$a^2+b^2=(a+b)(a+b)$

$a^2-b^2=(a-b)(a+b)$

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Q $\frac{d^2y}{dx^2}+a^2y=\tan ax$

$e^{i\theta}=\cos\theta+i\sin\theta$

$e^{-i\theta}=\cos\theta-i\sin\theta$

$(D^2+a^2)y=\tan ax$

$m^2+a^2=0$

$m=\pm ai=\alpha\pm i\beta$

$\boxed{a^2+b^2=(a+b)(a+b)}$

$C.f.=\{c_1\cos ax+c_2\sin ax\}e^{0x}$

$P.I.=\frac{1}{(D^2+a^2)}(\tan ax)$

$P.I.=\frac{1}{(D-ai)(D+ai)}\tan ax$

$P.I.=\frac{1}{(D-ai)(D+ai)}=\frac{A}{(D-ai)}+\frac{B}{(D+ai)}$, $A=\frac{1}{2ai}$, $B=-\frac{1}{2ai}$

$L>0$, $D=ai$; $L>0$, $D=-ai$

$P.I.=\left\{\frac{1/2ai}{(D-ai)}+\frac{(-1/2ai)}{(D+ai)}\right\}\tan ax$

$P.I=\frac{1}{2ai}\left\{\frac{1}{(D-ai)}\tan ax-\frac{1}{(D+ai)}\tan ax\right\}$ —— (i)

for

$\frac{1}{D-ai}\tan ax=\frac{1}{(D-ai)}1\times\tan ax=\frac{1}{(D-ai)}e^{aix-aix}\tan ax$

$=\frac{1}{(D-ai)}e^{aix}\cdot e^{-aix}\tan ax$ $\left[\frac{1}{f(D)}e^{ax}\phi(x)=e^{ax}\frac{1}{f(D+a)}\phi(x)\right]$

$=e^{aix}\frac{1}{(D+ai-ai)}e^{-aix}\tan ax=e^{aix}\frac{1}{D}\int e^{-aix}\tan ax\,dx$

$=e^{aix}\int e^{-aix}\tan(ax)\,dx$ $\{\because e^{-i\theta}=\cos\theta-i\sin\theta\}$

$=e^{aix}\int\{\cos ax-i\sin ax\}\tan(ax)\,dx$

$=e^{aix}\int\{\cos ax-i\sin ax\}\frac{\sin ax}{\cos ax}\,dx$

$$= e^{ai x}\int\left\{\sin ax - \frac{i\sin^2 ax}{\cos ax}\right\}dx$$

$$= e^{ai x}\int\left\{\sin ax - \frac{i(1-\cos^2 ax)}{\cos ax}\right\}dx$$

$$= e^{ai x}\int\{\sin ax - i(\sec ax - \cos ax)dx\}$$

$$= e^{ai x}\left[-\cos ax - i\left\{\frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a}\right\}\right] \quad \text{--- (ii)}$$

for $\frac{1}{D+ai}\tan ax$

$$= \frac{1}{D+ai} 1 \times \tan(ax)$$

$$= \frac{1}{D+ai} e^{ai x - ai x}\tan(ax)$$

$$= \frac{1}{D+ai} e^{ai x}\cdot e^{-ai x}\tan(ax)$$

$$= \frac{1}{D} e^{-ai x}\frac{1}{(D-ai+ai)} e^{ai x}\tan(ax)$$

$$= e^{-ai x}\frac{1}{D} e^{ai x}\tan(ax)$$

$$= e^{-ai x}\int e^{ai x}\tan(ax)\,dx$$

$$= e^{-ai x}\int\{\cos ax + i\sin ax\}\tan(ax)\,dx \quad \{\because e^{i\theta} = \cos\theta + i\sin\theta\}$$

$$= e^{-ai x}\int\{\cos ax + i\sin ax\}\frac{\sin ax}{\cos ax}dx$$

$$= e^{-ai x}\int\left\{\sin ax + \frac{i\sin^2 ax}{\cos ax}\right\}dx$$

$$= e^{-ai x}\int\left\{\sin ax + \frac{i(1-\cos^2 ax)}{\cos ax}\right\}dx$$

$$= e^{-aix}\int\{\sin ax + i(\sec ax - \cos ax)\}dx$$

$$= e^{-aix}\left[-\cos ax + i\left\{\frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a}\right\}\right] \quad \text{--- (iii)}$$

Put (ii) & (iii) in Eq^n (i)

$$P.I. = \frac{1}{2ai}\left\{\left(e^{aix}\left[-\cos ax - i\left\{\frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a}\right\}\right]\right) - \left(e^{-aix}\left[-\cos ax + i\left\{\frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a}\right\}\right]\right)\right\}$$

Complete solution $y = C.f. + P.I$

$$y = \{c_1\cos ax + c_2\sin ax\}e^{0x} + \frac{1}{2ai}\left[\left(e^{aix}\left[-\cos ax - i\left\{\frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a}\right\}\right]\right) - \left(e^{-aix}\left[-\cos ax + i\left\{\frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a}\right\}\right]\right)\right]$$

Ans