PROBLEMS FOR ALGEBRA SECTION

1. Around test

Problem 1. For p any prime and $n \in \mathbb{N}$, we let $\zeta_{p^n} = \exp(2\pi i/p^n)$.

(1) When p is an odd prime, show that there is a unique subfield $K \subset \mathbb{Q}(\zeta_{p^n})$ that is quadratic over $\mathbb{Q}$.

(2) When p is an odd prime, determine which quadratic field K is in $\mathbb{Q}(\zeta_{p^n})$? (Hint: you may consider the sum $g = \sum_{a \in \mathbb{Z}/p\mathbb{Z}} \zeta_p^{a^2}$.)

(3) When $p = 2$ and $n \geq 3$, how many quadratic subfields does $\mathbb{Q}(\zeta_{p^n})$ have? What are they?