10/March/25 Monday

"SYLLABUS" Eng. Physics

UNIT-I

WAVE NATURE OF PARTICLES AND THE SCHRODINGER EQ^n

TOPICS

i. Introduction to Quantum mechanics,
ii. Wave nature of particles,
iii. Operators,
iv. Time (dependent and Independent),
v. schrodinger eq^n for wavefunct^n,
vi. Particle in a one dimensional Box,
vii. Born Interpretation,
viii. V_g & V_p relat^n Uncertaintly principle,

UNIT-III

INTRODUCTION OF SOLID

(i) free electron theory of metals
(ii) band theory of solids
(iii) fermilevel of intrinsic & Extrinsic semiconductor
(iv) froning-Pemey Model
(v) Density of states
(vi) V-I characteristics of PN Junc^n diode & Zener Diode
(vii) V-I characteristics of Solid cell
(viii) Hall Effect

UNIT-II

WAVE OPTICS

TOPICS

i. Huygen Principle,
ii. Principle of Superposition,
iii. Interference of light,
iv. Young's double slit Experiment,
v. Newton's rings,
vi. Michelson interferometer
vii. Machzehnder Interferometer
viii. single & Double - slit Frauhofer diffraction
ix - N-slit Diffraction & Grating
x - Rayleigh Criteria
xi - Resolving Power of Grating.

UNIT-IV

LASER

(i) Properties of laser
(ii) Principle of laser
(iii) Extinction coefficient
(iv) Ruby Laser
(v) He-Ne laser
(vi) CO_2 laser
(vii) optical fibre
(viii) Mono-chromaticity
(ix) directionality and brightness
(x) Laser speckles
(xi) Application laser in science
(xii) Numericals.

UNIT-V ELECTROSTATICS IN VACUUM

(i) Coulomb's law
(ii) Electric field
(iii) Gauss's theorem
(iv) Electric Potential (V)
(v) capacitance & capacitors
(vi) Energy stored in capacitor
(vii) Electric Dipole
(viii) P-N Junc^n Diode
(ix) Solar cell

11/MAR/25 ~~Mond~~ Tuesday

(Laser light always ~~travel at straight~~)

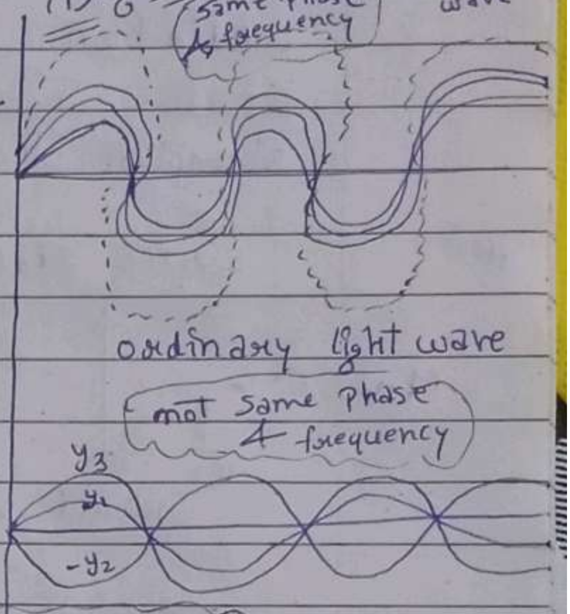
UNIT - ~~IV~~

LASER {light amplification by ~~stimulated~~ emission of radiation}

Properties of laser light

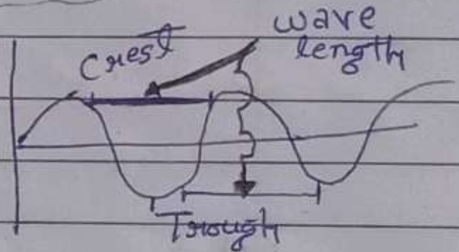
$\vec{y} = \vec{y_1} + \vec{y_2} + \vec{y_3}$

① Coherent :- laser light waves are coherent wave (having same Phase & frequency) where as ordinary light waves are not Coherent



② Monochromatic :- laser light waves are monchromatic, where as ordinary light wave are not Monochromatic

Means — (~~Single wave length~~ Closed)



③ Directionality :- laser ~~I~~ light does not diverge with distance, where as ordinary light diverges.

④ Intensity :- Intensity of laser light is very high as compare to ordinary light.

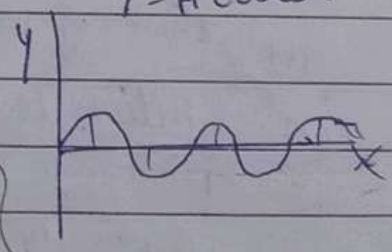
$I \propto A^2$

Displacement y

dispo eqn

$y = A\sin\omega t$

$Y = A\cos\omega t$



⑤ Distance :- laser light travel longer distance as compare to ordinary light.

Phenomenon Absorption, Spontaneous, Stimulated Radiation Application

(Hilium Leon laser used for Bar code reading)

12/Mar/25 WEDNESDAY

Carbon dioxide (Medical field) Surgery

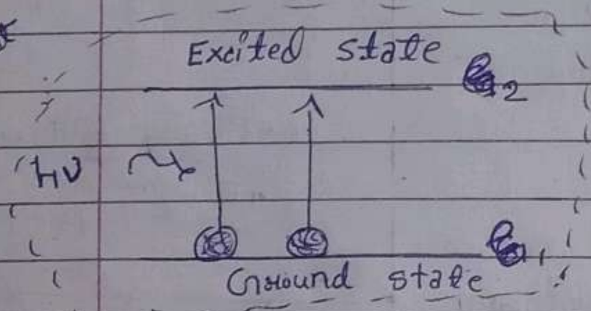
NdYAG Welding (Temperory Close)

Laser Shows, Skin Tightening, Industries

① Absorption Radiation

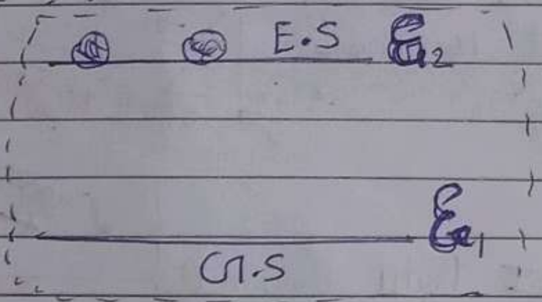
⇒ Atoms of the ground state absorb radiation and raise to the Excited State. This phenomenon is called absorption of radiation

(Excited state is unstable state)



$E = h\nu$ — frequency; Energy Electron Charge; Plank's constant

(ground state → Excited state पर जाएगा या नहीं Probability)



Surrounding Temp.

$15^\circ C \xrightarrow{e} 100^\circ C$
water g-state → E-state

(Energy given to the volume)

② Probability of Absorption Radiation (P_{12})

Probability of radiation depends on Energy density that is $P_{12} \propto u(\nu)$

($u(\nu)$ funcⁿ)

$$P_{12} = B_{12}\, u(\nu)$$

B_{12} = Einstein Coefficient of absorption radiation

Application of Laser

1. Helium leon laser
2. Distance measure
3. Surgery
4. Medical field CO_2 Nd YAG
5. Laser shows
6. Skin Tightening
7. Industries
8. Welding

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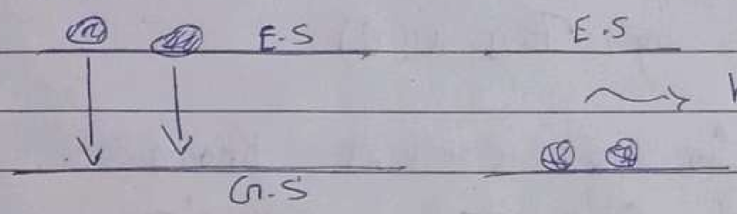
excited state realises photon to the ground state

② **Spontaneous emission :-**

Atoms of excited state realises photon and Come down to the ground state this Phenomenon is called spontaneous emission



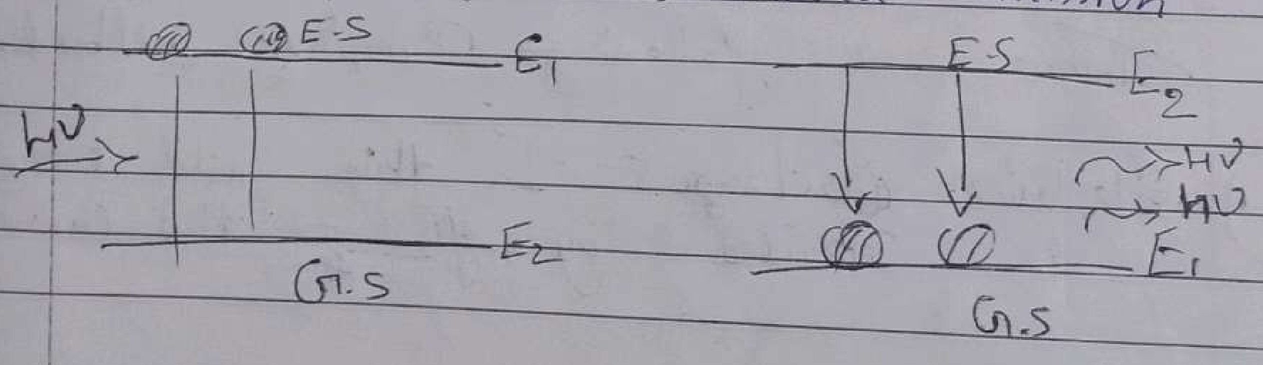
⇒ **Probability of Spontaneous emission :-**

Probability of spontaneous emission does not depend on Energy density

Probability of spontaneous emission is represented by A_{21} & it is called Einstein Coefficient of spontaneous emission.

Stimulated Emission :-

When atom is at higher energy state and other photon are incident on it, then whenever this atom comes down to ground state, it emits two photon of same frequency and phase, this phenomenon is called stimulated emission



Probability of stimulated emission

→ It is directly proportional to energy density $u(\nu)$.
it is given by $B_{21}\, u(\nu)$

where B_{21} is a constant known as Einstein coefficient of stimulated emission.

⇒ Total probability of an atom of higher energy state to come down to the ground state is given by

$$P_{21} = A_{21} + B_{21}\, u(\nu)$$

Q1. What do you mean by Population inversion and also write the pumping method.

As:-

Accumulation of atoms at higher energy state is known as population inversion.

[Process of achieving population inversion is called pumping]

There are following Pumping method.

1) Electrical discharge :- In this mtd energy is provided by flowing the current

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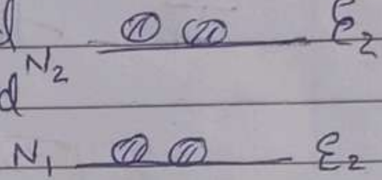
2) OPTICAL PUMPING :- Energy is provided by flashing the lamp.

3) Collision :- Energy is Provided by Colliding the atoms.

Relation between A_{21} & B_{21} Coefficient

Let N_1 atoms are at the ground state and N_2 atoms are at excited state Probability of N_1 atoms to move to excited state



$$N_1 P_{12} = N_1 B_{12}\, u(\nu) \quad \text{--- (1)}$$

Probability of N_2 atoms to come down to ground state

$$N_2 P_{21} = N_2 \left[A_{21} + B_{21}\, u(\nu) \right] \quad \text{--- (2)}$$

In thermodynamic equilibrium condition

$$N_1 P_{12} = N_2 P_{21}$$

$$N_1 B_{12}\, u(\nu) = N_2 \left[A_{21} + B_{21}\, u(\nu) \right]$$

$$N_1 B_{12}\, u(\nu) = N_2 A_{21} + N_2 B_{21}\, u(\nu)$$

$$(N_1 B_{12} - N_2 B_{21})\, u(\nu) = N_2 A_{21}$$

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Carbon Black

Black Body Absorbed & Entered radiation

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$$u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}$$

$$u(\nu) = \frac{A_{21}}{\left[\frac{N_1}{N_2} B_{12} - B_{21}\right]}$$

$$u(\nu) = \frac{A_{21}}{B_{21}\left[\frac{N_1}{N_2}\frac{B_{12}}{B_{21}} - 1\right]}$$

In thermodynamic equilibrium condition

$$\boxed{B_{12} = B_{21}}$$

~~In~~ $u(\nu) = \frac{A_{21}}{B_{21}\left[\frac{N_1}{N_2} - 1\right]}$ — (3)

By Maxwell Boltzmann

$$N \propto e^{-E/KT}$$

where K is Boltzmann constant and ~~T~~ T is Temperature

So

$N_1 \propto e^{-E_1/KT}$ and $N_2 \propto e^{-E_2/KT}$

$$\frac{N_1}{N_2} = \frac{e^{-E_1/KT}}{e^{-E_2/KT}} = e^{(E_2 - E_1)/KT}$$

Eq - photo electric

$E = h\nu$

$E_2 - E_1 = h\nu$

$$\frac{N_1}{N_2} = e^{h\nu/KT}$$

By placing value in Eqn (3)

$$u(\nu) = \frac{A_{21}}{B_{21}\left[e^{h\nu/KT} - 1\right]} \quad \text{— (4)}$$

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Resonance
amplitude
Basic Construction

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~~By Comparing Eqn 4~~

According to Plank

$$u(\nu) = \frac{8\pi h\nu^3}{c^3[e^{h\nu/kT}-1]} \quad -(5)$$

where 8π is constant
& h is plank's constant
or c = speed of light

By Comparing Eqn (4) & (5)

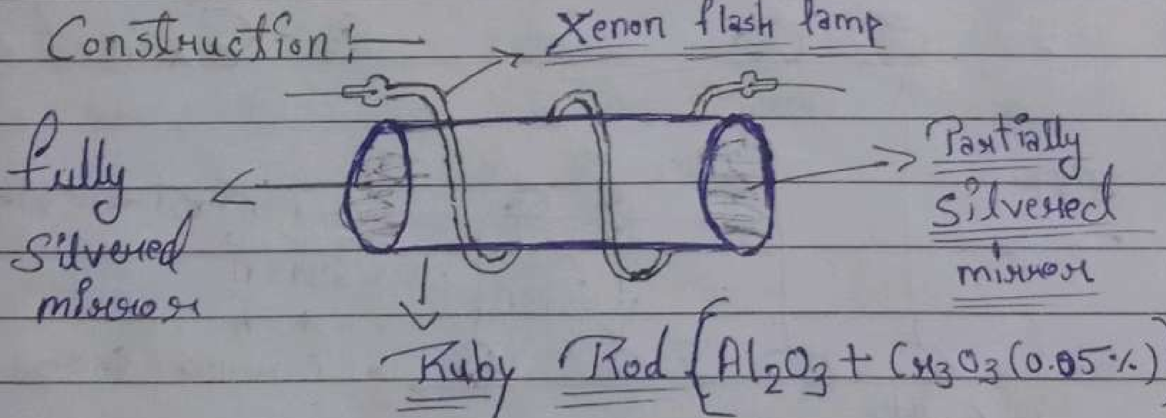
$$\frac{A_{21}}{B_{21}[e^{h\nu/kT}-1]} = \frac{8\pi h\nu^3}{c^3[e^{h\nu/kT}-1]}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \Rightarrow \frac{A_{21}}{B_{21}} \propto \nu^3$$ — for other term constant

: Rubey laser :—

Q21: Explain Construction & Working of Ruby laser?
Write any thre defficiency of laser.

Construction :—



5500Å Chromium ~~ratone~~ Catalyst E.S E_2 M.S Metastable state

green Colour E_2 E_1

Steel conducting material Cr.S

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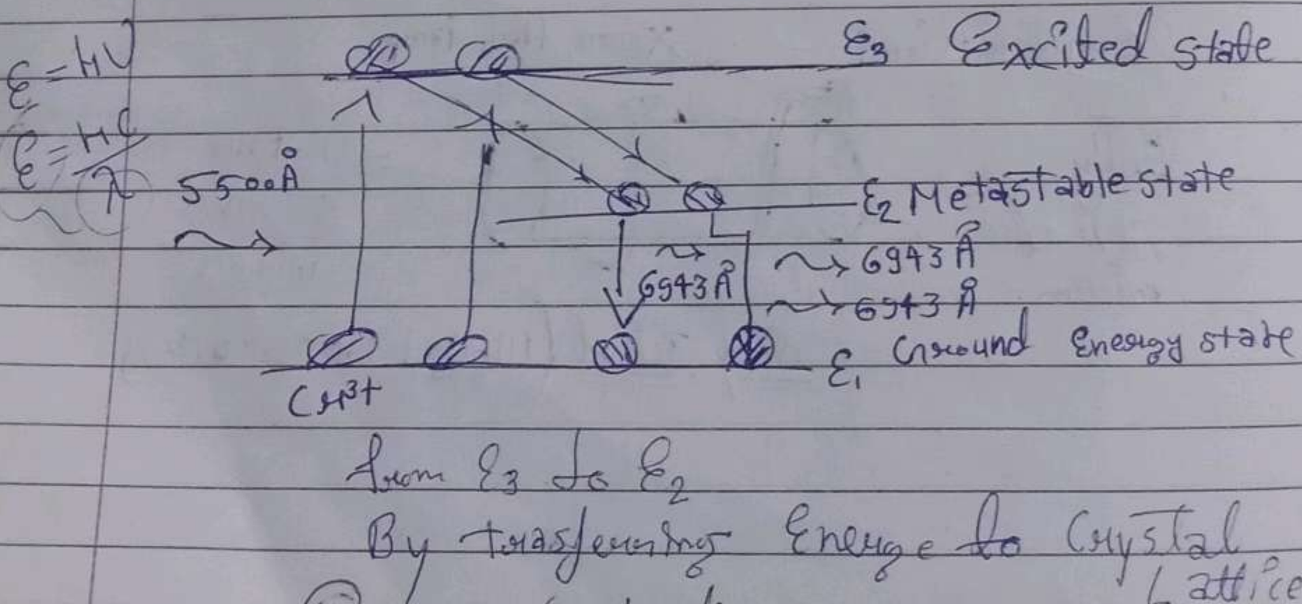
\# This laser ~~it~~ is consist of ~~the~~ a ruby rod which is made by Al_2O_3 and it is ~~deo~~ doped with Cr_2O_3 (0.05%)

\# A Xenon flash lamp is bounded on a ruby rod to provide optical Pumping

\# There is a fully silvered mirror at one end and partially silver mirror the other end of the ruby rod.

\# It is three Energy level

\# It is a ~~cristlying~~ Crystalline laser {Crystaline is one laser where crystal material is used}.



from E_3 to E_2

By transferring Energe to Crystal Lattice

Energy level diagram.

$E_3 < E_2$ Ruby laser — Pink Dim Beam

$1s$ $3s$ 25/March/25

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Initially Cr^{3+} ions are at the ground state & then by absorbing the photon of wavelength 5500 Å this Cr^{3+} ions raise to the excited state.

zenon flash lamp is provide optical pumping

from E_3 Energy state (Excited state) Cr^{3+} ions transfer some of their energy to crystal lattice and comedown to meta-stable state (E_2)

From E_2 Energy state Initially Cr^{3+} ion comes down to ground state by spontaneous emission of wave length 6943Å

If this photon's find other metastable state Cr^{3+} ion then the other will come down to ground state by stimulated emission

When this photon strikes on the transparent section they comeout in the form a laser beam

~~Def~~ # Defficiencies of the laser

(1) It is not operating in ~~Continuous~~ contineous mode

(2) High Pumping Power is required because ony green component of light is utalized

25 | Marks 25

95% and Hoe laser

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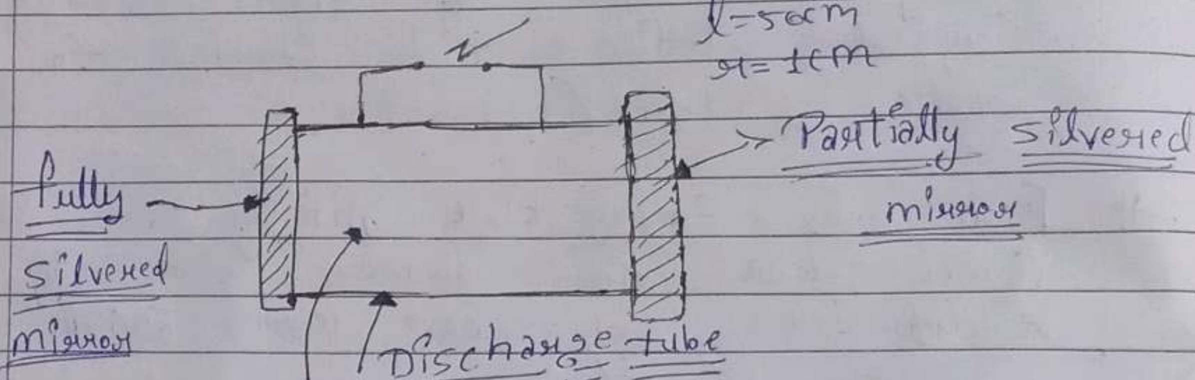
(3) Efficiency of this laser is lower than <1 % (Helium Neon laser)

5 Marks (2) He-Ne Laser :-

MST

Q.1 Explain He-Ne laser under the following Heads

(1) Construction, (2) Working, (3) Application



filled with He & Ne gases

He : Ne

{10 : 1}

It is a gaseous laser
It consist of a discharge tube of length 50cm & diameter 1cm
This tube is filled with He & Ne gases in the ratios of 10:1
Their is a fully silvered mirror at one end and partially silver mirror on the other end of the discharge tube.
two electrodes are connected with discharge tube to provide electrical discharge.

E4
E3 → $1^{st}, 2^{nd}$ metastable state
E2
E1 Pink-Red-Colour

6328
6328

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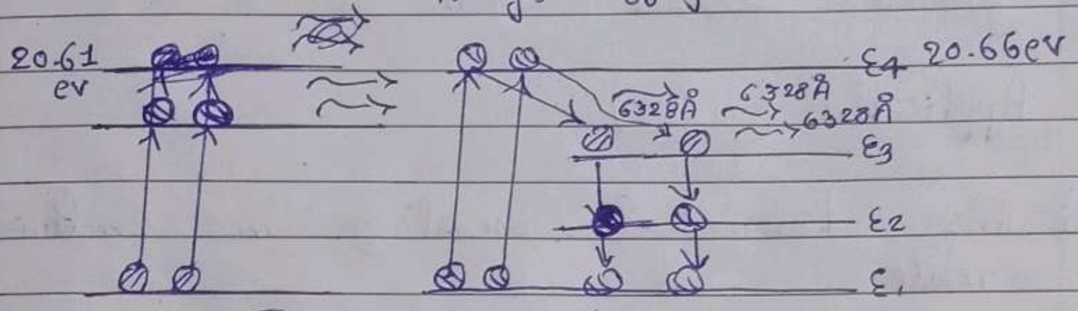
Q.1 =

It is four energy level laser.

26 Marks 10^{-9} sec Nenosec

Working of He-Ne laser :-

Transfer energy by collision to Ne atoms



Energy level diagram

⇒ Initially, He & Ne atoms are at the ground state. then we flow current through the mixture of He & Ne gases.

⇒ By Absorbing the energy He & Ne atoms are raised to excited state.

⇒ Ne atoms are Excited by two ways, One due to flow of the current and second due to collision by He atoms.

⇒ From Excited state E4 (20.66eV). Ne atoms come down to E3 energy level by spontaneous emission of photon wavelength 6328Å. If this ~~strikes~~ strikes on other E4 energy level Ne-atom then the other will come down by stimulated emission of two photons. (6328Å)

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⇒ From E_3 Energy level Ne-atoms come down to E_2 level and then E_1 level by transferring their energy to other ground state atom

Application (USES) :-

⇒ This Laser is operating in Continueous mode
⇒ This laser is mainly use for Bar code Reading
⇒ Efficiency of this laser is higher than Ruby laser.
⇒ This laser is used for targeting an object
⇒ This laser is used for Industrial work and Research or study work.

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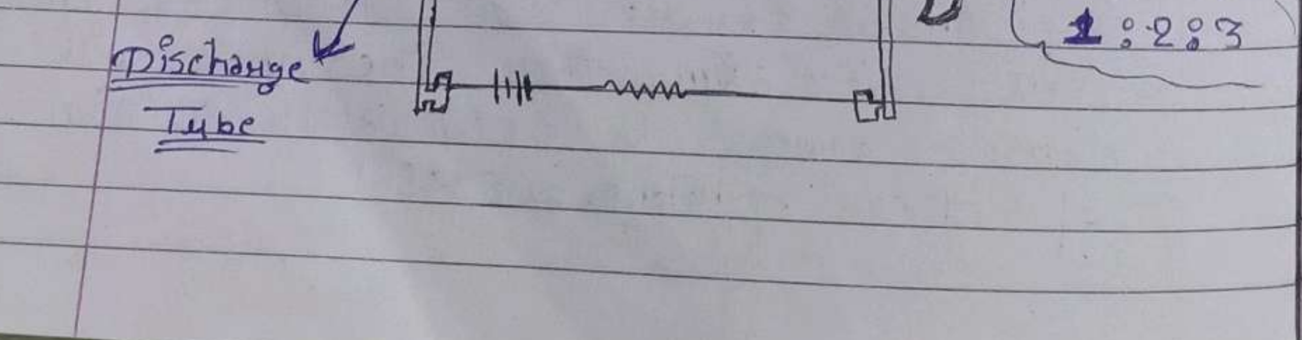
CO_2 LASER

⇒ Construction



⇒ It is a gaseas laser.
⇒ It Consist of a discharge tube of length 26cm and cross section area in about $1.5\ mm^2$
→ This tube is filled with Carbon dioxide (CO_2), Nitrogen(N_2) and Helium (He) gases in the ratio of 1:2:3
⇒ Electrical discharge method is used for Pumping of CO_2 and N_2 molecules
⇒ This is four energy level laser.

WORKING:-

(1) When Energy is given to the CO_2 molecules then it start vibration in different modes such as stetch mode, Bending mode & Assymetric mode. To Comeback in normal state CO_2 molecules Execute infrared radiation.

(2) When we flow the current through the mixture of CO_2, N_2 and He gases their molecules absorbed energy and raise ~~to~~ the Excited state.

(3) CO_2 molecules are Excited by two ways one - by collision through N_2 Molecules and second by the flow of current

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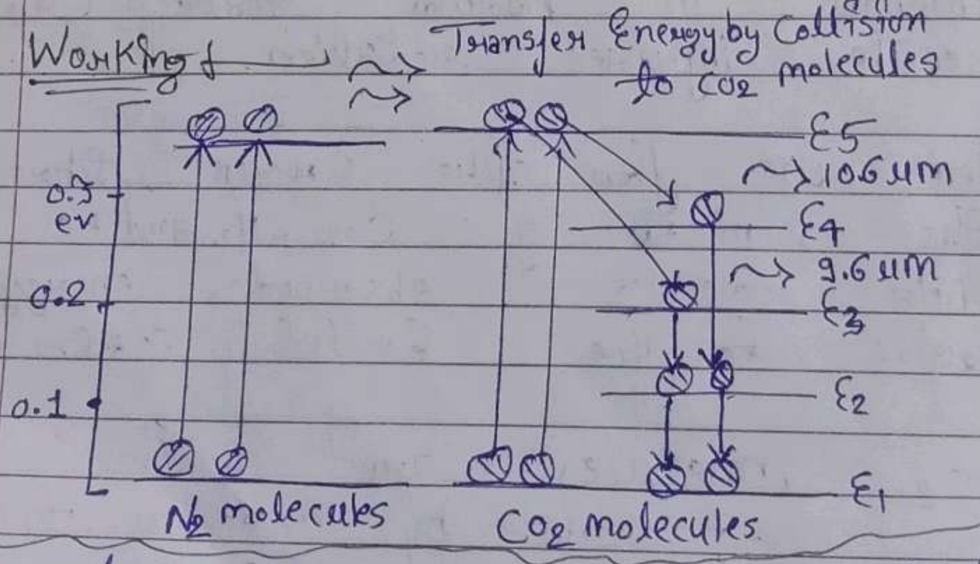
(4) ~~A~~T E_5 Energy level CO_2 molecules ~~steal~~ releases photon of wavelength ~~10~~ 10.6 µm or 9.6 µm and comedown ~~to~~ E_4 Energy level or E_3 Energy level

(5) ~~Kno~~ Now from here CO_2 molecules come down to E_2 level and then E_1 Energy level by transfering their Energy to other ground state molecules and He atoms.

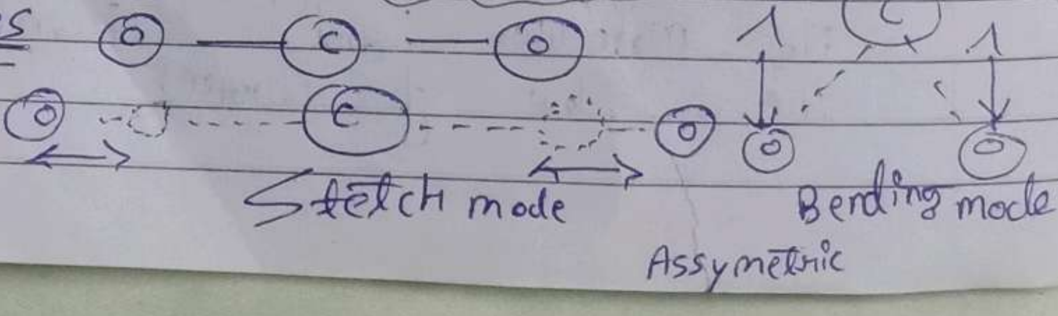
Application of CO_2-N_2-He laser.

This laser is mainly ~~las~~ used for Surgery work. Efficiency of this laser is about 45 % and which is more than Ruby and He-Ne laser.

Working :



Modes



19640 Å — Energyband

Nd:YAG LASER
Ruby laser
Solid state laser

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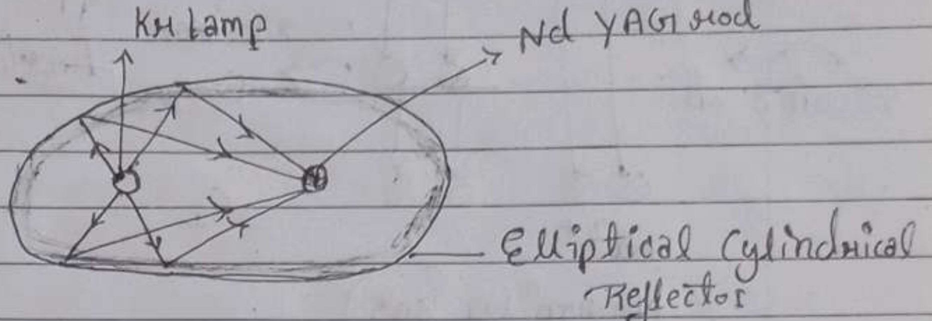
Nd: YAG LASER — Crystaline laser

Construction

Diagram



⇒ It is a Solid state laser where Nd stands for ~~neodinium~~ Neodymium. YAG stands for Yittrium aluminum garnet

⇒ It is Consist of an elliptical Cylindrical reflector

⇒ Krypton (Kr) lamp is used to provide optical pumping

⇒ Krypton (Kr) lamp is fixed at one focus and Nd:YAG Rod on the other focus of the Elliptical Cylindrical reflector

⇒ It is four energy level laser, Having energyband 6. at the Excited state

Working:— Infrared radiation to come to E_3 to E_2 (and E_4 to E_3 is radiation less)

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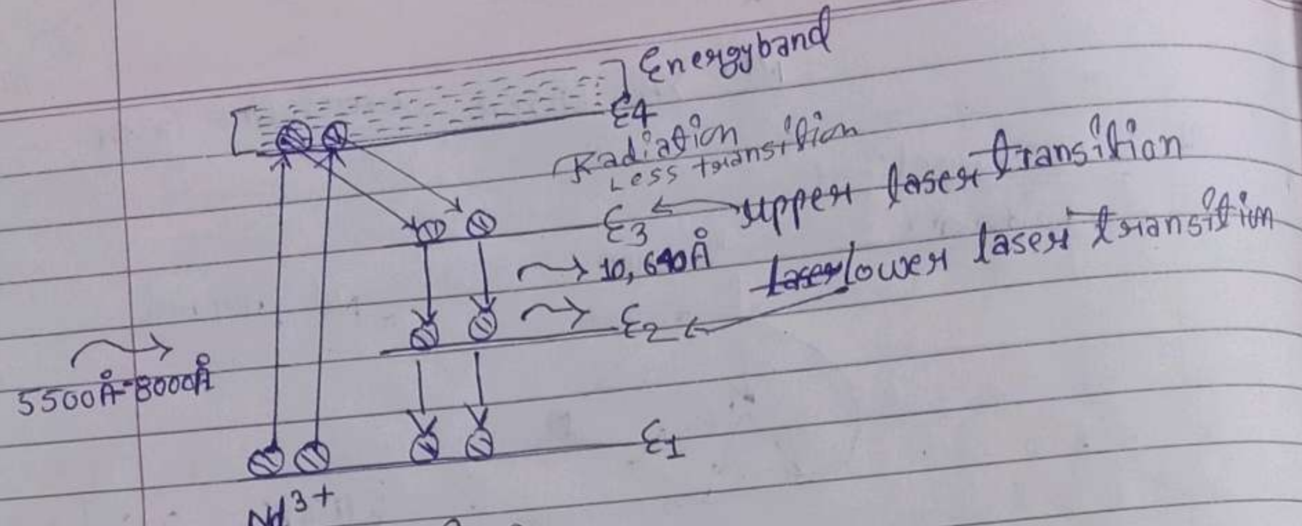


fig. 1.4 Energy level diagram

Working :-

1. When we flash krypton lamp neodymium ions (Nd^{3+}) absorbed radiation of wavelength 5500Å - 8000Å and raise to the Excited state.
2. from E_4 Energy state Nd^{3+} ions Execute radiation less transition and Comedown to E_3 energy level.
3. Laser transition takes place between the E_3 and E_2 Energy level by the infra red emission of wave length 10,640 Å. ~~wavelength~~
4. from E_2 Energy level Nd^{3+} ions are Comedown to the ground state by transferring their Energy to the ground state ions.

Application :-

\# This laser is mainly use in Medical field for Cosmetic Surgery work, like Skin tightening, Unwanted hair removal, & tatoo removal

01/04/25 Critical angle $\phi c < \angle i$ Rare Denser

~~tatoo~~ removal — Based on Principle of total internal reflection

Optical fibre

Total Internal Reflection (TIR)

Their are two Conditions required two Execute (TIR)

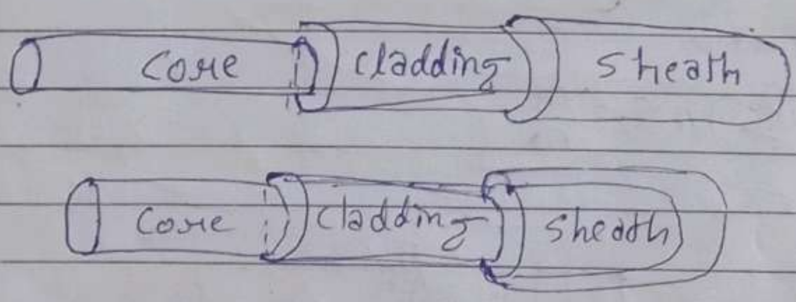
Light should be travel from denser medium to rare medium.

Angle of incidance should be more than critical angle

Optical fibre :

It is made by glass ~~or~~ plastic pipe. It is very thin like a human Hair.

It is based on the Principle (TIR)



Optical Fibre has three sections.

I Core

II Cladding

III Sheath

$H_2O = 1.5$
air $\mu = 1$
$\mu = \frac{\sin i}{\sin r}$

~~04~~ 03/April/25 UNIT — 4

C.C Interference

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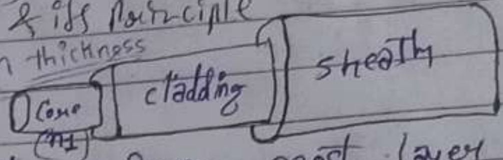
$R.I = (n_1)$

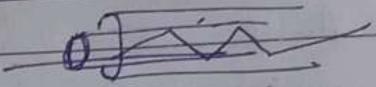
Describe structure optical fibre & its principle

(1) **Core:-** 50 µm thickness



This is the inner most layer of an optical fibre. Refractive Index of core is represented by (n_1) ~~Clad~~

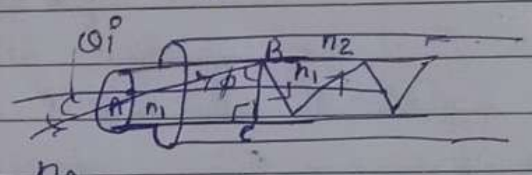
(2) **Cladding:-**
Cladding is the next layer to the core and it's refractive index (n_2)
- where ~~n_1~~ $n_1 > n_2$



(3) **Sheath:-**
This is the outer most layer of the optical fibre. And it protects from outside disturbance.

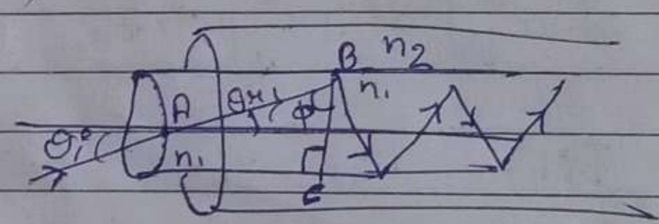
or Derive.

Q.1 Describe acceptance angle and Numerical aperture of an optical fibre.



outer medium — n_0

~~$\sin \theta_i = \frac{n_1}{n_0}$~~



$x^2+10x-90=0$ Solve   Snel's law ${}_1\mu_2 = \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

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let $n_0, n_1 \& n_2$ be the refractive index of outside medium, core, cladding respectively. then by Applying Snel's law at point A.

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_1}{n_0}$$

$$\sin\theta_i = \frac{n_1}{n_0}\sin\theta_r \quad \text{--- (i)}$$

By ΔABC $\quad \theta_r + \phi + 90 = 180°$

$$\theta_r = 90 - \phi$$

By placing this value in Eqn (i)

$$\sin\theta_i = \frac{n_1}{n_0}\sin(90-\phi)$$

$$\sin\theta_i = \frac{n_1}{n_0}\cos\phi \quad \text{--- (2)}$$

when $\theta_i = \theta_{max}$ than $\phi = \phi_c$ $\quad$ c. is critical angle

$$\sin\theta_{max} = \frac{n_1}{n_0}\cos\phi_c \quad \text{--- (3)}$$

At point B by applying Snel's law

$$\frac{\sin\phi_c}{\sin 90} = \frac{n_2}{n_1}$$

$\sin 90 = 1$

$$\sin\phi_c = \frac{n_2}{n_1}$$

$$\cos\phi_c = \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\cos\phi_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

By placing this value in Eqn (3)

$$\sin\theta_{max} = \frac{n_1}{n_0}\cdot\frac{\sqrt{n_1^2-n_2^2}}{n_1}$$

$$\sin\theta_{max} = \frac{\sqrt{n_1^2-n_2^2}}{n_0}$$

If first medium is

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By placing this value in Eqn (3)

$$\sin\theta_{max} = \frac{n_1}{n_0} \cdot \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

$$\sin\theta_{max} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

If first medium is air $n_0 = 1$

$$\sin\theta_{max} = \sqrt{n_1^2 - n_2^2} \longrightarrow \text{Numerical aperture}$$

$$\theta_{max} = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$

acceptance angle

<u>04/04/25</u>

V-number: It is a ~~count~~ quantity that characterise an optical fibre as single mode or multimode. If v-number is less than 2.405 then optical fiber is single mode

V - Number $< 2.405 \longrightarrow$ Single Mode

V - Number $> 2.405 \longrightarrow$ Multimode

where,

2.405 is called cut off value.

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

$\lambda \rightarrow$ wavelength

$n_1 \rightarrow$ Refractive index of core

$n_2 \rightarrow$ Refractive index of cladding

where,

$a \rightarrow$ Radius of the core.

calculator

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Q. Calculate the numerical operature, acceptance angle and v- number of an optical fibre. Given that (n_1) refractive Index of core = 1.45
(n_2) Refractive Index of cladding = 1.40
(n_3) wavelength of light = 6000Å = 6000×10^{-10} m
(a) Core radius = 2mm = 2×10^{-3} m

$$n.A = \sqrt{n_1^2 - n_2^2}$$
$$n.A = \sqrt{(1.45)^2 - (1.40)^2}$$
$$n.A = \sqrt{(0.05)(2.85)}$$
$$n.A = 0.37$$

Acceptance

Angle $\Rightarrow \theta = \sin^{-1}\sqrt{n_1^2 - n_2^2}$

$$\theta = \sin^{-1}(0.37)$$

$$\theta = 21.71^\circ$$

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

~~$V = \frac{2\pi a}{\lambda}$~~

$$V = \frac{2 \times \frac{22}{7} \times 2 \times 10^{-3} \times 0.37}{6000 \times 10^{-10}}$$

$V = 2$

wave optics — light — wave
quantum mechanics — light — wave nature & particle

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Unit - Quantum mechanics

Dual nature of light :-

Light shows wave nature as well as particle nature this phenomenon is known as dual nature of light.

Energy of light is concentrated to small regions which is known as photon. So photons are energy particle

Louis - de Broglie Hypothesis :-

According to the de-Broglie every ~~movement~~ moving particle is surrounded by a wave and wavelength of this wave depends on the mass and velocity of the particle and the relation is given by

$$\boxed{\lambda = \frac{h}{mv}}$$

$$\boxed{h = 6.63 \times 10^{-34}\ Js}$$

where, $h \rightarrow$ Planck's constant
$m \rightarrow$ mass of particle
$v \rightarrow$ velocity of particle

$$\boxed{\lambda = \frac{h}{p}} \qquad p = mv = \text{momentum}$$

moving particle - electron

this wave is called ~~metal~~ matter wave.

Imp

Properties of matter wave :-

09/04/25 — ~~Charge~~ oscillating particle
Electromagnetic waves

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(1) Wavelength of ~~heaviest~~ particle is less

(1) Wavelength of a matter wave for havier particle is less as compared to lighter particle.

(2) Wavelength of faster moving particle is less as compared to slower moving particle

(3) If the particle is at ~~les~~ Rest then the wave does not exist

(4) Matter wave is independent of charge and every moving particle produces matter wave.

(5) This waves are not electromagnetic waves.

Displacement Eqn $y = A\sin(\omega t - Kx)$ ω-angular velocity

or

$y = A\cos(\omega t - Kx)$ $K \to$ wave no. $K = \frac{2\pi}{\lambda}$

Max^m displa - A

Two crest Trough — wavelength

$(\omega t - Kx) \to$ phase angle

y-displacement on y-axis t-time

$A \to$ Amplitude x-displacement on x-axis

In ~~Con~~ Quantum mechanics wave eqn is given by

$y = A\sin(\omega t - Kx)$

$y = A\cos(\omega t - Kx)$

where y = displacement on y-axis

A - Amplitude

~~K~~ $(\omega t - Kx) \to$ phase

$K = \frac{2\pi}{\lambda}$ $\quad \lambda \lessgtr {m \atop v} \quad \lambda = \frac{h}{mv} \rightarrow p \quad \left(\frac{h}{p} = \lambda\right)$

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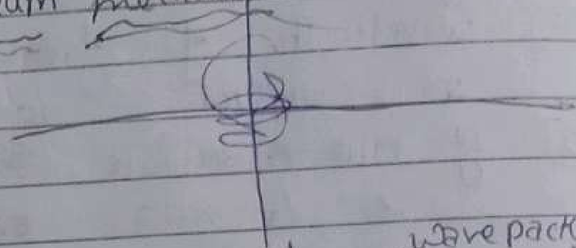
$y = A\sin(\omega t - Kx)$

$\cancel{y = A\cos(\omega t - Kx)}$

wave packet

Every moving particle is surrounded by group of wave these wave forms a packet which is known as wave packet

11/04/25 Quantum mechanics

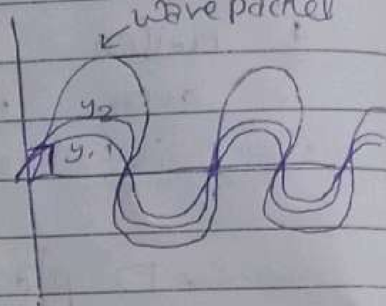


Wave Eqn

$y = A\cos(\omega t - Kx)$

or

$y = A\sin(\omega t - Kx)$



$y \rightarrow$ Displacement on y-axis

$(\omega t - Kx) \rightarrow$ Phase

$K \rightarrow$ wave no. $= \frac{2\pi}{\lambda}$

$x \rightarrow$ Displacement on x-axis.

Wave packet:

Every moving Particle is surrounded by group of wave that forms a packet which is known as wave packet.

Q.1 PYQ

Group velocity & Phase velocity & relation b/w them

Group velocity: velocity of the wave packet is called Group velocity

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$\cos A + \cos B = 2\cos\frac{A+B}{2} \cdot \cos\frac{A-B}{2}$

A —velocity→ G.V → Average

$\frac{\text{Amplitude}}{\text{G.V}}$ $\frac{\text{Phase}}{\text{P.V}}$

Second definition :-
(or)

→ velocity of the overall amplitude of resultant wave is called Group velocity

★ Phase velocity :- $\left(\frac{d\theta}{dt}\right)$

→ The rate at which the Phase of wave changes with time. is Known as phase ~~wav~~ velocity.

$\frac{d\theta}{dt}$

→ Let a moving particle is surrounded by two waves and their displacement Eqn's are given by y_1 & y_2

$$y_1 = A\cos(\omega_1 t - K_1 x)$$
$$y_2 = A\cos(\omega_2 t - K_2 x)$$

After superposition displacement eqn of resultant wave

$$\vec{y} = \vec{y_1} + \vec{y_2}$$

$$y = A\left[\cos(\omega_1 t - K_1 x) + \cos(\omega_2 t - K_2 x)\right]$$

$\left(\cos A + \cos B = 2\cos\frac{A+B}{2} \cdot \cos\frac{A-B}{2}\right)$

$$y = 2A\left[\cos\left(\frac{(\omega_1+\omega_2)}{2}t - \frac{(K_1+K_2)x}{2}\right) \cdot \cos\left(\frac{(\omega_1-\omega_2)}{2}t - \frac{(K_1-K_2)x}{2}\right)\right] \quad (1)$$

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let $\frac{\omega_1+\omega_2}{2} = \omega$, $\frac{K_1+K_2}{2} = K$

$\omega_1 - \omega_2 = \Delta\omega$, $K_1 - K_2 = \Delta K$

By Placing value in Eq. (1)

$$y = 2A\cos(\omega t - Kx).\cos\left(\frac{\Delta\omega t}{2} - \frac{\Delta K x}{2}\right) \quad (2)$$

By rearranging the Eqn displacement Eqn of resultant wave is given by

$$y = \underbrace{2A\cos\left(\frac{\Delta\omega t}{2} - \frac{\Delta K x}{2}\right)}_{\text{Amplitude}} . \underbrace{\cos(\omega t - Kx)}_{\text{(Average) phase.}} \quad (3)$$

Group velocity

$$\frac{\Delta\omega t}{2} - \frac{\Delta K x}{2} = \text{Constant}$$

Differentiate with respect to t

$$\frac{\Delta\omega}{2} - \frac{\Delta K}{2}\frac{dx}{dt} = 0$$

$$\frac{\Delta\omega}{2} = \frac{\Delta K}{2}\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{\Delta\omega}{\Delta K}$$

$$V_g = \frac{\Delta\omega}{\Delta K} \quad (4)$$

$\therefore V_g = \frac{\Delta\omega}{\Delta K}$ group velocity

Phase velocity

$$\omega t - Kx = \text{constant}$$

diff. w.r.to t

$$\omega - K\frac{dx}{dt} = 0$$

$$\omega = K\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{\omega}{K} \quad (5)$$

$$V_p = \frac{\omega}{K}$$ phase velocity

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$V_g = V_p - \lambda \frac{dV_p}{d\lambda}$

for small ω can be written by

Relation b/w V_g & V_p

$\frac{2\pi}{\lambda} \frac{dV_p}{d} \frac{\lambda}{2\pi}$

from Eqn 4 ~~& 5~~

$V_g = \frac{\Delta\omega}{\Delta K}$ - (4) , $\boxed{V_p = \frac{\omega}{K}}$ - (5)

$V_g = \frac{\Delta\omega}{\Delta K} \approx \frac{d\omega}{dK}$

from Eqn (5)

$V_g = \frac{d}{dK}(KV_p)$

$V_g = V_p + K\frac{dV_p}{dK}$

we know that $K = \frac{2\pi}{\lambda}$

$V_g = V_p + K\frac{dV_p}{d\lambda}\frac{d\lambda}{dK}$

$V_g = V_p + K\frac{dV_p}{d\lambda} \cdot \frac{d(2\pi/K)}{dK}$

$V_g = V_p + K\frac{dV_p}{d\lambda}\left(-\frac{2\pi}{K^2}\right)$

$V_g = V_p - \frac{2\pi}{K} \cdot \frac{dV_p}{d\lambda}$

$V_g = V_p - \lambda\frac{dV_p}{d\lambda}$ Ans