# 7. PROBLEMS ON NUMBERS

In this section, questions involving a set of numbers are put in the form of a puzzle. You have to analyze the given conditions, assume the unknown numbers and form equations accordingly, which on solving yield the unknown numbers.

## SOLVED EXAMPLES

**Ex.1. A number is as much greater than 36 as is less than 86. Find the number**.

**Sol.** Let the number be x. Then, x - 36 = 86 - x => 2x = 86 + 36 = 122 => x = 61.
Hence, the required number is 61.

**Ex. 2. Find a number such that when 15 is subtracted from 7 times the number, the Result is 10 more than twice the number**. (Hotel Management, 2002)

**Sol.** Let the number be x. Then, 7x - 15 = 2x + 10 => 5x = 25 =>x = 5.
Hence, the required number is 5.

**Ex. 3. The sum of a rational number and its reciprocal is 13/6. Find the number.**
(S.S.C. 2000)

**Sol.** Let the number be x.
Then, $x + (1/x) = 13/6 \Rightarrow (x^2 + 1)/x = 13/6 \Rightarrow 6x^2 - 13x + 6 = 0$
$\Rightarrow 6x^2 - 9x - 4x + 6 = 0 \Rightarrow (3x - 2)(2x - 3) = 0$
⇨ x = 2/3 or x = 3/2
Hence the required number is 2/3 or 3/2.

**Ex. 4. The sum of two numbers is 184. If one-third of the one exceeds one-seventh of the other by 8, find the smaller number.**

**Sol.** Let the numbers be x and (184 - *x).* Then,
(X/3) - ((184 – x)/7) = 8 => 7x – 3(184 – x) = 168 => 10x = 720 => x = 72.
So, the numbers are 72 and 112. Hence, smaller number = 72.

**Ex. 5. The difference of two numbers is 11 and one-fifth of their sum is 9. Find the numbers**.

**Sol.** Let the number be x and y. Then,
x – y = 11 ----(i) and 1/5 *(x + y)* = 9 => *x + y* = 45 ----(ii)
Adding (i) and (ii), we get: 2x = 56 or x = 28. Putting x = 28 in (i), we get: y = 17.
Hence, the numbers are 28 and 17.

**Ex. 6. If the sum of two numbers is 42 and their product is 437, then find the absolute difference between the numbers**. (S.S.C. 2003)

**Sol.** Let the numbers be x and y. Then, x + y = 42 and xy = 437
*x - y* = sqrt[$(x + y)^2$ - 4xy] = sqrt[$(42)^2$ - 4 x 437 ] = sqrt[1764 – 1748] = sqrt[16] = 4.
Required difference = 4.

**Ex. 7. The sum of two numbers is 16 and the sum of their squares is 113. Find the numbers.**

**Sol**. Let the numbers be x and (15 - *x).*

Then, $x^2 + (15 - x)^2 = 113$ => $x^2 + 225 + X^2 - 30x = 113$

=> $2x^2 - 30x + 112 = 0$ => $x^2 - 15x + 56 = 0$

=> $(x - 7)(x - 8) = 0$ => x = 7 or x = 8.

*So,* the numbers are 7 and 8.

**Ex. 8. The average of four consecutive even numbers is 27. Find the largest of these numbers.**

**Sol.** Let the four consecutive even numbers be x, x + 2, x + 4 and x + 6.

Then, sum of these numbers = (27 x 4) = 108.

*So,* x + *(x* + 2) + *(x* + 4) + *(x* + 6) = 108 or *4x* = 96 or x = 24.

:. Largest number = *(x* + 6) = 30.

**Ex. 9. The sum of the squares of three consecutive odd numbers is 2531.Find the numbers.**

**Sol.** Let the numbers be x, x + 2 and x + 4.

Then, $X^2 + (x + 2)^2 + (x + 4)^2 = 2531 => 3x^2 + 12x - 2511 = 0$

=> $X^2 + 4x - 837 = 0$ => *(x* - 27) *(x* + 31) = 0 => x = 27.

Hence, the required numbers are 27, 29 and 31.

**Ex. 10. Of two numbers, 4 times the smaller one is less then 3 times the 1arger one by 5. If the sum of the numbers is larger than 6 times their difference by 6, find the two numbers**.

**Sol**. Let the numbers be x and *y,* such that x > y

Then, *3x* - 4y = 5 ...(i) and *(x + y)* - 6 *(x - y)* = 6 => *-5x* + 7y = 6 …(ii)

Solving (i) and *(ii),* we get: x = 59 and *y* = 43.

Hence, the required numbers are 59 and 43.

**Ex. 11. The ratio between a two-digit number and the sum of the digits of that number is 4 : 1.If the digit in the unit's place is 3 more than the digit in the ten's place, what is the number?**

**Sol.** Let the ten's digit be x. Then, unit's digit = *(x* + 3).

Sum of the digits = x + *(x* + 3) = *2x* + 3. Number = l0x + *(x* + 3) = llx + 3.

11x+3 / 2x + 3 = 4 / 1 => 1lx + 3 = 4 (2x + 3) => 3x = 9 => *x* = 3.

Hence, required number = *11x* + 3 = 36.

**Ex. 12. A number consists of two digits. The sum of the digits is 9. If 63 is subtracted from the number, its digits are interchanged. Find the number.**

**Sol.** Let the ten's digit be x. Then, unit's digit = (9 - *x).*

Number = l0x + (9 - x) = *9x* + 9.

Number obtained by reversing the digits = 10 (9 - x) + x = 90 - 9x.

therefore, *(9x* + 9) - 63 = 90 - *9x* => *18x* = 144 => x = 8.

*So,* ten's digit = 8 and unit's digit = 1.

Hence, the required number is 81.

**Ex. 13. A fraction becomes 2/3 when 1 is added to both, its numerator and denominator.**

**And ,it becomes 1/2 when 1 is subtracted from both the numerator and denominator. Find the fraction.**

**Sol**. Let the required fraction be x/y. Then,

$x+1 / y+1 = 2 / 3 \Rightarrow 3x - 2y = -1$ …(i) and $x - 1 / y - 1 = 1 / 2$

$\Rightarrow 2x - y = 1$ …(ii)

Solving (i) and (ii), we get : $x = 3$ , $y = 5$

therefore, Required fraction= 3 / 5.

**Ex. 14. 50 is divided into two parts such that the sum of their reciprocals is 1/ 12.Find the two parts.**

**Sol.** Let the two parts be x and (50 - *x).*

Then, $1 / x + 1 / (50 - x) = 1 / 12 \Rightarrow (50 - x + x) / x ( 50 - x) = 1 / 12$

$\Rightarrow x^2 - 50x + 600 = 0 \Rightarrow (x - 30) ( x - 20) = 0 \Rightarrow x = 30$ or $x = 20$.

So, the parts are 30 and 20.

**Ex. 15. If three numbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers )**

**Sol**. Let the numbers be x, *y* and z. Then,

$x + y = 10$ ...*(i)* $y + z = 19$ ...*(ii)* $x + z = 21$ …(iii)

Adding (i) ,*(ii)* and *(iii),* we get: $2 (x + y + z) = 50$ or $(x + y + z) = 25$.

Thus, $x = (25 - 19) = 6$; $y = (25 - 21) = 4$; $z = (25 - 10) = 15$.

Hence, the required numbers are 6, 4 and 15.