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MEng Computing – Final year project

Machine Learning for the Prediction of Professional Tennis Matches

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Abstract

Extensive research has been conducted into the modelling of professional tennis matches. Most current approaches take advantage of the hierarchical structure of the tennis scoring system to define stochastic models, based on Markov chains. These models use only the probability of each of the players winning a point on their serve to compute their respective probabilities of winning the match. Consequently, a variety of factors that contribute to the outcome of a match are ignored. We propose a supervised machine learning approach that uses historical player performance across a wide variety of statistics to predict match outcomes. We define a novel method of extracting 22 features from raw historical data, including abstract features, such as player fatigue and injury. Using the resulting dataset, we develop and optimise models based on two machine learning algorithms: logistic regression and artificial neural networks. When evaluated on a test set of 6315 ATP matches played in the years 2013-2014, our models outperform Knottenbelt's Common-Opponent model, the current state-of-the-art in stochastic modelling. Our neural network generates a return on investment of 4.35% when in competition with the betting market, an improvement of about 75%. We believe that the use of machine learning will lead to innovation in the field of tennis modelling.

Acknowledgements

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Chapter 1

Introduction

Tennis is undoubtedly among the world's most popular sports. The Association of Tennis Professionals (ATP) features over 60 professional tennis tournaments in 30 countries every year, drawing immense numbers of spectators. Andy Murray's historic defeat of Novak Djokovic in the 2013 Wimbledon final was the most watched television broadcast of the year in Great Britain, with an audience of 17.3 million. The growth of the popularity of the sport, paired with the expansion of the online sports betting market, has led to a large increase in tennis betting volume in recent years. The same Murray-Dojokovic Wimbledon final saw £48 million traded on Betfair, the world's largest betting exchange. The potential profit, as well as academic interest, has fuelled the search for accurate tennis match prediction algorithms.

The scoring system in tennis has a hierarchical structure, with a match being composed of sets, which in turn are composed of games, which are composed of individual points. Most current state-of-the-art approaches to tennis prediction take advantage of this structure to define hierarchical expressions for the probability of a player winning the match. By assuming that points are independently and identically distributed (iid)[1], the expressions only need the probabilities of the two players winning a point on their serve. From this basic statistic, easily calculated from historical data available online, we can deduce the probability of a player winning a game, then a set, and finally the match. Barnett [1] and O'Malley [18] both defined such hierarchical models, and Knottenbelt [13] refined the models to calculate the probabilties of winning a point on serve using only matches with the common opponents of the players, instead of all past opponents. This reduces the bias resulting from the players having historically had different average opponents. Madurska [16] further extended the Common-Opponent model to use different probabilities of winning on serve for different sets, challenging the iid assumption and allowing the model to reflect the way a player's performance varies over the course of the match. Knottenbelt's Common-Opponent model and Madurska's Set-By-Set model are the current state-of-the-art, claiming a return on investment of 6.8% and 19.6%, respectively, when put into competition with the betting market on matches in the 2011 WTA Grand Slams.

While elegant, this mathematical approach is not perfect. By representing the quality of players using only a single value (service points won), the method is unable to act upon the more subtle factors that contribute to the outcome of a match. For example, a player's susceptibility to a particular playing strategy (e.g., attacking the net), the time since their last injury, or accumulated fatigue from previous matches would only indirectly affect match prediction. Furthermore, the characteristics of the match itself (location, weather conditions, etc.) would have no effect on the prediction. Considering the availability of an immense amount of diverse historical tennis data, an alternative approach to tennis prediction could be based on machine learning. The features of players and the features of the match, paired with the match result, could form a set of labelled training examples. A supervised ML algorithm could use these examples to infer a function for predicting the results of new matches.

Despite machine learning being a natural candidate for the tennis match prediction problem, the approach seems to have had little attention in comparison with the stochastic hierarchical approaches. Most past attempts made use of logistic regression. For example, Clarke and Dyte [6] fit a logistic regression model to the difference in the ATP rating points of the two players for predicting the outcome of a set. A

[1] Klaasen and Magnus [12] show that points are neither independent nor identically distributed. However, they find that deviations from iid are small, and using this assumption often provides good approximations.

simulation then was run to predict the result of several men's tournaments in 1998 and 1999, producing reasonable results. Ma, Liu and Tan [15] used logistic regression with 16 variables related to characteristics of the players and the match. Investigating a different ML algorithm, Somboonphokkaphan [22] trained an artificial neural network (ANN) using the match surface and several features of both players (winning percentages on first serve, second serve, return, break points, etc.) as training parameters. The authors claim an accuracy of about 75% in predicting the matches in the Grand Slam tournaments in 2007 and 2008.

The goal of the project is to investigate the applicability of machine learning methods to the prediction of professional tennis matches. We begin by developing an approach for extracting a set of relevant features from raw historical data (Chapter 3). Next, we train a logistic regression model on the constructed dataset (Chapter 4). Seeking further improvement, we train two higher-order models (logistic regression with interaction features and an artificial neural network) in Chapter 5. We then evaluate the performance of the models on an independent dataset of 6135 ATP matches played during the years 2013-2014, using three different betting strategies (Chapter 7). We find that our most profitable machine learning model generates a 4.35% return on investment, an improvement of approximately 75% over the current state-of-the-art stochastic models. This shows that a machine learning approach is is well worth pursuing. We propose some extensions to our work in Chapter 8.

Chapter 2

Background

2.1 The Game of Tennis

Tennis is a racquet sport that can be played either against a single opponent (singles) or between two teams of two players (doubles). For simplicity, we will focus only on modelling singles tennis matches.

At any point in a match, one of the players is designated to be the *server*, and the other is the *receiver*. The players stand on opposite sides of the tennis *court*, a rectangular area with a net stretched across its width. Various court surfaces are used in different tournaments, including clay, grass, or hard. After a legal service by the server (for which they have two attempts), the players alternate hitting the ball, until eventually one wins the rally, earning a *point*. (We omit the full details of the tennis rules for brevity, the official rules are published online by the International Tennis Federation[1]).

A *game* consists of a sequence of points with the same player serving. The first player to win at least four points and at least two more than the opponent wins the game. The points are counted in an unusual way, in the sequence 0, 15, 30, 40. If the score reaches 40-40, this is called a *deuce*. Whichever player wins the next point has the *advantage*, because winning another point will result in them winning the game. After each game, the players alternate at serving. The first player to win at least six games and at least two more than the opponent wins the *set*. However, if the set score reaches 6-6, in most tournaments, a *tiebreaker* is played, a special game in which the first player to have won at least seven points and at least two more than the opponent wins the set. A match is won when a player wins the majority of a specified number of sets, either three or five (this depends on the tournament).

Professional tournaments take place 11 months of the year, and are organised by the Association of Tennis Professionals (ATP) and the Women's Tennis Association (WTA) for men's and women's tournaments, respectively. We will further restrict our focus on predicting the results of ATP matches. This will impose no loss of generality on our model, since the only major distinction is the additional possibility of best-of-five matches for men, which do not occur in women's professional tennis.

2.2 The Tennis Dataset

Historical tennis data is widely available online. Tennis websites such as `atpworldtour.com` provide access to information about players, the outcomes of matches and statistics related to player performance in particular matches. Some sources, such as `tennis-data.co.uk`, provide historical data in structured form (CSV or Excel files). More complex datasets with a longer historical timespan and higher accuracy are available for purchase online. One such dataset is provided by the OnCourt system[2], which will provide the basis for data used throughout the project. OnCourt has results for over 500 thousand ATP matches since 1990, of which over 40 thousand include betting odds. Table 2.1 summarises the most relevant data available from OnCourt.

[1] Official Rules of Tennis. `http://www.itftennis.com/officiating/rulebooks/rules-of-tennis.aspx`

[2] `www.oncourt.info`

Table 2.1: OnCourt dataset

Player details
Name
Date of birth
Country of birth
Prize money
ATP rating points over time
ATP rank over time
Match details
Tournament name
Tournament type (e.g., Grand Slam)
Surface
Location (country, lat/lon)
Date
Result (scoreline)
Prize money
Odds (Marathonbet, Pinnacle)
Per-match stats for both players
First serve percentage
Aces
Double faults
Unforced errors
Percentage of points won on first serve
Percentage of points won on second serve
Percentage of receiving points won
Winners
Break points (won, total)
Net approaches (won, total)
Total points won
Fastest serve
Average first serve speed
Average second serve speed
Odds (Marathonbet, Pinnacle)

Some data which may be relevant for tennis modelling but is unavailable through OnCourt includes per-set statistics for players and the details of how matches progressed point-by-point. This can be obtained for some matches by scraping websites such as `flashscore.com`. It is worth noting that for many tournaments, data of a much finer granularity is captured through HawkEye ball-tracking technology, including the location of the ball and players at any point in the match. However, this data is owned by the management group behind the ATP and is not licensed to third parties.

2.3 Tennis Betting

There are two main categories of tennis betting: *pre-game* and *in-game*, with the distinction that pre-game bets cannot be placed after the game commences. Furthermore, it is usually possible to bet on a variety of factors, such as the winner of the match, the score of different sets, the total number of games, etc. We will focus on pre-game bets on the winner of the match, as the odds for this bet type are most available historically, allowing us to perform a more comprehensive evaluation of the performance of our model against the betting market.

Bets on tennis matches can be placed either with *bookmakers* or on *betting exchanges*. Traditional bookmakers (e.g., Pinnacle Sports) set *odds* for the different outcomes of a match, and a bettor competes against the bookmakers. In the case of betting exchanges (e.g., Betfair), customers can bet against odds set by other customers. The exchange matches the customers' bets to earn a risk-free profit by charging a commission on each bet matched.

2.3.1 Betting Odds and Implied Probability

Betting odds represent the return a bettor receives from correctly predicting the outcome of an event. For example, if a bettor correctly predicts the win of a player for whom the odds are $3/1$, they will receive £3 for every £1 staked (in addition to their staked amount, which is returned). If the bettor mis-predicts the match, they will lose their stake of £1. This profit or loss resulting from a bet is called the *return on investment* (ROI), and will the main metric used to assess our model. Measuring the performance of the model based on the ROI generated from competition against the historical betting market has been common in past research on the subject (including [13, 16]).

Betting odds give an *implied probability* of the outcome of a match, the bookmaker's estimate of the true probability. For odds X/Y for a player winning a match, the implied probability p of the win is:

$$p = \frac{Y}{Y+X} \tag{2.1}$$

2.3.2 Betting Strategies

Given the betting odds and a predicted probability of a match outcome, a bettor has various methods of deciding if, and how much, to stake in a bet. Needless to say, different strategies will result in a different return on investment. We will consider three different strategies for evaluating the profitability of our model. In the following, define:

$$\begin{aligned}
s_i &= \text{amount to stake on player } i \\
p_i^{\text{bettor}} &= \text{bettor's estimate of probability of player } i \text{ winning} \\
b_i &= \text{net odds received when betting on player } i\text{, calculated as } \frac{X}{Y} \text{ for odds X/Y} \\
p_i^{\text{implied}} &= \text{implied probability of player } i \text{ winning, calculated as } \frac{Y}{Y+X} \text{ for odds X/Y}
\end{aligned}$$

1. **Betting on the predicted winner**

   In the simplest strategy, the bettor always stakes a fixed amount q on the player which they expect to win:

   $$s_i = \begin{cases} q, & \text{if } p_i^{\text{bettor}} > 0.5 \\ 0, & \text{otherwise} \end{cases}$$

2. **Betting on the predicted winner at better odds**

   A bettor may increase their returns by only betting a fixed amount q on matches where they have an *edge* over the bookmakers, i.e., their probability estimate of player i winning is greater than the probability implied by the betting odds. In other words, this strategy avoids betting on the predicted winner when the odds do not sufficiently compensate for the risk of the bet.

   $$s_i = \begin{cases} q, & \text{if } p_i^{\text{bettor}} > p_i^{\text{implied}} \\ 0, & \text{otherwise} \end{cases}$$

3. **Betting on the predicted winner using the Kelly criterion**

   In the previous strategy, the bettor staked a fixed amount on a bet if they believed they had an edge, regardless of the size of the edge. The Kelly criterion, described by John Kelly in 1956 [11], can be used to determine the optimal size of a bet based on a bettor's edge, and is guaranteed to perform better than any other essentially different betting strategy in the long run. The bettor now bets a fraction of a maximum bet size q on the predicted winner if they believe they have an edge:

$$s_i = \begin{cases} q \cdot \dfrac{p_i^{\text{bettor}}(b_i + 1) - 1}{b_i}, & \text{if } p_i^{\text{bettor}} > p_i^{\text{implied}} \\ 0, & \text{otherwise} \end{cases}$$

In practice, the maximum bet size q is often a fraction of the bettor's bankroll, and therefore varies over time, depending on the success of the bettor's previous bets. For model evaluation, we fix q to be a constant so that all bets contribute equally to the overall return on investment, regardless of their temporal order.

Note that in all three strategies, a bet is never placed on both players. Also, while the first strategy will bet on every match (provided that the estimated probability is never exactly 0.5), for the latter two strategies, it is possible for no bet to be placed on a match.

2.4 Statistical Models

Current state-of-the-art models for tennis prediction make use of hierarchical stochastic expressions based on Markov chains. A comparison with such models will be used for the evaluation of our model, and this section gives an overview of their fundamental concepts.

2.4.1 Markov Models

Klaasen and Magnus [12] show that points in tennis are approximately independent and indentically distributed (iid). This finding allows us to assume that for any point played during the match, the point outcome does not depend on any of the previous points. Let's further assume that we know the probability of each player winning a point on their serve. Namely, let p be the probability that player A wins a point on their serve, and q the probability that player B wins a point on their serve. Using the iid assumption and the point-winning probabilities, we can formulate a Markov chain describing the probability of a player winning a game.

Formally, a *Markov chain* is a system which undergoes transitions between different *states* in a *state space*. An important property is the system's *lack of memory*, meaning that the next state of the system depends only on the current state, not on the preceeding sequence of states. If we take the different scores in a game to be our state space, and the transitions between the states to be probabilities of a point being won or lost by player A, the resulting Markov chain will reflect the stochastic progression of the score in a game. Figure 2.1 depicts the Markov chain for a game diagrammatically, where player A is serving. Assuming their probability of winning a point on serve is p, due to the iid assumption, all transitions representing the win of a point by player A have this probability, and all transitions representing the loss of a point happen with probability $1 - p$.

As described in Section 2.1, scoring in tennis has a hierarchical structure, with sets being composed of games, and a match composed of sets. Additional Markov chains are constructed in a similar fashion, modelling the progression of scores in tiebreakers, sets and matches. For example, in the model of a match, there would be two out-going transitions from each non-terminal state, labelled with the probabilities of the player winning and losing a single set. Diagrams for the remaining models can be found in [16].

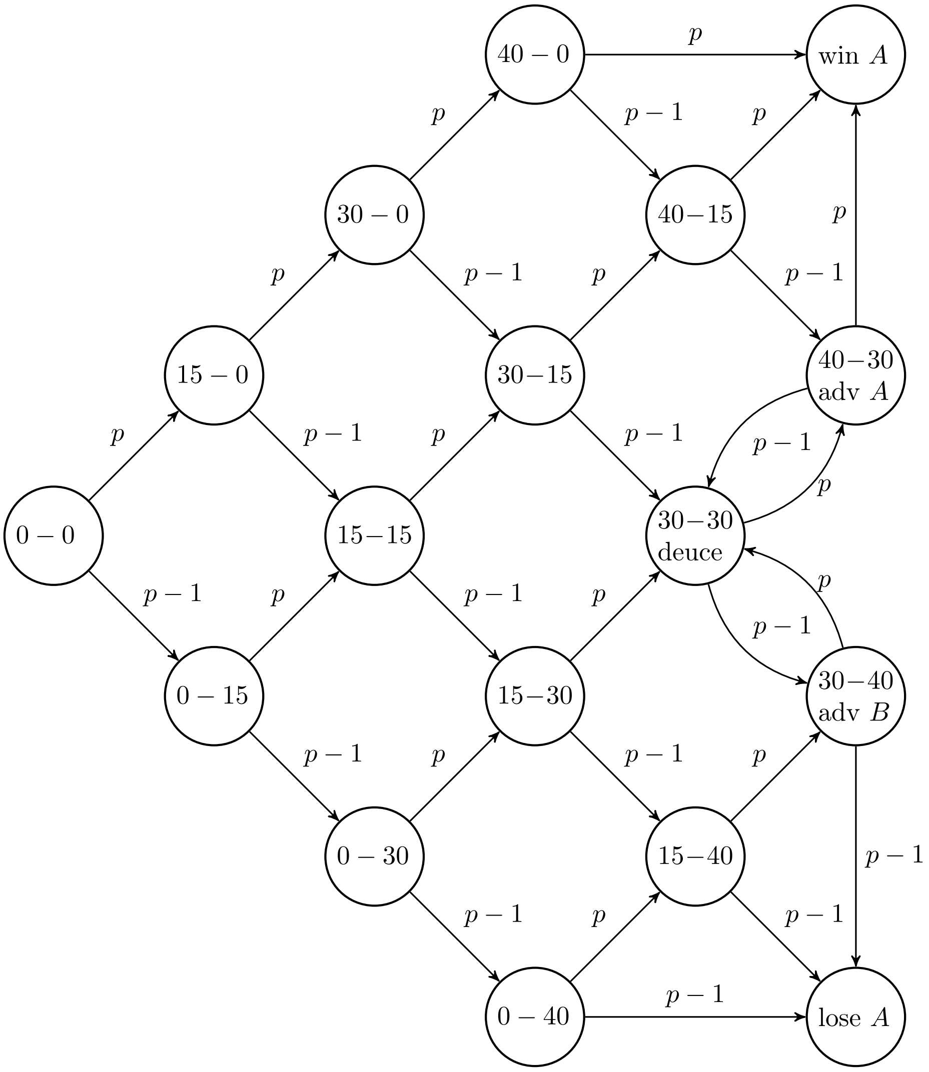
2.4.2 Hierarchical Expressions

Based on the idea of modelling tennis matches with Markov chains, both Barnett and Clarke [1] and O'Malley [18] have developed hierarchical expressions for the probability of a particular player winning an entire tennis match.

Barnett and Clark express the probability of player A winning a game on their serve P_{game} using the following recursive definition:

$$P_{game}(x, y) = p \cdot P_{game}(x + 1, y) + (1 - p) \cdot P_{game}(x, y + 1) \tag{2.2}$$

Figure 2.1: Markov chain for a game in a singles match, player A serving



The boundary values are defined as follows:

$$
\begin{aligned}
P_{game}(x,y) &= 1 \text{ when } x = 4, x - y \geq 2 \\
P_{game}(x,y) &= 0 \text{ when } y = 4, y - x \geq 2 \\
P_{game}(x,y) &= \frac{p^2}{p^2 + (1-p)^2} \text{ when } x = 4, x - y \geq 2
\end{aligned}
$$

In the above, p is the probability of player A winning a point on their serve, and x and y represent the number of points won by players A and B, respectively. This expression clearly corresponds to the Markov chain depicted in Figure 2.1.

Barnett and Clark further define a similar expression for the set-winning probability, based on the probabilities of players winning individual games (given by equation 2.2) and for tiebreakers (which also depend on the serve-winning probabilities of the players). Finally, the match-winning probability can be expressed in terms of the previously defined expressions. The all-important realisation is that the resulting expression for the match-winning probability is dependent *only* on the probabilities of both players winning a point on their serve.

2.4.3 Estimating Serve Winning Probabilities

Given the probabilities of both players winning a point on their serve, we can use the hierarchical expressions derived by Barnett and Clark (described in Section 2.4.2) to find the match-winning probability. The question remains of how to estimate these serve-winning probabilities for matches that have not yet been played. Barnett and Clark [1] give an efficient method for estimating these probabilities from historical player statistics:

$$
\begin{aligned}
f_i &= a_i b_i + (1 - a_i) c_i \\
g_i &= a_{av} d_i + (1 - a_{av}) e_i
\end{aligned}
\tag{2.3}
$$

Where:

$$
\begin{aligned}
f_i &= \text{ percentage of points won on serve for player } i \\
g_i &= \text{ percentage of points won on return for player } i \\
a_i &= \text{ first serve percentage of player } i \\
a_{av} &= \text{ average first serve percentage (across all players)} \\
b_i &= \text{ first serve win percentage of player } i \\
c_i &= \text{ second serve win percentage of player } i \\
d_i &= \text{ first service return points win percentage of player } i \\
e_i &= \text{ second service return points win percentage of player } i
\end{aligned}
$$

Now, for a match between players A and B, we can estimate the probabilities of player A and B winning a point on their serve as f_{AB} and f_{BA}, respectively, using the following equation:

$$
f_{AB} = f_t + (f_i - f_{av}) - (g_j - g_{av}) \tag{2.4}
$$

Where:

$$
\begin{aligned}
f_t &= \text{ average percentage of points won on serve for tournament} \\
f_{av} &= \text{ average percentage of points won on serve (across all players)} \\
g_{av} &= \text{ average percentage of points won on return (across all players)}
\end{aligned}
$$

2.4.4 Current State-of-the-Art

Current state-of-the-art tennis prediction models are based on the hierarchical stochastic expressions described in the previous sections. Knottenbelt [13] adapted the way the serve-winning probabilities of players are calculated before being supplied to the Barnett formulas. Instead of finding historical averages of statistics for the players across all opponents, only the players' performance against common opponents is considered. The modified serve-winning probabilities more accurately reflect the quality of two players if they have historically had different average opponents. Madurska [16] further modified Knottenbelt's Common-Opponent model to allow for different serve-winning probabilities in different sets. This weakens the iid assumption to cover only points and games, and enables the model to account for a player's typical change in performance from set to set. The Common-Opponent and Set-by-Set models claim an ROI of 6.8% and 19.6%, respectively, when put into competition with the betting market on matches in the 2011 WTA Grand Slams. The Common-Opponent model was also tested on a larger and more diverse test set, generating an ROI of 3.8% over 2173 ATP matches played during 2011. We will therefore use the Common-Opponent model as a reference for the evaluation of our model.

2.5 Machine Learning Models

2.5.1 Machine Learning in Tennis

Machine learning is a field of artificial intelligence (AI) that studies algorithms which learn from data. A *supervised* machine learning system has the task of inferring a function from a set of labelled training examples, where a labelled example is a pair consisting of an input vector and the desired output value.

In the context of tennis, historical tennis data can be used to form the set of training examples. For a particular match, the input vector can contain various features of the match and the players, and the output value can be the outcome of the match. The selection of relevant features is one of challenges of the construction of successful machine learning algorithms, and is described further in Section 2.5.5

Different machine learning algorithms exist to solve different types of problems. We can approach the tennis prediction problem in two ways:

1. As a **regression** problem, in which the output is real-valued. The output may represent the match-winning probability directly, but true match-winning probabilities are unknown for historical matches, forcing us to use discrete values for training example labels (e.g., 1 for match won, 0 for match lost). Alternatively, we can predict the probabilities of the players winning a point on their serve, and feed this into Barnett's or O'Malley's hierarchical expressions to find the match-winning probability (see Section 2.4).
2. As a **binary classification** problem, in which we can attempt to classify matches into either a 'winning' or a 'losing' category. Some classification algorithms, such as logistic regression (described in Section 2.5.2), also give some measure of the certainty of an instance belonging to a class, which can be used as the match-winning probability.

We now present several machine learning algorithms which have either been applied to tennis match prediction in the past, or are expected to produce good results by the author.

2.5.2 Logistic Regression

Despite its name, logistic regression is in fact a classification algorithm. The properties of the logistic function are central to the algorithm. The logistic function $\sigma(t)$ is defined as:

$$\sigma(t) = \frac{1}{1+e^{-t}} \tag{2.5}$$

As can be seen in Figure 2.2, the logistic function maps real-valued inputs between $-\infty$ and $+\infty$ to values between 0 and 1, allowing for its output to be interpreted as a probability.

A logistic regression model for match prediction consists of a vector of n match features $\mathbf{x} = (x_1, x_2, \cdots, x_n)$ and a vector of $n+1$ real-valued model parameters $\boldsymbol{\beta} = (\beta_0, \beta_1, \cdots, \beta_n)$. To make a prediction using the model, we first project a point in our n-dimensional feature space to a real number:

$$z = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n$$

Figure 2.2: Logistic function $\sigma(t)$

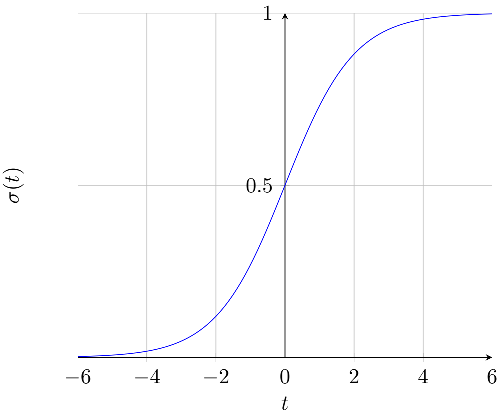
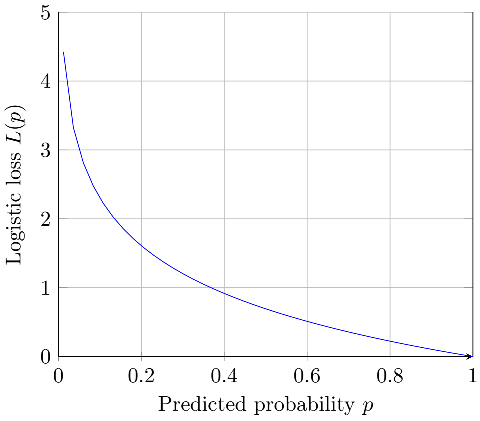


Figure 2.3: Logistic loss in predicting a won match



Now, we can map z to a value in the acceptable range of probability (0 to 1) using the logistic function defined in equation 2.5:

$$p = \sigma(z) = \frac{1}{1 + e^{-z}} \tag{2.6}$$

The training of the model consists of optimising the parameters $\boldsymbol{\beta}$ so that the model gives the best reproduction of match outcomes for the training data. This is done by minimising the *logistic loss* function (equation 2.7), which gives a measure of the error of the model in predicting outcomes of matches used for training.

$$L(p) = -\frac{1}{N} \sum_{i=1}^{N} p_i \log(y_i) + (1 - p_i) \log(1 - y_i) \tag{2.7}$$

Where:

$$\begin{aligned} N &= \text{ number of training matches} \\ p_i &= \text{ predicted probability of a win for match } i \\ y_i &= \text{ actual outcome of match } i \text{ (0 for loss, 1 for win)} \end{aligned}$$

Figure 2.3 shows the logistic loss incurred due to a single match for different predicted probabilities, assuming the match resulted in a win. Any deviation from the most correct prediction of $p = 1.0$ is penalised.

Depending on the number of training examples, one of two methods of training (i.e., minimising the logistic loss) is chosen:

1. **stochastic gradient descent** - an slower iterative method suitable to large datasets
2. **maximum likelihood** - a faster numerical approximation, cannot deal with large datasets

Most published ML-based models make use of logistic regression. Clarke and Dyte [6] fit a logistic regression model to the difference in the ATP rating points of the two players for predicting the outcome of a set. In other words, they used a 1-dimensional feature space $\mathbf{x} = (\textit{rankdiff})$, and optimised β_1 so that the function $\sigma(\beta_1 \cdot \textit{rankdiff})$ gave the best predictions for the training data. The parameter β_0 was omitted from the model on the basis that a *rankdiff* of 0 should result in a match-winning probability of 0.5. Instead of predicting the match outcome directly, Clark and Dyte opted to predict the set-winning probability and run a simulation to find the match-winning probability, thereby increasing the size of the dataset. The model was used to predict the result of several men's tournaments in 1998 and 1999, producing reasonable results (no precise figures on the accuracy of the prediction are given).

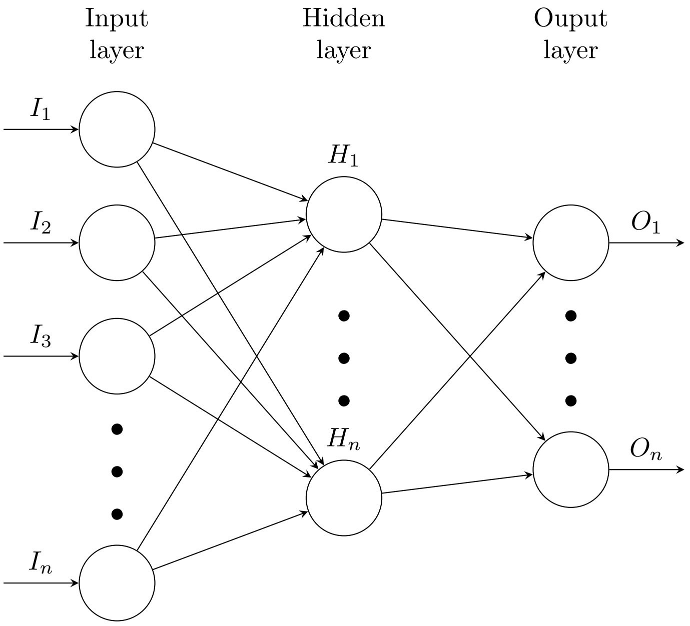
Ma, Liu and Tan [15] used a larger feature space of 16 variables belonging to three categories: player skills and performance, player characteristics and match characteristics. The model was trained with matches occurring between 1991 and 2008 and was used to make training recommendations to players (e.g., "more training in returning skills").

Logistic regression is attractive in the context of tennis prediction for its speed of training, resistance to overfitting (described in Section 2.5.5), and for directly returning a match-winning probability. However, without additional modification, it cannot model complex relationships between the input features.

2.5.3 Artificial Neural Networks

An artificial neural network is a system of interconnected "neurons", inspired by biological neurons. Each neuron computes a value from its inputs, which can then be passed as an input to other neurons. A *feed-forward* network is a directed, acyclic graph (DAG). ANNs are typically structured to have several layers, with a neuron in each non-input layer being connected to all neurons in the previous layer. A three-layer network is illustrated in Figure 2.4.

Figure 2.4: A three-layer feed-forward neural network



Associated with each connection in the network is a *weight*. A neuron uses its inputs and their weights to calculate an output value. A typical composition method is a non-linear weighted sum:

$$f(x) = K\left(\sum_i w_i x_i\right), \qquad \text{where } w_i \text{ is the weight of input } x_i \tag{2.8}$$

The non-linear *activation function K*, allows the network to compute non-trivial problems using only a small number of neurons. The logistic function defined in equation 2.5 is one of several *sigmoid* functions commonly used for this purpose.

Match prediction can be done by passing the values of player and match features to the neurons in the input layer and propagating values through the network. If a logistic activation function is used, the output of the network can represent the match-winning probability. There are many different training algorithms, which aim to optimise the network's weights to generate the best outputs for a set of training examples. For example, the back-propagation algorithm uses gradient descent to reduce the mean-square error between the target values and the network outputs.

Somboonphokkaphan [22] trained a three-layer feed-forward ANN for match prediction with the back-propagation algorithm. Several different networks with different sets of input features were trained and compared. The best-performing network had 27 input nodes, representing features of both players and the match, and had an average accuracy of about 75% in predicting the outcomes of matches in the 2007 and 2008 Grand Slam tournaments.

ANNs can detect complex relationships between the various features of the match. However, they have a "black box" nature, meaning that the trained network gives us no additional understanding of the system, as it is too difficult to interpret. Furthermore, ANNs are prone to overfitting and therefore necessitate a large amount of training data. Also, ANN model development is highly empirical, and the selection of the hyperparameters of the model (discussed in Section 2.5.5) often requires a trial and error approach. However, due to its success in the above-mentioned experiment, this approach is clearly deserves further investigation.

2.5.4 Support Vector Machines

Support vector machines (SVMs), just like the other machine learning models described in this section, are supervised learning models. An SVM is built by mapping examples to points in space, and finding a maximum-margin hyperplane which separates them into the categories with which they are labelled (as before, these can be the ‘winning’ and ‘losing’ categories). An unseen example, such as a future match, can then be mapped to the same space and classified according to which side of the margin it falls on.

To the best of the author’s knowledge, no work has yet been published on applying SVMs to tennis match prediction. SVMs have several advantages over ANNs in this context. Firstly, the training never results in a local minimum, as is frequent with ANNs. Also, SVMs typically out-perform ANNs in prediction accuracy, especially when the ratio of features to training examples is high. However, the training time for SVMs tends to be much higher, and the models tend to be difficult to configure.

2.5.5 Machine Learning Challenges

Overfitting

As described in Section 2.2, a considerable amount of historical data is available for the training of the models described above. However, it is important to note that the performance of players in an upcoming match will need to be estimated based on their past matches. Only recent matches on the same surface against similar opponents accurately reflect the expected performance of the players. For this reason, tennis modelling inherently suffers from a lack of data. The lack of data often results in *overfitting* of the model, meaning that the model describes random error or noise in the data, instead of the underlying relationship. ANNs are particularly prone to overfitting, especially when the number of hidden layers/neurons is large relative to the number of examples.

To overcome the overfitting problem, only the most relevant features of matches will be used for training. The process by which these features are selected is called *feature selection*, for which various algorithms exist. Removing irrelevant features will also improve training times.

Hyperparameter optimisation

The training of a model optimises the model parameters, such as the weights in an ANN. However, models commonly also have *hyperparameters*, which are not learned and must be provided. For example, the number of hidden layers and the number of neurons in each layer are some of the configurable hyperparameters of an ANN. The process of arriving at optimal hyperparameters for a given model tends to be empirical. The traditional algorithmic approach, grid search, involves exhaustively searching through a pre-defined hyperparameter space. A successful tennis prediction model will necessitate a careful selection of hyperparameters.

Chapter 3

Feature Extraction

3.1 Tennis Match Representation

A supervised machine learning algorithm requires a set of labelled examples for training. In the context of tennis prediction, each training example corresponds to a single historical tennis match, and is composed of two elements:

1. A **vector of input features** ($\mathbf{X}$), representing the characteristics of the players and the match
2. The **target value** (y), corresponding to the outcome of the match

The trained model can then be used to predict the outcome of a future match, provided that the set of input features can be constructed for the match.

3.1.1 Match Outcome Representation

Two players participate in every singles tennis match, and are labelled as Player 1 and Player 2. The target value can be defined as follows:

$$y = \begin{cases} 1, & \text{if Player 1 won} \\ 0, & \text{if Player 1 lost} \end{cases} \tag{3.1}$$

Incomplete matches are not used for training, so no other outcome is possible.

3.1.2 Symmetric Match Feature Representation

Any effective tennis prediction model must consider the characteristics of *both* players participating in a match. Consequently, we must have two values for each variable of interest, one for each player. We construct a feature by taking the difference between these two values. For example, consider a simple model based only on the ATP ranks of the two players. In this case, we construct a single feature $\mathbf{RANK} = \mathbf{RANK}_1 - \mathbf{RANK}_2$, where $\mathbf{RANK}_1$ and $\mathbf{RANK}_2$ are the ranks of players 1 and 2 at the time of the match, respectively. Clarke and Dyte [6] used precisely the rank difference as the sole feature in their logistic regression model.

An alternative way of representing match features would be to include both values of a variable (one for each player) as two distinct features. For example, we could include $\mathbf{RANK}_1$ and $\mathbf{RANK}_2$ as two independent features in our model. Arguably, this approach would preserve more information about the two players, allowing for a more accurate model. However, in practice, the difference in a variable for the two players is often a sufficiently informative measure. For example, O'Malley [18] showed that in the hierarchical model (Section 2.4), the match outcome depends on the difference between the serve-winning probabilities of the two players, and the individual probabilities are not essential.

An important advantage of using the differences in variables as features is the possibility of a *symmetric* model. We define a symmetric model as one which would produce an identical match outcome prediction, even if the labels of the players were swapped (i.e., if Player 1 was Player 2 and vice versa). An asymmetric model may, due to noise in the data, assign more importance to a feature for Player 1 than for Player 2, resulting in different predictions depending on the labelling of the players. For example, a logistic regression model may give a higher absolute weight to $\mathbf{RANK}_1$ than to $\mathbf{RANK}_2$. We avoid any bias by having a single **RANK** feature, representing their difference.

Using variable differences as features halves the number of features, reducing the *variance* of the model (the model's sensitivity to small variations in the training dataset). This helps prevent overfitting (see section 2.5.5).

3.2 Historical Averaging

In the previous section, we described how the variables representing the qualities of two players are combined into features. Although some values of variables, such as the ranks of the players, are easily accessible before a match, others must be estimated based on the performance of the players in their previous matches. For example, a player's success on return varies from match to match. In order to construct a feature representing the difference in the players' average winning on return percentage (**WRP**), we would use the past matches of the players to find their average winning on return percentages ($\mathbf{WRP}_1$ and $\mathbf{WRP}_2$), and then take their difference. Table 3.1 illustrates how the winning on return percentage and the aces per game would be estimated for Federer's match on June 1, 2014. An identical process would be performed to find any other features which must be estimated from the previous matches. We would then perform a similar calculation for his opponent, and take the difference between the estimates to generate features for training. Note that we average over *all* prior matches, resulting in higher estimates for Federer's performance than if we had only used the past several matches.

Table 3.1: Estimating Federer's performance based on past matches

	Win on return %	**Aces per game**	···	**Match date**
	⋮	⋮		⋮
	0.27	0.26		2014-01-24
	0.33	0.19		2014-03-16
Statistics in past matches	0.41	0.23		2014-03-26
	0.30	0.10		2014-04-20
	0.35	0.31		2014-05-14
Average performance	0.41	0.30		

Although simple, this method of estimating the performance of the players has several shortcomings. Firstly, the players may have historically had very different average opponents. If Player 1 has played against more difficult opponents than Player 2, the resulting estimates will be biased towards Player 2. We discuss a method of removing this bias in Section 3.2.1.

Furthermore, naive historical averaging overlooks the fact that not all of a player's past matches are equally relevant in predicting their performance. We can address this by taking weighted averages, and giving a higher weight to past matches which we think are more relevant for predicting the upcoming match. Sections 3.2.2 and 3.2.3 describe approaches to determining these weights.

3.2.1 Common Opponents

The simple averaging of player performance across all past matches described in the previous section is biased if two players have had different average opponents. Knottenbelt [13] proposed a method for a fair comparison of players by using their common opponents. Although the technique was developed as a means of estimating the serve and return winning percentages of players for use in a hierarchical Markov model, the same idea can be applied to our use case.

First, a set of *common opponents* of the two players is found (the players which both players have played against). Next, we take each common opponent in turn, and find the average performance of both players against the common opponent. Finally, we average the performance values for each player across all common opponents. In this way, performance estimates for an upcoming match are based on the same set of opponents for both players.

Figure 3.1: Estimating the **WRP** feature using common opponents

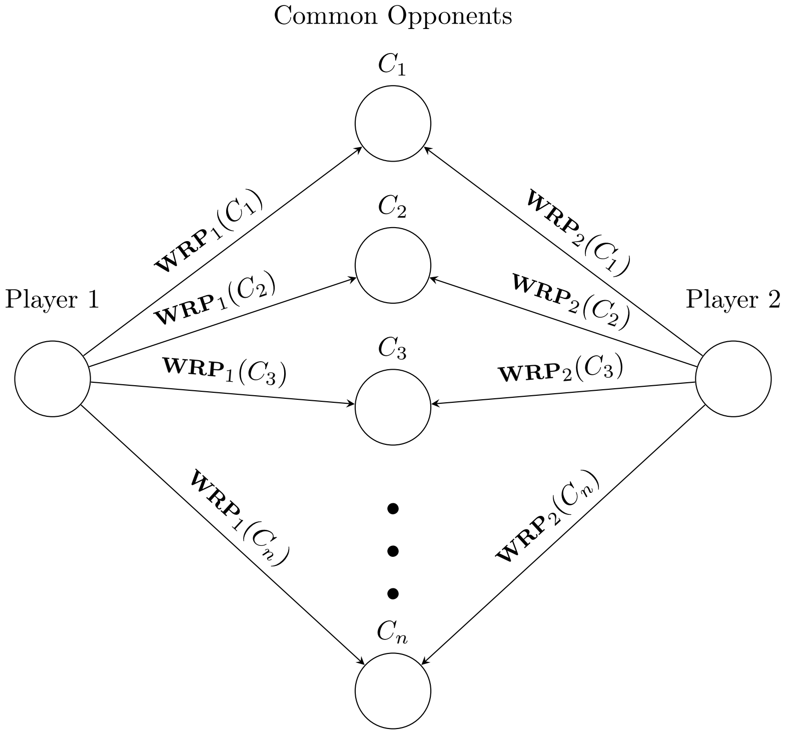


Figure 3.1 shows how we would estimate the winning on return percentages for two players using their common opponents. The common opponents are labelled as C_1 to C_n. For player i, $\mathbf{WRP}_i(C_j)$ is their average winning on return percentage in all matches against common opponent C_j. We need to average these values to obtain an estimate for each player:

$$\mathbf{WRP}_i = \frac{\sum_{j=0}^{n} \mathbf{WRP}_i(C_j)}{n}$$

Finally, we construct the **WRP** feature by taking the difference of the estimates for the two players (as discussed in Section 3.2):

$$\mathbf{WRP} = \mathbf{WRP}_1 - \mathbf{WRP}_2$$

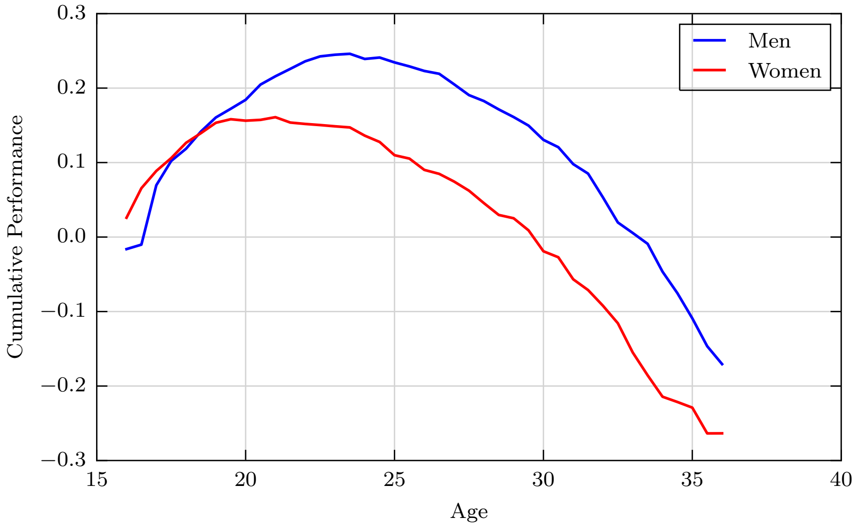
We can perform a similar computation to find the other match features. Clearly, this method will be accurate only if a sufficient number of common opponents exists for the two players.

3.2.2 Time Discounting

There are many factors that affect a player's performance over time. In general, a player's performance improves as they gather strength and experience in the first part of their career and later declines due to the physiological effects of ageing, as shown in Figure 3.2. However, injury may also have a long-term impact on a player's performance, as well as events in their private lives. For example, Farrelly and Nettle [8] found that professional tennis players suffer a significant decrease in ranking points during the year after their marriage.

Although many of these factors are difficult to model, we can assume that a player's recent matches more accurately reflect their current state than older matches. For example, the matches that a 35-year-old

Figure 3.2: Tennis ageing curves



Source: `SBNation.com`

player has played in the past year are likely to yield better estimates of their performance than matches played in their 20s. We reflect this using *time discounting*, giving higher weights to more recent matches when estimating features. We assign the weights using an exponential function:

$$W(t) = \min(f^t, f) \tag{3.2}$$

Where:

$$\begin{aligned} t &= \text{ time since the match (years)} \\ f &= \text{ discount factor} \end{aligned}$$

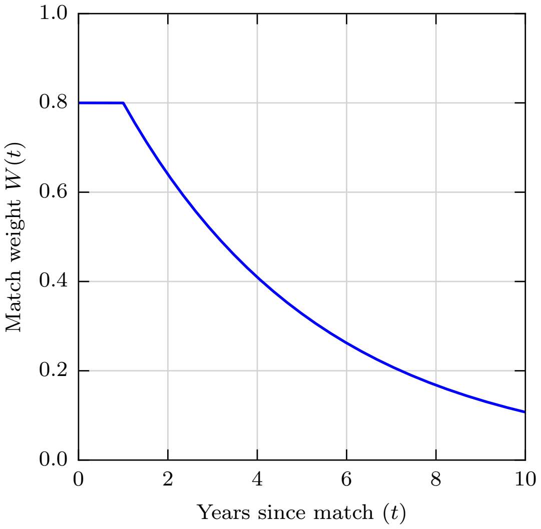
The discount factor f can be any real number between 0 and 1, and determines the magnitude of the effect of time discounting. If f is small, older matches have very little significance. Figure 3.3 shows the weights assigned to historical matches when a discount factor of 0.8 is used. Due to the min function in Equation 3.2, all matches in the past year are assigned the same weight. Otherwise, very recent matches would be assigned extremely large weights. Note that the discount factor is a hyperparameter in the model, and needs to be optimised (see Section 2.5.5).

3.2.3 Surface Weighting

Tennis is played on a variety of court surfaces (clay, hard, grass, etc.), each having a different impact on the bounce of the ball. Grass is the fastest surface, while clay is the slowest, and hard is somewhere in between. A player is likely to perform differently depending on how the characteristics of the surface affect their playing style. An analysis of the highest ranked men and women by Barnett [2] confirms that players' performances are affected by the court surface. Furthermore, he deduces fundamental relationships between players' performances across different surfaces. For example, if a player's optimal surface is grass, they are likely to perform better on hard court than clay.

Clearly, for predicting an upcoming match on a particular surface, a player's past matches on the same surface will be more informative than those on other surfaces. As in time discounting, we can assign a weight to past matches, depending on their surface. In the simplest approach, we can consider only past matches played on the same surface as the match we are predicting, by assigning them a weight of 1 and giving all other matches a weight of 0. We will refer to this surface weighting strategy as **splitting by surface**. The drawback of splitting by surface is that it significantly reduces the amount of data used for estimating the features of the match. For example, it is likely that two players have no common opponents on grass, since few tournaments use this surface.

Figure 3.3: Time discount function ($f = 0.8$)



We could run an optimisation to find the best weighting of other surfaces for each surface. However, the search space is too large, and this is computationally infeasible. Instead, we can use the dataset to find the correlations in player performance across different surfaces. First, for each player, we can find the percentage of matches won across different surfaces during their career. For every pair of surfaces (a, b), we then calculate correlations in performance across all players:

$$\rho_{a,b} = \frac{\sum_{i=1}^{n}(a_i - \overline{a})(b_i - \overline{b})}{(n-1)s_a s_b} \tag{3.3}$$

Where

$a_i =$ percentage of matches won by player i on surface a

$b_i =$ percentage of matches won by player i on surface b

$s_a =$ standard deviation of percentage of matches won on surface a

$s_b =$ standard deviation of percentage of matches won on surface b

$\overline{a} =$ mean percentage of matches won on surface a

$\overline{b} =$ mean percentage of matches won on surface b

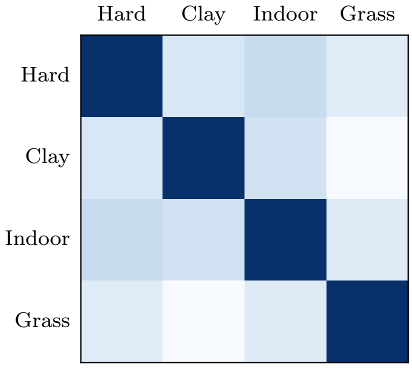
$n =$ number of players

Computing Equation 3.3 for all pairs of surfaces on ATP matches in the years 2004 - 2010 (our training set) yields the correlation matrix shown in Figure 3.4. We can see that all correlations are positive, i.e., a player that tends win on one surface will also tend to win on another, but perhaps not as often. Our findings support Barnett's results. For example, there is a much higher correlation between performance on grass and hard courts than between grass and clay courts.

As correlation is a measure of dependence between two sets of data, it can be used to provide the weights for past matches when estimating features for an upcoming match. We refer to this as **weighting by surface correlation**. This approach makes use of a larger amount of historical data than splitting by surface, and could therefore allow for a more accurate comparison of players. Furthermore, we avoid any optimisation process, using the values in the correlation matrix directly when computing averages.

Figure 3.4: Correlation matrix of player performance across surfaces

	Hard	**Clay**	**Indoor**	**Grass**
Hard	1.00			
Clay	0.28	1.00		
Indoor	0.35	0.31	1.00	
Grass	0.24	0.14	0.25	1.00



3.3 Uncertainty

Time discounting and surface weighting (described in Sections 3.2.2 and 3.2.3, resp.) assign weights to a player's past matches when computing estimates of their performance in an upcoming match. The weights of the player's past matches can be used to give a measure of *uncertainty* with respect to the player's performance estimates and consequently the match features. Such a quantity is useful for removing noise prior to training and for obtaining a level of confidence in a match outcome prediction. The calculation is slightly different depending on whether we use the common opponent approach or not, and we describe both methods below.

3.3.1 Uncertainty For Simple Averaging

To find the match feature uncertainty for the simple averaging approach (without the use of common opponents), we first find the total weight of past matches for player i:

$$S_i = \sum_{m \in P_i} W(m)$$

Where

$$\begin{aligned} W(m) &= \text{ Weight of match } m \\ P_i &= \text{ Set of past matches of player } i \end{aligned}$$

We define the overall uncertainty of the features of the match as the inverse of the product of the total weights for the two players:

$$U = \frac{1}{S_1 \cdot S_2} \tag{3.4}$$

This implies that we will only be confident in the accuracy of the features of the match if the performance estimates for both players are based on a sufficiently large amount of data.

3.3.2 Uncertainty For Common Opponents

If match features are found using the common opponents of the players, we first find the total weight for each player's estimates with respect to each common opponent:

$$S_i(C_j) = \sum_{m \in P_i(C_j)} W(m)$$

Where

$$\begin{aligned} W(m) &= \text{ Weight of match } m \\ P_i(C_j) &= \text{ Set of past matches of player } i \text{ against opponent } C_j \end{aligned}$$

The overall uncertainty is computed using the sum of the weights across all common opponents:

$$U = \frac{1}{\sum_j S_1(C_j) \cdot S_2(C_j)} \tag{3.5}$$

This means that we expect the quality of match features to increase with the number of common opponents. However, a smaller number of common opponents with strong relationships with both players will result in a lower uncertainty than a large number of common opponents with low weights.

3.4 New Feature Construction

As discussed in Section 2.2, the OnCourt system provides the dataset from which features are extracted for each match. In general, there are two approaches to extracting features:

1. If we are certain about the value of a feature at the time of the match (e.g., the difference in the rank of the players), we use the value directly
2. If we do not know the value, we must use the historical matches of the players to estimate it. This approach is used for all match statistics in Table 2.1 (first serve percentage, aces per game, winning on return percentage, etc.). During the averaging, we can apply any combination of the averaging methods described in the previous section (common opponents, time discounting, and surface weighting).

In addition to features produced using these two methods, we apply our knowledge of the tennis domain to generate additional features which we expect to be relevant in match prediction.

3.4.1 Combining Statistics

Adding combinations of the estimates of players' performance statistics as features may improve the prediction accuracy of a machine learning algorithm. In fact, higher-order learning algorithms (such as neural networks with at least one hidden layer) attempt to discover patterns in the weighted *combinations* of input features. However, we can use our knowledge of the game to include the most relevant combinations directly, allowing them to also be used in simpler models (e.g., logistic regression). Higher-order models are discussed in Chapter 5.

Overall winning on serve percentage

The OnCourt dataset provides the winning percentage on first and second serves as separate values. When combined with the first serve accuracy, we can calculate an overall winning on serve percentage for a player i:

$$\mathbf{WSP}_i = \mathbf{W1SP}_i \cdot \mathbf{FS}_i + \mathbf{W2SP}_i(1 - \mathbf{FS}_i)$$

We expect this aggregate statistic to be more consistent for players across different matches. As for all features, the **WSP** feature is computed by taking the difference of the values for the two players:

$$\mathbf{WSP} = \mathbf{WSP}_1 - \mathbf{WSP}_2$$

Completeness

The very best of tennis players have few weaknesses, and are strong in both offensive and defensive playing styles. For example, Roger Federer is a considered by many to be the greatest all-court player of all time. We can attempt to measure the *completeness* of a player by combining their serve and return winning percentages:

$$\mathbf{COMPLETE}_i = \mathbf{WSP}_i \cdot \mathbf{WRP}_i$$

The multiplicative relationship ensures that a player has high completeness if they are strong in both offensive and defensive aspects of the game.

Advantage on serve

So far, all features discussed were generated using performance estimates computed *independently* for the two players. However, a performance estimate for one player can also rely on some statistic of the other player. For example, instead of comparing the players' winning on return percentages directly, we may want to gauge one player's serve strength against the other's return strength. We call the resulting feature a player's advantage on serve (**SERVEADV**):

$$\begin{aligned}
\mathbf{SERVEADV}_1 &= \mathbf{WSP}_1 - \mathbf{WRP}_2 \\
\mathbf{SERVEADV}_2 &= \mathbf{WSP}_2 - \mathbf{WRP}_1 \\
\mathbf{SERVEADV} &= \mathbf{SERVEADV}_1 - \mathbf{SERVEADV}_2
\end{aligned}$$

Arguably, this feature is much more informative of the outcome of a match than the **WSP** and **WRP** features taken on their own, since a player's performance when serving clearly has a strong dependence on the quality of the opponent's return play. Directly comparing the serve strengths of the players, without accounting for the opponents' return strength, does not account for this relationship.

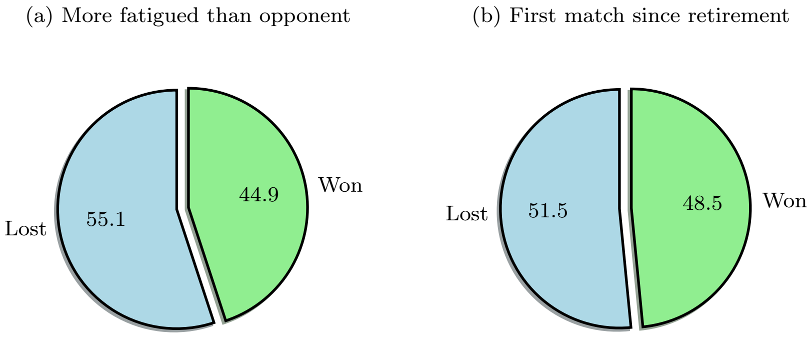
We could attempt to construct many other features by comparing different characteristics of players. However, this requires an understanding of the sport and of the semantic relationships between different player statistics.

3.4.2 Modelling Fatigue

The physical form of a player before entering a match is likely to have a strong impact on their performance. A common explanation for the under-performance of a player in a match is the accumulated fatigue from previous matches. We therefore represent fatigue as the number of *games* a player have played in the past three days. The contribution of each day is weighted in a similar fashion to the time-discounting of matches (Section 3.2.2), using a discount factor of 0.75. For example, if a player contested in a 50-game match two days ago, their fatigue score would be $50 \cdot 0.75^2 = 28$.

The size of the time window (three days) and the discount factor were found by experimentation. Figure 3.5a shows the distribution of the outcome of the match for the player who entered with a higher fatigue score (as defined above), using all matches in our training set. Clearly, a less fatigued player has an improved chance of winning.

Figure 3.5: Match outcome for player with impaired form (ATP matches 2004 - 2010)


(a) More fatigued than opponent

(b) First match since retirement

3.4.3 Modelling Injury

A player's form is also affected by any recent injuries. Although the OnCourt dataset does not provide any specific information regarding player injuries, we can deduce from the match results whether a player retired from a match. A player is said to *retire* from a match if they withdraw during the match, usually do to injury, and forfeit their place in a tournament. We can thus use retirement as an approximation for injury. This is only an approximation, since a player may retire for other reasons (e.g., to conserve strength for more important upcoming matches), or they may instead injure themselves during training, which we have no knowledge of.

Initially, we considered using the time since a retirement as the measure of the severity of an injury. However, a player that has not competed for longer has also had more time to recover, so the relationship is unclear. Also, the effect of the retirement is only a significant factor in the match immediately following the retirement. If a player has retired but has already competed since, we can assume that they have sufficiently recovered. For these reasons, we define the retirement of player i as a *binary* variable:

$$\textbf{RETIRED}_i = \begin{cases} 1, & \text{if first match since player } i \text{ retired} \\ 0, & \text{otherwise} \end{cases}$$

Figure 3.5b confirms that retirement has a negative impact on the outcome of the match (although the effect is smaller than for fatigue).

3.4.4 Head-to-head Balance

The outcomes of the matches directly between two players, also known as their head-to-head balance, is an important factor in the prediction of an upcoming match. Some players routinely struggle against a particular opponent despite being the favourite. One such surprising result is Federer's 11-8 head-to-head balance against David Nalbandian, who was consistently ranked lower than Federer throughout his career. If the two were to compete today (which is unlikely, since Nalbandian has retired from professional tennis), the head-to-head statistic would lower our predicted probability of Federer winning. Another example would be Rafael Nadal's 5-6 head-to-head standing against Nikolay Davydenko.

We represent the head-to-head relationship between player using the **DIRECT** feature (direct total matches won), computed as follows:

$$\textbf{DIRECT} = H2H(1,2) - H2H(2,1) \tag{3.6}$$

Where

$$H2H(i,j) = \text{ percentage of matches won by player } i \text{ against player } j$$

Note that the computation of this feature is the same, irrespective of whether common opponents are used. Also, if either (or both) of time discounting or surface weighting is used, the mutual matches of the players are also be weighted in a similar fashion. The **DIRECT** feature thus assigns a higher weight to the more relevant matches between the two players.

3.5 Data Preparation

3.5.1 Data Cleansing

The OnCourt dataset is imperfect, as some statistics for matches are innacurate or corrupt. *Data cleansing* is the process of detecting and removing such statistics. We perform data cleansing on our training dataset, to prevent any innacuracies from degrading the quality of match outcome predictions.

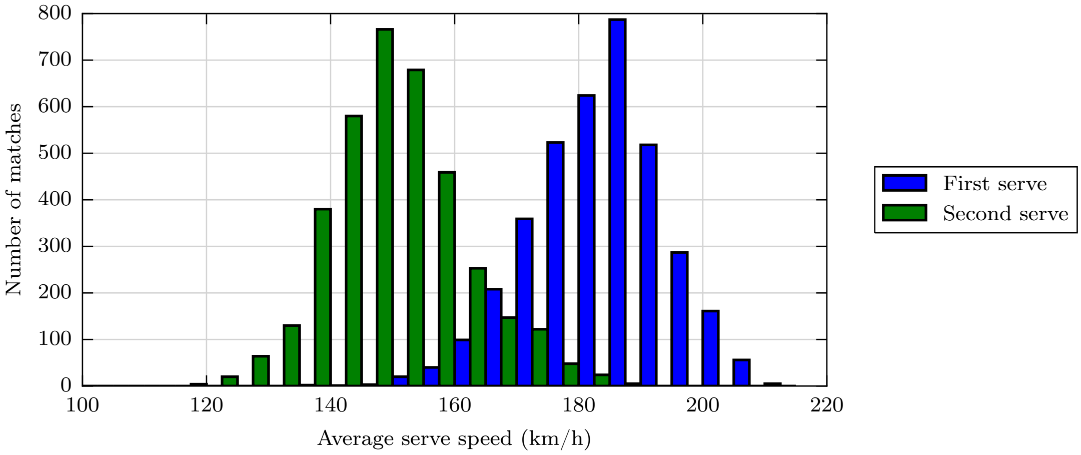
Invalid Percentages

All values representing percentages m must be real numbers between 0 and 1. The dataset contains some matches that have a value of greater than 1 for the first serve success rate, the winning on first serve percentage, the winning on second serve percentage, or the winning on return percentage. The percentages are then marked as invalid and ignored in any averages. The dataset contains 54 such records (from a total of about 80 000 matches with statistics).

Improbable Average Serve Speeds

By inspecting the dataset, we notice that for a few matches, the average serve speeds have a value of zero. This is clearly the result of an error in the generation of the data – missing serve speeds should be marked as invalid, not given the value of zero. Furthermore, some matches have highly unlikely values for serve speeds. From the distribution of ATP serve speeds shown in Figure 3.6, we can see that average serve speeds of less than 120 and 100 for first and second serves, respectively, must correspond to some sort of inaccuracy. In this case, we also set the values to invalid (approximately 40 matches are affected).

Figure 3.6: Average serve speeds (ATP)



Extreme Percentages After Averaging

If there are only a few matches used in performance estimates for a player (e.g., if the player has only participated in several ATP matches during their career), some estimates may result in extreme values. For example, if a player has a defensive playing style and they have only approached the net once in all their past matches, but succeeded in this one attempt, they will have an expected net approach success rate of 100%. This is, however, only due to the lack of data, and does not accurately reflect the quality of the player's net game.

To alleviate this problem, we can use the measure of uncertainty defined in Section 3.3. For matches with high uncertainty (i.e, those for which the feature estimates are less reliable), we can ignore features that are likely to be inaccurate. Features more prone to inaccuracy are those for which fewer observations are made. Specifically, break points, net approaches, and total matches won are all based on only a few observations per match (or a single observation, in the case of total matches won). We ignore these features if the uncertainty is below a specified threshold.

Despite the filtering based on uncertainty, some percentages retain extreme values (exactly 0 or 1). These values signify a lack of data, and as they are uncharacteristic of the performance of players, they are also ignored. For example, regardless of the number of matches used in generating an estimate of a player's break point win percentage, if the estimate is 100%, the actual probability of a player winning a break point in an upcoming match is certainly less than 100%.

Figure 3.7: Distribution of estimated break point winning percentages

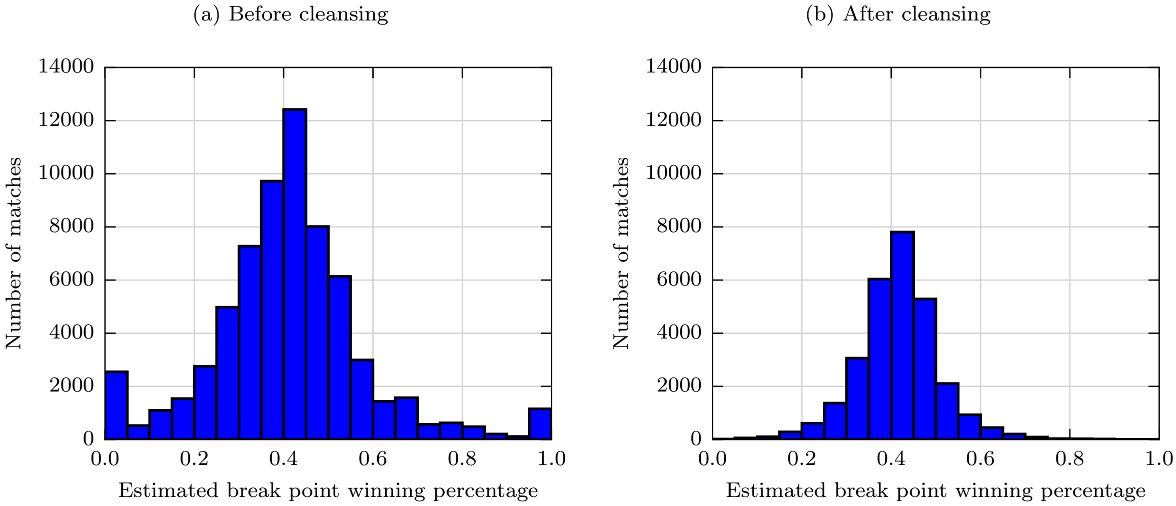


Figure 3.7 shows the effect of the filtering of break point winning percentage estimates with high uncertainty (in this case, we used a threshold of 1.0) and extreme percentage values. Firstly, we see than the filtering significantly reduces the number of matches with break points as a valid feature. However, the distribution now bears a closer resemblance to the normal distribution, with a more regular curve and no clusters at 0 and 1.

Finally, the **DIRECT** feature (Section 3.4.4) describes the head-to-head balance between the two players. However, an insufficient number of mutual matches can result in inaccurate predictions. As before, we only use the feature if its uncertainty (based on the number / relevance of mutual matches) is below a specified threshold. We do not, however, filter out the extreme values of 0 and 1. If a player has defeated another player in every match they have played, provided that they have played a sufficient number of matches, keeping this information is likely to improve our prediction accuracy.

3.5.2 Feature Scaling

By inspecting the distributions for performance estimates, we see that most are approximately normally distributed (e.g., the break point winning percentage in Figure 3.7). The match features (formed by taking the difference in performance estimates of the two players) are usually also normally distributed. In fact, of the features discussed so far, only the following do not seem to follow a normal distribution:

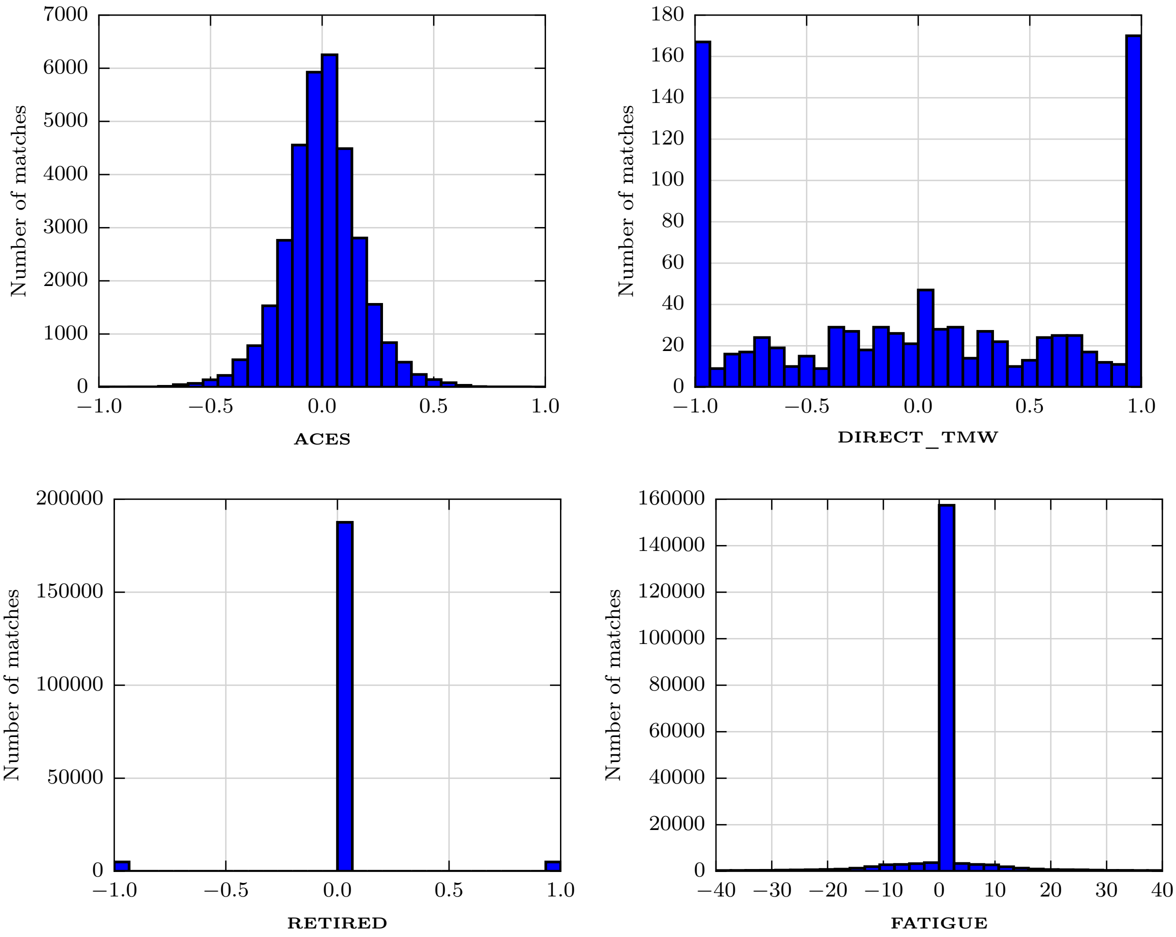
- **DIRECT** - the distribution of the head-to-head balance has clusters at 0 and 1, since it is very common for one player to always win / lose against another
- **FATIGUE** - in many matches, most players have a fatigue score of zero, resulting in a large spike in the middle of the distribution
- **RETIRED** - since the underlying variable is *binary*, the feature can only take on values in the set $\{-1, 0, 1\}$

Figure 3.8 shows the distribution of the **ACES** feature, which resembles the general shape of the distributions of most of our features. We also show the distributions of the three features described above, which are not normally distributed. For all features except these three, we perform *standardisation*, the scaling to unit variance. We standardise a feature $\mathbf{x}$ by dividing it by its standard deviation σ:

$$\mathbf{x}_{\text{standardised}} = \frac{\mathbf{x}}{\sigma} \tag{3.7}$$

We do *not* mean-center the features, since we expect the features to already have a mean of zero. This is a consequence of our symmetric approach to feature construction (Section 3.1.2). Since players are labelled

Figure 3.8: Feature distributions



as Player 1 and Player 2 *arbitrarily* for each match, there is no bias towards either of the players, and we expect the averaged difference in their performance estimates to be zero. This is confirmed in Figure 3.8 - all the distributions are already centered at zero. Mean-centering the features would therefore only have the effect of introducing bias due to random noise in the data.

Standardisation is an implicit requirement for many machine learning algorithms. For the algorithms we employ, logistic regression and neural networks, standardisation is not, in theory, a requirement. There are nonetheless several advantages of this pre-processing step:

1. If the features have unit variance, the weights assigned by logistic regression can be used to compare the relative significance of different features in determining the match outcome.
2. In both algorithms, *regularisation* has a stronger effect on features with greater values, so large differences in the standard deviations of feature distributions could penalise some features more than others (regularisation is explained in the following chapter).
3. Standardisation typically improves training times for neural networks [14].

Despite the feature **FATIGUE** not conforming to a normal distribution, its standard deviation of approximately 14.4 is a reasonable scaling factor, so we standardise this feature as well. The remaining two non-normal features (**DIRECT** and **RETIRED**) are left unscaled, as they tend to already take on values of the same order of magnitude as the scaled features.

3.6 Summary of Features

In Table 3.2, we provide a summary of all extracted features. Note that all features are *differences* in values for the two players, as discussed in Section 3.1.2. Also, the features **ACES**, **DF**, **UE**, **WIS** and **TPW** are normalised to per-game averages (by dividing by the number of games in a match), instead of the per-match averages provided in the OnCourt dataset (Table 2.1).

Table 3.2: Summary of all extracted features

Feature	Explanation	Cleansing	Standardised
RANK	ATP rank		Yes
POINTS	ATP points		Yes
FS	First serve success percentage	P, E	Yes
W1SP	Winning on first serve percentage	P, E	Yes
W2SP	Winning on second serve percentage	P, E	Yes
WSP	Overall winning on serve percentage		Yes
WRP	Winning on return percentage	P, E	Yes
TPW	Percentage of all points won	P, E	Yes
TMW	Percentage of all matches won	E, U	Yes
ACES	Average number of aces per game		Yes
DF	Average number of double faults per game		Yes
UE	Average number of unforced errors per game		Yes
WIS	Average number of winners per game		Yes
BP	Percentage of break points won	P, E, U	Yes
NA	Percentage of net approaches won	P, E, U	Yes
A1S	Average first serve speed	S	Yes
A2S	Average second serve speed	S	Yes
FATIGUE	Fatigue from matches in past 3 days		Yes
RETIRED	Whether first match since retirement		No
COMPLETE	Player completeness		Yes
SERVEADV	Advantage when serving		Yes
DIRECT	Head-to-head balance	U	No

P –	prior to averaging, remove if value is not in range $[0, 1]$
S –	prior to averaging, remove if serve speed is below 120 / 100 km/h for first / second serves
E –	after averaging, remove if value is exactly 0 or 1
U –	after averaging, remove if the uncertainty is above threshold (e.g., 1.0)

Chapter 4

Logistic Regression Model

4.1 Dataset Division

In the previous chapter, we presented our approach to the extraction of match features from OnCourt data. We form a machine learning dataset by extracting a vector of features for all matches, and pairing them with the corresponding match outcomes. The dataset contains ATP matches for the past 11 years (2004 to 2014). We use common opponents, time discounting and surface weighting in unison during feature extraction.

We split the dataset in the following way:

- **Training set (2004-2010)**

  Training data is used by logistic regression (or any other supervised learning algorithm) to train a model that minimises the error in the prediction of the outcomes of the training matches. In other words, the parameters of the model are adjusted to create the most optimal mapping from the training match feature vectors to training match outcome predictions. For logistic regression, the logistic loss function is used as a measure of error during training (see Section 2.5.2).

- **Validation set (2011-2012)**

  The validation set is used for tuning the hyperparameters of the model (those parameters not optimised by the training algorithm). We train a model on the training data using various combinations of values for the hyperparameters and assess each combination using the validation set. The selection of a feature selection strategy is also done using the validation set.

- **Test set (2013-2014)**

  After a model's hyperparameters have been optimised, we evaluate its predictive performance on the test data, giving us a measure of how well the model generalises to unseen data. In this case, we use all other data (training and validation) to train the model. Test data is set aside and *never* used before the evaluation phase.

The division of the data follows a 7-2-2 ratio of the time-ranges of different splits (7 years for the training set and 2 years each for the other two sets). However, we filter out a large portion of the training matches prior to training (Section 4.5.2). Also, many of the matches in the training and validation sets do not have betting odds, which we require for evaluating the model. Consequently, the number of matches in each of the dataset splits is approximately 39 000, 6 200 and 12 600 for the training, validation and test sets, respectively. We have chosen to split the dataset in this way despite unequal validation and test set sizes. Each year of professional tennis has a fixed structure, with certain tournaments always being played at particular times of the year. Splitting the dataset by complete years results in the same distribution of tournaments across the different splits, which we believe makes the evaluation more accurate.

We have chosen *not* to use cross-validation for evaluating our model. *Cross-validation* is often used in place of a test set in assessing how well a model generalises to an independent dataset. This method involves partitioning the entire dataset into k equal-sized subsets (folds), of which one is retained as the validation set and the others are used for training. The process is then repeated k times, using a different fold as the validation set each time. Cross-validation has the advantage of lower variance in model evaluation, in comparison to when a single test set is used. However, the entire model fitting procedure (feature selection, hyperparameter optimisation, etc.) would have to be performed separately for each fold. As will become apparent in the following sections, this would be computationally very expensive. Furthermore, our dataset has a *time-series* element: the matches are ordered by time. The most recent years are chosen as the test set, because these are most representative of the current state of tennis prediction. As the field progresses, it is becoming increasingly more difficult to compete against the bookmakers. Using only the past couple of years will yield a more accurate assessment of profitability of the model. Finally, our dataset is sufficiently large for cross-validation to be considered unnecessary.

When evaluating the model, we will only consider predictions for the first 50% of the matches, when ordered by uncertainty. In Section 3.3, we defined uncertainty for a match based on the weights assigned by time discounting and surfaces weighting during feature extraction. As we are more confident in the accuracy of features for matches with lower uncertainty, we expect to make a greater profit when betting on these matches. Therefore, we will not place bets on matches with high uncertainties. We will aim to maximise the profitability of the model for the most certain 50% of the matches. By ignoring half of the matches in this way, we obtain a more realistic evaluation of our model, unaffected by the noise induced by high uncertainty matches (on which bets would not be placed).

4.2 Evaluation Metrics

In feature selection and hyperparameter optimisation (Sections 4.4 and 4.5, respectively), we require metrics for evaluating the performance of our model. Our overall goal is to maximise the return on investment (ROI) when placing bets based on the predictions generated by our model. Various betting strategies result in a different ROI, so we consider different strategies when assessing our model. We have selected three strategies, discussed in Section 2.3.2. Of the three, betting on the predicted winner using the Kelly criterion seems to be most profitable, so this will serve as the main betting strategy used for evaluating a model.

A metric commonly used to assess predictor performance in literature is *accuracy*, the proportion of match outcomes correctly predicted. However, accuracy does not consider the deviations of the predicted probabilities from the actual match outcomes. The more successful betting strategies rely on these probabilities. For this reason, we instead employ logistic loss as an evaluation metric, which penalises any deviations from actual match outcomes (see Section 2.5.2). Although logistic loss is used to guide the training process of a logistic regression predictor, it can also serve as a metric for comparing the quality of different predictors. In fact, it is a *proper scoring rule*[1] for the assessment of probability predictions. Other popular scoring rules exist (e.g., the quadratic Brier score). However, logistic loss has been shown to have superior performance in certain conditions (see comparison of scoring rules conducted by Bickel [3]).

Perhaps surprisingly, we have found that logistic loss and ROI often give contradictory results. The model with the minimal logistic loss is not always the most profitable (note that we *minimise* logistic loss, but *maximise* profit). We therefore consider both metrics in our decisions regarding the optimisation of a model.

4.3 Model Symmetry

As described in Section 2.5.2, logistic regression optimises a vector of $n+1$ parameters $\boldsymbol{\beta}$ to obtain the best mapping from the n input features $\mathbf{x}$ to a match result (for the matches in the training dataset). In this generic formulation of the model, the probability of a win is computed by Equation 4.1.

[1] A proper scoring rule is one which gives the best score when the true probability distribution is predicted

$$P(\text{Player 1 wins}) = \sigma(z) = \frac{1}{1+e^{-z}} \tag{4.1}$$

Where

$$z = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n, \;\; \beta_i \text{ is the weight for feature } x_i$$

To maintain a symmetric model (Section 3.1.2), we remove the bias term β_0. This ensures that when the features are all zero (i.e., the players are expected to have identical performance), the predicted probability of a win will be 0.5. Also, we will obtain the same prediction of the match outcome, regardless of the order of the labelling of the players.

4.4 Feature Selection

Feature selection is the process of selecting a subset of all available features to use within a model. The main motivation for applying this technique to tennis match prediction is the possibility of improving prediction accuracy through the removal of irrelevant features. A model with fewer features has lower variance, which prevents overfitting to the training set. In addition, feature selection will allow us to gain insight into the relative importance of different features in predicting match outcomes.

4.4.1 Ignoring Rank Information

There is considerable evidence that ATP ratings do not accurately reflect a player's current form. Both Clarke [5] and Dingle [7] constructed alternative rating systems, which had better predictive power than the official ATP ratings. These authors argue that one of the biggest weaknesses of ATP ratings is their disregard for the quality of a player's opponents or the margin by which a match is won or lost. Instead, the ratings are a cumulative measure of a player's progression through tournaments. In fact, players are awarded points for a match even if they win due to a walkover (non-attendance by the opponent). Furthermore, a player has a single rating across all surfaces, preventing us from taking into account any difference in performance across surfaces.

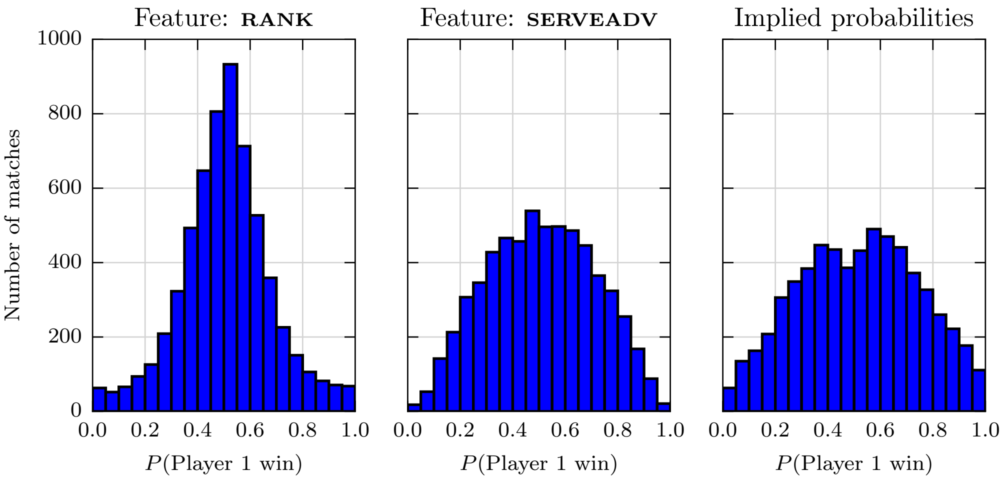
A difference in the ATP ranking of two players has a relatively strong correlation with the match outcome. In fact, as demonstrated by Clarke and Dyte [6], using **RANK** as the sole feature is sufficient to obtain a prediction accuracy of about 65%. However, these features are poor in predicting the true probabilities of match outcomes. As shown in Figure 4.1, using **RANK** as the sole feature in a logistic regression predictor results in a much narrower distribution of predicted probabilites than when **SERVEADV** is used. We can approximate the true probability distribution by the implied probabilites derived from betting odds, as shown in the right-most histogram in Figure 4.1. Clearly, the distribution resulting from using **SERVEADV** as the sole feature bears a much closer resemblance to the true distribution. The more profitable betting strategies require accurate probability estimates. For this reason, we have decided to henceforth exclude the **RANK** and **POINTS** features from our feature set, as they would be likely to distort the predictions.

4.4.2 Feature Selection Algorithms

A simple approach to feature selection would be to rank features according to their (absolute) correlation with the match outcome. However, as demonstrated by Guyon and Elisseeff [9], features influence each other when used in a machine learning algorithm. For example, two variables that are useless by themselves may be useful together. Therefore, instead of evaluating features separately, we select the *subset* of features which performs best.

We have extracted 22 features (Table 3.2), and after removing the **RANK** and **POINTS** features, 20 features remain, allowing for more than a million different subsets (2^{20}). It is computationally infeasible to perform an exhaustive search for the best subset. We implemented several techniques which use different heuristics for searching this space, as outlined by Guyon and Elisseeff [9]. The different feature selection techniques are applied to the training set. The validation set is then used to select the best-performing technique. During feature selection, the model hyperparameters are set to those that performed best with all features selected (hyperparameter optimisation in discussed in Section 4.5).

Figure 4.1: Distributions of probabilities for **RANK** and **SERVEADV**



Wrapper Methods

Wrapper methods use the underlying predictor as a black box for evaluating the performance of different subsets. There are two main flavours of greedy wrapper feature selection algorithms: *forward selection* and *backward elimination*. In forward selection, the features which cause the greatest improvement in the evaluation metric are progressively added, until all features have been added or no improvement is gained by adding additional features. Conversely, backward elimination begins will the full set of features and removes those whose elimination results in the greatest improvement in the evaluation metric. Algorithms 1 and 2 give the pseudocode for forward selection and backward elimination, respectively.

Algorithm 1 Forward Selection

```
procedure ForwardSelection($A, E$)    ▷ Given: set of features $A$ and evaluation metric $E$
    $F \leftarrow \emptyset$    ▷ Initialise the current set of selected features
    $F_B \leftarrow \emptyset$    ▷ Initialise the current best set of selected features
    while $|F| < |A|$ do    ▷ While more features may be added
        $x \leftarrow \text{argmax}_{x \in A-F} E(\{x\} + F)$    ▷ Select next best feature $x$ from remaining features
        $F \leftarrow \{x\} + F$    ▷ Add selected feature to feature set
        if $E(F) > E(F_B)$ then
            $F_B \leftarrow F$    ▷ Update best feature set
        end if
    end while
    return $F_B$    ▷ Return the best set of features
end procedure
```

Algorithm 2 Backward Elimination

```
procedure BackwardElimination($A, E$)    ▷ Given: set of features $A$ and evaluation metric $E$
    $F \leftarrow A$    ▷ Initialise the current set of selected features
    $F_B \leftarrow A$    ▷ Initialise the current best set of selected features
    while $|F| > 1$ do    ▷ While there are features remaining
        $x \leftarrow \text{argmax}_{x \in F} E(F - \{x\})$    ▷ Select next best feature $x$ from current features
        $F \leftarrow F - \{x\}$    ▷ Remove selected feature from feature set
        if $E(F) > E(F_B)$ then
            $F_B \leftarrow F$    ▷ Update best feature set
        end if
    end while
    return $F_B$    ▷ Return the best set of features
end procedure
```

The assessment of a subset of features proceeds by first training the predictor on a portion of the training data (years 2004-2008), and then evaluating its performance on another portion (2009-2010). Both algorithms require an evaluation metric for this purpose. The advantage of wrapper methods is the freedom of choosing a custom evaluation metric. Therefore, in addition to logarithmic loss, we also perform feature selection using ROI.

Embedded Methods

An alternative to wrapper methods are *embedded* feature selection methods, which incorporate feature selection as part of the training process of the predictor. One such method is *Recursive Feature Elimination* (we use the name adopted by the machine learning library Scikit-Learn [19]). In RFE, a logistic regression predictor is first trained using the set of all features. Then, the feature which is assigned the lowest absolute weight in the predictor is removed, and this is repeated until a single feature remains (see Algorithm 3). RFE uses the heuristic that a lower absolute weight is likely to correspond to a less important feature. The validation set is used to determine the optimal number of features to remove. As with the wrapper approaches, we can optimise both the logistic loss and the ROI.

RFE is often preferred over the wrapper methods due to its lower time complexity. The wrapper methods must train and evaluate the predictor many times at each step in the algorithm. Specifically, Step 5 in Algorithms 1 and 2 requires the evaluation of n subsets if there are n features remaining (to be added or removed). Although we perform this step in parallel across multiple cores, the time complexity is still quadratic in the number of features. On the other hand, RFE only requires the predictor to be trained once each time a feature is removed. However, for our current size of the feature set, efficiency is not yet a limiting factor.

Algorithm 3 Embedded Recursive Feature Elimination

```
procedure RFE($A, E$)                          ▷ Given: set of features $A$ and evaluation metric $E$
    $F \leftarrow A$                           ▷ Initialise the current set of selected features
    $F_B \leftarrow A$                         ▷ Initialise the current best set of selected features
    while $|F| > 1$ do                         ▷ While there are features remaining
        $x \leftarrow \text{argmin}_{x \in F} |weight(x)|$   ▷ Select feature with lowest absolute weight
        $F \leftarrow F - \{x\}$               ▷ Remove selected feature from feature set
        if $E(F) > E(F_B)$ then
            $F_B \leftarrow F$                 ▷ Update best feature set
        end if
    end while
    return $F_B$                               ▷ Return the best set of features
end procedure
```

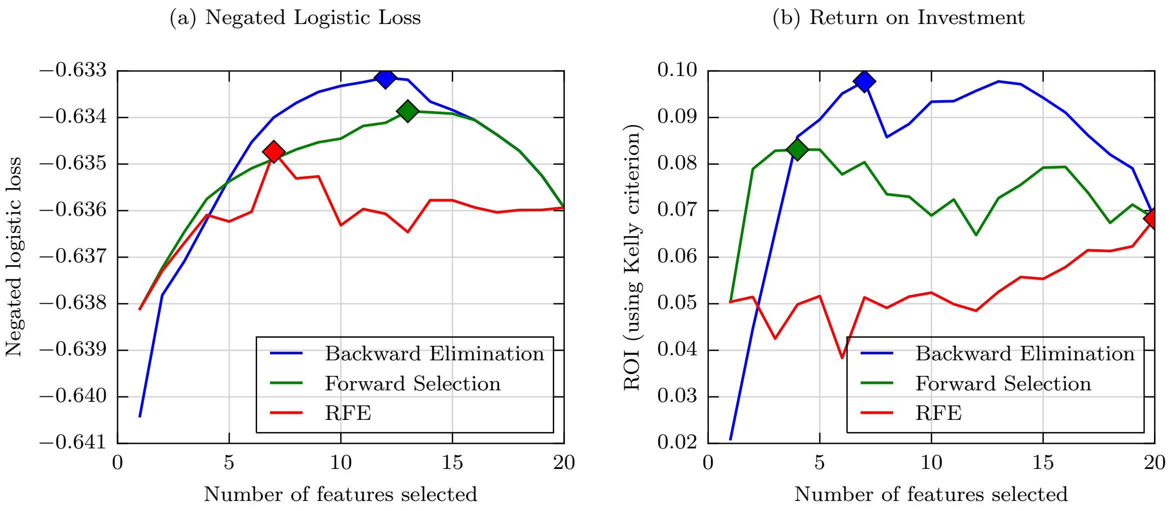
4.4.3 Results

Figure 4.2 shows the optimal number of features selected by the different feature selection approaches discussed in the previous sections. Each approach selects a different optimal number of features (marked by a diamond). For example, backward elimination selected 12 features when using logistic loss for the comparison of different subsets.

It would appear that backward elimination finds the most optimal subsets. However, we need to assess how well each strategy generalises to the validation set. For this reason, we evaluate the performance of the subset selected by each feature selection strategy using the validation set. From Table 4.1, we can see that all approaches that use the logistic loss evaluation metric have relatively similar performance on the validation set. However, the only approach that outperforms the benchmark (i.e., using all 20 features) in terms of logistic loss is forward selection. This strategy also offers the best ROI of the three, improving upon the benchmark by 1.9%.

When ROI is used as the evaluation metric for RFE, it chooses to retain all features. In the remaining two cases, logistic loss is increased. Forward selection with ROI as the evaluation metric results in the greatest ROI of all strategies (11.3%). However, this is considerably greater (by about 3%) than its ROI on the training set. Backward elimination using ROI, on the other hand, suffers a significant drop in performance when evaluated on the validation set. It appears that using ROI for optimisation results in unpredictable performance on unseen data. Also, Figure 4.2b shows that different adjacent subsets (those with one feature added or removed) have very large differences in profit, confirming the high volatility of ROI. Logistic loss appears to be a much more stable metric. As we want to achieve the best performance on the test set, it is imperative that the feature selection strategy we use will generalise well. For this reason, we select forward selection with logistic loss as the optimal feature selection strategy.

Figure 4.2: Optimal feature subset sizes using different approaches


(a) Negated Logistic Loss

(b) Return on Investment

Table 4.1: Evaluation of feature selection approaches

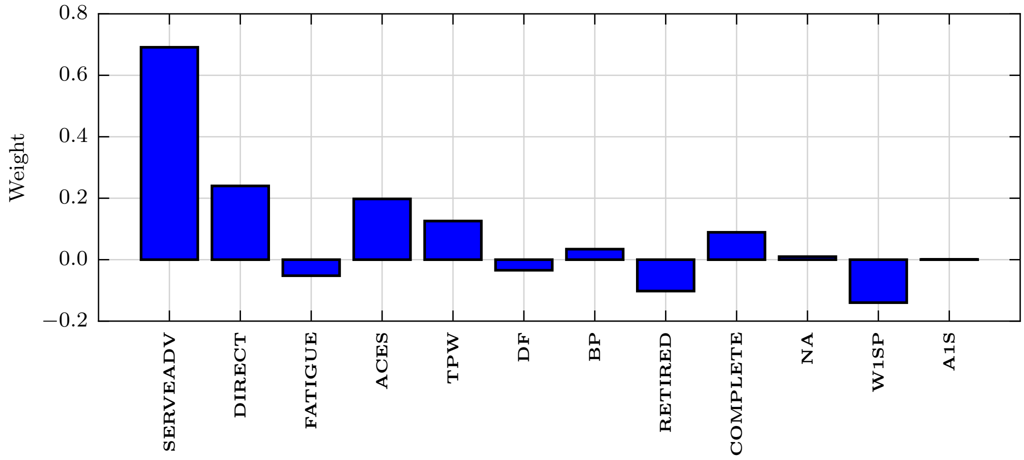
Approach	**Evaluation Metric**	**Features Selected** (in order of importance)	**Performance on Validation Set**	
			Log-loss	ROI % (Kelly)
Backward Elimination	log-loss	**COMPLETE**, **WRP**, **W1SP**, **ACES**, **W2SP**, **FATIGUE**, **DIRECT**, **NA**, **DF**, **WIS**, **A2S**, **TPW**	0.5764	7.1
Forward Selection	log-loss	**SERVEADV**, **FATIGUE**, **DF**, **DIRECT**, **TMW**, **WIS**, **TPW**, **A2S**, **NA**, **ACES**, **COMPLETE**, **WSP**, **W1SP**	0.5743	9.7
RFE	log-loss	**SERVEADV**, **TMW**, **DIRECT**, **FS**, **COMPLETE**, **W2SP**, **ACES**	0.5765	7.0
Backward Elimination	ROI	**COMPLETE**, **FS**, **WSP**, **UE**, **FATIGUE**, **WIS**, **A1S**	0.5803	8.2
Forward Selection	ROI	**SERVEADV**, **W2SP**, **BP**, **A2S**	0.5799	11.3
RFE	ROI	**SERVEADV**, **TMW**, **DIRECT**, **FS**, **COMPLETE**, **W2SP**, **ACES**, **WRP**, **W1SP**, **UE**, **NA**, **BP**, **RETIRED**, **FATIGUE**, **WSP**, **A1S**, **DF**, **TPW**, **A2S**, **WIS**	0.5762	7.8
None		All features	0.5762	7.8

Having settled on the feature selection strategy, we now use all training and validation data (2004-2012) to generate the **final feature set**:

SERVEADV, **DIRECT**, **FATIGUE**, **ACES**, **TPW**, **DF**, **BP**, **RETIRED**, **COMPLETE**, **NA**, **W1SP**, **A1S**

The final set of features has a large overlap with the subset selected when only the training set was used. In fact, 9 of the 12 selected features were also selected previously. This suggests that the strategy will select a relative stable subset of features as the dataset grows. We can also visualise the weights assigned to each feature by the training process. Figure 4.3 shows that **SERVEADV** has the greatest impact on the outcome of the match, followed by **DIRECT** and **ACES**. As we would expect, **FATIGUE**, **RETIRED**, and **DF** all have negative weights. It may come as a surprise that the difference in the estimated winning on first serve percentage (**W1SP**) is also assigned a negative weight. However, as mentioned previously, features are not trained independently, and affect one another. Therefore, **W1SP** might be given a negative weight to balance out the effect of one or more of the features with positive weights.

Figure 4.3: Weights assigned to final feature set



4.5 Hyperparameter Optimisation

The process of training a logistic regression predictor tunes the regression coefficients, i.e., the internal weights for each feature. However, the model also has parameters which are not optimised by the training process, termed *hyperparameters*. In order to achieve the best performance of the model, we optimise these parameters using the validation set (years 2011-2012 of the dataset). To reiterate, we optimise only according to the performance of the least uncertain 50% of the matches in the validation set (see Section 4.1).

4.5.1 Optimisation Approach

The most common approach to hyperparameter optimisation is *grid search*, a brute-force method that exhaustively searches the entire space of different hyperparameter configurations. Most of our hyperparameters are real-valued, and the tuning is fine-grained. A sufficiently fine-grained grid search would be prohibitively expensive for our purposes.

We instead proceed by a greedy heuristic search, which at any point optimises the parameter that, when altered, will cause the greatest improvement in an evaluation metric (e.g., logistic loss or ROI). We iterate until all parameters have settled in their global maxima. Since hyperparameters are not necessarily independent, it is entirely possible that this approach will terminate in a local maximum, or may not terminate at all. However, we found that this process worked very well for our use case.

Note that this process is intentionally *not* automatic, but guided by human decisions. As shown during feature selection, we must often trade off the optimisation of logistic loss and the optimisation of ROI.

We expect to achieve the best results by reasoning about this trade-off for each hyperparameter and making a conscious decision using our knowledge of the strengths and weaknesses of each metric.

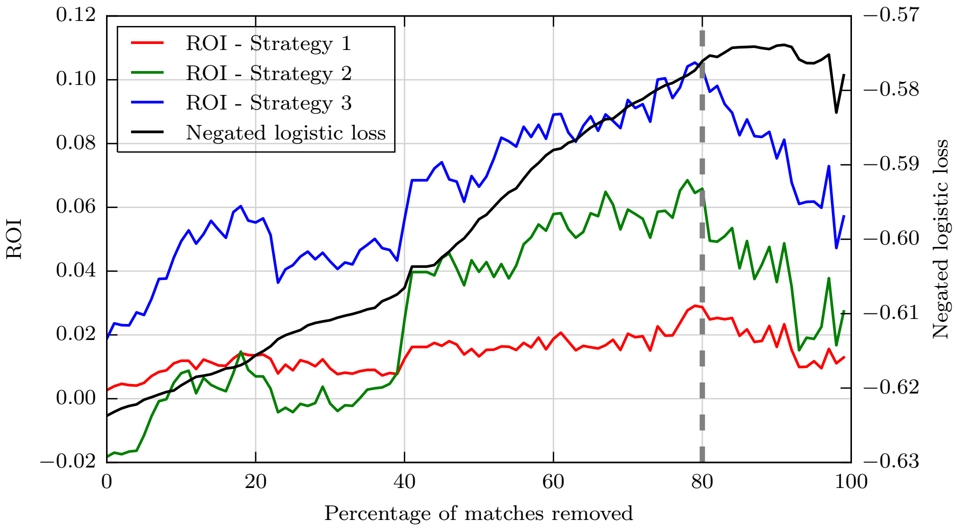
The set of features used for prediction may also be considered a model hyperparameter. Consequently, feature selection (described in the previous section) is also a part of the optimisation process. Clearly, we do not want to select features based on the performance of a very sub-optimal model. Therefore, we incorporate feature selection into the process as follows:

1. Perform an initial optimisation of hyperparameters using all features (except for **RANK** and **POINTS**, as discussed in Section 4.4.1)
2. Run feature selection to obtain a new feature set
3. Re-optimise using the new feature set

4.5.2 Noise Removal

Our training set contains matches with varying degrees of uncertainty. Matches with very high uncertainty (e.g., those where the two players have very few common opponents) can be treated as *noise* in the input data. By removing these matches, the predictor will be able to more accurately model the true underlying relationships in the dataset. We order all matches in the training set by their uncertainty, and run an optimisation to find the best percentage of higher-uncertainty matches to remove. Figure 4.4 shows our results. There is a clear improvement in both ROI and logistic loss as noisy matches are removed. We fix the hyperparameter value at 80%, the peak of the ROI. Although the logistic loss keeps improving past 80%, there is a sharp undesirable drop in the ROI after this point.

Figure 4.4: ROI and logistic loss when removing uncertain matches

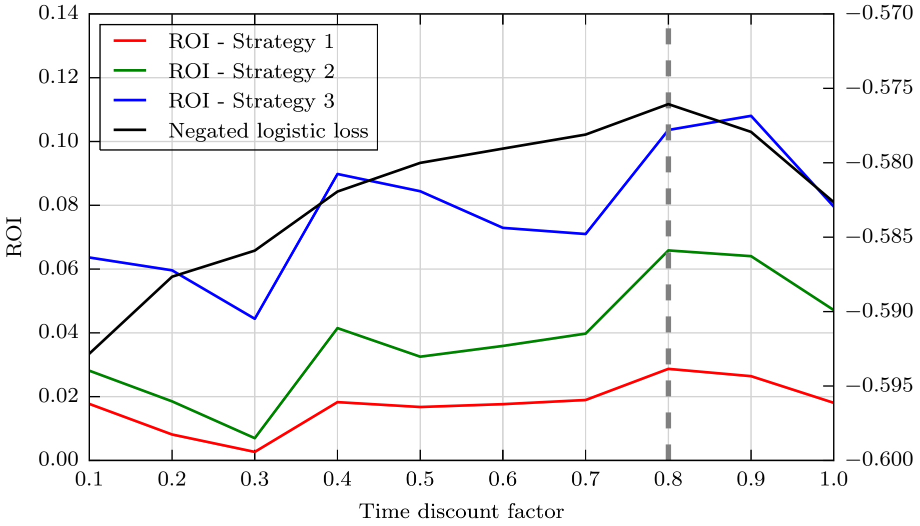


We have also considered removing training matches with noise in their output values. The betting odds for a match could be used to identify matches that had very surprising results. We could, for example, remove all matches for which the winner had an implied probability of winning of less than 30%. However, we found that a very small number of matches were affected by such filtering, and it thus had no considerable effect on the ROI or the logistic loss.

4.5.3 Time Discount Factor

Time discounting of past matches while extracting features requires a discount factor, which is a hyperparameter in our model. Essentially, the higher the discount factor, the lesser the effect of time discounting. In Figure 4.5, we see that the logistic loss peaks at a discount factor of 0.8, which we select as the optimal value. Although the ROI is maximised by using a factor of 0.9, the difference is small enough to be attributed to the volatility of ROI. The graph shows large fluctuations in profit for small changes in the discount factor (e.g., more than 2% for factors 0.3 and 0.4).

Figure 4.5: ROI and logistic loss for different time discount factors

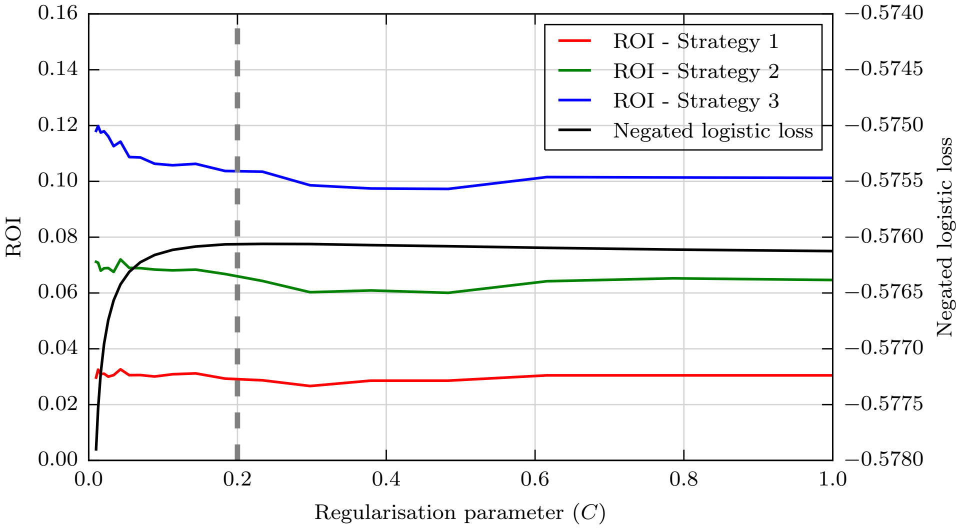


The optimal value of 0.8 for the discount factor has an interesting practical interpretation. It signifies the rate at which the performance of players changes over time. For example, as a player ages, we would expect matches played in the preceding year to be 80% as good an approximation of their current form as matches played this year.

4.5.4 Regularisation Parameter

Regularisation prevents overfitting to training data by penalising large weights when training a logistic regression predictor. The effect of regularisation is controlled using a regularisation parameter C. The lower the value of C, the stronger the effect of regularisation (the default value is 1.0). Figure 4.6 shows that increased regularisation improves logistic loss (although the effect is very minor – note that the y-axis on the right has very small increments). Conversely, there seems to be a slight increase in ROI as C is made smaller. Therefore, we choose $C = 0.2$, which appears to give reasonable results for both evaluation metrics.

Figure 4.6: ROI and logistic loss for different values of C



Chapter 5

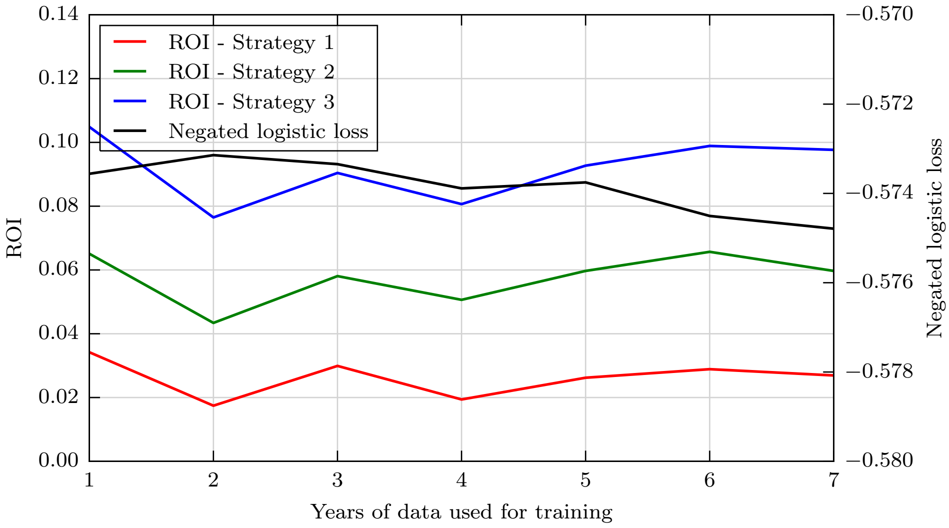
Higher Order Models

5.1 Bias and Variance

Errors in the predictions of tennis match outcomes can be classified into two categories: those due to *bias* and those due to *variance*. Bias results from erroneous assumptions of the model, i.e., its inability to accurately capture the relationships in the data (under-fitting). For example, we may be missing some important match features, and for this reason our predictions may be systematically wrong. On the other had, variance is the sensitivity of the model to small variations in the dataset. A model with high variance does not generalise well to unseen data (overfitting). There is always a trade-off between bias and variance. A more complex model can more accurately capture relationships in the data, but it will not generalise as well.

Model variance can typically be reduced by using a larger dataset, since the model will then be less likely to overfit to noise in the data. Figure 5.1 shows that the predictive power of our logistic regression model is relatively constant when we vary the size of the training dataset. The best return on investment oscillates between 8% and 10% and the error, as measured by logistic loss, actually grows (although very slightly) with more training data. These results suggest that our model has low variance, and to improve its performance, we could attempt to reduce the bias. The following sections present two different approaches to increasing the expressiveness of the model.

Figure 5.1: ROI and logistic loss on validation set for different training set sizes



5.2 Logistic Regression with Interaction Features

5.2.1 Constuction of Interaction Features

Logistic regression is a generalised *linear* model. As described in Section 2.5.2, it computes the probability of a match outcome using the weighted sum of the values of the match features:

$$P(\text{Player 1 wins}) = \sigma(z) = \frac{1}{1+e^{-z}}$$

Where

$$z = \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n, \ \ \beta_i \text{ is the weight for feature } x_i$$

For this reason, the model can only fit a linear decision boundary to the feature space and higher-order relationships between the features cannot be represented. A common approach to allowing a non-linear decision boundary is the introduction of *interaction* terms, the weighted *products* of features. After adding interaction terms, z becomes:

$$z = \overbrace{\beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n}^{\text{original terms}} + \overbrace{\beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \cdots + \beta_{(n-1)n} x_{n-1} x_n}^{\text{interaction terms}}$$

In total, there are $\binom{n}{2}$ additional features, one for each unique product of two of the original features. This model can now identify higher-order relationships in the data, resulting in lower model bias. However, increasing the complexity of the model will increase its variance, making it more prone to overfitting. Also, both the training and optimisation of the model will be more computationally expensive.

We have already extracted one interaction feature: **COMPLETE**. *Completeness* for player i was defined as the product of their serve and return strengths ($\mathbf{WSP}_i \cdot \mathbf{WRP}_i$). Notice that we compute the product *before* taking the difference in the values of the two players. This is a distinction between our approach and the standard application of interaction features. In general, the interaction feature **A_B**, based on features **A** and **B**, is computed as follows:

$$\mathbf{A\_B}_i = \mathbf{A}_i \cdot \mathbf{B}_i$$
$$\mathbf{A\_B} = \mathbf{A\_B}_1 - \mathbf{A\_B}_2$$

We have decided to exclude any interaction features formed using the **RETIRED** value for a player, due to its binary nature (it would either invert the sign of the other multiplicand or have no effect). This gives us with a total of $\binom{19}{2} + 20 = 173$ features.

5.2.2 Model Optimisation

A larger feature set increases the likelihood of our model overfitting to noise in the training data. As before, we can run a feature selection algorithm to select a subset of features with the best performance. It is essential that the chosen algorithm generalises well, so we re-evaluate all three approaches (backward elimination, forward selection, and RFE) using the new, higher-order model. As evaluating subsets using ROI previously failed to generalise well, we only use logistic loss to compare different subsets. Figure 5.2 shows the results of running the three feature selection strategies on the training set. Backward elimination selects a far greater number of features than the other approaches (57), but this subset has the best performance on the training set. Note that during feature selection, the model hyperparameters are set to the most optimal values derived for the basic logistic regression model in Section 4.5.

Table 5.1 shows that the feature subsets selected by the three approaches result in similar logistic loss when evaluated on the validation set. The only approach that improves upon the benchmark (i.e., using all features) in terms of logistic loss is backward elimination. Although the best ROI is achieved by the benchmark, the difference is very minor in comparison to backward elimination. Therefore, we select backward elimination as the feature selection strategy.

The final feature set for use by the model is selected by running backward elimination on all training and validation data. This time, only 19 features are selected. Furthermore, as shown in Figure 5.3, all

Figure 5.2: Feature selection with polynomial features

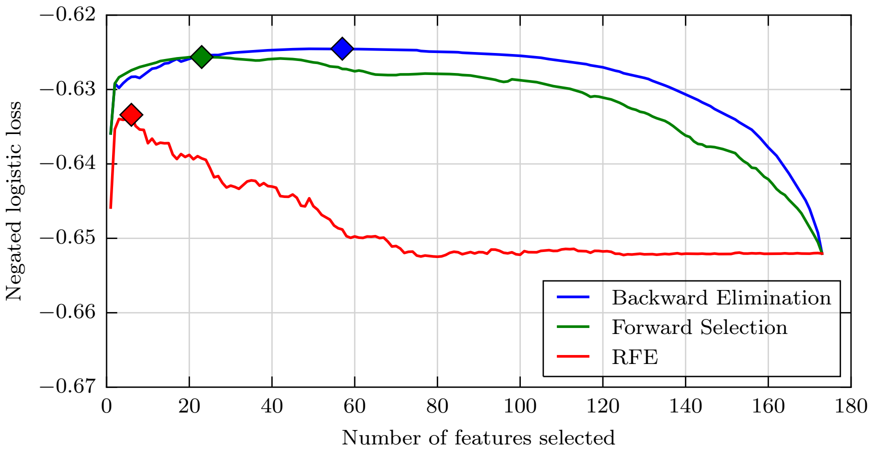


Table 5.1: Evaluation of feature selection approaches

Approach	**Number of features selected**	**Performance on Validation Set**	
		Log-loss	ROI % (Kelly)
Backward Elimination	57	0.5737	8.2
Forward Selection	23	0.5753	7.8
RFE	6	0.5775	6.3
None	All features (173)	0.5745	8.4

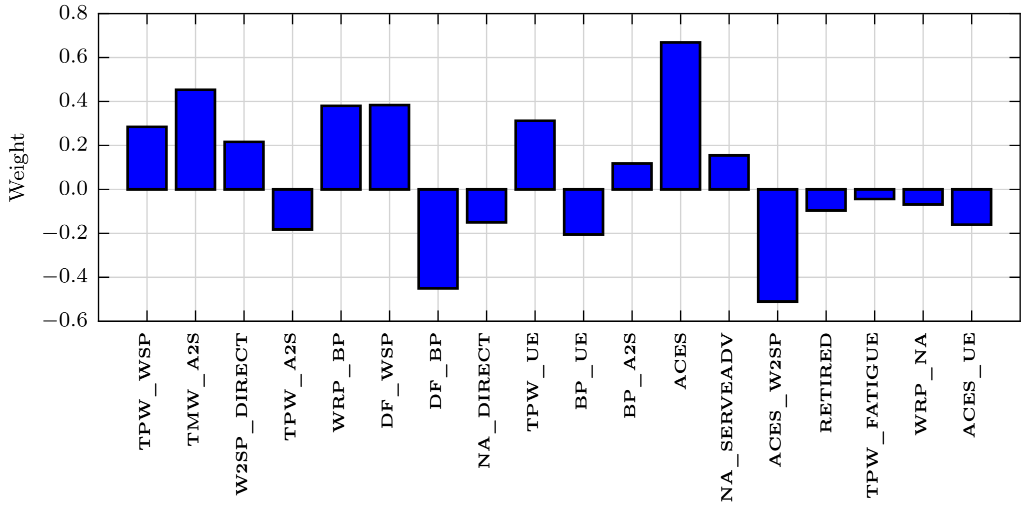
but two of the selected features are interaction features (**ACES** and **RETIRED** are the only two "original" features in the selected subset). Interestingly, **ACES** has replaced **SERVEADV** as the feature with the greatest weight. The meaning of many selected interaction features is difficult to grasp, and some seem completely non-sensical. For example, the large negative weight assigned to **ACES_W2SP** suggests that a player that hits many aces and has strong performance on their second serve is less likely to win. However, the weights of different features should not be considered independently, as they affect one another. For example, it is entire possible that **ACES_W2SP** would be assigned a positive weight if some additional features were removed. With the addition of interaction features, the model has become too complex to allow for an interpretation of the assigned weights.

A different feature set may require the re-optimisation of the model hyperparameters. However, the optimal value of only a single hyperparameter affected by the introduction of interaction features: the regularisation parameter C. We decided to decrease the value of C from 0.2 to 0.1, implying that the higher-order model performs slightly better with stronger regularisation. This is consistent with our expectations. Regularisation reduces the variance of a model, so a more variable model, as obtained by the introduction of interaction features, will benefit from stronger regularisation.

5.3 Artificial Neural Network

In the previous section, we have attempted to decrease the model bias by introducing additional features in a logistic regression predictor. These features allow the representation of more complex relationships in the data. However, the approach is clearly not scalable. Modelling interactions between triples of features would necessitate $\binom{19}{3} = 969$ additional features. Such a model would be highly susceptible to overfitting, and our feature selection approaches would no longer be feasible.

Figure 5.3: Weights assigned to final feature set (logistic regression with interaction features)



An alternative approach involves the use of an artificial neural network (described in Section 2.5.3). ANNs can model highly complex functions of the input features. The output values of neurons in the hidden layer may be influenced by many (or all) of the features. Such a representation may uncover significant relationships between the features, which would be ignored in a lower-dimensional model. However, the use of ANNs brings new challenges. Firstly, ANNs take significantly longer to train than logistic regression models. On our dataset, a logistic regression model took at most several seconds to train, while a neural network takes 10 minutes or more, depending on the hyperparameters. The model configuration is also more difficult, and forms an active research area. There are many hyperparameters to optimise (structure of the network, learning rate, momentum, etc.), and many parameters have strong dependencies on the values of other parameters.

The task of training a neural network for tennis match prediction has been attempted by Somboonphokkaphan et al. [22]. There are some essential differences in comparison to our model. Firstly, the authors used a simple averaging approach to feature extraction, without surface weighting or time discounting. Instead, the surface was fed as an additional input feature to the network (i.e., a binary input node for each possible surface). More importantly, separate input features were used for the average statistics of the two players, in contrast to our features of *differences* (Section 3.1.2), introducing asymmetry into the model. In addition, there is no mention of feature standardisation and mean-centering (which would be beneficial in this case). Although the authors claim an average accuracy of about 75% in predicting the matches in the Grand Slam tournaments in 2007 and 2008, there is no assessment of the actual probabilities predicted (using ROI or a scoring rule such as logarithmic loss). We have attempted to replicate the experiment as described in the paper, but we were unable to reproduce the results.

5.3.1 Network structure

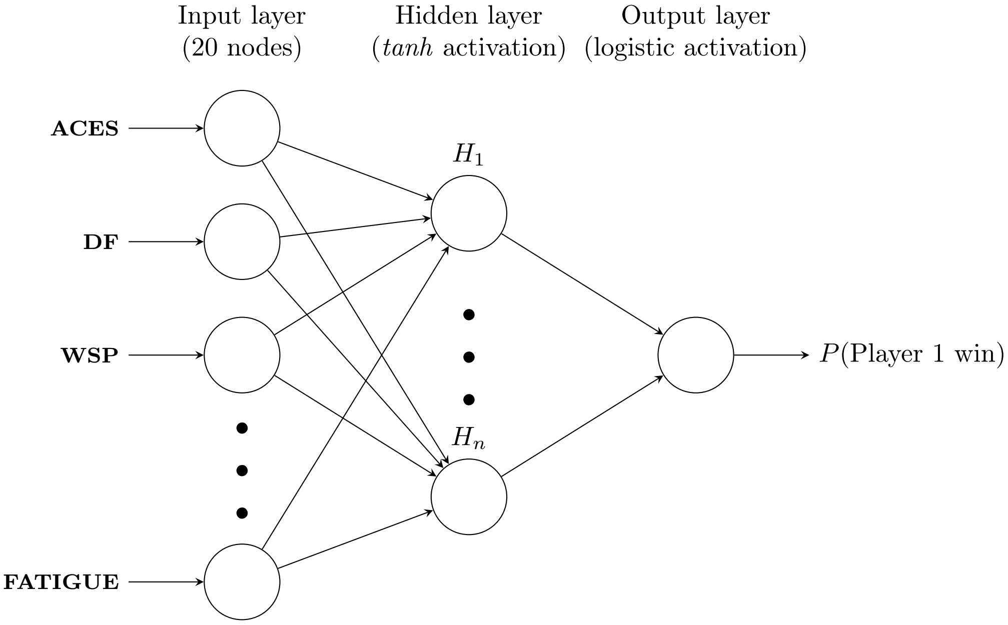
To reduce the size of the hyperparameter space, we fix some aspects of the structure of the network heuristically. The overall architecture of the network is that of a multilayer perceptron (MLP), a feed-forward network trained by backpropagation. We use all features (except for rank-related information) as inputs. The filtering of training data based on uncertainty and the time discount factor are set to their optimal values for logistic regression (80% and 0.8, respectively).

We use a single hidden layer. Hornik [10] showed that a single hidden layer with a finite number of neurons can approximate any continuous function, provided that a sufficient number of hidden neurons is used. It is generally accepted that a single layer is sufficient for most networks. However, as shown in Figure 5.4, the number of neurons in the hidden layer remains a hyperparameter that we must optimise.

The most common activation functions are sigmoid “squashing” functions. One such function is the logistic function, which has a range of $[0, 1]$, and is also used in logistic regression. An alternative is the hyperbolic tangent (*tanh*) function, with a range of $[-1, 1]$. LeCun [14] argues that symmetric sigmoids such as the *tanh* function result in lower training times, so we use the *tanh* activation function in our hidden neurons. However, the final value returned by an activation of the network (i.e., the output of the single neuron in the output layer) must be interpretable as a valid probability. For this reason, we use the logistic function for activation in the output neuron (an alternative approach would be to use *tanh* in the output neuron, and then remap the outputs to valid probabilities).

Conventionally, a network contains a *bias neuron*, which is connected to every non-input node in the network. The bias neuron always emits a constant value (e.g., 1), and allows a horizontal shift in the activation functions of individual neurons. However, as explained in Section 3.1.2, we strive to obtain a symmetric model, i.e., one that would give the same prediction if the players were labelled in reverse. By excluding the bias neuron, we are ensuring that the network is unable to give an unfair advantage to either player. In other words, if all input features are zero (the players are expected to have identical performance), the network output (the probability of Player 1 winning) is guaranteed to be exactly 0.5, regardless of the weights assigned by backpropagation. Furthermore, the exclusion of bias reduces the complexity of the network and thus helps prevent overfitting.

Figure 5.4: ANN architecture



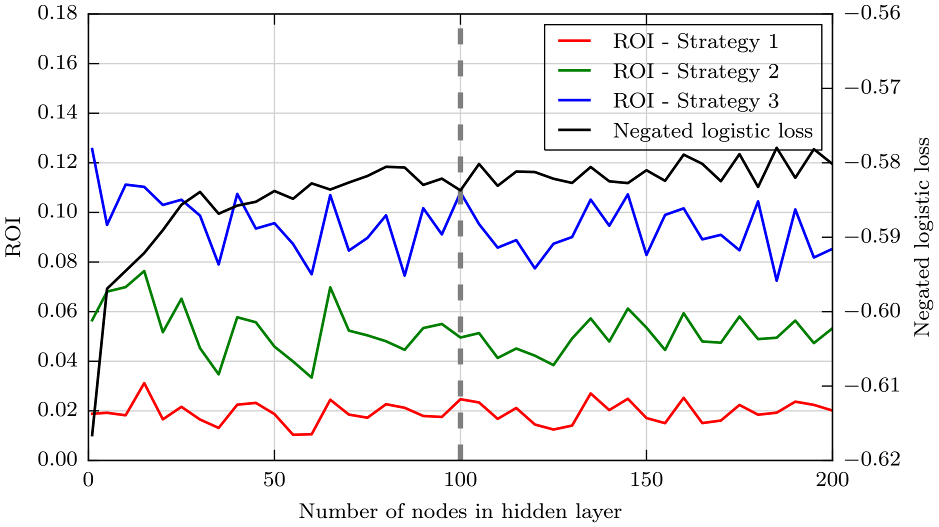
5.3.2 Model optimisation

We adopt the same approach to hyperparameter optimisation as for logistic regression: at each step, we optimise a single parameter (based on its performance on the validation set) and then re-run the analysis for the remaining parameters. This iterative process is more difficult this time, since the hyperparameters have stronger dependencies. For example, changing the learning rate requires re-calibrating regularisation, momentum, etc.

Number of hidden nodes

The number of hidden nodes affects the generalisation ability of a network. A network with a high number of hidden nodes has higher variance, and is therefore more likely to overfit to the training data. Too few hidden nodes, on the other hand, can result in high bias. There are various heuristics for selecting the number of hidden nodes, based on the number of training examples and the number of input / output nodes. Different sources advocate different “rules of thumb”, and there seems to be little consensus in the matter. Sarle [21] claims that these rules are “non-sense”, since they ignore the amount of noise in the data and the complexity of the function being trained. They also do not take into consideration the amount of regularisation and whether early stopping is used. Sarle suggests that in most situations, the only way determine the optimal number of hidden units is by training several networks and comparing their generalisation errors.

Figure 5.5: ROI and logistic loss for different numbers of nodes in hidden layer

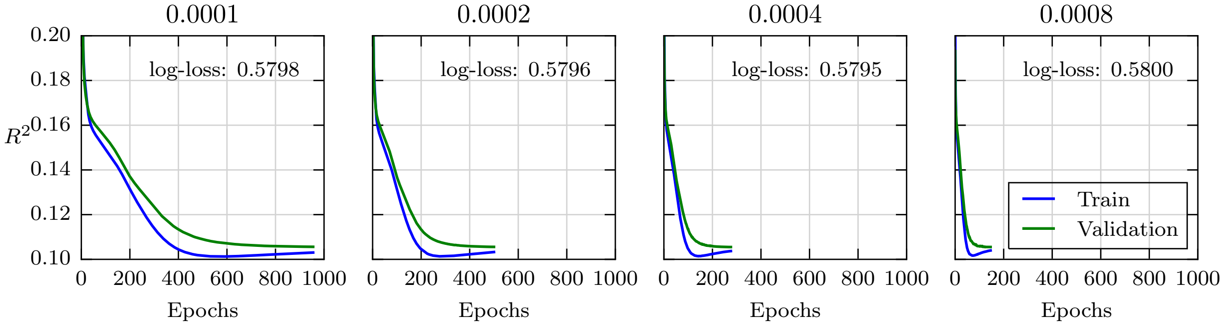


As shown in Figure 5.5, the profit is highly variable for networks of different sizes, oscillating between 8% and 10% when betting with Strategy 3. However, the logistic loss seems to improve with the number of nodes, especially when the number of nodes is less than 50. There does not seem to any significant benefit of having more than 100 nodes, and since it is in our interest to keep the network as small as possible (to reduce variance and training time), we fix the value of this parameter at 100.

Learning rate

The learning rate parameter determines the extent to which the current training set error affects the weights in the network at each training epoch. A higher learning rate results in faster convergence, but due to the coarser granularity of the weight updates, it may prevent the learning algorithm from converging to the optimal value. We can illustrate this by plotting *learning curves* for different learning rates, which show the evolution of the training and validation set errors during the training process (Figure 5.6). A learning rate of 0.0001 takes more than three times as long to converge as a learning rate of 0.0004. Also, the error on the validation set, as measured by logistic loss, actually slightly decreases from 0.5798 from 0.5795 when 0.0004 is used. A further increase in the learning rate (to 0.0008) results in a less significant improvement in training time and an increase in logistic loss. Therefore, we select 0.0004 as the learning rate.

Figure 5.6: Learning curves for different learning rates

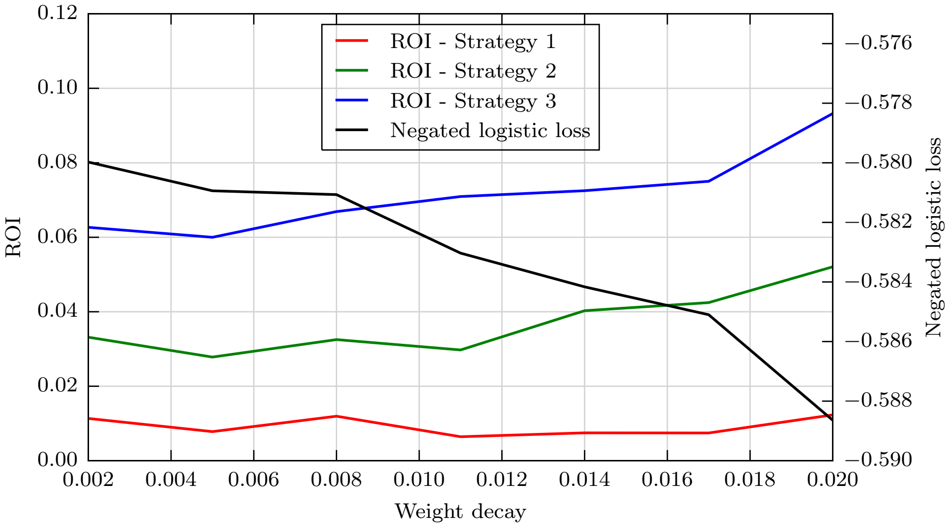


The learning rate does not need to remain constant during training. An *adaptive* learning rate can change during training so as to take smaller steps when converging to a value. One such approach is *learning rate decay*, which exponentially shrinks the learning rate during training. Although we have not attempted to incorporate adaptive learning rates into the model (to minimise the hyperparameter search space), this is a possible future improvement of the model.

Regularisation

Regularisation in neural networks can be achieved through *weight decay*, the exponential shrinkage of weights during training. For example, a weight decay parameter of 0.01 means that that the updated weights are shrunk by one percent during each training epoch. This prevents weights from becoming very large, which helps avoid overfitting. The approach is analogous to the C parameter in logistic regression.

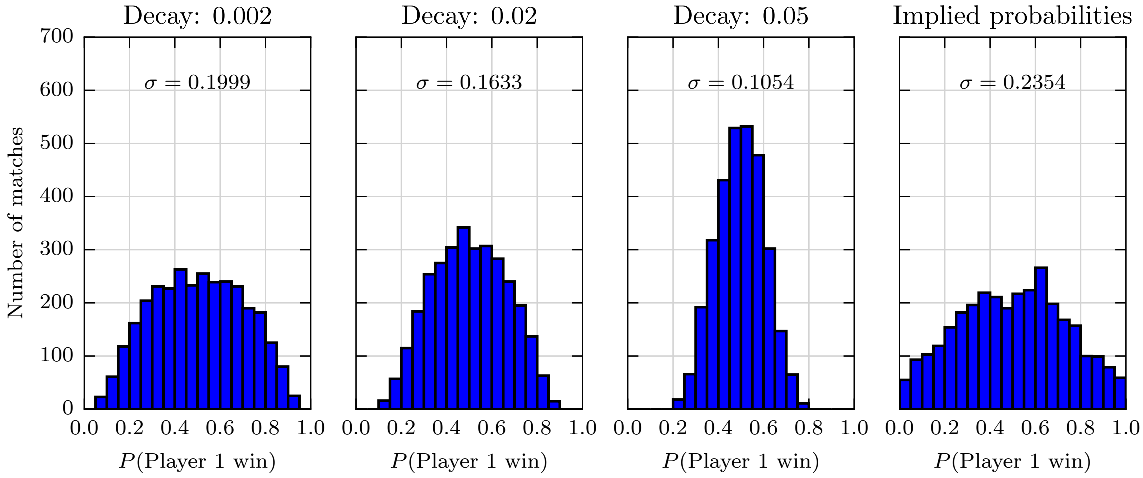
Figure 5.7: ROI and logistic loss for varying weight decay



In Figure 5.7, we can see that the error on the validation set grows with increased regularisation. This means that the decay of weights prevents the predictor for modelling relationships in the dataset as accurately. However, we can also see a clear upward trend in the return on investment. As we increase the weight decay from 0.002 to 0.02, the ROI grows by over 3%. Further analysis shows that the trend continues even for unreasonably large values of weight decay: stronger regularisation results in higher profits, despite greater logistic loss. This counter-intuitive phenomenon can be explained as follows: strong regularisation forces the weights in the network to be smaller and therefore the predicted probabilities are more moderate (i.e., closer to 0.5). As a result, bets are only placed on matches with a greater mis-pricing of the odds, resulting in greater profits. Notice that Strategy 1 (betting on the predicted winner) is unaffected by the magnitude of the probability values and thus remains constant with stronger regularisation. This graph would suggest maximising regularisation to achieve the maximum returns. However, stronger regularisation also reduces the number of bets placed by Strategies 2 and 3, and the amount wagered by Strategy 3. If an investor re-invests their profits into subsequent matches, a higher frequency of bets results in *exponentially* higher returns. This notion of *compounding* is not measurable by ROI, which assumes a fixed bet size, regardless of change in the investor's bankroll.

Our goal is to predict the most accurate probabilities for the outcomes of matches. In this situation, optimising for the greatest ROI degrades the quality of the predictions. We aim to obtain a distribution of predicted probabilites that is similar to the distribution of *true* probabilities. Figure 5.8 shows that increasing the regularisation makes the distribution of predicted probabilities narrower. If we approximate the true probability distribution by the distribution of probabilities implied by betting odds, we see that it has a larger standard deviation than any of our predicted probability distributions. To obtain the most realistic probability estimates, we should therefore *minimise* the weight decay. We find that a weight decay of less than 0.002 results in very inconsistent behaviour. Therefore, we choose 0.002 as the value for the weight decay parameter.

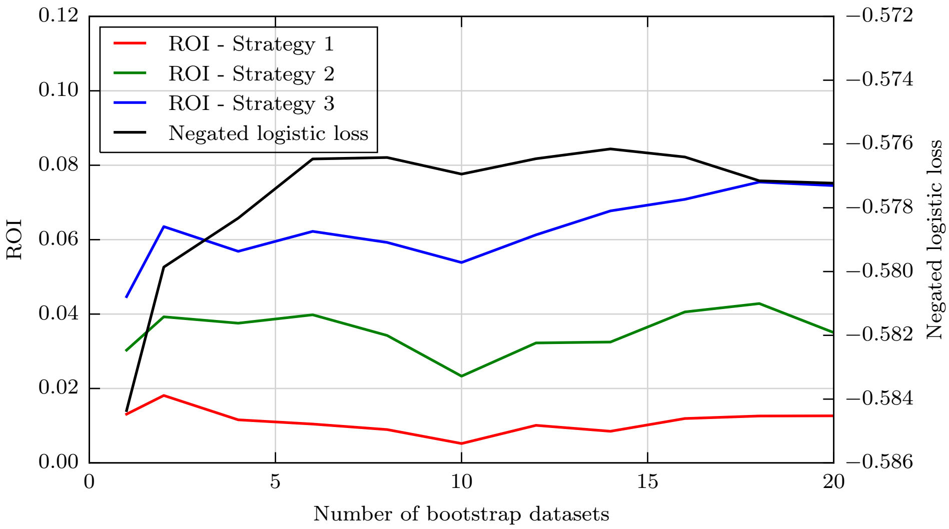
Figure 5.8: Distributions of predicted probabilities for varying weight decay



Bagging

The training process of a neural network begins with randomly initialised weights. Therefore, the same training dataset can produce networks with very different weights and thus different levels of performance. We wish to reduce this variability to ensure that the model performs well on the test set. *Bootstrap aggregating* (also known as *bagging*) is an approach for stabilising the performance of machine learning models by combining multiple versions of a predictor into a single aggregate predictor. First, we generate n bootstrap datasets from the original training dataset by sampling from the dataset uniformly and with replacement. Each bootstrap dataset has the same number of examples as the original, but only about $1-\frac{1}{e}$ of the examples are expected to be unique, with the rest being duplicates. We then train a different neural network using each bootstrap dataset. To predict the outcome of a match, we take the mean of the predictions of the n neural networks. Breiman [4] showed that bagging can provide significant gains in accuracy, especially when the underlying predictor is unstable.

Figure 5.9: ROI and logistic loss for different numbers of bootstrap datasets



It remains to decide the number of bootstrap datasets (n). Breiman used 25 datasets in his experiments, and most of the improvement in prediction accuracy was gained from only 10 datasets. By evaluating the performance of models with different numbers of bootrap datasets (Figure 5.9), we find that there is a significant improvement in performance when the number of bags is increased from 1 to 8. However, adding more than 10 bootstrap datasets appears to make little difference. Nevertheless, we expect the predictions to be more stable with a larger number of predictors, so we can train as many as possible, considering a reasonable amount of compute resources is used. In the evaluation, we train 20 individual predictors.

Other Parameters

We use the *online learning* variation of backpropagation, which updates weights immediately after being presented each training example. The alternative is *batch learning*, which only updates weights at the end of each epoch, having seen all training examples. In theory, batch learning should result in more accurate adjustments to weight, at the expense of longer training times. However, Wilson and Martinez [23] argue that this is a "widely held myth" in the neural network community, and that convergence can be reached significantly faster with online learning, with no apparent difference in accuracy. An investigation into batch learning could be conducted in the future.

Momentum can be added to the learning process to avoid local minima and, according to LeCun [14], speed up convergence. When momentum is used, a fraction of the previous weight update is incorporated in the current update during training. In this way, we avoid large fluctuations in the directions of weight updates. Although momentum did not have a significant impact on the prediction accuracy, we empirically found that a momentum coefficient of 0.55 resulted in much faster convergence.

It is necessary to define the *stopping criteria* for training. We use a common technique called *early stopping* [20], which uses a validation set to detect when overfitting begins to occur. Note that we do not use our validation set (years 2010-2011) for this purpose, but instead split the training set into two portions, one for training and one for early stopping. We conclude the training process when the error on the validation set does not achieve a new minimum for 10 consecutive epochs.

Chapter 6

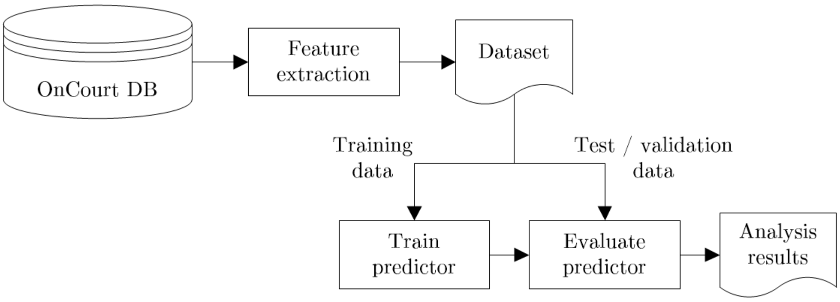
Implementation Overview

6.1 Data Flow

The first step in the implementation of a machine learning predictor for tennis match outcomes is the generation of the dataset. As discussed in Section 2.2, we obtain raw historical tennis data from the OnCourt system. SQL queries are run against the OnCourt MySQL database to retrieve all information necessary for feature extraction. Next, we process the data to generate the dataset, as described in Chapter 3. The generated dataset is persisted as a CSV (comma-separated values) file.

Each machine learning algorithm reads the dataset file prior to training. The dataset is divided as described in Section 4.1, and the model is first trained and then tested on separate parts of the dataset. Finally, the results of the evaluation are also saved in a file.

Figure 6.1: Data flow diagram



6.2 Technologies

All data processing components of the system were implemented in the **Python**[1] programming language. Python has several packages for scientific computing which have made the implementation succinct and efficient, in particular **NumPy**[2] and **Pandas**[3]. These two libraries provide a clean interface to in-memory manipulation of large datasets.

For logistic regression, we use the machine learning library **scikit-learn**[4]. This library also provides useful utility functions for machine learning, such as grid search. However, it does not have an implementation for artificial neural networks. For this purpose, we utilise **PyBrain**[5], the library recommended by scikit-learn. Both machine learning libraries are Python-based.

[1] http://www.python.org/

[2] http://www.numpy.org/

[3] http://pandas.pydata.org/

[4] http://scikit-learn.org/

[5] http://pybrain.org/

The data processing is done in an Ubuntu virtual machine hosted on the private cloud[6] of the Department of Computing at Imperial College. The VM has 8 × 3 GHz processors and 16 GB RAM. The entire system runs inside a **Docker**[7] container, for portability between VMs. **Git**[8] is used for version control.

6.3 Efficiency

We have no requirements for the efficiency of the system, provided that a prediction can be generated in time for an upcoming match with a reasonable amount of resources (compute power / memory).

The most demanding part of the data flow (by a large margin) is the generation of the dataset, which takes over 10 hours. The long processing time can be attributed to the common opponent feature extraction approach, which requires assessing the performance of both players in every match relative to all their common opponents. However, once generated, adding additional data points (i.e., new completed matches) is a matter of minutes, as is the training of a predictor. Therefore, the current architecture of the system is efficient enough to allow for betting on upcoming matches.

[6] `http://www.doc.ic.ac.uk/csg/services/cloud`

[7] `http://www.docker.com/`

[8] `http://git-scm.com/`

Chapter 7

Evaluation

7.1 Evaluation Method

Having trained different machine learning based models for the prediction of tennis matches, we now compare how they perform on unseen data. As described in Section 4.1, we have split our dataset into three divisions: training, validation, and test. We draw attention to the fact that the test data has not been used for any purpose before this point, and is therefore a valid approximation for new, upcoming matches.

Throughout the evaluation, as during the optimisation of the models, we only consider the models' performance on the least uncertain 50% of the matches. This gives us a more representative quantification of a model's performance, since the result is less affected by poor predictions caused by a lack of data. In reality, we would only bet on the matches whose features are based on a sufficient amount of data, so it makes sense to evaluate the performance of the models with respect to these matches.

We use odds from the bookmaker Pinnacle[1] for all ROI calculations. According to comparisons of different bookmakers[2], Pinnacle offers the best payout on almost all events, so the odds will give us a realistic ROI estimate. After removing 50% of the matches based on uncertainty, our test set contains **6315 ATP matches with Pinnacle odds, played during the years 2013-2014**.

We use Knottenbelt's Common-Opponent model [13] as the benchmark for comparison (see Section 2.4.4). The stochastic model represents the current state-of-the-art in tennis modelling, and it is our ambition to improve upon its performance.

7.2 Results

7.2.1 ROI and Logistic Loss

Figure 7.1 shows the return on investment when betting on the matches in the test set using different betting strategies (the three strategies are detailed in Section 2.3.2). All strategies are profitable for all models. Strategy 3, which bets on the predicted winner according to the Kelly criterion, has the best performance. Furthermore, all three machine learning models are **considerably more profitable than the benchmark** when this strategy is used, improving the ROI by about 75%. Interestingly, although their performance is very similar when using Strategy 3, the basic logistic regression model performs much better than the other ML models for the remaining strategies. This could be due to the subset of selected features used in this model, which differs from the others.

During hyperparameter optimisation, we noticed that the ROI was a very unstable evaluation metric (small changes in parameter configurations would result in large changes in ROI). For this reason, we also compare the prediction *error* of the different models (measured by logistic loss), which is less volatile.

[1] http://www.pinnaclesports.com/

[2] http://www.oddsportal.com/odds-quality/

Figure 7.1: Percentage return on investment on test set for different betting strategies

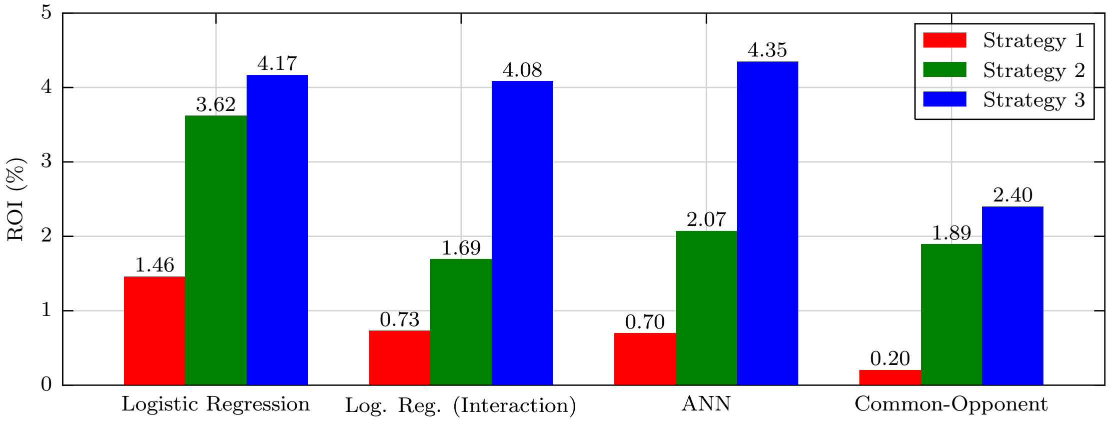
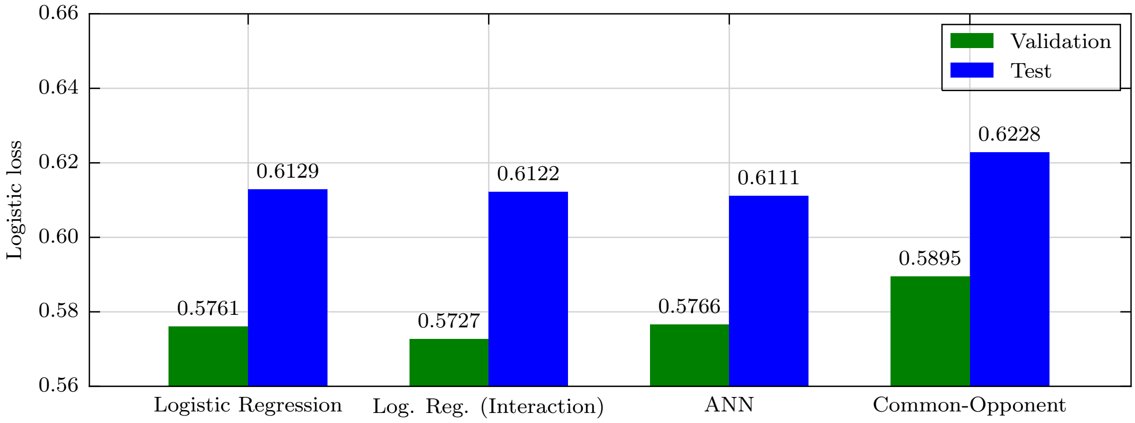


Figure 7.2 shows the error on both the validation and test sets for all models. As expected, the error is greater on the test set than on the validation set for the ML models, since their hyperparameters were optimised to achieve the best performance on the validation set, resulting in some overfitting. The model most prone to overfitting appears to be the logistic regression model with interaction features. The model had a much lower error on the validation set than the other models, but its performance on the test set is very similar to that of the basic logistic regression model. This overfitting is most likely a result of the feature selection process. Only 18 of 173 features were selected, based on the validation set. With the high dimensionality of the feature space, it is possible that some of these features have no true correspondence with match outcomes.

It is interesting to note that the error also increases for the Common-Opponent model. Since no optimisation was conducted for the Common-Opponent model using the validation set, we would expect it to perform just as well on both sets of data. The increased error of this model for matches played in the years 2013-2014 suggests that these matches might have simply been more difficult to predict (based on historical statistics) than those in the years 2011-2012. Perhaps players are becoming ever more inconsistent.

Figure 7.2: Logistic loss on the validation and test sets



If we consider the test set error in Figure 7.2, we see that for the ML models, the error slightly decreases with increased complexity. In other words, the ANN-based model, which can express the most complex relationships between the input features, also has the lowest test error (and the highest ROI). Due to the immense number of training examples, we can decrease the bias of a machine learning tennis predictor without a significant increase in variance. This suggests that further improvement could be achieved by an even greater increase in the model's complexity (e.g., by constructing additional relevant features).

7.2.2 Betting Volume

A limitation of ROI as an evaluation metric is its ignorance of the *betting volume* for different models. A model that bets only on 10 matches in the test set and has an ROI of 10% is clearly inferior to a model that bets on 20% of the matches and offers the same ROI. All three betting strategies use a fixed-size maximum bet. However, in practice, this bet limit can be adjusted according to the current size of a bettor's bankroll. In this way, the returns from previous matches can be re-invested into (larger) bets on future matches. A model which generates predictions that result in larger or more frequent bets will provide exponentially higher returns. It is therefore important to also compare this aspect of the models.

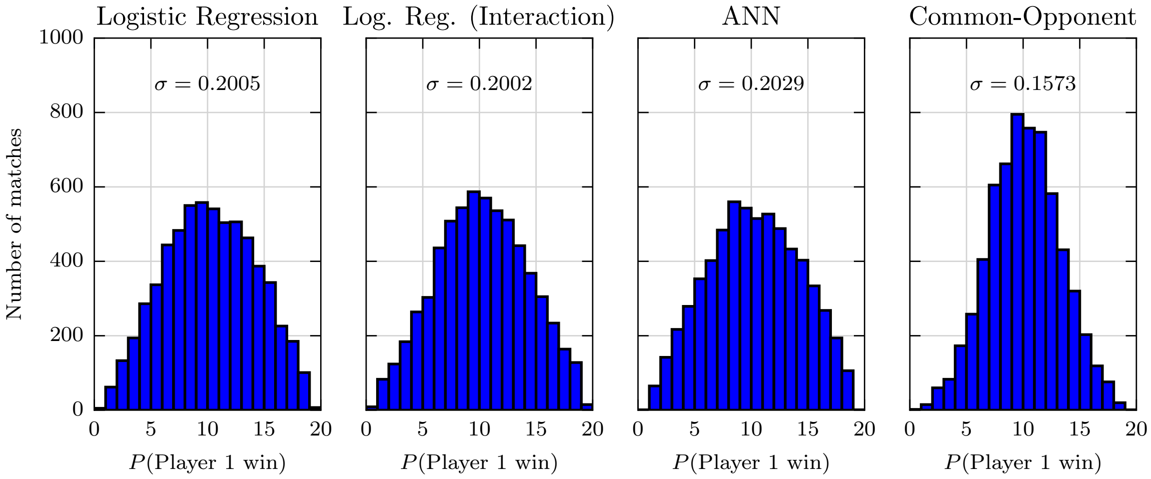
Table 7.1: Betting volume (based on Strategy 3)

Model	**Bets placed**	**Investment**	**Return**	**Net profit**
Logistic regression	3181	709.53	739.12	29.59
Logistic reg. (interaction features)	3140	670.69	698.08	27.39
Artificial neural network	3183	695.60	725.85	30.26
Common-opponent	2515	489.10	500.84	11.75

Table 7.1 shows that the models based on machine learning place a significantly larger number of bets when betting with Strategy 3 (the most profitable strategy for all four models). For example, the ANN bets on over 50% of the 6315 matches we are considering, while the benchmark bets on less than 40%. Furthermore, the amount invested is increased by about 43% when the ANN model is used. The ML models are considerably more effective at finding opportunities for placing bets, despite using the same betting strategy as the Common-Opponent model.

Why does the Common-Opponent model place fewer bets? Figure 7.3 reveals that the distribution of predicted probabilities for the Common-Opponent model has a much smaller standard deviation than the distributions of the other models (0.16% versus 0.20%, respectively). In other words, the stochastic model tends to assign less extreme probabilities to match outcomes. The betting strategy only places a bet if the predicted probability of a player winning a match is greater than the implied probability. Clearly, with lower probabilities, there will be fewer such cases. It may be possible to correct this systematic error in the Common-Opponent model (we have set all parameter values to those given in the publication). However, even if the Common-Opponent model had a wider distribution, we would still expect better performance from the ML models. All three ML models offer significantly higher returns even when Strategy 1 is used for betting (see Figure 7.1), and this strategy is unaffected by the standard deviation of the probability distribution.

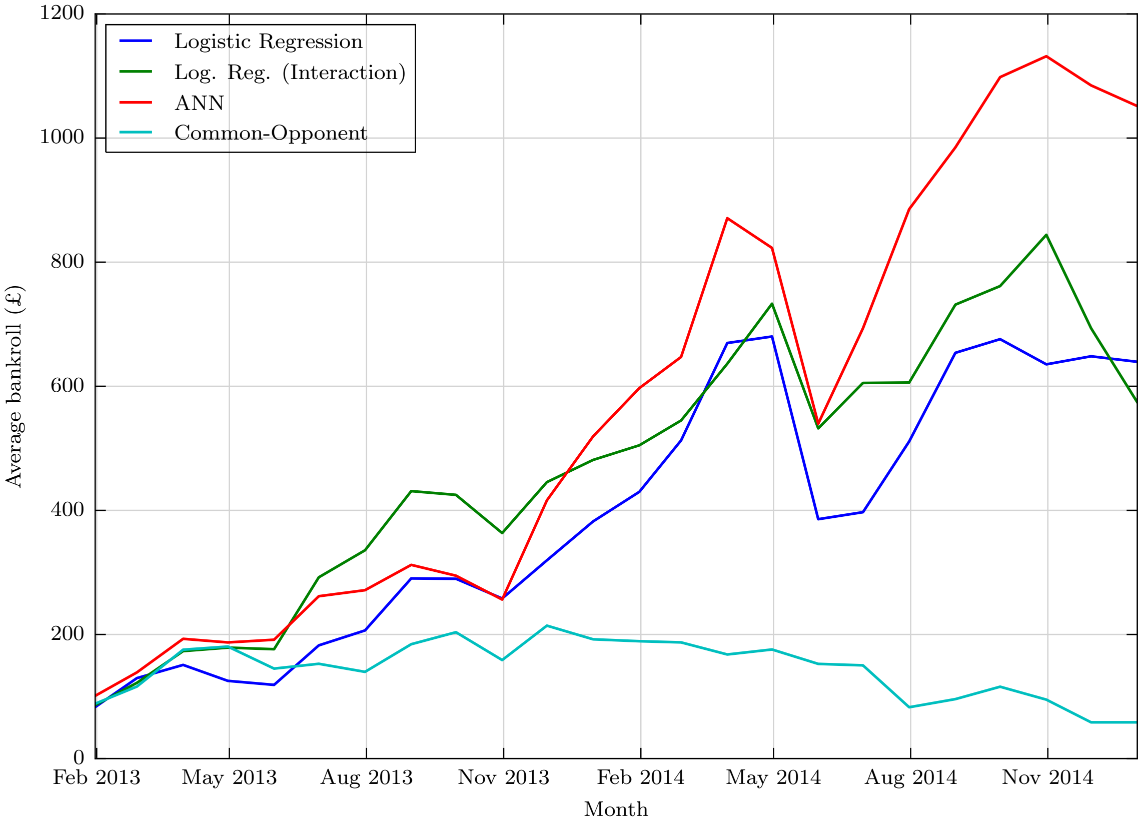
Figure 7.3: Predicted probability distributions for different models



7.2.3 Simulation

To demonstrate the profitability of the machine learning models, we can simulate the evolution of a bankroll (starting at £100) over the duration of the test set. At any point, we fix the maximum size of a bet to be 10% of the current bankroll, thereby compounding our profits from previous matches. Figure 7.4 shows the average bankroll for each month during this period. Firstly, although the Common-Opponent model is profitable in 2013, it is in fact loss-making in 2014, and completes the simulation with a bankroll of £58.41, which is lower than the initial one. On the other hand, all ML-based approaches make a significant profit. In particular, if bets are placed using predictions of the ANN model, we finish with a bankroll of £1051.55. This translates to a stunning average annual increase of 224% in the bankroll. However, the simulation also shows that the returns are very volatile. For example, May 2014 saw a large fall in the bankroll for all three ML models. Given our "edge" in the predictions, the profits should converge to exponential gains in the long term, but due to the large volatility, it is not possible to guarantee a return in the short term.

Figure 7.4: Monthly average bankroll during the years 2013-2014 (Strategy 3)



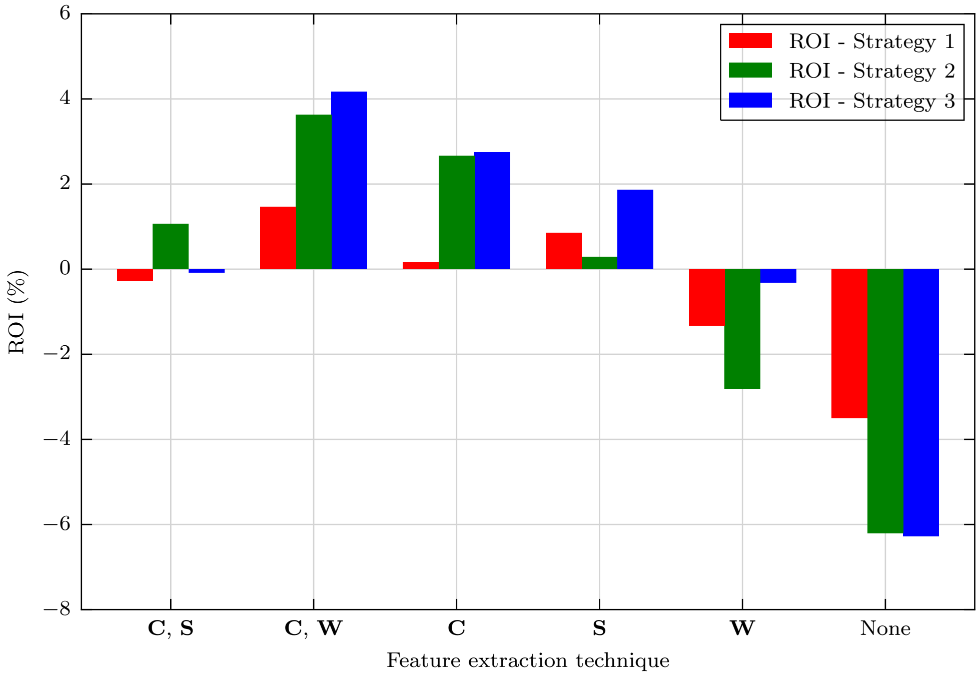
7.3 Tennis Insights

7.3.1 Relative Importance of Different Historical Matches

Our dataset was generated using all three feature extraction techniques described in Chapter 3: common opponents, surface weighting and time discounting. The effect of time discounting is controlled as a model hyperparameter, for which the optimal value was found to be 0.8 (Figure 4.5). This value is revealing of the effect of time on player performance. For example, matches that a player participated in three years ago are about half as relevant as ones played this year ($0.8^3 = 0.512$).

We can fix the time discount factor at its optimal value and assess the performance of the model for different configurations of the other techniques. Figure 7.5 shows that the best results are achieved when both common opponents and surface weighting by correlation are used together, as we have done. When no distinction is made between surfaces and averages are computed across all past matches, the logistic regression predictor has very poor performance, making a loss of over 6%. Interestingly, without common opponents, splitting by surface is more profitable than weighting by surface correlation. However, when combined with common opponents, splitting by surface performs worse. This is most likely because it results in very few matches being available for computing averages. For example, if a match is played on grass, it is entirely possible that the players have no (reasonably recent) common opponents on this surface. From these results, we can claim that a player's performance is heavily dependent on the surface, and a model is likely to have significantly better accuracy when the surface is taken into account.

Figure 7.5: Evaluation of feature extraction techniques (logistic regression)



C – Common opponents
S – Split by surface
W – Weight by surface correlations

7.3.2 Relative Importance of Different Features

As part of our investigation into tennis prediction with machine learning, we hoped to get an insight into the relevance of different features. By inspecting the weights assigned by logistic regression (Figure 4.3), we see that the single most important feature is **SERVEADV**, the difference in the players' advantage on serve. In fact, this is also the first feature to be chosen during feature selection using the forward selection algorithm. The importance of this feature is unsurprising, since it considers the most important qualities of the two players: their serve and return strengths. The weights in the logistic regression model also show that many of the features we constructed – **DIRECT**, **FATIGUE**, **RETIRED** and **COMPLETE** – affect the outcome of the match. The head-to-head balance of the players, modelled by the **DIRECT** feature, seems particularly important, and is not considered by the stochastic Common-Opponent model. For the author, it comes as a surprise that the **ACES** feature has a very high weight in both logistic regression models. This shows that powerful serves are essential to winning matches.

Unfortunately, the higher-order models give us little insight into the relative importance of different features. The limitations of such "black boxes" are discussed in the next section.

7.4 Limitations

7.4.1 Black Box Models

The stochastic tennis models use a single statistic about each of the two players in a tennis match to predict the match outcome: the probability of winning a point on their serve. Furthermore, the prediction is a result of the application of a set of well-understood mathematical formulas. For these reasons, it is possible to understand the decisions of the model. On the other hand, our machine learning approaches have a “black box” nature. The probability estimates of the ML models are difficult to justify, increasingly more so with higher model complexity. While the weights in a logistic regression model give us an intuition for the effects of different features, the predictions of the higher order models have to be blindly accepted.

7.4.2 In-play Prediction

Tennis betting expert Peter Webb claims that over 80% of the overall money wagered on tennis matches is bet *in-play*, i.e., during the course of the match.[3] The stochastic models can predict the match outcome probability from any starting score, allowing for in-play betting. Our ML models are not currently capable of adjusting a prediction according to the progression of the match. We could attempt to encode the current score as a match feature, but we doubt that this could compete with the structured hierarchical approaches.

7.4.3 Data Collection

As more features are added to the dataset, we will require multiple data sources, as not all information of interest is contained in the OnCourt database. It will most likely be necessary to scrape some information from tennis websites. This process is error-prone, and a substantial amount of resources must be invested to monitor the accuracy and consistency of the data. The stochastic models only require basic statistics, so the management of the dataset is much simpler.

[3] `http://www.sportspromedia.com/guest_blog/peter_webb_why_tennis_is_big_business_for_bookmakers`

Chapter 8

Conclusion

8.1 Innovation

Extensive research has been conducted into the prediction of tennis matches. Due to the hierarchical nature of the scoring system in tennis, most tennis prediction models are based on Markov chains. In this project, we explored the application of machine learning methods to this problem. All of our proposed ML models significantly outperform the current state-of-the-art stochastic models. In particular, the model based on artificial neural networks generated a return on investment of 4.4% when betting on 6315 ATP matches in 2013-2014, almost doubling the 2.4% ROI of the Common-Opponent model during the same period.

We have developed a novel method of extracting tennis match features from raw historical data. By finding player performance averages relative to a set of common opponents and by weighting historical matches by surface correlations and time discounting coefficients, we obtain features that more accurately model the differences in the expected performance of two players. Furthermore, we have constructed new features representing additional aspects of their form, such as fatigue accumulated from previous matches.

Two model evaluation metrics were used throughout the project: return on investment (ROI) and logistic loss. Although the ROI has a practical meaning, we warn against its use during model optimisation. We find that models tuned to generate a high ROI do not generalise well, and an error metric such as logistic loss should be used instead. Additionally, our results show that a betting strategy based on the Kelly criterion is consistently more profitable than more basic strategies for both the ML models and the Common-Opponent model.

Our method of weighting historical matches during feature extraction and our selection of the most relevant features could be used to refine the existing stochastic models. More generally, our investigation provides insights into ML-based modelling that are useful across a wide variety of sports, many of which have a similar dataset. Machine learning can model sports without a highly-structured scoring system, which is a necessity for hierarchical approaches based on Markov chains. Also, the proposed models may easily be extended with additional features, and may be altered to predict other aspects of the match (e.g., number of games in the match).

Due the profitability of the proposed models on the betting market, they offer potentially lucrative financial opportunities. It is not difficult to envision a fully-automatic bet-placing system, with a neural network at its core. Although the project is of an academic nature, it's practical applicability in sports betting is a powerful testament to its success.

8.2 Future Work

8.2.1 Additional Features

A majority of the features we constructed – such as player completeness, advantage on serve and pre-match fatigue – were shown to be influential in the predictions generated by our models. Professional bettors suggest additional factors to consider, such as motivation and home bias. Also, all of our features represent qualities of the players, not the conditions of the match. For example, the weather conditions (temperature, wind) may favour a particular playing style. Adding match-specific features may further reduce model bias.

8.2.2 Women's Tennis

We limited the scope of our investigation to ATP matches, due to a greater availability of betting odds for these matches in our dataset. Nonetheless, all our code is generic enough to accommodate predictions for WTA matches. However, as different features may be relevant for women, supporting women's tennis will require re-calibrating and re-evaluating the machine learning models.

8.2.3 Other ML Algorithms

We focused our efforts on two machine learning algorithms: logistic regression and artificial neural networks. Other approaches may produce better results. In particular, **support vector machines** (SVMs) often have greater accuracy than neural networks, at the expense of longer training times (see Section 2.5.4). We favoured ANNs as they are a natural extension of logistic regression and therefore likely to work well with the same features, but SVMs are certainly worth exploring. It is important to note that SVMs will require a calibration step to predict good probabilities, while this is not necessary for logistic regression or neural networks [17]. In addition, **Bayesian networks**, which model dependencies between different variables, could be used to predict match outcomes.

8.2.4 Set-by-Set Analysis

As demonstrated by Madurska [16], a set-by-set approach to tennis match prediction can be more accurate, as it allows the model to capture the change in a player's performance over the course of the match. For example, different players fatigue during a match in different ways. Although the OnCourt data does not include set-by-set statistics, these are partially available online (`flashscore.com`). The machine learning approach could be adapted to predict the outcome of a set, based on the result of the preceding set. This would allow for different values of features to be used for the prediction of different sets (e.g., a different in-match fatigue score).

8.2.5 Hybrid Model

Each model performs differently under different conditions. Machine learning could further be used to build a hybrid model, combining the output of many models. Essentially, the predictions of other models could be separate features, and the model could be trained to understand the strengths and weaknesses of each. For example, the predictions of our model could be combined with the Common-Opponent model, using the characteristics of the match to weight the relative influence of the two predictions.

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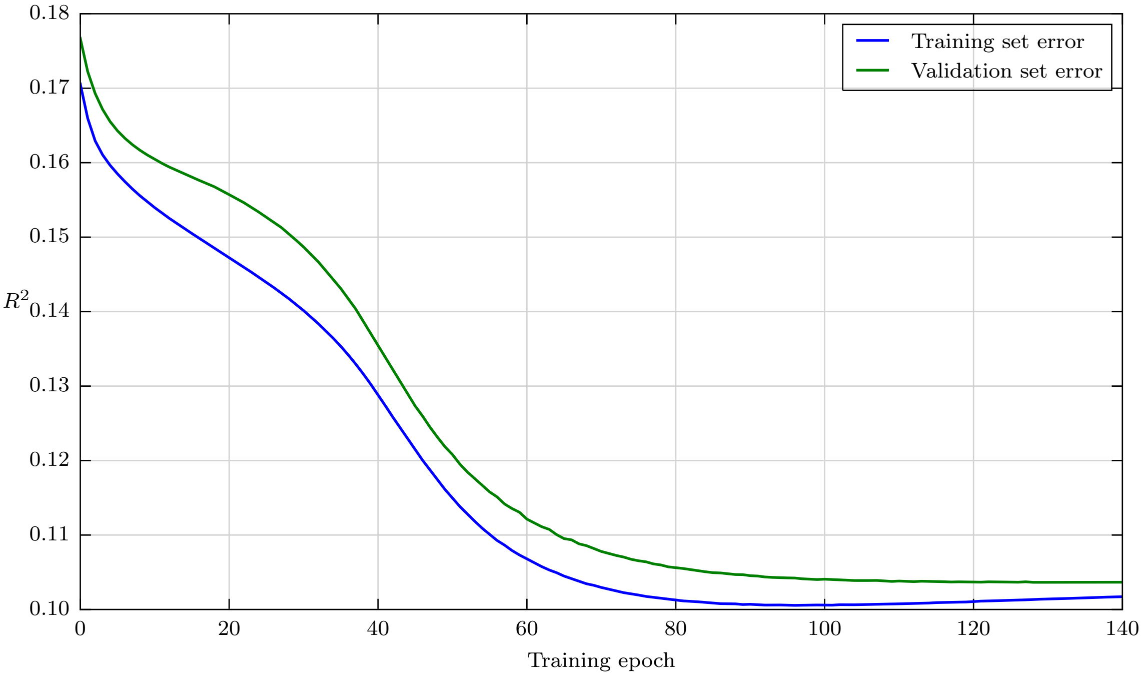
Appendix A

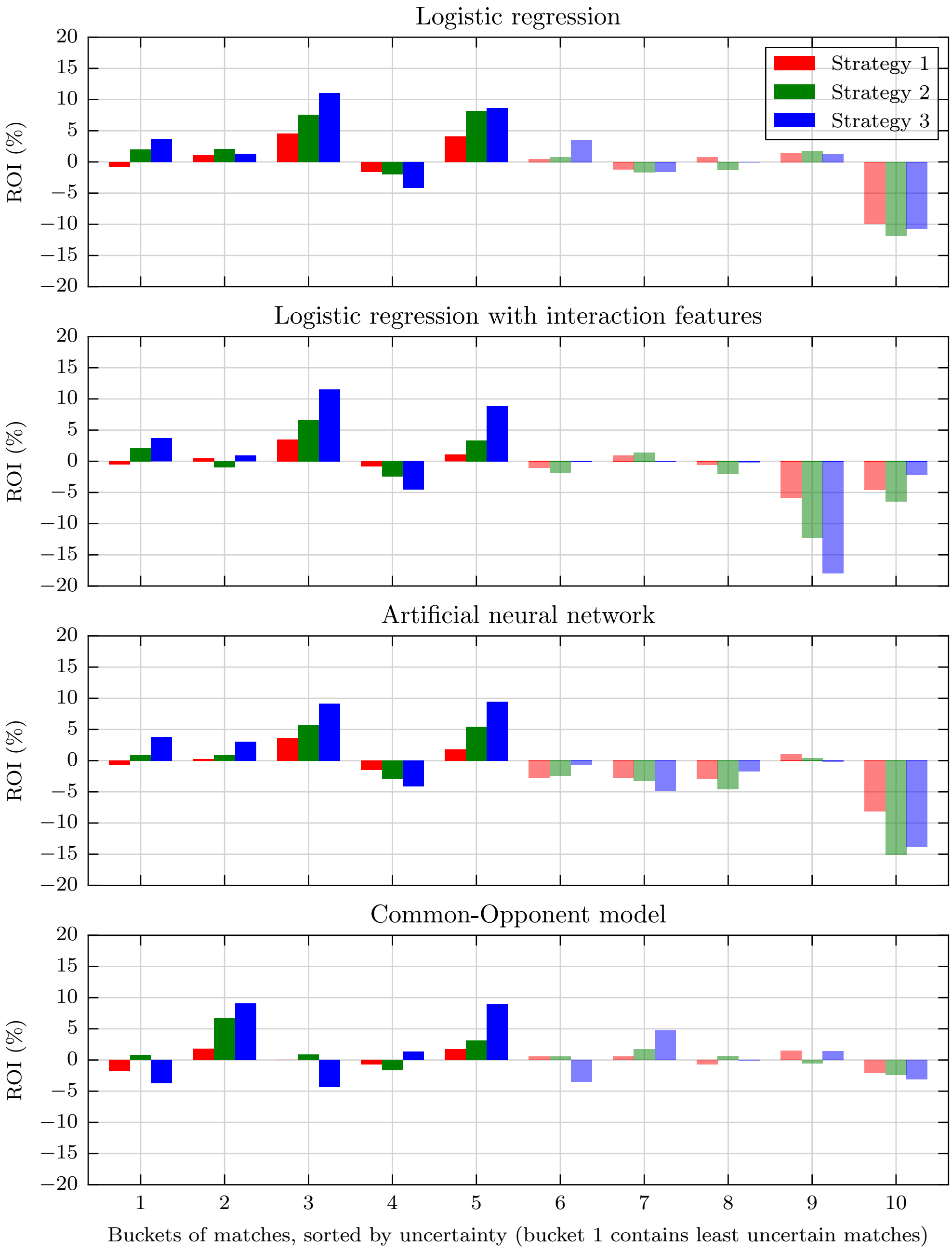
Additional Figures

Figure A.1: Learning curves for ANN



The figure shows the errors (R^2) on the training and validation sets at each training epoch of an artificial neural network, using the final hyperparameter configuration. The training stops when there is no improvement in the validation set error for 10 consecutive epochs, and in this case, 140 total epochs were necessary. Notice that the errors on the two sets move more or less in tandem, and the margin between then does not increase as training progresses. This is in partly due to the large amount of training data, and also due to regularisation (which prevents overfitting). The effect of regularisation can also be seen between epochs 100 and 140, where the training set error slightly grows due to the shrinkage of weights, while the error on the validation set improves.

Figure A.2: Return on investment across different uncertainty levels



The diagram shows the percentage return on investment for different groups of matches in the test set, when ordered by uncertainty. Note that we only evaluate the models with respect to their performance on the most certain 50% of the matches (we ignore the transparent columns). Also, the Common-Opponent model assigns uncertainty in a different manner (using only the number of common opponents), so a match present in some bucket in the Common-Opponent model could be present in another bucket in the other models.

Appendix B

Model Parameter Summary

For reproducibility of our results, we include a summary of all parameters used in the different models. See the sections on hyperparameter optimisation in the body of the report for justification.

Table B.1: Common Parameters

Parameter	**Value**
Time discount factor	0.8
Percentage of most uncertain matches removed prior to training	80%

Table B.2: Logistic Regression

Parameter	**Value**
Regularisation parameter C	0.2
Features	**SERVEADV**, **DIRECT**, **FATIGUE**, **ACES**, **TPW**, **DF**, **BP**, **RETIRED**, **COMPLETE**, **NA**, **W1SP**, **A1S**

Table B.3: Logistic Regression With Interaction Features

Parameter	**Value**
Regularisation parameter C	0.1
Features	**TPW_WSP**, **TMW_A2S**, **W2SP_DIRECT**, **TPW_A2S**, **WRP_BP**, **DF_WSP**, **DF_BP**, **NA_DIRECT**, **TPW_UE**, **BP_UE**, **BP_A2S**, **NA_SERVEADV**, **ACES_W2SP**, **RETIRED**, **ACES**, **TPW_FATIGUE**, **WRP_NA**, **ACES_UE**

Table B.4: Artificial Neural Network

Parameter	**Value**
Hidden neurons	100
Learning process	Online
Learning rate	0.0004
Weight decay	0.002
Momentum	0.55
Stopping criteria	No improvement in 10 epochs
Features	All features in Table 3.2 except for **POINTS** and **RANK**