

16. PIPES AND CISTERNS

IMPORTANT FACTS AND FORMULAE

1. Inlet: A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.

Outlet: A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an outlet.

2. (i) If a pipe can fill a tank in x hours, then : part filled in 1 hour = $1/x$

(ii) If a pipe can empty a full tank in y hours, then : part emptied in 1 hour = $1/y$

(iii) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $y > x$), then on opening both the pipes, the net part filled in 1 hour = $(1/x) - (1/y)$

(iv) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $x > y$), then on opening both the pipes, the net part emptied in 1 hour = $(1/y) - (1/x)$

SOLVED EXAMPLES

Ex. 1: Two pipes A and B can fill a tank in 36 hours and 46 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Sol: Part filled by A in 1 hour = $(1/36)$;

Part filled by B in 1 hour = $(1/45)$;

Part filled by (A + B) In 1 hour = $(1/36) + (1/45) = (9/180) = (1/20)$

Hence, both the pipes together will fill the tank in 20 hours.

Ex. 2: Two pipes can fill a tank in 10 hours and 12 hours respectively while a third, pipe empties the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time will the tank be filled?

Sol: Net part filled In 1 hour = $(1/10) + (1/12) - (1/20) = (8/60) = (2/15)$.

The tank will be full in $\frac{15}{2}$ hrs = 7 hrs 30 min.

Ex. 3: If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does it take the second pipe to fill the reservoir?

Sol: let the reservoir be filled by first pipe in x hours.

Then, second pipe fill it in $(x+10)$ hrs.

Therefore $(\frac{1}{x}) + (\frac{1}{x+10}) = (\frac{1}{12}) \Leftrightarrow (x+10+x)/(x(x+10)) = (\frac{1}{12})$.

$$\Leftrightarrow x^2 - 14x - 120 = 0 \Leftrightarrow (x-20)(x+6) = 0$$

$$\Leftrightarrow x = 20 \quad [\text{neglecting the negative value of } x]$$

so, the second pipe will take $(20+10)$ hrs. (i.e) 30 hours to fill the reservoir

Ex. 4: A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the 3 are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?

Sol: Work done by the waste pipe in 1 min

$$= (\frac{1}{20}) - (\frac{1}{12}) + (\frac{1}{15}) = -\frac{1}{10} \quad [\text{negative sign means emptying}]$$

therefore the waste pipe will empty the full cistern in 10 min

Ex. 5: An electric pump can fill a tank in 3 hours. Because of a leak in, the tank it took $3\frac{1}{2}$ hours to fill the tank. If the tank is full, how much time will the leak take to empty it?

Sol: work done by the leak in 1 hour $= (\frac{1}{3}) - (\frac{1}{(7/2)}) = (\frac{1}{3}) - (\frac{2}{7}) = (\frac{1}{21})$.

The leak will empty the tank in 21 hours.

Ex. 6. Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 minutes more to fill the cistern. When the cistern is full, in what time will the leak empty it?

Sol: Work done by the two pipes in 1 hour $= (1/14) + (1/16) = (15/112)$.

Time taken by these pipes to fill the tank $= (112/15)$ hrs $= 7$ hrs 28 min.

Due to leakage, time taken $= 7$ hrs 28 min $+ 32$ min $= 8$ hrs

Work done by (two pipes + leak) in 1 hour $= (1/8)$.

Work done by the leak in 1 hour $= (15/112) - (1/8) = (1/112)$.

Leak will empty the full cistern in 112 hours.

Ex. 7: Two pipes A and B can fill a tank in 36 min. and 45 min. respectively. A water pipe C can empty the tank in 30 min. First A and B are opened. after 7 min, C is also opened. In how much time, the tank is full?

Sol: Part filled in 7 min. $= 7 * ((1/36) + (1/45)) = (7/20)$.

Remaining part $= (1 - (7/20)) = (13/20)$.

Net part filled in 1 min. when A, B and C are opened $= (1/36) + (1/45) - (1/30) = (1/60)$.

Now, $(1/60)$ part is filled in one minute.

$(13/20)$ part is filled in $(60 * (13/20)) = 39$ minutes.

Ex.8: Two pipes A, B can fill a tank in 24 min. and 32 min. respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 min.?

Sol: let B be closed after x min. then ,

Part filled by (A+B) in x min. + part filled by A in $(18-x)$ min. $= 1$

Therefore $x * ((1/24) + (1/32)) + (18-x) * (1/24) = 1 \Leftrightarrow (7x/96) + ((18-x)/24) = 1$.

$\Leftrightarrow 7x + 4 * (18-x) = 96$.

Hence, B must be closed after 8 min.