SSY345 Project
Orientation estimation of smartphones

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1 Task 1

In the first case. it is a noise-free measurement so the input u_k is equal to angular velocity w_k.It is a good way to simplify our model when u_k is reliable. In the second case, we have the ability to estimate the bias of the gyroscope and accelerometer which allows the model to obtain better results. but the disadvantage is a high-complexity model and requires more computational resources. However, in cases where the output of the gyroscope y_k^ω is not as accurate as we expect, the state estimations can have deviations. In these cases, the gyroscope may not be a good choice of input, it would be better to include the angular velocities in the state vector and estimate them as well.

2 Task2

The measurements were collected by using the provided MATLAB function "filterTemplate.m" and collected data from a stationary mobile phone's accelerometer, gyroscope, and magnetometer. The gyroscope measures the angular velocity, the accelerometer measures the force acting on the sensor and the magnetometer measures the magnetic field. These measurements can be seen in Figures 1-3 where all the sensors have relatively low noise. The gyroscope's noise is close to zero, the accelerometer shows the g-force on the z-axis while about zero for the other two axes, and the magnetometer shows some initial magnetic field. The mean and covariances of the measurements are presented below. The distribution of the measurements can be seen in Figures 4-6 where it is reasonable to assume that all sensors have additive Gaussian noise.

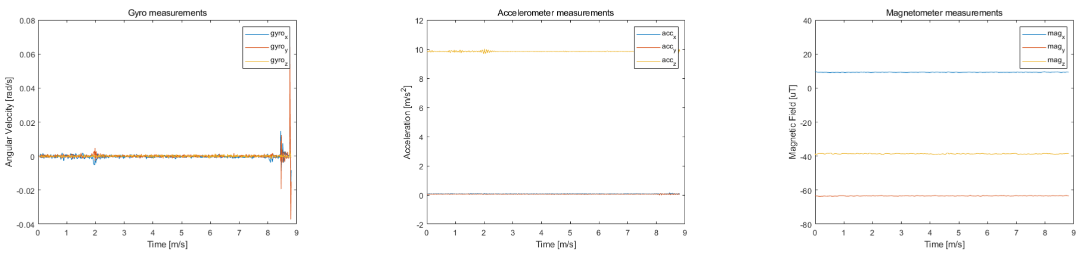

Figure 1: Histograms of accelerometer, gyroscope and magnetometer

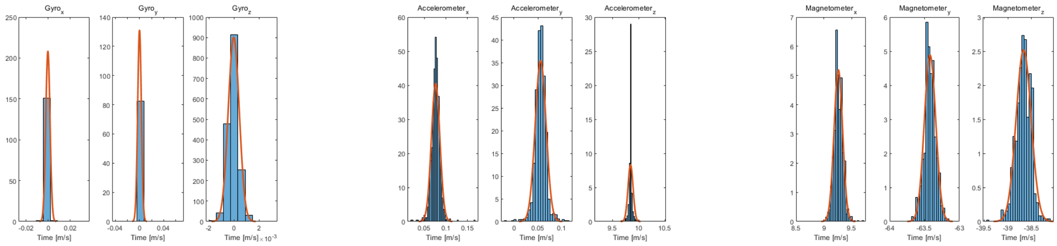

Figure 2: Data of accelerometer, gyroscope and magnetometer

Axis	Gyroscope	Accelerometer	Magnetometer
x	-0.0003156	0.1189	0
y	-0.0000125	-0.0243	10.4582
z	-0.0000216	9.8919	-54.5993

Table 1: Mean values

$$\operatorname{Cov}\left(y_k^{\omega}\right) = 10^{-5}\begin{bmatrix} 0.0009710 & 0.0000216 & 0.0001320 \\ 0.0000216 & 0.0001270 & 0.0000600 \\ 0.0001320 & 0.0000600 & 0.0023000 \end{bmatrix}$$

$$\operatorname{Cov}\left(y_k^{a}\right) = \begin{bmatrix} 0.0001 & 0.0001 & 0.0001 \\ 0.0000 & 0.0001 & 0.0001 \\ 0.0001 & 0.0001 & 0.0023 \end{bmatrix}$$

$$\operatorname{Cov}\left(y_k^{m}\right) = \begin{bmatrix} 0.0059 & 0.0006 & -0.0022 \\ 0.0006 & 0.0067 & -0.0027 \\ -0.0022 & -0.0027 & 0.0251 \end{bmatrix}$$

3 Task 3

We have

$$\dot{q}(t) = A_c q(t) = \frac{1}{2} S\left(\omega_{k-1} + v_{k-1}\right) q(t)$$

where A_c is the State Transition Equation in the continuous time, equal to $\frac{1}{2} S\left(\omega_{k-1} + v_{k-1}\right)$. Given the discretize-time State Transition Equation:

$$q_k = \underbrace{\left(e^{A_C t}\right)}_{A_d} q_{k-1}$$

where $v_k \sim \mathcal{N}(0, R_v)$.

When we have acquired an expression for A_d we calculate an expression of A_d by using the matrix exponential approximation and the relation $\exp(A) \approx I + A.$, $e^{A_C t} = I + A_C T$ and $S(\omega_1 + \omega_2) = S(w_1) + S(w_2)$ to rewrite the expression as:

$$
\begin{aligned}
A_d &= I + \frac{T}{2} S\left(\omega_{k-1} + v_{k-1}\right) \\
\Rightarrow A_d &= I + \frac{T}{2} S\left(\omega_{k-1}\right) + \frac{T}{2} S\left(v_{k-1}\right)
\end{aligned}
$$

we known the relation $S(\omega)q = \bar{S}(q)\omega$ So we can get F and G from the following formula:

$$
\begin{aligned}
\Rightarrow q_k &= \left(I + \frac{T}{2} S\left(\omega_{k-1} + v_{k-1}\right)\right) q_{k-1} \\
\Rightarrow q_k &= \left(I + \frac{T}{2} S\left(\omega_{k-1}\right)\right) q_{k-1} + \frac{T}{2} S\left(v_{k-1}\right) q_{k-1}
\end{aligned}
\Rightarrow q_k = \underbrace{\left(I + \frac{T}{2} S\left(\omega_{k-1}\right)\right)}_{F(\omega_{k-1})} q_{k-1} + \underbrace{\frac{T}{2} \bar{S}\left(q_{k-1}\right)}_{G(\hat{q}_{k-1})} v_{k-1}
$$

The Kalman filter is based on a linear system model. When we apply the Extended Kalman Filter (EKF) to nonlinear systems, it operates by linearizing the system about the current estimate and then applies the traditional Kalman filter to this linearized model. The term $G(q_{k-1})v_{k-1}$ is nonlinear in the state variable q_{k-1}. In the EKF process, this term would be linearized around the current estimate of the state. Hence, this is approximated as $G(\hat{q}k-1)vk-1$, where $\hat{q}_{k-1}$ is the estimated state from the previous time step.

4 Task 4

The EKF prediction step uses the input and the state to calculate the estimated state in the next time instance according to:

$$
\begin{aligned}
q_k &= F\left(\omega_{k-1}\right) q_{k-1} \\
P_k &= F\left(\omega_{k-1}\right) P_{k-1} F\left(\omega_{k-1}\right)^T + G\left(\hat{q}_{k-1}\right) R_w G\left(\hat{q}_{k-1}\right)^T
\end{aligned}
$$

When there is no input to use, the prediction step will instead use:

$$
\begin{aligned}
q_k &= q_{k-1} \\
P_k &= P_{k-1} + G\left(\hat{q}_{k-1}\right) R_w G\left(\hat{q}_{k-1}\right)^T
\end{aligned}
$$

Also in this scenario, we have to take into account when we get no measurements from the gyro. To fix this we can assume a random walk model into the system to handle these cases.

5 Task 5

The gyroscope only measures the angular velocities and can therefore not determine the absolute orientation. To estimate the orientation of the phone, it is possible to integrate the angular velocities. However, this will make the angles drift over time due to the nature of integrating approximations. In the case where I start the filter with the phone lying face up on the desk, it essentially establishes the initial orientation as the reference point. Then, all the relative changes in orientation measured by the gyroscope are accurate because they are small and are around this initial "zero" reference. The gyro drift might not be immediately visible in this case.

However, when you start the filter with the phone on its side, the gyroscope starts off with a non-zero initial orientation. As the gyroscope measures only the changes in orientation, it doesn't "know" about this initial offset. As the sensor data is integrated over time, the gyro drift will cause this initial offset to grow, leading to a growing disparity between your estimation and the true orientation.

When we shake the phone, the angular velocity is larger and it leads to more offset added in this scenario. Since the gyroscope has dynamic performance, it is only accurate in the short term and later it will drift. Therefore, our final estimation is not as good as Google's estimation.

As you can see in the Figure below with a side start it's observable how due to the rapid shifts in angular velocity the integration error of the gyro degenerates the positional values. In this case when returning to the original position we obtain around a 45-degree angle error.

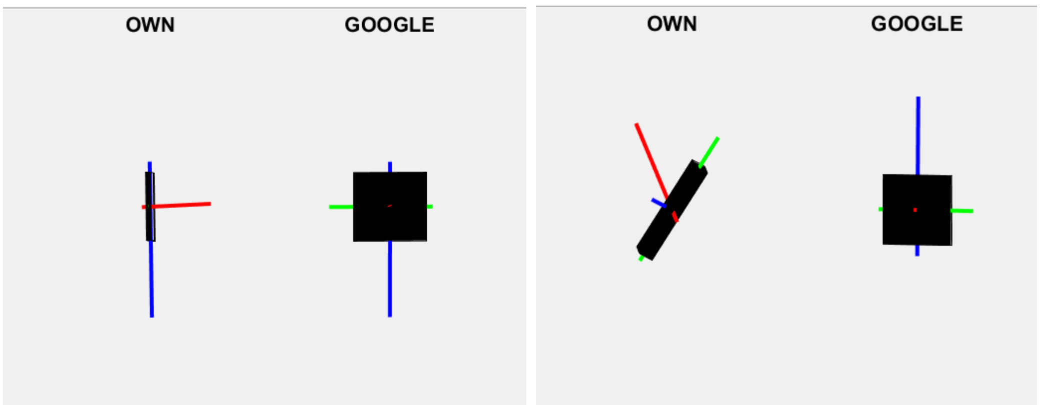

Figure 3: A side start where the phone is shaken and returned to original position

6 Task 6

In this task, we implement EKF updated with accelerometer measurements. The measurement model and update steps are shown below:

$$
\begin{aligned}
y_k^a &= Q^T\left(x_k\right)\left(g^0 + f_k^a\right) + e_k^a = Q^T\left(x_k\right) * g^0 + e_k^a, \textbf{where } f_k^a \textbf{ equal to 0.} \\
h\left(x_k\right) &= Q\left(x_k\right)^T g^0 \\
S_k &= h'\left(\hat{x}_{k|k-1}\right) P_{k|k-1} h'\left(\hat{x}_{k|k-1}\right)^T + R_a \\
K_k &= P_{k|k-1} h'\left(\hat{x}_{k|k-1}\right)^T S^{-1} \\
\hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k\left(y_k^a - h\left(\hat{x}_{k|k-1}\right)\right) \\
P_{k|k} &= P_{k|k-1} - K_k S_k K^T
\end{aligned}
$$

where

$$
h'(x) = \left[\frac{\partial Q^T}{\partial q_1} g_0, \frac{\partial Q^T}{\partial q_2} g_0, \frac{\partial Q^T}{\partial q_3} g_0, \frac{\partial Q^T}{\partial q_4} g_0\right]
$$

The equation of $h\left(x_k\right) = Q\left(x_k\right)^T g^0$ is the measurement model.

7 Task 7

The accelerometer is very close to zero for all the values due to the only nominal force acting upon the sensor being a gravitational force which has been taken to account for. Strong new forces acting upon the phone will be taken into account due to the high update frequency that the accelerometer has although this causes a high and constant noise for a shorter timespan.

8 Task 8

When the implementation of outlier rejection is introduced the phone more accurately disregards these and we let the other sensor such as the gyro which is better for short-term estimation be used for each time update step. For the outlier rejection the choice of having our mean around 9.9 and having a 10% deviation at most then the system rejects the accelerometer data. With the new outlier rejection, we get a pretty nice system which is quiet similar for small disturbances and a flat start on the table.

9 Task 9

The magnetometer measurements have a measurement model:

$$y_k^m = Q^T(q_k)\left(m^0 + f_k^m\right) + e_k^m, \textbf{where } f_k^a \textbf{ equal to 0 and } Q^T m^0 \textbf{ is h(x).}$$

In EKF, the model will be linearized as:

$$h'(x) = \left[\frac{\partial Q^T}{\partial q_1} m_0, \frac{\partial Q^T}{\partial q_2} m_0, \frac{\partial Q^T}{\partial q_3} m_0, \frac{\partial Q^T}{\partial q_4} m_0\right] \tag{1}$$

The update procedures are shown below:

$$\begin{aligned}
S_k &= h'\left(\hat{x}_{k|k-1}\right) P_{k|k-1} h'\left(\hat{x}_{k|k-1}\right)^T + R_m \\
K_k &= P_{k|k-1} h'\left(\hat{x}_{k|k-1}\right)^T S^{-1} \\
\hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k\left(y_k^m - h\left(\hat{x}_{k|k-1}\right)\right) \\
P_{k|k} &= P_{k|k-1} - K_k S_k K^T
\end{aligned}$$

10 Task 10

The earth's magnetic field is our reference point which we linearize the phone. When we introduce disturbances without outlier rejection as per the previous task we can assume that the orientation estimation will be incorrect. This is due to the fact that the weak magnetic field from the earth's poles is easily canceled or overtaken by similar magnetic fields such as from a computer or a TV.

11 Task 11

In order to mitigate magnetic anomalies, we draw upon a similar logical framework used to rectify the accelerometer's outliers, where g_0 served as the guide for outlier detection. We now substitute g_0 with m_0, our nominal value for Earth's magnetic field, which we assume is accurate barring any magnetic interference with the device.

With this filter in place, we can clearly observe from the graph below how the filter is triggered, disregarding measurements that deviate significantly from Earth's magnetic field and hence can no longer be linearized to it.

Throughout this project, we postulate that the Earth's magnetic field is represented by m_0. This forms the basis of our magnetometer measurement model, as

described in equation 7, where we assume that $f_k^m = 0$. This assumption only holds under the condition that no other magnetic fields, except for Earth's, are present.
Nevertheless, we anticipate a gradual drift in the expected magnitude even in the absence of magnetic disturbances. Thus, we implement the AR(1)-filter of $L_k = (1-\alpha)L\,(k_1) + \alpha\,|m_k|$ to estimate the drifting magnitude l_k based on each magnetometer measurement of m_k.
To further mitigate the influence of magnetic disturbances, we incorporate an outlier rejection algorithm into our filter, setting the outlier threshold to fall within $0.9\,|L_k|$ and $1.1\,|L_k|$. In essence, we permit a deviation error of approximately 10
The results of our experiments support the efficacy of this approach. After integrating the outlier rejection component, magnetic disturbances seem to have negligible impact. This is due to the fact that our predefined threshold effectively filters out measurements most severely affected.

12 Task 12

The figure shows the estimation by all three sensors. We can see that our filter performs pretty well as the Google filter. Most smartphones automatically calibrate their magnetometer measurements. To facilitate this process, we need to spin the phone gently in many different directions. Based on this, the magnetometer takes time to adapt to the model. Once it's calibrated, the estimation has as good a performance as the Google filter.

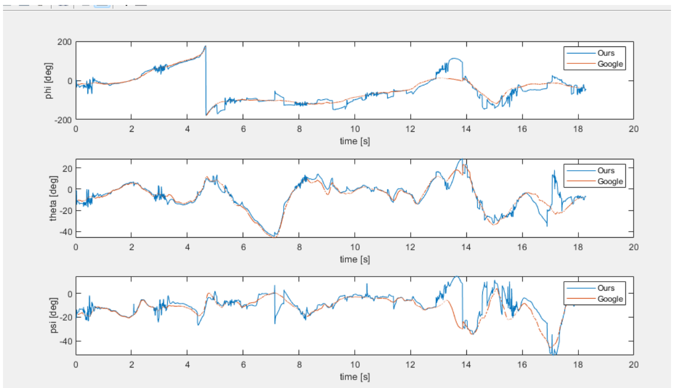

Figure 4: Orientation estimation with all three sensors along x, y, z dimensions

12.1 Accelerometer and magnetometer

if we only implement an accelerometer and magnetometer into our system, the estimation performance shows in Figure 5. The gyroscope is not applied in the filter, which means the angular rate measurements are missing. Our current motion model in the prediction step can be considered a random walk with measurement noise covariance $R_q = 0.01 - I_4$.We can observe that only the accelerometer and magnetometer has slow update rate and they can not follow the large angular change. So it causes the obvious delay. Therefore, the estimation has a bad performance.

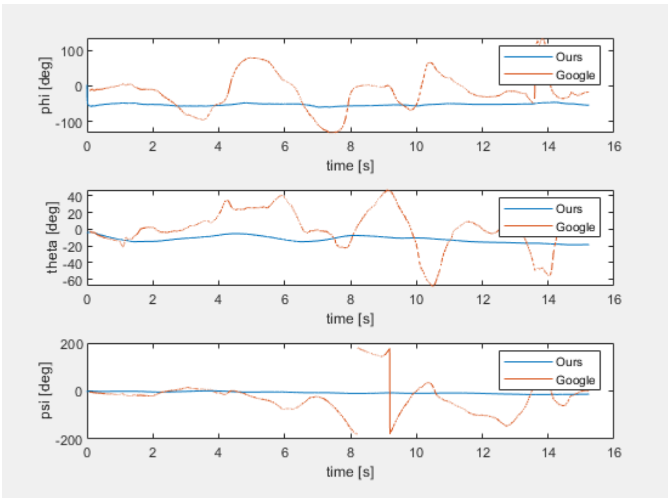

Figure 5: Orientation estimation without gyroscope along x, y, z dimensions

12.2 Accelerometer and gyroscope

In this scenario, the states are generated by a combination of accelerometer and gyroscope. Since the reference point is unknown when the magnetometer is missing. Therefore, the estimation has bad performance. The estimation generated by our filter is far from Google's estimation.

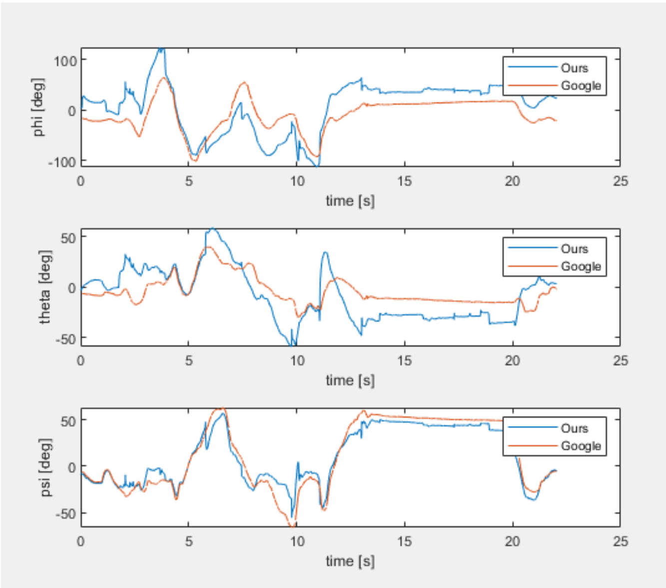

Figure 6: Orientation estimation without magnetometer along x, y, z dimensions

12.3 Magnetometer and gyroscope

In Figure 7 its observable how the estimation generated by the magnetometer and gyroscope is performing. Our filter performs worse compared to the Google filter. This is reasonable since there is no data provided by theaccelerometer, which is one of the most important for the orientation of the phone for changes.

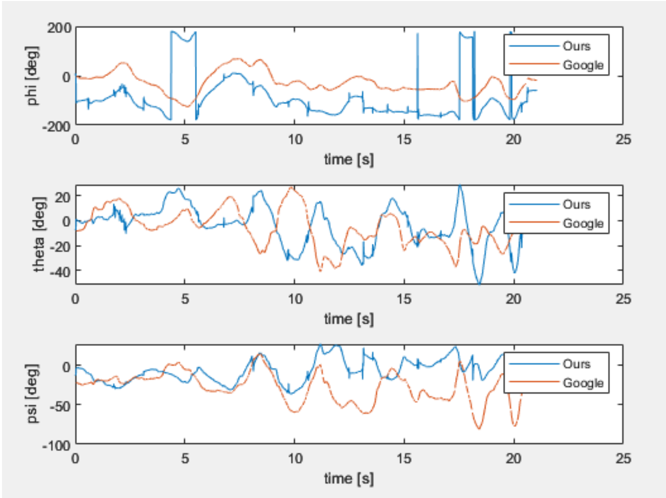

Figure 7: Orientation estimation without accelerometer along x, y, z dimensions

13 CONCLUSION

According to the estimation performance shown above, our filter estimation quite approximates to Google filter when implementing all three sensors. Although, the magnetometer takes time to adapt the model, once adapted, the difference is more or less non-existent. From the discussion above, we can conclude that among all the different combinations of sensors, the combination of accelerometer and gyroscope provides the best results than the other two combinations. Thus, we can say that the magnetometer is less important than the other two sensors to know how the phone is moving during short periods of time. Though magnetometer is still useful because it provides the estimation of the magnetic field, which can be used to determine the orientation relative to the Earth's magnetic field such as if the phone is upside down. The data provided by the accelerometer and gyroscope is more important to predict and update the state for the next time.