All Around of Geometry and Topology

1. Let $P_1 = (a_1, b_1), \cdots, P_n = (a_n, b_n)$ be n distinct points in $\mathbf{R}^2$. Define a 1-form ω on $\mathbf{R}^2 - \{P_1, \cdots, P_n\}$ by

$$\omega = \sum_{i=1}^{n} \frac{(x - a_i)dy - (y - b_i)dx}{(x - a_i)^2 + (y - b_i)^2}.$$

Let C be a simple closed curve containing $P_1, \cdots, P_n$ inside and rotate in the positive direction. Compute the line integral

$$\int_C \omega.$$

2.

    (a) Define complex projective space CP^n.

    (b) Compute the homology and cohomology of CP^n.