Geometry and Topology: Overall

(1) Show that a compact torus $T^n = (S^1)^n$ cannot admit a Riemannian metric of negative sectional curvature.

(2) Let M be an n-dimensional closed submanifold in the Euclidean space $\mathbb{R}^{n+p}$. Prove the following inequality

$$\int_M H^n dV \geq vol(S^n),$$

where H and dV are the mean curvature and the volume element of M, respectively, and S^n is the standard unit sphere of dimension n.

(3) Compute the index of a closed geodesic of length 4π on the standard unit 2-sphere.