

5.1 Compound Interest & Future Value

→ interest earned by an invested amount of money (principle) is reinvested so that it too earns interest (converted into principle)

ex. \$100 invested at rate of 5% compounded annually

$$100 + 100(0.05) = 100(1.05) = 105 \text{ at end of first year}$$

$$105 + 105(0.05) = 105(1.05) = 110.25$$

where \$110.25 = compounded amount

\$110.25 - \$100 = compound interest

$$\begin{aligned} * (FV) \quad S &= P(1+r)^n \\ &= P(1+r/k)^{kn} \end{aligned}$$

where S = Future Value / compounded amount

P = principle

n = at the end of n years

$S - P$ = compound interest r = rate compounded annually

k = # of interest periods per year

[remember your units!]

ex. \$1000 invested for 5 years at nominal rate 8% compounded quarterly

$$\text{rate per period} = \frac{0.08}{4}$$

$$\# \text{ of interest periods} = 5 \times 4 = 20$$

$$S = 1000 \left(1 + \frac{0.08}{4}\right)^{5 \times 4} = 1000(1.02)^{20} \approx 1485.95$$

ex. how long for \$600 to become \$900 at annual rate of 6% compounded quarterly?

$$r = \frac{0.06}{4} = 0.015$$

$$P = 600$$

$$S = 900$$

$$900 = 600(1 + 0.015)^n$$

$$\frac{900}{600} = (1.015)^n$$

$$\ln(1.5) = \ln(1.015)^n = n \cdot \ln(1.015)$$

$$n = \frac{\ln(1.5)}{\ln(1.015)}$$

$$\approx 27.23 \text{ quarter periods}$$

of interest periods

$$\text{so, } \frac{27.23}{4} \approx 6 \text{ years } 9 \text{ months}$$

must wait at interest period intervals since calculated quarterly

then, 7 years (28 quarter periods)

ex. \$1890 deposited for 7.5 years at 5.8% compounded quarterly

$$(FV) S = 1890 \left(1 + \frac{0.058}{4}\right)^{4 \times 7.5} \approx 2910.87$$

* (effective rate) $r_e = \left(1 + \frac{r}{n}\right)^n - 1$

→ approx rate of interest compounded annually that is actually earned

ex. effective rate equivalent to nominal rate of 6% compounded quarterly

$$r_e = \left(1 + \frac{0.06}{4}\right)^4 - 1 = (1.015)^4 - 1 \approx 0.0613$$

$$\text{so, } \sim 6.14\%$$

ex. how long to double a principle at 5% effective rate

$$2P = P(1 + 0.05)^t$$

$$2 = (1.05)^t$$

$$\ln(2) = \ln(1.05)^t = t \cdot \ln(1.05)$$

$$t = \frac{\ln(2)}{\ln(1.05)} \approx 14.206 \text{ years}$$

5.2 Present Value

* $P = S(1+r)^{-n}$

why → $S = P(1+r)^n$

$$P = \frac{S}{(1+r)^n} = S(1+r)^{-n}$$

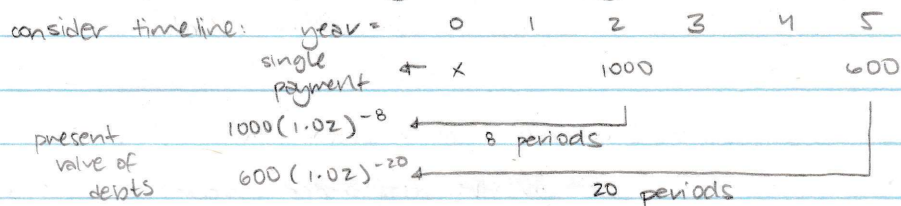
ex. present value of \$1000 years 3 years at 9% compounded monthly

$$S = 1000 \quad r = \frac{0.09}{12} = 0.0075 \quad n = 3 \times 12 = 36$$

$$P = 1000(1 + 0.0075)^{-36} \approx 764.15$$

* (equations of value) debt = payments

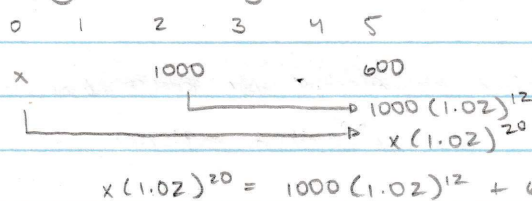
ex. suppose \$1000 due in 2 years, \$600 due in 5 years with interest rate of 8% compounded quarterly; want to pay total debt in single payment



so, payment = debt

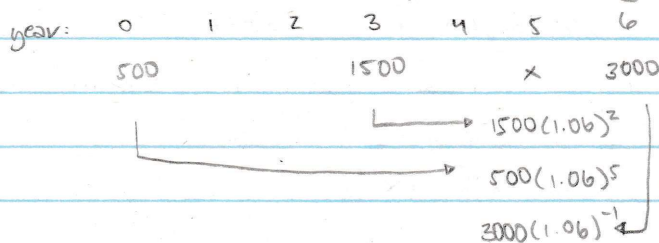
$$x = 1000(1.02)^{-8} + 600(1.02)^{-20} \approx 1257.27$$

consider payment on year 5



move forward
multiply equation
by $(1.02)^{20}$

ex. debt of \$3000 in 6 years paid in 3 payments of \$500 now, \$1500 in 3 years, and remaining on year 5. what is the remaining payment given interest rate of 6% compounded annually?



calculate
for values
at year 5

debt = payments

$$3000(1.06)^{-1} = x + 500(1.06)^5 + 1500(1.06)^2$$

$$475.68 \approx x$$