12. RATIO AND PROPORTION

IMPORTANT FACTS AND FORMULAE

I. RATIO: The ratio of two quantities a and b in the same units, is the fraction a/b and we write it as a:b. In the ratio a:b, we call a as the **first term or antecedent** and b, the **second term or consequent.**

Ex. The ratio 5: 9 represents 5/9 with antecedent = 5, consequent = 9.
Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.
Ex. 4: 5 = 8: 10 = 12: 15 etc. Also, 4: 6 = 2: 3.

2. **PROPORTION:** ***The equality of two ratios is called proportion.***
If *a:* b = c: d, we write, *a:* b:: c : d and we say that *a*, b, c, d are in proportion . Here *a* and d are called extremes, while b and c are called mean terms.
Product of means = Product of extremes.
Thus, a: b:: c : d <=> (b x c) = (a x d).

3. (i) Fourth Proportional: If *a* : b = c: d, then d is called the fourth proportional to *a*, b, c.
(ii) Third Proportional: If *a*: b = b: c, then c is called the third proportional to a and b.
(iii) Mean Proportional: Mean proportional between *a* and *b* is *square root of ab*

4. (i) COMPARISON OF RATIOS:
We say that *(a*: *b)* > (c: *d)* <=> (a/b)>(c /d).
(ii) COMPOUNDED RATIO:
The compounded ratio of the ratios (a: *b)*, (c: *d)*, (e : f) is *(ace*: bdf)

5. (i) *Duplicate ratio* of *(a* : b) is $(a^2 : b^2)$.
(ii) Sub-duplicate ratio of (a : *b)* is $(\sqrt{a} : \sqrt{b})$.
(iii)Triplicate ratio of (a : *b)* is $(a^3 : b^3)$.
(iv) Sub-triplicate ratio of (a : *b)* is $(a^{1/3} : b^{1/3})$.
(v) If (a/b)=(c/d), then ((a+b)/(a-b))=((c+d)/(c-d)) **(Componendo and dividendo)**

6. VARIATION:
(i) We say that x is directly proportional to *y*, if x = *ky* for some constant k and we write, $x \propto y$.
(ii) We say that x is inversely proportional to *y*, if xy = *k* for some constant k and we write, $x \propto (1/y)$

SOLVED PROBLEMS

Ex. 1. ***If*** **a : *b* = 5 : 9** ***and*** **b : c = 4: 7,** ***find*** **a : b : c.**

Sol. a:b=5:9 and b:c=4:7= (4X9/4): (7x9/4) = 9:63/4
a:b:c = 5:9:63/4 =20:36:63.

Ex. 2. ***Find:***
(i) ***the fourth proportional*** **to 4, 9, 12;**
(ii) ***the third proportional*** **to 16** ***and*** **36;**
iii) the **mean** ***proportional between 0.08 and 0.18.***

Sol.

i) Let the fourth proportional to 4, 9, 12 be x.

Then, 4 : 9 : : 12 : x ⇔4 x x=9x12 ⇔ X=(9 x 12)/14=27;
Fourth proportional to 4, 9, 12 is 27.

(ii) Let the third proportional to 16 and 36 be x.

Then, 16 : 36 : : 36 : x ⇔16 x *x* = 36 x 36 ⇔ x=(36 x 36)/16 =81
Third proportional to 16 and 36 is 81.

(iii) Mean proportional between 0.08 and 0.18
√0.08 x 0.18 =√8/100 x 18/100= √144/(100 x 100)=12/100=0.12

Ex. 3. ***If*** **x : *y* = 3 : 4,** ***find*** **(4x + *5y)* : (5x - *2y).***

Sol. X/Y=3/4 ⇔ (4x+5y)/(5x+2y)= (4(x/y)+5)/(5 (x/y)-2) =(4(3/4)+5)/(5(3/4)-2)

=(3+5)/(7/4)=32/7

Ex. 4. ***Divide Rs.*** **672** ***in the ratio*** **5 : 3**.

Sol. Sum of ratio terms = (5 + 3) = 8.

First part = Rs. (672 x (5/8)) = Rs. 420; Second part = Rs. (672 x (3/8)) = Rs. 252.

Ex. 5. ***Divide Rs.*** **1162 among** ***A, B,*** **C** ***in the ratio*** **35 : 28 :** ***20.***

Sol. Sum of ratio terms = (35 + 28 + 20) = 83.

A's share = Rs. (1162 x (35/83))= Rs. 490; B's share = Rs. (1162 x(28/83))= Rs. 392;

C's share = Rs. (1162 x (20/83))= Rs. 280.

Ex. 6. ***A bag contains*** **50** ***p,*** **25** ***P and*** **10** ***p coins in the ratio*** **5: 9: 4,** ***amounting*** **to** ***Rs. 206. Find the*** **number of** ***coins*** **of** ***each type.***

Sol. Let the number of 50 p, 25 P and 10 p coins be 5x, 9x and 4x respectively.

(5x/2)+(9x/ 4)+(4x/10)=206 ⇔ 50x + *45x* + 8x = 4120⇔1O3x = 4120 ⇔x=40.

Number of 50 p coins = (5 x 40) = 200; Number of 25 p coins = (9 x 40) = 360;
Number of 10 p coins = $(4 \times 40) = 160$.

Ex. 7. ***A*** **mixture** ***contains alcohol*** **and** ***water*** **in** ***the ratio*** **4 : 3.** ***If*** **5 litres of water** ***is*** **added** ***to the mixture, the ratio becomes*** **4: 5.** ***Find the quantity of alcohol*** **in the given mixture**

Sol. Let the quantity of alcohol and water be 4x litres and 3x litres respectively

4x/(3x+5)=4/5 ⇔20x=4(3x+5)⇔8x=20 ⇔x=2.5
Quantity of alcohol = (4 x 2.5) litres = 10 litres.