

$$\text{Q.1} \quad \frac{d^2y}{dx^2} + a^2 y = \sec ax$$

$$(D^2 + a^2)y = \sec ax$$

$\hookrightarrow$  Auxiliary Eqn  $D=M$

$$M^2 + a^2 = 0$$

$$M^2 = -a^2 \Rightarrow M = \pm a i = a \pm i \beta$$

$$\text{C. f.} = \{c_1 \cos ax + c_2 \sin ax\} e^{ix}$$

$$\text{P.I.} = \frac{1}{D^2 + a^2} \sec ax$$

$$\text{P.I.} = \frac{1}{(D - ai)(D + ai)} \sec ax$$

$$\text{P.I.} = \frac{1}{(D - ai)(D + ai)} = \frac{A}{(D - ai)} + \frac{B}{(D + ai)}$$

$$\begin{aligned} A &= \frac{1}{2ai} \\ B &= -\frac{1}{2ai} \\ D - ai &= 0 \quad D + ai = 0 \\ D &= ai \quad D = -ai \end{aligned}$$

$$\text{P.I.} = \left\{ \frac{\frac{1}{2}ai}{(D - ai)} + \frac{(-\frac{1}{2}ai)}{(D + ai)} \right\} \sec ax$$

$$\text{P.I.} = \frac{1}{2ai} \left\{ \frac{1}{(D - ai)} \sec ax - \frac{1}{(D + ai)} \sec ax \right\} - (i)$$

$$\text{For } \frac{1}{D - ai}$$

$$= \frac{1}{D - ai} \sec ax = \frac{1}{(D - ai)} x_1 \times \cancel{\sec ax} = \frac{1}{(D - ai)} e^{aix - aix} \times \cancel{\sec ax}$$

$$= \frac{1}{D - ai} e^{aix} \cdot e^{-aix} \sec ax \left[ \frac{1}{f(D)} e^{ax} \phi(x) = e^{ax} \frac{1}{f(D+a)} \phi(x) \right]$$

$$= e^{aix} \frac{1}{(D + ai - ai)} e^{-aix} \sec ax = e^{aix} \int e^{-aix} \sec(ax) dx$$

$$= e^{aix} \int \{ \cos ax - i \sin ax \} \sec(ax) dx \quad \left\{ \because e^{i\theta} = \cos \theta - i \sin \theta \right\}$$

$$= e^{ax} \int \{ \cos ax - i \sin ax \} \frac{1}{\cos ax} dx$$

$$= e^{ax} \left\{ \{ 1 - i \tan ax \} dx \right.$$

$$= e^{ax} \left[ x - i \frac{\log \sec ax}{a} \right] - \textcircled{II}$$

from  $\frac{1}{(D+ai)} \sec ax = \frac{1}{(D+ai)} ix \sec ax = \frac{1}{(D+ai)} e^{ax - aix} \sec ax$

$$= \frac{1}{D+ai} e^{ax} \cdot e^{-aix} \sec ax$$

$$= e^{ax} \frac{1}{D-a^2+i^2} e^{aix} \sec ax$$

$$= e^{-aix} \frac{1}{0} e^{aix} \sec ax = e^{aix} \int e^{aix} \sec ax$$

$$= e^{-aix} \int (\cos ax + i \sin ax) \sec ax \quad \left\{ \begin{array}{l} \textcircled{I} \\ e^{i\theta} = \cos \theta + i \sin \theta \end{array} \right.$$

$$= e^{-aix} \left\{ (1 + i \tan ax) \right\}$$

$$= e^{-aix} \left[ x + i \frac{\log \sec ax}{a} \right] - \textcircled{III}$$

Put  $\textcircled{II}$  &  $\textcircled{III}$  in Eqn  $\textcircled{I}$

$$\text{P.I.} = \frac{1}{2ai} \left\{ (e^{ax} \left[ x - i \frac{\log \sec ax}{a} \right]) - (e^{-aix} \left[ x + i \frac{\log \sec ax}{a} \right]) \right\}$$

Complete Sol'n

$$y = \text{C.F.} + \text{P.I.}$$

$$y = \{C_1 \cos ax + C_2 \sin ax\} e^{ax} + \frac{1}{2ai} \left[ (e^{ax} \left[ x - i \frac{\log \sec ax}{a} \right]) - (e^{-aix} \left[ x + i \frac{\log \sec ax}{a} \right]) \right]$$

Ans

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Q.2

$$\frac{d^2y}{dx^2} + 4y = \tan 2x$$

$$(D^2 + 4)y = \tan 2x$$

↪ Auxiliary Eqn

$$M^2 + 4 = 0$$

$$N^2 = -4 \Rightarrow M = \pm 2i = \alpha \pm \beta$$

$\alpha = 0$

$\beta = 2$

$$C.F. = \{C_1 \cos 2x + C_2 \sin 2x\} e^{0x}$$

$$P.I. = \frac{1}{D^2 + 4} \tan 2x = \left| \frac{1}{D^2 + 4} \right| \cancel{1 \times \tan 2x} \cancel{\frac{1}{D^2 + 4}} e^{\frac{0}{2}x - 2ix} \tan 2x$$

P.I.

$$P.I. = \frac{1}{D^2 + 4} \tan 2x$$

$\gamma = \frac{1}{4}i$

$\gamma = -\frac{1}{4}i$

$$= \frac{1}{(D-2i)(D+2i)} = \frac{A}{(D-2i)} + \frac{B}{(D+2i)}$$

$L=0$

$L=0$

$$D=2i \quad D=-2i$$

$$P.I. = \left\{ \frac{\frac{1}{4}i}{(D-2i)} + \frac{\left(-\frac{1}{4}i\right)}{(D+2i)} \right\} \tan 2x$$

$$P.I. = \frac{1}{4i} \left\{ \frac{1}{(D-2i)} \tan 2x - \frac{1}{(D+2i)} \tan 2x \right\} - (1)$$

$$\text{First} \quad \frac{1}{(D-2i)} \tan 2x = \frac{1}{(D-2i)} e^{2ix-2ix} \tan 2x = \frac{1}{(D-2i)} e^{2ix} \cdot e^{-2ix} \tan 2x$$

$$= e^{2ix} \frac{1}{(D+2i-2i)} \bar{e}^{-2ix} \tan 2x = e^{2ix} \frac{1}{D} \bar{e}^{-2ix} \tan 2x$$

$$= e^{2ix} \int \bar{e}^{-2ix} \tan 2x dx$$

$$= e^{2ix} \int [\cos 2x - i \sin 2x] \tan 2x dx$$

$$\left\{ e^{-i\theta} = \cos \theta - i \sin \theta \right\}$$

$$\begin{aligned}
 &= e^{\frac{i}{2}ix} \int \left[ (\cos 2x - i \sin 2x) \frac{\sin 2x}{\cos 2x} dx \right] \\
 &= e^{\frac{i}{2}ix} \int \left[ \sin 2x - i \frac{(\sin^2 2x)}{\cos 2x} \right] dx \\
 &= e^{\frac{i}{2}ix} \int \left[ \sin 2x - i \frac{(1 - \cos^2 2x)}{\cos 2x} \right] dx \\
 &= e^{\frac{i}{2}ix} \int \left[ \sin 2x - i (\sec 2x - \csc 2x) \right] dx \\
 &= e^{\frac{i}{2}ix} \left[ -\frac{\cos 2x}{2} - i \left( \log(\sec 2x + \tan 2x) - \frac{\sin 2x}{2} \right) \right] - \text{(ii)}
 \end{aligned}$$

for  $\frac{1}{D+2i} \tan 2x = \pm e^{\frac{i}{2}ix - 2i} \tan 2x = \frac{1}{(D+2i)} e^{\frac{i}{2}ix - 2i} \tan 2x$

$$= \frac{-e^{-2i} \pm 1}{(D-2i+2i)} e^{\frac{i}{2}ix + \tan 2x} - \frac{1}{D} e^{\frac{i}{2}ix + \tan 2x}$$

$$\boxed{e^{-2i} \pm 1} = e^{\frac{i}{2}ix} \int e^{\frac{i}{2}ix + \tan 2x} dx$$

$$= e^{-2i} \int \left[ \cos 2x + i \sin 2x \right] + \tan 2x dx$$

$$= \bar{e}^{-2i} \int \left[ \cos 2x + i \sin 2x \right] \frac{\sin 2x}{\cos 2x} dx$$

$$= \bar{e}^{-2i} \int \left[ \sin 2x + i \frac{(\sin^2 2x)}{\cos 2x} \right] dx$$

$$= \bar{e}^{-2i} \int \left[ \sin 2x + i \frac{(1 - \cos^2 2x)}{\cos 2x} \right] dx$$

$$= \bar{e}^{-2i} \int \left[ \sin 2x + i (\sec 2x - \csc 2x) \right] dx$$

$$= \bar{e}^{-2i} \left[ -\frac{\cos 2x}{2} + i \left( \log(\sec 2x + \tan 2x) - \frac{\sin 2x}{2} \right) \right] - \text{(iii)}$$

Put this value (ii) & (iii) in Eqn(i)

$$\text{P.J.} = \frac{1}{4i} \left\{ \left( e^{2ix} \left[ -\frac{\cos 2x}{2} - i \left( \log(\sec 2x + \tan 2x) - \frac{\sin 2x}{2} \right) \right] \right) - \right. \\ \left. \left( e^{-2ix} \left[ -\frac{\cos 2x}{2} + i \left( \log(\sec 2x + \tan 2x) - \frac{\sin 2x}{2} \right) \right] \right) \right\}$$

Complete Sol'n  $y = \text{C.f.} + \text{P.J.}$

$$y = \left\{ (c_1 \cos 2x + c_2 \sin 2x) e^{ix} + \frac{1}{4i} \left\{ \left( e^{2ix} \left[ -\frac{\cos 2x}{2} - i \left( \log(\sec 2x + \tan 2x) - \frac{\sin 2x}{2} \right) \right] \right) - \right. \right. \\ \left. \left. \left( e^{-2ix} \left[ -\frac{\cos 2x}{2} + i \left( \log(\sec 2x + \tan 2x) - \frac{\sin 2x}{2} \right) \right] \right) \right\} \right\}$$

Ans

$$(D-2i)(D+2i) = D^2 - 2^2 i^2$$

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$$\text{Q. 3 :- } \frac{d^2y}{dx^2} + 4y = \sec 2x$$

$$\text{Soln :- } (D^2 + 4)y = \sec 2x$$

$\hookrightarrow$  Auxiliary Eqn

$$M^2 + 4 = 0 \quad \begin{matrix} \nearrow d=0 \\ M^2 = -4 \Rightarrow M = \pm 2i = \alpha \pm \beta i \end{matrix} \quad \begin{matrix} \nearrow \alpha=0 \\ \beta=2 \end{matrix}$$

$$C.F. = \{C_1 \cos 2x + C_2 \sin 2x\} e^{0x}$$

$$P.I. = \frac{1}{D^2 + 4} \sec 2x \quad \begin{matrix} \nearrow \alpha = \frac{1}{4} \\ \nearrow \beta = -\frac{1}{4} \end{matrix}$$

$$P.I. = \frac{1}{(D-2i)(D+2i)} = \frac{A}{(D-2i)} + \frac{B}{(D+2i)} \quad \begin{matrix} \nearrow \alpha=0 \\ D=2i \\ \nearrow \beta=0 \\ D=-2i \end{matrix}$$

$$P.I. = \left\{ \frac{\frac{1}{4}i}{(D-2i)} + \frac{-\frac{1}{4}i}{(D+2i)} \right\} \sec 2x$$

$$P.I. = \frac{1}{4i} \left\{ \frac{1}{(D-2i)} \sec 2x - \frac{1}{(D+2i)} \sec 2x \right\} - \textcircled{i}$$

$$\text{form } \frac{1}{D-2i} \sec 2x = \frac{1}{D-2i} e^{2ix-2i2x} \sec 2x = \frac{1}{D-2i} e^{2ix} \cdot e^{-2i2x} \sec 2x$$

$$= \frac{1}{D+2i-2i} e^{2ix} \left\{ \frac{1}{e^{-2i2x}} \sec 2x - e^{2ix} \frac{1}{e^{-2i2x}} \sec 2x \right\}$$

$$= e^{2ix} \left\{ \frac{1}{e^{-2i2x}} \sec 2x dx \right\} \quad \left\{ \because e^{i\theta} = (\cos \theta - i \sin \theta) \right\}$$

$$= e^{2ix} \left\{ [\cos 2x - i \sin 2x] \sec 2x dx \right\}$$

$$= e^{2ix} \left\{ [1 - i \tan 2x] dx \right\}$$

$$= e^{2ix} \left[ x - i \frac{(\log \sec 2x)}{2} \right] - \textcircled{ii}$$

$$\begin{aligned}
 & \text{Eqn } \frac{1}{(D+2i)} \sec 2x = \frac{1}{(D+2i)} e^{2ix} - \frac{e^{2ix}}{\sec 2x} \\
 &= \frac{1}{(D+2i)} e^{2ix} \cdot \bar{e}^{2ix} \sec 2x \\
 &= \bar{e}^{-2ix} \frac{1}{(D-2i+2i)} \bar{e}^{2ix} \sec 2x \\
 &= \bar{e}^{-2ix} \frac{1}{D} e^{2ix} \sec 2x \\
 &- \bar{e}^{-2ix} \left[ e^{2ix} \sec 2x dx \right] \quad \left\{ \because e^{ix} = (\cos x + i \sin x) \right\} \\
 &= \bar{e}^{2ix} \int [1 + i \tan 2x] \sec 2x dx \\
 &= \bar{e}^{2ix} \left[ x + \frac{i(\log \sec 2x)}{2} \right] - \text{(iii)}
 \end{aligned}$$

Put (ii) & (iii) in Eqn (i)

Complete Soln  $y = C.F. + P.I.$

$$\begin{aligned}
 y &= \{c_1 \cos 2x + c_2 \sin 2x\} e^{ix} + \\
 P.I. &= \frac{1}{4i} \left\{ \left( \bar{e}^{2ix} \left[ x - \frac{i(\log \sec 2x)}{2} \right] \right) - \left( \bar{e}^{2ix} \left[ x + \frac{i(\log \sec 2x)}{2} \right] \right) \right\}
 \end{aligned}$$

Complete Soln  $y = C.F. + P.I.$

$$y = \{c_1 \cos 2x + c_2 \sin 2x\} e^{ix} + \frac{1}{4i} \left\{ \left( \bar{e}^{2ix} \left[ x - \frac{i(\log \sec 2x)}{2} \right] \right) - \left( \bar{e}^{2ix} \left[ x + \frac{i(\log \sec 2x)}{2} \right] \right) \right\} \text{ Ans}$$

Q.4  $\frac{d^2y}{dx^2} + a^2y = \cosec(ax)$

Soln  $\Rightarrow (D^2 + a^2)y = \cosec(ax)$

Auxiliary Eqn

$$M^2 + a^2 = 0$$

$$M^2 = -a^2 \Rightarrow M = \pm ai \Rightarrow \alpha = \pm \beta i \Rightarrow \beta = a$$

$$\text{C.f.} = \{C_1 \cos ax + C_2 \sin ax\} e^{ax}$$

$$\text{P.I.} = \frac{1}{D^2 + a^2} \cosec ax$$

$$\text{P.I.} = \frac{1}{(D - ai)(D + ai)} \cosec ax$$

$$= \frac{1}{(D - ai)(D + ai)} = \frac{A}{(D - ai)} + \frac{B}{(D + ai)}$$

$$\begin{cases} A = 0 \\ B = 0 \end{cases} \quad \begin{cases} D = ai \\ D = -ai \end{cases}$$

$$= \left\{ \frac{\frac{1}{2}ai}{(D - ai)} + \frac{\left(-\frac{1}{2}ai\right)}{(D + ai)} \right\} \cosec ax$$

$$= \frac{1}{2ai} \left\{ \frac{1}{(D - ai)} - \frac{1}{(D + ai)} \right\} \cosec ax$$

$$\text{P.I.} = \frac{1}{2ai} \left\{ \frac{1}{(D - ai)} \cosec ax - \frac{1}{(D + ai)} \cosec ax \right\} - (i)$$

$$\text{For } \frac{1}{(D - ai)} \cosec ax = \frac{1}{(D - ai)} e^{aix - ai^2x} \cosec ax = \frac{1}{(D - ai)} e^{aix - ai^2x} \cdot e^{-ai^2x} \cosec ax$$

$$= e^{aix} \frac{1}{(D + ai - ai^2)} e^{-ai^2x} \cosec ax = e^{aix} \frac{1}{D} e^{-ai^2x} \cosec ax$$

$$\begin{aligned}
 &= e^{aix} \int e^{-aix} \cosec ax dx \quad \left\{ \because -e^{i\theta} = [\cos \theta - i \sin \theta] \right\} \\
 &= e^{aix} \int [\cos ax - i \sin ax] \cosec ax dx \quad \cosec ax = \frac{1}{\sin ax} \\
 &= e^{aix} \int [-\cot ax - i(±)] dx \\
 &= e^{aix} \left[ \frac{\log(\sin ax)}{a} - ix \right] - (ii)
 \end{aligned}$$

For  $\frac{1}{(D+2i)} \cosec ax = \frac{1}{(D+2i)} e^{aix-aix} \cosec ax = \frac{1}{(D+2i)} e^{aix} \cdot e^{-aix} \cosec ax$

$$\begin{aligned}
 &= \bar{e}^{-aix} \frac{1}{(D-2i+2i)} e^{aix} \cosec ax = \bar{e}^{-aix} \frac{1}{D} e^{aix} \cosec ax \\
 &= \bar{e}^{-aix} \int e^{aix} \cosec ax dx \quad \left\{ \because e^{i\theta} = \cos \theta + i \sin \theta \right\} \\
 &= \bar{e}^{-aix} \int [\cos ax + i \sin ax] \cosec ax dx \\
 &= \bar{e}^{-aix} \int [\cot ax + i(±)] dx \\
 &= \bar{e}^{-aix} \left[ \frac{\log(\sin ax)}{a} + ix \right] - (iii)
 \end{aligned}$$

Put (ii) & (iii) in Eqn (i)

$$P.I. = \frac{1}{2ai} \left\{ (e^{aix} \left[ \frac{\log(\sin ax)}{a} - ix \right]) - (\bar{e}^{-aix} \left[ \frac{\log(\sin ax)}{a} + ix \right]) \right\}$$

Complete complete Soln  $y = C.F. + P.I.$

$$y = \{C_1 \cos ax + C_2 \sin ax\} e^{ax} + \frac{1}{2ai} \left\{ (e^{aix} \left[ \frac{\log(\sin ax)}{a} - ix \right]) - (\bar{e}^{-aix} \left[ \frac{\log(\sin ax)}{a} + ix \right]) \right\} -$$

$$\left. \left( \bar{e}^{-aix} \left[ \frac{\log(\sin ax)}{a} + ix \right] \right) \right\} \text{ Ans}$$

$$\text{Q.5} \Rightarrow \frac{d^2y}{dx^2} + 4y = \operatorname{cosec}(2x)$$

$$(D^2 + 4)y = \operatorname{cosec} 2x$$

$\leftarrow$  Auxiliary Eqn

$$M^2 + 4 = 0 \quad \begin{matrix} \alpha=0 \\ \beta=2 \end{matrix}$$

$$M^2 = -4 \Rightarrow M = \pm 2i = \alpha \pm \beta i$$

$$\text{C.F.} = \{C_1 \cos 2x + C_2 \sin 2x\} e^{0x}$$

$$\text{P.I.} = \frac{1}{D^2 + 4} \operatorname{cosec} 2x \quad \begin{matrix} \nearrow \frac{1}{4i} \\ \nearrow \frac{-1}{4i} \end{matrix}$$

$$= \frac{1}{(0-2i)(0+2i)} = \frac{A}{(0-2i)} + \frac{B}{(0+2i)} \quad \begin{matrix} \hookrightarrow = 0 \\ D = 2i \end{matrix} \quad \begin{matrix} \hookrightarrow = 0 \\ D = -2i \end{matrix}$$

$$\text{P.I.} = \left\{ \frac{1/4i}{(0-2i)} + \frac{(-1/4i)}{(0+2i)} \right\} \operatorname{cosec} 2x$$

$$\text{P.I.} = \frac{1}{4i} \left\{ \frac{1}{(0-2i)} - \frac{1}{(0+2i)} \right\} \operatorname{cosec} 2x$$

$$\text{P.I.} = \frac{1}{4i} \left\{ \frac{1}{(0-2i)} \operatorname{cosec} 2x - \frac{1}{(0+2i)} \operatorname{cosec} 2x \right\} - \textcircled{1}$$

$$\text{First} = \frac{1}{(0-2i)} \operatorname{cosec} 2x = \frac{1}{(0-2i)} e^{2ix-2ix} \operatorname{cosec} 2x = \frac{1}{(0-2i)} e^{2ix} \cdot e^{-2ix} \operatorname{cosec} 2x$$

$$= e^{2ix} \frac{1}{(0+2i-2i)} e^{-2ix} \operatorname{cosec} 2x = e^{2ix} \frac{1}{0} e^{-2ix} \operatorname{cosec} 2x$$

$$= e^{2ix} \int e^{-2ix} \operatorname{cosec} 2x dx \quad \left\{ \text{so } e^{-i\theta} = \cos \theta - i \sin \theta \right\}$$

$$= e^{2ix} \int [\cos 2x - i \sin 2x] \operatorname{cosec} 2x dx$$

$$= e^{2ix} \int [ \cot 2x - i(1) ] dx$$

$$= e^{2ix} \left[ \frac{\log \sin 2x}{2} - ix \right] - \textcircled{111}$$

$$\text{Post 1} \quad \text{Cosec } 2x = \frac{1}{(D+2i)} e^{\frac{2ix-2ix}{D}} \quad \text{Cosec } 2x = \frac{1}{(D+2i)} e^{\frac{2ix-2ix}{D}} e \cdot e^{\text{cosec } 2x}$$

$$= e^{2ix} \frac{1}{(D-2i+2i)} e^{\frac{2ix}{D}} \text{cosec } 2x$$

$$= e^{2ix} \frac{1}{D} e^{\frac{2ix}{D}} \text{cosec } 2x$$

$$= e^{2ix} \int e^{\frac{2ix}{D}} \text{cosec } 2x \, dx \quad \left\{ \because e^{i\theta} = \cos \theta + i \sin \theta \right\}$$

$$= e^{-2ix} \int [\cos 2x + i \sin 2x] \text{cosec } 2x \, dx$$

$$= e^{-2ix} \int [\cot 2x + P(\pm)] \, dx$$

$$= e^{-2ix} \left[ \frac{\log \sin 2x}{2} + ix \right] - \text{(iii)}$$

Put (ii) A (iii) in Eqn (i)

$$\text{P.I.} = \frac{1}{4i} \left\{ \left( e^{2ix} \left[ \frac{\log \sin 2x}{2} - ix \right] \right) - \left( e^{-2ix} \left[ \frac{\log \sin 2x}{2} + ix \right] \right) \right\}$$

Complete soln  $y = \text{C.F.} + \text{P.I.}$

$$y = \left\{ C_1 \cos 2x + C_2 \sin 2x \right\} e^{ix} + \left\{ \frac{1}{4i} \left[ \left( e^{2ix} \left[ \frac{\log \sin 2x}{2} - ix \right] \right) - \left( e^{-2ix} \left[ \frac{\log \sin 2x}{2} + ix \right] \right) \right] \right\} \text{Ans}$$

c2x

o}