Dynamics of secured and unsecured debt over the business cycle

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Motivation

- Standard macro-finance models often assume a **uniform debt structure** where collateralized credit is the main channel for **the propagation and amplification of economic shocks.**
- We start by documenting the empirical patterns of firms' debt structures using U.S. firm level data.
- it ignores the fact that firms are financed by different types of debt.
- **High credit quality firms** rely almost exclusively on **unsecured debt**.
- While **low credit quality firms** use a large share of **secured debt**, confirming the findings by Rauh and Sufi (2010).
- *How to explain the cyclicality of secured and unsecured debt?*

Indroduction

Set up

- we build a tractable dynamic stochastic general equilibrium model with debt heterogeneity.
- In the secured debt contract, the lender takes over the firm's assets in the event of default, and the borrower exits with nothing left.(BGG)
- In the unsecured debt contract, **the lender receives no payment in the event of default and the borrower keeps a fraction of revenue and keeps operating.**
  - unsecured debt borrower stays in business **with a penalty on their track** record and a loss of continuation value.
- **Firm's track record of default evolves endogenously over time and can be observed by lenders.**
  - firms with a default record will be punished by being excluded from using unsecured credit in the future and will only borrow in the secured debt market.
  - This is undesirable because we assume secured debt is costly to initiate, and borrowers prefer unsecured debt contracts.

Indroduction

Implication

- the introduction of unsecured debt contracts **amplifies the financial accelerator effect in BGG.**
- In the one-sector BGG model, the financial accelerator effect exists because debt falls just when a firm's net worth falls, which amplifies the effects of the original shock.
- in our model, in response to a negative shock, the increase in the leverage ratio in the unsecured debt market is rather limited, so the fall in debt is more pronounced.
- This leads to a quantitatively important amplification effect relative to BGG.
- Our results suggest that the standard one-sector BGG model may underestimate the amplification effects of the financial accelerator mechanism.

IMPORTANT STYLIZED FACTS

- Debt structure is closely related with a firm's credit quality.

1. **High credit quality firms** rely almost exclusively on **unsecured debt** while **low credit quality** firms have **a substantial share of secured debt.**
2. A firm's **leverage is countercyclical** and there is huge heterogeneity among leverage ratios across credit quality distributions. In particular, high credit quality firms operate with relatively low leverage while low credit quality firm use higher leverage.
3. Unsecured and secured debt show different dynamics along the business cycle: **unsecured debt is strongly procyclical, while secured debt is at best weakly procyclical.**

Data

- we use the item **"mortgages and other secured debt**" to measure **secured debt.**
- We then attribute the difference between **"long term debt + total current debt" and "mortgages and other secured debt" to unsecured debt.**
- leverage as total assets divided by total assets net of the sum of long-term debt and total current debt.

IMPORTANT STYLIZED FACTS

Table 1
Summary statistics on leverage.

Panel A: Sample summary statistics on leverage

Rated only		All observations	
Mean	Correlation with GDP	Mean	Correlation with GDP
1.78	−0.15	1.83	−0.37

Panel B: Leverage ratios across quality distribution

	Leverage ratio		Leverage ratio
AA and above	1.53	B- and below	1.95
BBB and above	1.62	CCC and below	2.13
BBB- and above	1.65	CC and below	2.31

This table reports summary statistics of firm leverage. Statistics are calculated for the Compustat sample of U.S. rated firms and all firms (both rated and non-rated) in Panel A. Panel B summarizes the leverage ratios across credit ratings.

Table 2
Debt volatilities and correlations with GDP.

	Rated only		All observations	
	Std. deviation	Corr. with GDP	Std. deviation	Corr. with GDP
Secured debt	10.16	0.06	10.07	0.15
Unsecured debt	13.94	0.48	15.29	0.50

This table reports the standard deviations and contemporaneous correlations of debt with GDP. The left panel shows rated firms only. The right panel shows all firms (both rated and non-rated). GDP is deflated by the GDP deflator. Debt is deflated by business gross valued index.

IMPORTANT STYLIZED FACTS

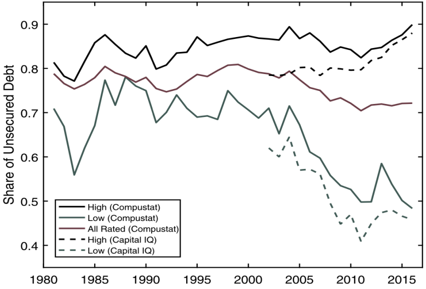


Fig.: This figure shows the share of unsecured debt for **public** and **private** U.S. firms by credit ratings. (Compustat, 1981-2016 and Capital IQ, 2001-2016.)

3.4. Firms

Firms produce with the following Cobb-Douglas production function:

$$Y_{jt}^{i} = A_t \left(\omega_{jt} K_{jt-1}^{i}\right)^{\alpha} \left(L_{jt}^{i}\right)^{1-\alpha} \tag{1}$$

- ω_{jt} denotes a privately observable **idiosyncratic shock** to the firm's capital quality.
- The variable ω_{jt} follows an exogenous log-normal distribution with mean 1 and variance σ_{t-1}^2, i.e., $\log\left(\omega_{jt}\right) \sim N\left(-\frac{1}{2}\sigma_{t-1}^2, \sigma_{t-1}^2\right)$.
- The cumulative distribution function and probability density function are $F\left(\omega_{jt}; \sigma_{t-1}\right)$ and $f\left(\omega_{jt}; \sigma_{t-1}\right)$ respectively.

FOC:

$$w_t L_{jt}^{i} = (1-\alpha) Y_{jt}^{i} \tag{2}$$

$$R_t^K \equiv \frac{r_t^K + (1-\delta) Q_t}{Q_{t-1}} \tag{3}$$

3.4. Firms

Secured debt contracts:

To borrow secured debt, **the firm is subject to an initialization cost, which equals a proportion κ of the firm's net worth.**

$$B^B_{jt-1} = Q_{t-1}K^B_{jt-1} - (1-\kappa)N^B_{jt-1} \tag{4}$$

- secured debt is costly to initiate, and **borrowers prefer unsecured debt contracts.**

It is helpful to define a default threshold, $\bar{\omega}^B_{jt}$, where

$$\bar{\omega}^B_{jt} R^K_t Q_{t-1} K^B_{jt-1} = Z^B_{jt} B^B_{jt-1}. \tag{5}$$

- When $\omega_{jt} \geq \bar{\omega}^B_{jt}$, the firm repays the promised amount $Z^B_{jt}B^B_{jt-1}$.
- If $\omega_{jt} < \bar{\omega}^B_{jt}$, the firm defaults and shuts down.
- In this situation the lender monitors the firm and gets to keep the net receipt $(1-\mu)\omega_{jt}R^K_t Q_{t-1}K^B_{jt-1}$, where μ is a proportional default cost.
- The payoff structure of secured debt is summarized in Table 3 .

3.4. Firms

Table 3
Payoff structure of secured debt.

	Defaults: ($\omega_{jt} < \bar{\omega}^B_{jt}$)	Does not default: ($\omega_{jt} \geq \bar{\omega}^B_{jt}$)
B firm	Gets nothing.	Repays debt and keeps profit.
Lender	Gets liquidation value of the firm.	Receives repayment.

Table 4
Payoff structure of unsecured debt.

	Defaults: ($\omega_{jt} < \tilde{\omega}^G_{jt}$)	Does not default: ($\omega_{jt} \geq \tilde{\omega}^G_{jt}$)
G firm	With Prob $= \zeta$, keeps assets and becomes B firm; With Prob $= 1 - \zeta$, gets nothing.	Repays debt and keeps profit.
Lender	Gets nothing.	Receives repayment.

3.4. Firms

Unsecured debt contracts:

- In the unsecured debt contract, the loan is not backed up by an underlying asset and lenders will not be paid in case of default.
- A firm borrows $B^G_{jt-1} = Q_{t-1}K^G_{jt-1} - N^G_{jt-1}$.
- The firm promises a gross non-default loan rate Z^G_{jt}.
- We similarly define a cutoff threshold $\bar{\omega}^G_{jt}$, where

$$\bar{\omega}^G_{jt} R^K_t Q_{t-1} K^G_{jt-1} = Z^G_{jt} B^G_{jt-1}. \tag{6}$$

3.4. Firms

Let $V_t^i\left(N_t^i\right)$ denote the firms' valuation function.
For B firms:

$$V_t^B\left(N_t^B\right)=\max_{K_t^B,\bar{\omega}_{t+1}^B} E_t\Lambda_{t,t+1}\int_{\bar{\omega}_{t+1}^B}\left\{\begin{array}{c}\theta V_{t+1}^B\left[\left(\omega-\bar{\omega}_{t+1}^B\right)R_{t+1}^K Q_t K_t^B\right]\\ +(1-\theta)\left(\omega-\bar{\omega}_{t+1}^B\right)R_{t+1}^K Q_t K_t^B\end{array}\right\}dF_t \tag{7}$$

The participation constraint of the secured lender is given by:

$$R_t^K Q_{t-1}K_{t-1}^B\left[\int_{\bar{\omega}_t^B}\bar{\omega}_t^B dF_{t-1}+(1-\mu)\int^{\bar{\omega}_t^B}\omega dF_{t-1}\right]\geq R_{t-1}B_{t-1}^B, \tag{8}$$

We guess the value functions are given by:

$$V_t^i\left(N_{jt}^i\right)=\lambda_t^i N_{jt}^i,\ \text{for } i\in\{G,B\},$$

where λ_t^G,λ_t^B are the marginal values of net worth in a G firm and a B firm respectively. We require that $\lambda_t^G>\lambda_t^B>1$ for all t. The first equality ensures that G firms have no incentives to borrow in the secured debt market.

3.4. Firms

We write down the Lagrangian as

$$
\begin{aligned}
V_t^B\left(N_{jt}^B\right) = & \max E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K Q_t K_{jt}^B \int_{\bar{\omega}_{jt+1}^B} \left(\omega - \bar{\omega}_{jt+1}^B\right) dF_t \\
& + Im_{jt}^B \left[\frac{R_{t+1}^K}{R_t} \frac{Q_t K_{jt}^B}{(1-\kappa)} \left(\int_{\bar{\omega}_{jt+1}^B} \bar{\omega}_{jt+1}^B dF_t + (1-\mu) \int^{\bar{\omega}_{jt+1}^B} \omega dF_t \right) - \frac{Q_t K_{jt}^B}{(1-\kappa)} + N_{jt}^B \right],
\end{aligned}
$$

where $\Omega_t^B \equiv \theta \lambda_t^B + 1 - \theta$, and lm_{jt}^B is the Lagrange multiplier. The envelope condition says that $\lambda_t^B = \mathrm{lm}_{jt}^B$. The first order condition for K_{jt}^B is:

$$
\begin{aligned}
K_{jt}^B : 0 = & E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K \left(\int_{\bar{\omega}_{jt+1}^B} \left(\omega - \bar{\omega}_{jt+1}^B\right) dF_t \right) \\
& + \lambda_t^B \left[\frac{R_{t+1}^K}{R_t} \frac{1}{(1-\kappa)} \left(\int_{\bar{\omega}_{jt+1}^B} \bar{\omega}_{jt+1}^B dF_t + (1-\mu) \int^{\bar{\omega}_{jt+1}^B} \omega dF_t \right) - \frac{1}{(1-\kappa)} \right].
\end{aligned}
$$

continue

Rearranging the first order condition for K_{jt}^{B}, we obtain:

$$\lambda_t^B = (1-\kappa)\phi_t^B E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K \int_{\bar{\omega}_{t+1}^B} \left(\omega - \bar{\omega}_{t+1}^B\right) dF_t.$$

The first order condition for $\bar{\omega}_{t+1}^B$ is given by:

$$\lambda_t^B = \frac{(1-\kappa) E_t \Lambda_{t+1} \Omega_{t+1}^B R_{t+1}^K \left[1 - F\left(\bar{\omega}_{t+1}^B\right)\right]}{E_t \frac{R_{t+1}^K}{R_t} \left[1 - F\left(\bar{\omega}_{t+1}^B\right) - \mu \bar{\omega}_{t+1}^B f\left(\bar{\omega}_{t+1}^B\right)\right]}.$$

3.4. Firms

Unsecured debt contracts:

The Bellman equation of G firms is

$$V_t^G\left(N_t^G\right)=\max_{K_t^G,\bar{\omega}_{t+1}^G} E_t\Lambda_{t,t+1}\int\max\left\{V_{t+1}^{G,ND},V_{t+1}^{G,D}\right\}dF_t, \tag{9}$$

where $V^{G,ND}$ denotes the firm value when it repays and $V^{G,D}$ denotes the firm value when it defaults. They are given by:

$$V_t^{G,ND}=\theta V_t^G\left[\left(\omega-\bar{\omega}_t^G\right)R_t^K Q_{t-1}K_{t-1}^G\right]+(1-\theta)\left(\omega-\bar{\omega}_t^G\right)R_t^K Q_{t-1}K_{t-1}^G, \tag{10}$$

and

$$V_t^{G,D}=\theta\zeta V_t^B\left[\omega R_t^K Q_{t-1}K_{t-1}^G\right]+(1-\theta)\zeta\omega R_t^K Q_{t-1}K_{t-1}^G. \tag{11}$$

The participation constraint of the unsecured lender is:

$$R_t^K Q_{t-1}K_{t-1}^G\left(\int_{\tilde{\omega}_t^G}\bar{\omega}_t^G dF_{t-1}\right)\geq R_{t-1}B_{t-1}^G$$

3.4. Firms

Strategic default decision of G firms:

Let $\xi_t \equiv \bar{\omega}_t^G / \tilde{\omega}_t^G$

$$\xi_t = 1 - \zeta \frac{\Omega_t^B}{\Omega_t^G} \leq 1 \tag{12}$$

where $\Omega_t^i \equiv \theta \lambda_t^i + 1 - \theta$ for $i \in \{B, G\}$ are the probability weighted average of the marginal values of net worth of continuing and exiting firms at $t+1$.

- Since $\lambda_t^G > \lambda_t^B$, (18) implies that $\xi \in (0, 1)$, i.e. **some G firms default strategically.**
- One can think of the ratio Ω_t^G / Ω_t^B as the **reputation** of being a G firm.
- When the ratio is large, it is costly to default strategically so we expect fewer strategic defaults. Indeed, (18) shows that ξ_t is increasing in Ω_t^G / Ω_t^B.

3.4. Firms

Unsecured debt contracts:

The marginal values of net worth for B firms and G firms are given by:

$$\lambda_t^B = (1-\kappa)\phi_t^B E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K \int_{\bar{\omega}_{t+1}^B} \left(\omega - \bar{\omega}_{t+1}^B\right) dF_t \tag{13}$$

$$\lambda_t^G = \phi_t^G E_t \Lambda_{t,t+1} \Omega_{t+1}^G R_{t+1}^K \left[(1-\xi_{t+1}) \int^{\tilde{\omega}_{t+1}^G} \omega dF_t + \int_{\tilde{\omega}_{t+1}^G} \left(\omega - \bar{\omega}_{t+1}^G\right) dF_t \right]. \tag{14}$$

The firms'optimal demand for secured and unsecured debt respectively are given by the following two conditions:

$$\lambda_t^B = \frac{(1-\kappa) E_t \Lambda_{t+1} \Omega_{t+1}^B R_{t+1}^K \left[1 - F\left(\bar{\omega}_{t+1}^B\right)\right]}{E_t \frac{R_{t+1}^K}{R_t} \left[1 - F\left(\bar{\omega}_{t+1}^B\right) - \mu \bar{\omega}_{t+1}^B f\left(\bar{\omega}_{t+1}^B\right)\right]} \tag{15}$$

$$\lambda_t^G = \frac{E_t \Lambda_{t+1} R_{t+1}^K \Omega_{t+1}^G \xi_{t+1} \left[1 - F\left(\tilde{\omega}_{t+1}^G\right)\right]}{E_t \frac{R_{t+1}^K}{R_t} \xi_{t+1} \left[1 - F\left(\tilde{\omega}_{t+1}^G\right) - \tilde{\omega}_{t+1}^G f\left(\tilde{\omega}_{t+1}^G\right)\right]}. \tag{16}$$

3.5. Aggregation and accumulation of net worth

We have:

$$N_t^G \phi_t^G = Q_t K_t^G \tag{17}$$

$$(1-\kappa) N_t^B \phi_t^B = Q_t K_t^B \tag{18}$$

It is helpful to define the aggregate leverage ratio of the economy as $\phi_t \equiv Q_t K_t / N_t$.
G firms' net worth evolves as follow:

$$N_t^G = \theta \int_{\tilde{\omega}_t^G} \left(\omega - \bar{\omega}_t^G\right) R_t^K Q_{t-1} K_{t-1}^G dF_{t-1} + \tau N_{t-1}^G, \tag{19}$$

Net worth of B firms evolves as follow:

$$N_t^B = \theta \int^{\tilde{\omega}_t^G} \zeta \omega R_t^K Q_{t-1} K_{t-1}^G dF_{t-1} + \theta \int_{\bar{\omega}_t^B} \left(\omega - \bar{\omega}_t^B\right) R_t^K Q_{t-1} K_{t-1}^B dF_{t-1} + \tau N_{t-1}^B. \tag{20}$$

The goods market clearing condition is given by:

$$Y_t = C_t + I_t + (1-\zeta) \int^{\tilde{\omega}_t^G} \omega dF_{t-1} R_t^K Q_{t-1} K_{t-1}^G + \mu \int^{\bar{\omega}_t^B} \omega dF_{t-1} R_t^K Q_{t-1} K_{t-1}^B + \kappa N_t^B \tag{21}$$

3.5. Aggregation and accumulation of net worth

Finally, the debt market clears:

$$D_t = B_t \tag{22}$$

3.6. Shocks

There are two shocks in the economy, namely a TFP shock and a shock to σ_t, the cross-sectional variance of ω, which we call a risk shock, following Christiano et al. (2014). These shocks follow exogenous AR(1) processes as follows:

$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon_{At}, \quad \epsilon_{At} \sim N\left(0, s_A^2\right) \tag{23}$$

$$\ln \sigma_t = (1 - \rho_\sigma) \ln \sigma + \rho_\sigma \ln \sigma_{t-1} + \epsilon_{\sigma t}, \quad \epsilon_{\sigma t} \sim N\left(0, s_\sigma^2\right) \tag{24}$$

1. Households

The problem of the representative household is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln\left(C_t - hC_{t-1}\right) - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right] \tag{25}$$

subject to the following budget constraint:

$$w_t L_t + R_{t-1} D_{t-1} + \Pi_t^K = C_t + D_t + tr_t \tag{26}$$

The consumption Euler equation and labor supply equation are:

$$1 = R_t E_t \left(\Lambda_{t,t+1}\right) \tag{27}$$

$$w_t = \chi L_t^{\varphi} U_{Ct}^{-1} \tag{28}$$

where $\Lambda_{t,t+1} = \beta U_{Ct+1} / U_{Ct}$ and $U_{Ct} = \left(C_t - hC_{t-1}\right)^{-1} - \beta h E_t \left(C_{t+1} - hC_t\right)^{-1}$.

2. Investors

- Investors collect deposits from households and lend to firms.
- They observe the credit quality of each firm and issue debt to them.
- Investors require a risk-free return R_t in every state of the world for both secured and unsecured debt.
- Investors do not play a meaningful role in the model other than making sure that households hold a diversified loan portfolio across firms.

3. Capital goods producers
The evolution of aggregate capital K_t is given by:

$$K_t = (1-\delta)K_{t-1} + (1 - Adj_t)\, I_t \tag{29}$$

Capital goods producers maximize the sum of discounted expected future profits, $E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \Pi_{t+s}^K$, where $\Pi_t^K = Q_t \left[K_t - \ (1-\delta)K_{t-1}\right] - I_t$. The first order condition for the optimal investment choice is:

$$1 = Q_t \left[1 - A_{dj_t} - \Psi^{I} \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1\right)\right] + E_t \left[\Lambda_{t,t+1} Q_{t+1} \Psi^{I} \left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right)\right] \tag{30}$$

4.1. Level of leverage ratios

Proposition 2. The cutoff value for secured debt contract, $\bar{\omega}_t^B$, satisfies:

$$E_t\left(\frac{R_{t+1}^K}{R_t}\right) = E_t\rho^B\left(\bar{\omega}_{t+1}^B;\sigma_t\right) \geq 1,$$

where the function $\rho^B\left(\bar{\omega}_{t+1}^B;\sigma_t\right)$ is increasing in the cutoff value $\bar{\omega}_{t+1}^B$, and increasing in the cross-sectional dispersion of idiosyncratic productivity σ_t.

Proposition 3. The cutoff value for the unsecured debt contract, $\tilde{\omega}_t^G$, satisfies:

$$E_t\left(\frac{R_{t+1}^K}{R_t}\right) = E_t\rho^G\left(\tilde{\omega}_{t+1}^G,\xi_{t+1};\sigma_t\right) \geq 1,$$

where the function $\rho^G\left(\tilde{\omega}_{t+1}^G,\xi_{t+1};\sigma_t\right)$ is increasing in the cutoff value $\tilde{\omega}_{t+1}^G$, decreasing in ξ_{t+1}, and increasing in the cross-sectional dispersion of idiosyncratic productivity σ_t.

Appendix Proposition 2.

To derive the function ρ^B, we first note that the evolution of λ_t^B, the optimal threshold $\bar{\omega}_{t+1}^B$ and the participation constraint can be written as:

$$\lambda_t^B = (1-\kappa)\phi_t^B E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K \left[1 - \Gamma\left(\bar{\omega}_{t+1}^B\right)\right],$$

$$\lambda_t^B = \frac{(1-\kappa) E_t \Lambda_{t+1} \Omega_{t+1}^B R_{t+1}^K \Gamma_\omega\left(\bar{\omega}_{t+1}^B\right)}{E_t \frac{R_{t+1}^K}{R_t}\left[\Gamma_\omega\left(\bar{\omega}_{t+1}^B\right) - \mu G_\omega\left(\bar{\omega}_{t+1}^B\right)\right]}$$

$$1 - \frac{1}{\phi_{t-1}^B} = \frac{R_t^K}{R_{t-1}}\left[\Gamma\left(\bar{\omega}_t^B\right) - \mu G\left(\bar{\omega}_t^B\right)\right] \tag{31}$$

$$E_t\left(\frac{R_{t+1}^K}{R_t}\right) = E_t \rho^B\left(\bar{\omega}_{t+1}^B; \sigma_t\right),$$

where

$$\rho^B\left(\bar{\omega}_{t+1}^B\right) \equiv \frac{\Gamma_\omega\left(\bar{\omega}_{t+1}^B\right)}{\left[1 - \Gamma\left(\bar{\omega}_{t+1}^B\right)\right]\left[\Gamma_\omega\left(\bar{\omega}_{t+1}^B\right) - \mu G_\omega\left(\bar{\omega}_{t+1}^B\right)\right] + \left[\Gamma\left(\bar{\omega}_{t+1}^B\right) - \mu G\left(\bar{\omega}_{t+1}^B\right)\right]\Gamma_\omega\left(\bar{\omega}_{t+1}^B\right)}.$$

Appendix Proposition 2.
Following the same procedures, we show that for the unsecured debt contract, we have,

$$E_t\left(\frac{R^K_{t+1}}{R_t}\right) = E_t\rho^G\left(\tilde{\omega}^G_{t+1}, \xi_{t+1}; \sigma_t\right),$$

where

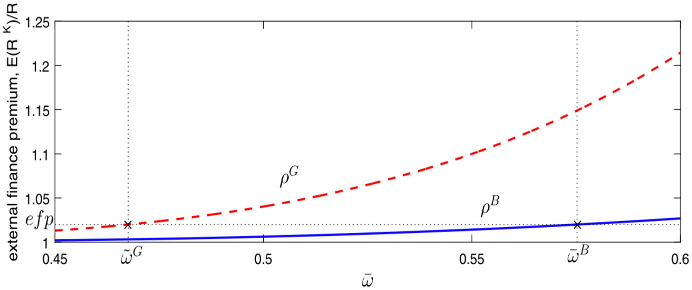
$$\rho^G\left(\tilde{\omega}^G_{t+1}, \xi_{t+1}\right) \equiv \frac{\Gamma_\omega\left(\tilde{\omega}^G_{t+1}\right)}{\left[1-\xi_{t+1}\Gamma\left(\tilde{\omega}^G_{t+1}\right)\right]\left[\Gamma_\omega\left(\tilde{\omega}^G_{t+1}\right) - G_\omega\left(\tilde{\omega}^G_{t+1}\right)\right] + \xi_{t+1}\left[\Gamma\left(\tilde{\omega}^G_{t+1}\right) - G\left(\tilde{\omega}^G_{t+1}\right)\right]\Gamma_\omega\left(\tilde{\omega}^G_{t+1}\right)}.$$

Appendix Proposition 4.

Proposition 4. For any $\xi_t \in (\mu, 1), \sigma_{t-1} > 0$ and $\bar{\omega}_t > 0$, we have

$$\frac{\partial \rho^G \left(\bar{\omega}_t, \xi_t; \sigma_{t-1}\right)}{\partial \bar{\omega}_t} > \frac{\partial \rho^B \left(\bar{\omega}_t; \sigma_{t-1}\right)}{\partial \bar{\omega}_t},$$

and $\rho^G \left(\bar{\omega}_t, \xi_t; \sigma_{t-1}\right) > \rho^B \left(\bar{\omega}_t; \sigma_{t-1}\right)$



We now turn to the lenders' side. It is useful to write the lenders' participation constraint in each market as a relationship between the cutoff threshold and the leverage ratio specific to the borrower type:

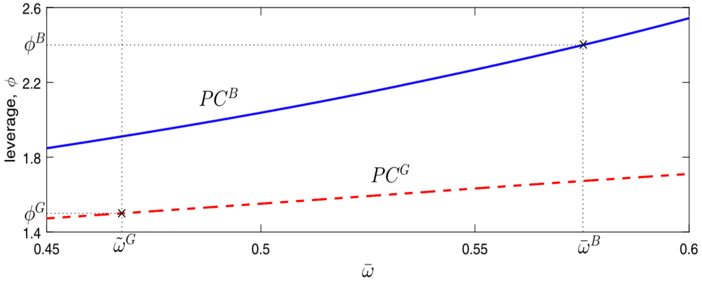
$$\phi^B_{t-1} = PC^B\left(\bar{\omega}^B_t, \frac{R^K_t}{R_{t-1}}; \sigma_{t-1}\right) \equiv \left\{1 - \frac{R^K_t}{R_{t-1}}\left[\int_{\bar{\omega}^B_t} \bar{\omega}^B_t dF_{t-1} + (1-\mu)\int^{\bar{\omega}^B_t} \omega dF_{t-1}\right]\right\}^{-1},$$

$$\phi^G_{t-1} = PC^G\left(\tilde{\omega}^G_t, \xi, \frac{R^K_t}{R_{t-1}}; \sigma_{t-1}\right) \equiv \left[1 - \frac{R^K_t}{R_{t-1}}\left(\xi_t \int_{\tilde{\omega}^G_t} \tilde{\omega}^G_t dF_{t-1}\right)\right]^{-1}.$$

Proposition 5. For any $\xi_t \in (\mu, 1), R_t^K / R_{t-1} > 1, \sigma_{t-1} > 0$ and $\bar{\omega}_t > 0$, we have

$$\frac{\partial PC^B\left(\bar{\omega}_t, \frac{R_t^K}{R_{t-1}}; \sigma_{t-1}\right)}{\partial \bar{\omega}_t} > \frac{\partial PC^G\left(\bar{\omega}_t, \xi_t, \frac{R_t^K}{R_{t-1}}; \sigma_{t-1}\right)}{\partial \bar{\omega}_t} > 0,$$

$$\text{and } PC^B\left(\bar{\omega}_t, \frac{R_t^K}{R_{t-1}}; \sigma_{t-1}\right) > PC^G\left(\bar{\omega}_t, \xi_t, \frac{R_t^K}{R_{t-1}}; \sigma_{t-1}\right).$$



4.2. Dynamics of leverage ratios

This subsection explains why our model is consistent with stylized fact 3, i.e. that unsecured debt has a higher correlation with output than secured debt. We start with a comparative static analysis to show the key intuition.

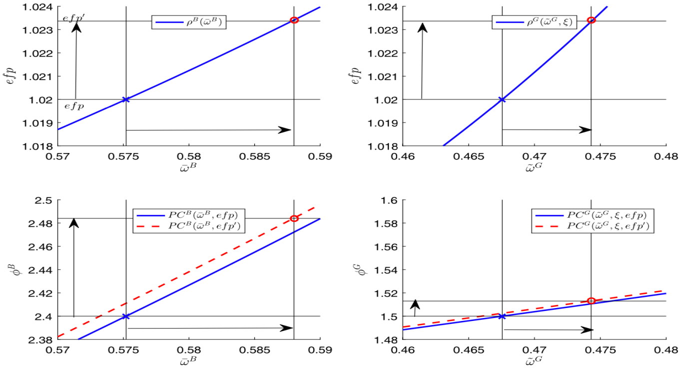


Fig. 3. This figure illustrates the relationships among cutoff values, the external finance premium and leverage. The top left and right panels plot the credit demand functions in secured and unsecured debt contracts. The bottom left and right panels plot the participation constraints in secured and unsecured debt contracts. All plots use the calibrated parameters in the benchmark calibration. Blue lines represent the steady-state relationships. Red lines show the relationships after the external finance premium increases by the initial jump in response to a one standard deviation negative TFP shock. The reputation

6.1. Impulse responses

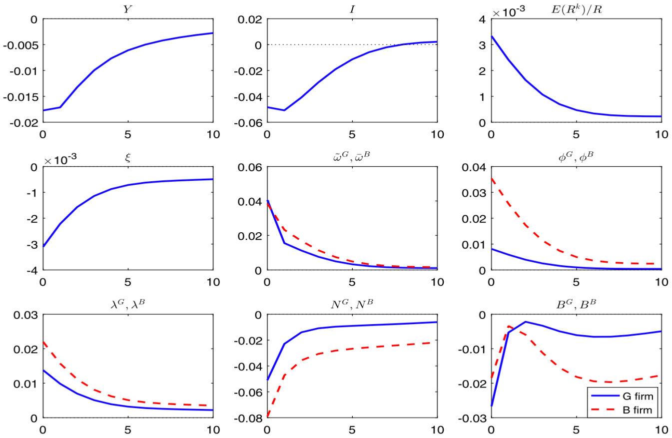


Fig. 4. Impulse response to a negative TFP shock. *Note*: The impulse response functions measure the response to a one standard deviation negative shock to the innovations in TFP as percent deviation from the steady state.

6.1. Impulse responses

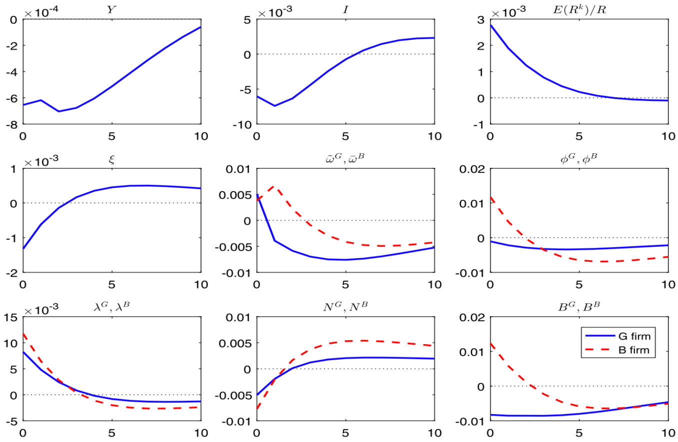


Fig. 5. Impulse response to a positive risk shock. *Note*: The impulse response functions measure the response to a one standard deviation increase in the innovations in the cross-sectional variance of the idiosyncratic shock as percent deviation from the steady state.

6.2. Comparing model with data

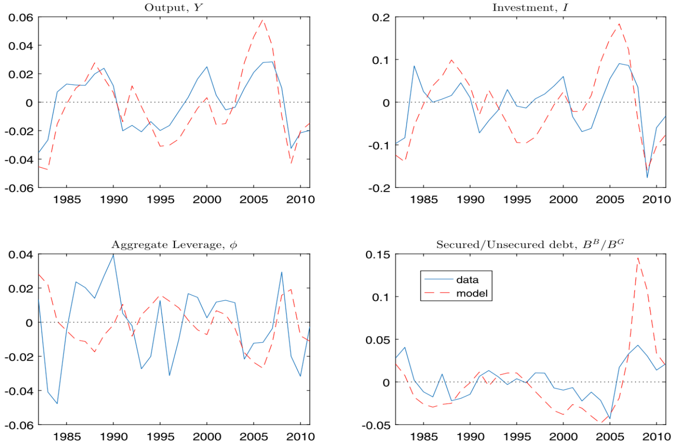


Fig. 6. Historical and model-generated series, all shocks. The historical series are in logs and HP-filtered with smoothing parameter 100. Model-generated series are log-deviations from steady state with smoothing parameter 100. Source: NBER, CES and Compustat.

6.2. Comparing model with data

Table 6
Moments.

	U.S. data	Benchmark model
Panel A: Standard deviation		
Output (Y)	1.81	1.81
Panel B: Standard deviation/std.(Y)		
Consumption (C)	0.90	0.60
Investment (I)	3.18	3.18
Unsecured debt (B^G)	7.68	1.35
Secured debt (B^B)	5.60	1.18
Total debt (B)	4.39	1.18
Panel C: Correlation with output		
Consumption (C)	0.94	0.97
Investment (I)	0.87	0.98
Unsecured debt (B^G)	**0.48**	**0.62**
Secured debt (B^B)	**0.06**	**0.22**
Total debt (B)	0.53	0.58

Moments of U.S. data are computed by using annual data from 1981 to 2016. The numbers from the model are theoretical moments based on the benchmark calibration. Panel A reports the standard deviation of output. Panel B reports the relative standard deviations with respect to output. Panel C reports the contemporaneous correlations with output. Model-generated series are HP-filtered with smoothing parameter 100.

7. Comparison with one-sector financial accelerator model

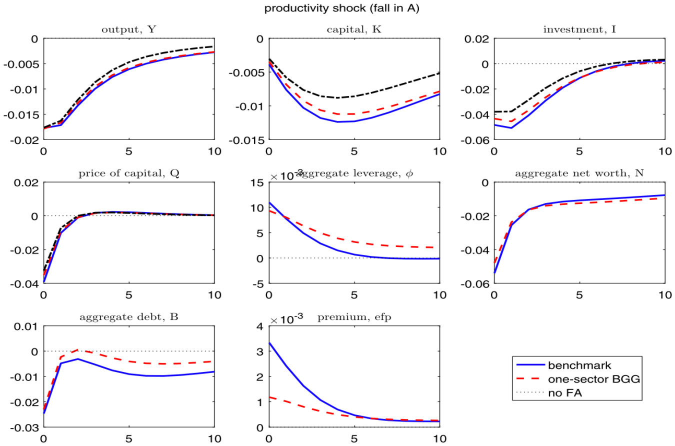


Fig. 7. Impulse response to a negative TFP shock. *Note*: The impulse response functions measure the response to a one standard deviation negative shock to the innovations in TFP as percent deviation from the steady state.

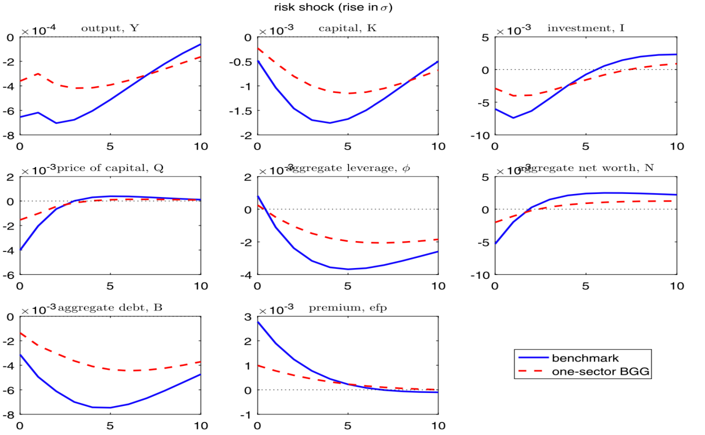


Fig. 8. Impulse response to a positive risk shock. *Note*: The impulse response functions measure the responses to a one standard deviation increase in the innovations in the cross-sectional variance of the idiosyncratic shock in the benchmark model as percent deviation from the steady state. The shock in the BGG model has the same size.

Question!

The conclusion of the working paper is just the **opposite**???