Propagation of financial shocks in an input-output economy with trade and financial linkages of firms *(Job Market Paper,2015)*

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Introduction

Job Market Paper Version!

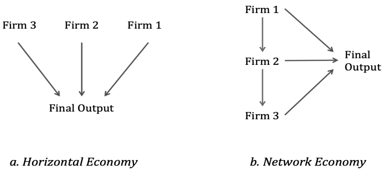
- Firms are connected through the production network.
- At the same time, the production linkages coincide with financial linkages because of **delays to input payments**.
- how these interconnected production and financial linkages lead to the propagation of financial shocks both upstream and downstream.
  - **financial shocks can propagate upstream**
  - based on the input-output matrix and the bond yield data in the U.S., upstream propagation of financial shocks is stronger than downstream propagation.
  - I elaborate a DSGE model –GK2011
  - credit policies would have a stronger impact if liquidity were transferred to downstream sectors after aggregate liquidity shocks.
  - *note that: Trade Credit and Bank Lending: A Two-Way Interaction and Its Macroeconomic Effect, coming soon*

Outline

2 A Simple Model

In my model here, firms are connected vertically in a production chain

Figure 1: Horizontal VS Network Economy



A horizontal economy with financial frictions

Intermediate sectors thus have the following production functions,

$$m_1 = z_1 l_1^{\alpha_1}, \quad m_2 = z_2 l_2^{\alpha_2}, \quad m_3 = z_3 l_3^{\alpha_3}$$

In this economy, the final output is

$$Y = \Xi m_1^{\zeta_1^h} m_2^{\zeta_2^h} m_3^{\zeta_3^h} \tag{1}$$

A representative household solves the following problem

$$\begin{array}{ll} \max_{C,L} & \log(C) - L \\ \text{s.t.} & C = wL + \Psi \end{array} \tag{2}$$

2 A Simple Model

A network economy with financial frictions

Each firm produces output using Cobb-Douglas:

$$m_1 = z_1 l_1^{\alpha_1}, \quad m_2 = z_2 l_2^{\alpha_2} m_{21}^{1-\alpha_2}, \quad m_3 = z_3 l_3^{\alpha_3} m_{32}^{1-\alpha_3}$$

The final output of the economy is:

$$Y = \zeta_1^{-\zeta_1} \zeta_2^{-\zeta_2} \zeta_3^{-\zeta_3} y_1^{\zeta_1} y_2^{\zeta_2} y_3^{\zeta_3}$$

The goods market clearing conditions are: $m_1 = m_{21} + y_1$, $m_2 = m_{32} + y_2, m_3 = y_3$, and $Y = C$. The labor market clearing condition is $L = \sum_{i=1}^{3} l_i$.

important setting

- Firms have a working capital requirement on labor and intermediate inputs.
- firms pay an exogenous borrowing fee R_i on each unit of the borrowed funds.
- Producers could, however, **defer** a proportion of their intermediate input payments using trade credit.
- A proportion θ_i of intermediate input payments is paid after production without borrowing cost.

What is Trade Credit? Some Facts:

Trade credit is a ST loan suppliers provides to its customer upon a purchase of its product.

- Empirical evidence shows that the implicit interest rate in a trade credit agreement is usually **very high** as compared with the rate on bank credit.

Several theories have been put forth to explain why suppliers provide credit to customers even though those firms cannot get additional banking credit.

- suppliers would like to keep business relationship as the cost of losing customers is high.
- suppliers may have an information advantage over banks and have a comparative advantage in liquidating collateral that the borrower may put up to secure the loan.
- the transaction argument states that trade credit reduces the transaction cost of paying bills.

Notably, a trade credit debtor in bankruptcy almost surely default on the claims held by its trade creditors (Boissay and Gropp (2007)).

- Shocks to the liquidity of some firms caused by the default of the customers, may in turn cause default or postponement of accounts payables on their suppliers and propagate upstream through the production chain.
- The trade credit propagation mechanism will amplify the impact of idiosyncratic shocks on aggregate output.
- Thus, this mechanism creates a big multiplier effect.

2 A Simple Model

Financial linkages of firms

- $(1-\theta_i)$ of their intermediate input purchases at the beginning of the production period
  - When $\theta_i = 0$, intermediate input costs have to be paid fully before production
  - when $\theta_i = 1$, there are no financial frictions in trade.
  - upstream firms provide additional credit to downstream firms, compared to which upstream firms naturally have more working capital.

Table 1: Financial linkages of firms in the network model

	Accounts-payable	Accounts-receivable	Net working capital from trade
Firm 1	0	$\theta_2 p_{21} m_{21}$	$(1-\theta_2)p_{21}m_{21}$
Firm 2	$\theta_2 p_{21} m_{21}$	$\theta_3 p_{32} m_{32}$	$-(1-\theta_2)p_{21}m_{21} + (1-\theta_3)p_{32}m_{32}$
Firm 3	$\theta_3 p_{32} m_{32}$	0	$-(1-\theta_3)p_{32}m_{32}$

- working capital of upstream firms is more sensitive to economic fluctuations
  - A shock to a downstream firm would impact the early payments to its upstream firms, which wouldin turn cause fluctuations in the working capital of these upstream firms.
- trade credit increases the correlation of firms; firms share liquidity with surrounding firms through trade credit.

The propagation of financial shocks

For example, Firm 2 solves the following problem,

$$\max_{l_2, m_{32}, m_{21}, y_2} \left[\left(1-\theta_3\right) R_2 + \theta_3 \right] p_{32} m_{32} + p_2 y_2 - w l_2 R_2 - \left[\left(1-\theta_2\right) R_2 + \theta_2 \right] p_{21} m_{21}$$

$$\text{s.t. } z_2 l_2^{\alpha_2} m_{21}^{1-\alpha_2} \geq m_{32} + y_2$$

Upstream propagation.

Two factors **affect the intermediate input demand** of Firm 2 after a positive R_2 shock, which factors are in turn related to the upstream propagation and the sensitivity of m_1 to R_2.

a the **income effect** decreases intermediate input demand, since production falls on account of the financial distress.
   - $m_{21} \downarrow$

b the **substitution effect** increases the intermediate input demand.
   - Firm 2 will **substitute labor** with intermediate inputs, **a positive R_2 shock.**
     - $m_{21} \uparrow$
     - The substitution effect **attenuates** the negative impact to m_1.
   - as θ_2 increases, this substitution effect becomes stronger and the upstream propagation effect is diminished.
   - $\theta_i = 1 \forall i$, there is no upstream propagation

For example, Firm 3 solves the following problem,

$$\max_{l_2, m_{32}, m_{21}, y_2} p_3 y_3 - w l_3 R_2 - \left[(1-\theta_3) R_2 + \theta_3 \right] p_{32} m_{32}$$
$$\text{s.t. } z_3 l_3^{\alpha_3} m_{32}^{1-\alpha_3} \geq y_3$$

Two factors affect **the product prices** of Firm 2 , which factors are in turn related to the downstream propagation and the sensitivity of m_3 to R_2.

Downstream propagation.

a the cost effect increases product price p_{32}.

b the discount effect decreases p_{32}

- When R_2 is high, Firm 2 gives a large discount to Firm 3, pays in advance
- The discount effect **attenuates** the negative impact to m_3.
- as θ_3 decreases, the discount effect becomes stronger and the downstream propagation effect is diminished.
- $\theta_i = 0 \forall i$, the downstream propagation effect disappears.

2 A Simple Model
equilibrium

The equilibrium sectoral outputs of the TC-network economy are,

$$m_1 = \frac{z_1}{R_1}\left\{\zeta_1 + (1-\alpha_2)\frac{[\theta_2+(1-\theta_2)R_1]}{[\theta_2+(1-\theta_2)R_2]}\left[\zeta_2+(1-\alpha_3)\zeta_3\frac{[\theta_3+(1-\theta_3)R_2]}{[\theta_3+(1-\theta_3)R_3]}\right]\right\}$$

$$\begin{aligned} m_2 =& z_2 z_1^{1-\alpha_2}\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}R_1^{-(1-\alpha_2)}R_2^{-\alpha_2} \\ & \left(\frac{[\theta_2+(1-\theta_2)R_1]}{[\theta_2+(1-\theta_2)R_2]}\right)^{1-\alpha_2}\left[\zeta_2+(1-\alpha_3)\zeta_3\frac{[\theta_3+(1-\theta_3)R_2]}{[\theta_3+(1-\theta_3)R_3]}\right] \end{aligned}$$

$$\begin{aligned} m_3 =& z_3 z_2^{1-\alpha_3} z_1^{(1-\alpha_2)(1-\alpha_3)}\alpha_3^{\alpha_3}(1-\alpha_3)^{1-\alpha_3}\alpha_2^{\alpha_2(1-\alpha_3)}(1-\alpha_2)^{(1-\alpha_2)(1-\alpha_3)}\zeta_3 \\ & R_1^{-(1-\alpha_2)(1-\alpha_3)}R_2^{-\alpha_2(1-\alpha_3)}R_3^{-\alpha_3}\left(\frac{[\theta_2+(1-\theta_2)R_1]}{[\theta_2+(1-\theta_2)R_2]}\right)^{(1-\alpha_2)(1-\alpha_3)}\left(\frac{[\theta_3+(1-\theta_3)R_2]}{[\theta_3+(1-\theta_3)R_3]}\right)^{1-\alpha_3} \end{aligned}$$

The labor allocation is,

$$l_1 = \left\{\zeta_1 + (1-\alpha_2)\tfrac{[\theta_2+(1-\theta_2)R_1]}{[\theta_2+(1-\theta_2)R_2]}\left[\zeta_2+(1-\alpha_3)\zeta_3\tfrac{[\theta_3+(1-\theta_3)R_2]}{[\theta_3+(1-\theta_3)R_3]}\right]\right\}\cdot\tfrac{\alpha_1}{R_1},$$

$$l_2 = \left\{\zeta_2+(1-\alpha_3)\zeta_3\tfrac{[\theta_3+(1-\theta_3)R_2]}{[\theta_3+(1-\theta_3)R_3]}\right\}\cdot\tfrac{\alpha_2}{R_2},$$

$$l_3 = \zeta_3\cdot\tfrac{\alpha_3}{R_3}$$

2 A Simple Model

why upstream propagation > downstream?

Proposition 2. *(a) Financial shocks generate strong upstream and downstream propagations in a network economy with trade credit. The elasticity of output i with respect to the borrowing cost of Firm 2 is,*

$$\frac{\partial m_i}{\partial R_2}\frac{R_2}{m_i} = f_i(R_1, R_2, R_3)$$

$$\frac{\partial m_1}{\partial R_2}\frac{R_2}{m_1} = \frac{(1-\alpha_2)\zeta_2[\theta_2+(1-\theta_2)R_1]}{[\theta_2+(1-\theta_2)R_2]^2}\frac{\left[-(1-\theta_2)+(1-\alpha_3)\zeta_3\frac{\theta_2-\theta_3}{\{\zeta_3+(1-\theta_3)R_3\}}\right]R_2}{\left\{\zeta_1+(1-\alpha_2)\frac{[\theta_2+(1-\theta_2)R_2]}{[\theta_2+(1-\theta_2)R_2]}\left[\zeta_2+(1-\alpha_3)\zeta_3\frac{[\theta_3+(1-\theta_3)R_2]}{[\theta_3+(1-\theta_3)R_3]}\right]\right\}}$$

$$\frac{\partial m_2}{\partial R_2}\frac{R_2}{m_2} = -\alpha_2-(1-\alpha_2)\frac{(1-\theta_2)R_2}{\theta_2+(1-\theta_2)R_2}+\frac{(1-\alpha_3)\zeta_3(1-\theta_3)R_2}{\zeta_2+(1-\alpha_3)\zeta_3\frac{\theta_3+(1-\theta_3)R_2}{\theta_3+(1-\theta_3)R_3}},$$

$$\frac{\partial m_3}{\partial R_2}\frac{R_2}{m_3} = -(1-\alpha_3)\left[\alpha_2+(1-\alpha_2)\frac{(1-\theta_2)R_2}{\theta_2+(1-\theta_2)R_2}-\frac{(1-\theta_3)R_2}{\theta_3+(1-\theta_3)R_2}\right].$$

- If $\theta_i = 0, \forall i$
  - $\frac{\partial m_1}{\partial R_2}\frac{R_2}{m_1} = -(1-\alpha_2)\frac{\zeta_2 z_1 \Upsilon_1}{R_2 m_1}$, $\quad \frac{\partial m_2}{\partial R_2}\frac{R_2}{m_2} = -\frac{\zeta_2 z_2 z_1^{1-\alpha_2}\Upsilon_2}{R_2 m_2}$, $\quad \frac{\partial m_3}{\partial R_2}\frac{R_2}{m_3} = 0.$
  - **there is no downstream propagation**
- If $\theta_i = 1, \forall i$
  - $\frac{\partial m_1}{\partial R_2}\frac{R_2}{m_1} = 0$, $\quad \frac{\partial m_2}{\partial R_2}\frac{R_2}{m_2} = -\alpha_2$, $\quad \frac{\partial m_3}{\partial R_2}\frac{R_2}{m_3} = -\alpha_2(1-\alpha_3)$
  - **there is no upstream propagation**

(b) The upstream propagation of a R_2 shock is more likely to dominate the downstream propagation as trade credit decreases.

- **Upstream propagation > downstream** , $\theta < \frac{(1-\alpha_2)\zeta_2\bar{R}}{(1-\alpha_2)\zeta_2\bar{R}+\alpha_2(1-\alpha_3)[\zeta_1+(1-\alpha_2)(\zeta_2+(1-\alpha_3)\zeta_3)]}$

2.2.4 The importance of firms - the influence vector

Consider a general input-output structure, where the production function is,

$$m_i = z_i l_i^{\alpha_i} \left(\Pi_{j=1}^N m_{ij}^{\omega_{ij}} \right)^{1-\alpha_i} \tag{3}$$

$$\Omega = \begin{bmatrix} (1-\alpha_1)\,\omega_{11} & (1-\alpha_1)\,\omega_{12} & \cdots & (1-\alpha_1)\,\omega_{1N} \\ (1-\alpha_2)\,\omega_{21} & (1-\alpha_2)\,\omega_{22} & \cdots & (1-\alpha_2)\,\omega_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ (1-\alpha_N)\,\omega_{N1} & (1-\alpha_N)\,\omega_{N2} & \cdots & (1-\alpha_N)\,\omega_{NN} \end{bmatrix} \tag{4}$$

Proposition 3. *in the competitive equilibrium of the network conomy, the log deviation of the aggregate output (GDP) from its steady state is given by*

$$\tilde{Y} = v'\mathbf{z} + \left(v^R\right)'\mathbf{R}$$

where v and v^R are N -dimensional vectors given by

$$\begin{aligned} v &\equiv \left(I - \Omega'\right)^{-1} \zeta \\ v^R &\equiv -\left(I_1 - I_2\Omega\right)' v \end{aligned} \tag{5}$$

2.3 Conclusions to be drawn from the simple model

- financial constraint generates a strongly **negative** impact on aggregate output.
- depending on the level of financial frictions in trade, the propagation of financial shocks could be either upstream or downstream
    - In particular, financial frictions in trade and financial linkages of firms can **generate strong upstream propagation of financial shocks.**
- Supply shocks, by contrast, always propagate downstream, and demand shocks always propagate upstream, regardless of the level of financial frictions in trade

the empirical findings ?

Similar *Acemoglu et al. (2015a)*, which takes the following form:

$$\Delta m_{it} = \beta^{up} UP_{it-1} + \beta^{\text{down}}\, DOWN_{it-1} + \beta^{si}\, \text{shock}\,_{it-1} + \beta^{m} \tilde{m}_{it-1} + \varepsilon_{it} \qquad (6)$$

3.3 Data sources

- The industry-level data for manufacturing was obtained from the Industrial Production Index by the Federal Reserve Board of the United States.
  - This index reports the level of production for the period 1986-2015 on a monthly basis.
- To measure the linkages among industries $(h_{ij}, \hat{h}_{ji})$, I use the Input-Output Table created by the Bureau of Economic Analysis.
- TRACE (the Trade Reporting and Compliance Engine) collects corporate bond tick data from 2002 to 2015, and this dataset is used to measure idiosyncratic financial shocks at the sectoral level (shock_{it}).
  - I regress monthly sectoral bond yield changes on Federal Funds rate change and Aaa bond index change; the residual of this regression is the instrument of sectoral idiosyncratic financial shocks.

3 Empirical Findings
3.4 Empirical results

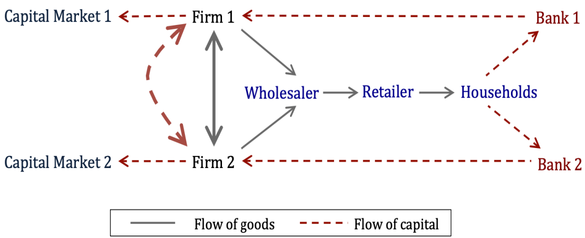
Table 2: UP vs. DOWN

	Panel	Cross Section	
	biannual (2003.01-2014.12)	6 months after Lehman Bankruptcy	
	(1)	(2)	(3)
UP$_i$	**-0.0929****	**-0.3042*****	**-0.1724****
	[0.0441]	[0.0733]	[0.0695]
DOWN$_i$	**0.1645**	**-0.1969**	**-0.0635**
	[0.1687]	[0.2715]	[0.2550]
Shock$_i$	-0.4395***	-0.4008*	-0.4469**
	[0.1221]	[0.2337]	[0.2022]
$\epsilon_{i,2008}^{demand}$			0.1835***
			[0.0383]
$\epsilon_{i,2008}^{supply}$			0.0305
			[0.0686]
Δm_{it-1}	0.0891*	0.6062***	0.5047***
	[0.0502]	[0.0543]	[0.0607]
Time Fixed Effect	X		
Sector Fixed Effect	X		
Observations	1357	61	61
# of sectors	59	-	-

4 A General Network Model With Trade Credit

- This section, I consider a DSGE model with an input-output structure in which firms are linked financially.

Figure 2: The model structure



4 A General Network Model With Trade Credit

4.1 Intermediate Goods Firms.

Each sector produces output using the following Cobb-Douglas production function,

$$m_{it} = z_{it}^{\alpha_i} l_{it}^{\alpha_i} \left(\xi_{it} k_{it-1}\right)^{\beta_i} \left(\Pi_{i=1}^{N} m_{ij,t}^{\omega_{ij}}\right)^{\gamma_i} \tag{13}$$

Thus, each firm solves the following problem,

$$\begin{aligned}
\max_{l_{it},\{m_{ijt}\}_{j=1}^{N},\{m_{jit}\}_{j=1}^{N},y_{it},\theta_{ijt}} & \sum_j \left[\left(1-\theta_{jit}\right) R_{iLt} + \theta_{jit}\right] p_{jit} m_{jit} + p_{it} y_{it} - w_t l_{it} R_{iLt} - u_{it} \xi_{it} k_{it-1} \\
& - \sum_j \left[\left(1-\theta_{ijt}\right) R_{iLt} + \theta_{ijt}\right] p_{ijt} m_{ijt} - \sum_j C\left(\theta_{ijt}, \bar{\theta}_i\right) p_{ijt} m_{ijt} \\
& + \Phi_{it} \left[\left(z_{it} l_{it}\right)^{\alpha_i} \left(\xi_{it} k_{it-1}\right)^{\beta_i} \left(\Pi_j m_{ijt}^{\omega_{ij}}\right)^{\gamma_i} - \sum_j m_{jit} - y_{it}\right].
\end{aligned} \tag{16}$$

Managers make two-step decisions.

- Initially, they decide the level of trade credit for each intermediate input purchase, θ_{ijt}.
- They then choose $\left\{l_{it}, m_{ijt}, m_{jit}, y_{it}\right\}$ given $\left\{\theta_{ijt}, \theta_{jit}\right\}$.

4 A General Network Model With Trade Credit
4.1 Intermediate Goods Firms.–Trade-off

Producers naturally value early payments, and a manager, recognizing this, chooses trade credit level θ_{ijt} in the **first step** so as to minimize the unit intermediate input cost.

tradeoff for adjusting trade credit

- The benefit of increasing θ_{ijt} is the reduction of banking loans and interest costs.
- The cost of increasing θ_{ijt} is the increase in intermediate input price p_{ijt} given that $\partial p_{ijt}/\partial \theta_{ijt} > 0$.
  - Notably, $p_{ijt} = p_{jt}/\left[\left(1-\theta_{ijt}\right) R_{jLt} + \theta_{ijt}\right]$.
  - p_{ijt} depends on θ_{ijt}.
  - The higher the trade credit, the more expensive the product price becomes.
- a trade credit adjustment cost per dollar purchases, $C\left(\theta_{ijt}, \bar{\theta}_i\right)$.

Thus, the manager solves the following problem,

$$\min_{\theta_{ijt}} \left[\left(1 - \theta_{ijt}\right) R_{iLt} + \theta_{ijt} \right] p_{ijt} + C\left(\theta_{ijt}, \bar{\theta}_i\right) p_{ijt}$$

$$\text{s.t.} \qquad p_{ijt} = p_{jt} / \left[\left(1 - \theta_{ijt}\right) R_{jLt} + \theta_{ijt} \right]$$

Therefore, the optimal level of trade credit is,

$$\theta_{ijt} = f\left(\varsigma, \bar{\theta}_i, R_{it}, R_{jt}\right) \tag{21}$$

with

$$\begin{aligned} \partial \theta_{ijt} / \partial R_{it} &> 0 \\ \partial \theta_{ijt} / \partial R_{jt} &< 0 \\ \left| \partial \theta_{ijt} / \partial \varsigma \right| &< 0 \\ \theta_{ijt} = \bar{\theta}_i, \quad \text{when} \quad R_{it} &= R_{jt}. \end{aligned}$$

Nonetheless, there is a quadratic trade credit adjustment cost

$$C\left(\theta_{ij}, \bar{\theta}_i\right) = \varsigma \left(\theta_{ij} - \bar{\theta}_i\right)^2 \tag{15}$$

4 A General Network Model With Trade Credit
4.1 Intermediate Goods Firms.

Proposition 4. *Trading parties share liquidity through the trade credit mechanism. θ_{ijt} is an increasing function of R_{iLt} and a decreasing function of R_{jLt}. Sectoral correlation is high when trade credit adjustment is flexible.*

- When firm *i* finds that bank loans are becoming costly, it increases trade credit.
  - $R_{it} \uparrow \rightarrow \theta_{ijt} \uparrow$
  - **financially distressed firm may postpone repaying trade credit to its suppliers**
  - If one firm has a liquidity problem, its accounts-payable would increase.
    - the financial stress is transmitted to its suppliers.
- When its suppliers are suffering financially, firm *i* shrinks its accounts-payable.
  - $R_{jt} \uparrow \rightarrow \theta_{ijt} \downarrow$
  - **reduce the provision of trade credit to its customers as discussed**
- sectoral correlation is even higher when trade credit adjustment is flexible.

4 A General Network Model With Trade Credit

4.1 Intermediate Goods Firms.–Trade-off

Accordingly, the stochastic return of a given financial intermediary's investment on a capital asset of firm i is,

$$R_{ik,t+1} = \xi_{i,t+1} \frac{u_{i,t+1} + Q_{it+1} - \delta}{Q_{it}} \tag{14}$$

The first-order conditions of the problem**(2step)**

$$\begin{aligned}
\partial m_{jit}: & \quad p_{jit}\left[\left(1-\theta_{jit}\right) R_{iLt}\theta_{jit}\right] = \Phi_{it} \\
\partial l_{it}: & \quad \alpha_i \Phi_{it} m_{it} = w l_{it} R_{iLt} \\
\partial m_{ijt}: & \quad \left[\left(1-\theta_{ijt}\right) R_{iLt} + \theta_{ijt}\right] p_{ijt} m_{ijt} = \Phi_{it}\gamma_i \omega_{ij} m_{it} \\
\partial y_{it}: & \quad \Phi_{it} = p_{it}
\end{aligned} \tag{7}$$

The wholesale output and the price are represented by:

$$Y_{wt} = \Pi_{i=1}^{N} \zeta_i^{-\zeta_i} y_{it}^{\zeta_i}, \quad P_{wt} = \Pi_{i=1}^{n} p_{it}^{\zeta_i}$$

The final output composite is,

$$Y_t = \left(\int_0^1 Y_{rt}^{\frac{\varepsilon-1}{\varepsilon}} dr \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

By tedious but straightforward derivation in Appendix rt D.1, I have

$$\frac{P_{rt}^*}{P_t} = \frac{\varepsilon}{\varepsilon-1} \frac{\sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} P_{wt+i} \left(\frac{P_{t+i}}{P_t} \right)^{\varepsilon} Y_{t+i}}{\sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} \left(\frac{P_{t+i}}{P_t} \right)^{\varepsilon-1} Y_{t+i}}$$

Thus, the innovation of the aggregate price level is,

$$P_t^{1-\varepsilon} = (1-\gamma) P_{rt}^{*1-\varepsilon} + \gamma (P_{t-1})^{1-\varepsilon} \tag{22}$$

They sell new capital kit to firms in sector i at the price Qit, so their problem is to,

$$\max_{I_{it}} Q_{it} I_{it} - \left[1 + \frac{\eta_I}{2}\left(\frac{I_{it}}{I_{i,t-1}} - 1\right)^2\right] I_{it} \tag{23}$$

Thus, the price of capital goods is equal to the marginal cost of investment goods production as follows,

$$\begin{aligned} Q_{it} = 1 + \frac{1}{2}\eta_I\left(\frac{I_{it}}{I_{i,t-1}} - 1\right)^2 + \frac{I_{it}}{I_{i,t-1}}\eta_I\left(\frac{I_{it}}{I_{i,t-1}} - 1\right) \\ -\mathbb{E}_t\left[\beta\Lambda_{t+1}\left(\frac{I_{i,t+1}}{I_{it}}\right)^2\eta_I\left(\frac{I_{i,t+1}}{I_{it}} - 1\right)\right] \end{aligned}$$

The capital innovation is:

$$k_{it} = e^{\psi_{it}}(1-\delta)k_{i,t-1} + I_{it}$$

The households'welfare function is,

$$\max \mathbb{E}_0 \sum_{i=0}^{\infty} \beta^i \left[\ln \left(C_{t+i} - hC_{t+i-1} \right) - \frac{\chi}{1+\psi} l_{t+i}^{1+\psi} \right] \tag{24}$$

The budget constraint they face is,

$$C_t = w_t l_t + \Pi_t + T_t + R_{t-1} B_{t-1} - B_t \tag{25}$$

Follows Gertler and Karadi (2011), but with the following **modifications**.

- N groups.
- banks of group i cannot finance firms in sector $j \neq i$
- there is no lending among banks.

The intermediary balance sheet of banks in group i is,

$$Q_{it}S_{it} + L_{it} = N_{it} + B_{it} \tag{26}$$

Their net worth then evolves according to

$$N_{it+1} = R_{ik,t+1}Q_{it}S_{it} + R_{iL,t}L_{it} - R_tB_{it} \tag{27}$$

Their net worth evolves according to:

$$N_{it+1} = \left(R_{ik,t+1} - R_t\right)Q_{it}S_{it} + \left(R_{iL,t} - R_t\right)L_{it} + R_tN_{it}$$

The financial intermediary's objective is to maximize:

$$V_{it}(N_t) = \max \mathbb{E}_t \sum_{j=0}^{\infty} (1-\tau)\tau^j \beta^{j+1} \Lambda_{t,t+1+j}\left(N_{i,t+1+j}\right) \tag{28}$$

Thus, it can be written as,

$$V_{it}(N_{it}) = v_{kit} Q_{it} S_{it} + v_{lit} L_{it} + \eta_{it} N_{it}$$

with

$$\begin{aligned} v_{kit} &= \mathbb{E}_t\left[(1-\tau)\beta\Lambda_{t,t+1}\left(R_{ik,t+1} - R_t\right) + \beta\Lambda_{t,t+1}\tau x_{kit,t+1} v_{kit+1}\right] \\ v_{lit} &= \mathbb{E}_t\left[(1-\tau)\beta\Lambda_{t,t+1}\left(R_{iLt} - R_t\right) + \beta\Lambda_{t,t+1}\tau x_{lit,t+1} v_{lit+1}\right] \end{aligned}$$

and

$$\eta_{it} \equiv \mathbb{E}_t\left[(1-\tau)\Lambda_{t,t+1} R_{t+1} + \beta\Lambda_{t,t+1}\tau z_{it,t+1}\eta_{it+1}\right]$$

in order to prohibit intermediaries expanding their assets indefinitely when risk premium is positive, I introduce the following incentive constraint,

$$V_{it} \geq \lambda_i (Q_{it} S_{it} + L_{it}) \tag{29}$$

The binding incentive constraint in equilibrium implies that the leverage ratio $\left(\frac{Q_{it} S_{it} + L_{it}}{N_{it}}\right)$ denoted by ϕ_{it} equals,

$$\phi_{it} = \frac{\eta_{it}}{\lambda_i - d_{it} \nu_{kit} - (1 - d_{it}) \nu_{lit}},$$

The total net worth of bankers in group i includes the net worth of existing bankers N_{iet} together with the net worth of new bankers N_{int} ,

$$N_{it} = N_{eit} + N_{nit}$$

where $N_{eit} = \tau\left[\left(R_{ikt} - R_{t-1}\right)d_{it-1}\phi_{it-1} + \left(R_{iLt-1} - R_{t-1}\right)\left(1 - d_{it-1}\right)\phi_{it-1} + R_{t-1}\right]N_{it-1}$
A financial shock represents an unexpected contraction of the existing bankers' net worth.
Assuming further that a fraction of $\omega_i/(1-\tau)$ of the total assets of exiting bankers $((1-\tau)\left(Q_{it}S_{it-1} + L_{it-1}\right))$ is transferred to new bankers, I have

$$N_{nit} = \omega_i\left(Q_{it}S_{it-1} + L_{it-1}\right)$$

4.6 Monetary policy and Government Expenditure

Monetary policy follows a simple Taylor rule with interest rate smoothing. The nominal interest rate follows,

$$i_t = (1-\rho)\left[i + \kappa_\pi \pi_t + \kappa_y (\log Y_t - \log Y_t^*)\right] + \rho\, i_{t-1} + \varepsilon_t^i \tag{32}$$

Government expenditure G is financed by lump sum taxes.

5 Quantitative Predictions

5.1 Calibration

5.2 The propagation of financial shocks

compare the propagation of financial shocks predicted by the model with my empirical findings.

Figure 3: Supply chain of a circle economy

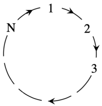


Table 4: Sectoral Level Parameters

sector	labor share α_i	capital share β_i	intermediate share γ_i	final share ζ_i	trade credit $\bar{\theta}_i$	leverage ϕ_i
Agriculture	0.24	0.18	0.58	0.01	0.43	1.70
Mining	0.23	0.39	0.37	0.01	1.00	1.53
Utility	0.17	0.41	0.42	0.02	0.49	3.24
Construction	0.39	0.13	0.48	0.07	0.43	2.35
Manufacturing	0.20	0.16	0.64	0.21	0.49	1.50
Wholesale	0.37	0.33	0.31	0.04	0.44	2.30
Retail	0.41	0.27	0.32	0.10	0.42	1.99
Transportation	0.35	0.16	0.50	0.02	0.30	2.33
Information	0.25	0.30	0.45	0.04	0.64	1.58
real estate	0.24	0.48	0.28	0.15	0.64	2.27
PBS	0.50	0.13	0.37	0.06	0.37	1.53
Education	0.53	0.08	0.40	0.17	0.25	1.77
Arts	0.38	0.18	0.44	0.07	0.21	1.92
Other services	0.49	0.13	0.38	0.04	0.37	2.33
Average	0.33	0.45	0.22	-	0.45	2.00

5.2 The propagation of financial shocks

I impose an unexpected banking net worth shock to bankers linked to Firm 5, ε_{Ne5}, namely a contraction of the existing bankers' net worth N_{e5};

- I assume that their net worth declines by one percent and that the decline is transferred to households.
- One immediate consequence is a rise in the cost of borrowing (R_{5Lt}) for Firm 5 .

Table 5: Model vs. Data: the propagation of financial shocks

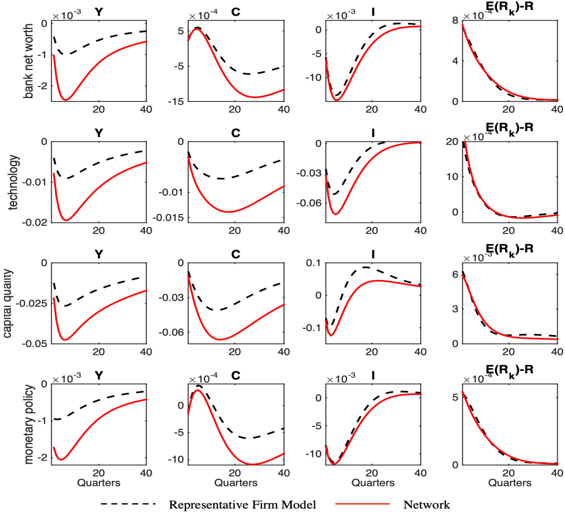
	Data	Model
$\text{UP}_{i,t=0}$	-0.1724**	-0.1979***
$\text{DOWN}_{i,t=0}$	-0.0635	-0.1294 ***
$\text{Shock}_{i,t=0}$	-0.4469**	-0.4360***

Notes: *** indicate statistical significance at the 1% levels. The dependent variable is the first 6-month output change $\tilde{m}_{i,t=6m}$ after the shock. $\text{Shock}_{i,t=0}$ is the borrowing cost R_{iL} change upon the impact of ϵ_{Ne5}. $\text{UP}_{i,t=0}$ and $\text{DOWN}_{i,t=0}$ follow Equations 11 and 12. The first column copies regression result (3) in Table 2.

5 Quantitative Predictions

5.4 Amplification effect of the network structure

Figure 5: Aggregate variables response to banking net worth (Ne), technology (z), capital quality (ξ) and monetary (i)



6 Credit Policy Implication

Suppose that, during a severe financial crisis, a policymaker is willing to facilitate lending.

the total value of intermediated assets of sector i is,

$$S_{it} = S_{it}^{p} + S_{it}^{g}$$
$$L_{it} = L_{it}^{p} + L_{it}^{g}$$

The total value of assets intermediated by the government would be

$$CP_t = \sum_{i}^{N} CP_{it} = \sum_{i}^{N} \left(Q_{it} S_{it}^{g} + L_{it}^{g} \right) \tag{33}$$

Suppose the credit policy follows that

$$CP_{t+1} = \rho_{CP} CP_t + \varepsilon_{t+1}^{CP}, \quad \text{with} \quad \varepsilon_t^{CP} = -\vartheta \varepsilon_t^{Ne} \tag{34}$$

6 Credit Policy Implication

Figure 6: Aggregate variables response to banking net worth (Ne) and credit policy (CP) shocks

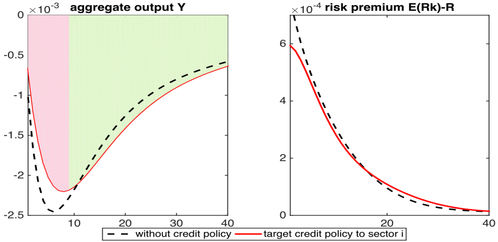
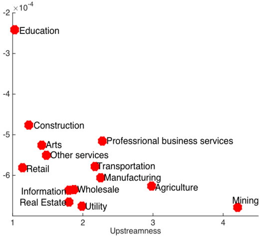
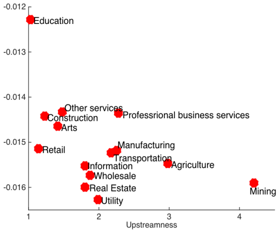


Figure 7: Y_{CPoni}

(a) Contemporaneous response



(b) 2-year cumulative change



7 Conclusions

This paper investigates how the interaction between the production network and trade credit network affect the propagation of financial shocks.

- Using the U.S. input-output matrix and the bond yield data, I find strong upstream propagation of financial shocks.
- To capture this pattern in an input-output model, it has been shown to be important to introduce financial friction in trade and interlocked balance sheets of trading parties.
- **credit policies have a greater impact when liquidity is supplied to downstream sectors after an aggregate liquidity shock.**
- **For downstream sectors are more important systemically, given that financial shocks propagate upstream.**
- ...

感谢大家的聆听！

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