$e^{z} + z + \sin z$ Ans.)

Q=13 ~~Real~~ $u = x^2y + y^2x + ye^x + xe^y$ then find analytic function.

$$\Rightarrow \frac{\partial u}{\partial x} = 2xy + y^2 + ye^x + e^y = \phi_1(x,y)$$

$$\phi_1(z,0) = (2 \times z \times 0) + 0 + 0 + e^0 = 1$$

$$\boxed{\phi_1(z,0) = 1}$$

$$\frac{\partial u}{\partial y} = x^2 + 2yx + e^x + xe^y = \phi_2(x,y)$$

$$\phi_2(z,0) = (z^2 + (2 \times 0) + e^z + (ze^0))$$

$$\boxed{\phi_2(z,0) = z^2 + e^z + z}$$

$$f(z) = \int \phi_1(z,0) - i\int \phi_2(z,0)$$

$$f(z) = \int 1\,dz - i\int (z^2+e^z+z)\,dz$$

$$f(z) = (z) - \left(\frac{z^3}{3} + e^z + \frac{z^2}{2}\right) i$$ Answer

Q=) 4 Imagenry part is $x^2\sin y + y^2\sin x +$ ~~$y^2\sin x$~~ $y\cos x + x\cos y$ then find analytic function.

Sol^n =)

$$v = x^2\sin y + y^2\sin x + y\cos x + x\cos y$$

$$\frac{\partial v}{\partial y} = x^2\cos y + 2y\sin x + \cos x - x\sin y \Rightarrow \psi_1(z,0)$$

$$\psi_1(z,0) = z^2\cos(0) + 2\times 0\times \sin z + \cos z - z\sin(0).$$

$$\boxed{\psi_1(z,0) = z^2 + \cos z}$$

$$\frac{\partial v}{\partial x} = 2x\sin y + y^2\cos x - y\sin x + \cos y \Rightarrow \psi_2(z,0)$$

$$\psi_2(z,0) = 2z\sin(0) + 0\times\cos z - 0\times\sin z + \cos(0)$$

$$\boxed{\psi_2(z,0) = 1}$$

$$f(z) = \int \psi_1(z,0) + i\int \psi_2(z,0)$$

~~$f(z) = \int (z^2+\cos z) + i\int$~~ ~~Answer~~

~~$f(z) = \int (z^2+\cos z)\,dz + i\int 1\,dz$~~

$$f(z) = \boxed{\left(\frac{z^3}{3} + \sin z\right) + zi}$$ Answer

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Q) If real part of analytic function $u = e^{x+y} - \cos(x+y)$ then find it's imaginary part.

Sol^n) $u = e^{x+y} - \cos(x+y)$

$$dv = -\frac{\partial u}{\partial y}\,dx + \int \frac{\partial u}{\partial x}\,dy$$

$$\frac{\partial u}{\partial y} = e^{x+y} + \sin(x+y)$$

$$\frac{\partial u}{\partial x} = e^{x+y} + \sin(x+y)$$

$$\int dv = -\int \{e^{x+y} + \sin(x+y)\}\,dx + \int \underbrace{\{e^{x+y} + \sin(x+y)\}}_{x\text{-absent}}\,dy$$

$$V = -(e^{x+y} - \cos(x+y)) + \int e^{y} + \sin(y)\,dy \quad 0$$

$$V = -(e^{x+y} - \cos(x+y)) + e^{y} - \cancel{\cos(y)}$$ Answer

$$\boxed{V = \cos(x+y) - e^{x+y}}$$ Answer

Q) If imaginary part of analytic function $v = ye^{x} + x\sin(xy)$ then find real part.

Sol^n) $v = \underset{\mathrm{I}}{ye^{x}} + \underset{\mathrm{II}}{x\sin(xy)}$

$$\frac{\partial v}{\partial x} = ye^{x} + \{xy\cos(xy) + \sin(xy)\}$$

$$\frac{\partial v}{\partial y} = e^{x} + x^{2}\cos(xy)$$

$$\int du = \int \underset{y\text{-constant}}{\frac{\partial v}{\partial y}\,dx} - \int \underset{x\text{-absent}}{\frac{\partial v}{\partial x}\,dy}$$

$$\int du = \int (\underset{I}{e^x} + \underset{II}{x^2}\cos(xy))\,dx - \int \{ye^x + (yx\cos(xy) + \sin(xy))\}dy$$

$$u = e^x + \left\{\frac{x^2\sin(xy)}{y} - \int \frac{2\overset{I}{x}.\sin(\overset{II}{xy})\,dx}{y}\right\} - 0$$

$$u = e^x + \left\{\frac{x^2\sin(xy)}{y} - \frac{2}{y}\left\{x.\left(\frac{-\cos(xy)}{y}\right)\right) - \int \frac{-\cos(xy)}{y}\,dx\right.$$

$$u = e^x + \left\{\frac{x^2\sin(xy)}{y} - \frac{2}{y^2}\left\{-x\cos(xy) + \frac{\sin(xy)}{y}\right\}\right.$$

Q=) If real part of analytic function ~~u~~ $u = x^2 + x^4y + y^3$ then find imaginary part.

Sol=)
$$u = x^2 + x^4y + y^3$$

$$\frac{\partial u}{\partial x} = 2x + 4x^3y + 0$$

$$\frac{\partial u}{\partial y} = 0 + x^4 + 3y^2$$

$$\int dv = \int \underset{y\text{-constant}}{-\frac{\partial u}{\partial y}\,dx} + \int \underset{x\text{-absent}}{\frac{\partial u}{\partial x}\,dy}$$

$$v = -\int (x^4 + 3y^2)\,dx + \int (2x + 4x^3y \cdot)\,dy$$

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$$V = -\left\{\left(\frac{x^5}{5}\right) + 3y^2x\right\}$$

$$\boxed{V = -\left\{\frac{x^5}{5} + 3y^2x\right\}}$$ Answer

Q.) If real part of analytic function is $u = x + ye^x + xe^y + x\sin y$ then find imaginary part.

Sol^n:)

$$u = x + ye^x + xe^y + x\sin y$$

$$\frac{\partial u}{\partial x} = 1 + ye^x + e^y + \sin y$$

$$\frac{\partial u}{\partial y} = 0 + e^x + xe^y + x\cos y$$

$$\int dV = \underbrace{\int -\frac{\partial u}{\partial y}\,dx}_{y\text{-constant}} + \underbrace{\int \frac{\partial u}{\partial x}\,dy}_{x\text{-absent}}$$

$$V = -\int (e^x + xe^y + x\cos y)\,dx + \int (1 + ye^x + e^y + \sin y)\,dy$$

$$V = -\left(e^x + \frac{x^2}{2}e^y + \frac{x^2}{2}\cos y\right) + (\cancel{e^x}\, y + 0 + e^y - \cos y)$$

$$\cancel{V = e^x - e^y - \cos y + y + e^y - \cos y}$$

$$\cancel{\boxed{V = y - e^x - 2\cos y}}$$ ~~Answer~~

$$\boxed{V = -\left(e^x + \frac{x^2}{2}e^y + \frac{x^2}{2}\cos y\right) + (y + e^y - \cos y)}$$ Answer