The Financial Accelerator in a Quantitative Business Cycle Framework

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contents

Abstract

- This chapter develops a dynamic general equilibrium model that is intended to help clarify the role of **credit market frictions** in business fluctuations, from both a qualitative and a quantitative standpoint.
- In particular, the framework exhibits a **"financial accelerator"**, in that endogenous developments in credit markets work to **amplify and propagate** shocks to the macroeconomy.
- In addition, we add several features to the model that are designed to **enhance the empirical relevance**.

  (i) **incorporate money and price stickiness**, which allows us to study how credit market frictions may influence the transmission of monetary policy.

  (ii) allow for **lags** in investment which enables the model to generate both hump-shaped output dynamics and a lead-lag relation between asset prices and investment.

  (iii) allow for **heterogeneity** among firms to capture the fact that borrowers have differential access to capital markets.
- the financial accelerator has a significant influence on business cycle dynamics.

Background

- RBC and Keynesian IS-LM：*Except for the term structure of real interest rates, which, together with expectations of future payouts, determines real asset prices,* in these models conditions in **financial and credit markets** do not affect the real economy.
  - *Modigliani-Miller(1958) theorem:*financial structure is both **indeterminate** and **irrelevant** to real economic outcomes. m
- **However**, *Gertler (1988):* there is a long-standing alternative tradition in macroeconomics, beginning with Fisher and Keynes, gives a more central role to credit-market conditions in the **propagation of cyclical fluctuations**.
- **In this alternative view**, deteriorating credit-market conditions - insolvencies and bankruptcies↑, real debt burdens ↑, collapsing asset prices, and bank failures are not simply passive reflections of a declining real economy, but are in themselves a major factor depressing economic activity. *[eg. Fisher (1933),Bernanke (1983).Bernanke and Lown (1992)]*

Purpose I

In order to show that credit-market imperfections can be incorporated into standard macroeconomic models in **a relatively straightforward yet rigorous way**.

- Introducing credit-market frictions into the standard models.
  (i) Can improve their ability to **explain even "garden-variety" cyclical fluctuations**.
    - Credit-market frictions may significantly **amplify** both real and nominal shocks to the economy*(modest changes in real interest rates induced by monetary policy, for example, or the small average changes in firm costs induced by even a relatively large movement in oil prices)*.
    - Credit-market frictions has**Added advantage** of permitting the standard models to explain a broader class of important **cyclical phenomena**, *such as changes in credit extension and the spreads between safe and risky interest rates*.

  (ii) Modern empirical research on the determinants of aggregate demand and *(to a lesser extent)* of aggregate supply has often ascribed an important role to various credit-market frictions.
    - **consumption** has emphasized the importance of limits on borrowing and the closely-related "buffer stock" behavior *[Mariger (1987), Zeldes (1989), Jappelli (1990), Deaton (1991), Eberly (1994), Gourinchas and Parker (1995), Engelhardt (1996), Carroll (1997), Ludvigson (1997), Bacchetta and Gerlach (1997)]*.

Purpose II

- In the **investment literature**, cash flow, leverage, and other balance-sheet factors also have a major influence on investment spending *[Fazzari, Hubbard and Petersen (1988), Hoshi, Kashyap and Scharfstein (1991), Whited (1992), Gross (1994), Gilchrist and Himmelberg (1995), Hubbard, Kashyap and Whited (1995)]*.
- the determinants of inventories and of employment *[Cantor (1990), Blinder and Maccini (1991), Kashyap, Lamont and Stein (1994), Sharpe (1994), Carpenter, Fazzari and Petersen (1994)]*.

Over the past twenty-five years, breakthroughs in the economics of **incomplete and asymmetric information** *[beginning with Akerlof (1970)]* and the extensive adoption of these ideas in corporate finance and other applied fields [e.g., Jensen and Meckling (1976)], have made possible more formal theoretical analyses of **credit-market imperfections**.

- In particular, it is now well understood that **asymmetries of infonnalion** play a key role in borrower-lender relationships
- Lending institutions and financial contracts typically take the forms that they do in order to **reduce the costs** of gathering information and to **mitigate principal-agent problems** in credit markets;

Purpose III

- The common feature of most of the diverse problems that can occur in credit markets is a **worsening** of informational asymmetries and **increases** in the associated agency costs.

Model Innovation

This chapter we develop a dynamic general equilibrium model that we hope will be useful for understanding the role of credit-market frictions in cyclical fluctuations.

- Framework exhibits a "financial accelerator" *[Bernanke, Gertler and Gilchrist (1996)]*, in that endogenous developments in credit markets work to propagate and amplify shocks to the macroeconomy.
- The key mechanism involves the link between **"external finance premium"** and the **net worth** of potential borrowers and both are **inversely**.
- To the extent that borrowers' **net worth** is **procyclical** *(because of the procyclicality of profits and asset prices, for example)*, the **external finance premium** will be **countercyclical**, **enhancing the swings** in borrowing and thus in investment, spending, and production.
- Enhancing the value of the model for quantitative analysis
  (i) Incorporating **price stickiness and money**;
  (ii) **Lags in investment**,to generate both hump-shaped output dynamics and a lead-lag relationship between asset prices and investment;
  (iii) **Heterogeneity** among firms;

overview

- The baseline DNK model is essentially a stochastic growth model that **incorporates money, monopolistic competition, and nominal price rigidities**. Several reasons as following:
  (i) Widely accepted *[See Goodfriend and King (1997)]*.
  (ii) It is possible to illustrate how credit market imperfections influence the transmission of monetary policy *[see Bernanke and Gertler (1995)]*.
  (iii) The DNK model nests the real business cycle paradigm as a special case.
- There are three types of agents, called **households, entrepreneurs, and retailers**.
  - Households and entrepreneurs are distinct from one another in order to explicitly motivate lending and borrowing.
  - Adding retailers permits us to incorporate inertia in price setting in a tractable way, as we discuss.
  - Government conducts both fiscal and monetary policy.
- Households live forever; they work, consume, and save. They hold both real money balances and interest-bearing assets.

Entrepreneurs Assumptions

Entrepreneurs are assumed to be risk-neutral and have finite horizons:

- a constant probability γ of surviving to the next period (implying an expected lifetime of $\frac{1}{1-\gamma}$)

In each period t entrepreneurs acquire **physical capital***(Entrepreneurs who "die" in period t are not allowed to purchase capital, but instead simply consume their accumulated resources and depart from the scene.)*.

- Physical capital acquired in period t is used in combination with **hired labor to produce output** in period $t+1$, by means of a constant-returns to scale technology. Acquisitions of capital are financed by entrepreneurial wealth, or **"net worth"**, and borrowing.

The net worth comes from two sources: **profits** *accumulated from previous capital investment* and **income** *from supplying labor*.

- Higher levels of net worth allow for increased self-financing (equivalently, collateralized external finance)

Introducing a conflict of interest between a **borrower** and his respective **lenders**.

Other Assumptions

- Entrepreneurs produce wholesale goods in competitive markets, and then sell their output to retailers who are monopolistic competitors.
- Retailers do nothing other than buy goods from entrepreneurs, differentiate them (costlessly), then re-sell them to households.
- Having described the general setup of the model, we proceed in two steps.
  - (i) Deriving the key **microeconomic** relationship： *the dependence of a firm's demand for capital on the potential borrower's net worth.* the firm's (entrepreneur's) partial equilibrium problem of jointly determining its demand for capital and terms of external finance in negotiation with a competitive lender (e.g., a financial intermediary).
  - (ii) Embing these relationships in an conventional DNK model. *Our objective is to show how fluctuations in borrowers' net worth can act to amplify and propagate exogenous shocks to the system.*

The demand for capital I

We now study the capital investment decision at the firm level, taking as given the price of capital goods and the expected return to capital.

- The return to capital is sensitive to both aggregate and **idiosyncratic** risk.
- The ex post gross return on capital : $\omega^j R^k_{t+1}$
  - ω^j is *i.i.d.* with a continuous and once-differentiable c.d.f., $F(\omega)$, over a non-negative support, and $E\{\omega^j\} = 1$.
  - hazard rate $h(\omega)$:

$$\frac{\partial(\omega h(\omega))}{\partial\omega} > 0, \quad \textit{where} \quad h(\omega) = \frac{\mathrm{d}F(\omega)}{1 - F(\omega)} \tag{3.1}$$

- Borrow an amount satisfied:

$$B^j_{t+1} = Q_t K^j_{t+1} - N^j_{t+1} \tag{3.2}$$

Because entrepreneurs are **risk-neutral** and households are **risk-averse**, the loan contract the intermediary signs has entrepreneurs absorb any aggregate risk, as we discuss below.

The demand for capital II

- Assuming a "costly state verification" (CSV)-*[Townsend (1979)]*
  - Lenders must pay a fixed "**auditing cost**" in order to observe an individual borrower's **realized return**
  - Motivating why uncollateralized external finance may be more **expensive** than internal finance without imposing arbitrary restrictions on the contract structure.
- Following the CSV approach, lender must pay a cost if he or she wishes to observe the borrower's realized return on capital.
  - The auditing cost: *the cost of bankruptcy(including for example auditing, accounting, and legal costs, as well as losses associated with asset liquidation and interruption of business).*
  - **The monitoring cost:** $\mu\omega^j R^k_{t+1} Q_t K^j_{t+1}$.

Contract terms when there is no aggregate risk I

Absent any aggregate uncertainty, the optimal contract under costly state verification looks very much like standard risky debt (see *Appendix A* for a detailed analysis of the contracting problem):

- A threshold value of the idiosyncratic shock ω^j, call it $\bar{\omega}^j$, defined by

$$\bar{\omega}^j R^k_{i+1} Q_i K^j_{i+1} = Z^j_{t+1} B^j_{t+1} \tag{3.3}$$

- Case 1: $\omega^j \geqslant \bar{\omega}^j$
  - Under the optimal contract the entrepreneur repays the lender the promised amount $Z^j_{t+1} B^j_{t+1}$.
  - Keeps the difference, equal to $\omega^j R^k_{t+1} Q_t K^j_{t+1} - Z^j_{t+1} B^j_{t+1}$.
- Case 2: $\omega^j < \bar{\omega}^j$
  - He entrepreneur cannot pay the contractual return and thus declares default.
  - The intermediary's net receipts are $(1-\mu)\omega R^k_{t+1} Q_t K^j_{t+1}$.
  - A defaulting entrepreneur receives nothing.

Contract terms when there is no aggregate risk II

The loan contract must satisfy:

$$\left[1-F\left(\bar{\omega}^{j}\right)\right] Z_{t+1}^{j} B_{t+1}^{j}+(1-\mu) \int_{0}^{\bar{\omega}^{j}} \omega R_{t+1}^{k} Q_{t} K_{t+1}^{j} \mathrm{d} F(\omega)=R_{t+1} B_{t+1}^{j} \tag{3.4}$$

where the left-hand side is the expected gross return on the loan to the entrepreneur and the right side is the intermediary's opportunity cost of lending. *Note that $F\left(\bar{\omega}^{j}\right)$ gives the probability of default.*

Combining Equations (3.2) and (3.3) with Equation (3.4) yields the following expression for $\bar{\omega}^{j}$:

$$\left[1-F\left(\bar{\omega}^{j}\right)\right] \bar{\omega}^{j}+(1-\mu) \int_{0}^{\bar{\omega}^{j}} \omega \mathrm{d} F(\omega) R_{t+1}^{k} Q_{t} K_{t+1}^{j}=R_{t+1}\left(Q_{t} K_{t+1}^{j}-N_{t+1}^{j}\right) \tag{3.5}$$

Note that: there are two effects of changing $\bar{\omega}^{j}$ on the expected return, and they work in **opposite** directions.

- A **rise** in $\bar{\omega}^{j}$ **increases** the non-default payoff;

Contract terms when there is no aggregate risk III

- it also **raises** the default probability, which lowers the expected payoff.

Note that: the expected return reaches a maximum at an unique interior value of $\bar{\omega}^j$ *(Appendix A provides details)*:

- As $\bar{\omega}^j$ **rises above this value** the expected return **declines** due to the increased likelihood of default.
- For values of $\bar{\omega}^j$ **below the maxinm**m, the function is **increasing** and **concave**.

  - *(This is footnote:)* The change in the expected payoff fl'om a unit increase $\bar{\omega}^j$ is $\left\{\left[1-F\left(\bar{\omega}^j\right)\right]-\mu\bar{\omega}^j \mathrm{d}F\left(\bar{\omega}^j\right)\right\} R_{t+1}^k Q_t K_{t+1}^j$. The first term in the expression in brackets reflects the rise in the non-default payoff. The second term reflects the rise in expected default costs. Note that we can rewrite this expression as $\left\{1-\mu\bar{\omega}^j h\left(\bar{\omega}^j\right)\right\}\left[1-F\left(\bar{\omega}^j\right)\right] R_{i+1}^k Q_i K_{t+1}^j$, where $h(\omega) \equiv \frac{dF(\omega)}{1-F(\omega)}$ is the hazard rate. Given Equation (3.1) the derivative of this expression is negative for values of $\bar{\omega}^j$ below the maximum one feasible, implying that the expected payoff is concave in this range.

Contract terms when there is no aggregate risk IV

- If the lender's opportunity cost is so large that there does not exist a value of $\bar{\omega}^j$ that generates the required expected return, then the borrower is "rationed" from the market.
- For simplicity, in what follows, we consider only equilibria without rationing, i.e., equilibria in which the equilibrium value of $\bar{\omega}^j$ always lies below the maximum feasible value.

Contract terms when there is aggregate risk I

Because he cares only about the mean return on his wealth, the entrepreneur is willing to bear all the aggregate risk. Thus he is willing to guarantee the lender a return that is free of any systematic risk, ie.,

- Conditional on the ex post realization of R^k_{t+1}
- The borrower offers a (state-contingent) non-default payment that guarantees the lender a return equal in expected value to the riskless rate.
- Diversification by intermediaries implies that households earn the riskless rate on their saving.

Descriptively, the existence of aggregate uncertainty effectively ties the risky loan rate Z^j_{t+1} macroeconomic conditions. **The loan rate adjusts countercyclically.**

- $R^k_{t+1} < Z^j_{t+1}$, to compensate for the increased default probability due to the low average return to capital, the non-default payment must rise.
- In turn implies $\bar{\omega}^j \uparrow$.
- That default probabilities and default premia rise when the aggregate return to capital is lower than expected.

Net worth and the optimal choice of capital I

Given the state-contingent debt form of the optimal contract, the expected return to the entrepreneur may be expressed as

$$E\left\{\int_{\bar{\omega}^j}^{\infty}\omega R_{t+1}^k Q_t K_{t+1}^j \mathrm{d}F(\omega)-\left(1-F\left(\bar{\omega}^j\right)\right)\bar{\omega}^j R_{t+1}^k Q_t K_{t+1}^j\right\} \tag{3.6}$$

Combining this relation with Equation (3.5) allows us to simplify the entrepreneur's objective to maximization of

$$E\left\{\left[1-\mu\int_0^{\bar{\omega}^j}\omega \mathrm{d}F(\omega)\right]U_{t+1}^{rk}\right\}E\left\{R_{t+1}^k\right\}Q_t K_{t+1}^j - R_{t+1}\left(Q_t K_{t+1}^j - N_{t+1}^j\right) \tag{3.7}$$

Where $U_{t+1}^{rk}\equiv R_{t+1}^k/E\left\{R_{t+1}^k\right\}$ is the ratio of the realized return to capital to the expected return.

Net worth and the optimal choice of capital II

Let $s_t \equiv E\left\{R^k_{t+1}/R_{t+1}\right\}$ be the expected discounted return to capital. Given $s_t \geqslant 1$, the first-order necessary conditions yield the following relation for optimal capital purchases *(see Appendix A for details)*:

$$Q_t K^j_{t+1} = \psi\left(s_t\right) N^j_{t+1}, \quad \text{with} \quad \psi(1) = 1, \psi'(\cdot) > 0 \tag{3.8}$$

- It shows that capital expenditures by each firm are proportional to the net worth of the owner/entrepreneur, with a proportionality factor that is increasing in the expected discounted return to capital, s_t.
- a rise in the expected discounted return to capital reduces the expected default probability. As a consequence, the entrepreneur carl take on more debt and expand the size of his firm.
- He is constrained from raising the size of the firm indefinitely by the fact that expected default costs also rise as the ratio of borrowing to net worth increases.

Net worth and the optimal choice of capital III

An equivalent way of expressing Equation (3.8) is

$$E\left\{R^k_{t+1}\right\} = s\left(\frac{N^j_{t+1}}{Q_t K^j_{t+1}}\right) R_{t+1}, \quad s'(\cdot) < 0 \tag{3.9}$$

- not fully self-financed, the return to capital = the marginal cost of external finance.
- s:the discounted return to capital, or the external finance premium.

Net worth and the optimal choice of capital IV

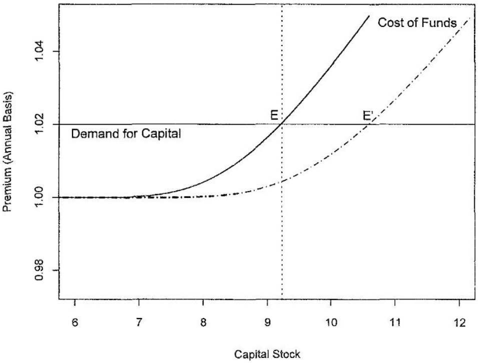

图: 1.Effect of an increase in net worth.

- Case 1: $K < 4.6$,be financed entirely by the entrepreneur's net worth, *cost of funds = the risk free rate*.
- Case 2: As $K \uparrow$, expected default costs $\uparrow$, higher ratio of debt to net worth.
- Case 3: Point E, $K = 9.2$: marginal cost of funds = the expected return to capital yields, leverage ratio is 50 $\%$.
- **Note that**$(E \Rightarrow E')$:$N^j_{t+1} \uparrow 15\% \Rightarrow$ the premium for external finance $\downarrow \Rightarrow$ the expected default probability $\downarrow$.

The entrepreneurial sector I

We specify the aggregate production function relevant to any given period t as

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{4.1}$$

The aggregate capital stock evolves according to

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) K_t + (1-\delta) K_t \tag{4.2}$$

Aggregate investment expenditures of I_t yield a gross output of new capital goods $\Phi\left(I_t/K_t\right) K_t$, where $\Phi(\cdot)$ is **increasing and concave** and $\Phi(0) = 0$.
As in Kiyotaki and Moore (1997), the idea is to have **asset price variability** contribute to **volatility in entrepreneurial net worth**. The price of a unit of capital in terms of the numeraire good, Q_t, is given by

$$Q_t = \left[\Phi'\left(\frac{I_t}{K_t}\right)\right]^{-1} \tag{4.3}$$

The entrepreneurial sector II

Assume that entrepreneurs sell their output to retailers. The rent paid to a unit of capital in $t+1$ (for production of wholesale goods) is

$$\frac{1}{X_{t+1}}\frac{\alpha Y_{t+1}}{K_{t+1}}$$

Note that: X_t is the gross **markup** of retail goods over wholesale goods.
The expected gross return to holding a unit of capital from t to $t+1$ can be written

$$E\{R_{t+1}^k\} = E\left\{\frac{\frac{1}{X_{t+1}}\frac{\alpha Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta)}{Q_t}\right\} \tag{4.4}$$

The supply curve for investment finance is obtained by aggregating Equation (3.8) over firms, and inverting to obtain:

$$E\{R_{t+1}^k\} = s\left(\frac{N_{t+1}}{Q_t K_{t+1}}\right) R_{t+1} \tag{4.5}$$

The entrepreneurial sector III

The function $s(\cdot)$ is the ratio of the costs of external and internal finance; it is decreasing in N_{t+1}/Q_tK_{t+1} for $N_{t+1} < Q_tK_{t+1}$.
Following *Bernanke and Gertler (1989)* and *Carlstrom and Fuerst (1997)*, we assume that, in addition to operating firms, entrepreneurs supplement their income by working in the general labor market:

$$L_t = H_t^{\Omega} \left(H_t^e\right)^{1-\Omega} \tag{4.6}$$

Note that: total labor input L_t is taken to be the following composite of household labor, H_t, and "entrepreneurial labor", H_t^e.
Then aggregate entrepreneurial net worth at the end of period t, N1+1, is given by

$$N_{t+1} = \gamma V_t + W_t^e \tag{4.7}$$

Where V_t is entrepreneurial equity (i.e., wealth accumulated by entrepreneurs from operating firms), W_t^e denote the entrepreneurial wage.

The entrepreneurial sector IV

- Where γV_t is the equity held by entrepreneurs at $t-1$ who are **still in busines** at t.
- Entrepreneurs who **fail** in t consume the residual equity $(1-\gamma)V_t$. That is, $C_t^e = (1-\gamma)V_t$.
- The premium for external finance: $\frac{\mu \int_0^{\bar{\omega}_t} \omega R_t^k Q_{t-1} K_t \mathrm{d}F(\omega)}{Q_{t-1}K_t - N_{t-1}}$.

with

$$V_t = R_t^k Q_{t-1} K_t - \left(R_t + \frac{\mu \int_0^{\bar{\omega}_t} \omega R_t^k Q_{t-1} K_t \mathrm{d}F(\omega)}{Q_{t-1}K_t - N_{t-1}} \right) (Q_{t-1}K_t - N_{t-1}) \tag{4.8}$$

Clearly, this equity may be highly sensitive to unexpected shifts in asset prices, especially if firms are leveraged. To illustrate:

- Let $U_t^{rk} \equiv R_t^k - E_{t-1}\left\{R_t^k\right\}$ be the unexpected shift in the gross return to capital.
- Let $U_t^{dp} \equiv \int_0^{\bar{\omega}_t} \omega Q_{t-1} K_t \mathrm{d}F(\omega) - E_{t-1}\left\{\int_0^{\bar{\omega}_t} \omega Q_{t-1} K_t \mathrm{d}F(\omega)\right\}$ be the unexpected shift in the conditional (on the aggregate state) default costs.

The entrepreneurial sector V

We can express V_t as

$$V_t = \left[U_t^{rk}\left(1 - \mu U_t^{dp}\right)\right] Q_{t-1}K_t + E_{t-1}\left\{V_t\right\} \tag{4.9}$$

We can obtain that:

$$\frac{\partial V_t / E_{t-1}\left\{V_t\right\}}{\partial U_t^{rk} / E_{t-1}\left\{R_t^k\right\}} = \frac{E_{t-1}\left\{R_t^k\right\} Q_{t-1}K_t}{E_{t-1}\left\{V_t\right\}} \geqslant 1 \tag{4.10}$$

- This ratio $\geqslant 1$, implying a magnification effect of unexpected asset returns on entrepreneurial equity.
- That unexpected movements in asset prices can have a substantial effect on firms' financial positions.

The entrepreneurial sector VI

Note that, there is a kind of multiplier effect, unanticipated rise in asset prices $\uparrow \rightarrow$ net worth $\uparrow$ more than proportionately $\rightarrow$ stimulates investment $\rightarrow$ asset prices.

We next obtain demand curves for household and entrepreneurial labor, found by equating marginal product with the wage for each case:

$$(1-\alpha)\Omega\frac{Y_t}{H_t} = X_t W_t \tag{4.11}$$

$$(1-\alpha)(1-\Omega)\frac{Y_t}{H_t^e} = X_t W_t^e \tag{4.12}$$

- W_t is the real wage for household labor
- W_t^e is the real wage for entrepreneurial labor.

The entrepreneurial sector VII

Combining Equations (4.1), (4.7), (4.8), and (4.12) and imposing the condition that entrepreneurial labor is fixed at unity, yields a difference equation for N_{t+1}:

$$\begin{aligned} N_{t+1} = & \gamma \left[R_t^k Q_{t-1} K_t - \left(R_t + \frac{\mu \int_0^{\bar{\omega}_t} \omega \mathrm{d} F(\omega) R_t^k Q_{t-1} K_t}{Q_{t-1} K_t - N_t} \right) (Q_{t-1} K_t - N_t) \right] \\ & + (1-\alpha)(1-\Omega) A_t K_t^{\alpha} H_t^{(1-\alpha)\Omega} \end{aligned} \tag{4.13}$$

Note that: Equation (4.13) and the supply curve tbr investment funds, Equation (4.5), are **the two basic ingredients of the financial accelerator.**
Add the household, retail, and government sectors**(details to Appendix B)**.

The complete log-linearized model I

- **Aggregate demand**

$$y_l = \frac{C}{Y}c_l + \frac{I}{Y}i_t + \frac{G}{Y}g_t + \frac{C^e}{Y}c_t^e + \cdots + \phi_t^y \tag{4.14}$$

$$c_t = -r_{t+1} + E_t\left\{c_{t+1}\right\} \tag{4.15}$$

$$c_t^e = n_{t+1} + \cdots + \phi_t^{c^e} \tag{4.16}$$

$$E_t\left\{r_{t+1}^k\right\} - r_{t+1} = -v\left[n_{t+1} - \left(q_t + k_{t+1}\right)\right] \tag{4.17}$$

$$r_{t+1}^k = (1-\epsilon)\left(y_{t+1} - k_{t+1} - x_{t+1}\right) + \epsilon q_{t+1} - q_t \tag{4.18}$$

$$q_t = \varphi\left(i_t - k_t\right) \tag{4.19}$$

- **Aggregate Supply**

$$y_t = a_t + \alpha k_t + (1-\alpha)\Omega h_t \tag{4.20}$$

$$y_t - h_t - x_t - c_t = \eta^{-1} h_t \tag{4.21}$$

$$\pi_t = E_{t-1}\left\{\kappa\left(-x_t\right) + \beta\pi_{t+1}\right\} \tag{4.22}$$

The complete log-linearized model II

- **Evolution of State Variables**

$$k_{t+1} = \delta i_t + (1-\delta) k_t \tag{4.23}$$

$$n_{t+1} = \frac{\gamma RK}{N} \left(r_t^k \cdot r_t \right) + r_t + n_t + \cdots \phi_t^n \tag{4.24}$$

- **Monetary Policy Rule and Shock Processes**

$$r_t^n = \rho r_{t-1}^n + \zeta \pi_{t-1} + \varepsilon_t^{rn} \tag{4.25}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{4.26}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{4.27}$$

The complete log-linearized model III

with

$$\phi_t^v \equiv \frac{DK}{Y}\left[\log\left(\mu\int_0^{\bar{\omega}_t}\omega \mathrm{d}F(\omega)R_t^k Q_{t-1}K_t/DK\right)\right]$$

$$D \equiv \mu\int_0^{\bar{\omega}}\omega \mathrm{d}F(\omega)R^k$$

$$\phi_t^{c^e} = \log\left(\frac{1-C_{t+1}^e/N_{t+1}}{1-C^e/N}\right)$$

$$\phi_t^n \equiv \frac{\left(R^k/R-1\right)K}{N}\left(r_t^k+q_{t-1}+k_t\right)+\frac{(1-\alpha)(1-S2)(Y/X)}{N}y_t - x_t$$

$$v \equiv \frac{\psi\left(R^k/R\right)}{\psi'\left(R^k/R\right)}, \quad \epsilon \equiv \frac{1-\delta}{(1-\delta)+\alpha Y/(XK)}$$

$$\varphi \equiv \frac{\left(\Phi(I/K)^{-1}\right)'}{\left(\Phi(I/K)^{-1}\right)''}, \quad \kappa \equiv \left(\frac{1-\theta}{\theta}\right)(1-\theta\beta)$$

Two extensions of the baseline model I

Two modifications that we consider are:

(i) Allowing for delays in investment: *to generate the kind of hump-shaped output dynamics that are observed in the data.*

(ii) Allowing for firms with differential access to credit: *to increase descriptive realism.*

Inoestment delays

The output response to a monetary policy shock*[see, e.g., Christiano, Eichenbaum and Evans (1996) and Bernanke and Mihov (1998)].*
consumption expenditures are determined two periods in advance*[Rotemberg and Woodford (1997)].*
We assume that it is **investment expenditures** rather than consumption expenditures that are determined in advance. Two reasons:

(i) take time to plan is highly plausible*[Christiano and Todd (1996)].*

(ii) movements in consumption lead movements in investment over the cycle*[Bernanke and Gertler (1995) and Christiano and Todd (1996)].*

Two extensions of the baseline model II

Suppose that investment expenditure are chosen j periods in advance. Equation (4.3), is modified to

$$E_t \left\{ Q_{t+j} - \left[\Phi' \left(\frac{I_{t+j}}{K_{t+j}} \right) \right]^{-1} \right\} = 0 \tag{4.28}$$

Log-linearized:

$$E_t \left\{ q_{t+j} - \varphi \left(i_{t+j} - k_{t+j} \right) \right\} = 0 \tag{4.29}$$

In our simulations, we take $j = 1$.

Heterogeneous firms

Heterogeneity among firms along many dimensions, in particular in access to credit*[see, e.g., the discussion in Gertler and Gilchrist (1994)]*.

Two extensions of the baseline model III

we assume that there are two types of intermediate goods which are combined into a single wholesale good via a CES aggregator. Production of the intermediate good is given by

$$Y_{it} = A_{it} K_{it}^{\alpha} H_{it}^{\Omega} \left(H_i^e\right)^{(1-\alpha-\Omega)}, \quad i = 1, 2 \tag{4.30}$$

Aggregate wholesale output is composed of sectoral output according to

$$Y_t = \left[a Y_{1t}^{\rho} + (1-a) Y_{2t}^{\rho}\right]^{(1/\rho)} \tag{4.31}$$

there are costs of adjusting the capital stock within each sector:

$$K_{i,t+1} - K_{it} = \phi\left(I_{it}/K_{it}\right) K_{it} - \delta K_{it} \tag{4.32}$$

Let *ji* denote the number of periods in advance that investment expenditures must be chosen in sector *i (note that the lag may differ across sectors)*:

$$E_t \left\{ Q_{i,t+ji} - \left[\Phi' \left(\frac{I_{i,t}, t+ji}{K_{i,t+ji}} \right) \right]^{-1} \right\} = 0$$

Two extensions of the baseline model IV

Note that the price of capital may differ across sectors, but that arbitrage requires that each sector generate the same expected return to capital

$$E_t\left\{\left[R^k_{1,t+1} - R^k_{2,t+1}\right]\beta C_t/C_{t+1}\right\} = 0$$

where

$$R^k_{i,t+1} = \left(\frac{1}{X_{t+1}}\frac{P_{it}}{P^W_t}\frac{\alpha Y_{i,t+1}}{K_{i,t+1}} + Q_{i,t+1}(1-\delta)\right)/Q_{it}$$

and

$$\frac{P_{1l}}{P^W_t} = a\left(\frac{Y_{1t}}{Y_t}\right)^{\rho-1}, \quad \frac{P_{2t}}{P^W_t} = (1-a)\left(\frac{Y_{2t}}{Y_t}\right)^{\rho-1}$$

are the relative (wholesale) prices of goods produced in sectors 1 and 2 respectively.

Results

In our experiments we consider four types of aggregate shocks:

(i) a monetary policy shock
(ii) a technology shock
(iii) a government expenditure shock
(iv) a one-time, unanticipated transfer of wealth from households to entrepreneurs.

We first study the response of the economy to these shocks in our model, excluding and including the financial accelerator.

We then consider the implications of allowing for investment delays and heterogeneous firms.

Response to a monetary policy shock I

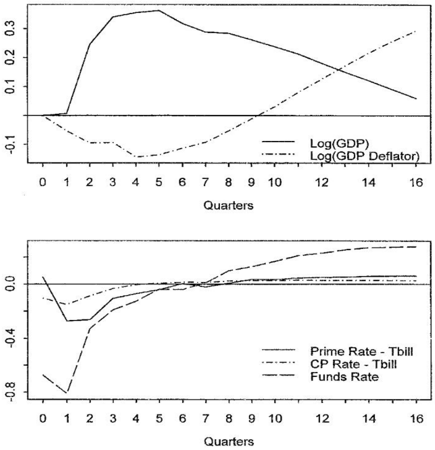

Fig.2. Impulse response to a funds rate shock

- Figure 2 illustrates the impulse responses of several variables to a **negative** innovation in the federal funds rate.
- The output **decline** persists well after the funds rate reverts to trend.
- Each of the spread variables **rises** fairly quickly, leading the downturn in output.

Response to a monetary policy shock II

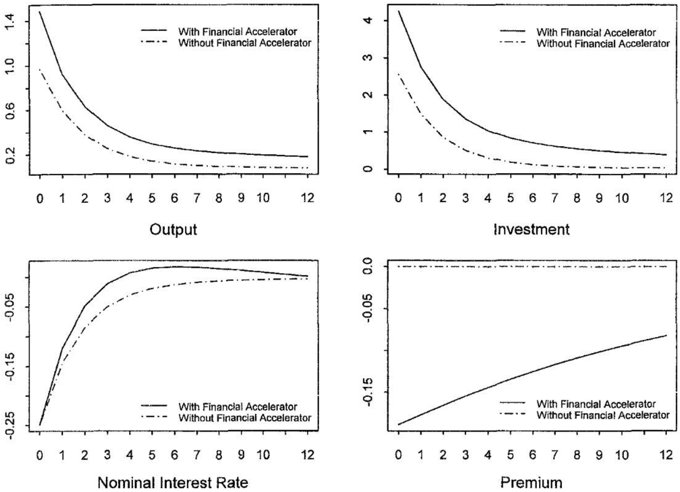

Fig.3. Monetary shock - no investment delay.

Response to a monetary policy shock III

Unanticipated 25 basis point (on an annual basis) $\downarrow$ in the nominal interest rate. Find that:

- With the financial accelerator included, the initial response of output to a given monetary impulse is about 50% greater, and the effect on investment is nearly twice as great.
- the persistence of the real effects is substantially greater in the presence of the credit-market factors
  - *e.g., relative to trend, output and investment in the model with credit-market imperfections after four quarters are about where they are in baseline model after only two quarters.*
- The unanticipated the funds rate $\downarrow \Rightarrow$ *stimulates* demand for capital $\Rightarrow$ investment$\uparrow$, the price of capital$\uparrow$.
- The unanticipated asset prices$\uparrow \Rightarrow$ net worth$\uparrow \Rightarrow$ the external finance premium$\downarrow \Rightarrow$ *stimulates* investment.
  - **multiplier effect arises: investment$\uparrow \Rightarrow$ asset prices and net worth$\uparrow \Rightarrow$ investment.**

Response to a monetary policy shock IV

- Note that: the **countercyclical** movement in the premium for external funds (which is the essence of the financial accelerator) serves to flatten the marginal cost curve.
- Overall, supports to *Bernanke and Gertler(1995)*, that credit-market effectscan help explain both the strength of
    - the economy's response to monetary policy
    - the tendency for policy effects to linger even after interest rates have returned to normal.

Shock to technology, demand, and wealth I

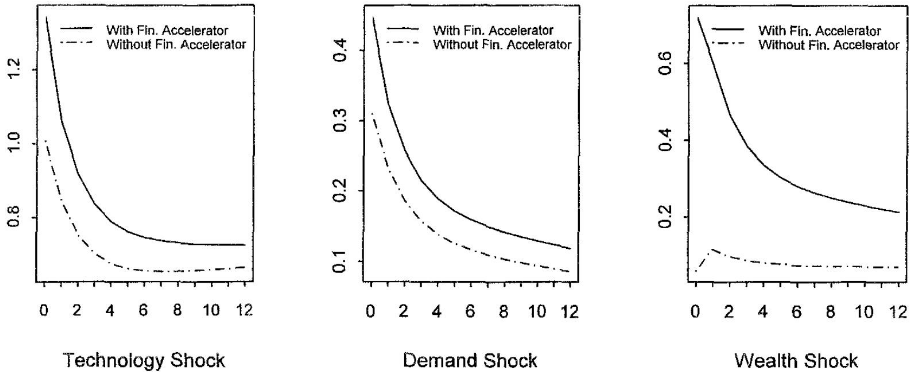

图: 4.Output response alternative shocks.

Find that:

Shock to technology, demand, and wealth II

- the financial accelerator magnifies and propagates both the technology and demand shocks.
- the central mechanism: asset prices$\uparrow\Rightarrow$net worth$\uparrow$ & the external finance premium$\downarrow$.
- A positive shock to entrepreneurial wealth:
  - has essentially **no effect** in the baseline model
  - **but** has both **significant impact and propagation effects** when **credit-market frictions** are present.
- The transfer of wealth drives up the demand for investment goods, which raises the price of capital and thus entrepreneurs' wealth, initiating a positive feedback loop.
- Credit-market effects$\uparrow\Rightarrow$ possibility(relatively small changes in entrepreneurial wealth could be an important source of cyclical fluctuations)$\uparrow$.
  - it is reminiscent of *Fisher's (1933)* **"debt-deflation" argument**: between creditors and debtors arising from unanticipated price changes can have important real effects.

Investment delays

An expansionary monetary policy shock

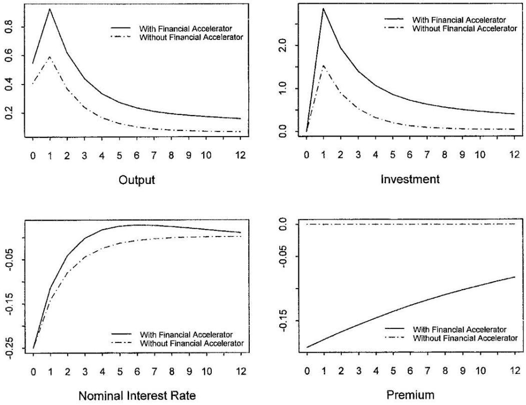

图: 5.Monetary shock one period investment delay.

- hump-shaped **accentuated** with the financial accelerator.
- the persistence of the response of output is **considerably greater** than in the case without investment delays.
- there remains an immediate response of the external funds premium, Because asset prices↑immediately,in anticipation of the investment boom.

heterogeneous firms

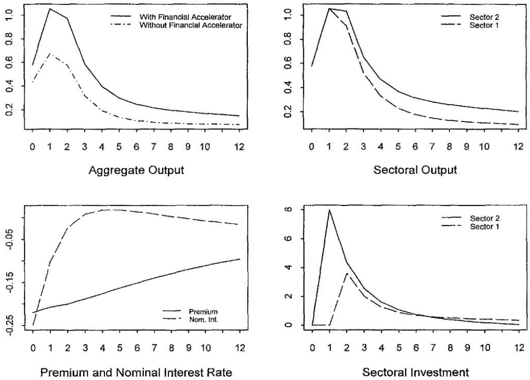

图: 6.Monetary shock multisector model with investment delays. Aggregate output: models with and without financial accelerator; other panels: model with financial accelerator

- the differential investment delays across sectors smooth out the hump-shaped response of output, adding to the overall persistence of the output response.
- investment by firms with relatively poor access to external credit markets rises by nearly three times as much as the investment of firms with better access to credit.

Directions for future work

In subsequent research we hope to consider several extensions to the work so far:

- We have not addressed the role of banks in cyclical fluctuations.
- There are several ways to incorporate a nontrivial role for banks into our framework.
  - (i) Allow the **financial intermediaries** which lend to entrepreneurs to face financial frictions in raising funds themselves.
  - (ii) It would be relatively easy to **incorporate nominal contracting** into this model, in order to evaluate whether the redistributions among debtors and creditors associated with unanticipated changes in the price level are of quantitative significance.
  - (iii) It would be interesting to extend the analysis to the **open economy.**
  - (iv) We have restricted the credit-market frictions to the investment sector.

Households I

The household's objective is given by

$$\max E_t \sum_{k=0}^{\infty} \beta^k \left[\ln\left(C_{t+k}\right) + \zeta \ln\left(M_{t+k}/P_{t+k}\right) + \xi \ln\left(1 - H_{t+k}\right) \right]$$

The individual household budget constraint is given by

$$C_t = W_t H_t - T_t + \Pi_t + R_t D_t - D_{t+1} + \frac{\left(M_{t-1} - M_t\right)}{P_t}$$

First order conditions

Consumption/saving, labor supply, and money holdings:

$$\frac{1}{C_t} = E_t \left\{ \beta \frac{1}{C_{t+1}} \right\} R_{t+1}$$

$$W_i \frac{1}{C_t} = \xi \frac{1}{1 - H_t}$$

Households II

$$\frac{M_t}{P_t} = \zeta C_t \left(\frac{R^n_{t+1} - 1}{R^n_{t+1}} \right)^{-1}$$

where R^n_{t+1} is the gross nominal interest, i.e.,

$$i_{t+1} \equiv R^n_{t+1} \frac{P_{t+1}}{P_t} - 1$$

Household deposits at intermediaries equal total loanable funds supplied to entrepreneurs:

$$D_t = B_t$$

The Retail and Government Sector I

The Retail Sector and Price Setting

Total final usable goods Y_t^f :

$$Y_t^f = \left[\int_0^1 Y_t(z)^{(\epsilon-1)/\epsilon}\mathrm{d}z\right]^{\epsilon/(\epsilon-1)}$$

with $\epsilon > 1$. The corresponding price index is given by

$$P_t = \left[\int_0^1 P_t(z)^{(1-\epsilon)}\mathrm{d}z\right]^{1/(1-\epsilon)}$$

the economy-wide resource constraint is given by

$$Y_l^f = C_t + C_t^e + I_t + G_t + \mu \int_0^{\bar{\omega}_t} \omega \mathrm{d}F(\omega) R_t^k Q_{t-1} K_t$$

The Retail and Government Sector II

Given $Y_t^f = \left[\int_0^1 Y_t(z)^{(\epsilon-1)/\epsilon} \mathrm{d}z\right]^{\epsilon/(\epsilon-1)}$, the demand curve facing each retailer is given by

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\epsilon} Y_t^f$$

Retailer z chooses his price to maximize expected discounted profits, given by

$$\sum_{k=0}^{\infty} \theta^k E_{t-1}\left[\Lambda_{t,k} \frac{P_t^* - P_{t+k}^w}{P_{t+k}} Y_{t+k}^*(z)\right]$$

the optimally set price satisfies

$$\sum_{k=0}^{\infty} \theta^k E_{t-1}\left\{\Lambda_{t,k}\left(\frac{P_t^*}{P_{t+k}}\right)^{-c} Y_{t+k}^*(z)\left[\frac{P_t^*}{P_{t+k}} - \left(\frac{\epsilon}{\epsilon-1}\right)\frac{P_{t+k}^w}{P_{t+k}}\right]\right\} = 0$$

The Retail and Government Sector III

the aggregate price evolves according to

$$P_t = \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)\left(P_t^*\right)^{(1-\epsilon)}\right]^{1/(1-\epsilon)}$$

Government Sector

government expenditures are financed by lump-sum taxes and money creation as follows:

$$G_t = \frac{M_t - M_{t-1}}{P_t} + T_t$$