

10. PERCENTAGE

IMPORTANT FACTS AND FORMULAE

1. **Concept of Percentage :** By a certain percent, we mean that many hundredths. Thus x percent means x hundredths, written as $x\%$.

To express $x\%$ as a fraction : We have, $x\% = x/100$.

Thus, $20\% = 20/100 = 1/5$; $48\% = 48/100 = 12/25$, etc.

To express a/b as a percent : We have, $a/b = ((a/b) \times 100)\%$.

Thus, $1/4 = [(1/4) \times 100] = 25\%$; $0.6 = 6/10 = 3/5 = [(3/5) \times 100]\% = 60\%$.

2. If the price of a commodity increases by $R\%$, then the reduction in consumption so as not to increase the expenditure is

$$[R/(100+R)] \times 100\%.$$

If the price of the commodity decreases by $R\%$, then the increase in consumption so as to decrease the expenditure is

$$[R/(100-R)] \times 100\%.$$

3. **Results on Population :** Let the population of the town be P now and suppose it increases at the rate of

$R\%$ per annum, then :

1. Population after n years $= P [1 + (R/100)]^n$.

2. Population n years ago $= P / [1 + (R/100)]^n$.

4. **Results on Depreciation :** Let the present value of a machine be P . Suppose it depreciates at the rate

$R\%$ per annum. Then,

1. Value of the machine after n years = $P[1-(R/100)]^n$.
2. Value of the machine n years ago = $P/[1-(R/100)]^n$.

5. If A is R% more than B, then B is less than A by

$$[(R/(100+R))*100]\%.$$

If A is R% less than B, then B is more than A by

$$[(R/(100-R))*100]\%.$$

SOLVED EXAMPLES

Ex. 1. Express each of the following as a fraction :

- (i) 56% (ii) 4% (iii) 0.6% (iv) 0.008%

sol. (i) $56\% = 56/100 = 14/25$. (ii) $4\% = 4/100 = 1/25$.
 (iii) $0.6 = 6/1000 = 3/500$. (iv) $0.008 = 8/1000 = 1/1250$.

Ex. 2. Express each of the following as a Decimal :

- (i) 6% (ii) 28% (iii) 0.2% (iv) 0.04%

Sol. (i) $6\% = 6/100 = 0.06$. (ii) $28\% = 28/100 = 0.28$.
 (iii) $0.2\% = 0.2/100 = 0.002$. (iv) $0.04\% = 0.04/100 = 0.0004$.

Ex. 3. Express each of the following as rate percent :

- (i) 23/36 (ii) $6\frac{3}{4}$ (iii) 0.004

Sol. (i) $23/36 = [(23/36)*100]\% = [575/9]\% = 63\frac{8}{9}\%$.

(ii) $0.004 = [(4/1000)*100]\% = 0.4\%$.

(iii) $6\frac{3}{4} = 27/4 = [(27/4)*100]\% = 675\%$.

Ex. 4. Evaluate :

- (i) 28% of 450 + 45% of 280

(ii) 16 2/3% of 600 gm- 33 1/3% of 180 gm

Sol. (i) $28\% \text{ of } 450 + 45\% \text{ of } 280 = [(28/100)*450 + (45/100)*280] = (126+126) = 252.$

(iii) $16 \frac{2}{3}\% \text{ of } 600 \text{ gm} - 33 \frac{1}{3}\% \text{ of } 180 \text{ gm} = [((50/3)*(1/100)*600) - ((100/3)*(1/3)*280)] \text{ gm} = (100-60) \text{ gm} = 40 \text{ gm}.$

Ex. 5.

(i) 2 is what percent of 50 ?

(ii) 1/2 is what percent of 1/3 ?

(iii) What percent of 8 is 64 ?

(iv) What percent of 2 metric tonnes is 40 quintals ?

(v) What percent of 6.5 litres is 130 ml?

Sol.

(i) Required Percentage = $[(2/50)*100]\% = 4\%.$

(ii) Required Percentage = $[(1/2)*(3/1)*100]\% = 150\%.$

(iii) Required Percentage = $[(84/7)*100]\% = 1200\%.$

(iv) 1 metric tonne = 10 quintals.

Required percentage = $[(40/(2 * 10)) * 100]\% = 200\%.$

(v) Required Percentage = $[(130/(6.5 * 1000)) * 100]\% = 2\%.$

Ex. 6.

Find the missing figures :

(i) ?% of 25 = 2.125 (ii) 9% of ? = 63

(iii) 0.25% of ? = 0.04

Sol.

(i) Let $x\%$ of 25 = 2.125. Then , $(x/100)*25 = 2.125$
 $X = (2.125 * 4) = 8.5.$

(ii) Let 9% of $x = 6.3$. Then , $9*x/100 = 6.3$
 $X = [(6.3*100)/9] = 70.$

(iii) Let 0.25% of $x = 0.04$. Then , $0.25*x/100 = 0.04$
 $X = [(0.04*100)/0.25] = 16.$

Ex. 7.

Which is greatest in $16\frac{2}{3}\%$, $2/5$ and 0.17 ?

Sol. $16\frac{2}{3}\% = [(50/3) * (1/100)] = 1/6 = 0.166$, $2/5 = 0.133$. Clearly, 0.17 is the greatest.

Ex. 8.

If the sales tax reduced from $3\frac{1}{2}\%$ to $3\frac{1}{3}\%$, then what difference does it make to a person who purchases an article with market price of Rs. 8400 ?

Sol. Required difference = $[3\frac{1}{2}\% \text{ of Rs.8400}] - [3\frac{1}{3}\% \text{ of Rs.8400}]$
 $= [(7/20 - (10/3))\% \text{ of Rs.8400}] = 1/6\% \text{ of Rs.8400}$
 $= \text{Rs. } [(1/6) * (8400/100)] = \text{Rs. } 14.$

Ex. 9. An inspector rejects 0.08% of the meters as defective. How many will be examine to project ?

Sol. Let the number of meters to be examined be x.

Then, 0.08% of x = 2

$$[(8/100) * (1/100) * x] = 2$$

$$x = [(2 * 100 * 100) / 8] = 2500.$$

Ex. 10. Sixty five percent of a number is 21 less than four fifth of that number. What is the number ?

Sol. Let the number be x.

$$\text{Then, } 4x/5 - (65\% \text{ of } x) = 21$$

$$4x/5 - 65x/100 = 21$$

$$5x = 2100$$

$$x = 140.$$

Ex.11. Difference of two numbers is 1660. If 7.5% of the number is 12.5% of the other number , find the number ?

Sol. Let the numbers be x and y. Then , 7.5 % of x = 12.5% of y

$$X = 125*y/75 = 5*y/3.$$

Now, $x - y = 1660$

$$5*y/3 - y = 1660$$

$$2*y/3 = 1660$$

$$y = [(1660*3)/2] = 2490.$$

One number = 2490, Second number = $5*y/3 = 4150$.

Ex. 12.

In expressing a length 810472 km as nearly as possible with three significant digits , find the percentage error.

Sol. Error = $(81.5 - 81.472)\text{km} = 0.028$.

Required percentage = $[(0.028/81.472)*100]\% = 0.034\%$.

Ex. 13.

In an election between two candidates, 75% of the voters cast thier thier votes, out of which 2% of the votes were declared invalid. A candidate got 9261 votes which were 75% of the total valid votes. Find the total number of votes enrolled in that election.

Sol.

Let the number of votes enrolled be x . Then ,

Number of votes cast = 75% of x . Valid votes = 98% of (75% of x).

75% of (98% of (75% of x)) = 9261.

$$[(75/100)*(98/100)*(75/100)*x] = 9261.$$

$$X = [(9261*100*100*100)/(75*98*75)] = 16800.$$

Ex.14. Shobha's mathematics test had 75 problems i.e.10 arithmetic, 30 algebra and 35 geometry problems. Although she answered 70% of the arithmetic ,40% of the algebra, and 60% of the geometry problems correctly. she did not pass the test because she got less than 60% of the problems right. How many more questions she would have to answer correctly to earn 60% of the passing grade?

Sol. Number of questions attempted correctly = $(70\% \text{ of } 10 + 40\% \text{ of } 30 + 60\% \text{ of } 35)$
 $= 7 + 12 + 21 = 45$

questions to be answered correctly for 60% grade = $60\% \text{ of } 75 = 45$

therefore required number of questions = $(45-40) = 5$.

Ex.15. if 50% of $(x-y) = 30\%$ of $(x+y)$ then what percent of x is y ?

Sol. 50% of $(x-y) = 30\%$ of $(x+y) \Leftrightarrow (50/100)(x-y) = (30/100)(x+y)$

$$\Leftrightarrow 5(x-y) = 3(x+y) \Leftrightarrow 2x = 8y \Leftrightarrow x = 4y$$

$$\text{therefore required percentage} = ((y/x) \times 100)\% = ((y/4y) \times 100) = 25\%$$

Ex.16. Mr. Jones gave 40% of the money he had to his wife. he also gave 20% of the remaining amount to his 3 sons. half of the amount now left was spent on miscellaneous items and the remaining amount of Rs.12000 was deposited in the bank. how much money did Mr. Jones have initially?

Sol. Let the initial amount with Mr. Jones be Rs. x then,

Money given to wife = Rs. $(40/100)x = \text{Rs. } 2x/5$. Balance = Rs. $(x - (2x/5)) = \text{Rs. } 3x/5$.

Money given to 3 sons = Rs. $(3 \times ((20/100) \times (3x/5))) = \text{Rs. } 9x/5$.

Balance = Rs. $((3x/5) - (9x/25)) = \text{Rs. } 6x/25$.

Amount deposited in bank = Rs. $(1/2 \times 6x/25) = \text{Rs. } 3x/25$.

$$\text{Therefore } 3x/25 = 12000 \Leftrightarrow x = ((12000 \times 25)/3) = 100000$$

So Mr. Jones initially had Rs. 1,00,000 with him.

Short-cut Method : Let the initial amount with Mr. Jones be Rs. x

Then, $(1/2)[100 - (3 \times 20)]\%$ of $x = 12000$

$$\Leftrightarrow (1/2) \times (40/100) \times (60/100) \times x = 12000$$

$$\Leftrightarrow x = ((12000 \times 25)/3) = 100000$$

Ex 17 10% of the inhabitants of village having died of cholera, a panic set in, during which 25% of the remaining inhabitants left the village. The population is then reduced to 4050. Find the number of original inhabitants.

Sol:

Let the total number of original inhabitants be x .

$$((75/100) \times (90/100) \times x) = 4050 \Leftrightarrow (27/40) \times x = 4050$$

$$\Leftrightarrow x = ((4050 \times 40)/27) = 6000.$$

Ex.18 A salesman's commission is 5% on all sales upto Rs.10,000 and 4% on all sales exceeding this. He remits Rs.31,100 to his parent company after deducting his commission. Find the total sales.

Sol:

Let his total sales be Rs. x . Now $(\text{Total sales}) - (\text{Commission}) = \text{Rs. } 31,100$

$$x - [(5\% \text{ of } 10000 + 4\% \text{ of } (x - 10000))] = 31,100$$

$$x - [(5/100) \times 10000 + (4/100) \times (x - 10000)] = 31,100$$

$$\Leftrightarrow x - 500 - ((x - 10000)/25) = 31,100$$

$$\Leftrightarrow x - (x/25) = 31200 \Leftrightarrow 24x/25 = 31200 \Leftrightarrow x = [(31200 * 25)/24] = 32,500.$$

Total sales = Rs.32,500

Ex .19 Raman`s salary was decreased by 50% and subsequently increased by 50%.How much percent does he lose?

Sol:

Let the original salary = Rs.100

New final salary = 150% of (50% of Rs.100) =

Rs.((150/100)*(50/100)*100) = Rs.75.

Decrease = 25%

Ex.20 Paulson spends 75% of his income. His income is increased by 20% and he increased his expenditure by 10%.Find the percentage increase in his savings .

Sol:

Let the original income = Rs.100 . Then , expenditure = Rs.75 and savings = Rs.25

New income = Rs.120 , New expenditure =

Rs.((110/100)*75) = Rs.165/2

New savings = Rs.(120 - (165/2)) = Rs.75/2

Increase in savings = Rs.((75/2) - 25) = Rs.25/2

Increase % = ((25/2)*(1/25)*100)% = 50%.

Ex21. The salary of a person was reduced by 10% .By what percent should his reduced salary be raised so as to bring it at par with his original salary ?

Sol:

Let the original salary be Rs.100 . New salary = Rs.90.

Increase on 90 = 10 , Increase on 100 = ((10/90)*100)%

= (100/9)%

Ex.22 When the price fo a product was decreased by 10% , the number sold increased by 30%. What was the effect on the total revenue ?

Sol:

Let the price of the product be Rs.100 and let original sale be 100 pieces.

Then , Total Revenue = Rs.(100*100) = Rs.10000.

New revenue = Rs.(90*130) = Rs.11700.

Increase in revenue = ((1700/10000)*100)% = 17%.

Ex 23 . If the numerator of a fraction be increased by 15% and its denominator be diminished by 8% , the value of the fraction is 15/16. Find the original fraction.

Sol:

Let the original fraction be x/y.

Then (115% of x)/(92% of y) = 15/16 => (115x/92y) = 15/16

$\Rightarrow ((15/16)*(92/115)) = 3/4$

Ex.24 In the new budget , the price of kerosene oil rose by 25%. By how much percent must a person reduce his consumption so that his expenditure on it does not increase ?

Sol:

$$\text{Reduction in consumption} = \left[\left(\frac{R}{100+R} \right) * 100 \right] \%$$

$$\Rightarrow \left[\left(\frac{25}{125} \right) * 100 \right] \% = 20\%.$$

Ex.25 The population of a town is 1,76,400 . If it increases at the rate of 5% per annum , what will be its population 2 years hence ? What was it 2 years ago ?

Sol:

$$\text{Population after 2 years} = 176400 * [1 + (5/100)]^2$$

$$= [176400 * (21/20) * (21/20)]$$

$$= 194481.$$

$$\text{Population 2 years ago} = 176400 / [1 + (5/100)]^2$$

$$= [176400 * (20/21) * (20/21)] = 160000.$$

Ex.26 The value of a machine depreciates at the rate of 10% per annum. If its present is Rs.1,62,000 what will be its worth after 2 years ? What was the value of the machine 2 years ago ?

Sol.

Value of the machine after 2 years

$$= \text{Rs.} [162000 * (1 - (10/100))^2] = \text{Rs.} [162000 * (9/10) * (9/10)]$$

$$= \text{Rs.} 131220$$

Value of the machine 2 years ago

$$= \text{Rs.} [162000 / (1 - (10/100)^2)] = \text{Rs.} [162000 * (10/9) * (10/9)] = \text{Rs.} 200000$$

Ex.27. During one year, the population of town increased by 5% . If the total population is 9975 at the end of the second year , then what was the population size in the beginning of the first year ?

Sol:

Population in the beginning of the first year

$$= 9975 / [1 + (5/100)] * [1 - (5/100)] = [9975 * (20/21) * (20/19)] = 10000.$$

Ex.28 If A earns 99/3% more than B, how much percent does B earn less than A ?

Sol:

$$\text{Required Percentage} = \left[\left(\frac{100/3}{100 + (100/3)} \right) * 100 \right] \%$$

$$= \left[\left(\frac{100}{400} \right) * 100 \right] \% = 25\%$$

Ex. 29 If A's salary is 20% less than B's salary , by how much percent is B's salary more than A's ?

Sol:

$$\text{Required percentage} = \left[\frac{20 * 100}{100 - 20} \right] \% = 25\%.$$

Ex30 .How many kg of pure salt must be added to 30kg of 2% solution of salt and water to increase it to 10% solution ?

Sol:

Amount of salt in 30kg solution = $[(20/100)*30] \text{kg} = 0.6 \text{kg}$

Let x kg of pure salt be added

Then , $(0.6+x)/(30+x) = 10/100 \Leftrightarrow 60+100x=300+10x$

$\Leftrightarrow 90x=240 \Leftrightarrow x=8/3.$

Ex 31. Due to reduction of 25/4% in the price of sugar , a man is able to buy 1kg more for Rs.120. Find the original and reduced rate of sugar.

Sol:

Let the original rate be Rs.x per kg.

Reduced rate = Rs. $[(100-(25/4))*(1/100)*x]$ = Rs.15x/16 per kg

$120/(15x/16) - (120/x) = 1 \Leftrightarrow (128/x) - (120/x) = 1$

$\Leftrightarrow x=8.$

So, the original rate = Rs.8 per kg

Reduce rate = Rs. $[(15/16)*8]$ per kg = Rs.7.50 per kg

Ex.32 In an examination , 35% of total students failed in Hindi , 45% failed in English and 20% in both . Find the percentage of those who passed in both subjects .

Sol:

Let A and B be the sets of students who failed in Hindi and English respectively .

Then , $n(A) = 35$, $n(B)=45$, $n(A \cap B)=20.$

So , $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 35 + 45 - 20 = 60.$

Percentage failed in Hindi and English or both = 60%

Hence , percentage passed = $(100-60)\% = 40\%$

Ex33. In an examination , 80% of the students passed in English , 85% in Mathematics and 75% in both English and Mathematics. If 40 students failed in both the subjects , find the total number of students.

Sol:

Let the total number of students be x .

Let A and B represent the sets of students who passed in English and Mathematics respectively .

Then , number of students passed in one or both the subjects

$= n(A \cup B) = n(A) + n(B) - n(A \cap B) = 80\% \text{ of } x + 85\% \text{ of } x - 75\% \text{ of } x$

$= [(80/100)x + (85/100)x - (75/100)x] = (90/100)x = (9/10)x$

Students who failed in both the subjects = $[x - (9x/10)] = x/10.$

So, $x/10 = 40$ of $x = 400$.

Hence ,total number of students = 400.