

Staged Specification Logic

Higher-order Imperative Programs + Algebraic Effects

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My Research

- PhD (2018 Aug – 2023 May)

Thesis: Symbolic Temporal Verification Techniques with Extended Regular Expressions

Keywords: Modularly (Scalability), Expressive Specification, Hoare-style Verification

Applications

- Event-based reactive systems [ICFEM 2020]
- Synchronous languages like Esterel [VMCAI 2021]
- User-defined algebraic effects and handlers [APLAS 2022]
- Real-time systems [TACAS 2023]

- Research Fellow (2023 – now)

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- Temporal Property Guided Bug Detection and Repair [FSE 2024]
- Staged Specification Logic:**



Higher-order Imperative Programs [FM 2024]

Unrestricted Algebraic Effects and Handling [ICFP 2024]

Staged Specification Logic for Verifying Higher-Order Imperative Programs

Darius Foo, Yahui Song, Wei-Ngan Chin

Challenges with Effectful Higher-order Functions

- Existing (automated) verifier varies greatly:
 - Pure only: Dafny, WhyML, Cameleer
 - Type system (Rust) guarantees: Creusot, Prusti
 - Interactive: Iris, CFML, Pulse/Steel (F*)
- When supported, specifications are *imprecise*
- Is there a ***precise and general*** way to support effectful higher-order functions in ***automated program verifiers?***

Specification in Iris

Some clients may want to operate
only on certain kinds of lists

f must preserve the invariant

$$\forall P, Inv, f, xs, l. \left\{ \begin{array}{l} (\forall x, a', ys. \{P x * Inv ys a'\} f(x, a') \{r. Inv (x::ys) r\}) \\ * isList l xs * all P xs * Inv [] a \end{array} \right\}$$

(Separation logic) Invariant

foldr should not change the list *foldr f a l*

relating input to result

$$\{r. isList l xs * Inv xs r\}$$

```
let rec foldr f a l =
  match l with
  | [] => a
  | h :: t =>
    f h (foldr f a t)
```

The Use of Abstract Properties

$$\forall P, Inv, f, xs, l. \left\{ \begin{array}{l} (\forall x, a', ys. \{P x * Inv ys a'\} f(x, a') \{r. Inv (x::ys) r\}) \\ * isList l xs * all P xs * Inv [] a \\ foldr f a l \\ \{r. isList l xs * Inv xs r\} \end{array} \right\}$$

- `foldr` commits to an *abstraction* of `f`'s behavior
- The abstraction may not be precise enough for a given client

The specification of `foldr` is higher-order in the sense that it involves nested Hoare triples (here in the precondition). The reason being that `foldr` takes a function f as argument, hence we can't specify `foldr` without having some knowledge or specification for the function f . Different clients may instantiate `foldr` with some very different functions, hence it can be hard to give a specification for f that is reasonable and general enough to support all these choices. In particular knowing when one has found a good and provable specification can be difficult in itself.

<https://iris-project.org/tutorial-pdfs/iris-lecture-notes.pdf> (page 32)

Difficult to handle cases like:

- Problem 1: Mutable list

```
let foldr_ex1 l = foldr (fun x r -> let v = !x  
                           in x := v+1; v+r) l 0
```

- Problem 2: Strengthened precondition

```
let foldr_ex2 l = foldr (fun x r -> assert(x+r>=0); x+r) l 0
```

- Problem 3: Exceptions/effects

```
let foldr_ex3 l = foldr (fun x r -> if x>=0 then x+r  
                           else raise Exc()) l 0
```

$$\forall P, Inv, f, xs, l. \left\{ \begin{array}{l} (\forall x, a', ys. \{P x * Inv ys a'\} f(x, a') \{r. Inv (x::ys) r\}) \\ * isList l xs * all P xs * Inv [] a \end{array} \right\}$$

foldr f a l
 $\{r. isList l xs * Inv xs r\}$

Difficult to handle cases like:

- Problem 1: Mutable list

```
let foldr_ex1 l = foldr (fun x r -> let v = !x  
                           in x := v+1; v+r) l 0
```

- Problem 2: Strengthened precondition

let **Can we get rid of abstraction
when designing spec for HO-functions?**

```
let foldr_ex3 l = foldr (fun x r -> if x < 0 then x + r  
                                         else raise Exc()) l 0
```

$$\forall P, Inv, f, xs, l. \left\{ \begin{array}{l} (\forall x, a', ys. \{P x * Inv ys a'\} f(x, a') \{r. Inv (x::ys) r\}) \\ * isList l xs * all P xs * Inv [] a \end{array} \right\}$$

foldr f a l
 $\{r. isList l xs * Inv xs r\}$

We Propose “Staged Specification Logic”

$$\begin{array}{c} \text{Sequencing} \quad \text{(Un)interpreted relations} \\ \varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi \\ D, P, Q ::= \sigma \wedge \pi \quad \quad \quad \sigma ::= \mathit{emp} \mid x \mapsto y \mid \sigma * \sigma \mid \dots \end{array}$$

1. *Sequencing and uninterpreted relations*
2. *Recursive formulae*
3. *Re-summarization of recursion/lemmas*
4. *Compaction via bi-abduction*

⇒ *Defer abstraction until appropriate*

1. Sequencing and uninterpreted relations

$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$

let hello f x y = \rightarrow *hello(f, x, y, res) =*
x := !x + 1; $\exists a. \mathbf{req} x \mapsto a; \mathbf{ens} x \mapsto a+1;$
let r = f y **in** $\exists r. f(y, r);$
let r2 = !x + r **in** $\exists b. \mathbf{req} x \mapsto b * y \mapsto -;$
y := r2; $\mathbf{ens} x \mapsto b * y \mapsto res \wedge res = b + r$
r2

We can no longer assume anything about y at this point.

We also cannot assume anything about x!

2. Recursive formulae

$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$

```
let rec foldr f a l =
  match l with
  | [] => a
  | h :: t =>
    f h (foldr f a t)
```

$$\begin{aligned} foldr(f, a, l, res) = \\ \mathbf{ens} \ l = [] \wedge res = a \\ \vee \exists r, l_1; \mathbf{ens} \ l = x :: l_1; \\ foldr(f, a, l_1, r); f(x, r, res) \end{aligned}$$

$$\forall P, Inv, f, xs, l. \left\{ \begin{array}{l} (\forall x, a', ys. \{P x * Inv ys a'\} f(x, a') \{r. Inv (x :: ys) r\}) \\ * isList l xs * all P xs * Inv [] a \\ foldr f a l \\ \{r. isList l xs * Inv xs r\} \end{array} \right\}$$

$$\begin{aligned} foldr(f, a, l, res) = \\ \exists P, Inv, xs. \mathbf{req} List(l, xs) * Inv([], a) \wedge all(P, xs) \\ \wedge f(x, a', r) \sqsubseteq (\exists ys. \mathbf{req} Inv(ys, a') \wedge P(x); \mathbf{ens}[r] Inv(x :: ys, r)); \\ \mathbf{ens}[res] List(l, xs) * Inv(xs, res) \end{aligned}$$

$$\pi ::= \dots \mid \varphi \sqsubseteq \varphi$$

3. Re-summarization of recursion via lemmas

$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$

```
let foldr_sum_state x xs init  
  
= let g c t = x := !x + c; c + t in foldr g xs init
```

3. Re-summarization of recursion via lemmas

$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$

```
let foldr_sum_state x xs init
foldr_sum_state(x, xs, init, res) =
     $\exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens}[res] x \mapsto i+r \wedge res = r + init \wedge sum(xs, r)$ 
= let g c t = x := !x + c; c + t in foldr g xs init
```

$$sum(li, res) =$$

$$l = [] \wedge res = 0$$

$$\vee \exists r, l_1. l = x :: l_1 \wedge sum(l_1, r) \wedge res = x + r$$

$$foldr(f, a, l, res) =$$

$$\mathbf{ens} l = [] \wedge res = a$$

$$\vee \exists r, l_1; \mathbf{ens} l = x :: l_1;$$

$$foldr(f, a, l_1, r); f(x, r, res)$$

$$\forall x, xs, init, res. \quad \frac{}{\subseteq \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i+r \wedge res = r + init \wedge r = sum(xs)}$$

$$foldr(g, xs, init, res)$$

3. Re-summarization of recursion via lemmas

$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$

$$D, P, Q ::= \sigma \wedge \pi \quad \sigma ::= \mathit{emp} \mid x \mapsto y \mid \sigma * \sigma \mid \dots$$

- Recovering abstraction: proving entailments

$$\begin{array}{c}
 \frac{}{x \mapsto i \vdash \exists i. x \mapsto i * \mathit{emp}} \text{SL} \quad \frac{x \mapsto i + r + h \wedge res = h + r + init \wedge r = sum(t)}{\vdash \exists r. x \mapsto i + r \wedge res = r + init \wedge r = sum(h :: t)} \text{SL} \\
 \frac{}{\exists i. \mathbf{req} x \mapsto i; \exists r, h, t. \mathbf{ens} x \mapsto i + r + h \wedge res = h + r + init \wedge r = sum(t) \wedge xs = h :: t} \text{ENTAIL} \\
 \frac{\subseteq \exists i. \mathbf{req} x \mapsto i; \exists r. \mathbf{ens} x \mapsto i + r \wedge res = r + init \wedge r = sum(xs)}{\exists r_1, h, t. \mathbf{ens} xs = h :: t; \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i + r \wedge r_1 = r + init \wedge r = sum(t); \exists b. \mathbf{req} x \mapsto b; \mathbf{ens} x \mapsto b + h \wedge res = h + r_1 \subseteq \dots} \text{NORMALIZE} \\
 \frac{\exists r_1, h, t. \mathbf{ens} xs = h :: t; \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i + r \wedge r_1 = r + init \wedge r = sum(t); g(h, r_1, res) \subseteq \dots}{(\dots \vee \exists r_1, h, t. \mathbf{ens} xs = h :: t; foldr(g, t, init, r_1); g(h, r_1, res)) \subseteq \dots} \text{UNFOLD} \\
 \frac{}{\forall x, xs, init, res. \quad \frac{foldr(g, xs, init, res)}{\subseteq \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i + r \wedge res = r + init \wedge r = sum(xs)}} \text{UNFOLD}
 \end{array}$$

Inductive
Step



3. Re-summarization of recursion via lemmas

$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$

$$D, P, Q ::= \sigma \wedge \pi \quad \sigma ::= \mathit{emp} \mid x \mapsto y \mid \sigma * \sigma \mid \dots$$

- Recovering abstraction: proving entailments

Bi-abduction

$$\frac{D_a * D_1 \vdash D_2 * D_f}{\mathbf{ens} D_1; \mathbf{req} D_2 \implies \mathbf{req} D_a; \mathbf{ens} D_f}$$

$$\begin{array}{c}
 \frac{}{x \mapsto i \vdash \exists i. x \mapsto i * \mathit{emp}} \text{SL} \quad \frac{x \mapsto i + r + h \wedge res = h + r + init \wedge r = sum(, \\ \vdash \exists r. x \mapsto i + r \wedge res = r + init \wedge r = sum(h :: t))}{\exists i. \mathbf{req} x \mapsto i; \exists r, h, t. \mathbf{ens} x \mapsto i + r + h \wedge res = h + r + init \wedge r = sum(t) \wedge xs = h :: t} \text{ EN-ML} \\
 \subseteq \exists i. \mathbf{req} x \mapsto i; \exists r. \mathbf{ens} x \mapsto i + r \wedge res = r + init \wedge r = sum(xs) \\
 \frac{\exists r_1, h, t. \mathbf{ens} xs = h :: t; \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i + r \wedge r_1 = r + init \wedge r = sum(t);}{\exists b. \mathbf{req} x \mapsto b; \mathbf{ens} x \mapsto b + h \wedge res = h + r_1} \subseteq \dots} \text{ NORMALIZE} \\
 \frac{\exists r_1, h, t. \mathbf{ens} xs = h :: t; \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i + r \wedge r_1 = r + init \wedge r = sum(t); g(h, r_1, res) \subseteq \dots}{(\dots \vee \exists r_1, h, t. \mathbf{ens} xs = h :: t; \mathbf{foldr}(g, t, init, r_1); g(h, r_1, res)) \subseteq \dots} \text{ UNFOLD} \\
 \frac{}{\forall x, xs, init, res. \quad \mathbf{foldr}(g, xs, init, res)} \text{ REWRITE} \\
 \subseteq \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i + r \wedge res = r + init \wedge r = sum(xs)} \text{ UNFOLD}
 \end{array}$$

3. Re-summarization of recursion via lemmas

$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$

$$D, P, Q ::= \sigma \wedge \pi \quad \sigma ::= \mathit{emp} \mid x \mapsto y \mid \sigma * \sigma \mid \dots$$

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 \frac{}{\exists i. \mathbf{req} x \mapsto i; \exists r, h, t. \mathbf{ens} x \mapsto i + r + h \wedge res = h + r + init \wedge r = sum(t) \wedge xs = h :: t} \text{ENTAIL} \\
 \frac{\subseteq \exists i. \mathbf{req} x \mapsto i; \exists r. \mathbf{ens} x \mapsto i + r \wedge res = r + init \wedge r = sum(xs)}{\exists r_1, h, t. \mathbf{ens} xs = h :: t; \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i + r \wedge r_1 = r + init \wedge r = sum(t); \exists b. \mathbf{req} x \mapsto b; \mathbf{ens} x \mapsto b + h \wedge res = h + r_1 \sqsubseteq \dots} \text{NORMALIZE} \\
 \frac{\exists r_1, h, t. \mathbf{ens} xs = h :: t; \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i + r \wedge r_1 = r + init \wedge r = sum(t); g(h, r_1, res) \sqsubseteq \dots}{(\dots \vee \exists r_1, h, t. \mathbf{ens} xs = h :: t; foldr(g, t, init, r_1); g(h, r_1, res)) \sqsubseteq \dots} \text{UNFOLD} \\
 \frac{}{\forall x, xs, init, res. \quad \frac{foldr(g, xs, init, res)}{\sqsubseteq \exists i, r. \mathbf{req} x \mapsto i; \mathbf{ens} x \mapsto i + r \wedge res = r + init \wedge r = sum(xs)}} \text{REWRITE}
 \end{array}$$

Solutions with Staged Logic

- Problem 1: Mutable list

```
let foldr_ex1 l = foldr (fun x r -> let v = !x  
                           in x := v+1; v+r) l 0
```

$$\begin{aligned} foldr_ex1(l, res) \sqsubseteq & \exists xs . \mathbf{req} \text{List}(l, xs); \\ & \exists ys . \mathbf{ens} \text{List}(l, ys) \wedge \text{mapinc}(xs) = ys \wedge \text{sum}(xs) = res \end{aligned}$$

- Problem 2: Strengthened precondition

```
let foldr_ex2 l = foldr (fun x r -> assert(x+r>=0); x+r) l 0
```

$$foldr_ex2(l, res) \sqsubseteq \mathbf{req} \text{allSPos}(l); \mathbf{ens} \text{sum}(l) = res$$

- Problem 3: Exceptions/effects

```
let foldr_ex3 l = foldr (fun x r -> if x>=0 then x+r  
                           else raise Exc()) l 0
```

More about
Effects later!

$$foldr_ex3(l, res) \sqsubseteq \mathbf{ens} \text{allPos}(l) \wedge \text{sum}(l) = res \vee (\mathbf{ens}[_] \neg \text{allPos}(l); \text{Exc}())$$

Implementation & Evaluation

- 5K LoC on top of OCaml 5
- Reasonably low verification time
- Feasibility & increased expressiveness over existing systems

Benchmark	Heifer				Cameleer [21]			Prusti [27]		
	LoC	LoS	T	T_P	LoC	LoS	T	LoC	LoS	T
map	13	11	0.66	0.58	10	45	1.25	-	-	
map_closure	18	7	1.06	0.77		X		-	-	
fold	23	12	1.06	0.87	21	48	8.08	-	-	
fold_closure	23	12	1.25	0.89		X		-	-	→ inexpressible
iter	11	4	0.40	0.32		X		-	-	
compose	3	1	0.11	0.09	2	6	0.05		-	→ incomparable
compose_closure	23	4	0.44	0.32		X		X	-	
closure [24]	27	5	0.37	0.27		X		13	11	6.75
closure_list	7	1	0.15	0.09		X		-	-	
applyN	6	1	0.19	0.17	12	13	0.37	-	-	
blameassgn [11]	14	6	0.31	0.28		X		13	9	6.24
counter [16]	16	4	0.24	0.18		X		11	7	6.37
lambda	13	5	0.25	0.22		X		-	-	
	197	73			45	112		37	27	
	LoS/LoC = 0.37				= 2.49			= 0.73		

Summary

$$\varphi ::= \mathbf{req} P \mid \mathbf{ens}[r] Q \mid \varphi; \varphi \mid f(x, r) \mid \exists y. \varphi \mid \varphi \vee \varphi$$

- Staged logic for effectful higher-order programs
 1. *Sequencing and uninterpreted relations*
 2. *Recursive formulae*
 3. *Re-summarization of recursion/lemmas*
 4. *Compaction via bi-abduction*
⇒ *Defer abstraction until appropriate*
- Heifer – a new automated verifier: <https://github.com/hipsleek/heifer>

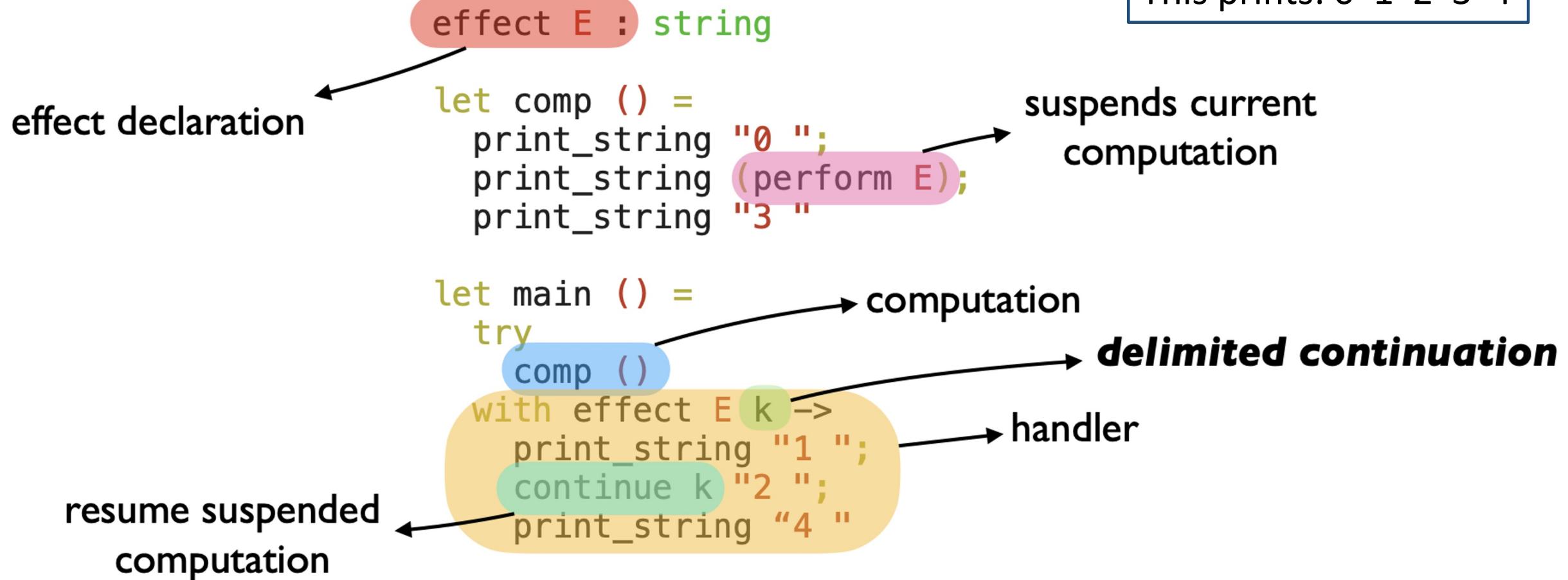
*Higher-order Effectful Imperative Function Entailments and Reasoning

Specification and Verification for Unrestricted Algebraic Effects and Handling

Yahui Song, Darius Foo, Wei-Ngan Chin



User-defined Effects and Handlers



Motivation Example

```
1 effect Label: int
2 (* User-defined effect, which will be resumed with int values *)
3
4 let callee () : int
5 = let x = ref 0 in (* initialize x to zero *)
6   let ret = perform Label in (* the handler has no access to x *)
7     x := !x + 1; (* increment x from zero to one *)
8     assert (!x = 1); (* x now contains one *)
9     ret + 2 (* return the resumed value + 2 *)
```

- Zero-shot handlers: abandon the continuation, just like exception handlers;
- One-shot handlers: resume the continuation once, the assertion on line 8 succeeds;
- Multi-shot handlers: resume the continuation more than once, the assertion on line 8 fails.

Motivation Example

```
1 effect Label: int
2 (* User-defined effect, which will be resumed with int values *)
3
4 let callee () : int
5 = let x = ref 0 in (* initialize x to zero *)
6   let ret = perform Label in (* the handler has no access to x *)
7     x := !x + 1; (* increment x from zero to one *)
8     assert (!x = 1); (* x now contains one *)
9     ret + 2 (* return the resumed value + 2 *)
```

Existing verification techniques:

- multi-shot continuations + pure setting, e.g. [Song et al. 2022];
- heap manipulation + one-shot continuations, e.g. [de Vilhena and Pottier 2021];
- multi-shot + heap-manipulation, under a restricted frame rule, e.g. [de Vilhena 2022].

Protocol Based Approach [de Vilhena and Pottier 2021]

- Hazel & Maze: Model client-handler interactions in the form of protocols
- Globally define the effects that clients perform and the replies they receive from handlers
- Global assumptions to provide explicit (or early) interpretation effects

$$ABORT \triangleq ! () \{ \text{True} \}. ? y (y) \{ \text{False} \}$$

$$CXCHG \triangleq ! x x' (x') \{ \ell \mapsto x \}. ? (x) \{ \ell \mapsto x' \}$$

$$AXCHG \triangleq ! x x' (x') \{ I x \}. ? (x) \{ I x' \}$$

$$\begin{aligned} READWRITE \triangleq & \quad \text{read } \# ! x () \{ I x \}. ? (x) \{ I x \} \\ & + \text{write } \# ! x x' (x') \{ I x \}. ? () \{ I x' \} \end{aligned}$$

Motivation Example

```
1 effect Label: int
2 (* User-defined effect, which will be resumed with int values *)
3
4 let callee () : int
5 = let x = ref 0 in (* initialize x to zero *)
6   let ret = perform Label in (* the handler has no access to x *)
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Specification and Verification for **Unrestricted Algebraic Effects and Handling**

Yahui Song, Darius Foo, Wei-Ngan Chin



4th Sep @ ICFP 2024, Milan, Italy

Our Solution: Effectful Specification Logic (ESL)

- Fully modular per-method verification (no global assumption)
 - Sequencing, $\varphi_1 ; \varphi_2$
 - Uninterpreted relations for **unhandled effects** and unknown functions, $E(x, r)$
 - Reducible **try-catch logic constructs**
 - Normalization: compact each sequence of pre/post stages, via bi-abduction
 - Use **re-summarization** (lemma) when handling recursive generated effects
-
- The diagram shows two yellow boxes labeled 'result' and 'input'. An arrow points from 'result' to the $E(x, r)$ term in the ESL definition. Another arrow points from 'input' to the same term.

$$(ESL) \quad \varphi ::= \text{req } P \mid \text{ens}[r] Q \mid \varphi ; \varphi \mid \varphi \vee \varphi \mid \exists x^* ; \varphi \mid \\ E(x, r) \mid f(x^*, r) \mid \text{try}[\delta](\varphi) \text{ catch } \mathcal{H}_\Phi$$

$$D, P, Q ::= \sigma \wedge \pi$$

$$\sigma ::= \text{emp} \mid x \mapsto y \mid \sigma * \sigma \mid \dots$$

We propose ESL

(ESL) $\varphi ::= \text{req } P \mid \text{ens}[r] Q \mid \varphi; \varphi \mid \varphi \vee \varphi \mid \exists x^*; \varphi \mid \textcolor{red}{E(x, r)} \mid f(x^*, r) \mid \text{try}[\delta](\varphi) \text{ catch } \mathcal{H}_\Phi$

```
1 effect Label: int
2 (* User-defined effect, which will be resumed with int values *)
3
4 let callee () : int
5 = let x = ref 0 in (* initialize x to zero *)
6   let ret = perform Label in (* the handler has no access to x *)
7     x := !x + 1; (* increment x from zero to one *)
8     assert (!x = 1); (* x now contains one *)
9     ret + 2 (* return the resumed value + 2 *)
```

→ $\text{callee}(r_c) = \exists x \cdot \text{ens } x \mapsto 0;$ // Line 5
 $\exists ret \cdot \text{Label}(ret);$ // Line 6
 $\exists z \cdot \text{req } x \mapsto z \wedge \text{z+1=1} \text{ ens}[r_c] x \mapsto z+1 \wedge r_c = (\text{ret}+2)$ // Lines 7-9

We propose ESL

(ESL) $\varphi ::= \text{req } P \mid \text{ens}[r] Q \mid \varphi; \varphi \mid \varphi \vee \varphi \mid \exists x^*; \varphi \mid \textcolor{red}{E(x, r)} \mid f(x^*, r) \mid \text{try}[\delta](\varphi) \text{ catch } \mathcal{H}_\Phi$

```

1  effect Label: int
2  (* User-defined effect, which will be resumed with int values *)
3
4  let callee () : int
5  = let x = ref 0 in          (* initial value *)
6    let ret = perform Label in   (* the handle *)
7      x := !x + 1;           (* increment *)
8      assert (!x = 1);        (* x now 1 *)
9      ret + 2                (* return value *)

```

Bi-abduction:

$$\begin{aligned} &\exists z; \text{req } x \rightarrow z; \\ &\quad \text{ens } x \rightarrow z + 1; \\ &\exists b; \text{req } x \rightarrow b \wedge b = 1 \end{aligned}$$

→ $\text{callee}(r_c) = \exists x \cdot \text{ens } x \mapsto 0; \quad // \text{ Line 5}$
 $\exists ret \cdot \text{Label}(ret); \quad // \text{ Line 6}$
 $\exists z \cdot \text{req } x \mapsto z \wedge z + 1 = 1 \text{ ens}[r_c] x \mapsto z + 1 \wedge r_c = (ret + 2) \quad // \text{ Lines 7-9}$

We propose ESL

(ESL) $\varphi ::= \text{req } P \mid \text{ens}[r] Q \mid \varphi; \varphi \mid \varphi \vee \varphi \mid \exists x^*; \varphi \mid \textcolor{red}{E(x, r)} \mid f(x^*, r) \mid \text{try}[\delta](\varphi) \text{ catch } \mathcal{H}_\Phi$

```

1  effect Label: int
2  (* User-defined effect, which will be resumed with int values *)
3
4  let callee () : int
5  = let x = ref 0 in          (* initial value *)
6    let ret = perform Label in   (* the handle *)
7      x := !x + 1;           (* increment *)
8      assert (!x = 1);        (* x now 1 *)
9      ret + 2                (* return the resumed value + 2 *)

```

Bi-abduction:

$$\exists z; \text{req } x \rightarrow z \wedge z+1=1; \\ \text{ens } x \rightarrow z+1$$

→ $\text{callee}(r_c) = \exists x \cdot \text{ens } x \mapsto 0; \quad // \text{ Line 5}$
 $\exists ret \cdot \text{Label}(ret); \quad // \text{ Line 6}$
 $\exists z \cdot \text{req } x \mapsto z \wedge z+1=1 \text{ ens}[r_c] x \mapsto z+1 \wedge r_c=(ret+2) // \text{ Lines 7-9}$

Try-Catch Reduction (Examples)

$$\begin{aligned} callee(r_c) = & \exists x \cdot \mathbf{ens} \ x \mapsto 0 ; && // \text{ Line 5} \\ & \exists ret \cdot \mathbf{Label}(ret) ; && // \text{ Line 6} \\ & \exists z \cdot \mathbf{req} \ x \mapsto z \wedge z+1=1 \ \mathbf{ens}[r_c] \ x \mapsto z+1 \wedge r_c=(ret+2) && // \text{ Lines 7-9} \end{aligned}$$

```
let zero_shot () : int
(* zero_shot(r_z) =  $\exists x ; \mathbf{ens}[r_z] \ x \mapsto 0 \wedge r_z=-1$  *)
= match callee () with
| effect Label k -> -1
```

```
let one_shot () : int
(* one_shot(r_o) =  $\exists x ; \mathbf{ens}[r_o] \ x \mapsto 1 \wedge r_o=5$  *)
= match callee () with
| effect Label k -> resume k 3
```

Try-Catch Reduction (Examples)

$$\begin{aligned} callee(r_c) &= \exists x \cdot \mathbf{ens} \ x \mapsto 0 ; && // \text{ Line 5} \\ &\quad \exists ret \cdot \mathbf{Label}(ret) ; && // \text{ Line 6} \\ &\quad \exists z \cdot \mathbf{req} \ x \mapsto z \wedge z+1=1 \ \mathbf{ens}[r_c] \ x \mapsto z+1 \wedge r_c=(ret+2) && // \text{ Lines 7-9} \end{aligned}$$

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= match callee () with
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```

```
let multi_shot () : int
(* multi_shot(r_m) = req false *)
= match callee () with
| effect Label k ->
  let _ = resume k 4 in resume k 5
```

Try-Catch Reduction (Examples)

$$\begin{aligned} callee(r_c) = & \exists x \cdot \mathbf{ens} \ x \mapsto 0 ; && // \text{ Line 5} \\ & \exists ret \cdot \mathbf{Label}(ret) ; && // \text{ Line 6} \\ & \exists z \cdot \mathbf{req} \ x \mapsto z \wedge z+1=1 \ \mathbf{ens}[r_c] \ x \mapsto z+1 \wedge r_c=(ret+2) && // \text{ Lines 7-9} \end{aligned}$$

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let multi_shot () : int
(* multi_shot(r_m) = req false *)
= match callee () with
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  let _ = resume k 4 in resume k 5
```

Intuition:

- Explicit access to continuation
- Modular verification:
 - try-catch reduction
 - normalization via bi-abduction

Try-Catch Reduction (Selected Rules)

- The base case:

$$\frac{(x \rightarrow \Phi_n) \in \mathcal{H}_\Phi}{\text{try}[\delta](\mathcal{N}[r]) \text{ catch } \mathcal{H}_\Phi \rightsquigarrow \mathcal{N}[r] ; \Phi_n[r/x]} \quad [\mathcal{R}\text{-Normal}]$$

- When handling an effect, first reason about the behaviours of its continuation

$$\frac{\mathcal{E} = \mathcal{N} ; E(x, r) \quad E \in \text{dom}(\mathcal{H}_\Phi) \quad \text{try}[d](\theta) \text{ catch } \mathcal{H}_\Phi \rightsquigarrow \Phi}{\text{try}[d](\mathcal{E} ; \theta) \text{ catch } \mathcal{H}_\Phi \rightsquigarrow \text{try}[d](\mathcal{E} \# \Phi) \text{ catch } \mathcal{H}_\Phi} \quad [\mathcal{R}\text{-Deep}]$$

- Instantiate the high-order predicate k using the continuation's specification

$$\frac{\mathcal{E} = \mathcal{N} ; E(x, r) \quad (E(y)k \rightarrow \Phi) \in \mathcal{H}_\Phi \quad \Phi' = \Phi[x/y, (\lambda(r, r_c) \rightarrow \Phi[r_c])/k]}{\text{try}[\delta](\mathcal{E} \# \Phi[r_c]) \text{ catch } \mathcal{H}_\Phi \rightsquigarrow \mathcal{N} ; \Phi'} \quad [\mathcal{R}\text{-Eff-Handle}]$$

Try-Catch Reduction (Selected Rules)

- The base case:

$$\frac{(x \rightarrow \Phi_n) \in \mathcal{H}_\Phi}{\mathbf{try}[\delta](\mathcal{N}[r]) \mathbf{catch} \mathcal{H}_\Phi \rightsquigarrow \mathcal{N}[r] ; \Phi_n[r/x]} \quad [\mathcal{R}\text{-Normal}]$$

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effect-free (wrt \mathcal{H}_Φ) after $\#$

- Instantiate the high-order predicate k using the continuation's specification

$$\frac{\mathcal{E} = \mathcal{N} ; E(x, r) \quad (E(y)k \rightarrow \Phi) \in \mathcal{H}_\Phi \quad \Phi' = \Phi[x/y, (\lambda(r, r_c) \rightarrow \Phi[r_c])/k]}{\mathbf{try}[\delta](\mathcal{E} \# \Phi[r_c]) \mathbf{catch} \mathcal{H}_\Phi \rightsquigarrow \mathcal{N} ; \Phi'} \quad [\mathcal{R}\text{-Eff-Handle}]$$

Binding the effect free continuation to k

Higher-Order Function meets Unresolved Try-Catch Construct

```
let foo f : int  
(* foo(f, r) = try (exists res. f(res)) catch { Label k -> k(5, r) } *)  
= match f() with  
| effect Label k -> resume k 5
```

Keep the try-catch constructs
in the specification, which allows
modular verification.

```
let goo () : int (* goo(r) = exists x. ens[r] x->0 ∧ r=15 *)  
= let f = (fun () -> let x = ref 0 in (perform Label) + 10)  
in foo f
```

Instantiate the
unknown function

```
goo(r) = exists f. ens f(res) = (exists x, y. ens x->0 ; Label(y); ens[res] res=y+10) ; foo(f, r)  
~> try exists res, x, y. ens x->0 ; Label(y); ens[res] res=y+10 catch { Label k -> k(5, r) }  
~>* exists x. ens[r] x->0 ∧ r=15
```

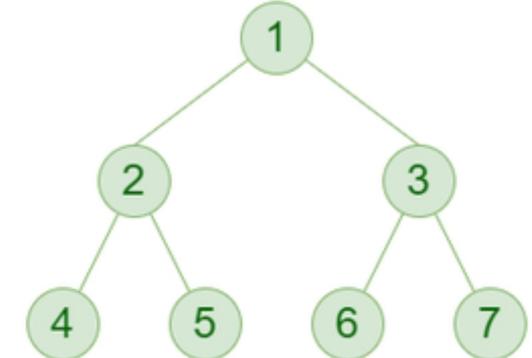
Reducing the
try-catch construct

Inductive Proofs via Lemmas

```

1  effect Flip : bool
2
3  let tossN n
4    (*  $tossN(n, res) = \exists r_0; \text{ens } n=1; \text{Flip}(r_0); \text{ens}[res] res=r_0 \vee$ 
   *  $\exists r_1; \text{ens } n>1; \text{Flip}(r_1); \exists r_2; tossN(n-1, r_2); \text{ens}[res] res=(r_1 \wedge r_2)$  *)
5  = match n with
6  | 1 -> perform Flip
7  | n -> let r1 = perform Flip in
8    let r2 = tossN (n-1) in r1 && r2
9
10 let all_results counter n      Conjunct each Flip result
11  (*  $all\_results(n, r) = \exists z; \text{req } counter \mapsto z \wedge n>0 \text{ ens}[r] \text{ counter} \mapsto z+(2^{n+1}-2) \wedge r=1$  *)
12 = match tossN n with
13 | x -> if x then 1 else 0
14 | effect Flip k ->
15   counter := !counter + 1;          (* increase the counter *)
16   let res1 = resume k true in       (* resume with true *)
17   counter := !counter + 1;          (* increase the counter *)
18   let res2 = resume k false in       (* resume with false *)
19   res1 + res2                      (* gather the results *)

```



$n=1, \text{counter} = 2, \text{res} = 1$

$n=2, \text{counter} = 6, \text{res} = 1$

$n=3, \text{counter} = 14, \text{res} = 1$

...

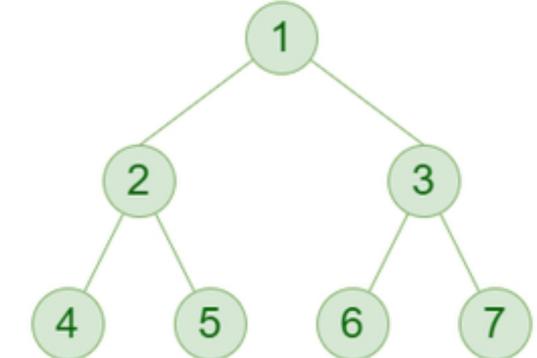
$\dots, \text{counter} = 2^{n+1}-2, \text{res} = 1$

Inductive Proofs via Lemmas

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...

..., $\text{counter} = 2^{n+1}-2, \text{res} = 1$

Sum up how many back tracking branches leads to all true

Inductive Proofs via Lemmas

```
1  effect Flip : bool
```

```
try  $\exists res; tossN(n, res) \# \exists r; \text{ens}[r] (acc \wedge res) \wedge r=1 \vee \neg(acc \wedge res) \wedge r=0$  catch  $\mathcal{H}_\Phi \sqsubseteq \exists r; \Phi_{inv}(n, acc, r)$ 
```

```
 $\Phi_{inv}(n, acc, r) = \exists w; \text{req } counter \mapsto w \text{ ens}[r] counter \mapsto w + (2^{n+1} - 2) \wedge (acc \wedge r=1 \vee \neg acc \wedge r=0)$ 
```

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7  | n -> let r1 = perform Flip in
8      let r2 = tossN (n-1) in r1 && r2
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```

- Proving via applying lemmas
- Lemmas are proved based on:
 - ✓ Try-catch reduction
 - ✓ Unfolding and rewriting (entailment rules)

Implementation and Evaluation

- 5K LoC on top of OCaml 5
- Benchmark programs with features: (Ind) proof is inductive, (MultiS)multi-shot handlers, (ImpureC) impure continuations, (HO) program is higher-order.

#	Program	Ind	MultiS	ImpureC	HO	LoC	LoS	Total(s)	AskZ3(s)
1	State monad	✗	✗	✓	✗	126	16	8.54	6.21
2	Inductive sum	✓	✗	✓	✗	41	11	1.68	1.28
3	Flip-N (Deep Right Rec) (Fig. 7)	✓	✓	✓	✗	39	10	2.09	1.52
4	Flip-N (Deep Left Rec)	✓	✓	✓	✗	45	13	2.03	1.53
5	Flip-N (Shallow Right Rec)	✗	✓	✓	✗	37	11	5.08	3.18
6	Flip-N (Shallow Left Rec)	✓	✓	✓	✗	64	23	6.75	4.26
7	McCarthy's amb operator (Fig. 25)	✓	✓	✓	✓	109	45	7.71	5.34
Total		-	-	-	-	461	129	33.88	23.32

LoS/LoC < 30%

Summary

- ✓ **Scope:** Zero/one/multi-shots + impure continuations, deep/shallow handlers, left/right recursion
- ✓ **Effectful Specification Logic:** Staged specifications + unhandled effects + try-catch logic constructs
- ✓ **Hoare-style Verifier:** ML-like language + imperative higher-order + algebraic effects.
- ✓ **The Back-end Checker for ESL:** Normalization rules + reduction process of try-catch constructs.
- ✓ **Prototype (Multicore OCaml):** Proven correctness, report on experimental results, and case studies.

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Take Away:

Thanks!

- 1) **Try not to assume**, for both HO functions and effects!
- 2) Modular specifications without global assumptions: try-catch constructs.
- 3) Explicit access to the continuations, which can be composed as needed.

My Research

- PhD (2018 Aug – 2023 May)

Thesis: Symbolic Temporal Verification Techniques with Extended Regular Expressions

Keywords: Modularly (Scalability), Expressive Specification, Hoare-style Verification

Applications { Event-based reactive systems [ICFEM 2020]
Synchronous languages like Esterel [VMCAI 2021]
User-defined algebraic effects and handlers [APLAS 2022]
Real-time systems [TACAS 2023]

- Research Fellow (2023 – now)

{ Temporal Property Guided Bug Detection and Repair [FSE 2024]
Staged Specification Logic [FM 2024] [ICFP 2024]



Future

➤ Staged Specification Logic

- 1) Summary/Lemma Inference;
- 2) Non-terminating/liveness behaviours of effects handling;
- 3) Apply staged specification to delimited control operators, e.g., shift/reset;
- 4) Combine staged specification with type system.

➤ Temporal Logic Based Bug Finding and Repair

- 1) Least Fixpoint & Greatest Fixpoint defined analysis : CTL + Datalog;
- 2) Liveness Checking: Termination + Safety checking + Fairness Assumption;
- 3) Temporal Logic Augmented LLM.

➤ ...

- PhD (2018 Aug – 2023 May)

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Applications

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