S.-T. Yau College Student Mathematics Contest, 2018
Applied Mathematics, All-around

Let $\mathbf{u}(t) = (u_1(t), u_2(t), u_3(t))$ be a solution of the ODE

$$\frac{d\mathbf{u}}{dt} = \mathbf{a} \times \mathbf{u}.$$

where $\times$ denotes the cross product and $\mathbf{a} = (a_1, a_2, a_3) \neq 0$.
Consider a forward difference scheme in time

$$\mathbf{u}^{n+1} = \mathbf{u}^n + \Delta t \, (\mathbf{a} \times \mathbf{u}^n).$$

1. Show that the scheme is always unstable.

2. Discuss ways to improve the stability of the scheme.