MATA32 - TUT0017

week02

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5.1 Compound Interest & Future Value

→ interest earned by an invested amount of money (principle) is reinvested so that it too earns interest (converted into principle)

ex. \$100 invested at rate of 5% compounded anually

$100 + 100(0.05) = 100(1.05) = 105$ at end of first year

$105 + 105(0.05) = 105(1.05) = 110.25$

where \$110.25 = compounded amount

\$110.25 - \$100 = compound interest

* (FV) $S = P(1+r)^n$
$= P(1 + r/k)^{kn}$

where S = Future value / compounded amount

P = principle n = at the end of n years

S - P = compound interest r = rate compounded <u>anually</u>

k = # of interest periods per year

[remember your units!]

ex. \$1000 invested for 5 years at nominal rate 8% compounded quarterly

rate per period = $\frac{0.08}{4}$ # of interest periods = 5×4

$S = 1000\left(1 + \frac{0.08}{4}\right)^{5\times4} = 1000(1.02)^{20} \approx 1485.95$

ex. how long for \$600 to become \$900 at annual rate of 6% compounded quarterly?

$r = \frac{0.06}{4} = 0.015$ $P = 600$ $S = 900$

$900 = 600(1 + 0.015)^n$ where $n = 4t$ calculate # of quarterly periods

$\frac{900}{600} = (1.015)^n$

$\ln(1.5) = \ln(1.015)^n = n \cdot \ln(1.015)$

$n = \frac{\ln(1.5)}{\ln(1.015)}$

≈ 27.23 quarter periods

so, $\frac{27.233\ldots}{4}$ (# of interest periods) $\approx$ 6 years 9 months

must wait at interest period intervals since calculated quarterly

then, 7 years (28 quarter periods)

ex. \$1890 deposited for 7.5 years at 5.8% compounded quarterly

$$(FV)\ S = 1890\left(1 + \frac{0.058}{4}\right)^{4 \times 7.5} \approx 2910.87$$

* (effective rate) $r_e = (1 + r/n)^n - 1$

→ approx rate of interest compounded anually that is actually earned

ex. effective rate equivalent to nominal rate of 6% compounded quarterly

$$r_e = \left(1 + \frac{0.06}{4}\right)^4 - 1 = (1.015)^4 - 1 \approx 0.0613$$

so, ~ 6.14%

ex. how long to double a principle at 5% effective rate

$$2P = P(1 + 0.05)^t$$

$$2 = (1.05)^t$$

$$\ln(2) = \ln(1.05)^t = t \cdot \ln(1.05)$$

$$t = \frac{\ln(2)}{\ln(1.05)} \approx 14.206 \text{ years}$$

5.2 Present Value

* $P = S(1+r)^{-n}$ why → $S = P(1+r)^n$

$$P = S/(1+r)^n = S(1+r)^{-n}$$

ex. present value of \$1000 ~~years~~ 3 years at 9% compounded monthly

$S = 1000$ $r = \frac{0.09}{12} = 0.0075$ $n = 3 \times 12 = 36$

$$P = 1000(1 + 0.0075)^{-36} \approx 764.15$$

* (equations of value) debt = payments

ex. suppose \$1000 due in 2 years, \$600 due in 5 years with interest rate of 8% compounded quarterly; want to pay total debt in single payment

consider timeline:

year =	0	1	2	3	4	5
single payment	x		1000			600

present value of debts

$1000(1.02)^{-8}$ ← 8 periods

$600(1.02)^{-20}$ ← 20 periods

so, payment = debt

$$x = 1000(1.02)^{-8} + 600(1.02)^{-20} \approx 1257.27$$

consider payment on year 5

0	1	2	3	4	5
x		1000			600

→ $1000(1.02)^{12}$

→ $x(1.02)^{20}$

$$x(1.02)^{20} = 1000(1.02)^{12} + 600$$

move forward
multiply equation
by $(1.02)^{20}$

ex. debt of \$3000 in 6 years paid in 3 payments of \$500 now, \$1500 in 3 years, and remaining on year 5. what is the remaining payment given interest rate of 6% compounded anually?

year:	0	1	2	3	4	5	6
	500			1500		x	3000

calculate
for values
at year 5

→ $1500(1.06)^2$

→ $500(1.06)^5$

$3000(1.06)^{-1}$ ←

debt = payments

$$3000(1.06)^{-1} = x + 500(1.06)^5 + 1500(1.06)^2$$

$$475.68 \approx x$$