Solve By change of Independent variable

$$\cos x \frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} - 2\cos^3 x\, y = 2\cos^5 x$$

$$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} - 2\cos^2 x\, y = 2\cos^4 x$$

$P = \tan x$, | $Q = -2\cos^2 x$ | $R = 2\cos^4 x$

Put $\left(\frac{dz}{dx}\right)^2 = |Q| = |-2\cos^2 x| = 2\cos^2 x$

$$\frac{dz}{dx} = \sqrt{2}\cos x$$

$$\int dz = \int \sqrt{2}\cos x\, dx$$

$$z = \sqrt{2}\sin x$$

$$\frac{z}{\sqrt{2}} = \sin x$$

$$\frac{d^2z}{dx^2} = \sqrt{2}\sin x$$

$$P_1 = \frac{-\sqrt{2}\sin x + \tan x(\sqrt{2}\cos x)}{2\cos^2 x} \Rightarrow P_1 = 0$$

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} = \frac{-2\cos^2 x}{2\cos^2 x} = -1 \Rightarrow Q_1 = -1$$

$$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2} = \frac{2\cos^4 x}{2\cos^2 x} = \cos^2 x \Rightarrow R_1 = \cos^2 x$$

Put in $\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$

$$\frac{d^2y}{dz^2} + 0\frac{dy}{dz} - y = \cos^2 x$$

$$\frac{d^2y}{dz^2} - y = \cos^2 x$$

$$\frac{d^2y}{dz^2} - y = 1 - \sin^2 x$$

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$$\frac{d^2y}{dz^2} - y = \left(1 - \frac{z^2}{2}\right) \quad \left\{\because \sin x = \frac{z}{\sqrt{2}}\right\}$$

$$(D^2-1)y = (1 - z^2/2)$$

$$m^2 - 1 = 0 \Rightarrow m = \pm 1$$

$$C.f. = C_1 e^{-z} + C_2 e^{z}$$

$$P.I = \frac{1}{(D^2-1)}\{1 - z^2/2\}$$

$$P.I. = \frac{1}{(D^2-1)} e^{0z} - \frac{1}{(D^2-1)} z^2/2$$

$$P.I = \frac{1}{0-1} e^{0z} - \frac{1}{(D^2-1)} z^2/2$$

$$P.I. = -e^{0z} - \frac{1}{-(1-D^2)} z^2/2$$

$$P.I = -1 + \frac{1}{2}(1-D^2)^{-1} z^2$$

$$(1+p)^n = 1 + np + \frac{n(n-1)}{\lfloor 2} p^2 + \frac{n(n-1)(n-2)}{\lfloor 3} p^3 + \cdots$$

$$\cancel{P.I. = -1 + \frac{1}{2}\left[1 + (-1)(-D^2) + \frac{(-1-1)}{}\right.}$$

$$P.I = -1 + \frac{1}{2}\left[1 + (-1)(-D^2) + \frac{(-1)(-1-1)(-D^2)^2}{\lfloor 2} + \cdots\right] z^2$$

$$P.I = -1 + \frac{1}{2}\left[1 + D^2 + D^4\right] z^2$$

$$P.I = -1 + \frac{1}{2}\left[z^2 + D^2 z^2 + D^4 z^2\right]$$

$$P.I = -1 + \frac{1}{2}\left[z^2 + 2\right]$$

$$P.I. = -1 + \frac{z^2}{2} + 1$$

$$P.I. = \frac{z^2}{2}$$

$$P.I. = \frac{(\sqrt{2}\sin x)^2}{2} \quad \{\because z = \sqrt{2}\sin x\}$$

$$P.I. = \frac{2\sin^2 x}{2}$$

$$\boxed{P.I = \sin^2 x}$$

Complete Soln

$$y = C.f. + P.I.$$

$$\boxed{y = C_1 e^{-z} + C_2 e^{z} + \sin^2 x}$$

Ans