

Q3

$$\frac{d^2y}{dx^2} + 2x \frac{dy}{dx} + (x^2 + 1)y = (x^3 + 3x)$$

Let,

$$P = 2x \quad | \quad Q = (x^2 + 1) \quad | \quad R = (x^3 + 3x)$$

$$U = e^{\int P dx} = e^{\frac{-1}{2} \int 2x dx} = e^{\frac{-1}{2} x^2} = e^{-\frac{1}{2} x^2}$$

$$\left. \begin{aligned} \frac{d^2V}{dx^2} + IV &= R/U \\ I &= Q - \frac{1}{2} \frac{dP}{dx} - \frac{1}{4} P^2 \end{aligned} \right| \quad \textcircled{1}$$

$$\frac{dP}{dx} = 2$$

$$I = (x^2 + 1) - \frac{1}{2}(2) - \frac{1}{4}(2x)^2$$

$$I = x^2 + 1 - 1 - x^2$$

$$(I = 0)$$

Put in \textcircled{1}

$$\frac{d^2V}{dx^2} + 0V = \frac{(x^3 + 3x)}{e^{-\frac{1}{2}x^2}}$$

$$\frac{d^2V}{dx^2} = \int (x^3 + 3x) \cdot e^{\frac{1}{2}x^2}$$

$$\frac{dV}{dx} = (x^3 + 3x) \frac{e^{\frac{1}{2}x^2}}{\frac{1}{2}}$$

$$\frac{dV}{dx} = \int x^3 e^{\frac{1}{2}x^2} dx + \int 3x e^{\frac{1}{2}x^2} dx + C_1$$

$$\frac{dV}{dx} = \int x^2 \cdot x e^{\frac{1}{2}x^2} dx + 3 \int x e^{\frac{1}{2}x^2} dx + C_1$$

$$\frac{dV}{dx} = \int x^2$$

Put $\frac{1}{2}x^2 = t \Rightarrow x^2 = 2t$
on diff ~~on diff~~

$$\frac{1}{2} \cdot 2x \, dx = dt + \\ x \, dx = dt$$

$$\frac{dv}{dx} = \int 2t e^t dt + 3 \int e^t dt + C_1$$

$$\frac{dv}{dx} = 2 \int t e^t dt + 3 \int e^t dt + C_1$$

$$\int x e^{\frac{1}{2}x^2} dx$$

Put $\frac{1}{2}x^2 = t$

$$\frac{dv}{dx} = 2(t e^t - e^t) + 3 e^t + C_1$$

$$\frac{1}{2} \cdot 2x \, dx = dt \\ x \, dx = dt$$

$$\frac{dv}{dx} = 2 \frac{1}{2} x e^{\frac{1}{2}x^2} - e^{\frac{1}{2}x^2} + 3 e^{\frac{1}{2}x^2} + C_1$$

$$\frac{dv}{dx} = x^2 e^{\frac{1}{2}x^2} - 2 e^{\frac{1}{2}x^2} + 3 e^{\frac{1}{2}x^2} + C_1$$

$$\int e^t dt$$

e^t
 $(e^{\frac{1}{2}x^2})$

$$\frac{dv}{dx} = x^2 e^{\frac{1}{2}x^2} + e^{\frac{1}{2}x^2} + C_1$$

$$v = \int_I x(x e^{\frac{1}{2}x^2}) dx + \int_{II} e^{\frac{1}{2}x^2} dx + C_1 dx + C_2$$

$$v = x \left[(x e^{\frac{1}{2}x^2}) - \left[\frac{d}{dx} \int x e^{\frac{1}{2}x^2} dx \right] \right] + \int e^{\frac{1}{2}x^2} dx$$

$$v = x e^{\frac{1}{2}x^2} - \int x e^{\frac{1}{2}x^2} dx + \int e^{\frac{1}{2}x^2} dx + C_1 x + C_2 + C_1 \int x dx$$

$$v = x e^{\frac{1}{2}x^2} + (C_1 x + C_2) \quad \text{--- (2)}$$

$$y = u \cdot v$$

$$y = e^{-\frac{1}{2}x^2} (x e^{\frac{1}{2}x^2} + (C_1 x + C_2))$$

$$y = x e^{-\frac{1}{2}x^2 + \frac{1}{2}x^2} + (C_1 x e^{-\frac{1}{2}x^2} + C_2 e^{-\frac{1}{2}x^2})$$

$$y = x + (C_1 x + C_2) e^{-\frac{1}{2}x^2}$$

Ans