ALL ROUND TEST: ALGEBRA

Let C be a double cover over \mathbb{P}^1 ramified at 6 points.

- (1) Classify all unramified 7-cyclic covers D over C.
- (2) Prove that all such D are Galois over \mathbb{P}^1 with Galois group $\mathbb{Z}/7\mathbb{Z} \rtimes \mathbb{Z}/2\mathbb{Z}$.
- (3) Classify all intermediate curves between D and \mathbb{P}^1 .

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