

$$a^2 + b^2 = (a+b)(a+b)$$

$$a^2 - b^2 = (a-b)(a+b)$$

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$$\phi \frac{d^2y}{dx^2} + a^2 y = \tan ax$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$e^{-i\theta} = \cos \theta - i \sin \theta$$

$$(D^2 + a^2)y = \tan ax$$

$$a^2 + b^2 = (a+b)(a+b)$$

$$m^2 + a^2 = 0$$

$$D^2 + a^2 = p$$

$$m = \pm a i = \alpha \pm i\beta$$

$$C.F. = \{C_1 \cos ax + C_2 \sin ax\} e^{ax}$$

$$P.I. = \frac{1}{(D^2 + a^2)} (\tan ax)$$

$$P.I. = \frac{1}{(D - ai)(D + ai)} \tan ax$$

$$P.I. = \frac{1}{(D - ai)(D + ai)} = \frac{A}{(D - ai)} + \frac{B}{(D + ai)} = \frac{\frac{1}{2ai}}{D - ai} + \frac{\frac{1}{2ai}}{D + ai}$$

$\stackrel{L>0}{D = ai}$ $\stackrel{L>0}{D = -ai}$

$$P.I. = \left\{ \frac{\frac{1}{2ai}}{(D - ai)} + \frac{\frac{(-1)}{2ai}}{(D + ai)} \right\} \tan ax$$

$$P.I. = \frac{1}{2ai} \left\{ \frac{1}{(D - ai)} \tan ax - \frac{1}{(D + ai)} \tan ax \right\} \quad (1)$$

Post

$$\frac{1}{D - ai} \tan ax \mid_{(D - ai)} \times \tan ax = \frac{1}{(D - ai)} e^{aix} - \tan ax$$

$$= \frac{1}{(D - ai)} e^{aix} \cdot e^{-aix} \tan ax \left[\frac{1}{f(D)} e^{aix} f(x) = e^{ax} \mid_{f(D+a)} f(x) \right]$$

$$= e^{aix} \frac{1}{(D + ai - ai)} \tan ax = e^{aix} \int_{(2)}^{-aix} \tan ax dx$$

$$= e^{aix} \int_a^{-aix} \tan ax dx$$

$$\left\{ \therefore e^{i\theta} = \cos \theta + i \sin \theta \right.$$

$$= e^{aix} \int \{ \cos ax - i \sin ax \} \tan ax dx$$

$$= e^{aix} \int \{ \cos ax - i \sin ax \} \frac{\sin ax}{\cos ax} dx$$

$$= e^{\frac{a}{2}ix} \int \left\{ \sin ax - i \sin^2 ax \right\} dx$$

$$= e^{\frac{a}{2}ix} \int \left\{ \sin ax - i(1 - \cos^2 ax) \right\} dx$$

$$= e^{\frac{a}{2}ix} \int \left\{ \sin ax - i(\sec ax - \cos ax) dx \right\}$$

$$= e^{\frac{a}{2}ix} \left[-\cos ax - i \left\{ \frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a} \right\} \right] - (1)$$

$$\text{Post } \frac{1}{D+ai} \tan(ax)$$

$$= \frac{1}{D+ai} \frac{1}{1+x \tan(ax)}$$

$$= \frac{1}{D+ai} e^{\frac{a}{2}ix - aix} \tan(ax)$$

$$= \frac{1}{D+ai} e^{\frac{a}{2}ix - aix} - \tan(ax)$$

$$= \frac{1}{D} e^{-aix} \frac{1}{(D - ai + ai)} e^{\frac{a}{2}ix} \tan(ax)$$

$$= \frac{e^{-aix}}{D} \frac{1}{e^{\frac{a}{2}ix}} e^{\frac{a}{2}ix} \tan(ax)$$

$$= e^{-aix} \int e^{\frac{a}{2}ix} \tan(ax) dx$$

$$= e^{-aix} \int \{ \cos ax + i \sin ax \} - \tan(ax) dx \quad \left\{ \because e^{\frac{a}{2}ix} = \cos \theta + i \sin \theta \right\}$$

$$= e^{-aix} \int \{ \cos ax + i \sin ax \} \frac{\sin ax}{\cos ax} dx$$

$$= e^{-aix} \int \{ \sin ax + i \sin^2 ax \} dx$$

$$= e^{-aix} \int \left\{ \sin ax + i \frac{(1 - \cos^2 ax)}{\cos ax} \right\} dx$$

$$= e^{ax} \int \{ \sin ax + i(\sec ax - \cos ax) \} dx$$

$$= e^{ax} \left[-\cos ax + i \left\{ \frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a} \right\} \right] \quad \text{(iii)}$$

Put (ii) & (iii) in eqn (i)

$$\text{P.I.} = \frac{1}{2ai} \left\{ \left(e^{ax} \left[-\cos ax - i \left\{ \frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a} \right\} \right] \right) - \left(e^{ax} \left[-\cos ax + i \left\{ \frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a} \right\} \right] \right) \right\}$$

Complete solution $y = \text{C.F.} + \text{P.I.}$

$$y = \{c_1 \cos ax + c_2 \sin ax\} e^{ax} +$$

$$\frac{1}{2ai} \left\{ \left(e^{ax} \left[-\cos ax - i \left\{ \frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a} \right\} \right] \right) - \left(e^{ax} \left[-\cos ax + i \left\{ \frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a} \right\} \right] \right) \right\}$$

$$\left(\underline{\text{Ans}} \right)$$