}$$

2 The missing Si in the Earth's mantle

2.1 Question 01

The missing Si composition in BSE is:

$$C_{Si,missing} = \frac{M_{Si,CI}}{M_{Mg,CI}} \cdot M_{Mg,BSE} - M_{Si,BSE} = 3.53 \ wt\% \tag{5}$$

2.2 Qustion 02

The Si concentration in the core using eq(5) can be calculated as:

$$M_{Si,missing} = C_{Si,missing} \cdot M_{BSE} \tag{6}$$

$$C_{Si,core} = \frac{M_{Si,missing}}{M_{core}} = C_{Si,missing} \cdot \frac{M_{BSE}}{M_{core}} = 3.53 * \frac{4.03}{1.94} = 7.33\ wt\% \tag{7}$$

3 Two-layer model of Earth

3.1 Question (a)

Note $R_1 = 3480km$, $R_2 = 6371km$. The mean density of Earth is:

$$\bar{\rho_E} = \frac{M_E}{V_E} = \frac{3M_E}{4\pi R_2^3} = 5.51 \times 10^3\ kg/m^3 \tag{8}$$

The surface gracity:

$$g_S = \frac{GM_E}{R_2^2} = 9.81\ m/s^2 \tag{9}$$

3.2 Question (b)

Note ρ_1 to be core density and ρ_2 to be mantle density, so the Earth's total mass can be expressed as:

$$M_S = (\rho_1 - \rho_2) \cdot \frac{4}{3}\pi R_1^3 + \rho_2 \cdot \frac{4}{3}\pi R_2^3 \tag{10}$$

The Earth's moment of inertia can be expressed as:

$$I_E = \frac{8\pi}{3}\int_0^{R_2} \rho(r) r^4 dr = \frac{8\pi}{3}\int_0^{R_1} \rho_1 r^4 dr + \frac{8\pi}{3}\int_{R_1}^{R_2} \rho_2 r^4 dr = \frac{8\pi}{15}(\rho_1 R_1^5 + \rho_2 R_2^5 - \rho_2 R_1^5) \tag{11}$$

From (10) and (11), the density can be derived:

$$\begin{cases} \rho_1 = 12.4 \times 10^3\ kg/m^3 \\ \rho_2 = 4.17 \times 10^3\ kg/m^3 \end{cases} \tag{12}$$

3.3 Question (c)

The mass distribution is:

$$M(r) = \begin{cases} \frac{4}{3}\rho_1 \pi r^3 & , r \leq R_1 \\ \frac{4}{3}(\rho_1 - \rho_2)\pi R_1^3 + \frac{4}{3}\rho_2 \pi r^3 & , R_1 < r \leq R_2 \end{cases} \tag{13}$$

So, P_{CMB} can be derived as:

$$\begin{aligned} P_{CMB} &= \int_{R_2}^{R_1} -\frac{GM(r)}{r^2}\rho(r)dr \\ &= \int_{R_1}^{R_2} \frac{4}{3}\pi G(\rho_1-\rho_2)\rho_2 R_1^3 \cdot \frac{1}{r^2}dr + \int_{R_1}^{R_2} \frac{4}{3}\pi G\rho_2^2 \cdot r dr \\ &= \frac{4}{3}\pi G(\rho_1-\rho_2)\rho_2 R_1^3(\frac{1}{R_1}-\frac{1}{R_2}) + \frac{2}{3}\pi G\rho_2^2(R_2^2-R_1^2) = 9.89\times 10^{10}\ Pa \end{aligned} \tag{14}$$

Then, P_{center} can be derived the same way as:

$$P_{center} - P_{CMB} = \frac{2}{3}\pi G\rho_1^2 R_1^2; \tag{15}$$

$$P_{center} = 3.59\times 10^{11}\ Pa \tag{16}$$

3.4 Question (d)

From Question (b) and Question (c), we can derive that

$$g(r) = \frac{GM(r)}{r^2} = \begin{cases} \frac{4G\rho_1\pi}{3}r & ,r\leq R_1 \\ \frac{4G(\rho_1-\rho_2)\pi}{3}\frac{R_1^3}{r^2} + \frac{4G\rho_2\pi}{3}r & ,R_1 < r \leq R_2 \end{cases} \tag{17}$$

$$P(r) = \begin{cases} \frac{2G\rho_1^2\pi}{3}(R_1^2-r^2) + P_{CMB} & ,r\leq R_1 \\ \frac{4G(\rho_1-\rho_2)\rho_2\pi}{3}(\frac{R_1^3}{r}-\frac{R_1^3}{R_2}) + \frac{2G\rho_2^2\pi}{3}(R_2^2-r^2) & ,R_1 < r\leq R_2 \end{cases} \tag{18}$$

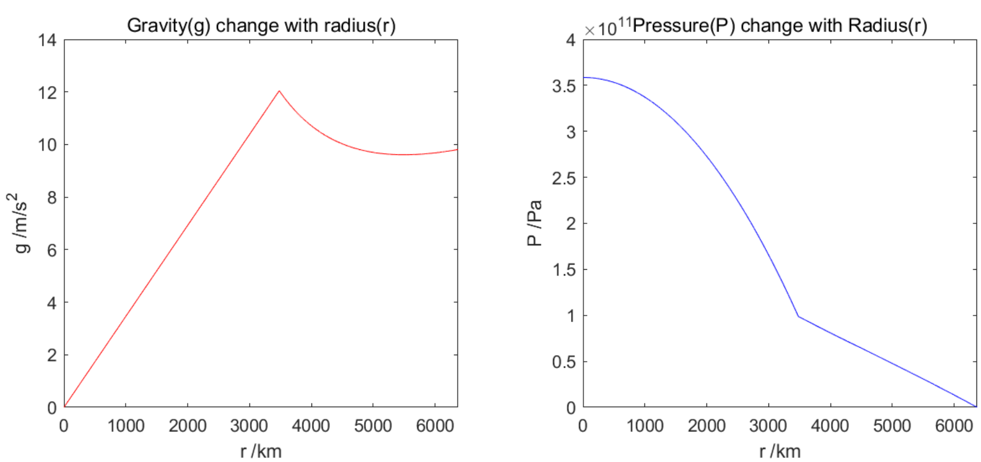

Figure 1: plot of gravity and pressure change with radius r

4 Gravity anomalies

The observation gravity is:

$$g_m = 9.803243\ m/s^2 \tag{19}$$

The theory gravity is:

$$g_n = 9.780327(1 + 0.0053024 sin^2(\phi) - 0.0000059 sin^2(2\phi)) = 9.804876\ m/s^2 \tag{20}$$

The free air correction is:

$$\Delta g_{FA} = 0.3086 \times 10^{-5} \times h = 1.6735 \times 10^{-3}\ m/s^2 \tag{21}$$

The Bouguer correction is:

$$\Delta g_{BP} = 0.112 \times 10^{-5} \times h = 6.074 \times 10^{-4}\ m/s^2 \tag{22}$$

So, the free-air and Bouguer gravity anomalies are:

$$\Delta g_F = g_m - g_n + \Delta_{FA} = 4.05 \times 10^{-5}\ m/s^{-2} \tag{23}$$

$$\Delta g_B = g_m - g_n + \Delta_{FA} - \Delta_{BP} = -5.669 \times 10^{-4}\ m/s^{-2} \tag{24}$$