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Q.1 $\dfrac{d^2y}{dx^2} + a^2 y = \sec ax$

$(D^2 + a^2)y = \sec ax$

$\hookrightarrow$ Auxiliary Eqn $D = M$

$M^2 + a^2 = 0$

$M^2 = -a^2 \Rightarrow M = \pm ai = \alpha \pm i\beta$

$C.f. = \{c_1 \cos ax + c_2 \sin ax\} e^{0x}$

$P.I. = \dfrac{1}{D^2 + a^2} \sec ax$

$P.I. = \dfrac{1}{(D-ai)(D+ai)} \sec ax$

$\cancel{P.I.} = \dfrac{1}{(D-ai)(D+ai)} = \dfrac{\overset{=\frac{1}{2ai}}{A}}{\underset{\substack{=0 \\ D=ai}}{(D-ai)}} + \dfrac{\overset{=\frac{-1}{2ai}}{B}}{\underset{\substack{=0 \\ D=-ai}}{(D+ai)}}$

$P.I. = \left\{ \dfrac{1/2ai}{(D-ai)} + \dfrac{(-1/2ai)}{(D+ai)} \right\} \sec ax$

$P.I. = \dfrac{1}{2ai} \left\{ \dfrac{1}{(D-ai)} \sec ax - \dfrac{1}{(D+ai)} \sec ax \right\}$ — (i)

for $\cancel{\dfrac{1}{D-ai}}$

$= \dfrac{1}{D-ai} \sec ax = \dfrac{1}{(D-ai)} \times 1 \times \sec ax = \dfrac{1}{(D-ai)} e^{aix-aix} \times \sec ax$

$= \dfrac{1}{D-ai} e^{aix} \cdot e^{-aix} \sec ax \quad \left[\dfrac{1}{f(D)} e^{ax} \phi(x) = e^{ax} \dfrac{1}{f(D+a)} \phi(x) \right]$

$= e^{aix} \dfrac{1}{(D+ai-ai)} e^{-aix} \sec ax = e^{aix} \int e^{-aix} \sec(ax)\, dx$

$= e^{aix} \int \{\cos ax - i \sin ax\} \sec(ax)\, dx \quad \{\because e^{-i\theta} = \cos\theta - i\sin\theta\}$

$$= e^{aix}\int \{\cos ax - i\sin ax\}\frac{1}{\cos ax}\,dx$$

$$= e^{aix}\int \{1 - i\tan ax\}\,dx$$

$$= e^{aix}\left[x - i\frac{\log \sec ax}{a}\right] \quad \text{--- (ii)}$$

for $\frac{1}{(D+ai)}\sec ax = \frac{1}{(D+ai)}1\times \sec ax = \frac{1}{(D+ai)}e^{aix-aix}\sec ax$

$$= \frac{1}{D+ai}e^{aix}\cdot e^{-aix}\sec ax$$

$$= e^{-aix}\frac{1}{D-ai+ai}e^{aix}\sec ax$$

$$= e^{-aix}\frac{1}{D}e^{aix}\sec ax = e^{-aix}\int e^{aix}\sec ax$$

$$\{\because e^{i\theta} = \cos\theta + i\sin\theta\}$$

$$= e^{-aix}\int(\cos ax + i\sin ax)\sec ax$$

$$= e^{-aix}\int(1 + i\tan ax)$$

$$= e^{-aix}\left[x + \frac{i\log \sec ax}{a}\right] \quad \text{--- (iii)}$$

Put (ii) & (iii) in Eqn (i)

$$P.I. = \frac{1}{2ai}\left\{\left(e^{aix}\left[x - \frac{i\log \sec ax}{a}\right]\right) - \left(e^{-aix}\left[x + \frac{i\log \sec ax}{a}\right]\right)\right\}$$

Complete Soln

$$y = C.F. + P.I.$$

$$y = \{C_1\cos ax + C_2\sin ax\}e^{0x} + \frac{1}{2ai}\left[\left(e^{aix}\left[x - \frac{i\log \sec ax}{a}\right]\right) - \left(e^{-aix}\left[x + \frac{i\log \sec ax}{a}\right]\right)\right]$$

Ans

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Q.2 $\frac{d^2y}{dx^2}+4y=\tan 2x$

$(D^2+4)y=\tan 2x$

→ Auxiliary Eqn

$M^2+4=0$

$M^2=-4 \Rightarrow M=\pm 2i=\alpha\pm\beta$; $\alpha=0$, $\beta=2$

$C.f.=\{C_1\cos 2x+C_2\sin 2x\}e^{0x}$

$P.I.=\frac{1}{D^2+4}\tan 2x = \cancel{\frac{1}{D^2+4}1\times\tan 2x = \frac{1}{D^2+4}e^{2ix-2ix}\tan 2x}$

$P=\frac{1}{D^2+4}\tan 2x$

$=\frac{1}{(D-2i)(D+2i)}=\frac{A}{(D-2i)}+\frac{B}{(D+2i)}$; $A=\frac{1}{4i}$, $B=-\frac{1}{4i}$

$D-2i=0$, $D=2i$; $D+2i=0$, $D=-2i$

$P.I.=\left\{\frac{\frac{1}{4i}}{(D-2i)}+\frac{\left(-\frac{1}{4i}\right)}{(D+2i)}\right\}\tan 2x$

$P.I.=\frac{1}{4i}\left\{\frac{1}{(D-2i)}\tan 2x-\frac{1}{(D+2i)}\tan 2x\right\}$ — (i)

for $\frac{1}{(D-2i)}\tan 2x=\frac{1}{(D-2i)}e^{2ix-2ix}\tan 2x=\frac{1}{(D-2i)}e^{2ix}\cdot e^{-2ix}\tan 2x$

$=e^{2ix}\frac{1}{(D+2i-2i)}e^{-2ix}\tan 2x=e^{2ix}\frac{1}{D}e^{-2ix}\tan 2x$

$=e^{2ix}\int e^{-2ix}\tan 2x\,dx$

$=e^{2ix}\int[\cos 2x-i\sin 2x]\tan 2x\,dx$ $\{e^{-i\theta}=\cos\theta-i\sin\theta\}$

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$$= e^{2ix}\int\left[\cos 2x - i\sin 2x\right]\frac{\sin 2x}{\cos 2x}\,dx$$

$$= e^{2ix}\int\left[\sin 2x - i\frac{(\sin^2 2x)}{\cos 2x}\right]dx$$

$$= e^{2ix}\int\left[\sin 2x - i\frac{(1-\cos^2 2x)}{\cos 2x}\right]dx$$

$$= e^{2ix}\int\left[\sin 2x - i(\sec 2x - \cos 2x)\right]dx$$

$$= e^{2ix}\left[-\frac{\cos 2x}{2} - i\left(\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2}\right)\right] \quad \text{— (ii)}$$

for $\frac{1}{(D+2i)}\tan 2x = \frac{1}{(D+2i)}e^{2ix-2ix}\tan 2x = \frac{1}{(D+2i)}e^{2ix}\cdot e^{-2ix}\tan 2x$

$$= e^{-2ix}\frac{1}{(D-2i+2i)}e^{2ix}\tan 2x = e^{-2ix}\frac{1}{D}e^{2ix}\tan 2x$$

$$= e^{-2ix}\int e^{2ix}\tan 2x\,dx$$

$$= e^{-2ix}\int\left[\cos 2x + i\sin 2x\right]\tan 2x\,dx$$

$$= e^{-2ix}\int\left[\cos 2x + i\sin 2x\right]\frac{\sin 2x}{\cos 2x}\,dx$$

$$= e^{-2ix}\int\left[\sin 2x + i\frac{(\sin^2 2x)}{\cos 2x}\right]dx$$

$$= e^{-2ix}\int\left[\sin 2x + i\frac{(1-\cos^2 2x)}{\cos 2x}\right]dx$$

$$= e^{-2ix}\int\left[\sin 2x + i(\sec 2x - \cos 2x)\right]dx$$

$$= e^{-2ix}\left[-\frac{\cos 2x}{2} + i\left(\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2}\right)\right] \quad \text{— (iii)}$$

Put this value (ii) & (iii) in Eqn (i)

$$P.I. = \frac{1}{4i}\left\{\left(e^{2ix}\left[-\frac{\cos 2x}{2} - i\left(\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2}\right)\right]\right) - \left(e^{-2ix}\left[-\frac{\cos 2x}{2} + i\left(\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2}\right)\right]\right)\right\}$$

Complete Sol^n $y = C.F. + P.I.$

$$y = \{C_1\cos 2x + C_2\sin 2x\}e^{x} + \frac{1}{4i}\left\{\left(e^{2ix}\left[-\frac{\cos 2x}{2} - i\left(\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2}\right)\right]\right) - \left(e^{-2ix}\left[-\frac{\cos 2x}{2} + i\left(\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2}\right)\right]\right)\right\}$$

Ans

$$(D-2i)(D+2i) = D^2-(2i)^2 = D^2-2i^2 = D^2+4$$

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Q.3: $\dfrac{d^2y}{dx^2}+4y=\sec 2x$

Solⁿ → $(D^2+4)y=\sec 2x$

→ Auxiliary eqⁿ

$M^2+4=0$

$M^2=-4 \Rightarrow M=\pm 2i = \alpha \pm \beta i$; $\alpha=0$, $\beta=2$

$C.F.=\{C_1\cos 2x + C_2 \sin 2x\}e^{0x}$

$$P.I.=\frac{1}{D^2+4}\sec 2x$$

$$P.I.=\frac{1}{(D-2i)(D+2i)}=\frac{A}{(D-2i)}+\frac{B}{(D+2i)}$$

$A=\frac{1}{4i}$, $B=\frac{-1}{4i}$

$D-2i=0 \Rightarrow D=2i$; $D+2i=0 \Rightarrow D=-2i$

$$P.I.=\left\{\frac{\frac{1}{4i}}{(D-2i)}+\frac{\frac{-1}{4i}}{(D+2i)}\right\}\sec 2x$$

$$P.I.=\frac{1}{4i}\left\{\frac{1}{(D-2i)}\sec 2x-\frac{1}{(D+2i)}\sec 2x\right\} \quad \text{— (i)}$$

for $\dfrac{1}{D-2i}\sec 2x = \dfrac{1}{D-2i}e^{2ix-2ix}\sec 2x = \dfrac{1}{D-2i}e^{2ix}.e^{-2ix}\sec 2x$

$$= e^{2ix}\frac{1}{D+2i-2i}e^{-2ix}\sec 2x = e^{2ix}\frac{1}{D}e^{-2ix}\sec 2x$$

$$= e^{2ix}\int e^{-2ix}\sec 2x\,dx \qquad \{\because e^{-i\theta}=(\cos\theta-i\sin\theta)\}$$

$$= e^{2ix}\int[\cos 2x - i\sin 2x]\sec 2x\,dx \qquad \sin 2x=\frac{1}{\cos 2x}$$

$$= e^{2ix}\int[1-i\tan 2x]\,dx$$

$$= e^{2ix}\left[x-i\frac{(\log \sec 2x)}{2}\right] \quad \text{— (ii)}$$

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for $\frac{1}{(D+2i)} \sec 2x = \frac{1}{(D+2i)} e^{2ix - 2ix} \sec 2x$

$= \frac{1}{(D+2i)} e^{2ix} \cdot e^{-2ix} \sec 2x$

$= e^{-2ix} \frac{1}{(D-2i+2i)} e^{2ix} \sec 2x$

$= e^{-2ix} \frac{1}{D} e^{2ix} \sec 2x$

$= e^{-2ix} \int e^{2ix} \sec 2x \, dx$ $\{ \because e^{i\theta} = (\cos\theta + i\sin\theta) \}$

$= e^{-2ix} \int [\cos 2x + i \sin 2x] \sec 2x \, dx$

$= e^{-2ix} \int [1 + i \tan 2x] dx$

$= e^{-2ix} \left[x + i \frac{(\log \sec 2x)}{2} \right]$ — (iii)

Put (ii) & (iii) in Eqn (i)

~~Complete Soln $y = C.F. + P.I.$~~

~~$y = \{ C_1 \cos 2x + C_2 \sin 2x \} e^{0x} +$~~

$P.I = \frac{1}{4i} \left\{ \left(e^{2ix} \left[x - i \frac{(\log \sec 2x)}{2} \right] \right) - \left(e^{-2ix} \left[x + i \frac{(\log \sec 2x)}{2} \right] \right) \right\}$

Complete Soln $y = C.f. + P.I.$

$y = \{ C_1 \cos 2x + C_2 \sin 2x \} e^{0x} + \frac{1}{4i} \left\{ \left(e^{2ix} \left[x - i \frac{(\log \sec 2x)}{2} \right] \right) - \left(e^{2ix} \left[x + i \frac{(\log \sec 2x)}{2} \right] \right) \right\}$ Ans

Q4 $\frac{d^2y}{dx^2} + a^2y = \operatorname{cosec}(ax)$

Soln $\Rightarrow (D^2 + a^2)y = \operatorname{cosec}(ax)$

$\hookrightarrow$ Auxiliary Eqn

$M^2 + a^2 = 0$

$M^2 = -a^2 \Rightarrow M = \pm ai = \alpha \pm \beta i$ $\quad \alpha = 0,\ \beta = a$

$$\boxed{C.f. = \{c_1 \cos ax + c_2 \sin ax\} e^{0x}}$$

$$P.I. = \frac{1}{D^2 + a^2} \operatorname{cosec} ax$$

$$P.I. = \frac{1}{(D-ai)(D+ai)} \operatorname{cosec} ax$$

$$\frac{1}{(D-ai)(D+ai)} = \frac{A}{(D-ai)} + \frac{B}{(D+ai)} \qquad A \to \frac{1}{2ai},\quad B \to -\frac{1}{2ai}$$

$\hookrightarrow = 0$ $\quad \hookrightarrow = 0$

$D = ai$ $\quad D = -ai$

$$= \left\{ \frac{\frac{1}{2ai}}{(D-ai)} + \frac{\left(\frac{-1}{2ai}\right)}{(D+ai)} \right\} \operatorname{cosec} ax$$

$$= \frac{1}{2ai} \left\{ \frac{1}{(D-ai)} - \frac{1}{(D+ai)} \right\} \operatorname{cosec} ax$$

$$P.I. = \frac{1}{2ai} \left\{ \frac{1}{(D-ai)} \operatorname{cosec} ax - \frac{1}{(D+ai)} \operatorname{cosec} ax \right\} \quad \text{— (i)}$$

for $\frac{1}{(D-ai)} \operatorname{cosec} ax = \frac{1}{(D-ai)} e^{aix - aix} \operatorname{cosec} ax = \frac{1}{(D-ai)} e^{aix} \cdot e^{-aix} \operatorname{cosec} ax$

$$= e^{aix} \frac{1}{(D + \cancel{ai} - \cancel{ai})} e^{-aix} \operatorname{cosec} ax = e^{aix} \frac{1}{D} e^{-aix} \operatorname{cosec} ax$$

$$= e^{2ix}\int e^{-2ix}\,\mathrm{cosec}\,ax\,dx \qquad \left\{\because e^{-i\theta} = (\cos\theta - i\sin\theta)\right\}$$

$$= e^{2ix}\int [\cos ax - i\sin ax]\,\mathrm{cosec}\,ax\,dx \qquad \mathrm{cosec}\,ax = \frac{1}{\sin ax}$$

$$= e^{2ix}\int [\cot ax - i(1)]\,dx$$

$$= e^{2ix}\left[\frac{\log(\sin ax)}{a} - ix\right]. \quad \text{— (ii)}$$

for $\frac{1}{(D+2i)}\,\mathrm{cosec}\,ax = \frac{1}{(D+2i)}\,e^{2ix-2ix}\,\mathrm{cosec}\,ax = \frac{1}{(D+2i)}\,e^{2ix}\cdot e^{-2ix}\,\mathrm{cosec}\,ax$

$$= e^{-2ix}\frac{1}{(D-2i+2i)}e^{2ix}\,\mathrm{cosec}\,ax = e^{-2ix}\frac{1}{D}e^{2ix}\,\mathrm{cosec}\,ax$$

$$= e^{-2ix}\int e^{2ix}\,\mathrm{cosec}\,ax\,dx \qquad \left\{\because e^{i\theta} = \cos\theta + i\sin\theta\right\}$$

$$= e^{-2ix}\int [\cos ax + i\sin ax]\,\mathrm{cosec}\,ax\,dx$$

$$= e^{-2ix}\int [\cot ax + i(1)]\,dx$$

$$= e^{-2ix}\left[\frac{\log(\sin ax)}{a} + ix\right] \quad \text{— (iii)}$$

Put (ii) & (iii) in Eqn (i)

$$P.I. = \frac{1}{2ai}\left\{\left(e^{2ix}\left[\frac{\log(\sin ax)}{a} - ix\right]\right) - \left(e^{-2ix}\left[\frac{\log(\sin ax)}{a} + ix\right]\right)\right\}$$

~~Complete~~ complete Soln $y = C.F. + P.I.$

$$y = \{c_1\cos ax + c_2\sin ax\}e^{0x} + \frac{1}{2ai}\left\{\left(e^{2ix}\left[\frac{\log(\sin ax)}{a} - ix\right]\right) - \left(e^{-ax}\left[\frac{\log(\sin ax)}{a} + ix\right]\right)\right\}$$

Ans

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Q.5 ⇒ $\frac{d^2y}{dx^2} + 4y = \text{Cosec}(2x)$

$(D^2+4)y = \text{Cosec}\,2x$

⇒ Auxiliary Eqn

$M^2 + 4 = 0$

$M^2 = -4 \Rightarrow M = \pm 2i = \alpha \pm \beta i$ $\quad \alpha = 0,\ \beta = 2$

$C.f. = \{c_1 \cos 2x + c_2 \sin 2x\} e^{0x}$

$P.I. = \frac{1}{D^2+4} \text{Cosec}\,2x$

$= \frac{1}{(D-2i)(D+2i)} = \frac{A}{(D-2i)} + \frac{B}{(D+2i)}$ $\quad A \to \frac{1}{4i},\ B \to \frac{-1}{4i}$

$\quad D-2i = 0,\ D = 2i \qquad D+2i = 0,\ D = -2i$

$P.I. = \left\{ \frac{1/4i}{(D-2i)} + \frac{(-1/4i)}{(D+2i)} \right\} \text{Cosec}(2x)$

$P.I. = \frac{1}{4i} \left\{ \frac{1}{(D-2i)} - \frac{1}{(D+2i)} \right\} \text{Cosec}(2x)$

$P.I. = \frac{1}{4i} \left\{ \frac{1}{(D-2i)} \text{Cosec}\,2x - \frac{1}{(D+2i)} \text{Cosec}\,2x \right\}$ — (i)

for $\frac{1}{(D-2i)} \text{Cosec}\,2x = \frac{1}{(D-2i)} e^{2ix-2ix} \text{Cosec}\,2x = \frac{1}{(D-2i)} e^{2ix} \cdot e^{-2ix} \text{Cosec}\,2x$

$= e^{2ix} \frac{1}{(D+2i-2i)} e^{-2ix} \text{Cosec}\,2x = e^{2ix} \frac{1}{D} e^{-2ix} \text{Cosec}\,2x$

$= e^{2ix} \int e^{-2ix} \text{Cosec}\,2x \, dx$ $\qquad \{\because e^{-i\theta} = \cos\theta - i\sin\theta\}$

$= e^{2ix} \int [\cos 2x - i \sin 2x] \text{Cosec}\,2x \, dx$

$= e^{2ix} \int [\cot 2x - i(1)] \, dx$

$= e^{2ix} \left[\frac{\log \sin 2x}{2} - ix \right]$ — (ii)

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for $\frac{1}{(D+2i)}\operatorname{Cosec}2x = \frac{1}{(D+2i)}e^{2ix-2ix}\operatorname{Cosec}2x = \frac{1}{(D+2i)}e^{2ix}\cdot e^{-2ix}\operatorname{Cosec}2x$

$= e^{-2ix}\frac{1}{(D-2i+2i)}e^{2ix}\operatorname{Cosec}2x$

$= e^{-2ix}\frac{1}{D}e^{2ix}\operatorname{Cosec}2x$

$= e^{-2ix}\int e^{2ix}\operatorname{Cosec}2x\,dx \qquad \left\{\because e^{i\theta} = \cos\theta + i\sin\theta\right\}$

$= e^{-2ix}\int[\cos 2x + i\sin 2x]\operatorname{Cosec}2x\,dx$

$= e^{-2ix}\int[\cot 2x + i(1)]\,dx$

$= e^{-2ix}\left[\frac{\log\sin 2x}{2} + ix\right]$ — (iii)

Put (ii) & (iii) in Eqn (i)

$P.I. = \frac{1}{4i}\left\{\left(e^{2ix}\left[\frac{\log\sin 2x}{2} - ix\right]\right) - \left(e^{-2ix}\left[\frac{\log\sin 2x}{2} + ix\right]\right)\right\}$

Complete Sol^n $y = C.f. + P.I.$

$y = \left\{C_1\cos 2x + C_2\sin 2x\right\}e^{0x} + \left\{\frac{1}{4i}\left[\left(e^{2ix}\left[\frac{\log\sin 2x}{2} - ix\right]\right) - \left(e^{-2ix}\left[\frac{\log\sin 2x}{2} + ix\right]\right)\right]\right\}$ Ans