Q.1 28/05/25

$$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial y^2} = y^2x + x$$

$$(D^2 + 4D'^2)z = y^2x + x$$

$$D = \frac{\partial}{\partial x}, \quad D' = \frac{\partial}{\partial y}$$

Auxiliary Eqn $D = M, D' = 1$

$$M^2 + 4 = 0$$

$$M = \pm 2i$$

$$C.f. = \phi_1(y + 2ix) + \phi_2(y - 2ix)$$

$$P.I. = \frac{1}{D^2 + 4D'^2}(y^2x + x)$$

$$P.I. = \frac{1}{4D'^2\left(1 + \frac{D^2}{4D'^2}\right)}(y^2x + x)$$

$$P.I. = \frac{1}{4D'^2}\left[1 + \frac{D^2}{4D'^2}\right]^{-1}(y^2x + x)$$

$$(1+P)^n = 1 + nP + \frac{n(n-1)}{\lfloor 2}P^2 + \frac{n(n-1)(n-2)}{\lfloor 3}P^3 + \cdots$$

$$P.I. = \frac{1}{4D'^2}\left[1 + (-1)\left(\frac{D^2}{4D'^2}\right) + \frac{(-1)(-1-1)}{\lfloor 2}\left(\frac{D^2}{4D'^2}\right)^2 + \cdots\right](y^2x + x)$$

$$P.I. = \frac{1}{4D'^2}\left[(y^2x + x) - \frac{D^2(y^2x + x)}{4D'^2} + \frac{D^4(y^2x + x)}{4D'^2} + \cdots\right]$$

$$P.I. = \frac{1}{4D'^2}\left[(y^2x + x) - 0 + 0\right]$$

$$\left\{\iint = \frac{1}{D'^2}\right\} \quad D' = \frac{\partial}{\partial y}$$

$$P.I. = \frac{1}{4}\iint\left[y^2x + x\right]$$

$$P.I. = \frac{1}{4}\left[\frac{y^4x}{4\times 3} + \frac{xy^2}{2}\right] \Rightarrow P.I. = \frac{y^4x}{48} + \frac{xy^2}{8}$$

$$Z = C.f. + P.I.$$

$$Z = \phi_1(y + 2ix) + \phi_2(y - 2ix) + \frac{y^4x}{48} + \frac{xy^2}{8}$$ Ans

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Q2 $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x^2$ 28/05/25

$[D^2 + 2DD' + 2D'^2] z = x^2$

$D = \frac{\partial}{\partial x} \mid D' = \frac{\partial}{\partial y}$

Auxiliary Eqn

$D = M \ , \ D' = 1$

$M^2 + 2M + 2 = 0$

$M = -1 \pm i \ \{ m_1 = (-1+i), m_2 = (-1-i) \}$

$C.f. = \phi_1 (y (-1+i)$

$C.f. = \phi_1 \{ y + (-1+i)x \} + \phi_2 (y + (-1-i)x)$

$P.I. = \frac{1}{D^2 + 2DD' + 2D'^2} [x^2]$

$P.I. = \frac{1}{2D'^2 \left[1 + \frac{D^2 + 2DD'}{2D'^2} \right]} [x^2]$

$P.I. = \frac{1}{2D'^2} \left[1 + \frac{D^2 + 2DD'}{2D'^2} \right]^{-1} [x^2]$

$[1+P]^n = 1 + nP + \frac{n(n-1)P^2}{\lfloor 2} + - -$

$P.I. = \frac{1}{2D'^2} \left[1 + (-1) \left(\frac{D^2 + 2DD'}{2D'^2} \right) + \frac{(-1)(-1-1)}{\lfloor 2} \left(\frac{D^2 + 2DD'}{2D'^2} \right)^2 + - - \right] x^2$

$P.I. = \frac{1}{2D'^2} \left[x^2 - \frac{D^2(x^2) - 2DD'(x^2)}{2D'^2} + \frac{D^4(x^2) + 4D^2D'^2(x^2) + 4D^3D'(x^2)}{4D'^4} + - - \right]$

$P.I. = \frac{1}{2D'^2} \left[x^2 - \frac{2 - 0}{2D'^2} + 0 \right]$

$\frac{1}{D'^2} = \iint \Rightarrow D' = \frac{\partial}{\partial y}$

$P.I. = \frac{1}{2D'^2} \left[x^2 - \frac{y^2}{2} \right]$

$P.I. = \frac{1}{2} \left[\frac{x^2y^2}{2} - \frac{y^4}{4 \times 3 \times 2} \right]$

$Z = C.f. + P.I$

$Z = \phi_1 \{ y + (-1+i)x \} + \phi_2 \{ y + (-1-i)x \} + \frac{x^2y^2}{4} - \frac{y^4}{48}$

Ans

$\frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2(1)}$

$\frac{-2 \pm \sqrt{4-8}}{2}$

$\frac{-2 \pm 2i}{2}$

$\frac{-1 \pm i}{1}$

$-1 \pm i$

$1 \quad -1 \quad -1$

$\frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2(1)}$

$\frac{1 \pm \sqrt{1+4}}{2}$

$\frac{1 \pm \sqrt{5}}{2}$

Q.

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Q.3 $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - \frac{\partial^2 z}{\partial y^2} = (x+y^2)$

$D^2 - DD' - D'^2 = x + y^2$

$\left\{ D = \frac{\partial}{\partial x} \;\middle|\; D' = \frac{\partial}{\partial y} \right.$

Auxiliary Eqn $D = M \mid D' = 1$

$M^2 - M - 1 = 0$

$M = \frac{1 \pm \sqrt{5}}{2} \left\{ m_1 = \frac{1+\sqrt{5}}{2}, m_2 = \frac{1-\sqrt{5}}{2} \right\}$

$C.f. = \phi_1 \left\{ y + \left(\frac{1+\sqrt{5}}{2}\right)x \right\} + \phi_2 \left\{ y - \left(\frac{1-\sqrt{5}}{2}\right)x \right\}$

$P.I. = \frac{1}{D^2 - DD' - D'^2} [x+y^2]$

$P.I. = \frac{1}{-D'^2\left[1 + \frac{DD' - D^2}{D'^2}\right]} [x+y^2]$

$P.I. = \frac{1}{-D'^2} \left[1 + \frac{DD' - D^2}{D'^2}\right]^{-1} [x+y^2]$

$\left\{ (1+p)^n = 1 + np + \frac{n(n-1)}{\lfloor 2} p^2 + \cdots \right.$

$P.I. = \frac{1}{-D'^2} \left[1 + (-1)\left(\frac{DD' - D^2}{D'^2}\right) + \frac{(-1)(-1-1)}{\lfloor 2}\left(\frac{DD' - D^2}{D'^2}\right)^2 + \cdots \right](x+y^2)$

$P.I. = \frac{1}{-D'^2} \left[(x+y^2) - \frac{DD'(x+y^2) + D^2(x+y^2)}{D'^2} + \frac{D^2D'^2(x+y^2) + D^4(x+y^2) - 2D^3D'(x+y^2)}{D'^4} \right]$

$P.I. = \frac{1}{-D'^2} \left[(x+y^2) - \frac{0+0}{D'^2} + \frac{0+0+0}{D'^4} \right]$

$P.I. = \frac{1}{-D'^2} \left[x + y^2 \right]$

$P.I. = -1 \left(\frac{xy^2}{2} + \frac{y^4}{4 \times 3 \times 2} \right)$

$\boxed{P.I. = -\frac{xy^2}{2} - \frac{y^4}{24}}$

$Z = C.f + P.I.$

$Z = \phi_1\left(y + \left(\frac{1+\sqrt{5}}{2}\right)x\right\} + \phi_2\left\{y - \left(\frac{1-\sqrt{5}}{2}\right)x\right\} - \frac{xy^2}{2} - \frac{y^4}{24}$ Ans

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Q.

$$\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = xy$$

$$1 \quad 3 \quad -2$$

$$\frac{-3 \pm \sqrt{9-4(1)(-2)}}{2}$$

$$\frac{-3 \pm \sqrt{9+8}}{2}$$

$$\frac{-3 \pm \sqrt{17}}{2}$$

$$[D^2 + 3DD' - 2D'^2]z = xy$$

$D = \frac{\partial}{\partial x}$ | $D' = \frac{\partial}{\partial y}$

Auxiliary form $D = M$, $D' = 1$

$$M^2 + 3M - 2 = 0$$

$$M = \frac{-3 \pm \sqrt{17}}{2} \quad \left\{ m_1 = \frac{-3+\sqrt{17}}{2}, \; m_2 = \frac{-3-\sqrt{17}}{2} \right\}$$

$$C.F. = \phi_1\left(y + \left(\frac{-3+\sqrt{17}}{2}\right)x\right) + \phi_2\left(y + \left(\frac{-3-\sqrt{17}}{2}\right)x\right)$$

$$P.I. = \frac{1}{D^2 + 3DD' - 2D'^2}[xy]$$

$$P.I. = \frac{1}{-2D'^2\left[1 - \frac{D^2 + 3DD'}{2D'^2}\right]}[xy]$$

$$P.I. = \frac{1}{-2D'^2}\left[1 - \frac{D^2 + 3DD'}{2D'^2}\right]^{-1}[xy]$$

$$(1+p)^n = 1 + np + \frac{n(n-1)p^2}{\lfloor 2} + \cdots$$

$$P.I. = \frac{1}{-2D'^2}\left[1 + (-1)\left(-\frac{D^2+3DD'}{2D'^2}\right) + \frac{(-1)(-1-1)}{\lfloor 2}\left(-\frac{D^2+3DD'}{2D'^2}\right)^2 + \cdots\right]xy$$

$$P.I. = \frac{1}{-2D'^2}\left[xy + \frac{D^2(xy) + 3DD'(xy)}{2D'^2} + \frac{D^4(xy) + 9D^2D'^2(xy) + 6D^3D'(xy)}{4D'^4}\right]$$

$$P.I. = \frac{1}{-2D'^2}\left[xy + \frac{0+3}{2D'^2} + 0\right]$$

$$P.I. = \frac{1}{-2D'^2}\left[xy + \frac{3}{2}\frac{y^2}{2}\right]$$

$$P.I. = -\frac{1}{2}\left[\frac{xy^3}{3\times 2} + \frac{3}{4}\frac{y^4}{4\times 3}\right]$$

$$P.I. = \left[\frac{-xy^3}{12} - \frac{y^4}{32}\right]$$

$$z = C.F. + P.I.$$

$$z = \phi_1\left(y + \left(\frac{-3+\sqrt{17}}{2}\right)x\right) + \phi_2\left(y + \left(\frac{-3-\sqrt{17}}{2}\right)x\right) - \frac{xy^3}{12} - \frac{y^4}{32}$$

Ans

Q.5

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = y\sin x$$

$$[D^2 - D'^2]z = y\sin x$$

$$m^2 - 1 = 0 \quad \{(m-1)(m+1) = 0\}$$

$$m = \pm 1$$

$$C.f. = \phi_1(y-x) + \phi_2(y+x)$$

$$P.I. = \frac{1}{D^2 - D'^2} y\sin x$$

$$P.I. = \frac{1}{(D+D')(D-D')} y\sin x$$

$$P.I. = \frac{1}{(D+D')} \times \frac{1}{(D-D')} y\sin x$$

$\Rightarrow$ Put $y + x = c_1$

$$y = c_1 - x$$

$$P.I. = \frac{1}{(D+D')} \int \underset{I}{(c_1 - x)} \underset{II}{\sin x}\, dx$$

$$P.I. = \frac{1}{(D+D')} \left[(c_1 - x)\bar{\cos x} - \int (-1)(-\cos x) \right]$$

$$P.I. = \frac{1}{(D+D')} \left[-(c_1 - x)\cos x - \int (-1)(-\cos x) \right]$$

$$P.I. = \frac{1}{(D+D')} \left[-y\cos x - \sin x \right]$$

$\Rightarrow$ Put $y - x = c_2$

$$y = c_2 + x$$

$$P.I. = -\int \left[\underset{I}{(c_2 + x)} \underset{II}{\cos x} + \sin x \right]$$

$$P.I. = -\left[(c_2 + x)\sin x - \int 1.\sin x - \cos x \right]$$

$$P.I. = -\left[(c_2 + x)\sin x + \cos x - \cos x \right]$$

$$P.I. = -(c_2 + x)\sin x$$

$$P.I. = -y\sin x$$

$$z = C.f. + P.I.$$

$$z = \phi_1(y-x) + \phi_2(y+x) - y\sin x$$

Ans