Choose 2 of the following 3 problems.

Problem 1. Let $\{X_n\}$ be a sequence of i.i.d. random variables with mean zero and variance 1 and $S_n = X_1 + \cdots + X_n$. Consider the sequence $Z_n = S_n/n^{\alpha}$. For what values of α does the sequence $\{Z_n\}$ converge

(1) almost surely?

(2) in distribution, but not almost surely (or in probability)?

Problem 2. Someone says that "primes play a game of chance". Try to make some sense of the following statements from a probabilistic perspective.

(1) The "events" of a randomly chosen natural number being divisible by a prime p and by another prime q are "independent".

(2) Determine the "probability" of two randomly chosen natural numbers being coprime (relatively prime).

Problem 3. Let $X_1, X_2, \cdots$ be independent observations. Assume that $X_k \sim U[0, e^k\theta]$ (uniform distribution).

(1) Find an unbiased estimator of θ.

(2) Derive its asymptotic distribution.