Determining Consumers' Discount Rates with Field Studies

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Abstract: Determining Consumers' Discount Rates With Field Studies

Because utility/profits, state transitions and discount rates are confounded in dynamic models, discount rates are typically fixed for the purpose of identification. We propose a strategy of identifying discount rates. The identification rests upon imputing the utility/profits using decisions made in a context where the future is inconsequential, the objective function is concave, and the decision space is continuous; and then using these utilities/profits to identify discount rates in contexts where dynamics become material. We exemplify this strategy using a field study wherein cellphone users transitioned from a linear to three-part-tariff pricing plan.

We find that the estimated discount rate corresponds to a weekly discount factor (0.90), lower than the value typically assumed in empirical research (0.995). When using a standard 0.995 discount factor, we find the price coefficient is underestimated by 16%. Moreover, the predicted inter-temporal substitution pattern and demand elasticities are biased, leading to a 29% deterioration in model fit; and suboptimal pricing recommendations that would lower potential revenue gains by 76%.

Keywords: dynamic structural model, identification, forward-looking consumers, nonlinear pricing.

1 Introduction

1.1 Overview

Individuals often face situations where they must choose between engaging in consumption in the present or waiting to consume at a future time. A rich stream of recent literature has adopted dynamic structural models to study intertemporal consumption, yielding deep insights into consumer behavior, such as Rust (1987), Erdem and Keane (1996), Hendel and Nevo (2006), and Sun (2005).

Albeit the increasingly ubiquitous use of structural models to study dynamic consumption behavior, the identification of these models are problematic (Rust, 1994). To identify consumer utility functions, it is often necessary to assume or fix the discount factor at a given value. In contrast, we develop a dynamic structural model in order to identify and measure consumer discount rates using field data. The identification strategy relies on the intuition that one can impute consumers' utility when their current decisions are inconsequential for their future decisions, the objective function is concave, and the decision space is continuous. Then, conditioned on the estimated utilities under this static setting, it becomes possible to identify discount rates when future outcomes became material to these consumers' decisions. We outline a proof of identification and use a Monte Carlo simulation to show the sampling properties of the identification approach.

The identification strategy has general applications in contexts where researchers have repeated observations of firms or consumers making continuous decisions under both static and dynamic settings. These include observations of consumer behavior of reward points usage decision when facing loyalty programs with expiration dates, usage consumption decisions for Internet or cell phone data access plans with a monthly fixed allowance that does not rollover, firms' markdown pricing for end of season clearance inventory, and seller's pricing decision on perishable goods such as ticket sales. In auctions, for example, consumers may face posted prices for some goods and auctions for others. When consumers face identical items that are auctioned off sequentially, the intertemporal tradeoff is consequential and affects the bidding behavior (e.g., Zeithammer, 2006). The posted price context where consumers make static decisions can then be used to infer preferences. Likewise, in the context of firms' markdown pricing of seasonal goods such as swimwear or skis where styles change each year, intertemporal substitution between the final markdown period and the future is minimized in that period; hence the pricing decision in final periods become static and can be used to infer firm's objective function. What these contexts have in common is that there are multiple observations of decision making in periods that are independent of dynamics. The repeated observations of static data provide information needed for identifying the preferences, which can then be used in the dynamic contexts to infer discount rates.

To exemplify this approach, we apply it to consumer cellphone minute usage data during a field study that involves switching consumer pricing plans. In our data, consumers were initially under a linear "pay-per-minute" plan; hence usage decisions involved no intertemporal trade-offs as usage early in the month had no bearing on prices paid later in the month. Subsequently, the cellphone service provider switched consumers to a nonlinear three-part tariff plan.[1] The plan switch induced intertemporal substitution trade-offs as consumers' minute usage decisions early in the month had consequences for the rates they faced later in the month. Results indicate that consumers have weekly discount factors in our data of around 0.9 with a 95% confidence interval of (0.78, 0.97).[2] The 0.995 discount factor implies a consumer would accept 1 minute and 1 second at the end of the month as a substitute for an equally priced minute at the beginning of the month; instead we compute that consumers value a minute now more closely to 1 minute and 30 seconds at the end of the month.

The remainder of the paper is organized as follows. In section 1.2, we overview the relevant literature to differentiate our paper from past research. Next, we detail our identification strategy. To exemplify our identification strategy, we then detail our modeling context. Next, we report a Monte Carlo simulation to demonstrate the validity of the identification strategy and then apply

[1] A three-part tariff plan contains three components: an access fee, a certain amount of allowance minutes, and a marginal price if the consumer's usage exceeds the allowance within a billing cycle. As a result, the consumer may be subject to extra fees when the consumption exceed the allowance (overage) and overpays if the consumption falls below the allowance (underage). Due to the existence of the allowance and the high marginal price, a consumer needs to decide how to allocate her consumption across time within a billing cycle, intending to avoid overage and underage so as to maximize her total utility.

[2] The implied weekly discount rates are around 1.11 with a 95% confidence interval of (1.03,1.28).

the approach to a field setting. Subsequently, we present and discuss the results and sensitivity analyses. We conclude with some managerial implications and future research directions.

1.2 Relevant Literature

Given that discount rates are not typically identified, several approaches have emerged to contend with the problem, including i) assuming a fixed value for the discount rate, ii) functional identification via structural assumptions and/or estimation via exclusion restrictions, and iii) experimental approaches. Table 1 overviews a sample of these approaches and their resulting discount values converted to their weekly equivalents. Table 1 makes it apparent that discount rates vary considerably across studies. The mean weekly discount factor is 0.98 with a standard deviation of 0.032. The corresponding weekly discount rates average 2.09% with a large standard deviation of 3.66%. In short, there is no clear consensus regarding the value of discount factors, partially due to the fact that discount rates are typically not identified.

[Insert Table 1 about here.]

First, most studies assume or fix the discount factors to certain values, typically between 0.995 to 1.0. For the purpose of identification, it is also a common practice to assume the discount factor is the same across individuals which might be, in some instances, a strong assumption (Frederick et al., 2002).

A second approach to the identification of discount rates includes the imposition of structure on the model such as assuming the distribution of the model errors, individuals knowing the state transition probability, and no unobserved heterogeneity (e.g., Hotz and Miller, 1993, Goettler and Clay, 2010). However, these identification assumptions may be difficult to validate in some contexts (e.g., homogeneous consumers, full information of state transition probability for new products or markets, etc.). Moreover, the data needed to achieve parametric identification is often prohibitive. One needs repeated decisions for each combination of states to attain parametric identification. When the state space increases and is continuous, the likelihood of observing multiple observations at each combination of states quickly becomes negligible. A related stream of research in the descriptive (non-structural) literature imputes discount factors from data wherein consumers face trade-offs between immediate returns/costs and future flows of returns/costs (e.g., the current capital cost of a more energy efficient car with its future gasoline costs). Examples of this approach include Allcott and Wozny (2011), Busse et al. (2011), Harrison et al. (2002), Warner and Pleeter (2001), Dubin and McFadden (1984), and Hausman (1979). As in the structural literature, it is typically necessary to invoke certain restrictions to identify the discount factor. For example, to quantify the future operating cost of a durable appliance, it is necessary to assume the consumer has perfect foresight about future prices of electricity. As a result, there is no uncertainty about the future state transition.

A third identification strategy is to rely on an exclusion restriction argument (Chung et al. (2010), Fang and Wang (2010), Lee (2010)). Exclusion restrictions involve specifying a set of exogenous variables that do not affect current utility but do affect state transitions. Accordingly, variation in these exogenous variables affects future utilities through their impact on the state transition but do not have an effect on current utility. By exploring how choices are made in light of changes in future utility when current utility remains fixed, the utility and the discount factor can potentially be identified together.[3] However, such exclusion restrictions are often unavailable in field data or are difficult to validate.

To alleviate these concerns, recent work by Dubé et al. (2010b) use experimental conjoint analysis data to identify dynamic model in the context of durable goods adoption. In particular, Dubé et al. (2010b) manipulate consumers' beliefs about state transitions by informing them alternative future market situations in the experiments. As a result, they are able to identify utility and discount factors. This approach is most similar to ours in that it uses data rather than invoking assumptions to infer discount rates. Though an important step forward, it is often difficult to replicate

[3]In a discrete choice context, Magnac and Thesmar (2002) specify an exclusion restriction wherein exogenous state variable(s) do not affect the "current value function" (defined as the difference between the expected values of (1) choosing a specific alternative followed by the outside option, and behaving optimally thereafter and (2) choosing the outside option now and in the next period, and behaving optimally thereafter (p.807, equation 8)). This exclusion restriction, coupled with the normalization of the future value function of the outside option to zero, enables identification of the *current value function* and the discount factor (Proposition 4, p.809). We thank Robin Lee for the insight regarding this identification strategy.

dynamic choices in lab settings owing to demand artifacts and contracted durations. For example, Dubé et al. (2010b) consider annual budget tradeoffs in a lab experiment that lasts one session. It would therefore be desirable to supplement this research using a field context with choices made in practice and over extended periods.

Following a similar logic, we advance the research in dynamic structural models by identifying discount factors using field data. Our identification strategy is to first identify consumers' utilities and the distribution of random consumption shocks using data that have no dynamics involved. Then we further recover their discount factor when the dynamic structure was exogeneously imposed. By using the static data to pin down inference regarding utility and the distribution of random shocks, the discount rate can then be inferred from the dynamic decision making context.

Our contributions are fourfold. First, we show that it is possible to recover discount rates by supplementing dynamic decision data with static decision data in cases where the decision variables are continuous and the objective function is concave. Second, we provide an empirical illustration wherein we use field data to exemplify this identification strategy. Third, we explore the potential for biased parameter estimates in this sample as a result of mis-specifying discount rates as well as the potentially suboptimal marketing decision making. Finally, our research also advances the empirical literature on the non-linear pricing of telephony or Internet services (e.g., Narayanan et al. 2007, Lambrecht 2006, Lambrecht et al. 2007, Iyengar et al. 2007, Grubb and Osborne 2011). Most previous empirical studies are based on aggregate usage data, limiting their ability to investigate consumers' intertemporal substitution in consumptions.[4] Since our data are at the disaggregate level, we are able to evaluate the tradeoff of consumptions across time and the corresponding managerial implications for the firm's pricing strategy.

2 Identification of Discount Factors with Static Data

In this section we formalize our logic that the discount factor can be identified when (1) the utility and random shock distribution are known, (2) the objective function is concave, and (3) the controls

[4]Grubb and Osborne (2011), who focus on plan choice under uncertainty and learning, is a notable exception. Our emphasis lies instead on the allocation of minutes used within month.

are continuous. We focus the discussion on the finite horizon case, which is consistent with the empirical application detailed below. In the Appendix we extend the logic of the identification strategy to the infinite horizon with continuous controls. We also discuss our conjecture about the case of discrete actions in the Appendix.

In this finite horizon case, at any given period of $t < T$, the optimization problem of consumption can be written as

$$\max_{x_t} u(x_t; s_t, \nu_t) + \sum_{k=t+1}^{T} \delta^{k-t} \mathbf{E} u(x_k; g(s_{k-1}, x_{k-1}), \nu_k) \tag{1}$$

where $u(\cdot)$ are the utility function (or profit function); $x.$ are continuous control variables (e.g., the minutes of cell phone consumption in our application); $s.$ are the state variable (the cumulative minutes consumed in our application); $\nu.$ are the random shock, the expectation is taken over the random shock; δ is the discount factor; and $g(\cdot, \cdot)$ is the state transition equation such that $s_{t+1} = g(s_t, x_t), \forall t$ (in our case, $g(s_t, x_t) = s_t + x_t$).

As discussed in Rust (1994) (p.3090), under weak regularity conditions, the optimal decision rule $\Sigma_1^* = \{\sigma_1^*, \sigma_{t+2}^*, ..., \sigma_T^*\}$ exists and can be computed using backward induction such that:

$$\begin{aligned}
\sigma_T^*(s_T, \nu_T) &= \arg\max_{x_T} u(x_T; s_T, \nu_T) \qquad (2)\\
\sigma_{T-1}^*(s_{T-1}, \nu_{T-1}; \Sigma_T^*) &= \arg\max_{x_{T-1}} u(x_{T-1}; s_{T-1}, \nu_{T-1}) + \delta \mathbf{E} u(\sigma_T^*; g(s_{T-1}, x_{T-1}), \nu_T)\\
&\vdots\\
\sigma_1^*(s_1, \nu_1; \Sigma_2^*) &= \arg\max_{x_1} u(x_1; s_1, \nu_1) + \sum_{k=2}^{T} \delta^{k-1} \mathbf{E} u(\sigma_k^*; g(s_{k-1}, x_{k-1}), \nu_k)
\end{aligned}$$

In particular, because the utility is concave in $x.$ and the linear combination of concave functions are also concave, backward induction leads to one unique optimal level of $x.$ for each period, starting with the last period. If we take the first order condition of equation 1 with respect to x_t,

$\forall t < T$, we have the Euler equation

$$u_x(x_t^*; s_t, \nu_t) + \delta \mathbf{E} u_g(x_{t+1}^*; g(s_t, x_t^*), \nu_{t+1}) = 0$$

where u_x and u_g are the partial derivatives with respect to x and g, respectively.

Since the Euler Equation is linear in δ, given that the utility and the distribution of ν are known, and $x_\cdot^*$ are the unique and observed optimal consumption levels, the discount factor δ can then be uniquely identified from the data. Below, we apply this identification strategy to show it is possible to recover the discount rates from a finite sample and to explore the implications of these rates for inference and managerial policy decisions.

3 Illustrative Application

3.1 Consumer Usage Data and Carrier Tariff Structure

The data used to illustrate the sampling properties of our identification strategy are supplied by a major mobile phone service provider in China, covering the period from September 2004 to January 2005. The data provider accounted for more than 70% of the market share of Chinese mobile phone service market during that time. Initially, this firm used only linear pricing schedules, i.e., consumers were billed on a pay-per-minute basis. In November 2004, the firm offered three-part tariff plans to a randomly selected set of consumers. The firm divided these consumers into multiple groups based on their past usage volumes. The firm then offered each group a respective three-part tariff plan. A consumer could choose the three-part tariff plan or remain on the original pay-per-minute plan.

3.1.1 Tariff Structure

Table 2 depicts the pricing structure of the most popular three-part tariff plans, covering 90% of the consumer base. Consumers who enroll in one of the listed plans are allowed a fixed number of free calling minutes in a given calendar month by paying the monthly access fee.

[Insert Table 2 about here.]

When a given consumer places or receives a call, the minutes of the phone call are deducted from the allowance and the consumer does not need to pay for that usage. However, when the monthly allowance is exhausted, the consumer is billed the marginal price for each minute of usage beyond the allowance. There is no "roll-over" for these plans, i.e., unused allowance minutes cannot be carried over to next month. At the beginning of next month, the consumer's allowance of free minutes is replenished after paying the new month's access fee. The consumers have different linear rates before the switch. The mean linear rate is 0.27 with a standard deviation of 0.09.

3.1.2 Usage Data

For the first four months (from September 2004 to December 2004), we observe each consumer's aggregate monthly minute usage and expenditures. However, in the last month (January 2005), we observe call level consumer records, including the starting time, duration, and expense of each phone call. The data also include some demographic information, including the age, gender, and zip code of each consumer.

Table 3 summarizes the consumers' average usage levels (normalized by their allowance level) and demographic information. The average usage under both the linear and three-part tariff plans are close.

[Insert Table 3 about here.]

3.1.3 Overage and Underage

Underage occurs when consumers do not use all the allowance at the end of a month. In this case, consumers are overpaying in the sense that they have been charged for minutes they do not use. In comparison, overage occurs when usage exceeds the allowance. In this case, consumers again overpay inasmuch as a plan with more allowance minutes normally has a lower average price per allowance minute (Iyengar et al. (2007)). As a result, consumers who strategically manage the minutes should evidence less underage or overage.

In Figure 1, we plot the histogram of the ratios of minutes used to minutes allowed for the last month of data. The average ratio is close to 1 (0.96) under the three-part tariff, suggesting that consumers on average tend to avoid overage or underage. Yet this average behavior belies a large standard deviation (0.35). Hence, we next consider whether and how users manage their minutes over the month to comport with the allowance; to the extent this behavior changes as the allowance becomes more salient, evidence is afforded for the strategic use of minutes.

[Insert Figure 1 about here.]

3.1.4 Strategic Minute Usage within a Month

We consider some model free evidence that consumers are strategic in their usage of allowance minutes. This evidence is predicated on the notion that minute consumption changes as the distance between minutes used and the allowance becomes small; in particular, consumers start to conserve minutes as the number of free minutes dwindles and the overage potential increases.

Dividing the last month of the data into five 6-day periods, $t = 1, ..., 5$,[5] we compute the ratio of cumulative minutes used to the allowance for each consumer at the end of each six-day period. Figure 2 portrays a scatter plot of this ratio and its lag value for each period $t = 2, ..., 5$. The line in this figure depicts a nonparametric function relating the ratio and its lag and the gray band indicates the 95% confidence interval for this function.[6] A key insight from this figure is that this function is concave when the cumulative usage is within quota (the ratio in period $t - 1$ is less than 1). In contrast, when usage exceeds quota (the ratio in period $t - 1$ is greater than 1), the line is almost linear. The concavity of the line pre-quota suggests that people decelerate usage as they approach the quota, that is, they start to ration their minutes to avoid overage. Moreover, those who are far from the quota appear to accelerate usage to avoid underage. In comparison, consumers who have already exceeded the quota do not decelerate (or accelerate) their usage. Instead, they follow some relatively stable usage rates, as might be expected were they no longer face an intertemporal tradeoff in usage. Misra and Nair (2009) and Chung et al. (2010) use similar methods to investigate

[5] To test the model's sensitivity to the definition of the 6-day period, we also consider the specifications of 10-day, 5-day, and 3-day periods. The implied weekly discount factors and price coefficients are statistically equivalent across these different specifications.

[6] We consider both spline and local regression methods. The results are similar. The figure presented shows the results from spline method.

the effect of quota on salesperson's allocation of efforts across time. They found analogous patterns of dynamic effort allocation in salesforce due to the existence of quota.

[Insert Figure 2 about here.]

To further elaborate upon these insights arising from Figure 2, we consider how usage acceleration (deceleration) changes as individuals approach their allowance/quota. This acceleration can be summarized by the statistic (Usage during period t)/(Usage during period $t-1$). This ratio is analogous to the slope of the line in Figure 2. When the ratio is one, consumers are neither decelerating or accelerating use. When the ratio is greater than one, usage is accelerating. When the ratio is less than one, usage is decelerating. We compute this ratio for each person in each period and then, in Figure 3, present a histogram of this minute acceleration measure across persons and periods conditioned on users distance to quota. For example, the upper leftmost histogram shows the distribution of usage acceleration observations conditioned upon consumer usage at time $t-1$ being less than 20% of their allowance. Figure 3 indicates that consumers' usage decelerates as they approach their allowance. Further, when the consumer reaches an overage situation (where there is no longer an intertemporal tradeoff in usage), the slopes average around 1, indicating a stable usage rate. These observations are consistent with Figure 2. Overall, we conclude that there exists some model free evidence of strategic behavior on the part of consumers.

[Insert Figure 3 about here.]

3.2 Model

3.2.1 Utility under the Linear Pricing Plan

In this section, we first specify the consumer utility for consumption under a linear pricing plan and derive the optimal level of consumption. We then extend the analysis to the case of the three-part tariff plan.

Similar to Lambrecht et al. (2007), we begin by assuming that consumer i derives utility from phone usages and the consumption of a composite outside product (numeraire). To be specific,

$$
\begin{aligned}
u_{it}(x_{it}, z_{it}) &= (\frac{d_{it}}{b}x_{it} - \frac{1}{2b}x_{it}^2) + z_{it}, \quad (3)\\
s.t. \quad & z_{it} = y_i - p_{i0}x_{it}, \quad (4)\\
& d_{it}, b > 0
\end{aligned}
$$

where $t = 1, 2, ..., T$ are the periods within a month;[7] x_{it} is the minutes of phone usage during period t; p_{i0} is the linear price rate of consumer i before switching to the three-part tariff; z_{it} is the consumption of the numeraire;[8] y_i is the income; d_{it}/b is the linear component of minute usage utility while $(-1/2b)$ is its quadratic effect.

Consumer i chooses the optimal levels of phone usage x_{it} and numeraire consumption z_{it} so as to maximize her total utility subject to the budget constraint. Substituting the budget constraint equation 4 into equation 3, the utility function can be rewritten as

$$u_{it}(x_{it}, z(x_{it})) = \frac{d_{it}x_{it}}{b} - \frac{x_{it}^2}{2b} + y_i - p_{i0}x_{it} \quad (5)$$

Solving the maximization problem of equation 5 yields the optimal consumption

$$x_{it}^* = \begin{cases} 0, & \text{if } d_{it} - bp_{i0} < 0 \\ d_{it} - bp_{i0}, & \text{if } d_{it} - bp_{i0} \geq 0 \end{cases} \quad (6)$$

The foregoing equation clarifies the interpretation of (1) b as the price sensitivity and (2) d_{it} as the baseline consumption level under the linear pricing plan as it represents a fixed shift in the demand

[7] We use t to index periods within a month and τ to index months.

[8] The budget constraint in equation 4 normalizes the price of the outside good to one, effectively turning it into the numeraire. The purpose of this normalization is twofold. First, it normalizes the marginal utility of income (or numeraire) to one. Because we do not observe consumer churning or variation in plan choices due to income effect in the data, the identification of the marginal utility of income is infeasible. Such a normalization treatment for the purpose of identification is similar to Narayanan et al. (2007) and Ascarza et al. (2010). Second, it transforms the consumption of minutes into a dollar metric in comparison to the numeraire, making its interpretation more meaningful.

curve as well as the minute consumption level when $p_{i0} = 0$ (Lambrecht et al., 2007). Following Narayanan et al. (2007) and Lambrecht et al. (2007), we further allow baseline consumption, d_{it}, to be affected by consumer characteristics, D_i, and a random shock ν_{it},

$$d_{it} = \exp(D_i^{'}\alpha) + \nu_{it} \tag{7}$$

where α is a vector of parameters and $\nu_{it} \sim N(0, \zeta^2)$ is exogeneously i.i.d. across consumers and periods.[9] Sources of the shock may include (1) technical problems with the consumer's phoneset or coverage which limit the phone usage; (2) unexpected events that require extra communications with others, and so on. Though the random shocks are unknown to the researchers, the consumer observes the shocks at the beginning of each period, *before* deciding her usage levels accordingly. Given all consumers in the dataset are experienced users, we assume the distribution of the shocks is known to the consumer.[10] Summing optimal period consumptions within the same month τ yields the optimal total minutes consumed within a month as

$$q_{i\tau} = \sum_{t'=1}^{T} x_{it'}^{*}$$

3.2.2 Utility under the Three-part Tariff Plan

The three-part tariff plan can be described as the triple $\{F, A, p\}$, where F is the fixed access fee, A is the allowance amount, and p is the marginal price after the consumer exhausts the allowance.

[9]The exponential function ensures that, on average $d_{it} > 0$. One related concern with the use of a normal distribution assumption for the random shocks is that the baseline demand, d_{it}, may become negative. One approach is to consider a truncated normal distribution. However this is computationally costly. Hence we instead assume that the magnitude of ν_{it} (standard deviation) is small compared to $\exp(D_i^{'}\alpha)$ so a normal distribution is a good approximation of a truncated normal distribution. This assumption is consistent with the estimation results.

[10]Though this assumption is not material for the static model because the error is revealed prior to the usage decision, the assumption becomes important under the context of the three-part tariff when future shocks become relevant to current period consumptions.

Period Utility and Budget Constraint At period t during a given month, consumer i has a utility level

$$
\begin{aligned}
u_{it}(x_{it}, z(x_{it}); s_{it}, \nu_{it}) = \frac{d_{it}x_{it}}{b} - \frac{x_{it}^2}{2b} + z_{it}(x_{it}), \qquad\qquad (8)\\
s.t.\ z_{it}(x_{it}) = y_i - C(x_{it}) \\
C(x_{it}) = \begin{cases} (\sum_{k=1}^{t-1} x_{ik} + x_{it} - A)pI_{\sum_{k=1}^{t-1} x_{ik}+x_{it}>A}, & \text{if } \sum_{k=1}^{t-1} x_{ik} < A \\ px_{it} & \text{if } \sum_{k=1}^{t-1} x_{ik} \geq A \end{cases} \\
d_{it} = \exp(D_i^{'}\alpha) + \nu_{it}
\end{aligned}
$$

where s_{it} is a vector containing state variables at period t that include (1) cumulative usage up to period $t-1$, $\sum_{k=1}^{t-1} x_{ik}$, and (2) period t (or the distance to the terminal period). Among these state variables, the cumulative usage is endogenous and the period t is exogenous. $I_{\sum_{k=1}^{t-1} x_{ik}+x_{it}>A}$ is an indicator, which takes the value of 1 if $\sum_{k=1}^{t-1} x_{ik} + x_{it} > A$ and 0 otherwise. Note that the fixed access fee F does not enter the period budget constraint since it is essentially a sunk cost. It does not affect the optimal decision at period t as long as the choice is not a corner solution.

Substituting the budget constraint to equation 8, we may rewrite the period utility as

$$u_{it}(x_{ijt}, y_i - C(x_{it}); s_{it}, \nu_{it}) = (\frac{d_{it}x_{it}}{b} - \frac{x_{it}^2}{2b}) + y_i - C(x_{it}) \qquad (9)$$

Similarly to the period utility under the linear pricing plan, we assume that d_{it} is affected by the random shock ν_{it} and $\nu_{it} \sim N(0, \zeta^2)$. ν_{it} is observed by consumer i at the beginning of period t, *before* making the decision of minute consumption.

Total Discounted Utility As a consumer's current minute consumption may affect her future marginal price, the consumer aims to maximize her total discounted utility by optimizing her consumption over time. In particular, the total discounted utility at period $t \leq T-1$ can be presented as

$$U_{it}(u_{it}, u_{i(t+1)}, ..., u_{iT}) \equiv \mathbf{E}(u_{it} + \sum_{k=1}^{T-t} \delta^k u_{i(t+k)})$$

where $\delta \in [0, 1]$, representing the discount factor.[11]

We model the consumer's minute usage decision as the dynamic optimization problem of a Markov Decision Process (MDP) such that the decision rule of minute usage of period t only depends on the then-current state vector s_{it} (Rust (1994)) and the random shock ν_{it}. To facilitate the exposition, we first define $x_{it} = \sigma_{it} \equiv \sigma_{it}(s_{it}, \nu_{it})$ as the decision rule of consumer i at period t, depending on the state variables s_{it} and random shock. We also define $\Sigma_{it} \equiv (\sigma_{it}, \sigma_{i(t+1)}, ..., \sigma_{iT})$ as a decision rule profile for this MDP from period t onwards; this profile includes a set of decision rules that dictate current and future consumptions. Also denote $V_{it}(s_{it}; \Sigma_{it})$ as the expected continuation utility at period t conditioned on s_{it} and Σ_{it}. Because of the finite horizon of this MDP, V_{it} can be defined recursively as follows:

The expected utility of the terminal period T for a given s_{iT} is

$$V_{iT}(s_{iT}; \Sigma_{iT}) \equiv \mathbf{E} u_{iT}(\sigma_{iT}, y_i - C(\sigma_{iT}); s_{iT}, \nu_{iT}) \tag{10}$$

$$\text{where } C(\sigma_{iT}) = \begin{cases} (\sum_{k=1}^{T-1} x_{ik} + \sigma_{iT} - A) p I_{\sum_{k=1}^{T-1} x_{ik} + \sigma_{iT} > A}, & \text{if } \sum_{k=1}^{T-1} x_{ik} < A \\ p\sigma_{iT} & \text{if } \sum_{k=1}^{T-1} x_{ik} \geq A \end{cases} \tag{11}$$

where the expectation is taken over the random shock ν_{iT}.

Then the continuation utility function V_{it} at period $t < T$ can be written recursively as

$$V_{it}(s_{it}; \Sigma_{it}) = \mathbf{E} u_{it}(\sigma_{it}, y_i - C(\sigma_{it}); s_{it}, \nu_{it}) + \delta[V_{i(t+1)}(s_{i(t+1)}; \Sigma_{i(t+1)}) | s_{it}, \sigma_{it}] \tag{12}$$

$$\text{where } C(\sigma_{it}) = \begin{cases} (\sum_{k=1}^{t-1} x_{ik} + \sigma_{it} - A) p I_{\sum_{k=1}^{t-1} x_{ik} + \sigma_{it} > A}, & \text{if } \sum_{k=1}^{t-1} x_{ik} < A \\ p\sigma_{it}, & \text{if } \sum_{k=1}^{t-1} x_{ik} \geq A \end{cases} \tag{13}$$

[11] We also estimate a model with an additional hyperbolic discount factor. We do not find strong support for the existence of hyperbolic discounting in our context. Such a result is consistent with Chevalier and Goolsbee (2005) and Dubé et al. (2010b). Although time-inconsistent preferences and hence hyperbolic discounting exist (Angeletos et al., 2001), they may not be universal in all contexts.

where the expectation is taken over the random shock ν_{it}. Further, given s_{it}, ν_{it} and $\sigma_{it}(s_{it}, \nu_{it})$, the state transition $\pi(s_{i(t+1)}|s_{it}, \sigma_{it})$ is deterministic such that $\sum_{k=1}^{t} x_{ik} = \sum_{k=1}^{t-1} x_{ik} + \sigma_{it}$, and the consumer is one period closer to the terminal period T.

We further recursively define the optimal decision rule profile $\Sigma_{it}^* \equiv (\sigma_{it}^*, \sigma_{i(t+1)}^*, ..., \sigma_{iT}^*)$, starting with the terminal period:

$$\sigma_{iT}^* = \arg\max_{\sigma_{iT}} u_{iT}(\sigma_{iT}, y_i - C(\sigma_{iT}); s_{iT}, \nu_{iT}) \tag{14}$$

and optimal decision rule of period $t < T$ is defined recursively as

$$\sigma_{it}^* = \arg\max_{\sigma_{it}} u_{it}(\sigma_{it}, y_i - C(\sigma_{it}); s_{it}, \nu_{it}) + \delta[V_{i(t+1)}(s_{i(t+1)}; \Sigma_{i(t+1)}^*)|s_{it}, \sigma_{it}] \tag{15}$$

4 Estimation

4.1 Static Decisions

Consumers make static consumption decisions under (1) the linear pricing plans and (2) the terminal period of the three-part tariff plan. We discuss the likelihood function separately for each case.

4.1.1 Minutes Usage under the Linear Pricing Plans

For a given month τ under the linear pricing plans, we observe consumer i's characteristics D_i. In our specific application, D_i includes (1) age (2) the consumer's tenure with the firm (3) gender and (4) whether the consumer lives in a rural or urban area.

We also observe individual consumer monthly aggregate usage $q_{i\tau} = \sum_t x_{it}^*$.[12] As shown in Appendix, while there is no closed form for the distribution of $q_{i\tau}$, the distribution can be approximated by a normal distribution. As a result, we can write down the likelihood function of

[12]Note that for the linear pricing plans, we only observe $q_{i\tau}$ but not the individual x_{it}^*'s.

consumer i for the minutes usage under linear pricing plans.

$$L_{i \cdot Linear} = \prod_{\tau} \widetilde{f}(q_{i\tau}|\Omega) \tag{16}$$

where $\widetilde{f}(\cdot)$ is the approximation density function of $q_{i\tau}$ detailed in the Appendix; $\Omega \equiv \{\alpha, b, \zeta\}$, i.e., the utility parameters and the distribution of random shocks.

4.1.2 Minutes Usage in Terminal Period T under the Three-part Tariff

In the terminal period T, the consumption becomes a static decision given the allowance will be reset next month. Hence we may solve the optimal minute consumption strategy σ^*_{iT} such that

$$\sigma^*_{iT} = \begin{cases} d_{iT} - bp, & \text{if } \sum_{t=1}^{T-1} x_{it} + d_{iT} - bp > A \\ d_{iT}, & \text{if } \sum_{t=1}^{T-1} x_{it} + d_{iT} < A \\ A - \sum_{t=1}^{T-1} x_{it}, & \text{if } \sum_{t=1}^{T-1} x_{it} + d_{iT} - bp \leq A \leq \sum_{t=1}^{T-1} x_{it} + d_{iT} \end{cases} \tag{17}$$

The first component of equation 17 accounts for the situation under which the consumer faces a positive marginal price after her cumulative usage exceeds the allowance. The second component represents the situation when the consumer's cumulative usage is less than the allowance and the marginal price is zero. The third component represents the situation when the cumulative usage under the optimal σ^*_{iT} exceeds the allowance at a zero marginal price but falls below the allowance with a positive marginal price. We follow Lambrecht et al. (2007) and set the optimal usage under such a situation at a mass point $\sigma^*_{iT} = A - \sum_{t=1}^{T-1} x_{it}$.

According to equation 17, the density of each observed consumption level x_{iT} conditioned on σ^*_{iT} can be written as

$$f_T(x_{iT}|\sigma^*_{iT},\Omega) = \begin{cases} f(x_{iT} = d_{it} - bp) & \text{if } \sum_{t=1}^{T} x_{it} > A \\ f(x_{iT} = d_{iT}) & \text{if } \sum_{t=1}^{T} x_{it} < A \\ \Pr(\sum_{t=1}^{T-1} x_{it} + d_{iT} - bp \leq A \leq \sum_{t=1}^{T-1} x_{it} + d_{iT}) & \text{if } \sum_{t=1}^{T} x_{it} = A \end{cases} \quad (18)$$

and the likelihood for consumer i in the terminal period T is

$$L_{i \cdot Terminal} = f_T(x_{iT}|\sigma^*_{iT},\Omega) \quad (19)$$

4.2 Dynamic Decisions

Under the three-part tariff, decisions become dynamic for periods $t < T$. As the result of the dynamic nature of the consumption decision, the discount factor δ enters the data generating process and hence the estimation. Since there is no closed form solution to the optimal decision rule Σ^*_{it} in the dynamic decision context, a likelihood function based on observed x_{it} and Σ^*_{it} becomes infeasible. Instead, we implement a numerical approximation method to establish a simulated likelihood function for estimation. This approximation method contains two steps: (1) using Gauss-Hermite quadrature method to approximate the value function V_{it} at a grid of state points and interpolating V_{it} at the remaining state points using regression; (2) simulating the density for each observed x_{it} using V_{it} from the previous step. We elaborate each step below.

4.2.1 Approximating and Interpolating $V_{it}(s_{it};\Sigma^*_{it})$

Using backward recursion and simulation, it is possible to numerically evaluate the value function under the optimal strategy Σ^*_{it} specified in equations 15 and 17. To be specific, starting with the terminal period T:

1. Pick $ns = 250$ random draws of state points $\sum_{k=1}^{T-1} x_{ik}$ to construct a grid, i.e., the cumulative minute usage at the beginning of period T.

2. For each grid point that we draw, conditioned on Ω, the continuation value function $V_{iT}(s_{iT})$ is approximated as $\widetilde{V}_{iT}(s_{iT})$ using Gauss-Hermite quadrature method with $nr = 15$ nodes (Judd (1998)); on each of the nodes, the optimal minute consumption level $x^*_{iT}(s_{iT}, \nu_{iT})$ is calculated using equation 17.

3. For state points that are not drawn, based on the value functions obtained on the grid from last step, we use a spline interpolation to approximate their values.

Then for period $t < T$, we have the following backward recursion steps:

4. Pick $ns = 250$ random draws of state points $\sum_{k=1}^{t-1} x_{ik}$ to construct a grid, i.e., the cumulative minute usage at the beginning of period t.

5. For each grid point, conditioned on Ω, δ and the $\widetilde{V}_{i(t+1)}$, the continuation value function $V_{it}(s_{it})$ can be approximated as $\widetilde{V}_{it}(s_{it})$ using Gauss-Hermite quadrature method with $nr = 15$ nodes; on each of the nodes, the optimal minute consumption level $x^*_{it}(s_{it}, \nu_{it})$ is calculated using the following equation

$$x^*_{it}(s_{it}, \nu_{it}) = \arg\max_{x_{it}} u_{it}(x_{it}, y_i - C(x_{it}); s_{it}, \nu_{it}) + \delta \widetilde{V}_{i(t+1)}(s_{i(t+1)} | s_{it}, x_{it})$$

6. For state points that are not drawn, based on the continuation functions obtained on the grid from last step, we use a spline interpolation to approximate their values.

4.2.2 Simulating the Density of Observed x_{it}, $\widetilde{f}_{it}(x_{it} | s_{it}, \Omega, \delta)$

For each x_{it} observed in the data and its corresponding state point s_{it}, we use the following steps to simulate its density:

1. First draw $nr_{density} = 100$ random shocks ν_{it};

2. For each random draw of ν_{it} and the observed s_{it}, calculate the optimal minute consumption by solving the following equation.

$$x_{it}^*(s_{it}, \nu_{it}) = \arg\max_{x_{it}} u_{it}(x_{it}, y_i - C(x_{it}); s_{it}, \nu_{it}) + \delta \widetilde{V}_{i(t+1)}(s_{i(t+1)}|s_{it}, x_{it})$$

3. Using the calculated $nr_{density} = 100$ optimal $x_{it}^*(s_{it}, \nu_{it})$'s, simulate $\widetilde{f}_{it}(\cdot)$, the density of the observed x_{it}, using Gaussian kernel density estimator.

With the simulated densities for all observed x_{it}, we are able to write a likelihood function for consumer i such that

$$L_i = \prod_t \widetilde{f}_{it}(x_{it}|s_{it}, \Omega, \delta)$$

We use MLE to estimate the parameters. The total likelihood function is

$$L = \prod_i L_i \cdot L_{i \cdot Terminal} \cdot L_{i \cdot Linear}$$

4.3 Inference of the Utility Parameters

In this section we provide an informal discussion of the inference of the utility parameters; Appendix B outlines a more detailed discussion regarding the identification of the discount factor.

Parameters that construct our model can be categorized into two sets. The first set of parameters appear under both the linear pricing plan and the three-part tariff plan, including $\Omega \equiv \{\alpha, b, \zeta\}$, i.e., the utility parameters and the distribution of random shocks. The second set of parameters, δ, only affect the demand under the dynamic setting. In essence, the parameters $\Omega \equiv \{\alpha, b, \zeta\}$ are identified from choices under the linear plan and the terminal period of the three-part tariff, where there are no dynamics involved. Conditioned on Ω, we then recover the discount factor δ.

The consumption decisions under the linear plans and the terminal period of the three-part tariff have no dynamics involved. Besides individual consumption across time, we further observe the following information under the linear plan and the terminal period of the three-part tariff.

- Different linear prices across individual consumers.
- Depending on whether a consumer has exhausted her allowance at the beginning of the terminal period, there is variation in marginal prices across individuals.
- Variation in demographic characteristics across individuals.

The variation in consumption across and within individuals over time, conditioned on the variation in prices and demographics, enable us to identify α and b. Together, α, b, prices and demographics determine the mean levels of consumptions of each individual over time. The observed deviations from these mean levels across individuals and time identify ζ, the standard deviation of the random shocks ν_{it}'s.

5 Monte Carlo Simulation

To evidence the identification approach detailed in section 2 and show it is possible to recover discount rates from sample data, we employ a Monte Carlo simulation. Using the same consumption model as in our empirical illustration, we simulate three data sets with 50, 75, and 100 customers, respectively. To be consistent with the empirical application, in this simulation, each customer makes consumption decisions over five periods. We set the satiation level parameter $d = 100$, the price sensitivity $b = 1$, the standard deviation of the random shock $\zeta = 0.5$, and the discount factor $\delta = 0.9$. Each dataset has both static data and dynamic data.

The results are reported in Table 4, Table 5, and Table 6. From the results, we observe the following:

- Estimating utility with static and dynamic data (Column 1). Using the complete data, estimates of the discount factor and the preference coefficients are statistically indistinguishable from their true values, well within the 95% confidence interval of the true values.

- Estimating preference parameters using only static data (Column 2). Using only the terminal period of three-part tariff and the linear pricing data, we assess whether preferences can be identified without dynamic usage data. Results indicate that the preference estimates are statistically indistinguishable from their true values, suggesting the static data are sufficient to infer utility.

- Estimating the preference parameters and discount factors using a two-step approach (Column 3). In this analysis we illustrate the identification strategy using a two-step approach. Specifically, we estimate the preference parameters using linear pricing data and the last period of three-part tariff (as in the preceding step). Conditioned on the preference estimates, in the second step we then estimate the discount factor using the three-part tariff data excluding the last period. The preference parameters and discount factor in this two-step approach are statistically equivalent to those estimated jointly in one single step (Column 1). However, the one-step approach is more efficient than two-step approach (Column 3).

- Estimating the preference parameters and discount rates using only a portion of the static data. Building upon the insights obtained from the last two steps, this analysis further clarifies the role of static data in identification. Specifically, we estimate the model by pooling static data and dynamic data together. However, for the static data, we either use i) the linear pricing data or ii) the last period of the three-part tariff data (as opposed to using both). The results are listed in Column 4 and Column 5. We find that the preference and discount factor in both cases are statistically equivalent to those estimated with all static data. This finding suggests the discount factor can be recovered when there are at least some static data to augment the dynamic data. With more data, the estimate of the discount factor becomes more efficient (Column 4 and 5 vs. Column 1 and 2).

- In cases where the preference and discount factor parameter are identified, more observations under both static and dynamic settings lead to more efficient estimates (Column 1 through 5).

Moreover, pooling static and dynamic data yields more precise estimates for utility (Column 2 vs. Column 3).

- The model cannot recover the discount factor when no static data are available (Column 6).
  - One of the simulations does not converge (sample size=75, Table 5).
  - When the model converges, the price coefficient and discount factor are insignificant, the satiation level and random shock have greater sampling variance relative to the identified models.
  - In contrast to the case where the discount factor is identified, more data cannot improve the efficiency of estimates.
- To assess potential parameter biases when the discount factor is misspecified to be 0.995, we re-estimate our model using this rate on the simulated data for $N = 100$. The satiation level d and random shock standard deviation ζ are still statistically indistinguishable from the true value ($\hat{d} = 99.33, s.e.(\hat{d}) = 0.33$; $\hat{\zeta} = 0.47, s.e.(\hat{\zeta}) = 0.02$). However, the price parameter b is underestimated ($\hat{b} = 0.53, s.e. = 0.18$). The intuition behind the bias is as follows. The higher discount factor implies consumers excessively substitute future consumption for current within allowance consumption. Given future over-allowance consumption is costly, the model compensates by lowering price sensitivity to generate the same level of overall utility for the future consumption occasion. Consequently, the smaller price coefficients imply that the overage charge has less impact on future utility; as a result there is no need for the consumer to make the tradeoff between consumptions across time.

In conclusion, the simulations exemplify the validity of the proposed identification strategy and the potential for misspecification of the discount rate to induce parameter bias.

[Insert Table 4 about here.]

[Insert Table 5 about here.]

[Insert Table 6 about here.]

6 Results

In this section we detail the results of our model estimation in the field context described in Section 3. Whereas the foregoing simulation strategy illustrates the validity of our identification strategy and its ability to uncover the discount factor, the field application is useful in showing that i) we can estimate discount factors in an actual field setting and ii) the estimated rate is different than commonly assumed in our context. Moreover, as we discuss in Section 7 this difference can have a material consequence for the inference of the utility parameters and that these biases in utility estimates affect implied policy decisions of firms.

Our sample includes the 284 consumers (50% of the observations) who select the three-part tariff plan with a monthly access fee of 98RMB (about $14) an allowance of 450 minutes, and a marginal price of 0.4RMB (See Table 2). We first report the results of our base model and then explore how these estimates are affected by the availability of static data.[13]

As robustness checks, we conclude this section by considering i) alternative specifications to our continuous specification of heterogeneity, and ii) an alternative model of pricing dynamics wherein consumers account for minute usage when billed as opposed to when used.

6.1 Parameter Estimates

Table 7 reports the parameter estimates. Considering the utility estimates first, our price estimates imply that a user would cut monthly consumption by about 11 minutes for each unit price increase. We also find that rural consumers value the minute consumption marginally higher than urban residents, probably due to their relatively limited access to alternative communication methods such as landlines and the Internet. We also find that female consumers have a higher baseline consumption rate than male consumers.

[Insert Table 7 about here.]

[13] A separate analysis using 83 consumers (access fee 168RMB, allowance 800 minutes) suggests our key results are not an artifact of the sample selected. See Table 9 for additional details.

Turning to the estimate for discount factor, the estimated weekly discount factor is 0.9 with a 95% confidence interval of (0.78, 0.97) .[14] The corresponding weekly interest rate is 1.11 with a 95% confidence interval of (1.03,1.28). This discount factor is statistically lower than those typically assumed in structural modeling studies (mean=0.983, see Table 1). Placing this result in perspective, our estimates indicate a consumer values an immediate one minute phone call at the start of the month the same as a future call of one minute and 34 seconds at the end of the month. In contrast, a weekly discount factor of 0.995 implies that a consumer values a one-minute phone call at the beginning of the month to be equal to a one-minute and one additional second at the end of the month (under the assumption of a constant pricing rate). The implication in our model is that consumers would be reticent to make the latter minutes trade, but willing to make the former.[15]

6.2 The Effect of Static Data on Inference

As in the case of the simulation, we consider how static data affect inference in our sample of field data. Overall, the findings are consistent with those in our simulation, namely that it is hard to infer discount factors without static data and that more information improves the efficiency of our estimates. Specifically, we consider the following sequence of sensitivity checks and Table 8 and the corresponding results. As is evidenced by the table, our findings are consistent with the findings from the simulation in Section 5.

In particular, we note that the model cannot recover the discount factor when no static data are available (column 5). Reflective of identification issues, when estimating the model on three-part tariff data without the last period or linear data, the estimation algorithm takes a much longer time to converge even though less data are used in estimation (about 2.5 times than the pooling case). More important, the discount factor estimate has a much larger variance (Column 5 in Table 8). The discount factor takes the median of 0.95 and a standard error of 0.52. We implement two LR

[14] We reparametrize the discount factors as $\delta = \exp(\pi)/(1+\exp(\pi))$ during estimation.

[15] This estimate corresponds to a high annual interest rate of 22640%. Consumers evidence high levels of impatience in some settings. Warner and Pleeter (2001) found that annual discount factors vary between 0 and 0.58 (annual interest rate$>$ (1×10^{14})%) in a field study by observing how agents trade off current and future payoffs. In experimental settings, researchers also find discount factors may approach zero (e.g., Kirby and Marakovic (1995), Petry and Casarella (1999), Madden et al. (1997), etc.). For a more detailed discussion, see Frederick et al. (2002).

tests for H0: δ=0 and H0: δ=1, respectively. In both tests, we cannot reject the null hypotheses. That is, we cannot determine the discount factor without the static data.

[Insert Table 8 about here.]

6.3 Robustness Checks

In this section we consider several robustness checks to assess how inference regarding the discount rate in our sample might be affected by the interval used for aggregating the data, our assumptions about heterogeneity and the timing of the payment.

6.3.1 Data Aggregation

We consider the potential effect of temporal aggregation of data on our model estimates. This robustness check demonstrates the effect of data aggregation on the identification of discount rates and preferences (10 day, 5 day and 3 day). As the 5 day and 3 day results are similar to the results reported in this paper, we focus upon the 10 day results. As indicated in Columns 6 and 7 of Table 8, we can not recover preferences and discount rates using the 10 day intervals unless we use both the static and dynamic data. These results suggest that (1) data aggregation, via reducing the number of observations and the concavity evidenced in Figure 2 (from which we infer intertemporal tradeoffs in demand), leads to a decrease in the efficiency of the parameter estimates, (2) this problem can be addressed to some degree via the use of the static data.

6.3.2 Heterogeneity

As another robustness check, we explore the effect of our parametric assumption regarding consumer utility on the inferred discount factor. To do this, we estimate the model consumer by consumer (by combining static and dynamic data), thereby allowing all users to have different preferences and discount factors. We find the median discount factor across consumers to be 0.87 (sd=0.26), similar to the level estimated under the homogenous model.[16] We also consider a la-

[16]Recognizing the potential for small sample problems arising from ML estimation in our panel (we have limited observations per customer for inference), we use simulated data to explore this potential. Specifically, we generate data using the parameters estimated from our homogeneous model. In one case we generate 1000 static observations per consumer. In another case, we use three static observations (as observed in the data) to supplement the dynamic

tent class mixture model. This analysis yielded two segments. The first comprised of 85% of the consumers, evidenced a discount rate of 0.89 (s.e.=0.04) while the smaller segment rate was estimated to be 0.92 (s.e.=0.05). Overall, it appears that our discount factor estimates are robust to our specification of heterogeneity.

6.3.3 Payment Timing

Note that Equation 5 and Equation 9 presume consumers account for minutes at the time they are transacted as opposed to when they are billed. To explore this assumption, we consider some model-free tests. Specifically, when the "mental accounting" of the expense is concurrent with the billing date and the pricing plan is linear, the usage decision now becomes dynamic in the early periods of the month. This is because the consumption x_{it} in period t will affect the total payment in period T, but not the period in which the usage occurs. This implies a downward trend in optimal consumption within the month as future payments are discounted more steeply in earlier periods. If the payment only materializes in period T, the utilities in periods t and T become

$$\begin{aligned} u_{it} &= \frac{d_{it}x_{it}}{b} - \frac{x_{it}^2}{2b} + y_i \\ u_{iT} &= \frac{d_{iT}x_{iT}}{b} - \frac{x_{iT}^2}{2b} + y_i - p_{i0}\sum_{k=1}^{T} x_{ik} \end{aligned}$$

The optimization problem of period t is

$$x_{it}^* = \arg\max_{x_{it}} u_{it} + \delta^{T-t}\mathbf{E}_d u_{iT}$$

data. We then proceed to estimate our model consumer by consumer for both simulated data sets. The estimates from the larger sample are statistically equivalent to those in the paper (price median=2.17, sd=0.10; discount median=0.91, sd=0.11). We further find a small degree of small sample bias for the three-observation case. The price coefficients are slightly overestimated (median=2.99, sd=0.79) and the discount factors are slightly underestimated (median=0.84, sd=0.28); but they are both still statistically equivalent to those in the paper (2.15 and 0.90). The simulation suggests that the small bias is not sufficient to change our conclusion that the static data enable identification of discount rates. This is consistent with our Monte Carlo simulation results.

Taking first order condition to solve x^*_{it}, it can be shown that

$$x^*_{it} = d_{it} - b\delta^{T-t}p_{i0}$$

This implies that on average the minute usage should drop over time were the payment only materialized in the utility function of period T. This observation suggests some model-free tests of whether consumers budget for minutes at the point of consumption or at the payment:

- Model Free Evidence I: Though our data for the first couple of months do not include call-by-call observations under the linear pricing plan, sales agents contacted consumers in month 3 to offer the three-part tariff option. The timing of the call for each consumer was random and the consumers had no foresight of the call. Importantly for our immediate concern, data were collected regarding the total minute consumption under the linear plan prior to the offer. These data enable us to calculate average weekly minute consumption across consumers up to the week of the switch. If the consumers account for payments at the terminal period T, usage rates over time should decrease on average. Figure 5 is a box-plot that depicts the averages of weekly usages of consumers who switched during the same week. It indicates no downward trend in consumption. This pattern is inconsistent with a mental accounting of expense at the time of billing.

- Model Free Evidence II: In Figure 3 subfigure 6, when consumers exceed the quota, the three-part tariff becomes a linear plan. If there is a downward trend in consumption, we should likewise see the histogram skewed towards 0, reflective of usage deceleration. However, according to Figure 3, the consumption rate is centered around 1, which is also inconsistent with the mental accounting of expense at the time of billing.

- Model Comparison: In addition to these model-free tests, we further consider a third test based on the three-part tariff data. We estimate a dynamic model similar to the base model but with the payment only entering in period T. Results indicate that the price sensitivity is not significant and the discount factor, though similar to the estimate in the paper, has a very large standard error. Model fit deteriorates as measured by BIC (9470.12 vs 9511.30) and Mean Absolute Percentage Error (0.14 vs. 0.23).

Accordingly, the data seem to be more consistent with the "mental accounting" of payment at time of consumption than time of billing.

6.3.4 Selection

Our data include only consumers who switch price schedules. Though the static and dynamic data are sufficient to identify the preference and discounting as shown in the Monte Carlo simulation, to the extent a consumer's adoption decision could be endogenous, the estimates in this field application may not be generalized to the overall population due to potential selection bias.

To explore the potential selection effects, we estimate the model separately for consumers who chose two different three-part tariff plans (450 minutes plan vs. 800 minutes plan). Table 9 presents the results.

[Insert Table 9 about here.]

Of note, the discount factors for consumers selecting each plan are statistically similar (0.90 vs. 0.88). In contrast, the preference estimates are different. In particular, those who chose the 450-minute plan have a lower baseline consumption (a smaller constant estimate) and lower price sensitivity. This is intuitive because (1) consumers with lower baseline consumption needs are more likely to enroll in a lower allowance plan, and (2) consumers with lower price sensitivity are more likely to enroll in a lower allowance plan since they care less about overage charges.

This robustness check merely shows that the estimates are conditional on a set of consumers that self-select into a pricing scheme that induces a forward-looking incentive (as in Goettler and Clay (2010)). On the other hand, however, one key implication of this exercise is that the findings are consistent with the conjecture that selection plays a greater role in consumption utility than discounting.

7 Managerial Implications

The dynamics of consumer behavior may have substantial impact on firm strategic decisions and revenues. In this section, we build on the broad literature pertaining to plan choice over months for telephony or Internet services (e.g., Narayanan et al. 2007, Lambrecht 2006, Lambrecht et al. 2007, Iyengar et al. 2007, Grubb and Osborne 2011) to explore i) the implications of within month pricing policy on carrier revenue and ii) how discount rates affect these policies.

7.1 Usage Prediction and Intertemporal Substitution Pattern

7.1.1 Biased Price Effects

To assess the potential bias in model estimates arising from specifying the commonly employed discount factor of 0.995 rather than using the estimated discount factor, we re-estimate the model by fixing the discount factor to 0.995. While there is little impact on most estimates, we find the price coefficients to be underestimated (have smaller absolute magnitudes) by 16%. The price coefficient becomes 1.81 (vs. 2.15 in Table 7), with the standard error of 0.03. This underestimation of price effects is consistent with the findings of the Monte Carlo simulation in Section 5.

7.1.2 Biased Forecasts

To ascertain how well the model fits the data and resulting intertemporal substitution pattern, we calculate the in-sample mean absolute percentage error (MAPE), BIC, and mean percentage error (MPE) across time under both the estimated discount rate and under the assumed weekly discount factor of 0.995. The MAPE measures a model's overall accuracy of fitting the data while the MPE indicates bias in model predictions. Table 10 and Table 11 depict the results.

[Insert Table 10 about here.]

According to Table 10, the fit under the 0.995 discount factor is universally worse than the fit under the estimated coefficients across time as measured by both MAPE and BIC. To develop a better sense of why the higher discount factor performs more poorly, we next turn to the MPE.

[Insert Table 11 about here.]

Based on Table 11, there is no obvious forecasting bias from using the higher discount rate when summing across all periods, yet aggregating across time obscures the patterns in intertemporal substitution. When setting the discount factor at 0.995, the demand in the first 4 periods is under-estimated; and the demand in the last period is over-estimated. This bias occurs because that consumers are more impatient than what $\delta = 0.995$ implies. As a result, in the early periods, when the allowance has not been exhausted, impatient consumers consume more than predicted under the 0.995 discount factor. Further, consumers are more price sensitive than the 0.995 discount factor case implies (recall that the price coefficient is underestimated under the 0.995 discount factor). As a result, consumers in overage (roughly coincident with the last period) evidence lower consumption than predicted under the 0.995 discount factor.

Although the difference between the estimated discount factor (mean 0.90 with a 95% confidence interval of (0.78, 0.97)) and 0.995 seems negligible, the effect on the forecast results are highly significant. This is because the joint distributions of all of the coefficients differ considerably under the two scenarios. The inflated discount factor 0.995 also causes biased estimates of preference, especially downward biased price sensitivity (2.15 vs. 1.81).

7.2 Elasticities

To ascertain how consumers' minutes usage varies under alternative allowance and the marginal price levels, we compute their monthly minutes demand changes for both the estimated discount factor and 0.995. Table 12 presents the results.

[Insert Table 12 about here.]

The elasticities in Table 12 suggest that the 0.995 discount factor leads to an underestimation of the effect of allowances and price on usage (that is, users are not as price sensitive as it implies when the discount factor is set to 0.995). Were consumers to actually have a discount factor of 0.995, they would be more forward looking than they were under the actual discount rates we estimate. As a result, the more forward looking consumers implied by 0.995 should conserve minutes so as not to pay overage in later periods. Because they do not actually conserve minutes, the model with a 0.995 discount factor needs to rationalize the observed overage. It does so by estimating a relative lower sensitivity to price and allowance; a lower price and allowance sensitivity means that consumers do not mind paying overage as much and have lower elasticities.

7.3 Alternative Pricing Schedule

Based on our communication with the data provider, their process of picking the three-part tariffs in this field study is ad hoc. There was no price optimization consideration. As a result, the focal three-part tariff is not likely to be optimal for the firm in terms of maximizing its revenue. To access the potential for revenue improvement, we create a grid of alternative allowance and price levels. For each combination of allowance and price, we calculate the percentage of revenue change at the end of the month.[17] Table 13 reports the results.[18]

[Insert Table 13 about here.]

Table 13 includes the current plan (allowance=450 minutes, price=RMB0.40). Surrounding the current plan, each column from left to right represents a 2-cent change in the marginal price for minutes in overage and each row from top to bottom stands for a 25-minute change in the allowance. A lower allowance enhances the possibility of overage; and a moderately decreased price tends to increase the consumption level under the overage situation. The optimal combination of allowance and marginal price is 400 minutes and 0.36RMB. The revenue of the firm would increase by 1.11% under this alternative pricing schedule. To the extent that similar exercises can be implemented across all groups of consumers, the revenue increase would be considerable.

[17] Note that we do not model plan choice since there are no plan choice data available. To ensure that the presented price/allowance changes do not lead to consumers leaving the company or opt to a different plan, we calculate consumer welfare for each point on the grid as measured by consumers' total discounted utility. We then compare it with the welfare level under the original plan. None of the welfare changes is significantly different from zero, hence we do not believe that the recommended policies will result in substantial plan switching even though these changes benefit the firm. Further, the company had a significant market share and there was no cellphone number portability in China until October, 2010 (ChinaTechNews.com, 2010). Consequently, we conclude (1) that the new price structures in the Table are unlikely causing large consumer churning and plan switching, and (2) that the effect of competitive response is likely to be modest.

[18] To account for sampling error when computing the revenue changes, we sample 100 sets of values from the estimates distribution. We then calculate the revenues using these sampled estimates. The table presents the median values.

As shown earlier, under the discount factor of 0.995, the model may lead to biased estimates of coefficients and elasticities. To see whether such biases may lead to inaccurate policy recommendations, we re-create the same grid but calculate the revenue changes using the estimates under the 0.995 discount factor. Table 14 reports the results. As indicated in the Table, with the 0.995 discount factor, the model generates notably different pricing plan recommendations. Since the assumed discount factor is much higher, to enhance consumers' likelihood of overage, the allowance levels would be much lower. As a result, under the 0.995 discount factor, the optimal allowance level becomes 325 instead of 400. Further, since the price sensitivities are underestimated, the optimal price would be higher. This effect leads to the optimal price changes from 0.36 to 0.38. In short, the firm sets its allowance too low and its marginal price somewhat high, thereby overcharging its consumers when using the standard practice of setting discount rates.

[Insert Table 14 about here.]

The predicted revenue gains also significantly differ between the two scenarios, $\delta = 0.90$ vs. $\delta = 0.995$. For each grid point we calculate the predicted revenue difference between the two scenarios. As shown in Figure 4, the percentage differences can be substantial. By implementing the pricing plans as suggested by the model with $\delta = 0.995$, the firm would forego revenue gains. For example, with an allowance of 400 and a price of 0.36 (where $\delta = 0.90$), the firm's revenue improvement would be 1.11% (95% confidence interval (0.80%, 1.43%)). However, if the firm used $\delta = 0.995$, it would predict that the revenue could only improve by 0.23% (95% confidence interval (-0.05%, 0.67%)). In comparison, with $\delta = 0.995$ the firm would adopt the "optimal" plan with an allowance of 325 and a price of 0.38 and expect a revenue improvement of 0.96% (95% confidence interval (0.66%, 1.23%). However, the actual revenue improvement would only be 0.27% (95% confidence interval (-0.01%, 0.53%). Such a suboptimal pricing level would reduce the potential gains by 76% relative to the optimal pricing level.

[Insert Figure 4 about here.]

8 Conclusion

Owing to the ability to capture the trade-off between long-term and short-term goals, the application of dynamic structural models to consumer and firm decision making has become increasingly widespread. However, dynamic structural models face a fundamental identification problem, namely, the preference, the state transition, and the discount factor are confounded and become difficult to identify simultaneously. Should the rate be misspecified, inferences about agent behavior might be misleading and the implied policies for improving agent welfare might be suboptimal.

We advance the literature on identifying discount factors by using field data to measure them. Specifically, we estimate a dynamic structural model using consumers' cellphone usage data. The data contain observations of consumers' cellphone consumptions under both static setting and dynamic setting. Using the static data, we first identify consumers' utilities and the distribution of random consumption shocks. Conditioned on the identified utilities and random shocks, we then recover the discount factor using the dynamic data. The identification strategy proposed may have more general applications in contexts beyond the cellphone pricing strategy we consider herein. The identification of discount factors is possible if 1) researchers observe consumers making continuous decisions under both static and dynamic settings. Such contexts may include loyalty programs, markdown pricing during clearance and seasonal sales, some finite duration auctions, and perishable goods such as ticket sales.

Findings suggest that discount factor in our specific context (0.90) is significantly below those commonly assumed in the literature (0.995). As a consequence, price effects are underestimated in our application. Moreover, the higher rate leads to a mistaken presumption that more minutes would be saved for later use, leading to a 29% increase in the mean absolute percentage error in model fit. The attendant consequences for pricing policy are notable, leading to pricing recommendations that are generally too high and would lower potential revenue gains by 76%.

The inherent complexity of dynamic structural models often requires simplifications that correspondingly represent future research opportunities. Our model is no exception. First, our study focuses on a specific consumption context with a small focal group of consumers over a specific duration. Therefore, the results may not generalize to other contexts involving different consumers, decisions or decision durations. Hence, more research is necessary to generalize the degree to which discount rates are an inherent trait or the degree to which they are context dependent. For example, it would be fruitful to consider how discount rates might vary in practice when one considers different contexts of intertemporal consumption; do consumers invoke the same level of patience when making choices over years as they do when making decisions over days?

Second, two potential sources of selection bias exist in our field study. The first arises from the firm's choices of consumers to participate in the plans. Per our discussion with the firm's managers, consumer selection was randomized so this form of selection bias is not germane. The second selection bias arises from the consumer's decision of whether to adopt the three-part tariff plan that was offered. The decision to adopt the plan can be correlated with both preferences and discounting behavior. As such, our estimates should be interpreted conditional upon plan selection (as in Goettler and Clay (2010)), limiting the generalizability of estimated rates to the overall consumer population of the company. As a result, another area of interest is to extend our research into the domain of plan choice to broaden insights regarding the distribution of discount rates across the population.

Third, our identification strategy is based on a dynamic model with continuous controls. We discuss the set identification conjecture for discrete decisions in Appendix. A more formal investigation on the identification on discrete decision dynamic models will be a challenging but significant extension.

In sum, this paper is the first (to our knowledge) to provide field-study based evidence regarding the nature of discount rates that obviate the need for structural assumptions or exclusion restrictions to identify discount rates. Consistent with Dubé et al. (2010b) and Ishihara (2010), we find evidence that discount rates are substantially lower than those used in practice and that this difference is material from a policy perspective. Given the widespread use of dynamic models in marketing and economics, we hope our analysis will spark future work to pin down how such intertemporal trade-offs are made in practice.

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Table 1: Example Discount Rates in Empirical Studies

Study	Choice Domain	Approach[1]	Discount factor	Period	Weekly Discount Factor	Implied Weekly Interest Rate (%)
Rust (1987)	Bus engine replacement	F	0.9999	month	$\rightarrow 1$	0.002
Hotz and Miller (1993)	Children/sterilization	E	0.65 (0.68)	year	0.992	0.83
Erdem and Keane (1996)	Laundry detergent	F	0.995	week	0.995	0.50
Ackerberg (2003)	Yogurt	E	0.98 (0.02)	week	0.98	2.04
Hartmann (2006)	Golf	F	0.99	day	0.932	7.29
Gordon (2009)	Personal computers	F	0.98	month	0.995	0.47
Kim et al. (2010)	Camcorders	F	1.0	minutes	1.0	0
Goettler and Clay (2010)	Online grocery service	E	0.974 (0.001)	week	0.974	2.67
Dubé et al. (2010a)	Video game consoles	F	0.9	month	0.976	2.49
Dubé et al. (2010b)	HD DVD players	P	0.7	year	0.993	0.69
Hartmann and Nair (2010)	Razors and blades	F	0.998	week	0.998	0.20
Chung et al. (2010)[2]	Salesforce compensation	E	0.95	month	0.988	1.20
Fang and Wang (2010)[2]	Mammography exams	E	0.72 (0.09) 0.80 (0.03)	two years	0.997 0.998	0.32 0.21
Ishihara (2010)	Video games	E	0.885 (0.006)	week	0.885	12.99
Overall Mean (S.D.)					0.980 (0.032)	2.088 (3.663)
Mean of Fixed Discount Rates (S.D.)					0.983 (0.026)	1.825 (2.824)
Mean of Estimated Discount Rates (S.D.)					0.973 (0.037)	2.932 (4.972)

Notes:

1. F in the "Approach" column indicates an assumed fixed value for the discount factor. The study labeled P estimates the discount rate using experimental data. E indicates the discount parameter is estimated by functional restrictions and/or the use of exclusion restrictions. The standard errors of the estimates are reported in the parentheses.

2. Chung et al. (2010) and Fang and Wang (2010) also consider hyperbolic discounting. We only report their results of exponential discount factors. Fang and Wang (2010) use two specifications in their estimation, hence we report two discount factors. Chung et al. (2010) obtain the discount rate via grid search so there is no sampling error to report. Note that the grid search approach yields estimated parameter distributions that are conditionally marginal with respect to the discount rate, which can lead to inefficient estimates.

Table 2: Three-part Tariff Plans

Access Fee (CNY)	Allowance (Minutes)	Marginal Price (CNY)	Number of Enrollees	Percentage	Average Linear Rate Before Switch (S.D.)
98	450	0.40	284	50.35	0.28 (0.10)
128	600	0.40	111	19.68	0.27 (0.09)
168	800	0.40	83	14.72	0.27 (0.07)
218	1100	0.36	50	5.92	0.25 (0.07)
288	1500	0.36	21	2.86	0.24 (0.07)
388	2500	0.30	15	2.31	0.28 (0.04)

Table 3: Summary Statistics

	Mean	Std. Dev.	Min.	Max.
Monthly Usage under Linear Plan/Allowance Level	0.92	0.46	0.02	2.22
Monthly Usage under Three-part Tariff/Allowance Level	0.96	0.35	0.01	1.63
Female	0.16	-	0	1
Rural Residency	0.41	-	0	1
Age (years)	36.18	7.45	19	58
New Customer (enrolled less than 12 months)	0.16	0.36	0	1

Table 7: Estimates of Model Parameters

	Estimate (S.E.)
Satiation	
Constant	**4.85 (0.13)**
Price (cent)	**2.15 (0.02)**
New Customer	-0.03 (0.02)
Age	0.04 (0.04)
Age^2	-0.01 (0.04)
Rural Residency	**0.03 (0.01)**
Female	**0.06 (0.02)**
Std. Dev. of Shocks	**7.46 (2.05)**
Weekly Discount Factor	**0.90 (0.05)**

Note: Bold fonts indicate the estimates being significant at 95% level.

Table 4: Monte Carlo Simulation, $N = 50$

	One-step with All Data	Static Data Only	Two-step	Three-part Tariff Data Only	Linear Pricing Data and Three-part Tariff w/o Last Period Data	Three-part Tariff w/o Last Period Data
Satiation $d = 100$	**99.58 (0.79)**	**100.55 (0.87)**	**100.55 (0.87)**	**101.10 (1.29)**	**100.88 (1.03)**	**150.55 (21.54)**
Price $b = 1$	**0.94 (0.18)**	**0.92 (0.20)**	**0.92 (0.20)**	**0.90 (0.30)**	**0.91 (0.28)**	2.11 (1.54)
Std. Dev. of Shocks $\zeta = 0.5$	**0.51 (0.03)**	**0.48 (0.03)**	**0.48 (0.03)**	**0.55 (0.10)**	**0.55 (0.08)**	**1.02 (0.50)**
Discount Factor $\delta = 0.9$	**0.88 (0.10)**	-	**0.88 (0.14)**	**0.89 (0.17)**	**0.92 (0.15)**	0.71 (0.37)

Note: Bold fonts indicate the estimates being significant at 95% level.

Table 5: Monte Carlo Simulation, $N = 75$

	One-step with All Data	Static Data Only	Two-step	Three-part Tariff Data Only	Linear Pricing Data and Three-part Tariff w/o Last Period Data	Three-part Tariff w/o Last Period Data (Does Not Converge)
Satiation $d = 100$	**99. 89(0.66)**	**99.59 (0.83)**	**99.59 (0.83)**	**101.00 (1.08)**	**100.60 (0.99)**	**-**
Price $b = 1$	**0.95 (0.15)**	**1.01 (0.19)**	**1.01 (0.19)**	**0.93 (0.25)**	**0.94 (0.22)**	-
Std. Dev. of Shocks $\zeta = 0.5$	**0.51 (0.02)**	**0.51 (0.02)**	**0.51 (0.02)**	**0.54 (0.10)**	**0.53 (0.07)**	**-**
Discount Factor $\delta = 0.9$	**0.91 (0.07)**	-	**0.89 (0.11)**	**0.92 (0.12)**	**0.91 (0.11)**	-

Note: Bold fonts indicate the estimates being significant at 95% level.

Table 6: Monte Carlo Simulation, $N = 100$

	One-step with All Data	Static Data Only	Two-step	Three-part Tariff Data Only	Linear Pricing Data and Three-part Tariff w/o Last Period Data	Three-part Tariff w/o Last Period Data
Satiation $d = 100$	**100.15 (0.60)**	**100.41 (0.72)**	**100.41 (0.72)**	**100.95 (0.98)**	**100.59 (0.95)**	**133.11 (25.80)**
Price $b = 1$	**0.99 (0.13)**	**0.99 (0.15)**	**0.99 (0.15)**	**0.95 (0.21)**	**0.97 (0.20)**	2.01 (1.21)
Std. Dev. of Shocks $\zeta = 0.5$	**0.49 (0.02)**	**0.49 (0.02)**	**0.49 (0.02)**	**0.52 (0.09)**	**0.52 (0.05)**	**0.70 (0.55)**
Discount Factor $\delta = 0.9$	**0.89 (0.06)**	-	**0.89 (0.08)**	**0.91 (0.10)**	**0.91 (0.08)**	0.88 (0.50)

Note: Bold fonts indicate the estimates being significant at 95% level.

Table 9: Robustness Check: Selection

	Plan 800 Minutes	Plan 450 Minutes (estimates in the paper)
Constant	**5.05 (0.33)**	**4.85 (0.13)**
Price (cent)	**2.91 (0.04)**	**2.15 (0.02)**
New Customer	**-0.10 (0.05)**	-0.03 (0.02)
Age	0.05 (0.11)	0.04 (0.04)
Age^2	-0.02 (0.05)	-0.01 (0.04)
Rural Residency	**0.10 (0.03)**	**0.03 (0.01)**
Female	**0.19 (0.07)**	**0.06 (0.02)**
Std. Dev. of Shocks	**8.85 (2.01)**	**7.46 (2.05)**
Weekly Discount Factor	**0.88 (0.06)**	**0.90 (0.05)**

Note: Bold fonts indicate the estimates being significant at 95% level.

Table 10: Mean Absolute Percentage Error (MAPE) Comparison

		Mean Absolute Percentage Error
Estimated Discount Factors (BIC: 9470.12)	First 4 Periods	0.14
	Final Period	0.16
	Monthly Aggregate	0.14
Discount Factor Set to 0.995 (BIC: 9499.95)	First 4 Periods	0.17
	Final Period	0.21
	Monthly Aggregate	0.18

LR χ^2=37.42, $H_0 : \delta = 0.995$ is rejected.

Table 11: Mean Percentage Error (MPE) Comparison

		Mean Percentage Error
Estimated Discount Factors	First 4 Periods	0.02
	Final Period	$\rightarrow 0$
	Monthly Aggregate	0.02
Discount Factor Set to 0.995	First 4 Periods	-0.09
	Final Period	0.02
	Monthly Aggregate	-0.03

Table 8: Robustness Check: Identification

	Static Data Only (1)	Two-step (2)	Three-part Tariff Data Only (3.1)	Linear Pricing Data and Three-part Tariff w/o Last Period Data (3.2)	Three-part Tariff w/o Last Period Data (4)	Three-part Tariff Data Only with 10-day period definition (5.1)	All Data with 10-day period definition (5.2)
Constant	**4.92 (0.19)**	**4.92 (0.19)**	**5.15 (0.52)**	**5.07 (0.45)**	**3.99 (1.26)**	**9.12 (1.26)**	**7.89 (0.81)**
Price (cent)	**2.21 (0.05)**	**2.21 (0.05)**	**2.03 (0.14)**	**2.13 (0.09)**	1.50 (1.00)	1.73 (1.41)	**2.09 (0.10)**
New consumer	-0.05 (0.03)	-0.05 (0.03)	-0.01 (0.11)	-0.02 (0.10)	0.10 (0.10)	-0.02 (0.08)	-0.04 (0.07)
Age	0.02 (0.05)	0.02 (0.05)	0.06 (0.24)	0.05 (0.23)	0.11 (0.30)	0.02 (0.14)	0.01 (0.10)
Age^2	-0.004 (0.06)	-0.004 (0.06)	-0.09 (0.26)	-0.05 (0.21)	-0.01 (0.29)	-0.05 (0.15)	-0.05 (0.16)
Rural Residency	**0.04 (0.01)**	**0.04 (0.01)**	0.03 (0.04)	0.01 (0.02)	-0.03 (0.08)	0.04 (0.07)	**0.05 (0.03)**
Female	**0.04 (0.02)**	**0.04 (0.02)**	**0.091 (0.046)**	**0.11 (0.05)**	0.21 (0.39)	**0.33 (0.08)**	0.10 (0.09)
Std. Dev. of Shocks	**7.75 (2.28)**	**7.75 (2.28)**	**7.50 (3.81)**	**7.59 (2.91)**	**9.91 (3.60)**	**9.58 (4.32)**	**9.99 (3.90)**
Weekly Discount Factor	-	**0.89 (0.09)**	**0.92 (0.13)**	**0.92 (0.11)**	0.95 (0.52)	0.85 (0.55)	**0.91 (0.11)**

Note: Bold fonts indicate the estimates being significant at 95% level.

Table 12: Demand Elasticities

	Demand Elasticity w.r.t. Price Overall	Demand Elasticity w.r.t. Price after Overage
Estimated Discount Factors	-0.07 (-0.09, -0.05)	**-0.19 (-0.21, -0.16)**
Discount Factor Set to 0.995	-0.06 (-0.07, -0.04)	**-0.13 (-0.14, -0.11)**
	Demand Elasticity w.r.t. Allowance	
Estimated Discount Factors	**0.24 (0.21, 0.26)**	-
Discount Factor Set to 0.995	**0.18 (0.15, 0.19)**	-

Note 1: 95% confidence intervals are in parentheses.

Note 2: **Bold** fonts indicate the biases are significant ($p < 0.05$).

Table 13: Revenue Percentage Change under Alternative Pricing Schedules ($\delta = 0.90$)

	Marginal Price						
Allowance (minutes)	0.34	0.36	0.38	0.40	0.42	0.44	0.46
300	-0.11	-0.06	-0.05	-0.03	-0.09	-0.15	-0.08
325	0.05	0.19	0.27	0.20	0.07	-0.01	-0.05
350	0.05	0.23	0.30	0.36	0.29	0.25	0.18
375	0.89	0.93	0.98	0.72	0.69	0.55	0.40
400	0.92	**1.11**	0.97	0.96	0.83	0.77	0.61
425	0.82	0.95	0.94	0.80	0.73	0.68	0.51
450	0.01	0.27	0.22	0	0.16	0.11	0.07
475	-0.18	0.12	-0.15	-0.22	-0.30	-0.33	-0.51
500	-0.21	-0.14	-0.23	-0.25	-0.33	-0.48	-0.61

Table 14: Revenue Percentage Change under Alternative Pricing Schedules ($\delta = 0.995$)

	Marginal Price						
Allowance (minutes)	0.34	0.36	0.38	0.40	0.42	0.44	0.46
300	0.59	0.79	0.92	0.72	0.61	0.55	0.39
325	0.65	0.80	**0.96**	0.88	0.77	0.73	0.64
350	0.48	0.57	0.71	0.94	0.83	0.80	0.75
375	0.43	0.47	0.54	0.68	0.58	0.44	0.39
400	0.11	0.23	0.33	0.49	0.44	0.40	0.36
425	0.08	0.14	0.19	0.21	0.21	0.18	0.17
450	-0.11	-0.07	-0.02	0	-0.01	-0.05	-0.08
475	-0.22	-0.21	-0.17	-0.23	-0.31	-0.34	-0.38
500	-0.39	-0.36	-0.33	-0.41	-0.46	-0.51	-0.57

Figure 1: Histogram of Total Usage vs Allowance

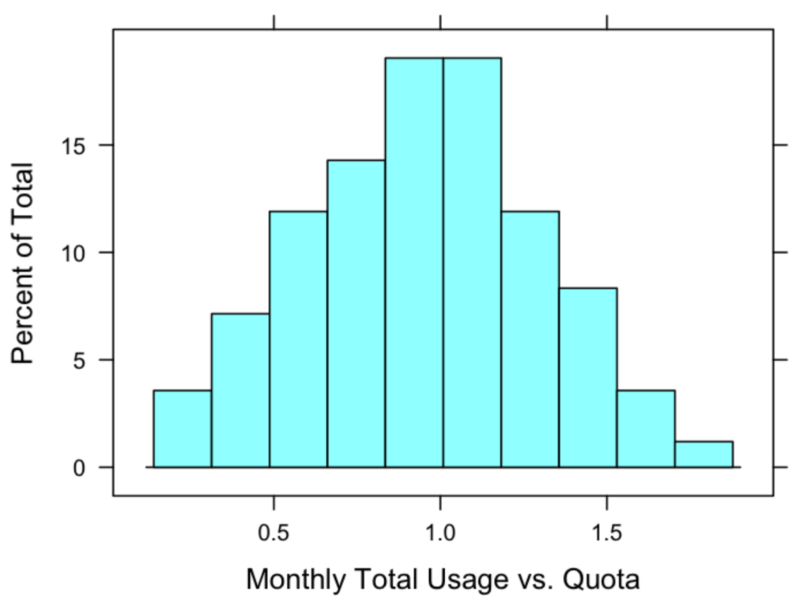


Figure 2: The Effect of Allowance on Minute Usage over Time

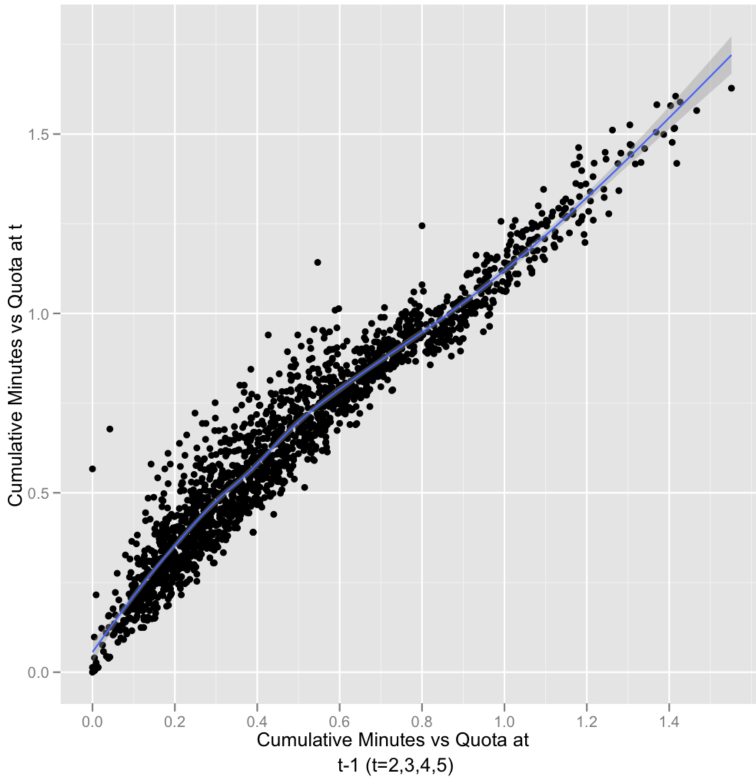


Figure 3: Usage Change over Time

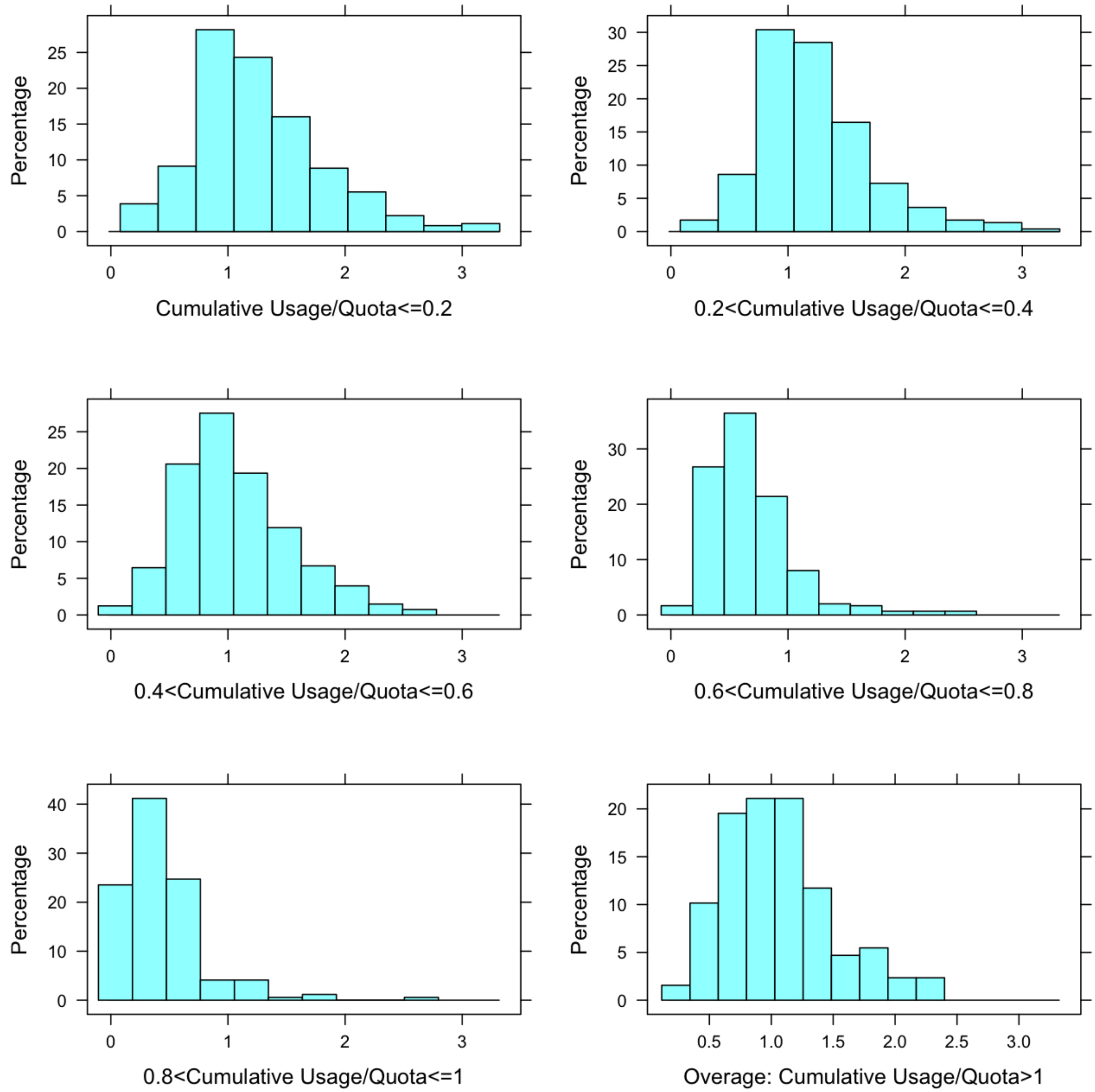


Figure 4: Revenue Prediction Differences: $\delta = 0.90$ vs. $\delta = 0.995$

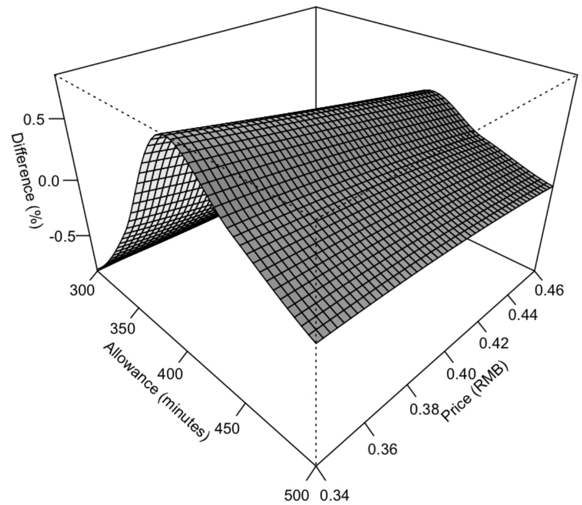
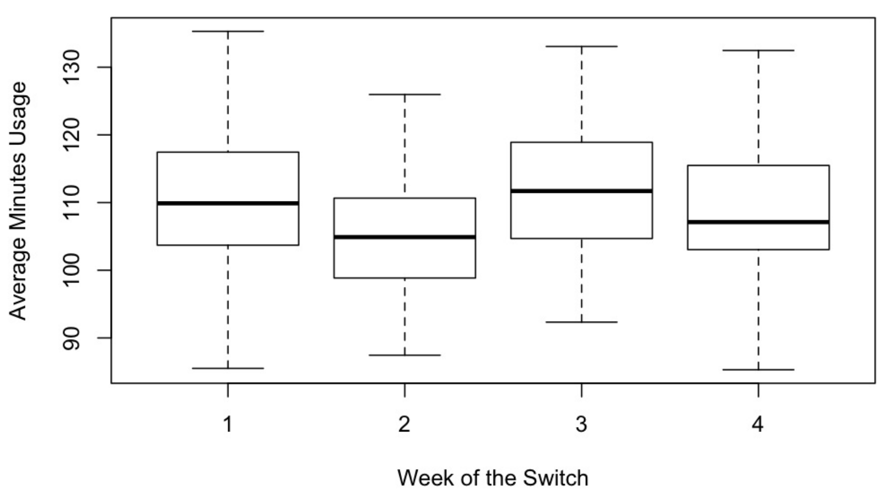


Figure 5: Average Usage over Weeks



Appendix

A The Distribution of Monthly Minutes Usage $q_{i\tau}$ under Linear Pricing Plan

Since we only observe the monthly minute consumption $q_{i\tau} = \sum_t x^*_{it}$ but not each respective x^*_{it}, we have to find the likelihood of $q_{i\tau}$.

As discussed in equation 6, the optimal minutes usage at period t, x^*_{it}, may take two values, 0 (if $d_{it} - bp_{i0} \leq 0$) and $d_{it} - bp_{i0}$ (if $d_{it} - bp_{i0} > 0$). Since $d_{it} = \exp(D'_i\alpha) + \nu_{it}$ and $\nu_{it} \sim N(0, \zeta^2)$, x^*_{it} follows a normal distribution $N(\exp(D'_i\alpha) - bp_{i0}, \zeta^2)$ that is truncated at zero. Thus the density of x^*_{it} is

$$f(x^*_{it}) = \frac{1}{\zeta}\phi(\frac{x^*_{it} - \mu_i}{\zeta})/[1 - \Phi(-\frac{\mu_i}{\zeta})] \qquad \text{(A1)}$$

$$\text{where } \mu_i = \exp(D'_i\alpha) - bp_{i0}$$

The monthly minute consumption $q_{i\tau} = \sum_t x^*_{it}$ can then be written as the summation of a series truncated normal random variables with the same truncation at zero. Although there is no closed form for the distribution of $q_{i\tau}$, if the occurrence of zero minute consumption for any period is nearly zero ($\Pr(x^*_{it} > 0) \to 1, \forall t$), $q_{i\tau}$ can be approximated well by a normal density function that has the mean as $T \cdot \mu_i$ and the variance as $T \cdot \zeta^2_g$.[19] Intuitively, although x^*_{it} is a truncated normal r.v., if $\Pr(x^*_{it} > 0)$ is nearly one, the truncation becomes moot and the distribution of x^*_{it} can be approximated well by a normal distribution. The summation of a series of i.i.d. normal r.v.'s is also normally distributed.

We implement a Monte Carlo simulation using the approximation mentioned above. The parameters are recovered with reasonable accuracy. As a robustness check to this approximation, we also use a Kernel estimator to compute the density in the Monte Carlo simulation (Härdle and Lin-

[19]The analytical proof of the validity of this approximation and Monte Carlo simulation results can be obtained from the authors.

ton (1994)). The results are similar to the ones using the approximation but the Kernel estimation is much more computationally demanding.

B The Identification of The Discount Factor

This appendix details the identification of the discount factor when the utility and random shock distribution are known. We extend the discussion of finite horizon case in section 2 to a more general infinite horizon case. We show that, conditioned on the utility and random shock distribution are identified by the static data, the discount factor is identified. We also provide a conjecture about the identification of discrete choice dynamic decision process with finite horizon.

B.1 Infinite Horizon with Continuous Control

For the infinite horizon case, at any given period of t, the optimization problem of continuous control can be written as

$$\max_{x_t} u(x_t; s_t, \nu_t) + \sum_{k=t+1}^{\infty} \delta^{k-t} \mathbf{E} u(x_k; g(s_{k-1}, x_{k-1}), \nu_k) \tag{A2}$$

where $x.$ are the continuous control; $s.$ is the state variable; $\nu.$ is the random shock and the expectation is taken over the random shock; $g(\cdot, \cdot)$ is the state transition equation.

Because the backward induction is no longer an option for the infinite horizon case, we need to instead establish that the sequence of $x_t, \forall t$ converges to a unique limit, which is the optimal decision rule of equation A2. To show that the solution to the optimization problem exists and is unique, we first rewrite the dynamic problem of equation A2 in the form of the Bellman's equation as (we drop the subscriptions of t):

$$V(s, \nu) = \max_{x} u(x; s, \nu) + \delta \mathbf{E} V(g(s, x), \nu') \tag{A3}$$

where $V(\cdot)$ is the value function.

Next we check the Blackwell's sufficient conditions to show that equation A3 is a contraction mapping with modulus δ (Stokey and Lucas (1989), p.55):

1. (Monotonicity) Take $V_1 \geq V_2$. Then:

$$\begin{aligned} TV_1 &= \max_x u(x; s, \nu) + \delta \mathbf{E} V_1(g(s, x), \nu') \\ &\geq \max_x u(x; s, \nu) + \delta \mathbf{E} V_2(g(s, x), \nu') \\ &= TV_2 \end{aligned}$$

2. (Discounting) For a real number $a \geq 0$,

$$\begin{aligned} T(V + a) &= \max_x u(x; s, \nu) + \delta \{\mathbf{E} V(g(s, x), \nu') + a\} \\ &= \max_x u(x; s, \nu) + \delta \mathbf{E} V(g(s, x), \nu') + \delta a \\ &= TV + \delta a \end{aligned}$$

Since both conditions are satisfied, V is a contraction mapping and there is a unique solution to the optimization problem. Denote the corresponding optimal policy rule as σ^*. It follows that at any period t, $x_t^* = \sigma^*(s_t, \nu_t)$ must be optimal. Examining equation A2, this means that for any period t, x_t^* must solve

$$\max_{x_t} u(x_t; s_t, \nu_t) + \delta \mathbf{E} u(x_{t+1}; g(s_t, x_t), \nu_{t+1})$$

which implies the Euler Equation of the optimization problem in equation A2:

$$u_x(x_t^*; s_t, \nu_t) + \delta \mathbf{E} u_g(x_{t+1}^*; g(s_t, x_t^*), \nu_{t+1}) = 0$$

where u_x and u_g are the partial derivatives with respect to x and g, respectively. Note that the Euler Equation can also be derived using the first order condition of equation A3 combined with Envelope Theorem. Given that the utility and the distribution of ν are known, and $x_\cdot^*$ are the unique and observed optimum, the discount factor δ can then be uniquely identified from the data.

B.2 Finite Horizon with Discrete Choice

The identification of discrete choice dynamic decision process is beyond the scope of the current study. Instead of providing a formal argument, we conjecture that set identification of the discount factor is possible with parametric restrictions on the utility shock.

First, for static discrete choice model, the distribution of random shock in the utility function is normally unidentified. It is a common practice to impose additional assumptions, including (1) one alternative's utility is normalized to 0 (e.g., the outside option); (2) distribution assumptions regarding the random shock (e.g., Probit and Logit assume the distribution and normalize the standard deviation to fixed values). Given the assumptions, the utility parameters may be identified.

Second, given the additional assumptions and the utility identified in the static period, the optimization problem of an agent can be written as

$$\max_{a_t \in A} u(a_t; s_t, \nu_t) + \sum_{k=t+1}^{T} \delta^{k-t} \mathbf{E} u(a_k; g(s_{k-1}, a_{k-1}), \nu_k) \tag{A4}$$

where A is the set of available alternative actions; a_t is the action chosen in period t; s_t is the state; ν_t is the random shock and the expectation is taken over the random shock; $g(\cdot, \cdot)$ is the state transition function such that $s_{t+1} = g(s_t, x_t), \forall t$.

Now we define the Bellman equation based on equation A4 for $t < T$

$$V_t(s_t, \nu_t) = \max_{a_t \in A} u(a_t; s_t, \nu_t) + \delta \mathbf{E} V_{t+1}(g(s_t, a_t), \nu_{t+1}) \tag{A5}$$

Conditional on the assumed distribution of random shocks, normalized outside option, identified utility, and observed action a_t^*, $\mathbf{E} V_{t+1}(g(s_t, a_t^*), \nu_{t+1})$ is known. The observed a_t^* must satisfy

$$\begin{aligned} a_t^* &= \arg\max_{a_t \in A} V_t(s_t, \nu_t) \\ &= \{a_t : u(a_t; s_t, \nu_t) + \delta \mathbf{E} V_{t+1}(g(s_t, a_t), \nu_{t+1}) \\ &\qquad \geq u(a_t'; s_t, \nu_t) + \delta \mathbf{E} V_{t+1}(g(s_t, a_t'), \nu_{t+1}), \forall a_t' \in A\} \end{aligned} \tag{A6}$$

i.e., a_t^* results in the highest value function among all alternatives. However, the optimal decision rule $a_t^* = \sigma^*(s_t, \nu_t; \delta)$ may not be injective with respect to ν_t and δ. For example, for two different discount factors, $\delta \neq \delta'$, there are two different sets of values of ν_t and ν_t' such that a^* satisfies equation A6, respectively. But the two sets of ν_t and ν_t' can overlap. As a result, for a given pair of observed (s_t, a_t^*), both δ and δ' satisfy equation A6. On the other hand, it is possible for δ to be set identified. For example, both δ and δ' are in that set. When there are more observations, potentially the identified set of δ becomes smaller and the inference pertaining to δ becomes more accurate.

In contrast, when static data are not available, parametric restrictions on the random shock, the utility and discount factor are not sufficient for identification in a nonparametric sense (e.g., Rust, 1994; Magnac and Thesmar, 2002).