Homework Set 4, CPSC 8420, Spring 2022

Last Name, First Name

Due 04/17/2022, Sunday, 11:59PM EST

Problem 1

Considering soft margin SVM, where we have the objective and constraints as follows:

$$\min \frac{1}{2} ||w||_2^2 + C \sum_{i=1}^m \xi_i$$
s.t. $y_i(w^T x_i + b) \ge 1 - \xi_i \ (i = 1, 2, ...m)$

$$\xi_i \ge 0 \ (i = 1, 2, ...m)$$
(1)

Now we formulate another formulation as:

$$\min \frac{1}{2} ||w||_2^2 + \frac{C}{2} \sum_{i=1}^m \xi_i^2$$
s.t. $y_i(w^T x_i + b) \ge 1 - \xi_i \ (i = 1, 2, ...m)$ (2)

- 1. Different from Eq. (1), we now drop the non-negative constraint for ξ_i , please show that optimal value of the objective will be the same when ξ_i constraint is removed.
- 2. What's the generalized Lagrangian of the new soft margin SVM optimization problem?
- 3. Now please minimize the Lagrangian with respect to w, b, and ξ .
- 4. What is the dual of this version soft margin SVM optimization problem? (should be similar to Eq. (10) in the slides)
- 5. Please analysis bias-variance trade-off when C increases.

Problem 2

Recall vanilla SVM objective:

$$L(w, b, \alpha) = \frac{1}{2} ||w||_2^2 - \sum_{i=1}^m \alpha_i [y_i(w^T x_i + b) - 1] \quad s.t. \quad \alpha_i \ge 0$$
(3)

If we denote the margin as γ , and vector $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_m]$, now please show $\gamma^2 * \|\alpha\|_1 = 1$.