Solution to Apostol's Calculus

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I. INTRODUCTION

Part 1. Historical Introduction

I 1.4 Exercises

Exercise 1 *Modify the region in Figure I.3 by assuming that the ordinate at each x is $2x^2$ instead of x^2. Draw the new figure. Check through the principal steps in the foregoing section and find what effect this has on the calculation of the area. Do the same if the ordinate at each x is (b) $3x^2$, (c) $\frac{1}{4}x^2$, (d) $2x^2+1$, (e) ax^2+c.*

Solution 1

Exercise 2 *Modify the region in Figure I.3 by assuming that the ordinate at each x is x^3 instead of x^2. Draw the new figure.*

(a) Use a construction similar to that iluustrated in Figure I.5 and show that the outer and inner sums S_n and s_n are given by

$$S_n = \frac{b^4}{n^4}\left(1^3+2^3+\cdots+n^3\right), \quad s_n = \frac{b^4}{n^4}\left(1^3+2^3+\cdots+(n-1)^3\right).$$

(b) Use the inequalities (which can be proved by mathematical induction; see Section I4.2)

$$1^3+2^3+\cdots+(n-1)^3 < \frac{n^4}{4} < 1^3+2^3+\cdots+n^3 \tag{I.12}$$

to show that $s_n < b^4/4 < S_n$ for every n, and prove that $b^4/4$ is the only number which lies between s_n and S_n for every n.

(c) What number takes the place of $b^4/4$ if the ordinate at each x is ax^3+c?

Solution 2

Exercise 3 *The inequalities (I.5) and (I.12) are special cases of the more general inequalities*

$$1^k+2^k+\cdot+(n-1)^k < \frac{n^{k+1}}{k+1} < 1^k+2^k+\cdot+n^k \tag{I.13}$$

that are vaild for every integer $n \geq 1$ and every integer $k \geq 1$. Assume the validity of (I.13) and generalize the results of Exercise 2

Solution 3