ESS 5031 Physics of Earth and Planetary Interiors
Fall 2022

Homework Problem Set 1
(Due on 10/12/2022 in class)

1. Condensation of the solar nebula

In this problem, we estimate the relative masses between ices, rocks, and metals for a planet/satellite condensed outside the snowline of the solar nebula (that is, the nebula disk temperature, at which the planet forms, is lower than the condensation temperature of ices). From the observation of the Sun's photosphere, we know the number of O, Mg, Si, and Fe atoms in the solar nebula is roughly in the ratio of 24:1:1:1.

a) Assuming (1) a half of Fe is condensed as metallic Fe, and the other half as FeO; (2) the rock components condensed from the nebula are made of MgO, FeO, SiO_2; (3) ices condensed are made of only H_2O: **calculate the mass ratio between ice, rocks, and metal** for a fully condensed nebula.

b) Given the densities of metal, rock, and ice as 8 g/cm^3, 3 g/cm^3, and 1 g/cm^3, respectively, **calculate the mean density of an icy satellite** that is condensed from the solar nebula according to the assumptions described in (a).

2. The missing Si in the Earth's mantle

The Mg/Si ratio in the bulk silicate Earth (crust and mantle) does not match that of any undifferentiated meteorite. Compared to the undifferentiated meteorites, the bulk silicate Earth (BSE) appears to be relatively depleted in Si, that is, the Si concentration in the BSE is smaller than that of the undifferentiated meteorites.

(1) Assuming the bulk Earth was created from a material with a CI chondrite signature (i.e., Mg/Si is the same for the bulk Earth and CI chondrite) and using the Mg and Si concentrations in the table below, **calculate how much Si is missing in the bulk silicate Earth (in weight %)**.

(2) Assuming all missing Si entered into the core during the core formation process. **Calculate the Si concentration in the core.** The mass of the bulk silicate Earth and the core are 4.03×10^{24} kg and 1.94×10^{24} kg, respectively.

Note: The exercise here just provides one of the possible explanations for the missing Si. Other explanations exist and are debated. For example, some researchers suggest the missing Si may be hidden somewhere in the lower mantle as Si-rich domains while others suggest the missing Si might not have condensed to the silicate Earth due to its slight volatility.

CI chondrite and model bulk silicate Earth compositions

Element	CI Chondrite[1]	Bulk Silicate Earth[2]
Mg	9.89 wt%	22.8 wt%
Si	10.64 wt%	21 wt%

[1]Anders and Grevesse (1989)
[2]McDonough and Sun (1995)

3. Two-layer model of Earth

Assuming Earth is a non-rotating sphere with a radius of 6371 km and has a two-layer structure including a spherical core with a radius of 3480 km in the center and an outer mantle layer. The Earth's total mass and moment of inertia are given as 5.97×10^{24} kg and 8.04×10^{37} kg m^2.

a) Determine the mean density of Earth and gravity on the surface of Earth;

b) Determine the mean densities of the Earth's mantle and core assuming the mantle and core have uniform densities;

c) Determine the pressure at the core-mantle boundary (CMB) and at the center of Earth assuming the core and mantle have uniform densities;

d) Plot the pressure and gravity inside Earth as functions of radius from the center to the surface (you will need to use a programming language/plotting software package for this task).

4. Gravity anomalies (D. Turcotte and G. Schubert, Geodynamics, Problem 5.20)

The surface gravity at a measuring site is 9.803243 ms^{-2}. The site has a latitude 43º32'16"N and an elevation of 542.3 m. Obtain the free-air and Bouguer gravity anomalies. (Hint: You will need to use the following International Gravity Formula to make the latitude correction. $g=9.780327(1+0.0053024\sin^2\phi-0.0000059\sin^2 2\phi)$ ms^{-2}, where ϕ is latitude. Pay attention to the unit of the latitude when using the sine function.)