34. HEIGHTS AND DISTANCES

IMPORTANT FACTS AND FORMULAE

1.We already know that:

In a rt.angled ΔOAB, where $\angle BOA = \theta$,

i)$\sin\theta$ = Perpendicular/Hypotenuse = AB/OB;

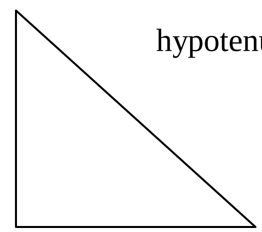
ii)$\cos\theta$ = Base/Hypotenuse = OA/OB;

iii)$\tan\theta$ = Perpendicular/Base = AB/OA;

iv)$\operatorname{cosec}\theta = 1/\sin\theta$ = OB/AB;

v)$\sec\theta = 1/\cos\theta$ = OB/OA;

vi)$\cot\theta = 1/\tan\theta$ = OA/AB.



2. **Trigonometrical identities:**

i)$\sin^2\theta + \cos^2\theta = 1$.

ii)$1+\tan^2\theta = \sec^2\theta$

iii)$1+\cot^2\theta = \operatorname{cosec}^2\theta$

3. **Values of T-ratios:-**

θ	0	30°	45°	60°	90°
Sin θ	0	½	$1/\sqrt{2}$	$\sqrt{3}/2$	1
Cos θ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	½	0
Tan θ	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not defined

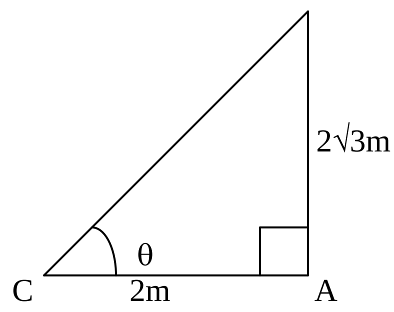
4. **Angle of Elevation:** Suppose a man from a point O looks up an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of elevation of P as seen from O.

$\therefore$ Angle of elevation of P from O = $\angle AOP$.

5. **Angle of Depression**: Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the angle of depression of P as seen from O.

SOLVED EXAMPLES

Ex.1.If the height of a pole is 2√3 metres and the length of its shadow is 2 metres, find the angle of elevation of the sun.



Sol. Let AB be the pole and AC be its shadow.

Let angle of elevation, ∠ACB=θ.

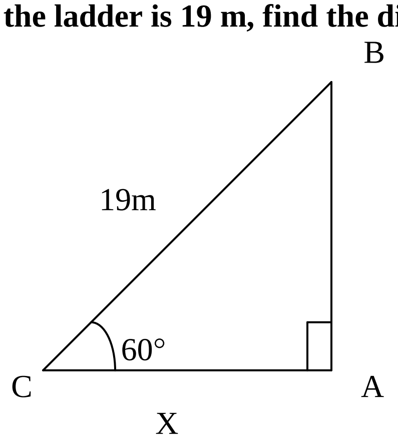
Then, AB = 2√3 m AC = 2 m.

Tan θ = AB/AC = 2√3/2 = √3 ⇒ θ = 60°

So, the angle of elevation is 60°

Ex.2. A ladder leaning against a wall makes an angle of 60° with the ground. If the length of the ladder is 19 m, find the distance of the foot of the ladder from the wall.

2)



Sol. Let AB be the wall and BC be the ladder.

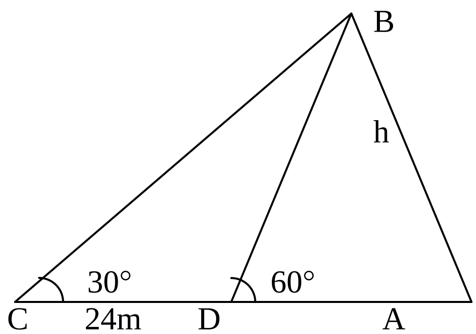
Then, ∠ACB = 60° and BC = 19 m.

Let AC = x metres

AC/BC = cos 60° ⇒ x/19 = ½ ⇒ x=19/2 = 9.5

$\therefore$ Distance of the foot of the ladder from the wall = 9.5 m

Ex.3. The angle of elevation of the top of a tower at a point on the ground is 30°. On walking 24 m towards the tower, the angle of elevation becomes 60°. Find the height of the tower.



Sol. Let AB be the tower and C and D be the points of observation. Then,

AB/AD = tan 60° = $\sqrt{3}$ => AD = AB/$\sqrt{3}$ = h/$\sqrt{3}$

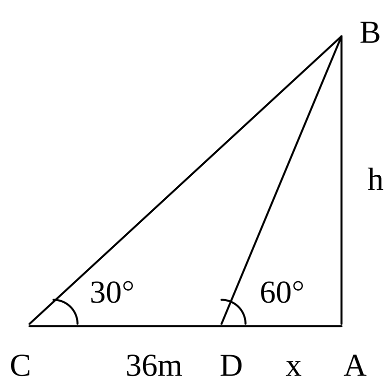
AB/AC = tan 30° = 1/$\sqrt{3}$ AC=AB x $\sqrt{3}$ = h$\sqrt{3}$

CD = (AC-AD) = (h$\sqrt{3}$–h/$\sqrt{3}$)

h$\sqrt{3}$-h/$\sqrt{3}$ = 24 => h=12$\sqrt{3}$ = (12×1.73) = 20.76

Hence, the height of the tower is 20.76 m.

Ex.4. A man standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he retires 36 m from the bank, he finds the angle to be 30°. Find the breadth of the river.



Sol. Let AB be the tree and AC be the river. Let C and D be the two positions of the man. Then,

$\angle ACB=60°$, $\angle ADB=30°$ and CD=36 m.

Let AB=h metres and AC=x metres.

Then, AD=(36+x)metres.

$AB/AD=\tan 30°=1/\sqrt{3}$ => $h/(36+x)=1/\sqrt{3}$

$h=(36+x)/\sqrt{3}$ .....(1)

$AB/AC=\tan 60°=\sqrt{3}$ => $h/x=\sqrt{3}$

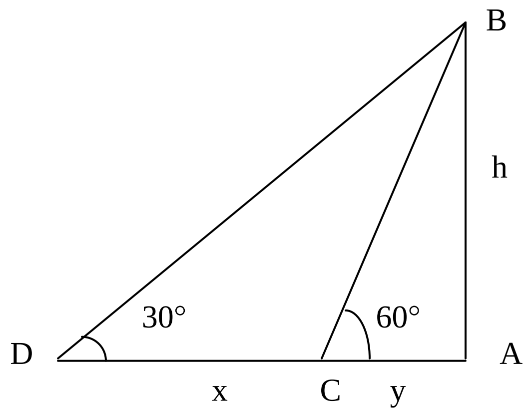
$h=\sqrt{3}x$ .....(2)

From (i) and (ii), we get:

$(36+x)/\sqrt{3}=\sqrt{3}x$ => x=18 m.

So, the breadth of the river = 18 m.

Ex.5. A man on the top of a tower, standing on the seashore finds that a boat coming towards him takes 10 minutes for the angle of depression to change from 30° to 60°. Find the time taken by the boat to reach the shore from this position.



Sol. Let AB be the tower and C and D be the two positions of the boat.

Let AB=h, CD=x and AD=y.

$h/y=\tan 60°=\sqrt{3}$ => $y=h/\sqrt{3}$

$h/(x+y)=\tan 30°$ = $1/\sqrt{3}$ => $x+y=\sqrt{3}h$

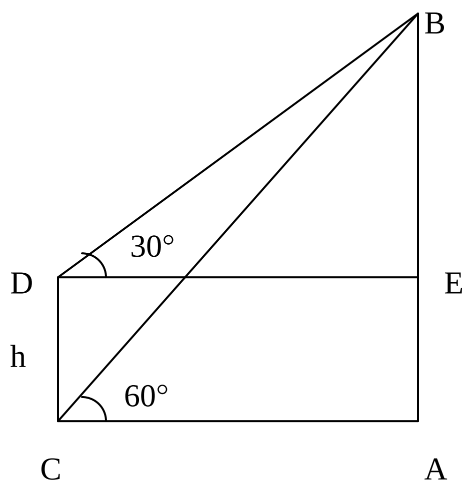
$x=(x+y)-y = (\sqrt{3}h-h/\sqrt{3})=2h/\sqrt{3}$

Now, $2h/\sqrt{3}$ is covered in 10 min.

$h/\sqrt{3}$ will be covered in $(10\times(\sqrt{3}/2h)\times(h/\sqrt{3}))=5$ min

Hence, required time = 5 minutes.

Ex 6. There are two temples, one on each bank of a river, just opposite to each other. One temple is 54 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple.



Sol. Let AB and CD be the two temples and AC be the river.

Then, AB = 54 m.

Let AC = x metres and CD=h metres.

∠ACB=60°, ∠EDB=30°

AB/AC=tan 60°=√3

AC=AB/√3=54/√3=(54/√3×√3/√3)=18m

DE=AC=18√3

BE/DE=tan 30°=1/√3

BE=(18√3×1/√3)=18 m

CD=AE=AB-BE=(54-18) m = 36 m.

So, Width of the river = AC = 18√3 m=18×1.73 m=31.14m

Height of the other temple = CD= 18 m.