

УНИВЕРСИТЕТ ИТМО
Факультет программной инженерии и компьютерной техники
Дисциплина «Дискретная математика»

Курсовая работа
Часть 2
Вариант 95

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Задание

Построить комбинационную схему реализующую функцию $C = A*B$ (C — 5 бит, A — 3 бита, B — 2 бита) $A \neq 0$ и $B \neq 0$.

Таблица истинности

| № | a_1 | a_2 | a_3 | b_1 | b_2 | c_1 | c_2 | c_3 | c_4 | c_5 |
|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 0 | 0 | 0 | d | d | d | d | d |
| 1 | 0 | 0 | 0 | 0 | 1 | d | d | d | d | d |
| 2 | 0 | 0 | 0 | 1 | 0 | d | d | d | d | d |
| 3 | 0 | 0 | 0 | 1 | 1 | d | d | d | d | d |
| 4 | 0 | 0 | 1 | 0 | 0 | d | d | d | d | 0 |
| 5 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 6 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 7 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 8 | 0 | 1 | 0 | 0 | 0 | d | d | d | d | 0 |
| 9 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 10 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 11 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 12 | 0 | 1 | 1 | 0 | 0 | d | d | d | d | 0 |
| 13 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 14 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 15 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 16 | 1 | 0 | 0 | 0 | 0 | d | d | d | d | 0 |
| 17 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 18 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 19 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 20 | 1 | 0 | 1 | 0 | 0 | d | d | d | d | 0 |
| 21 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 22 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 23 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 24 | 1 | 1 | 0 | 0 | 0 | d | d | d | d | 0 |
| 25 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 26 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 27 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 28 | 1 | 1 | 1 | 0 | 0 | d | d | d | d | 0 |
| 29 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| 30 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 31 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |

Минимизация булевых функций на картах Карно

| | | $b_1 b_2$ | | | |
|-----------|----|-----------|----|----|----|
| | | 00 | 01 | 11 | 10 |
| $a_2 a_3$ | 00 | d | d | d | d |
| | 01 | d | | | |
| | 11 | d | | | |
| | 10 | d | | | |

$a_1 = 0$

| | | $b_1 b_2$ | | | |
|-----------|----|-----------|----|----|----|
| | | 00 | 01 | 11 | 10 |
| $a_2 a_3$ | 00 | d | | | |
| | 01 | d | | | |
| | 11 | d | | 1 | |
| | 10 | d | | 1 | |

$a_1 = 1$

$$c_1 = a_1 a_2 b_1 b_2 \quad (S_Q = 4)$$

| | | $b_1 b_2$ | | | |
|-----------|----|-----------|----|----|----|
| | | 00 | 01 | 11 | 10 |
| $a_2 a_3$ | 00 | d | d | d | d |
| | 01 | d | | | |
| | 11 | d | | 1 | |
| | 10 | d | | | |

$a_1 = 0$

| | | $b_1 b_2$ | | | |
|-----------|----|-----------|----|----|----|
| | | 00 | 01 | 11 | 10 |
| $a_2 a_3$ | 00 | d | | 1 | 1 |
| | 01 | d | | 1 | 1 |
| | 11 | d | | | 1 |
| | 10 | d | | | 1 |

$a_1 = 1$

$$c_2 = a_1 \bar{b}_2 \vee a_1 \bar{a}_2 b_1 \vee \bar{a}_1 a_2 a_3 b_1 b_2 \quad (S_Q = 13)$$

| | | $b_1 b_2$ | | | |
|-----------|----|-----------|----|----|----|
| | | 00 | 01 | 11 | 10 |
| $a_2 a_3$ | 00 | d | d | d | d |
| | 01 | d | | | |
| | 11 | d | | | 1 |
| | 10 | d | | 1 | 1 |

$a_1 = 0$

| | | $b_1 b_2$ | | | |
|-----------|----|-----------|----|----|----|
| | | 00 | 01 | 11 | 10 |
| $a_2 a_3$ | 00 | d | 1 | 1 | |
| | 01 | d | 1 | 1 | |
| | 11 | d | 1 | 1 | 1 |
| | 10 | d | 1 | | 1 |

$a_1 = 1$

$$c_3 = a_1 \bar{b}_1 \vee a_2 \bar{b}_2 \vee a_1 a_2 a_3 \vee a_1 \bar{a}_2 b_2 \vee \bar{a}_1 \bar{a}_3 b_1 \quad (S_Q = 18)$$

| | | $b_1 b_2$ | | | |
|-----------|----|-----------|----|----|----|
| | | 00 | 01 | 11 | 10 |
| $a_2 a_3$ | 00 | d | d | d | d |
| | 01 | d | | 1 | 1 |
| | 11 | d | 1 | | 1 |
| | 10 | d | 1 | 1 | |

$a_1 = 0$

| | | $b_1 b_2$ | | | |
|-----------|----|-----------|----|----|----|
| | | 00 | 01 | 11 | 10 |
| $a_2 a_3$ | 00 | d | | | |
| | 01 | d | | 1 | 1 |
| | 11 | d | 1 | | 1 |
| | 10 | d | 1 | 1 | |

$a_1 = 1$

$$c_4 = a_2 \overline{b_1} \vee a_3 \overline{b_2} \vee a_2 \overline{a_3} b_2 \vee \overline{a_2} a_3 b_1 \quad (S_Q = 14)$$

| | | $b_1 b_2$ | | | |
|-----------|----|-----------|----|----|----|
| | | 00 | 01 | 11 | 10 |
| $a_2 a_3$ | 00 | d | d | d | d |
| | 01 | | 1 | 1 | |
| | 11 | | 1 | 1 | |
| | 10 | | | | |

$a_1 = 0$

| | | $b_1 b_2$ | | | |
|-----------|----|-----------|----|----|----|
| | | 00 | 01 | 11 | 10 |
| $a_2 a_3$ | 00 | | | | |
| | 01 | | 1 | 1 | |
| | 11 | | 1 | 1 | |
| | 10 | | | | |

$a_1 = 1$

$$c_5 = a_3 b_2 \quad (S_Q = 2)$$

Преобразование системы булевых функций

$$\begin{cases} c_1 = a_1 a_2 b_1 b_2 & (S_Q^{c_1} = 4) \\ c_2 = a_1 \bar{b}_2 \vee a_1 \bar{a}_2 b_1 \vee \bar{a}_1 a_2 a_3 b_1 b_2 & (S_Q^{c_2} = 13) \\ c_3 = a_1 \bar{b}_1 \vee a_2 \bar{b}_2 \vee a_1 a_2 a_3 \vee a_1 \bar{a}_2 b_2 \vee \bar{a}_1 \bar{a}_3 b_1 & (S_Q^{c_3} = 18) \\ c_4 = a_2 \bar{b}_1 \vee a_3 \bar{b}_2 \vee a_2 \bar{a}_3 b_2 \vee \bar{a}_2 a_3 b_1 & (S_Q^{c_4} = 14) \\ c_5 = a_3 b_2 & (S_Q^{c_5} = 2) \end{cases} \quad (S_Q = 51)$$

Проведем совместную декомпозицию системы.

$$c_5 = a_3 b_2$$

$$\begin{cases} c_5 = a_3 b_2 & (S_Q^{c_5} = 2) \\ c_1 = a_1 a_2 b_1 b_2 & (S_Q^{c_1} = 4) \\ c_2 = a_1 \bar{b}_2 \vee a_1 \bar{a}_2 b_1 \vee \bar{a}_1 a_2 b_1 c_5 & (S_Q^{c_2} = 12) \\ c_3 = a_1 \bar{b}_1 \vee a_2 \bar{b}_2 \vee a_1 a_2 a_3 \vee a_1 \bar{a}_2 b_2 \vee \bar{a}_1 \bar{a}_3 b_1 & (S_Q^{c_3} = 18) \\ c_4 = a_2 \bar{b}_1 \vee a_3 \bar{b}_2 \vee a_2 \bar{a}_3 b_2 \vee \bar{a}_2 a_3 b_1 & (S_Q^{c_4} = 14) \end{cases} \quad (S_Q = 50)$$

Проведем раздельную факторизацию системы.

$$\begin{cases} c_5 = a_3 b_2 & (S_Q^{c_5} = 2) \\ c_1 = a_1 a_2 b_1 b_2 & (S_Q^{c_1} = 4) \\ c_2 = a_1 (\bar{b}_2 \vee \bar{a}_2 b_1) \vee \bar{a}_1 a_2 b_1 c_5 & (S_Q^{c_2} = 12) \\ c_3 = a_2 \bar{b}_2 \vee a_1 (\bar{b}_1 \vee a_2 a_3 \vee \bar{a}_2 b_2) \vee \bar{a}_1 \bar{a}_3 b_1 & (S_Q^{c_3} = 17) \\ c_4 = a_2 \bar{b}_1 \vee a_3 (\bar{b}_2 \vee \bar{a}_2 b_1) \vee a_2 \bar{a}_3 b_2 & (S_Q^{c_4} = 14) \end{cases} \quad (S_Q = 49)$$

Проведем совместную декомпозицию системы.

$$\varphi_0 = \bar{b}_2 \vee \bar{a}_2 b_1$$

$$\begin{cases} \varphi_0 = \bar{b}_2 \vee \bar{a}_2 b_1 & (S_Q^{\varphi_0} = 4) \\ c_5 = a_3 b_2 & (S_Q^{c_5} = 2) \\ c_1 = a_1 a_2 b_1 b_2 & (S_Q^{c_1} = 4) \\ c_2 = \varphi_0 a_1 \vee \bar{a}_1 a_2 b_1 c_5 & (S_Q^{c_2} = 8) \\ c_3 = a_1 (\bar{b}_1 \vee a_2 a_3 \vee \bar{a}_2 b_2) \vee a_2 \bar{b}_2 \vee \bar{a}_1 \bar{a}_3 b_1 & (S_Q^{c_3} = 17) \\ c_4 = a_2 \bar{b}_1 \vee \varphi_0 a_3 \vee a_2 \bar{a}_3 b_2 & (S_Q^{c_4} = 10) \end{cases} \quad (S_Q = 45)$$

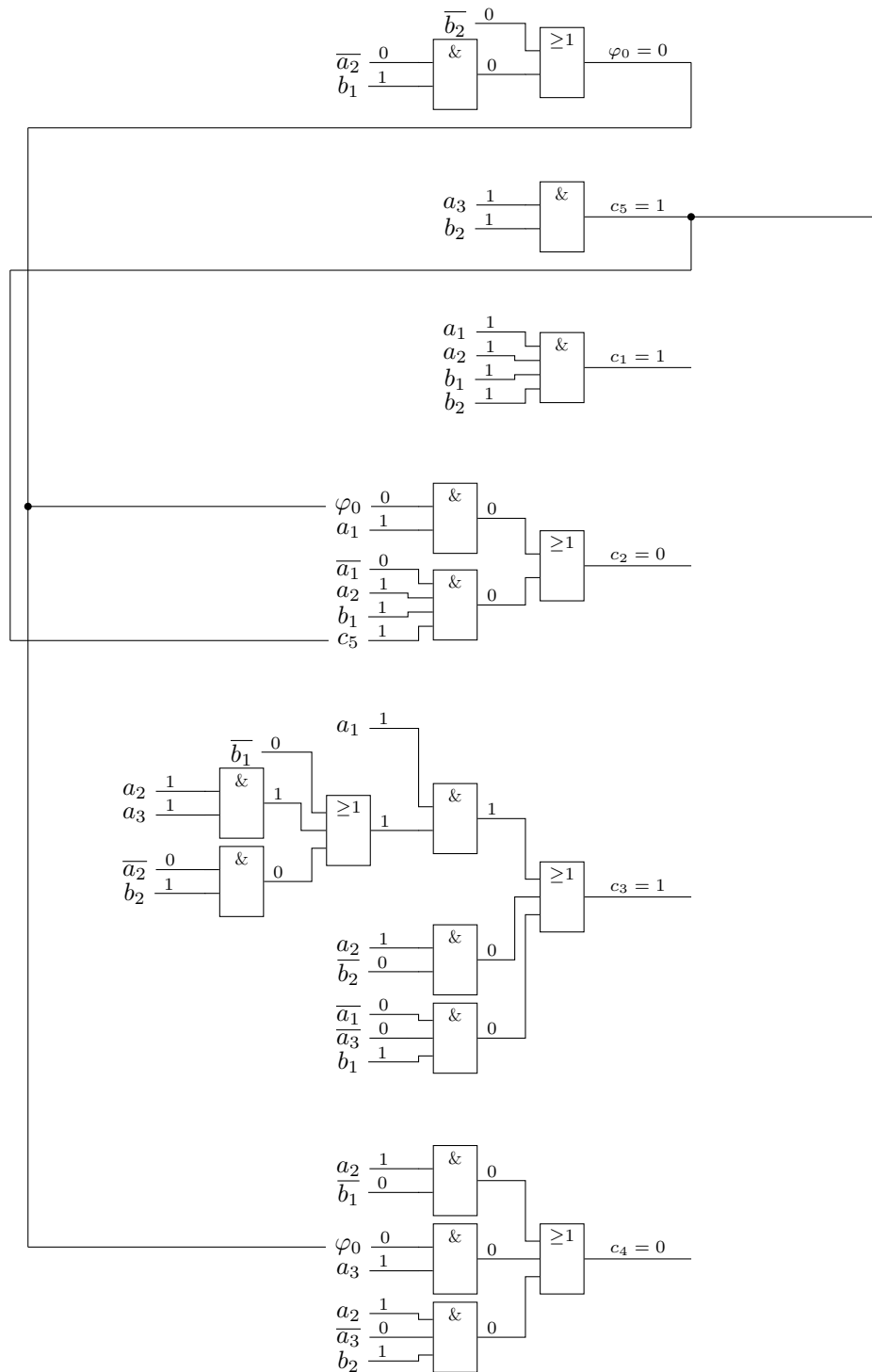
Синтез комбинационной схемы в булевом базисе

Будем анализировать схему на следующем наборе аргументов:

$$a_1 = 1, a_2 = 1, a_3 = 1, b_1 = 1, b_2 = 1$$

Выходы схемы из таблицы истинности:

$$c_1 = 1, c_2 = 0, c_3 = 1, c_4 = 0, c_5 = 1$$



Цена схемы: $S_Q = 45$. Задержка схемы: $T = 4\tau$.