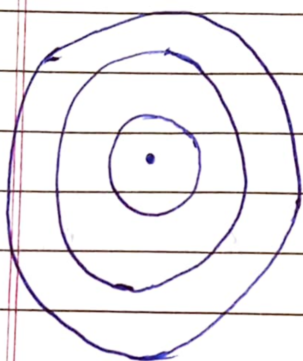
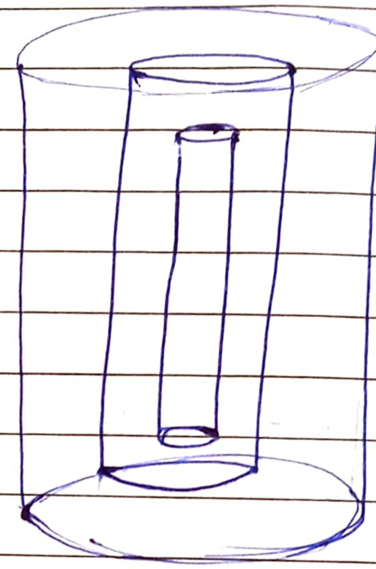
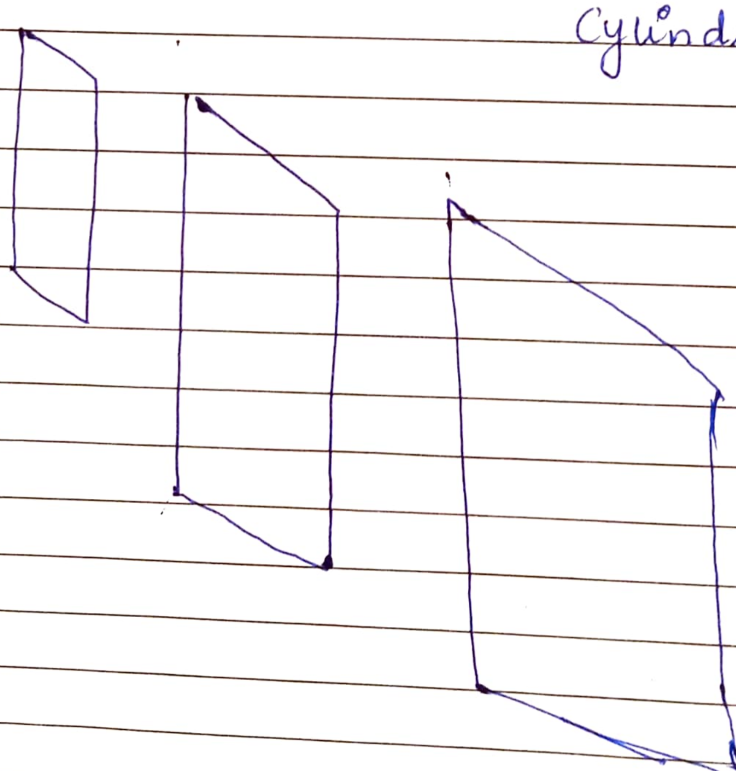
WAVE OPTICS

Huygen's Principle





Cylindrical wavefron



Plane wavefront

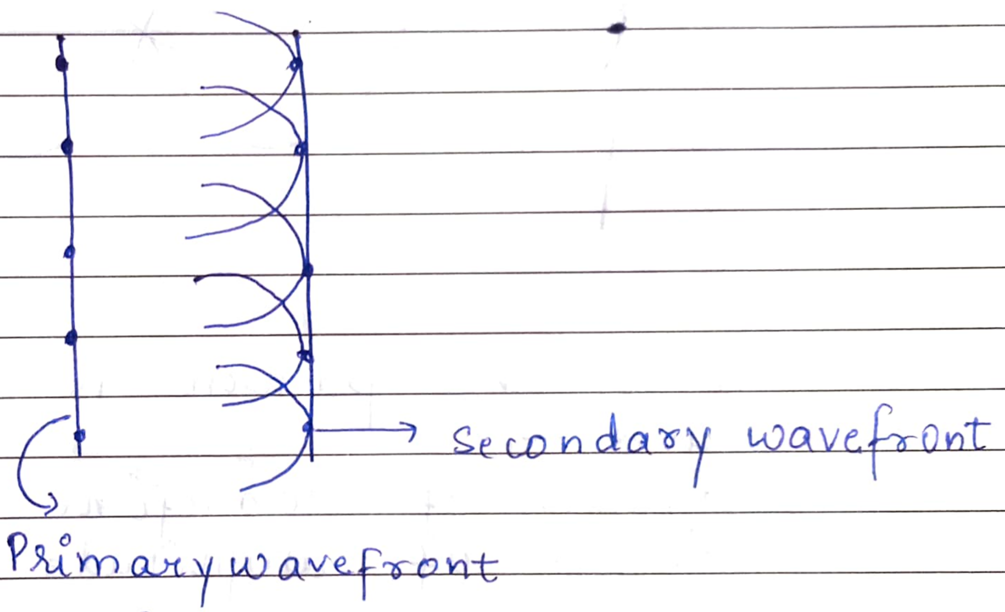
GoodLuck Page No.
Date

Wavefront

It is a collection of points where all the disturbance (vibration) are in same phase and in same frequency.

Huygen's Principle

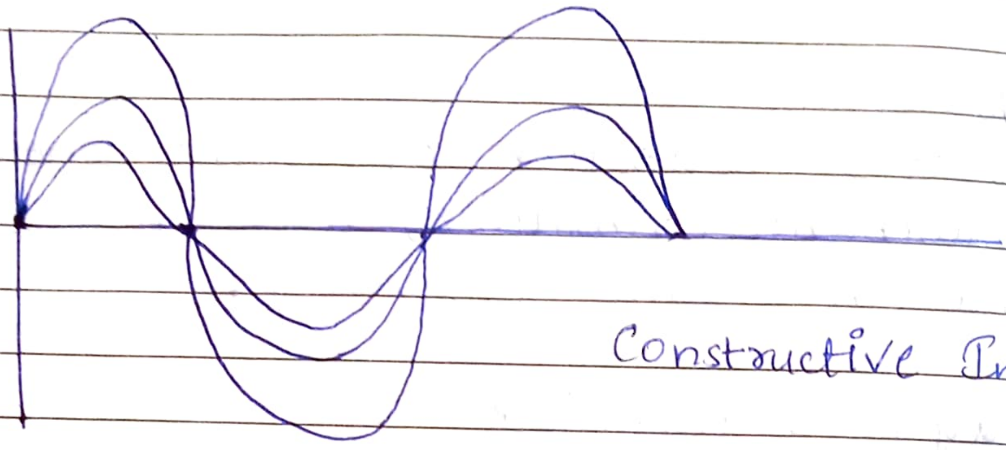
Every point on a propagating wavefront act as new sources of spherical secondary wavelets



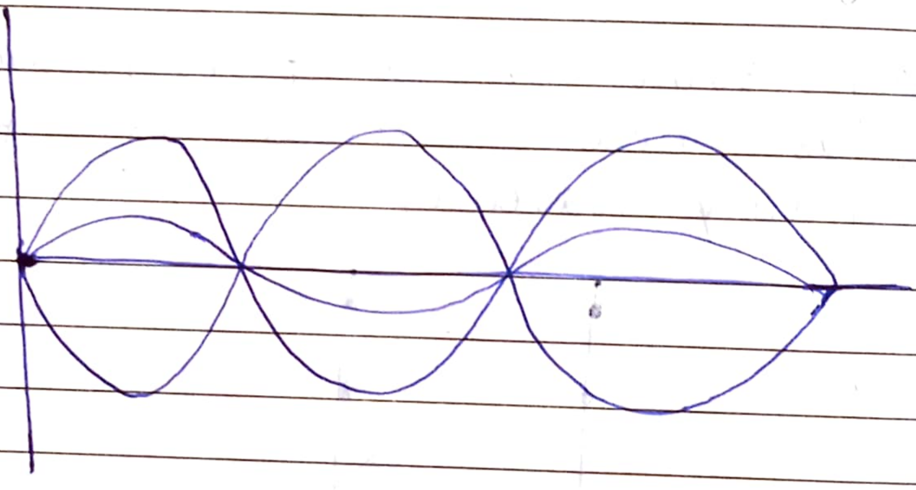
Interference

when two or more then two light waves having same frequency and constant phase difference superpose with each other, redistribution of energy occurs. This phenomenon is called interference.

At some points, intensity is maximum and interference is called constructive interference. At some other points, intensity is minimum and interference is called destructive interference.



Constructive Interfere



Interference pattern is produced by tw waves —

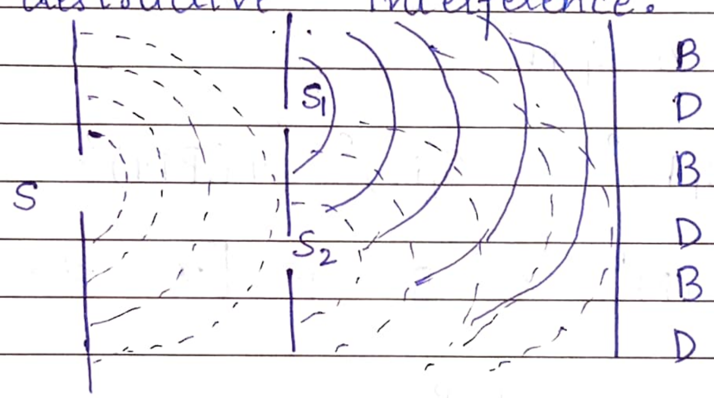
(1) Division of wavefront
(2) Division of amplitude

GoodLuck Page No.
Date

Condition for constructive and destructive interference -

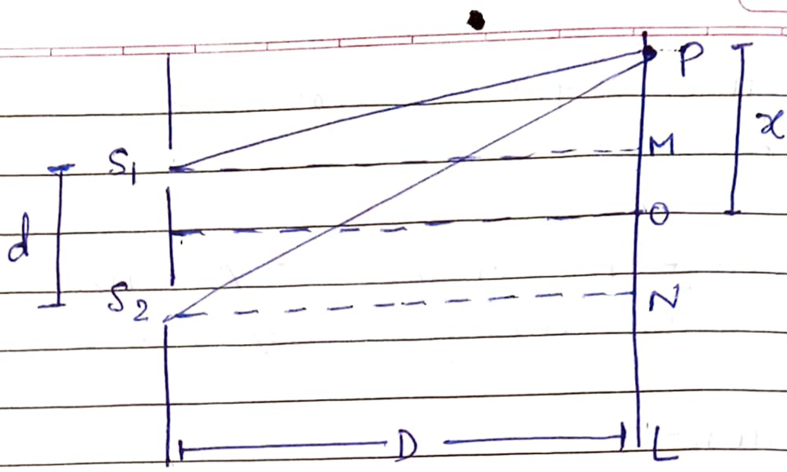
Con~~stant~~structive interference $\Rightarrow \Delta = n\lambda$
Distructive interference $\Rightarrow \Delta = (2n+1)\lambda/2$

Question-
Explain Young's double slit experiment and give the condition for constructive and destructive interference.



S is illuminated by monochromatic light wavefront coming out from slit S is incident on the two slit S_1 and S_2. This wavefront is divided into two wave-fronts. These wavefront reunite and gives interference pattern on the screen where the crest of the one lies over the crest of other, constructive interference takes place and where the crest of one lies over the trough of other destructive interference takes place.

Condition for constructive and Destructive Interference



$S_1P \simeq S_2P \simeq D$

In ΔS_1MP

$(S_1P)^2 = (S_1M)^2 + (PM)^2$

$(S_1P)^2 = D^2 + [x - d/2]^2$ —— (1)

In ΔS_2NP

$(S_2P)^2 = (S_2N)^2 + (PN)^2$

$(S_2P)^2 = D^2 + (x + d/2)^2$ —— (2)

By subtracting eq (1) from eq (2)

$(S_2P)^2 - (S_1P)^2 = xd + xd$

$(S_1P + S_2P)(S_2P - S_1P) = 2xd$

$S_2P - S_1P = \frac{2xd}{2D}$ $[S_1P \simeq S_2P \simeq D]$

$S_2P - S_1P = \frac{xd}{D}$

$\Delta = \frac{xd}{D}$

For Constructive interference

$$\Delta = n\lambda$$

$$\frac{xd}{D} = n\lambda$$

$$\boxed{x = \frac{Dn\lambda}{d}}$$

For Destructive interference

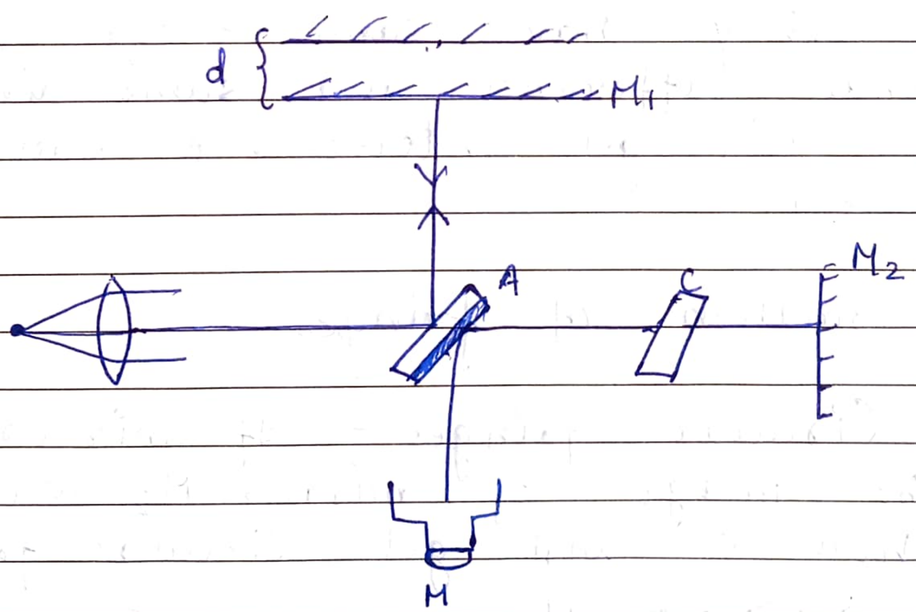
$$\cancel{\Delta = n\lambda}$$

$$\Delta = (2n+1)\lambda/2$$

$$\frac{xd}{D} = (2n+1)\lambda/2$$

$$\boxed{x = \frac{(2n+1)D\lambda}{2d}}$$

Question → Explain the formation of fringes by using Michelson's interferometer. Write its one application. (14)

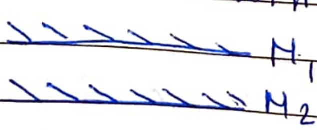


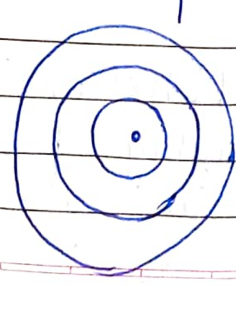
Michelson's interferometer consist of two highly polished mirrors M_1 and M_2 and two plane glass plates. A and C are parallel to each other. One side of glass plate A is half silvered so that the light coming from the source S is equally reflected and transmitted by it. The plate A is inclined at angle of 45°. One half of the energy of the incident ~~ser~~ beam is reflected by the plate A towards the mirror M_1 and M_2. These two ~~becomes~~ beams ~~relate~~ return to plate A. The beam reflected back by M_1 is transmitted through the glass plate A and the beam reflected back by M_2 is reflected by the glass A towards the eye. Therefore to complete the path, the plate C is used between the mirrors M_2 and A. Mirror M_1 is movable and the movement can be measured by using scale attached mirror M_1.

Waves reflected by the mirrors M_1 and M_2 are coming out from the same source so they have same frequency, constant phase difference and same direction so we will get interference pattern on the screen.

Formation of fringes

(i) Circular fringes – If mirror M_1 and the image of mirror M_2 are parallel then we will get circular fringes.



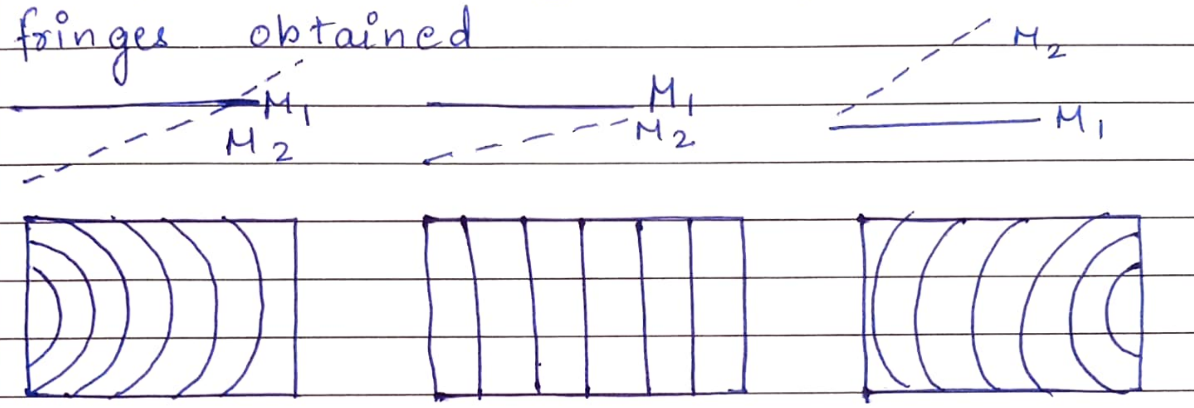


$\Delta = 2d$, $\Delta = N\lambda$

$2d = N\lambda$

$\lambda = \dfrac{2d}{N}$

(ii) Localized fringes - If mirror M_1 and virtual mirror M_2 are inclined localized fringes obtained



Application of Michelson's Interferometer -

Determination of wavelength

Let the apparatus is set for circular fringes. Now we move M_1 by a distance d due to that effective path difference

$$\Delta = 2d \quad \text{--- (1)}$$

Now let due to movement error were passes N fringes (say bright ~~and~~ fringe) then path difference becomes -

$$\Delta = N\cancel{d} \quad \text{--- (2)}$$

By (1) and (2)

$$\Delta = 2d$$

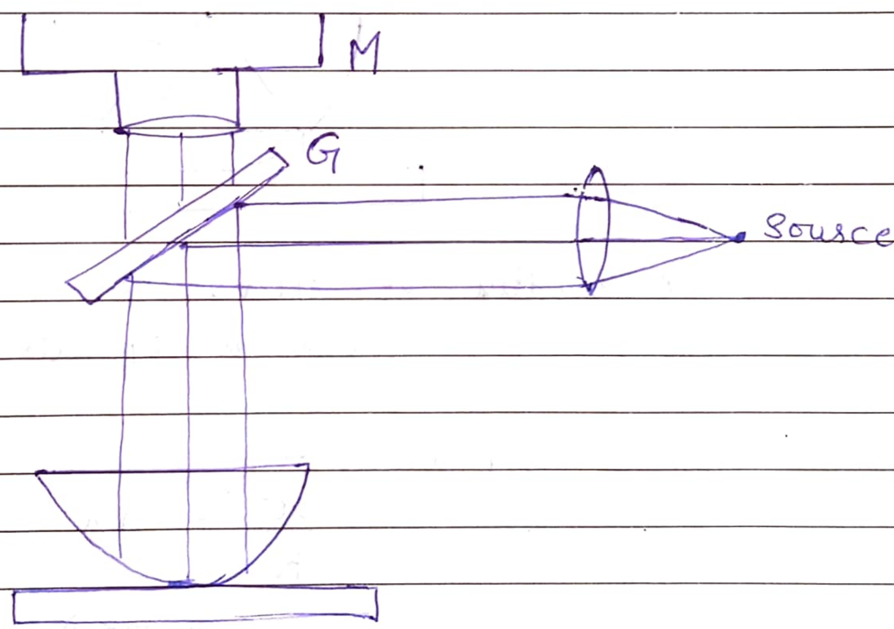
$$\Delta = N\cancel{d}\lambda$$

$$2d = N\lambda$$

$$\boxed{\lambda = \frac{2d}{N}}$$

Newton's Rings

When a plane convex lens of large focal length is placed on a plane glass plate, a thin film of air is formed between the convex surface of a lens and plane glass plate. The thickness of air film is zero at the point of contact and gradually increases as we move outwards from the centre. When monochromatic light falls on the film, then concentric dark and bright rings observed at microscope.



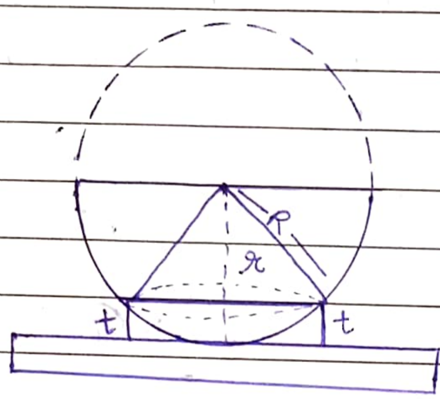
Light from the source incident on a glass plate inclined at an angle of 45°. This glass plate reflect light towards combination of plano convex lens. After reflection from plano convex lens light waves reaches to microscope. These wave's have same frequency and constant phase difference. So we will get interference position.

Path difference produced by the film -

$$\Delta = 2\mu t \cos r \pm \lambda/2 \quad \text{---} \quad (1)$$

For air film $[\mu = 1]$
For normal incidence $[r = 0]$

$$\Delta = 2t + \lambda/2 \quad \text{---} \quad (2)$$



The radius of curvature is R and radius of fringe r, thickness of air film is t then

From equation -

For bright fringe

$$\frac{r^2}{R} = n\lambda$$

$$r^2 = nR\lambda$$

$2t + \lambda/2 = n\lambda$

$$\left(\frac{D}{2}\right)^2 = nR\lambda$$

$2 \times \frac{r^2}{2R} + \lambda/2 = n\lambda$

$$\frac{D^2}{4} = nR\lambda$$

$r^2 = (2n-1)R\lambda/2$

$$D^2 = 4nR\lambda$$

$D^2 = (2n-1)\, 2R\lambda$

D is the diameter of n^{th} dark ring so it is denoted by -

$$\boxed{D_n^2 = 4nR\lambda} \quad \text{--- (5)}$$

formula for wavelength
For nth dark fringe

$$D_n^2 = 4nR\lambda \quad \text{--- (6)}$$

For $(n+p)^{th}$ dark ring

$$D_{n+p}^2 = 4(n+p)R\lambda \quad \text{--- (7)}$$

$$r^2 \simeq 2Rt \qquad t = \frac{r^2}{2R} \quad \text{--- (3)}$$

$$\left[\frac{D}{2}\right]^2 \simeq 2Rt$$

$$D^2 = 8Rt$$

For ~~bright~~ dark fringe

$$\Delta = 2t + \lambda/2 = (2n+1)\lambda/2$$

$$2t + \lambda/2 = n\lambda + \lambda/2$$

$$2t = n\lambda$$

By subtracting 6^{th} from 7^{th}

$$D_{n+p}^2 - D_n^2 = 4(n+p)R\lambda - 4nR\lambda$$

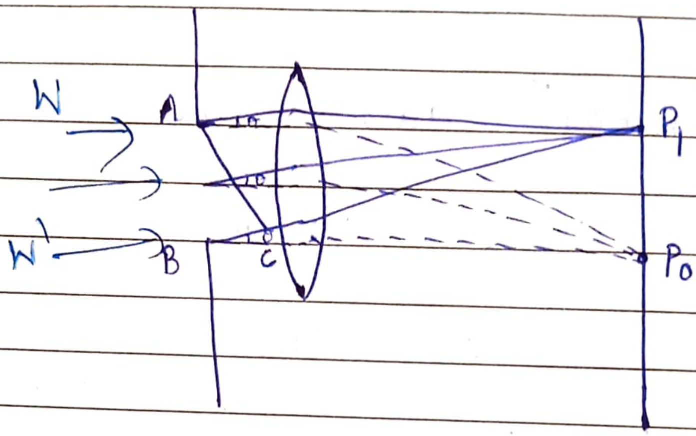
$$\frac{D_{n+p}^2 - D_n^2}{4PR} = \lambda$$

This is the formula for wavelength of a monochromatic light.

Diffraction

Bending of light towards the corner is known as diffraction.

Fraundhafer diffraction at single slit



Let a plane wavefront WW' is incid normally on the slit AB. Waves those are moving exactly perpendicular to th slit are focused at P_0 and the wa moving with θ angle are focused at point P_1. We have to find out inten of light at point P_1.

Let the two waves one is moving from A to P_1 and other is moving from B to P_1. There is a path difference BC between the two waves, Hence

Path difference $= \Delta = BC = AB\sin\theta$

Where AB is a width of slit
Let $AB = e$, then

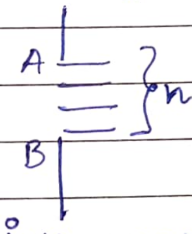
Path difference $= \Delta = e\sin\theta$

So, Phase difference

$$= \frac{2\pi}{\lambda} \times \text{Path difference}$$

$$\text{Phase difference} = \frac{2\pi}{\lambda} e\sin\theta$$

let n waves are propagating through the slit AB



let the amplitude of each wave is a, So the phase difference between two consecutive wave

$$\frac{1}{n}[\text{total phase}] = \frac{1}{n}\left[\frac{2\pi}{\lambda} e\sin\theta\right]$$

$$= d \text{ (say)}$$

The resultant amplitude R is given by-

$$R = \frac{a \sin nd/2}{\sin d/2}$$

$$R = \frac{a \sin n\left[\frac{1}{n}\left(\frac{2\pi}{\lambda \cdot 2} e\sin\theta\right)\right]}{\sin\left[\frac{\pi e\sin\theta}{n\lambda}\right]}$$

$$R = \frac{a \sin\left[\frac{\pi e\sin\theta}{\lambda}\right]}{\sin\left[\frac{\pi e\sin\theta}{n\lambda}\right]}$$

let $\frac{\pi e \sin\theta}{\lambda} = \alpha$ then,

$$R = \frac{a \sin\alpha}{\sin(\alpha/n)}$$

OR $R = \frac{a \sin\alpha}{\alpha/n}$ $\left[\text{as } \frac{\alpha}{n} \text{ is very sm}\right.$

$$R = \frac{na \sin\alpha}{\alpha}$$

$$R = \frac{A \sin\alpha}{\alpha}$$

$I \propto R^2$ so,

$$I = A^2 \left[\frac{\sin\alpha}{\alpha}\right]^2$$

For Principal ~~Momentum~~ Maximum

$\alpha = 0$ but $\sin\alpha \neq 0$

$$\frac{\pi e \sin\theta}{\lambda} = 0$$

$$\sin\theta = 0$$
$$\theta = 0^\circ$$

Minimum intensity position

$\sin\alpha = 0$ but $\alpha \neq 0$

$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi$ --- $m\pi$

$$\frac{\cancel{\pi} e\sin\theta}{\lambda} = \pm m\pi$$

$e\sin\theta = \pm m\lambda$ where $m = 1, 2, 3 \ldots$

Resolving Power of Grating

Ability of grating to form separate spectural pattern of two close wavelen-gth is called its resolving power. It is denoted by $\frac{\lambda}{d\lambda} = nN$

Diffraction at N slit (Grating)

Condition for principal maxima

$e\sin\theta = n\lambda$ where $\{n = 0, 1, 2, 3 \ldots\}$

Condition for minima

$Ne\sin\theta = m\lambda$ $\begin{cases} \text{where } m \neq N, 2N, 3N \ldots \\ m = 1, 2, \ldots N-1, N+1 \ldots \end{cases}$

Rayleigh's Criterion

According to this Criterion, two close wave-length are separable if the principale maxima of one lies over the first minima of the other.