Normalized Floating Point Numbers

- We are most interested in normalized floating-point numbers, a format which includes:
  - sign
  - significand (1.0 ≤ Significand < Radix)
  - integer power of the radix

Examples of Normalized Floating Point Numbers

These are normalized:

- $+1.23456789 \times 10^{1}$
- $-9.987654321 \times 10^{12}$
- $+5.0 \times 10^{0}$

These are ***<u>not</u>*** normalized:

- $+11.3 \times 10^{3}$ *significand > radix*
- -0.0002×10^{7} *significand < 1.0*
- -4.0×10^{½} *exponent not integer*

Converting From Binary To Decimal

$$\mathbf{1.00101_2} = 1 \times 2^0 + 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4} + 1 \times 2^{-5}$$

$$= 1 + 0/2 + 0/4 + 1/8 + 0/16 + 1/32$$

$$= 1 + 0.125 + 0.03125$$

$$= \mathbf{1.5625}$$

$$= 37/32 = 1.5625$$

Converting From Decimal To Binary

Let’s start with $3.4625 \times 10^1 = 34.625$

Let’s deal separately with the 34 (which equals 100010_2)

$2 \times .625 = 1.25$ (save the integer part)

$2 \times .25 = 0.5$ (no integer part to save)

$2 \times .50 = 1.00$ (save the integer part)

Let’s write them left to right in order:

$34.625_{10} = 100010.101_2$

Converting From Decimal To Binary – Another Example

$1.23125 \times 10^1 = 12.3125$

$12_{10} = 1100_2$

$2 \times .3125 = \mathbf{0}.625$

$2 \times .625 = \mathbf{1}.25$

$2 \times .25 = \mathbf{0}.50$

$2 \times .50 = \mathbf{1}.0$

$$\mathbf{12.3125_{10} = 1100.0101_2}$$

Normalizing Floating Point Data

Floating point data is normalized so that there is the significand is always one:

$100001.101_2 = 1.00001101 \times 2^5$

$1100.0101_2 = 1.1000101 \times 2^3$

Since the most significant bit is always 1, we can assume that it is implied and that we do not actually have to represent it.

Biased Exponents

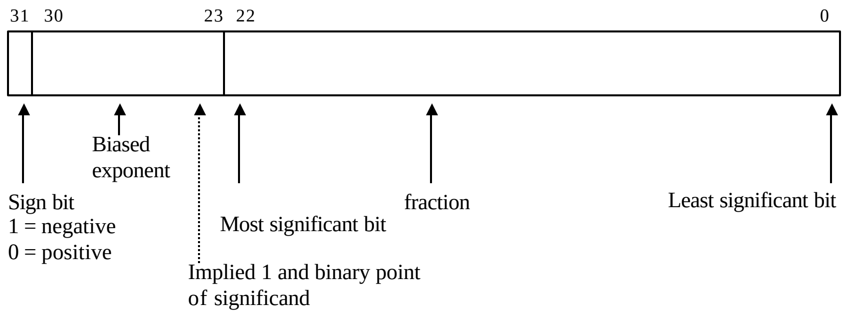
- Short floating point numbers uses 8-bits for the exponents, which we want to range from -128 to +127.
- A biased exponent uses some value other than 0 as the baseline, which must be subtracted to get the actual exponent value.
- Example (in short floating point):
  - exponent 135 = 135 - 127 = 2^8
  - exponent 120 = 120-127 = 2^{-7}

Representing Floating Point Values In Memory

There are three standard formats for representing floating-point numbers:

- 32-bit format (*single-precision*)
- 64-bit format (*double-precision*)
- 80-bit format (*extended precision*)

Short Floating Point Numbers



Representing Values

$-12.4375_{10} = -1100.0111_2$

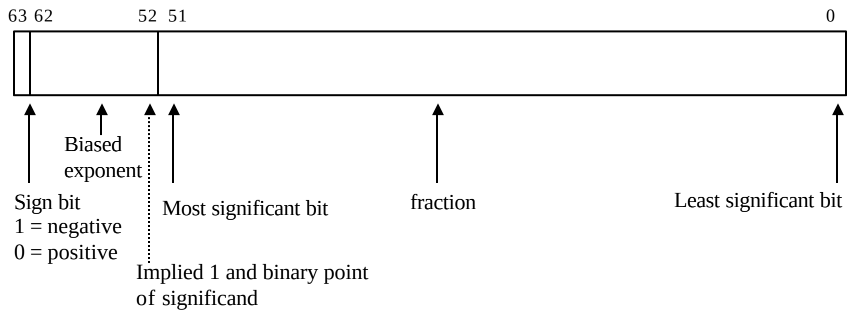
Short: $-1.10001110000\ldots 0000_2 \times 2^{3\ +127}$

1 10000010 10001110000… 0000

$1100\ 0001\ 0100\ 0111\ 0000\ \ldots\ 0000_2$

= C1470000h

Long Floating Point Numbers



Representing Values

$-12.4375_{10} = -1100.0111_2$

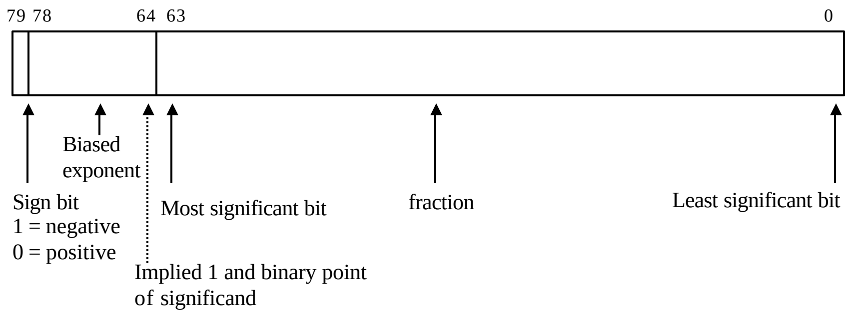
Long: $-1.10001110000\ldots 0000_2 \times 2^{3 \ +1023}$

1 10000000010 10001110000… 0000

$1100\ 0000\ 0010\ 1000\ 1110\ 0000\ \ldots\ 0000_2$

= C028E00000000000h

Extended Floating Point Numbers



Representing Values

$-12.4375_{10} = -1100.0111_2$

Extended: $-1.10001110000\ldots\ 0000_2 \times 2^{3\ +16383}$

1 100000000000010 10001110000… 0000

1100 0000 0000 0010 1100 0111 0000 … 0000_2

= C002C700000000000000h

Specifying Floating Point Data In Assembly Language

- We can use the:
  - **dd** (define doubleword) directive to allocate storage for single-precision floats
  - **dq** (define quadword) to allocate storage for double-precision floats and
  - **dt** (define tenbyte) for extended-precision floats.

Specifying Floating Point Data - An Example

- Allocating storage and initializing values

```
ShortOne  dd   1.0
LongOne   dq   1.0
Pi        dd   0.314159265E1
IntRate   dt   13.25E-1
```

Allocating storage without initializing:

```
Mass      dd   ?
CoefFric  dq   ?
Temp      dt   ?
```

Floating Point Operations

- Floating point operations include:
  - moving and rounding data
  - conversion
  - addition
  - subtraction
  - multiplication
  - division
  - remainder
  - comparison

Moving Floating Point Data

- Moving floating point data can be done using the standard **`mov`** instruction in Assembly language.
- If the source and destination are different length, care must be taken in conversion to ensure that exponent and significand are properly converted.

Data Conversion

- Integer and floating point data cannot be used interchangeably; data conversion is necessary and real-to-integer conversion is not without potential problems:
  - Underflow – a magnitude too small to represent as an integer.
  - Overflow – a magnitude too small to represent as an integer.
  - Inexact result – a loss of all of part of the fractional part of the floating-point fvalue.

Floating Point Addition

- To add two floating point values, they have to be aligned so that they have the same exponent.
- After addition, the sum may need to be normalized.
- Potential errors include overflow, underflow and inexact results.
- Examples:

$$\begin{array}{rr} & 2.34 \times 10^3 \\ + & 0.88 \times 10^3 \\ \hline & 3.22 \times 10^3 \end{array} \qquad \begin{array}{r} 6.22 \times 10^8 \\ + \; 3.93 \times 10^8 \\ \hline 10.15 \times 10^8 = 1.015 \times 10^8 \end{array}$$

Floating Point Subtraction

- Subtracting floating point values also requires re-alignment so that they have the same exponent.
- After subtraction, the difference may need to be normalized.
- Potential errors include overflow, underflow and inexact results, and the difference may have one signficant bit less than the operands..
- Examples:

$$\begin{array}{r} 2.34 \times 10^3 \\ \underline{-0.88 \times 10^3} \\ 1.46 \times 10^3 \end{array} \qquad \begin{array}{r} 6.44 \times 10^4 \\ \underline{-\ 6.23 \times 10^4} \\ 0.21 \times 10^4 = 2.1 \times 10^3 \end{array}$$

Floating Point Multiplication

- Multiplying floating point values does not requires re-alignment - realigning may lead to loss of significance.
- After multiplication, the product may need to be normalized.
- Potential errors include overflow, underflow and inexact results.
- Examples:

$$\begin{array}{r} 2.4 \times 10^{-3} \\ \times\ \underline{6.3 \times 10^{2}} \\ 15.12 \times 10^{1} = 1.512 \times 10^{2} \end{array}$$

Floating Point Division

- Dividing floating point values does not requires re-alignment.
- After division, the (floating point) quotient may need to be normalized – there is no remainder
- Potential errors include overflow, underflow, inexact results and attempts to divide by zero.
- Examples:

$$1.86 \times 10^{13} \div 7.44 \times 10^{5} \quad = \quad 0.25 \times 10^{8}$$
$$2.5 \times 10^{7}$$

Floating Point Remainder

- There is usually no remainder in floating point division, because the quotient can be a floating point value itself.
- Sometimes we want a remainder , i.e., the difference between the dividend and the product of the quotient rounded to the nearest integer) and the divisor:
- s REM t = s – t $\times$ NINT(s/t)
- Remainder will not produce inexact results, underflow or overflow but can lead to an attempt to divide by zero.

Floating Point Comparison

- There are usually three results that can happen as a result of floating point comparison:
  - less than
  - greater than
  - equal to
- In some instances, there is a fourth result: unordered, which occurs if one of the values is the result of an arithmetic error.
- These errors can result from adding or subtracting infinite values and are called ***NaN*s** (for ***N***ot ***a*** ***N***umber).

The Intel Floating Point Co-processors

- Early Intel processors (8088/8086, 80286, 80386) had no floating point capabilities; unless you wished to emulate floating point operations using software routines, you needed to add a co-processor (8087, 80287, 80387).
- 80486 and Pentium family processors include a floating point unit with an architecture that is the same as the coprocessors.