

4. SIMPLIFICATION

IMPORTANT CONCEPTS

I. 'BODMAS'Rule: This rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of a given expression.

Here, 'B' stands for 'bracket', 'O' for 'of', 'D' for 'division' and 'M' for 'multiplication', 'A' for 'addition' and 'S' for 'subtraction'.

Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order $()$, $\{\}$ and $[\]$.

After removing the brackets, we must use the following operations strictly in the order:
(1) of (2) division (3) multiplication (4) addition (5) subtraction.

II. Modulus of a real number : Modulus of a real number a is defined as

$$|a| = \begin{cases} a, & \text{if } a > 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Thus, $|5|=5$ and $|-5|=-(-5)=5$.

III. Virnaculum (or bar): When an expression contains Virnaculum, before applying the 'BODMAS' rule, we simplify the expression under the Virnaculum.

SOLVED EXAMPLES

Ex. 1. Simplify: (i) $5005-5000+10$ (ii) $18800+470+20$

Sol. (i) $5005-5000+10=5005-(5000/10)=5005-500=4505$.

$$(ii) 18800+470+20=(18800/470)+20=40/20=2.$$

Ex. 2. Simplify: $b-[b-(a+b)-\{b-(b-a-b)\}+2a]$

$$\begin{aligned} \text{Sol. Given expression} &= b-[b-(a+b)-\{b-(b-a-b)\}+2a] \\ &= b-[b-a-b-\{b-2b+a\}+2a] \\ &= b-[-a-\{b-2b+a+2a\}] \\ &= b-[-a-\{-b+3a\}]=b-[-a+b-3a] \\ &= b-[-4a+b]=b+4a-b=4a. \end{aligned}$$

Ex. 3. What value will replace the question mark in the following equation?

$$4\frac{1}{2}+3\frac{1}{6}+?+2\frac{1}{3}=13\frac{2}{5}.$$

Sol. Let $9/2+19/6+x+7/3=67/5$

$$\begin{aligned}\text{Then } x &= (67/5) - (9/2 + 19/6 + 7/3) \Leftrightarrow x = (67/5) - ((27+19+14)/6) = ((67/5) - (60/6)) \\ &\Leftrightarrow x = ((67/5) - 10) = 17/5 = 3 \frac{2}{5}\end{aligned}$$

Hence, missing fractions = $3 \frac{2}{5}$

Ex.4. 4/15 of 5/7 of a number is greater than 4/9 of 2/5 of the same number by 8. What is half of that number?

Sol. Let the number be x. then $4/15$ of $5/7$ of x - $4/9$ of $2/5$ of x = 8 $\Leftrightarrow 4/21x - 8/45x = 8$
 $\Leftrightarrow (4/21 - 8/45)x = 8 \Leftrightarrow (60-56)/315x = 8 \Leftrightarrow 4/315x = 8$
 $\Leftrightarrow x = (8 \times 315)/4 = 630 \Leftrightarrow 1/2x = 315$
Hence required number = 315.

Ex. 5. Simplify: $3 \frac{1}{4} \div \left\{ 1 \frac{1}{4} - \frac{1}{2} \left(2 \frac{1}{2} - \frac{1}{4} - \frac{1}{6} \right) \right\}$

Sol. Given exp. = $[13/4 \div \{5/4 - 1/2(5/2 - (3-2)/12)\}] = [13/4 \div \{5/4 - 1/2(5/2 - 1/12)\}]$
 $= [13/4 \div \{5/4 - 1/2((30-1)/12)\}] = [13/4 \div \{5/4 - 29/24\}]$
 $= [13/4 \div \{(30-39)/24\}] = [13/4 \div 1/24] = [(13/4) \times 24] = 78$

Ex. 6. Simplify: $108 \div 9 + 2 \frac{13}{5} \div 3 \frac{1}{4}$

Sol. Given exp. = $108 \div 9 + 2 \frac{13}{5} \div 3 \frac{1}{4} = \frac{108}{9} + \frac{13}{5} \div \frac{13}{4} = 12 + \frac{13}{5} \left(\frac{4}{13} \right) = 12 + \frac{4}{5} = 12 \frac{4}{5}$

Ex.7 Simplify: $\frac{(7/2) \div (5/2) \times (3/2)}{(7/2) \div (5/2) \text{ of } (3/2)} \div 5.25$

sol.

Given exp. $\frac{(7/2) \times (2/5) \times (3/2)}{(7/2) \div (15/4)} \div 5.25 = \frac{(21/10)}{(525/100)} = (21/10) \times (15/14) = 2 \frac{1}{2}$

Ex. 8. Simplify: (i) $12.05 \times 5.4 + 0.6$ (ii) $0.6 \times 0.6 + 0.6 \times 0.6$ (Bank P.O 2003)

Sol. (i) Given exp. = $12.05 \times (5.4/0.6) = (12.05 \times 9) = 108.45$
(ii) Given exp. = $0.6 \times 0.6 + (0.6 \times 6) = 0.36 + 0.1 = 0.46$

Ex. 9. Find the value of x in each of the following equation:

- (i) $[(17.28/x) / (3.6 \times 0.2)] = 2$
- (ii) $3648.24 + 364.824 + x - 36.4824 = 3794.1696$
- (iii) $8.5 - \{ 5 \frac{1}{2} - [7 \frac{1}{2} + 2.8]/x \} \times 4.25 / (0.2)^2 = 306$ (Hotel Management, 1997)

Sol. (i) $(17.28/x) = 2 \times 3.6 \times 0.2 \Leftrightarrow x = (17.28/1.44) = (1728/144) = 12.$
(ii) $(364.824/x) = (3794.1696 + 36.4824) - 3648.24 = 3830.652 - 3648.24 = 182.412.$

$$\Leftrightarrow x = (364.824/182.412) = 2.$$

$$\begin{aligned} \text{(iii)} \quad & 8.5 - \{5.5 - (7.5 + (2.8/x))\} * (4.25/0.04) = 306 \\ \Leftrightarrow & 8.5 - \{5.5 - \{(7.5x + 2.8)/x\}\} * (425/4) = 306 \\ \Leftrightarrow & 8.5 - \{(5.5x - 7.5x - 2.8)/x\} * (425/4) = 306 \\ \Leftrightarrow & 8.5 - \{(-2x - 2.8)/x\} * 106.25 = 306 \\ \Leftrightarrow & 8.5 - \{(-212.5x - 297.5)/x\} = 306 \\ \Leftrightarrow & (306 - 221)x = 297.5 \Leftrightarrow x = (297.5/85) = 3.5. \end{aligned}$$

Ex. 10. If $(x/y) = (6/5)$, find the value $(x^2 + y^2)/(x^2 - y^2)$

$$\begin{aligned} \text{Sol. } (x^2 + y^2)/(x^2 - y^2) &= (x^2/y^2 + 1)/(x^2/y^2 - 1) = [(6/5)^2 + 1] / [(6/5)^2 - 1] \\ &= [(36/25) + 1] / [(36/25) - 1] = (61*25)/(25*11) = 61/11 \end{aligned}$$

Ex. 11. Find the value of $4 - \frac{5}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{4}}}}$

$$\begin{aligned} \text{Sol. Given exp.} &= 4 - \frac{5}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{4}}}} = 4 - \frac{5}{1 + \frac{1}{3 + \frac{4}{9}}} = 4 - \frac{5}{1 + \frac{1}{(31/9)}} \\ &= 4 - \frac{5}{1 + \frac{9}{31}} = 4 - \frac{5}{(40/31)} = 4 - (5*31)/40 = 4 - (31/8) = 1/8 \end{aligned}$$

Ex. 12. If $\frac{2x}{1 + \frac{1}{1 + \frac{x}{1-x}}} = 1$., then find the value of x .

$$\begin{aligned} \text{Sol. We have : } \frac{2x}{1 + \frac{1}{\frac{(1-x)-x}{1-x}}}} &= 1 \Leftrightarrow \frac{2x}{1 + \frac{1}{[1/(1-x)]}} = 1 \Leftrightarrow \frac{2x}{1 + (1-x)} = 1 \\ \Leftrightarrow 2x &= 2-x \Leftrightarrow 3x = 2 \Leftrightarrow x = (2/3). \end{aligned}$$

Ex.13.(i) If $a/b = 3/4$ and $8a + 5b = 22$, then find the value of a.
(ii) if $x/4 - x - 3/6 = 1$, then find the value of x.

$$\text{Sol. (i) } (a/b) = 3/4 \Rightarrow b = (4/3) a.$$

$$\begin{aligned} \therefore 8a + 5b &= 22 \Rightarrow 8a + 5*(4/3) a = 22 \Rightarrow 8a + (20/3) a = 22 \\ \Rightarrow 44a &= 66 \Rightarrow a = (66/44) = 3/2 \end{aligned}$$

$$\text{(ii) } (x/4) - ((x-3)/6) = 1 \Leftrightarrow (3x - 2(x-3))/12 = 1 \Leftrightarrow 3x - 2x + 6 = 12 \Leftrightarrow x = 6.$$

Ex.14.If $2x+3y=34$ and $((x + y)/y)=13/8$,then find the value of $5y+7x$.

Sol. The given equations are:

$$2x+3y=34 \dots(i) \text{ and, } ((x + y)/y)=13/8 \Rightarrow 8x+8y=13y \Rightarrow 8x-5y=0 \dots(ii)$$

Multiplying (i) by 5,(ii) by 3 and adding, we get : $34x=170$ or $x=5$.

Putting $x=5$ in (i), we get: $y=8$.

$$\therefore 5y+7x=((5*8)+(7*5))=40+35=75$$

Ex.15.If $2x+3y+z=55$, $x-y=4$ and $y - x + z=12$,then what are the values of x , y and z ?

Sol. The given equations are:

$$2x+3y+z=55 \dots(i); x + z - y=4 \dots(ii); y - x + z=12 \dots(iii)$$

Subtracting (ii) from (i), we get: $x+4y=51 \dots(iv)$

Subtracting (iii) from (i), we get: $3x+2y=43 \dots(v)$

Multiplying (v) by 2 and subtracting (iv) from it, we get: $5x=35$ or $x=7$.

Putting $x=7$ in (iv), we get: $4y=44$ or $y=11$.

Putting $x=7,y=11$ in (i), we get: $z=8$.

Ex.16.Find the value of $(1-(1/3))(1-(1/4))(1-(1/5))....(1-(1/100))$.

Sol. Given expression = $(2/3)*(3/4)*(4/5) *.....* (99/100) = 2/100 = 1/50$.

Ex.17. Find the value of $(1/(2*3))+(1/(3*4))+(1/(4*5))+(1/(5*6))+.....+ ((1/(9*10)))$.

Sol. Given expression= $((1/2)-(1/3))+((1/3)-(1/4))+((1/4)-(1/5))+$

$((1/5)-(1/6))+.....+ ((1/9)-(1/10))$

$$=((1/2)-(1/10))=4/10 = 2/5.$$

Ex.18.Simplify: $99^{48}/_{49} * 245$.

Sol. Given expression = $(100-1/49) * 245=(4899/49) * 245 = 4899 * 5=24495$.

Ex.19.A board 7ft. 9 inches long is divided into 3 equal parts . What is the length of each part?

Sol. Length of board= $7\text{ft. } 9 \text{ inches}=(7*12+9)\text{inches}=93 \text{ inches}$.

\therefore Length of each part = $(93/3) \text{ inches} = 31 \text{ inches} = 2\text{ft. } 7 \text{ inches}$

20.A man divides Rs. Among 5 sons,4daughters and 2 nephews .If each daughter receives four times as much as each nephews and each son receives five times as much as each nephews ,how much does each daughter receive?

Let the share of each nephews be Rs.x.

Then,share of each daughter= $rs4x$;share of each son= $Rs.5x$;

$$\text{So, } 5*5x+4*4x+2*x=8600$$

$$\begin{aligned}
 25x + 16x + 2x &= 8600 \\
 = 43x &= 8600 \\
 x &= 200;
 \end{aligned}$$

21. A man spends $\frac{2}{5}$ of his salary on house rent, $\frac{3}{10}$ of his salary on food and $\frac{1}{8}$ of his salary on conveyance. If he has Rs.1400 left with him, find his expenditure on food and conveyance.

Part of salary left = $1 - (\frac{2}{5} + \frac{3}{10} + \frac{1}{8})$

Let the monthly salary be Rs. x

Then, $\frac{7}{40}$ of $x = 1400$

$$X = (1400 \times 40 / 7)$$

$$= 8600$$

Expenditure on food = Rs. $(\frac{3}{10} \times 8000) = \text{Rs. } 2400$

Expenditure on conveyance = Rs. $(\frac{1}{8} \times 8000) = \text{Rs. } 1000$

22. A third of Arun's marks in mathematics exceeds a half of his marks in English by 80. If he got 240 marks in two subjects together, how many marks did he get in English?

Let Arun's marks in mathematics and English be x and y

$$\text{Then } \frac{1}{3}x - \frac{1}{2}y = 80$$

$$2x - 3y = 160 \dots \dots (1)$$

$$x + y = 240 \dots \dots (2)$$

solving (1) and (2)

$$x = 180$$

$$\text{and } y = 60$$

23. A tin of oil was $\frac{4}{5}$ full. When 6 bottles of oil were taken out and four bottles of oil were poured into it, it was $\frac{3}{4}$ full. How many bottles of oil can the tin contain?

Suppose x bottles can fill the tin completely

$$\text{Then } \frac{4}{5}x - \frac{3}{4}x = 6 - 4$$

$$X/20 = 2$$

$$X = 40$$

Therefore required no. of bottles = 40

24. If $\frac{1}{8}$ of a pencil is black, $\frac{1}{2}$ of the remaining is white and the remaining $3\frac{1}{2}$ is blue, find the total length of the pencil?

Let the total length be x m

Then black part = $\frac{x}{8}$ cm

The remaining part = $(x - \frac{x}{8})$ cm = $\frac{7x}{8}$ cm

White part = $(\frac{1}{2} \times \frac{7x}{8}) = \frac{7x}{16}$ cm

Remaining part = $(\frac{7x}{8} - \frac{7x}{16}) = \frac{7x}{16}$ cm

$$\frac{7x}{16} = 3\frac{1}{2}$$

$$x = 8 \text{ cm}$$

25. In a certain office $\frac{1}{3}$ of the workers are women, $\frac{1}{2}$ of the women are married and $\frac{1}{3}$ of the married women have children. If $\frac{3}{4}$ of the men are married and $\frac{2}{3}$ of the married men have children, what part of workers are without children?

Let the total no of workers be x

No of women $=x/3$

No of men $=x-(x/3)=2x/3$

No of women having children $=1/3$ of $1/2$ of $x/3=x/18$

No of men having children $=2/3$ of $3/4$ of $2x/3=x/3$

No of workers having children $=x/8 + x/3=7x/18$

Workers having no children $=x-7x/18=11x/18=11/18$ of all workers

26. a crate of mangoes contains one bruised mango for every thirty mango in the crate. If three out of every four bruised mango are considerably unsaleable and there are 12 unsaleable mangoes in the crate then how many mango are there in the crate?

Let the total no of mangoes in the crate be x

Then the no of bruised mango $= 1/30 x$

Let the no of unsaleable mangoes $=3/4 (1/30 x)$

$1/40 x = 12$

$x=480$

27. a train starts full of passengers at the first station it drops $1/3$ of the passengers and takes 280 more at the second station it drops one half the new total and takes twelve more. On arriving at the third station it is found to have 248 passengers. Find the no of passengers in the beginning?

Let no of passengers in the beginning be x

After first station no passengers $=(x-x/3)+280=2x/3 +280$

After second station no passengers $=1/2(2x/3+280)+12$

$1/2(2x/3+280)+12=248$

$2x/3+280=2*236$

$2x/3=192$

$x=288$

28. if $a^2+b^2=117$ and $ab=54$ then find the value of $a+b/a-b$?

$(a+b)^2=a^2+b^2+2ab=117+2*54=225$

$a+b=15$

$(a-b)^2=a^2+b^2-2ab=117-2*54$

$a-b=3$

$a+b/a-b=15/3=5$

29. find the value of $(75983*75983 - 45983*45983)/30000$

Given expression $=(75983)^2-(45983)^2/(75983-45983)$

$=(a-b)^2/(a-b)$

$=(a+b)(a-b)/(a-b)$

$=(a+b)$

$=75983+45983$

$=121966$

30.find the value of
$$\left[\frac{343*343*343-113*113*113}{343*343+343*113+113*113} \right]$$

Given expression= $\frac{(a^3-b^3)}{a^2+ab+b^2}$
 $=(a-b)$

$=(343-113)$
 $=230$

31.Village X has a population of 68000,which is decreasing at the rate of 1200 per year.VillagY has a population of 42000,which is increasing at the rate of 800 per year .in how many years will the population of the two villages be equal?

Let the population of two villages be equal after p years
 Then, $68000-1200p=42000+800p$
 $2000p=26000$
 $p=13$

32.From a group of boys and girls,15 girls leave.There are then left 2 boys for each girl.After this,45 boys leave.There are then 5 girls for each boy.Find the number of girls in the beginning?

Let at present there be x boys.
 Then,no of girls at present= $5x$
 Before the boys had left:no of boys= $x+45$
 And no of girls= $5x$
 $X+45=2*5x$
 $9x=45$
 $x=5$
 no of girls in the beginning= $25+15=40$

33.An employer pays Rs.20 for each day a worker works and for feits Rs.3 for each day is ideal at the end of sixty days a worker gets Rs.280 . for how many days did the worker remain ideal?

Suppose a worker remained ideal for x days then he worked for $60-x$ days
 $20*(60-x)-3x=280$
 $1200-23x=280$
 $23x=920$
 $x=40$

Ex 34.kiran had 85 currency notes in all , some of which were of Rs.100 denaomination and the remaining of Rs.50 denomination the total amount of all these currency note was Rs.5000.how much amount did she have in the denomination of Rs.50?

Let the no of fifty rupee notes be x
 Then,no of 100 rupee notes $=(85-x)$
 $50x+100(85-x)=5000$

$$x+2(85-x)=100$$

$$x=70$$

so, required amount = Rs. (50*70) = Rs. 3500

Ex. 35. When an amount was distributed among 14 boys, each of them got rs 80 more than the amount received by each boy when the same amount is distributed equally among 18 boys. What was the amount?

Sol. Let the total amount be Rs. X the,

$$\frac{x}{14} - \frac{x}{18} = 80 \Leftrightarrow \frac{2x}{126} = 80 \Leftrightarrow \frac{x}{63} = 80 \Leftrightarrow x = 5040.$$

Hence the total amount is 5040.

Ex. 36. Mr. Bhaskar is on tour and he has Rs. 360 for his expenses. If he exceeds his tour by 4 days, he must cut down his daily expenses by Rs. 3. for how many days is Mr. Bhaskar on tour?

Sol. Suppose Mr. Bhaskar is on tour for x days. Then,

$$\frac{360}{x} - \frac{360}{x+4} = 3 \Leftrightarrow \frac{1}{x} - \frac{1}{x+4} = \frac{1}{120} \Leftrightarrow x(x+4) = 4 \times 120 = 480$$

$$\Leftrightarrow x^2 + 4x - 480 = 0 \Leftrightarrow (x+24)(x-20) = 0 \Leftrightarrow x = 20.$$

Hence Mr. Bhaskar is on tour for 20 days.

Ex. 37. Two pens and three pencils cost Rs 86. four Pens and a pencil cost Rs. 112. find the cost of a pen and that of a pencil.

Sol. Let the cost of a pen and a pencil be Rs. X and Rs. Y respectively.

Then, $2x + 3y = 86$ (i) and $4x + y = 112$.

Solving (i) and (ii), we get: $x = 25$ and $y = 12$.

Cost of a pen = Rs. 25 and the cost of a pencil = Rs. 12.

Ex. 38. Arjun and Sajal are friends . each has some money. If Arun gives Rs. 30 to Sajal, the Sajal will have twice the money left with Arjun. But, if Sajal gives Rs. 10 to Arjun, the Arjun will have thrice as much as is left with Sajal. How much money does each have?

Sol. Suppose Arun has Rs. X and Sajal has Rs. Y. then,

$$2(x-30) = y+30 \Rightarrow 2x-y=90 \dots(i)$$

$$\text{and } x+10 = 3(y-10) \Rightarrow x-3y = -40 \dots(ii)$$

Solving (i) and (ii), we get $x = 62$ and $y = 34$.

Arun has Rs. 62 and Sajal has Rs. 34.

Ex. 39. In a caravan, in addition to 50 hens there are 45 goats and 8 camels with some keepers. If the total number of feet be 224 more than the number of heads, find the number of keepers.

Sol. Let the number of keepers be x then,

$$\text{Total number of heads} = (50 + 45 + 8 + x) = (103 + x).$$

$$\text{Total number of feet} = (45 + 8) \times 4 + (50 + x) \times 2 = (312 + 2x).$$

$$(312 + 2x) - (103 + x) = 224 \Leftrightarrow x = 15.$$

Hence, number of keepers = 15.