Friot: A Functional Reactive Language for IoT Programs with Dependent Type-and-Effect System (Report)

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I. Type checking trees for function delay

```
delay t =
   if t == 0 then ev (LightUp)
   else ev (Tick);
        delay (t - 1);
```

$\tau_{\texttt{delay}} = \texttt{t} : \texttt{Int} \rightarrow (\texttt{Unit} \mathbin{\&} \Phi_{\texttt{delay}})$
$\Phi_{\texttt{delay}} = (\texttt{t} \geq 0 \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 0 \wedge \underline{\textbf{Tick}}^{\omega})$

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delay

$$\dfrac{\dfrac{\Phi = \underline{\textbf{LightUp}}}{\Gamma' \vdash \texttt{ev (LightUp)} : \texttt{Unit} \mathbin{\&} \underline{\textbf{LightUp}}} \text{(T-Event)} \quad \dfrac{\dfrac{\Phi = \underline{\textbf{Tick}}}{\Gamma' \vdash \texttt{ev (Tick)} : \texttt{Unit} \mathbin{\&} \underline{\textbf{Tick}}} \text{(T-Event)} \quad \textbf{A}}{\Gamma' \vdash (\texttt{ev (Tick); delay (t - 1);}) : \texttt{Unit} \mathbin{\&} (\underline{\textbf{Tick}} \cdot \Phi_{\texttt{delay}}^{(\texttt{t}-1)})} \text{(T-Let)}}{\dfrac{\Gamma' \vdash (\texttt{if t == 0 then ev (LightUp) else ev (Tick); delay (t - 1);}) : \texttt{Unit} \mathbin{\&} (\texttt{t} == 0 \wedge \underline{\textbf{LightUp}}) \vee (\texttt{t} \neq 0 \wedge \underline{\textbf{Tick}} \cdot \Phi_{\texttt{delay}}^{(\texttt{t}-1)})}{\dfrac{\textit{... (Effects Computation I) ...}}{\Gamma \vdash (\texttt{delay t = if ... then ... else ...}) : (\texttt{t} : \texttt{Int}) \rightarrow (\texttt{Unit} \mathbin{\&} (\texttt{t} \geq 0 \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 0 \wedge \underline{\textbf{Tick}}^{\omega}))} \text{(T-Fun)}} \text{(T-If)}}$$

$\Gamma' = \Gamma, \texttt{delay} : \tau_{\texttt{delay}}$
$\Phi_{\texttt{delay}}^{(\texttt{t}-1)} = (\texttt{t} \geq 1 \wedge \underline{\textbf{Tick}}^{\texttt{t-} 1} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 1 \wedge \underline{\textbf{Tick}}^{\omega})$

A:
$$\dfrac{\dfrac{\text{sty}(\Gamma'(\texttt{delay})) \in \rightarrow}{\Gamma' \vdash \texttt{delay} : (\texttt{t} : \texttt{Int}) \rightarrow (\texttt{Unit} \mathbin{\&} (\texttt{t} \geq 0 \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 0 \wedge \underline{\textbf{Tick}}^{\omega}))} \text{(T-VaF)} \quad \dfrac{\text{sty}(\Gamma'(\texttt{t})) = \texttt{Int}}{\Gamma' \vdash (\texttt{t - 1}) : \texttt{int}} \text{(T-Var, T-Op)}}{\Gamma' \vdash \texttt{delay (t - 1)} : (\texttt{t - 1} : \texttt{int}) \rightarrow (\texttt{Unit} \mathbin{\&} (\texttt{t} \geq 1 \wedge \underline{\textbf{Tick}}^{\texttt{t - } 1} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 1 \wedge \underline{\textbf{Tick}}^{\omega}))} \text{(T-App, S-Base)}$$

I:
$\quad (\texttt{t} == 0 \wedge \underline{\textbf{LightUp}}) \vee (\texttt{t} \neq 0 \wedge \underline{\textbf{Tick}} \cdot \Phi_{\texttt{delay}}^{(\texttt{t}-1)})$
$\equiv \quad (\texttt{t} == 0 \wedge \underline{\textbf{LightUp}}) \vee (\texttt{t} \neq 0 \wedge \underline{\textbf{Tick}} \cdot ((\texttt{t} \geq 1 \wedge \underline{\textbf{Tick}}^{\texttt{t-} 1} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 1 \wedge \underline{\textbf{Tick}}^{\omega})))$
$\equiv \quad (\texttt{t} == 0 \wedge \underline{\textbf{LightUp}}) \vee ((\texttt{t} \neq 0 \wedge \texttt{t} \geq 1) \wedge \underline{\textbf{Tick}} \cdot \underline{\textbf{Tick}}^{\texttt{t-} 1} \cdot \underline{\textbf{LightUp}}) \vee ((\texttt{t} \neq 0 \wedge \texttt{t} < 1) \wedge (\underline{\textbf{Tick}} \cdot \underline{\textbf{Tick}}^{\omega}))$
$\equiv \quad (\texttt{t} == 0 \wedge \underline{\textbf{LightUp}}) \vee (\texttt{t} \geq 1 \wedge \underline{\textbf{Tick}} \cdot \underline{\textbf{Tick}}^{\texttt{t-} 1} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 0 \wedge (\underline{\textbf{Tick}} \cdot \underline{\textbf{Tick}}^{\omega}))$
$\equiv \quad (\texttt{t} == 0 \wedge \underline{\textbf{LightUp}}) \vee (\texttt{t} \geq 1 \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 0 \wedge \underline{\textbf{Tick}}^{\omega})$
$\equiv \quad (\texttt{t} \geq 0 \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{LightUp}}) \vee (\texttt{t} < 0 \wedge \underline{\textbf{Tick}}^{\omega})$
$\equiv \quad \Phi_{\texttt{delay}}$

II. Type checking trees for function door_control

```
motion_sensor () =          door_control ()=
   if * then                   ev (Close);
     ev (Active)               motion_sensor ();
   else                        ev (Open);
     ev (Passive);             ev (Delay);
     motion_sensor ();         door_control ();
```

$\tau_{\texttt{motion\_sensor}} = \texttt{Unit} \rightarrow (\texttt{Unit} \mathbin{\&} \Phi_{\texttt{motion\_sensor}})$
$\Phi_{\texttt{motion\_sensor}} = \underline{\textbf{Passive}}^{*} \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^{\omega}$

$$\tau_{\texttt{door\_control}} \;=\; \texttt{Unit} \rightarrow (\texttt{Unit} \;\&\; \Phi_{\texttt{door\_control}})$$

$$\Phi_{\texttt{door\_control}} \;=\; (\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^\omega$$

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door_control

$$\cfrac{\Phi = \underline{\textbf{Close}}}{\Gamma' \vdash \texttt{ev (Close)} : \texttt{Unit} \;\&\; \underline{\textbf{Close}}} \text{(T-Event)} \quad \textbf{A} \quad \cfrac{\Phi = \underline{\textbf{Open}}}{\Gamma' \vdash \texttt{ev (Open)} : \texttt{Unit} \;\&\; \underline{\textbf{Open}}} \text{(T-Event)} \quad \cfrac{\Phi = \underline{\textbf{Delay}}}{\Gamma' \vdash \texttt{ev (Delay)} : \texttt{Unit} \;\&\; \underline{\textbf{Delay}}} \text{(T-Event)} \quad \textbf{B}$$

$$\cfrac{\rule{0pt}{0pt}}{\Gamma' \vdash (\texttt{ev (Close); motion\_sensor (); ev (Open); ev (Delay); door\_control ();}) : \texttt{Unit} \;\&\; (\underline{\textbf{Close}} \cdot \Phi_{\texttt{motion\_sensor}} \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}} \cdot \Phi_{\texttt{door\_control}})} \text{(T-If)}$$

$$\cfrac{... \text{ (Effects Computation I) } ...}{\Gamma \vdash (\texttt{door\_control = ... }) : \texttt{Unit} \rightarrow (\texttt{Unit} \;\&\; (\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^\omega)} \text{(T-Fun)}$$

$\Gamma' = \Gamma, \texttt{door\_control} : \tau_{\texttt{door\_control}}$

A:
$$\cfrac{\cfrac{\texttt{sty}(\Gamma'(\texttt{motion\_sensor})) \in \rightarrow}{\Gamma' \vdash \texttt{motion\_sensor} : \tau_{\texttt{motion\_sensor}}} \text{(T-VaF)} \quad \Gamma' \vdash () : \texttt{Unit}}{\Gamma' \vdash \texttt{motion\_sensor ()} : \texttt{Unit} \;\&\; \Phi_{\texttt{motion\_sensor}}} \text{(T-App)}$$

B:
$$\cfrac{\cfrac{\texttt{sty}(\Gamma'(\texttt{door\_control})) \in \rightarrow}{\Gamma' \vdash \texttt{door\_control} : \tau_{\texttt{door\_control}}} \text{(T-VaF)} \quad \Gamma' \vdash () : \texttt{Unit}}{\Gamma' \vdash \texttt{door\_control ()} : \texttt{Unit} \;\&\; \Phi_{\texttt{door\_control}}} \text{(T-App)}$$

I:
$$\begin{aligned}
&\underline{\textbf{Close}} \cdot \Phi_{\texttt{motion\_sensor}} \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}} \cdot \Phi_{\texttt{door\_control}} \\
\equiv\;& \underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}} \cdot ((\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^\omega) \\
\equiv\;& (\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^{\omega+1} \\
\equiv\;& (\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^\omega \\
\equiv\;& \Phi_{\texttt{door\_control}}
\end{aligned}$$

III. Type checking trees for function temperature_control

```
device_control t =
    if t < 20 then ev (CloseBoth)
    else if 20 <= t < 30
        then ev (Fan)
    else if 30 <= t < 40
        then ev (Alternate)
    else if 40 <= t < 50
        then ev (AC)
    else ev (OpenBoth);
```

```
temperature_control () =
    device_control (get_temp ());
    temperature_control ();
```

$$\tau_{\texttt{device\_control}} \;=\; \texttt{t} : \texttt{Int} \rightarrow (\texttt{Unit} \;\&\; \Phi_{\texttt{device\_control}})$$

$$\Phi_{\texttt{device\_control}} \;=\; (\texttt{t} < 20 \wedge \underline{\textbf{CloseBoth}}) \vee (20 \le \texttt{t} < 30 \wedge \underline{\textbf{Fan}}) \vee (30 \le \texttt{t} < 40 \wedge \underline{\textbf{Alternate}}) \vee (40 \le \texttt{t} < 50 \wedge \underline{\textbf{AC}}) \vee (50 \le \texttt{t} \wedge \underline{\textbf{OpenBoth}})$$

$$\tau_{\texttt{get\_temp}} \;=\; \texttt{Unit} \rightarrow \texttt{Int}$$

$$\tau_{\texttt{temperature\_control}} \;=\; \texttt{Unit} \rightarrow (\texttt{Unit} \;\&\; \Phi_{\texttt{temperature\_control}})$$

$$\Phi_{\texttt{temperature\_control}} \;=\; ((\texttt{t} < 20 \wedge \underline{\textbf{CloseBoth}}) \vee (20 \le \texttt{t} < 30 \wedge \underline{\textbf{Fan}}) \vee (30 \le \texttt{t} < 40 \wedge \underline{\textbf{Alternate}}) \vee (40 \le \texttt{t} < 50 \wedge \underline{\textbf{AC}}) \vee (50 \le \texttt{t} \wedge \underline{\textbf{OpenBoth}}))^\omega$$

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device_control

$$\Phi = \underline{\textbf{CloseBoth}}$$

$$\Gamma' \vdash \texttt{ev (CloseBoth)} : \texttt{Unit} \mathbin{\&} \underline{\textbf{CloseBoth}} \quad \text{(T-Event)}$$

$$\qquad (\underline{\textbf{Fan}}) \qquad (\underline{\textbf{Alternate}}) \qquad (\underline{\textbf{AC}})$$

$$\Phi = \underline{\textbf{OpenBoth}}$$

$$\Gamma' \vdash \texttt{ev (OpenBoth)} : \texttt{Unit} \mathbin{\&} \underline{\textbf{OpenBoth}} \quad \text{(T-Event)}$$

$$\text{(T-If)}$$

$$\Gamma' \vdash (\texttt{if t < 20 then ev (CloseBoth) else if ... ... else ev (OpenBoth);}) : \texttt{Unit} \mathbin{\&} (\Phi_1 \vee \Phi_2 \vee \Phi_3 \vee \Phi_4 \vee \Phi_5)$$

$$\text{... (Effects Computation \textbf{I}) ...}$$

$$\Gamma \vdash (\texttt{device\_control t = if ... }) : (\texttt{t} : \texttt{Int}) \rightarrow (\texttt{Unit} \mathbin{\&} \Phi_{\texttt{device\_control}}) \quad \text{(T-Fun)}$$

$\Gamma' = \Gamma, \texttt{device\_control} : \tau_{\texttt{device\_control}}$
$\Phi_1 = (\texttt{t} < 20 \wedge \underline{\textbf{CloseBoth}})$
$\Phi_2 = (20 \leq \texttt{t} < 30 \wedge \underline{\textbf{Fan}})$
$\Phi_3 = (30 \leq \texttt{t} < 40 \wedge \underline{\textbf{Alternate}})$
$\Phi_4 = (40 \leq \texttt{t} < 50 \wedge \underline{\textbf{AC}})$
$\Phi_5 = (50 \leq \texttt{t} \wedge \underline{\textbf{OpenBoth}})$

I:
$$\underline{\textbf{Close}} \cdot \Phi_{\texttt{motion\_sensor}} \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}} \cdot \Phi_{\texttt{door\_control}}$$
$$\equiv \quad \underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}} \cdot ((\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^\omega)$$
$$\equiv \quad (\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^{\omega+1}$$
$$\equiv \quad (\underline{\textbf{Close}} \cdot (\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot \underline{\textbf{Open}} \cdot \underline{\textbf{Delay}})^\omega$$
$$\equiv \quad \Phi_{\texttt{door\_control}}$$

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temperature_control

$$\texttt{sty}(\Gamma'(\texttt{temperature\_control})) \in \rightarrow$$

$$\Gamma' \vdash \texttt{temperature\_control} : \tau_{\texttt{temperature\_control}} \quad \text{(T-VaF)} \qquad \Gamma' \vdash () : \texttt{Unit}$$

$$\textbf{A} \qquad \Gamma' \vdash \texttt{temperature\_control ()} : \texttt{Unit} \mathbin{\&} \Phi_{\texttt{temperature\_control}} \quad \text{(T-VaF)}$$

$$\Gamma' \vdash (\texttt{device\_control (get\_temp ()); temperature\_control ();}) : \texttt{Unit} \mathbin{\&} (\Phi_{\texttt{device\_control}} \cdot \Phi_{\texttt{temperature\_control}}) \quad \text{(T-Let)}$$

$$\text{... (Effects Computation \textbf{II}) ...}$$

$$\Gamma \vdash (\texttt{temperature\_control = ... }) : \texttt{Unit} \rightarrow (\texttt{Unit} \mathbin{\&} \Phi_{\texttt{temperature\_control}}) \quad \text{(T-Fun)}$$

$\Gamma' = \Gamma, \texttt{temperature\_control} : \tau_{\texttt{temperature\_control}}$

A:
$$\texttt{sty}(\Gamma'(\texttt{device\_control})) \in \rightarrow$$

$$\texttt{sty}(\Gamma'(\texttt{get\_temp})) \in \rightarrow \quad \text{(T-VaF)}$$

$$\Gamma' \vdash \texttt{device\_control} : (\texttt{t} : \texttt{Int}) \rightarrow (\texttt{Unit} \mathbin{\&} \Phi_{\texttt{device\_control}}) \quad \text{(T-VaF)} \qquad \Gamma' \vdash \texttt{get\_temp} : \texttt{Unit} \rightarrow \texttt{Int} \qquad \Gamma' \vdash () : \texttt{Unit}$$

$$\Gamma' \vdash \texttt{get\_temp ()} : \texttt{Int} \quad \text{(T-App)}$$

$$\Gamma' \vdash \texttt{device\_control (get\_temp ())} : \texttt{Unit} \mathbin{\&} (\Phi_{\texttt{device\_control}}) \quad \text{(T-App)}$$

II:
$$(\Phi_{\texttt{device\_control}} \cdot \Phi_{\texttt{temperature\_control}})$$
$$\equiv \quad ((\texttt{t} < 20 \wedge \underline{\textbf{CloseBoth}}) \vee (20 \leq \texttt{t} < 30 \wedge \underline{\textbf{Fan}}) \vee (30 \leq \texttt{t} < 40 \wedge \underline{\textbf{Alternate}}) \vee (40 \leq \texttt{t} < 50 \wedge \underline{\textbf{AC}}) \vee (50 \leq \texttt{t} \wedge \underline{\textbf{OpenBoth}}))$$
$$\cdot ((\texttt{t} < 20 \wedge \underline{\textbf{CloseBoth}}) \vee (20 \leq \texttt{t} < 30 \wedge \underline{\textbf{Fan}}) \vee (30 \leq \texttt{t} < 40 \wedge \underline{\textbf{Alternate}}) \vee (40 \leq \texttt{t} < 50 \wedge \underline{\textbf{AC}}) \vee (50 \leq \texttt{t} \wedge \underline{\textbf{OpenBoth}}))^\omega$$
$$\equiv \quad ((\texttt{t} < 20 \wedge \underline{\textbf{CloseBoth}}) \vee (20 \leq \texttt{t} < 30 \wedge \underline{\textbf{Fan}}) \vee (30 \leq \texttt{t} < 40 \wedge \underline{\textbf{Alternate}}) \vee (40 \leq \texttt{t} < 50 \wedge \underline{\textbf{AC}}) \vee (50 \leq \texttt{t} \wedge \underline{\textbf{OpenBoth}}))^{\omega+1}$$
$$\equiv \quad ((\texttt{t} < 20 \wedge \underline{\textbf{CloseBoth}}) \vee (20 \leq \texttt{t} < 30 \wedge \underline{\textbf{Fan}}) \vee (30 \leq \texttt{t} < 40 \wedge \underline{\textbf{Alternate}}) \vee (40 \leq \texttt{t} < 50 \wedge \underline{\textbf{AC}}) \vee (50 \leq \texttt{t} \wedge \underline{\textbf{OpenBoth}}))^\omega$$
$$\equiv \quad \Phi_{\texttt{temperature\_control}}$$

IV. Type checking trees for function surveillance

$\tau_{\texttt{camera\_on}} = \texttt{Unit} \rightarrow (\texttt{Unit} \mathbin{\&} \underline{\textbf{10Mins}})$

$\tau_{\texttt{permission}} = \texttt{t} : \texttt{Int} \rightarrow \texttt{f} : (\texttt{x} : \texttt{Int} \rightarrow \{\texttt{u} : \texttt{Int} \mid \texttt{u} = \texttt{x - 1}\}) \rightarrow (\texttt{Unit} \mathbin{\&} \Phi_{\texttt{permission}})$

$\Phi_{\texttt{permission}} = (\underline{\textbf{Legal}} \vee ((\texttt{t}{<}0 \wedge \underline{\textbf{Tick}}^\omega) \vee (\texttt{t}{\geq}0 \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{10Mins}}))$

$\tau_{\texttt{surveillance}} = \texttt{t} : \texttt{Int} \rightarrow (\texttt{Unit} \mathbin{\&} \Phi_{\texttt{surveillance}})$

$\Phi_{\texttt{surveillance}} = ((\underline{\textbf{Passive}}^* \cdot \underline{\textbf{Active}} \vee \underline{\textbf{Passive}}^\omega) \cdot (\underline{\textbf{Legal}} \vee ((\texttt{t}{<}0 \wedge \underline{\textbf{Tick}}^\omega) \vee (\texttt{t}{\geq}0 \wedge \underline{\textbf{Tick}}^{\texttt{t}} \cdot \underline{\textbf{10Mins}}))) \cdot \underline{\textbf{CameraOff}})^\omega$

```
camera_on = ev (10Mins)
permission t f =
    if * then return ev (Legal)
    else if t == 0 then camera_on ()
    else
        ev (Tick);
        permission (f t) f
```

```
surveillance t =
    motion_sensor;
    permission t (\x -> x - 1);
    ev (CameraOff);
    surveillance t;
```

---

permission

$$
\cfrac{
  \cfrac{\Phi = \underline{\textbf{Legal}}}{\Gamma' \vdash \text{ev (Legal)} : \text{Unit} \;\&\; \underline{\textbf{Legal}}}\ \text{(T-Event)}
  \qquad
  \cfrac{
    \cfrac{
      \cfrac{\text{sty}(\Gamma'(\text{camera\_on})) \in\, \rightarrow}{\Gamma' \vdash \text{camera\_on} : \text{Unit} \rightarrow (\text{Unit} \;\&\; \underline{\textbf{10Mins}})}\ \text{(T-VaF)}
      \qquad
      \Gamma' \vdash () : \text{Unit}
    }{\Gamma' \vdash \text{camera\_on ()} : (\text{Unit} \;\&\; \underline{\textbf{10Mins}})}\ \text{(T-App)}
    \qquad \mathbf{A}
  }
}{\Gamma' \vdash (\text{if * then return ev (Legal) else if t == 0 then camera\_on () else ev (Tick); permission (f t) f;}) : \text{Unit} \;\&\; (\Phi_1 \vee (\Phi_2 \vee \Phi_3))}\ \text{(T-If)}
$$

$$
\cfrac{\dots\ (\textit{Effects Computation \textbf{I}})\ \dots}{\Gamma \vdash (\text{permission t f} = \text{if \dots}) : \text{t} : \text{Int} \rightarrow \text{f} : (\text{x} : \text{Int} \rightarrow \{\text{u} : \text{Int} \mid \text{u} = \text{x - 1}\}) \rightarrow (\text{Unit} \;\&\; \Phi_{\text{permission}})}\ \text{(T-Fun)}
$$

$\Gamma' = \Gamma,\ \text{permission} : \tau_{\text{permission}},\ \text{t} : \text{Int},\ \text{f} : (\text{x} : \text{Int} \rightarrow \{\text{u} : \text{Int} \mid \text{u} = \text{x - 1}\})$
$\Phi_1 = \underline{\textbf{Legal}}$
$\Phi_2 = \text{t==0} \wedge \underline{\textbf{10Mins}}$
$\Phi_3 = \text{t} \neq 0 \wedge (\underline{\textbf{Legal}} \vee \underline{\textbf{Tick}} \cdot ((\text{t} \geq 1 \wedge \underline{\textbf{Tick}}^{\text{t}-1} \cdot \underline{\textbf{10Mins}}) \vee (\text{t} < 1 \wedge \underline{\textbf{Tick}}^{\omega})))$

A:

$$
\cfrac{
  \cfrac{\Phi = \underline{\textbf{Tick}}}{\Gamma' \vdash \text{ev (Tick)} : \text{Unit} \;\&\; \underline{\textbf{Tick}}}\ \text{(T-Event)}
  \qquad
  \cfrac{
    \cfrac{\text{sty}(\Gamma'(\text{permission})) \in\, \rightarrow}{\Gamma' \vdash \text{permission} : \tau_{\text{permission}}}\ \text{(T-VaF)}
    \qquad \mathbf{B} \quad \mathbf{C}
  }{\Gamma' \vdash \text{permission (f t) f} : \text{Unit} \;\&\; \Phi^{\text{t}-1}_{\text{permission}}}\ \text{(T-App)}
}{\Gamma' \vdash (\text{ev (Tick); permission (f t) f;}) : \text{Unit} \;\&\; (\underline{\textbf{Tick}} \cdot \Phi^{\text{t}-1}_{\text{permission}})}\ \text{(T-Let)}
$$

$\Phi^{\text{t}-1}_{\text{permission}} = \underline{\textbf{Legal}} \vee (\text{t} \geq 1 \wedge \underline{\textbf{Tick}}^{\text{t}-1} \cdot \underline{\textbf{10Mins}}) \vee (\text{t} < 1 \wedge \underline{\textbf{Tick}}^{\omega})$

B:

$$
\cfrac{
  \cfrac{\text{sty}(\Gamma'(\text{f})) \in\, \rightarrow}{\Gamma' \vdash \text{f} : (\text{x} : \text{Int} \rightarrow \{\text{u} : \text{Int} \mid \text{u} = \text{x - 1}\}) \rightarrow \text{Int}}\ \text{(T-VaF)}
  \qquad
  \Gamma' \vdash \text{t} : \text{Int}
}{\Gamma' \vdash \text{f t} : \{\text{u} : \text{Int} \mid \text{u} = \text{t - 1}\}}\ \text{(T-App)}
$$

C:

$$
\cfrac{\text{sty}(\Gamma'(\text{f})) \in\, \rightarrow}{\Gamma' \vdash \text{f} : (\text{x} : \text{Int} \rightarrow \{\text{u} : \text{Int} \mid \text{u} = \text{x - 1}\}) \rightarrow \text{Int}}\ \text{(T-VaF)}
$$

I:

$\ (\Phi_1 \vee (\Phi_2 \vee \Phi_3))$
$\equiv\ \underline{\textbf{Legal}} \vee (\text{t==0} \wedge \underline{\textbf{10Mins}}) \vee (\text{t} \neq 0 \wedge (\underline{\textbf{Legal}} \vee \underline{\textbf{Tick}} \cdot ((\text{t} \geq 1 \wedge \underline{\textbf{Tick}}^{\text{t}-1} \cdot \underline{\textbf{10Mins}}) \vee (\text{t} < 1 \wedge \underline{\textbf{Tick}}^{\omega}))))$
$\equiv\ (\underline{\textbf{Legal}} \vee ((\text{t} < 0 \wedge \underline{\textbf{Tick}}^{\omega}) \vee (\text{t} \geq 0 \wedge \underline{\textbf{Tick}}^{\text{t}} \cdot \underline{\textbf{10Mins}}))$
$\equiv\ \Phi_{\text{permission}}$