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Q.1:- ~~where~~ Evaluate, $\iint_S \vec{F}\cdot\vec{ds} = ?$

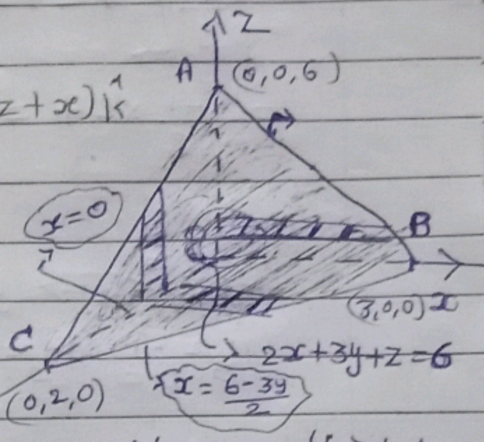
where $\vec{F} = (x+z)\hat{i} + (y+z)\hat{j} + (z+x)\hat{k}$

and (S) is bounded by

$S \Rightarrow x=0,\ y=0,\ z=0$

$2x+3y+z=6$

$\frac{x}{3} + \frac{y}{2} + \frac{z}{6} = 1$



$$\iint_S \vec{F}\cdot\vec{ds} = \iint_S \vec{F}\cdot\hat{n}\,ds = \iint_{S_1} \vec{F}\cdot\hat{n}_1 ds_1 + \iint_{S_2} \vec{F}\cdot\hat{n}_2 ds_2 + \iint_{S_3} \vec{F}\cdot\hat{n}_3 ds_3 + \iint_{S_4} \vec{F}\cdot\hat{n}_4 ds_4$$

(S_1) ABC $\Rightarrow 2x+3y+z=6$

(S_2) OBC $\Rightarrow z=0$

(S_3) OAB $\Rightarrow y=0$

(S_4) OAC $\Rightarrow x=0$

for $S_2 \Rightarrow z=0 \Rightarrow$ OBC

$\hat{n}_2 = -\hat{k}$, $ds_2 = \frac{dx\,dy}{|\hat{n}_2\cdot\hat{k}|} = \frac{dx\,dy}{|-\hat{k}\cdot\hat{k}|} = \frac{dx\,dy}{|-1|}$

$ds_2 = dx\,dy$

$$\iint_{S_2} \vec{F}\cdot\hat{n}_2 ds_2 = \iint_{S_2} \{(x+z)\hat{i} + (y+z)\hat{j} + (z+x)\hat{k}\}(-\hat{k})\,dx\,dy$$

$$= -\iint_{S_2} (z+x)\,dx\,dy = -\iint_{S_2} x\,dx\,dy = -\int_{y=0}^{2}\int_{x=0}^{x=\frac{6-3y}{2}} x\,dx\,dy$$

for $S_2 \Rightarrow z=0$

$$= -\int_{y=0}^{2}\left[\frac{x^2}{2}\right]_{x=0}^{\frac{6-3y}{2}} = -\int_{y=0}^{2}\left[\frac{\left(\frac{6-3y}{2}\right)^2}{2} - \frac{0^2}{2}\right]dy$$

$$= -\frac{1}{2}\int_{y=0}^{2}\left[\frac{36+9y^2-36y}{4}\right]dy$$

$$= -\frac{1}{8}\left[36y + \frac{9y^3}{3} - \frac{36y^2}{2}\right]_{y=0}^{2}$$

$$= -\frac{1}{8}\left[36(2) + 3(2)^3 - 18(2)^2 - 36(0) + 3(0)^3 - 18(0)^2\right]$$

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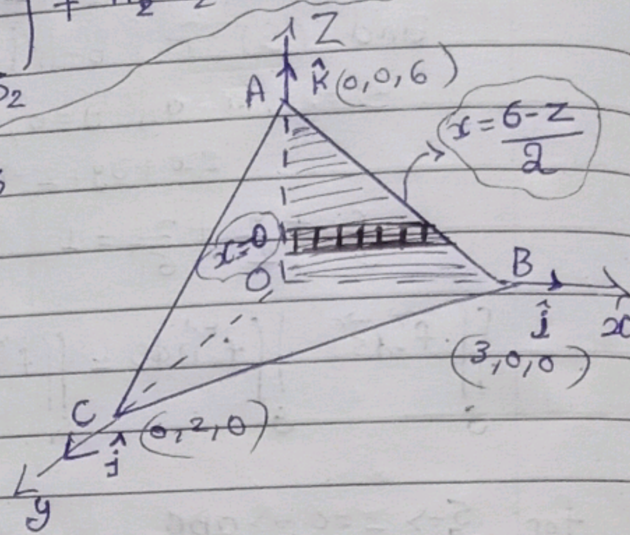
$$= \frac{-1}{8}\left[72 + 24 - 72\right]$$

$$= \frac{-1}{8} \times 24 = -3 \Rightarrow \iint_{S_2} \vec{f}\,\hat{n}_2\,dS_2 = -3$$



for $S_3 \Rightarrow y = 0 \Rightarrow OAB$

$\hat{n}_3 = -\hat{j}$ $\quad dS_3 = \frac{dx\,dz}{|\hat{n}_3 \cdot \hat{j}|}$

$$dS_3 = \frac{dx\,dz}{|-\hat{j}\cdot\hat{j}|} = \frac{dx\,dz}{|-1|} = dx\,dz$$

$$\iint_{S_3} \vec{f}\,\hat{n}_3\,dS_3 = \iint_{S_3} \left\{(x+z)\hat{i} + (y+z)\hat{j} + (z+x)\hat{k}\right\}(-\hat{j})\,dx\,dz$$

$$= -\iint_{S_3} (y+z)\,dx\,dz$$

for $S_3 \Rightarrow y = 0$

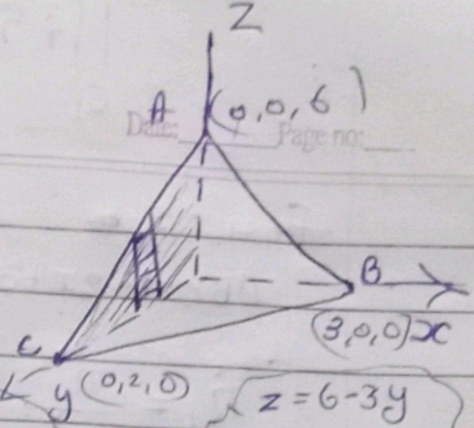
$$= -\iint_{S_3} (z)\,dx\,dz$$

$$= -\int_{z=0}^{6} \int_{x=0}^{x=\frac{6-z}{2}} z\,dx\,dz = -\int_{z=0}^{6} \left[\frac{zx}{2}\right]_{x=0}^{\frac{6-z}{2}} dz$$

$$= -\int_{z=0}^{6} \left[\frac{6z - z^2}{2} - z(0)\right]$$

$$= -\int_{z=0}^{6} \left[3z - \frac{z^2}{2}\right]$$

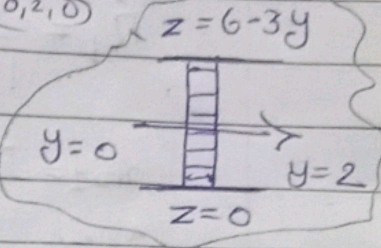
$$= -\left[\frac{3z^2}{2} - \frac{z^3}{2\times 3}\right]_{z=0}^{6} = -\left[\left(\frac{3(6)^2}{2} - \frac{(6)^3}{6}\right) - \frac{3(0)^2}{2} - \frac{(0)^3}{6}\right]$$



$= -\left[\frac{3(36)}{2} - 36\right]$

$= -\left[3\times 18 - 36\right]$

$= -18 \Rightarrow \iint_{S_3} \vec{f}\,\hat{n}_3\, ds_3 = -18$



For $S_4 \Rightarrow x=0 \Rightarrow OAC$

$\hat{n}_4 = -\hat{i},\ ds_4 = \frac{dydz}{|\hat{n}_4\cdot\hat{i}|} = \frac{dydz}{|-\hat{i}\cdot\hat{i}|} = \frac{dydz}{(-1)} = dydz$

$\iint_{S_4} \vec{f}\,\hat{n}_4\, ds_4 = \iint_{S_4} \{(x+z)\hat{i} + (y+z)\hat{j} + (z+x)\hat{k}\}(-\hat{i})\, dydz$

$= -\iint_{S_4} (x+z)\, dydz$

for $S_4 \Rightarrow x=0$

$= -\int_{y=0}^{2}\int_{z=0}^{z=6-3y} (z)\, dydz = -\int_{y=0}^{2}\left[\frac{z^2}{2}\right]_{z=0}^{6-3y} = -\int_{y=0}^{2}\left[\frac{(6-3y)^2}{2} - \frac{0^2}{2}\right]$

$= -\int_{y=0}^{2}\left[\frac{36+9y^2-36y}{2}\right]$

$= -\frac{1}{2}\left[36y + \frac{9y^3}{3} - \frac{36y^2}{2}\right]_{y=0}^{2}$

$= -\frac{1}{2}\left[\left(36(2)+3(2)^3-18(2)^2\right) - \left(36(0)+3(0)^3-18(0)^2\right)\right]$

$= \cancel{36}\ -\frac{1}{2}\left[\cancel{72} + 24 - \cancel{72}\right]$

$= -12 \Rightarrow \iint_{S_4} \vec{f}\,\hat{n}_4\, ds_4 = -12$

$$\iint_{S_1} \vec{F}\cdot\hat{n}_1 \, ds_1$$

for $S_1 \Rightarrow$

ABC $\Rightarrow 2x+3y+z=6$

$$\hat{n}_1 = \frac{\vec{n}_1}{|\vec{n}_1|} = \frac{2\hat{i}+3\hat{j}+\hat{k}}{\sqrt{2^2+3^2+1^2}} = \frac{2\hat{i}+3\hat{j}+\hat{k}}{\sqrt{14}}$$

$$ds_1 = \sqrt{14}\,dx\,dy$$

$$\hat{n}_1 ds_1 = \frac{2\hat{i}+3\hat{j}+\hat{k}}{\sqrt{14}}\sqrt{14}\,dx\,dy$$

$$\hat{n}_1 ds_1 = (2\hat{i}+3\hat{j}+\hat{k})\,dx\,dy$$

$$\iint_{S_1} \{(x+z)\hat{i}+(y+z)\hat{j}+(z+x)\hat{k}\}\cdot(2\hat{i}+3\hat{j}+\hat{k})\,dx\,dy$$

$$= \iint_{S_1} \{2(x+z)+3(y+z)+(z+x)\}\,dx\,dy$$

$$= \iint_{S_1} \{2x+2z+3y+3z+z+x\}\,dx\,dy$$

$$= \iint_{S_1} \{3x+3y+6z\}\,dx\,dy$$

~~for S_1 we take~~

z replace with S_1

$\Rightarrow 2x+3y+z=6$

$z = 6-2x-3y$

$$= \iint_{S_1} \{3x+3y+6(6-2x-3y)\}\,dx\,dy$$

$$= \iint_{S_1} \{3x+3y+36-12x-18y\}\,dx\,dy$$

$$= \iint_{S_1} \{36-9x-15y\}\,dx\,dy$$

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Know ~~in~~ ABC image on the xy

$z=0$

$\hookrightarrow \quad 2x+3y+z=6$

$$\boxed{2x+3y=6}$$

$x=0 \quad y=0$

at $x=0 \Rightarrow y=2$

at $y=0 \Rightarrow x=3$

~~(0,0)~~ ~~(0,~~ $(0,0),(3,0),(0,2)$

$$x=\frac{6-3y}{2}$$

$$= \int_{y=0}^{2} S_1 \int_{x=0}^{\frac{6-3y}{2}} \{36-9x-15y\}\,dx\,dy$$

$$= \int_{y=0}^{2}\left[36x-\frac{9x^2}{2}-15xy\right]_{x=0}^{x=\frac{6-3y}{2}} dy$$

$$= \int_{y=0}^{2}\left[36\left(\frac{6-3y}{2}\right)-\frac{9\left(\frac{6-3y}{2}\right)^2}{2}-15\left(\frac{6-3y}{2}\right)y\right]dy$$

$$= \int_{y=0}^{2}\left[18(6-3y)-\frac{9}{2}\left[\frac{36+9y^2-36y}{4}\right]-\frac{15}{2}(6y-3y^2)\right]dy$$

$$= \left[18\left(6y-\frac{3y^2}{2}\right)-\frac{9}{8}\left[36y+\frac{9y^3}{3}-\frac{36y^2}{2}\right]-\frac{15}{2}\left[\frac{6y^2}{2}-\frac{3y^3}{3}\right]\right]_{y=0}^{2}$$

$$= \left[18\left[6(2)-\frac{3(2)^2}{2}\right)-\frac{9}{8}\left(36(2)+9\frac{(2)^3}{3}-\frac{36(2)^2}{2}\right)-\frac{15}{2}\left(\frac{6(2)^2}{2}-\frac{3(2)^3}{3}\right)\right]$$

$$= \left[18(12-6)-\frac{9}{8}(\cancel{72}+\cancel{24}^{3}-\cancel{72})-\frac{15}{2}[12-8]\right]$$

$$= [108-27-30]$$

$$= 51 \Rightarrow \iint_{S_1} \vec{f}\cdot\hat{n}_1\,dS_1 = 51$$

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Final Answer is

$$\iint_S \vec{f}\cdot\vec{ds} = \iint_{S_1} \vec{f}\,\hat{n}_1\, ds_1 + \iint_{S_2} \vec{f}\,\hat{n}_2\, ds_2 + \iint_{S_3} \vec{f}\,\hat{n}_3\, ds_3 + \iint_{S_4} \vec{f}\,\hat{n}_4\, ds_4$$

$$\iint_S \vec{f}\cdot\vec{ds} = 51 - 3 - 18 - 12$$

$$\iint_S \vec{f}\,\vec{ds} = 18$$ Ans