

22.COMPOUND INTEREST

Compound Interest: Sometimes it so happens that the borrower and the lender agree to fix up a certain unit of time, say *yearly* or *half-yearly* or *quarterly* to settle the previous account.

In such cases, the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on.

After a specified period, the difference between the amount and the money borrowed is called the Compound Interest (abbreviated as C.I.) for that period.

IMPORTANT FACTS AND FORMULAE

Let Principal = P, Rate = R% per annum, Time = n years.

I. When interest is compound Annually:

$$\text{Amount} = P(1+R/100)^n$$

II. When interest is compounded Half-yearly:

$$\text{Amount} = P[1+(R/2)/100]^{2n}$$

III. When interest is compounded Quarterly:

$$\text{Amount} = P[1+(R/4)/100]^{4n}$$

IV. When interest is compounded Annually but time is in fraction, say $3(2/5)$ years.

$$\text{Amount} = P(1+R/100)^3 \times (1+(2R/5)/100)$$

V. When Rates are different for different years, say R₁%, R₂%, R₃% for 1st, 2nd and 3rd year respectively.

$$\text{Then, Amount} = P(1+R_1/100)(1+R_2/100)(1+R_3/100)$$

VI. Present worth of Rs.x due n years hence is given by :

$$\text{Present Worth} = x/(1+(R/100))^n$$

SOLVED EXAMPLES

Ex.1. Find compound interest on Rs. 7500 at 4% per annum for 2 years, compounded annually.

Sol.

$$\text{Amount} = \text{Rs } [7500 * (1 + (4/100))^2] = \text{Rs } (7500 * \frac{26}{25} * \frac{26}{25}) = \text{Rs. } 8112.$$

$$\text{therefore, C.I.} = \text{Rs. } (8112 - 7500) = \text{Rs. } 612.$$

Ex. 2. Find compound interest on Rs. 8000 at 15% per annum for 2 years 4 months, compounded annually.

Sol. Time = 2 years 4 months = $2(4/12)$ years = $2(1/3)$ years.

$$\text{Amount} = \text{Rs. } [8000 \times (1 + (15/100))^2 \times (1 + ((1/3) * 15/100))]$$

$$= \text{Rs. } [8000 * (23/20) * (23/20) * (21/20)]$$

$$= \text{Rs. } 11109.$$

$$\therefore \text{C.I.} = \text{Rs. } (11109 - 8000) = \text{Rs. } 3109.$$

Ex. 3. Find the compound interest on Rs. 10,000 in 2 years at 4% per annum, the interest being compounded half-yearly. (S.S.C. 2000)

Sol.

Principal = Rs. 10000; Rate = 2% per half-year; Time = 2 years = 4 half-years.

Amount =

$$\text{Rs } [10000 * (1 + (2/100))^4] = \text{Rs. } (10000 * (51/50) * (51/50) * (51/50) * (51/50))$$

$$= \text{Rs. } 10824.32.$$

$$\therefore \text{C.I.} = \text{Rs. } (10824.32 - 10000) = \text{Rs. } 824.32.$$

Ex. 4. Find the compound interest on Rs. 16,000 at 20% per annum for 9 months, compounded quarterly.

Sol. Principal = Rs. 16000; Time = 9 months = 3 quarters;
Rate = 20% per annum = 5% per quarter.

$$\text{Amount} = \text{Rs. } [16000 \times (1 + (5/100))^3] = \text{Rs. } 18522.$$

$$\text{C.I.} = \text{Rs. } (18522 - 16000) = \text{Rs. } 2522.$$

Ex. 5. If the simple interest on a sum of money at 5% per annum for 3 years is Rs. 1200, find the compound interest on the same sum for the same period at the same rate.

Sol.

Clearly, Rate = 5% p.a., Time = 3 years, S.I.= Rs. 1200. . .

So principal=RS $[100 \times 1200] / 3 \times 5 = \text{RS } 8000$

Amount = Rs. $8000 \times [1 + 5/100]^3 = \text{Rs. } 9261$.

.. C.I. = Rs. $(9261 - 8000) = \text{Rs. } 1261$.

Ex. 6. In what time will Rs. 1000 become Rs. 1331 at 10% per annum compounded annually? (S.S.C. 2004)

Sol.

Principal = Rs. 1000; Amount = Rs. 1331; Rate = 10% p.a. Let the time be n years. Then,
 $[1000 (1 + (10/100))^n] = 1331$ or $(11/10)^n = (1331/1000) = (11/10)^3$

$n = 3$ years.

Ex. 7. If Rs. 600 amounts to Rs. 683.20 in two years compounded annually, find the rate of interest per annum.

Sol. Principal = Rs. 500; Amount = Rs. 583.20; Time = 2 years.

Let the rate be R% per annum.. Then,

$$[500 (1 + (R/100))^2] = 583.20 \text{ or } [1 + (R/100)]^2 = 5832/5000 = 11664/10000$$

$$[1 + (R/100)]^2 = (108/100)^2 \text{ or } 1 + (R/100) = 108/100 \text{ or } R = 8$$

So, rate = 8% p.a.

Ex. 8. If the compound interest on a certain sum at $16 \frac{2}{3}\%$ to 3 years is Rs.1270, find the simple interest on the same sum at the same rate and for the same period.

Sol. Let the sum be Rs. x. Then,

$$\text{C.I.} = [x * (1 + ((50/3 \times 100)))^3 - x] = ((343x / 216) - x) = 127x / 216$$

$$127x / 216 = 1270 \text{ or } x = (1270 * 216) / 127 = 2160.$$

Thus, the sum is Rs. 2160

$$\text{S.I.} = \text{Rs } (2160 * (50/3) * 3 * (1/100)) = \text{Rs. } 1080.$$

Ex. 9. The difference between the compound interest and simple interest on a certain sum at 10% per annum for 2 years is Rs. 631. Find the sum.

Sol. Let the sum be Rs. x . Then,

$$C.I. = x \left(1 + \left(\frac{10}{100} \right) \right)^2 - x = 21x / 100 ,$$

$$S.I. = ((x * 10 * 2) / 100) = x / 5$$

$$(C.I.) - (S.I.) = ((21x / 100) - (x / 5)) = x / 100$$

$$(x / 100) = 632 \Rightarrow x = 63100.$$

Hence, the sum is Rs.63,100.

Ex. 10. *The difference between the compound interest and the simple interest accrued on an amount of Rs. 18,000 in 2 years was Rs. 405. What was the rate of interest p.c.p.a. ?*
(Bank P.O. 2003)

Sol. Let the rate be $R\%$ p.a. then,

$$[18000 \left(1 + \left(\frac{R}{100} \right)^2 \right) - 18000] - ((18000 * R * 2) / 100) = 405$$

$$18000 [\left(100 + \left(\frac{R}{100} \right)^2 / 10000 \right) - 1 - (2R / 100)] = 405$$

$$18000 [(100 + R)^2 - 10000 - 200R] / 10000 = 405$$

$$9R^2 / 5 = 405 \Rightarrow R^2 = ((405 * 5) / 9) = 225$$

$$R = 15.$$

Rate = 15%.

Ex. 11. *Divide Rs. 1301 between A and B, so that the amount of A after 7 years is equal to the amount of B after 9 years, the interest being compounded at 4% per annum.*

Sol. Let the two parts be Rs. x and Rs. $(1301 - x)$.

$$x(1 + 4/100)^7 = (1301 - x)(1 + 4/100)^9$$

$$x/(1301 - x) = (1 + 4/100)^2 = (26/25 * 26/25)$$

$$625x = 676(1301 - x)$$

$$1301x = 676 * 1301$$

$$x = 676.$$

So, the parts are rs.676 and rs.(1301-676) i.e rs.676 and rs.625.

Ex.12. *a certain sum amounts to rs.7350 in 2 years and to rs.8575 in 3 years.find the sum and rate percent.*

S.I on rs.7350 for 1 year=rs.(8575-7350)=rs.1225.

*Rate=(100*1225/7350*1)%=16 2/3%*

Let the sum be rs.x.then,

$$X(1 + 50/3 * 100)^2 = 7350$$

$$X * 7/6 * 7/6 = 7350$$

$$X = (7350 * 36/49) = 5400.$$

Sum=rs.5400.

Ex.13. a sum of money amounts to rs.6690 after 3 years and to rs.10,035 after 6 years on compound interest. find the sum.

Sol. Let the sum be rs.P. then

$$P(1+R/100)^3 = 6690 \dots (i) \text{ and } P(1+R/100)^6 = 10035 \dots (ii)$$

On dividing, we get $(1+R/100)^3 = 10025/6690 = 3/2$.

Substituting this value in (i), we get:

$$P * 3/2 = 6690 \text{ or } P = (6690 * 2/3) = 4460$$

Hence, the sum is rs.4460.

Ex.14. a sum of money doubles itself at compound interest in 15 years. in how many years will it become eight times?

$$P(1+R/100)^{15} = 2P$$

$$(1+R/100)^{15} = 2P/P = 2$$

$$\text{LET } P(1+R/100)^n = 8P$$

$$(1+R/100)^n = 8 = 2^3 = \{(1+R/100)^{15}\}^3 [\text{USING (I)}]$$

$$(1+R/100)^N = (1+R/100)^{45}$$

$$n = 45.$$

Thus, the required time = 45 years.

Ex.15. What annual payment will discharge a debt of Rs.7620 due in 3 years at 16 2/3% per annum interest?

Sol. Let each installment be Rs.x.

Then, (P.W. of Rs.x due 1 year hence) + (P.W of Rs.x due 2 years hence) + (P.W of Rs. X due 3 years hence) = 7620.

$$\therefore x/(1+(50/3*100)) + x/(1+(50/3*100))^2 + x/(1+(50/3*100))^3 = 7620$$

$$\Leftrightarrow (6x/7) + (936x/49) + (216x/343) = 7620.$$

$$\Leftrightarrow 294x + 252x + 216x = 7620 * 343.$$

$$\Leftrightarrow x = (7620 * 343 / 762) = 3430.$$

\therefore Amount of each installment = Rs.3430.