2. H.C.F. AND L.C.M. OF NUMBERS

IMPORTANT FACTS AND FORMULAE

I. Factors and Multiples : If a number a divides another number b exactly, we say that a is a factor of b. In this case, b is called a multiple of a.

II. Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.): The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.

There are *two methods* of finding the H.C.F. of a given set of numbers :

1. Factorization Method : Express each one of the given numbers as the product of prime factors.The product of least powers of common prime factors gives H.C.F.

2. Division Method: Suppose we have to find the H.C.F. of two given numbers. Divide the larger number by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is the required H.C.F.

Finding the H.C.F. of more than two numbers : Suppose we have to find the H.C.F. of three numbers. Then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given numbers.

Similarly, the H.C.F. of more than three numbers may be obtained.

III. Least Common Multiple (L.C.M.) : The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

1. Factorization Method of Finding L.C.M.: Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors,

2. Common Division Method {Short-cut Method) of Finding L.C.M.: Arrange the given numbers in a row in any order. Divide by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers,

IV. Product of two numbers =Product of their H.C.F. and L.C.M.

V. Co-primes: Two numbers are said to be co-primes if their H.C.F. is 1.

VI. H.C.F. and L.C.M. of Fractions:

1.H C F= $\dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$ 2.L C M = $\dfrac{\text{L.C.M of Numerators}}{\text{H.C.F. of Denominators}}$

VII. H.C.F. and L.C.M. of Decimal Fractions: In given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.

VIII. Comparison of Fractions: Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M. as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

SOLVED EXAMPLES

Ex. 1. Find the H.C.F. of 2^3 X 3^2 X 5 X 7^4, 2^2 X 3^5 X 5^2 X 7^3, 2^3 X 5^3 X 7^2

Sol. The prime numbers common to given numbers are 2,5 and 7.

H.C.F. = 2^2 x 5 x7^2 = 980.

Ex. 2. Find the H.C.F. of 108, 288 and 360.

Sol. 108 = 2^2 x 3^3, 288 = 2^5 x 3^2 and 360 = 2^3 x 5 x 3^2.

H.C.F. = 2^2 x 3^2 = 36.

Ex. 3. Find the H.C.F. of 513, 1134 and 1215.

Sol.

```
1134 ) 1215 ( 1
        1134
        ----------
          81 ) 1134 ( 14
                 81
                 324
                 324
                 ---
                  x
```

$\therefore$ H.C.F. of 1134 and 1215 is 81.

So, Required H.C.F. = H.C.F. of 513 and 81.

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81 )  513 ( 6
       486
      ------
        27) 81 ( 3
            81
           ----
             0
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H.C.F. of given numbers = 27.

Ex. 4. Reduce $\frac{391}{667}$ to lowest terms.

to lowest terms.

Sol. H.C.F. of 391 and 667 is 23.

On dividing the numerator and denominator by 23, we get :

$\frac{391}{667} = \frac{391 \div 23}{667 \div 23} = \frac{17}{29}$

Ex.5.Find the L.C.M. of 2^2 x 3^3 x 5 x 7^2, 2^3 x 3^2 x 5^2 x 7^4 , 2 x 3 x $5^{3 \text{ x } 7 \text{ x } 11.}$

Sol. L.C.M. = Product of highest powers of 2, 3, 5, 7 and 11 = 2^3 x 3^3 x 5^3 x 7^4 x 11

Ex.6. Find the L.C.M. of 72, 108 and 2100.

Sol. 72 = 2^3 x 3^2, 108 = 3^3 x 2^2, 2100 = 2^2 x 5^2 x 3 x 7.

L.C.M. = 2^3 x 3^3 x 5^2 x 7 = 37800.

Ex.7.Find the L.C.M. of 16, 24, 36 and 54.

Sol.

2	16 - 24 - 36 - 54
2	8 - 12 - 18 - 27
2	4 - 6 - 9 - 27
3	2 - 3 - 9 - 27
3	2 - 1 - 3 - 9
	2 - 1 - 1 - 3

$\therefore$ L.C.M. = 2 x 2 x 2 x 3 x 3 x 2 x 3 = 432.

Ex. 8. Find the H.C.F. and L.C.M. of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$.

Sol. H.C.F. of given fractions = $\frac{\text{H.C.F. of } 2,8,16,10}{\text{L.C.M. of } 3,9,81,27} = \frac{2}{81}$

L.C.M of given fractions = $\frac{\text{L.C.M. of } 2,8,16,10}{\text{H.C.F. of } 3,9,81,27} = \frac{80}{3}$

Ex. 9. **Find the H.C.F. and L.C.M. of 0.63, 1.05 and 2.1.**

Sol. Making the same number of decimal places, the given numbers are 0.63, 1.05 and 2.10.

Without decimal places, these numbers are 63, 105 and 210.

Now, H.C.F. of 63, 105 and 210 is 21.

H.C.F. of 0.63, 1.05 and 2.1 is 0.21.

L.C.M. of 63, 105 and 210 is 630.

L.C.M. of 0.63, 1.05 and 2.1 is 6.30.

Ex. 10. Two numbers are in the ratio of 15:11. If their H.C.F. is 13, find the numbers.

Sol. Let the required numbers be 15.x and llx.

Then, their H.C.F. is x. So, x = 13.

The numbers are (15 x 13 and 11 x 13) i.e., 195 and 143.

Ex. 11. TheH.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77,find the other.

Sol. Other number = $\frac{11 \text{ X } 693}{77} = 99$

Ex. 12. **Find the greatest possible length which can be used to measure exactly the lengths 4 m 95 cm, 9 m and 16 m 65 cm.**

Sol. Required length = H.C.F. of 495 cm, 900 cm and 1665 cm.

$495 = 3^2 \times 5 \times 11$, $900 = 2^2 \times 3^2 \times 5^2$, $1665 = 3^2 \times 5 \times 37$.

$\therefore$ H.C.F. = 32 x 5 = 45.

Hence, required length = 45 cm.

Ex. 13. **Find the greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively**.

Sol. Required number = H.C.F. of (1657 - 6) and (2037 - 5) = H.C.F. of 1651 and 2032

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1651 )  2032 ( 1 1651
        1651
         381 )  1651 ( 4
                1524
                 127 )  381 ( 3
                        381
                          0
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Required number = 127.

Ex. 14. Find the largest number which divides 62, 132 and 237 to leave the same remainder in each case.

Sol . Required number = H.C.F. of (132 - 62), (237 - 132) and (237 - 62)

= H.C.F. of 70, 105 and 175 = 35.

Ex.15.Find the least number exactly divisible by 12,15,20,27.

Sol.

3	12 -	15 -	20 -	27
4	4 -	5 -	20 -	9
5	1 -	5 -	5 -	9
	1 -	1 -	1 -	9

Ex.16.Find the least number which when divided by 6,7,8,9, and 12 leave the same remainder 1 each case

Sol. Required number = (L.C.M OF 6,7,8,9,12) + 1

3	6	- 7	- 8	- 9	- 12
4	2	- 7	- 8	- 3	- 4
5	1	- 7	- 4	- 3	- 2
	1	- 7	- 2	- 3	- 1

$\therefore$ L.C.M = 3 X 2 X 2 X 7 X 2 X 3 = 504.

Hence required number = (504 +1) = 505.

Ex.17. Find the largest number of four digits exactly divisible by 12,15,18 and 27.

Sol. The Largest number of four digits is 9999.

Required number must be divisible by L.C.M. of 12,15,18,27 i.e. 540.

On dividing 9999 by 540,we get 279 as remainder .

$\therefore$ Required number = (9999-279) = 9720.

Ex.18.Find the smallest number of five digits exactly divisible by 16,24,36 and 54.

Sol. Smallest number of five digits is 10000.

Required number must be divisible by L.C.M. of 16,24,36,54 i.e 432,

On dividing 10000 by 432,we get 64 as remainder.

$\therefore$ Required number = 10000 +(432 – 64) = 10368.

Ex.19.Find the least number which when divided by 20,25,35 and 40 leaves remainders 14,19,29 and 34 respectively.

Sol. Here,(20-14) = 6,(25 – 19)=6,(35-29)=6 and (40-34)=6.

$\therefore$ Required number = (L.C.M. of 20,25,35,40) – 6 =1394.

Ex.20.Find the least number which when divided by 5,6,7, and 8 leaves a remainder 3, but when divided by 9 leaves no remainder .

Sol. L.C.M. of 5,6,7,8 = 840.

$\therefore$ Required number is of the form 840k + 3

Least value of k for which (840k + 3) is divisible by 9 is k = 2.

$\therefore$ Required number = (840 X 2 + 3)=1683

Ex.21.The traffic lights at three different road crossings change after every 48 sec., 72 sec and 108 sec.respectively .If they all change simultaneously at 8:20:00 hours,then at what time they again change simultaneously .

Sol. Interval of change = (L.C.M of 48,72,108)sec.=432sec.

So, the lights will agin change simultaneously after every 432 seconds i.e,7 min.12sec

Hence , next simultaneous change will take place at 8:27:12 hrs.

Ex.22.Arrange the fractions $\frac{17}{18}, \frac{31}{36}, \frac{43}{45}, \frac{59}{60}$ in the ascending order.

Sol.L.C.M. of 18,36,45 and 60 = 180.

Now, $\frac{17}{18} = \frac{17 \text{ X } 10}{18 \text{ X } 10} = \frac{170}{180}$; $\frac{31}{36} = \frac{31 \text{ X } 5}{36 \text{ X } 5} = \frac{155}{180}$;

$\frac{43}{45} = \frac{43 \text{ X } 4}{45 \text{ X } 4} = \frac{172}{180}$; $\frac{59}{60} = \frac{59 \text{ X } 3}{60 \text{ X } 3} = \frac{177}{180}$;

Since, 155<170<172<177, so, $\frac{155}{180} < \frac{170}{180} < \frac{172}{180} < \frac{177}{180}$

Hence, $\frac{31}{36} < \frac{17}{18} < \frac{43}{45} < \frac{59}{60}$