Construction of Social Distancing Index using Principal Component Analysis

We choose to use the first component of a Principal Component Analysis to construct a social distancing index for our analysis.

Principal Component Analysis (PCA) is a statistical approach whose objective is to find a the most informative way to summarize information in many variables in a much smaller number of dimensions (Hotelling 1933). In our context, we take the first component of 4 individual metrics. In particular, let the vectors *FractStayHome*, *FullTimeWork*, *PartTimeWork*, and *StayHomeDuration* denote all observations of the demeaned stay-home percentage, full-time working percentage, part-time working percentage, and stay-home duration, respectively. Also define matrix $X = [FractStayHome, FullTimeWork, PartTimeWork, StayHomeDuration]$. To find the social distancing index, we need to first solve the following maximization problem by estimating the set of loading coefficients $\hat{\alpha} = (\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4)$:

$$\hat{\alpha} = \underset{\alpha}{\operatorname{argmax}} \frac{1}{n} [X\alpha]^T [X\alpha], subject\ to\ \alpha^T\alpha = 1$$

where n is the number of observations across counties and dates. The resulting $X\hat{\alpha}$ is the first principal component vector. Based on this PCA, the estimated loading coefficients are -0.53, 0.51, 0.61, and -0.31. Consequently, $X\hat{\alpha}$ is negatively correlated with stay-home ratio and stay-home duration, and positively correlated with two working ratio metrics. To make our measure easier to interpret, we take the negative of $X\hat{\alpha}$ and use it as the social distancing index so that a higher index means a higher level of social distancing, and vice versa. Accordingly, the vector of social distancing index is

$$SocialDistIndex = 0.53\ FractStayHome - 0.51\ FullTimeWork$$
$$-0.61\ \ PartTimeWork + 0.31\ StayHomeDuration$$

Reference:

Hotelling, H. (1933). Analysis of a complex of statistical variables into principal components. *Journal of Educational Psychology*, **24**, 417–441