MATA32 - TUT0017 S.V.

Week 03

5.3 Interest compounded continously

* $S = Pe^{rt}$ where S = FV (compounded amount) P = principle

t = after t years r = annual interest rate compounded continously

ex. \$100 invested at an annual rate of 5% compounded continously what is the compounded amount at the end of 5 years?

$$S = (100)(e)^{(0.05)(5)} = 100e^{0.25} \approx 128.40$$

* (effective rate continous) $r_e = e^r - 1$

ex. find the effective rate corresponding to 8% compounded continously

$$r_e = e^{0.08} - 1 \approx 0.83287\ldots \rightarrow 8.33\%$$

ex. convert 5% compounded continously to compounded monthly

$$Pe^{rt} = P(1+r/k)^{kt} \rightarrow Pe^{0.05t} = P(1+r/12)^{12t}$$

$$e^{0.05t} = (1+r/12)^{12t}$$

$$\ln(e)^{0.05t} = \ln(1+r/12)^{12t}$$

$\ln(e) = \log_e(e) = 1$

$$0.05t \cdot \ln(e) = 12t \cdot \ln(1+r/12)$$

$$0.05t = 12t \cdot \ln(1+r/12)$$

$$\ln(1+r/12) = 0.05/12$$

$e^{\ln(x)} = x$

$$e^{\ln(1+r/12)} = e^{0.05/12}$$

$$1 + r/12 = e^{0.05/12}$$

$$r/12 = e^{0.05/12} - 1$$

$$r = 12(e^{0.05/12} - 1)$$

ex. what annual rate r compounded continously is equivalent to an APR of 8% compounded semi-annually?

$$Pe^{rt} = P\left(1 + \frac{0.08}{2}\right)^{2t}$$

$$e^{rt} = (1.04)^{2t}$$

$$rt \cdot \ln(e) = 2t \cdot \ln(1.04)$$

$$r = 2 \cdot \ln(1.04) \approx 0.0784\ldots$$

$$r = 7.84\%$$

* $P = Se^{-rt}$ where P = present value (principle) S = compounded amount

t = after t years r = annual rate compounded continously

ex. want \$125,000 at the end of 20 years given continoos compounding at annual rate of 7%, what is the amount needed initially?

$$P = (25000)e^{-(0.07)(20)} = 25000e^{-1.4}$$

$$\approx 6164.92$$

5.4 Annuities

→ finite sequence of payments made at fixed periods over a given time interval

* (PV) $A = R \cdot \frac{1-(1+r)^{-n}}{r}$

gives present value A of an ordinary annuity (sum of present values of all paymen[ts]) with (\$) R per payment period for n periods at interest rate r per perio[d]

ex. find present value of an annuity of \$100 per month for 3.5 years at interest rate of 6% compounded monthly

$$R = 100 \qquad r = \frac{0.06}{12} = 0.005 \qquad n = (3.5)(12) = 42$$

$$A = 100\left(\frac{1-(1+0.005)^{-42}}{0.005}\right) \approx 3779.83$$

ex. find the present value of an annuity of \$100 at the end of each quarter for two years and \$200 afterwards at the end of each quarter for three years given a nominal rate of 4% compounded quarterly

(A) $A_1 = 100 \cdot \frac{1-\left(1+\frac{0.04}{4}\right)^{-4\times 2}}{\left(\frac{0.04}{4}\right)} = 100 \cdot \frac{1-(1.01)^{-8}}{0.01} \approx 765.17$ | year 0 – 2

(B) $A_2 = 200 \cdot \frac{1-\left(1+\frac{0.04}{4}\right)^{-4\times 3}}{\left(\frac{0.04}{4}\right)} = 200 \cdot \frac{1-(1.01)^{-12}}{0.01} \approx 2251.02$ | year 2 – 5

(C) present value of A_2 at year 0

$$S = 2251.02\left(1+\frac{0.04}{4}\right)^{-4\cdot 2} = 2251.02(1.01)^{-8} \approx 2078.78$$

(D) A_1 + PV of A_2 at year 0

$$\approx 765.17 + 2078.78 \approx 2843.95$$

* (annuity due) $A = R \cdot \left(\frac{1-(1+r)^{-n+1}}{r} + 1 \right)$ → payment at beginning

→ payment is due at beginning of each period

[keywords
"due now"
"at the beginning"
"in advance"]

ex. quarter periods: 0 1 2 3

50 50 50 50 (annuity over 3 not 4)

$\rightsquigarrow 50 + 50\left(\frac{1-(1+r)^{-3}}{r}\right) = 50\left(1 + \frac{1-(1+r)^{-3}}{r}\right)$

* (FV) $S = R \cdot \frac{(1+r)^n - 1}{r}$

gives future value S of an ordinary annuity (sum of future value of all payments) with (\$) R per payment period for n periods at interest rate r per period

can also use

A_{FV} AD_{FV}

A_{PV} AD_{PV}

ex. future value of ordinary annuity given \$600 per quarter for 4 years at 8% compounded quarterly

$R = 600$ $r = \frac{0.08}{4} = 0.02$ $n = (4)(4) = 16$

$A_{FV} = S = 600\left(1.02^{16} - 1 / 0.02\right) \approx 11\,183.57$

* (annuity due) $S = R \cdot \left(\frac{(1+r)^{n+1} - 1}{r} - 1 \right)$ → no payment on reaching last period

$R\left(\frac{(1+r)^{n+1} - 1}{r}\right) - R$

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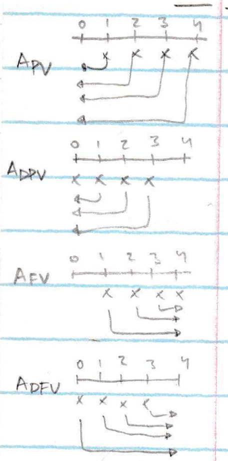

ex. (txt; 5.4 #20) machine purchased for \$3000 down and payments of \$250 at the end of every 6 months for six years. Given interest of 8% compounded semiannually, find cash price.

CP = price paid now

where 3000 = amount downpayment

$R = 250$ $t = 6$ years $k = 2$ periods (semi-annual)

$a = 8\%$ $r = \frac{0.08}{2} = 0.04$

$CP = 3000 + 250\left(\frac{1-(1.04)^{-12}}{0.04}\right)$

≈ 5346.27