$\log z \leq \sin z$

$\log(z) + \sin(z)$

11/04/25

~~Simetaneous~~ differential Eqn
simaltaneous

Solve $\frac{dx}{dt} - 7x + y = 0$

$\frac{dy}{dt} - 2x - 5y = 0$

$\left.\begin{array}{l} Dx - 7x + y = 0 \\ Dy - 2x - 5y = 0 \end{array}\right\} D = d/dt$

$(D-7)x + y = 0 \} \times 2$

$-2x + (D-5)y = 0 \} \times (D-7)$

$2(D-7)x + 2y = 0$

$-2(D-7)x + (D-5)(D-7)y = 0$

$((D-7)(D-5) + 2)y = 0$

$(D^2 - 12D + 37)y = 0$

$m^2 - 12m + 37 = 0$

$m = \frac{12 \pm \sqrt{144 - 4(1)(37)}}{2}$

$m = \frac{12 \pm 2i}{2} = 6 \pm i = \alpha \pm i\beta$

$C.f. = \{c_1 \cos t + c_2 \sin t\} e^{6t}$

$P.I. = \frac{1}{D^2 - 12D + 37}(0) = 0$

$y = \{c_1 \cos t + c_2 \sin t\} e^{6t}$

~~$(D-7)x + y = 0$~~ $\} \times (D-5)$

~~$-2x + (D-5)y = 0$~~ $\} \times 1$

$(D-7)(D-5)x + (D-5)y = 0$

$-2x + (D-5)y = 0$

$+$

$((D-7)(D-5) + 2)x = 0$

$(D^2 - 12D + 37)x = 0$

$m^2 - 12m + 37 = 0$

$m = 6 \pm i$

$C.f. = \{c_3 \cos t + c_4 \sin t\} e^{6t}$

$P.I. = \boxed{0}$

$x = C.f. + P.I.$

$x = \{c_3 \cos t + c_4 \sin t\} e^{6t}$

11/04/25

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Solve

$$\frac{dx}{dt} + 5x + y = e^t$$

$$\frac{dy}{dt} - x + 3y = e^{2t}$$

$$\left.\begin{aligned} Dx + 5x + y &= e^t \\ Dy - x + 3y &= e^{2t} \end{aligned}\right\} \quad D = \frac{d}{dt}$$

$$\left.\begin{aligned} (D+5)x + y &= e^t \\ -x + (D+3)y &= e^{2t} \end{aligned}\right\} \begin{aligned} &\times 1 \\ &\times (D+5) \end{aligned}$$

$$(D+5)\cancel{x} + y = e^t$$

$$-(D+5)x + (D+3)(D+5)y = e^{2t}(D+5)$$

$$\{(D+3)(D+5)+1\}y = e^t + (D+5)e^{2t}$$

$$(D^2+8D+16)y = e^t + De^{2t} + 5e^{2t}$$

$$(D^2+8D+16)y = e^t + 2e^{2t} + 5e^{2t}$$

$$(D^2+8D+16)y = e^t + 7e^{2t}$$

$$M^2 + 8M + 16 = 0$$

$$m = -4, -4$$

$$C.f. = (c_1 + c_2 t)e^{-4t}$$

$$P.I = \frac{1}{D^2+8D+16}\{e^t + 7e^{2t}\}$$

$$P.I. = \frac{1}{D^2+8D+16}e^t + \frac{7}{D^2+8D+16}e^{2t}$$

$$P.I = \frac{1}{1+8+16}e^t + \frac{7}{4+16+16}e^{2t}$$

$$P.I. = \frac{1}{25}e^t + \frac{7}{36}e^{2t}$$

$$y = C.f. + P.I.$$

$$\boxed{y = \{c_1 + c_2 t\}e^{-4t} + \frac{1}{25}e^t + \frac{7}{36}e^{2t}}$$

for x

$$\left.\begin{aligned} (D+5)x + y &= e^t \\ -x + (D+3)y &= e^{2t} \end{aligned}\right\} \begin{aligned} &\times (D+3) \\ &\times 1 \end{aligned}$$

$$(D+5)(D+3)x + \cancel{(D+3)y} = (D+3)e^t$$

$$\underset{+}{-}x \underset{-}{+} \cancel{(D+3)y} = \underset{-}{e^{2t}}$$

$$((D+5)(D+3)+1)x = De^t + 3e^t - e^{2t}$$

$$(D^2+8D+16)x = e^t + 3e^t - e^{2t}$$

$$(D^2+8D+16)x = 4e^t - e^{2t}$$

$$M^2 + 8D + 16 = 0$$

$$M = -4, -4$$

$$\boxed{C.f. = \{c_1 + c_2 t\}e^{-4t}}$$

$$P.I. = \frac{1}{D^2+8D+16}4e^t - e^{2t}$$

$$P.I. = 4\frac{1}{D^2+8D+16}e^t - \frac{1}{D^2+8D+16}e^{2t}$$

$$P.I. = 4\frac{1}{1+8+16}e^t - \frac{1}{4+16+16}e^{2t}$$

$$P.I. = 4\frac{1}{25}e^t - \frac{1}{36}e^{2t}$$

$$x = C.f. + P.I$$

$$\boxed{x = \{c_1 + c_2 t\}e^{-4t} + \frac{4}{25}e^t - \frac{1}{36}e^{2t}}$$

15/04/25

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Solve $\frac{dx}{dt} + y = \sin t$

$\frac{dy}{dt} + x = \cos t$

Given that $x = 2$, $y = 0$ at $t = 0$

$$\begin{aligned} 1\times \; & \{ Dx + y = \sin t \\ D\times \; & \{ x + Dy = \cos t \end{aligned} \quad D = d/dt$$

$Dx + y = \sin t$

$-\cancel{Dx} \pm D^2 y = D\cos t$

$(1 - D^2)y = \sin t - D\cos t$

$(1 - D^2)y = \sin t + \sin t$

$(1 - D^2)y = 2\sin t$

$1 - m^2 = 0$

$m^2 = 1$

$m = \pm 1$

$C.f = C_1 e^{t} + C_2 e^{-t}$

$P.I = \frac{1}{(1 - D^2)} 2\sin t$

$P.I. = 2 \frac{1}{(1 - D^2)} \sin t$

$D^2 = -(b)^2 = -(1^2) = -1$

$P.I = \frac{2}{1-(-1)} \sin t = \frac{2}{2} \sin t$

$y = C.f + P.I.$

$y = \{C_1 e^{t} + C_2 e^{-t}\} + \sin t \quad (1)$

$\{ Dx + y = \sin t \} \times D$

$\{ x + Dy = \cos t \} \times 1$

$D^2 x + Dy = D\sin t$

$x \pm \cancel{Dy} = \cos t$

$(D^2 - 1)x = D\sin t - \cos t$

$(D^2 - 1)x = \cos t - \cos t$

$(D^2 - 1)x = 0$

$M^2 - 1 = 0$

$m = \pm 1 \Rightarrow$

$C.f = C_3 e^{t} + C_4 e^{-t}$

$P.I = 0$

$x = C.f + P.I$

$x = \{C_3 e^{t} + C_4 e^{-t}\} \quad (2)$

Condition

at, $t = 0$, $y = 0$

$0 = C_1 1 + C_2 1 + 0$

$C_1 + C_2 = 0$

at, $t = 0$, $x = 2$

$2 = C_3 e^{0} + C_4 e^{-0}$

$2 = C_3 + C_4$

Unit - 2 15/04/25

Inspection method

$1+P+Q=0 \Rightarrow u=e^{x}$

$1-P+Q=0 \Rightarrow u=e^{-x}$

$$\frac{d^2y}{dx^2}+\frac{Pdy}{dx}+Qy=R$$ where P, Q, R are funcn of x

let $y=u.v$

P or Q की Highest degree same है तो

(1) $m^2+Pm+Q=0$
$\Rightarrow u=e^{mx}$

P or Q की diff. degree (1)

Inspection method

$m=1,2,3$

$m=-1,-2,-3$

(2) $m(m-1)+Pmx+Qx^2=0$
$\Rightarrow u=x^m$

⇒ Solution of differential Eqn of the type

$$\frac{d^2y}{dx^2}+\frac{Pdy}{dx}+Qy=R$$ where P, Q, R are funcn of x

let ~~y = u.v~~

Let its Soln $y=u.v$
where u, & v are called one of the integral part

(1) $m^2+Pm+Q=0$
$\Rightarrow u=e^{mx}$

P or Q की Highest degree same है तब

(2) $m(m-1)+Pmx+Qx^2=0$
$\Rightarrow u=x^m$

P or Q की degree में diff. (1) है

16/04/25

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Q.1 Solve:-

$$(3-x)\frac{d^2y}{dx^2}-(9-4x)\frac{dy}{dx}+(6-3x)y=0$$

$$\frac{d^2y}{dx^2}-\frac{(9-4x)}{(3-x)}\frac{dy}{dx}+\frac{(6-3x)}{(3-x)}y=0 \quad \text{—— (A)}$$

$$P=\frac{-(9-4x)}{(3-x)} \quad \Big| \quad Q=\frac{6-3x}{3-x} \quad \Big| \quad R=0$$

(1) $m^2+Pm+Q=0 \Rightarrow u=e^{mx}$

$\Rightarrow \quad m=1,2,3 \atop -1,-2,-3$ $\Big\}$ $m=1$

$$1+P+Q=1-\frac{(9-4x)}{3-x}+\frac{6-3x}{3-x} \Rightarrow e^{x}$$

let S.o.P of (A)

$y=u.v$

$y=e^x.v$ —— (1)

diff. w.r.t.x

$$\frac{dy}{dx}=e^x\frac{dv}{dx}+ve^x \quad \text{—— (2)}$$

Double diff. w.r.t.x

$$\frac{d^2y}{dx^2}=e^x\frac{d^2v}{dx^2}+\underline{\frac{dv}{dx}e^x}+ve^x+\underline{e^x\frac{dv}{dx}} \quad (3)$$

$$2e^x\frac{dv}{dx}$$

Put (1), (2), (3) in (A)

$$\left\{e^x\frac{d^2v}{dx^2}+\frac{dv}{dx}e^x+ve^x+e^x\frac{dv}{dx}\right\}-\frac{(9-4x)}{3-x}\left\{e^x\frac{dv}{dx}+ve^x\right\}+\frac{(6-3x)}{(3-x)}ve^x=0$$

$$(3-x)-(9-4x)+(6-3x)$$

$$\cancel{(3-x)+(9-4x)+} \quad (3-0) \quad \cancel{3-x-9+4x+6-3x}$$

$$\frac{3-x}{x}=0$$

$$\frac{d^2v}{dx^2}+\left\{2-\frac{(9-4x)}{(3-x)}\right\}\frac{dv}{dx}+\left\{1-\frac{(9-4x)}{3-x}+\frac{6-3x}{3-x}\right\}v=0$$

$$\frac{d^2v}{dx^2}+\left\{\frac{6-2x-9+4x}{3-x}\right\}\frac{dv}{dx}+\left\{\frac{3-x-9+4x+6-3x}{(3-x)}\right\}v=0$$

$$\frac{d^2v}{dx^2}+\left(\frac{2x-3}{3-x}\right)\frac{dv}{dx}=0$$

put $\frac{dv}{dx}=t \Rightarrow \frac{d^2v}{dx^2}=\frac{dt}{dx}$

$$\frac{dt}{dx}+\left\{\frac{(2x-3)}{(3-x)}\right\}t=0$$

$$\frac{dt}{dx}=-\frac{(2x-3)}{(3-x)}t$$

$$\int\frac{dt}{t}=-\int\frac{(2x-3)}{(3-x)}dx$$

$$\log t=\int\frac{(2x-3)}{(x-3)}dx=2\int\frac{(x-3/2)}{(x-3)}=2\int\frac{(x-3)+3-3/2}{(x-3)}$$

$$\log t=2\int\left\{\frac{(x-3)}{(x-3)}+\frac{3/2}{(x-3)}\right\}dx$$

$$\log t=2\int\left\{1+\frac{3/2}{(x-3)}\right\}dx=2\left\{x+3/2\log(x-3)\right\}$$

$$\log t=2x+3\log(x-3)$$

$$\log_e t=2x+\log(x-3)^3$$

$$t=e^{2x+\log(x-3)^3}$$

$$t=e^{2x}\cdot e^{\log(x-3)^3}$$

$$t=e^{2x}(x-3)^3 \qquad \frac{dv}{dx}=e^{2x}\cdot(x-3)^3$$

ILATE

$uv_1 - u'v_{11} + u''v_{111} + \ldots$

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$$\int dv = \int \underset{II}{e^{2x}} \underset{I}{(x-3)^3} dx$$

$$v = \left[(x-3)^3 \frac{e^{2x}}{2} - \int 3(x-3)^2 \cdot 1 \frac{e^{2x}}{2} dx\right] + c$$

$$v = \left[(x-3)^3 \frac{e^{2x}}{2} - \frac{3}{2}\int \underset{I}{(x-3)^2} \underset{II}{e^{2x}} dx\right] + c$$

$$v = \left\{\left[(x-3)^3 \frac{e^{2x}}{2}\right] - \left[\frac{3}{2}\left((x-3)^2 \frac{e^{2x}}{2} - \int 2(x-3)\frac{e^{2x}}{2} dx\right)\right] + c\right\}$$

$$v = \left\{\left[(x-3)^3 \frac{e^{2x}}{2}\right] - \left[\frac{3}{2}\left[(x-3)^2 \frac{e^{2x}}{2} - \left[(x-3)\frac{e^{2x}}{2} - \int 1 \cdot \frac{e^{2x}}{2} dx + c\right.\right.\right.$$

$$v = \left\{\left[(x-3)^3 \frac{e^{2x}}{2}\right] - \left[\frac{3}{2}\left[(x-3)^2 \frac{e^{2x}}{2} - (x-3)\frac{e^{2x}}{2} - \frac{e^{2x}}{4} + c\right]\right\}\right.$$

$$v = \left[(x-3)^3 \frac{e^{2x}}{2} - \frac{3(x-3)^2 e^{2x}}{4} + \frac{3(x-3)e^{2x}}{4} + \frac{3e^{2x}}{8} + c\right]$$

$$\frac{(x-3)^3 e^{2x}}{2} - \frac{3(x-3)^2 e^{2x}}{4} + \frac{6(x-3)e^{2x}}{8} + \frac{6e^{2x}}{16} + c$$

$$v = \frac{(x-3)^3 e^{2x}}{2} - \frac{3(x-3)^2 e^{2x}}{4} + \frac{3(x-3)e^{2x}}{4} + \frac{3e^{2x}}{8} + c$$

$$y = e^{x}\left[\frac{(x-3)^3 e^{2x}}{2} - \frac{3(x-3)^2 e^{2x}}{4} + \frac{3(x-3)e^{2x}}{4} + \frac{3e^{2x}}{8} + c\right]$$

$$\cancel{y = \frac{e^{3x}}{2}\left[e^{x} \cdot \frac{e^{2x}}{2}\right] x}$$

$$y = \frac{e^{3x}}{2}\left[(x-3)^3 - \frac{3}{2}(x-3)^2 + \frac{3}{2}(x-3) + \frac{3}{4} + c\right]$$ Ans

$$y = \frac{e^{3x}}{8}\left[4(x-3)^3 - 6(x-3)^2 + 6(x-3) + 3\right]$$ Ans

$$\frac{dv}{dx} = t \qquad \frac{d^2v}{dx^2} = \frac{dt}{dx}$$

$$\cancel{\frac{dv}{dx}}$$

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Solve

$$\frac{d^2y}{dx^2} - \cot x\frac{dy}{dx} - (1-\cot x)y = e^x \sin x \quad (A)$$

$P = -\cot x \mid Q = -(1-\cot x) \mid R = e^x \cdot \sin x$

(1) $m^2 + Pm + Q = 0$

$m = 1$

$1 + P + Q$

$1 - \cot x - (1 - \cot x) = 0$

$\Rightarrow u = e^{mx} = e^x$

let Solⁿ of (A) is $y = u \cdot v$

$y = e^x v$

$$\frac{dy}{dx} = e^x\frac{dv}{dx} + ve^x \quad (2)$$

$$\frac{d^2y}{dx^2} = e^x\frac{d^2v}{dx^2} + e^x\frac{dv}{dx} + e\frac{dv}{dx} + ve^x$$

$$\frac{d^2y}{dx^2} = e^x\frac{d^2v}{dx^2} + 2e^x\frac{dv}{dx} + ve^x \quad (3)$$

Put (1) (2) (3) in (A)

$$\left\{\cancel{e^x}\frac{d^2v}{dx^2} + 2\cancel{e^x}\frac{dv}{dx} + v\cancel{e^x}\right\} - \cot x\left\{\cancel{e^x}\frac{dv}{dx} + v\cancel{e^x}\right\}$$

$$-(1-\cot x)\, ve^x = e^x \sin x$$

$$\frac{d^2v}{dx^2} + \{2 - \cot x\}\frac{dv}{dx} + \{\cancel{1} - \cancel{\cot x} - \cancel{1} + \cancel{\cot x}\}v = \sin x$$

$$\frac{d^2v}{dx^2} + \{2 - \cot x\}\frac{dv}{dx} = \sin x$$

Put $\frac{dv}{dx} = t \Rightarrow \frac{d^2v}{dx^2} = \frac{dt}{dx}$

$$\frac{dt}{dx} + (2 - \cot x)t = \sin x$$

17(cal.5

$$\int e^{ax}\sin bx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2}$$

$$I.f = e^{\int P dx} = e^{\int(2-\cot x)dx} = e^{2x-\log\sin x} = e^{2x}\cdot e^{-\log\sin x}$$

$$= e^{2x}\frac{1}{\sin x}$$

solⁿ will be

$$t\times I.f = \int Q\times I.f + C$$

$$t\times\frac{e^{2x}}{\sin x} = \int\frac{\sin x\, e^{2x}}{\sin x} = \int e^{2x} = \frac{e^{2x}}{2}+C$$

$$\frac{t e^{2x}}{\sin x} = \frac{e^{2x}}{2}+C$$

$$\boxed{\int e^{ax}\sin bx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2}}$$

$$\frac{t e^{2x}}{\sin x} = \frac{e^{2x}}{2}+C$$

$$t = \frac{e^{2x}\sin x}{2e^{2x}} + C\frac{\sin x}{e^{2x}}$$

$$\frac{dv}{dx} = \frac{\sin x}{2} + Ce^{-2x}\sin x$$

$$\int dv = \int\frac{\sin x}{2}dx + C\int e^{-2x}\sin x\,dx$$

$$v = \frac{-\cos x}{2} + C\left\{\frac{e^{-2x}((-2)\sin x - 1.\cos x)}{(-2)^2+1^2}\right\}$$

$$v = \frac{-\cos x}{2} + C\left\{\frac{e^{-2x}((-2)\sin x - \cos x)}{5}\right]$$

$$y = u.v$$

$$y = e^{x}\left\{\frac{-\cos x}{2} + C\left(\frac{e^{-2x}((-2)\sin x - \cos x)}{5}\right)\right\}$$

$\frac{3}{2}-1$

$\frac{3-2}{2}$

$\frac{1}{2}$

ILATE

$I = \sin x \cdot \frac{e^{-2x}}{-2} - \int \cos x \frac{e^{-2x}}{2}$

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$$(1-x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y = x(1-x^2)^{3/2} \quad \text{—} \; A$$

$$\frac{d^2y}{dx^2}+\frac{x}{1-x^2}\frac{dy}{dx}-\frac{1}{(1-x^2)}y = \frac{x(1-x^2)^{3/2}}{(1-x^2)} \quad \text{—} \; A$$

$$\frac{d^2y}{dx^2}+\frac{x}{1-x^2}\frac{dy}{dx}-\frac{1}{(1-x^2)}y = x\sqrt{1-x^2}$$

$$P=\frac{x}{1-x^2},\ Q=\frac{-1}{1-x^2},\ R=x\sqrt{1-x^2}$$

$$m(m-1)+Pmx+Qx^2=0$$

$$\left.\begin{matrix} m=1,2,3 \\ -1,-2,-3 \end{matrix}\right\} \quad Px+Qx^2=\frac{x(x)}{1-x^2}-\frac{x^2}{1-x^2}=0$$

$$u=x^m=x$$

let solⁿ of (A) $y=u.v$

$$y=xv \quad \text{—} \; (1)$$

$$\frac{dy}{dx}=v+x\frac{dv}{dx} \quad \text{—} \; (2)$$

$$\frac{d^2y}{dx^2}=\frac{dv}{dx}+\frac{dv}{dx}+x\frac{d^2v}{dx^2}$$

$$\frac{d^2y}{dx^2}=x\frac{d^2v}{dx^2}+2\frac{dv}{dx} \quad \text{—} \; (3)$$

Put (1) (2) (3) in Eqn (A)

$$x\frac{d^2v}{dx^2}+2\frac{dv}{dx}+\frac{x}{1-x^2}\left\{v+x\frac{dv}{dx}\right\}-\frac{1}{(1-x^2)}\{xv\} = x(1-x^2)^{1/2}$$

$$\frac{d^2v}{dx^2}+\frac{2}{x}\frac{dv}{dx}+\frac{1}{1-x^2}\left\{v+x\frac{dv}{dx}\right\}-\frac{1}{1-x^2}(v) = (1-x^2)^{1/2}$$

$$\frac{d^2v}{dx^2}+\left\{\frac{2}{x}+\frac{x}{1-x^2}\right\}\frac{dv}{dx}+\left\{\cancel{\frac{v}{1-x^2}}-\cancel{\frac{v}{1-x^2}}\right\}v = (1-x^2)^{1/2}$$

$x(1-x^2)^{-1}$

$\frac{1}{1-x^2}$

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$$\frac{d^2v}{dx^2}+\left\{\frac{2}{x}+\frac{x}{1-x^2}\right\}\frac{dv}{dx}=(1-x^2)^{1/2}$$

Put $\frac{dv}{dx}=t \Rightarrow \frac{d^2v}{dx^2}=\frac{dt}{dx}$

$$\frac{dt}{dx}+\left\{\frac{2}{x}+\frac{x}{1-x^2}\right\}t=(1-x^2)^{1/2}$$

$$\left\{P=\frac{2}{x}+\frac{x}{1-x^2} \quad Q=(1-x^2)^{1/2}\right\}$$

$$I.f=e^{\int\frac{2}{x}+\frac{x}{1-x^2}dx}$$

$$I.f.=e^{2\log x+\frac{1}{-2}\int\frac{dt}{t}}$$

$1-x^2=t$

$-2x\,dx=dt$

$x\,dx=\frac{dt}{-2}$

$$I.f=e^{2\log x-\frac{1}{2}\log t}$$

$$I.f=e^{2\log x^2-\frac{1}{2}\log(1-x^2)}$$

$$I.f=e^{\log x^2-\log(1-x^2)^{1/2}}$$

$$I.f=e^{\log\frac{x^2}{(1-x^2)^{1/2}}}$$

$$I.f=\frac{x^2}{(1-x^2)^{1/2}}$$

Complete Soln

$$t\times I.f=\int Q.I.f\,dx+C$$

$$t\times\frac{x^2}{(1-x^2)^{1/2}}=\int\cancel{(1-x^2)^{1/2}}\frac{x^2}{\cancel{(1-x^2)^{1/2}}}dx+C$$

$$t\times\frac{x^2}{(1-x^2)^{1/2}}=\int x^2dx+C$$

$$t\times\frac{x^2}{(1-x^2)^{1/2}}=\frac{x^3}{3}+C$$

$$t=\frac{x^3}{3}\times\frac{(1-x^2)^{1/2}}{x^2}+\frac{C(1-x^2)^{1/2}}{x^2}$$

$$\frac{dv}{dx}=\frac{x}{3}\times(1-x^2)^{1/2}+C\frac{(1-x^2)^{1/2}}{x^2}$$

Put $1-x^2=t$

$-2x\,dx=dt$

$x\,dx=\frac{dt}{-2}$

$$\int 1.\,dv=\frac{1}{3}\int x(1-x^2)^{1/2}dx+C\int \underset{\text{II}}{x^{-2}}\,\underset{\text{I}}{(1-x^2)^{1/2}}dx$$

$$v=\frac{1}{3}\frac{1}{(-2)}\int t^{1/2}dt+C_1\left[(1-x^2)^{1/2}\frac{x^{-1}}{-1}-\int\left[\frac{1}{2}(1-x^2)^{-1/2}(-2x)\frac{x^{-1}}{-1}\right]dx\right]$$

Sol

$$\int u v\,dx = u\int v\,dx - \int[u'\int v\,dx]\,dx + C$$

$u = e$

$x^2 \frac{(1-x^2)^{3/2}}{3/2} - \int \cdots$

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$$v = \frac{1}{-6}\,\frac{t^{3/2}}{3/2} + c_1\left[\frac{-(1-x^2)^{1/2}}{x} - \int (1-x^2)^{-1/2}\,dx\right] + c_2$$

$$v = \frac{1}{-6}\times\frac{2}{3}\,t^{3/2} + c_1\left[-\frac{(1-x^2)^{1/2}}{x} - \int\frac{1}{\sqrt{1-x^2}}\,dx\right] + c_2$$

$$v = \frac{1}{-9}(1-x^2)^{3/2} - c_1\frac{(1-x^2)^{1/2}}{x} - c_1\sin^{-1}x + c_2$$

$$y = xv$$

$$y = x\left[\frac{1}{-9}(1-x^2)^{3/2} - c_1\frac{(1-x^2)^{1/2}}{x} - c_1\sin^{-1}x + c_2\right]$$

$$\boxed{y = \frac{x}{-9}(1-x^2)^{3/2} - c_1(1-x^2)^{1/2} - c_1 x\sin^{-1}x + c_2 x}$$

Ans

21/04/25

Removal of 1^{st} Derivative, ~~Soln~~ of

Solve → differential Eqn

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

Its Soln is $y = u.v$

where $u = e^{-\frac{1}{2}\int P\,dx}$ and v can ~~d~~ be fined

$$\frac{d^2v}{dx^2} + Iv = R/u$$

where $I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{1}{4}P^2$

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Solve

Solve $x^2\frac{d^2y}{dx^2} - 2(x^2+x)\frac{dy}{dx} + (x^2+2x+2)y = 0$

$y = u.v$ (1) $\quad u = e^{-\frac{1}{2}\int P\,dx}$

$$\frac{d^2v}{dx^2} + Iv = R/u$$

$$I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{1}{4}P^2$$

$$\frac{d^2y}{dx^2} - 2\left(1+\frac{1}{x}\right)\frac{dy}{dx} + \left(1+\frac{2}{x}+\frac{2}{x^2}\right)y = 0$$

$P = -2\left(1+\frac{1}{x}\right)$ | $Q = 1+\frac{2}{x}+\frac{2}{x^2}$ | $R = 0$

$$u = e^{-\frac{1}{2}\int P\,dx} = e^{-\frac{1}{2}\int\{-2(1+\frac{1}{x})\}dx}$$

$$= e^{x+\log x} = e^x \cdot e^{\log x} = e^x \cdot x$$

$$\frac{dP}{dx} = -2\left(0 - \frac{1}{x^2}\right) = \frac{2}{x^2}$$

$$I = 1 + \frac{2}{x} + \frac{2}{x^2} - \frac{1}{2}\left(\frac{2}{x^2}\right) - \frac{1}{4}\left\{-2\left(1+\frac{1}{x}\right)\right\}^2$$

$$I = \cancel{1} + \cancel{\frac{2}{x}} + \cancel{\frac{2}{x^2}} - \cancel{\frac{1}{x^2}} - \cancel{1} - \cancel{\frac{1}{x^2}} - \cancel{\frac{2}{x}}$$

$I = 0$ put $I = 0$ in $\frac{d^2v}{dx^2} + Iv = R/u$

$$\frac{d^2v}{dx^2} + 0v = 0/u$$

$$\int \frac{d^2v}{dx^2} + 0 = \int 0$$

$$\frac{dv}{dx} = c_1$$

$$\int dv = \int c_1\,dx$$

$$v = c_1x + c_2$$

Put this value in Eqn (1)

$$y = e^x \cdot x\,(c_1x + c_2)$$

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$e^{\log x} = x$ $\sin^2 x + \cos^2 x = 1$ $\sec^2 x - \tan^2 x = 1$

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Solve $\frac{d^2y}{dx^2} - 2\tan x \frac{dy}{dx} + 5y = \sec x \, e^x$

$y = uv$ $P = -2\tan x$ | $Q = 5$ | $R = \sec x \, e^x$

$u = e^{-\frac{1}{2}\int P dx} = e^{-\frac{1}{2}\int\{-2\tan x\}} = e^{\log \sec x} = \sec x$

$u = \sec x$

$\frac{dP}{dx} = -2\sec^2 x$

$I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{1}{4}P^2$

$I = 5 - \frac{1}{2}(-2\sec^2 x) - \frac{1}{4}(-2\tan x)^2$

$I = 5 + \sec^2 x - \tan^2 x$

$I = 5 + 1 = 6$ Put in

$\frac{d^2v}{dx^2} + Iv = R/u$

$\frac{d^2v}{dx^2} + 6v = \frac{\sec x \, e^x}{\sec x}$

$\frac{d^2v}{dx^2} + 6v = e^x$

$(D^2 + 6)v = e^x$

$m^2 + 6 = 0 \Rightarrow$

$m = \pm\sqrt{6}i$

$m^2 + 6 = 0$

$m^2 = -6$

$m = \pm\sqrt{-6}$

$m = \pm\sqrt{6}i$

$\pm\sqrt{6}i$

C.F. $= \{c_1 \cos\sqrt{6} + c_2 \sin\sqrt{6}\}$

P.I. $= \frac{1}{D^2+6}e^x = \frac{1}{1^2+6}e^x = \frac{1}{7}e^x$

$v = C.F. + P.I.$

$v = \{c_1 \cos\sqrt{6} + c_2 \sin\sqrt{6}\} + \frac{1}{7}e^x$

$y = \sec x\left\{\{c_1 \cos\sqrt{6} + c_2 \sin\sqrt{6}\} + \frac{1}{7}e^x\right\}$

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Q3 $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + (x^2+1)y = (x^3+3x)$

let, $y = u.v$ $\quad P = 2x \mid Q = (x^2+1) \mid R(x^3+3x)$

$u = e^{-\frac{1}{2}\int P dx} = e^{-\frac{1}{2}\int 2x dx} = e^{-\frac{1}{2} 2x^2/2} = e^{-\frac{1}{2}x^2}$

$$\boxed{\frac{d^2v}{dx^2} + Iv = R/u \quad I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{1}{4}P^2} \quad (1)$$

$\frac{dP}{dx} = 2$

$I = (x^2+1) - \frac{1}{2}(2) - \frac{1}{4}(2x)^2$

$I = x^2 + 1 - 1 - x^2$

$I = 0$

Put in (1)

$\frac{d^2v}{dx^2} + 0v = \frac{(x^3+3x)}{e^{-\frac{1}{2}x^2}}$

$\int \frac{d^2v}{dx^2} = \int (x^3+3x).e^{\frac{1}{2}x^2}$

$$\boxed{\frac{dv}{dx} = (x^3+3x)\frac{e^{\frac{1}{2}x^2}}{1/2}}$$

$\frac{dv}{dx} = \int x^3 e^{\frac{1}{2}x^2} dx + \int 3x e^{\frac{1}{2}x^2} dx + c_1$

$\frac{dv}{dx} = \int x^2.x e^{\frac{1}{2}x^2} dx + 3\int x e^{\frac{1}{2}x^2} dx + c_1$

$\frac{dv}{dx} = \int ^2$

Put $\frac{1}{2}x^2 = t \Rightarrow x^2 = 2t$

on diff w.r.t.

$\frac{1}{2} 2x\, dx = dt$

$x\, dx = dt$

$\frac{dv}{dx} = \int 2t e^t dt + 3\int e^t dt + C_1$

$\frac{dv}{dx} = 2\int t e^t dt + 3\int e^t dt + C_1$

$\frac{dv}{dx} = 2(te^t - e^t) + 3e^t + C_1$

$\frac{dv}{dx} = 2\frac{1}{2}x^2 e^{\frac{1}{2}x^2} - 2e^{\frac{1}{2}x^2} + 3e^{\frac{1}{2}x^2} + C_1$

$\frac{dv}{dx} = x^2 e^{\frac{1}{2}x^2} - 2e^{\frac{1}{2}x^2} + 3e^{\frac{1}{2}x^2} + C_1$

$\int \frac{dv}{dx} = \int x^2 e^{\frac{1}{2}x^2} + e^{\frac{1}{2}x^2} + C_1$

$V = \int \underset{I}{x}\underset{II}{(x e^{\frac{1}{2}x^2})} dx + \int e^{\frac{1}{2}x^2} dx + \int C_1 dx + C_2$

$V = x\int (x e^{\frac{1}{2}x^2}) - \int \left[\frac{d(x)}{dx}\int x e^{\frac{1}{2}x^2} dx\right] dx + \int e^{\frac{1}{2}x^2} dx + C_1\int 1 dx$

$V = x e^{\frac{1}{2}x^2} - \cancel{\int 1 \cdot e^{\frac{1}{2}x^2} dx} + \cancel{\int e^{\frac{1}{2}x^2} dx} + C_1 x + C_2$

$V = x e^{\frac{1}{2}x^2} + C_1 x + C_2$ — (2)

$\int x e^{\frac{1}{2}x^2} dx$

put

$\frac{1}{2}x^2 = t$

$\frac{1}{2} 2x\, dx = dt$

$x\, dx = dt$

$\int e^t dt$

e^t

$e^{\frac{1}{2}x^2}$

$y = u \cdot v$

$y = e^{-\frac{1}{2}x^2}\left(x e^{\frac{1}{2}x^2} + C_1 x + C_2\right)$

$y = x e^{-\frac{1}{2}x^2 + \frac{1}{2}x^2} + C_1 x e^{-\frac{1}{2}x^2} + C_2 e^{-\frac{1}{2}x^2}$

$y = x + (C_1 x + C_2) e^{-\frac{1}{2}x^2}$

Ans

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Mtd variation of Parameters

In this mtd 1st we find C.F

In c.f. constant C_1 and C_2 replaced by "A" & "B" where "A" and "B" are func^n of x

Take it as a complete so^n that is $y = C.f.$

Know find $\frac{dy}{dx}$ and In $\frac{dy}{dx}$

Put hole term of $\frac{dA}{dx}$ and $\frac{dB}{dx}$

$$\frac{dA}{dx} + \frac{dB}{dx} = 0$$

Know find $\frac{d^2y}{dx^2}$, Put $y, \frac{dy}{dx}, \frac{d^2y}{dx^2}$ in given eq^n. Know solve for $\frac{dA}{dx}$ & $\frac{dB}{dx}$

Solve By mtd of variation of Parameters.

$$\frac{d^2y}{dx^2} + 4y = \sin 2x \quad \text{— (A)}$$

$$(D^2+4)y = \sin 2x$$

$$m^2 + 4 = 0$$

$$m = \pm 2i$$

$$C.f. = (C_1 \cos 2x + C_2 \sin 2x)$$

m^2+4

$m = \pm 2i$

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$C.F. = (C_1 \sin 2x + C_2 \cos 2x)e^{0x}$

$y = C_1 \cos 2x + C_2 \sin 2x$ — (A)

~~when A, B a~~

$y = A\cos 2x + B\sin 2x$ — (1)

when A, B are func^n of 'x'

$$\frac{dy}{dy} = -2A\sin 2x + \cos 2x\frac{dA}{dx} + 2B\cos 2x + \sin 2x\frac{dB}{dx}$$

put $\cos 2x\frac{dA}{dx} + \sin 2x\frac{dB}{dx} = 0$ — (2)

$$\frac{dy}{dx} = -2A\sin 2x + 2B\cos 2x \text{ — (3)}$$

$$\frac{d^2y}{dx^2} = -4A\cos 2x - 2\sin 2x\frac{dA}{dx} - 4B\sin 2x + 2\cos 2x\frac{dB}{dx} \text{ — (4)}$$

$y, \frac{dy}{dx}, \frac{d^2y}{dx^2}$ from (1), (3), (4) in (A)

$$\left\{\cancel{-4A\cos 2x} - 2\sin 2x\frac{dA}{dx} - \cancel{4B\sin 2x} + 2\cos 2x\frac{dB}{dx}\right\}$$

$$+ 4\left\{\cancel{A\cos 2x} + \cancel{B\sin 2x}\right\} = \sin 2x$$

$$-2\sin 2x\frac{dA}{dx} + 2\cos 2x\frac{dB}{dx} = \sin 2x \text{ — (5)} \quad \times \cos 2x$$

$$\cos 2x\frac{dA}{dx} + \sin 2x\frac{dB}{dx} = 0 \text{ — (2)} \quad \times 2\sin 2x$$

$$\cancel{-2\sin 2x\cos 2x\frac{dA}{dx}} + 2\cos^2 2x\frac{dB}{dx} = \sin 2x\cos 2x$$

$$\cancel{2\sin 2x\cos 2x\frac{dA}{dx}} + 2\sin^2 2x\frac{dB}{dx} = 0$$

$$2\frac{dB}{dx} = \frac{\sin 4x}{2} \quad \frac{2\sin 2x\cos 2x}{2}$$

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$$2dB = \frac{\sin 4x}{2}\,dx$$

$$\int 2dB = \int \frac{\sin 4x}{2}\,dx \Rightarrow 2B = \frac{-\cos 4x}{8} = B = \frac{-\cos 4x}{16}$$

$$\left.\begin{aligned} -2\sin 2x\frac{dA}{dx} + 2\cos 2x\frac{dB}{dx} &= \sin 2x \\ \cos 2x\frac{dA}{dx} + \sin 2x\frac{dB}{dx} &= 0 \end{aligned}\right\} \begin{aligned} &\sin 2x \\ &2\cos 2x \end{aligned}$$

$1 - 2\sin^2 x = \cos 2x$

$\cos 4x = 1 - 2\sin^2 2x$

$2\sin^2 2x = 1 - \cos 4x$

$\sin^2 2x = \frac{1-\cos 4x}{2}$

$$-2\sin^2 2x\frac{dA}{dx} + 2\cos 2x\sin 2x\frac{dB}{dx} = \sin^2 2x$$

$$\underline{-\quad 2\cos^2 2x\frac{dA}{dx} + 2\cos 2x\sin 2x\frac{dB}{dx} = 0 \quad -}$$

$$-2\frac{dA}{dx} = \sin^2 2x$$

$$-2\int dA = \int \sin^2 2x\,dx$$

$$-2A = \int \frac{1}{2} - \int \frac{\cos 4x}{2}\,dx$$

$$-2A = \frac{1x}{2} - \frac{1}{2}\frac{\sin 4x}{4}$$

$$-2A = \frac{1}{2}x - \frac{\sin 4x}{8}$$

$$A = -\frac{1}{4}x + \frac{\sin 4x}{16}$$

$$y = \left(-\frac{1}{4}x + \frac{\sin 4x}{16}\right)(\cos 2x) + \left(-\frac{\cos 4x}{16}\right)(\sin 2x)$$

$$y = -\frac{x\cos 2x}{4} + \frac{\sin 4x\cos 2x}{16} - \frac{\cos 4x\sin 2x}{16}$$

Ans

Q.3 $\frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y = e^{2x}$

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Q.2 $\frac{d^2y}{dx^2} + 4y = \cancel{\tan 2x}\ \tan 2x$ — (A)

$(D^2+4)y = \tan 2x$

$m^2 = \pm 2i$

$C.F. = c_1\cos 2x + c_2\sin 2x$

$y = c_1\cos 2x + c_2\sin 2x$

$y = A\cos 2x + B\sin 2x$ — (1)

$\frac{dy}{dx} = \cancel{A} - 2A\sin 2x + \cos 2x\frac{dA}{dx} + 2B\cos 2x + \sin 2x\frac{dB}{dx}$

put $\cos 2x\frac{dA}{dx} + \sin 2x\frac{dA}{dx} = 0$ — (2)

$\frac{dy}{dx} = -2A\sin 2x + 2B\cos 2x$ — (3)

$\frac{d^2y}{dx^2} = -4A\cos 2x - 2\sin 2x\frac{dA}{dx} - 4B\sin 2x + 2\cos 2x\frac{dB}{dx}$ — (4)

Put (1), (3), & (4) in (A)

$-4A\cos 2x - 2\sin 2x\frac{dA}{dx} - 4B\sin 2x + 2\cos 2x\frac{dB}{dx} + 4(A\cos 2x + B\sin 2x)$

$-2\sin 2x\frac{dA}{dx} + 2\cos 2x\frac{dB}{dx} = \tan 2x$ — (5) $= \frac{\tan 2x}{\cos 2x}$

$\cos 2x\frac{dA}{dx} + \sin 2x\frac{dB}{dx} = 0$ — (2) $2\sin 2x$

$-2\sin 2x\cos 2x\frac{dA}{dx} + 2\cos^2 2x\frac{dB}{dx} = \sin 2x$

$2\sin 2x\cos 2x\frac{dA}{dx} + 2\sin^2 2x\frac{dB}{dx} = 0$

$2\frac{dB}{dx} = \sin 2x$

$\int 2\,dB = \int \sin 2x\,dx$

$2B = -\frac{\cos 2x}{2} \Rightarrow B = -\frac{\cos 2x}{4}$

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$$-2\sin 2x\frac{dA}{dx}+2\cos 2x\frac{dB}{dx}=\cancel{\sin 2}\tan 2x \quad \Big\} \sin 2x$$

$$\cos 2x\frac{dA}{dx}+\sin 2x\frac{dB}{dx}=0 \quad \Big\} 2\cos 2x$$

$$-2\sin^2 2x\frac{dA}{dx}+\cancel{2\cos 2x\sin 2x\frac{dB}{dx}}=\tan 2x\sin 2x$$

$$2\cos^2 2x\frac{dA}{dx}+\cancel{2\cos 2x\sin 2x\frac{dB}{dx}}=0$$

$$-2\frac{dA}{dx}=\tan 2x\sin 2x$$

$$-2\int dA=\int\tan 2x\sin 2x\,dx$$

$$-2A=\int\frac{\sin 2x\sin 2x}{\cos 2x}dx$$

$$-2A=\int\frac{\sin^2 2x}{\cos 2x}dx$$

$$-2A=\int\frac{1-\cos^2 2x}{\cos 2x}dx$$

$$-2A=\int(\sec 2x-\cos 2x)dx$$

$$-2A=\left\{\frac{\log(\sec 2x+\tan 2x)}{2}-\frac{\sin 2x}{2}\right\}$$

$$A=-\frac{1}{4}\left\{\log(\sec 2x+\tan 2x)-\sin 2x\right\}$$

$$y=-\frac{1}{4}\left\{\log\{\sec 2x+\tan 2x\}-\sin 2x\right\}\cos 2x+\left(-\frac{\cos 2x}{4}\right)\sin 2x$$

Q.3

Sol^n

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{2x} \quad (A)$$

Q.3 :- $\frac{d^2y}{dx^2} + 4y = \tan 2x$

Sol^n →

$(D^2 - 3D + 2)y = e^{2x}$

$M^2 - 3M + 2 = 0$

$m = 2, 1$

$C.f. = C_1 e^{2x} + C_2 e^x$

$y = C_1 e^{2x} + C_2 e^x$

$y = A e^{2x} + B e^x \quad (1)$

$\frac{dy}{dx} = A e^{2x} \cdot 2 + e^{2x}\frac{dA}{dx} + Be^x + e^x\frac{dB}{dx}$

$e^{2x}\frac{dA}{dx} + e^x\frac{dB}{dx} \quad (2)$

$\frac{dy}{dx} = 2Ae^{2x} + Be^x \quad (3)$

$\frac{d^2y}{dx^2} = 4Ae^{2x} + 2e^{2x}\frac{dA}{dx} + Be^x + e^x\frac{dB}{dx} \quad (4)$

Put value of (1), (3) & (4) in (A)

$4Ae^{2x} + e^{2x}\frac{dA}{dx} + Be^x + e^x\frac{dB}{dx} - 3(2Ae^{2x} + Be^x) + 2(Ae^{2x} + Be^x) = e^{2x}$

$2e^{2x}\frac{dA}{dx} + e^x\frac{dB}{dx} = e^{2x}$

$e^{2x}\frac{dA}{dx} + e^x\frac{dB}{dx} = 0$

$A = 0, \quad B = 0$

$y = 0 \leftarrow e^{2x}\frac{dA}{dx} = e^{2x}$

$\frac{dA}{dx} = 1$

$\int dA = \int 1\,dx$

$A = x$

$2e^{2x}\frac{dA}{dx} + e^x\frac{dB}{dx} = e^{2x} \quad \times 1$

$e^{2x}\frac{dA}{dx} + e^x\frac{dB}{dx} = 0 \quad \times 2$

$2e^{2x}\frac{dA}{dx} + e^x\frac{dB}{dx} = e^{2x}$

$2e^{2x}\frac{dA}{dx} + 2e^x\frac{dB}{dx} = 0$

$-e^x\frac{dB}{dx} = e^{2x}$

$\frac{dB}{dx} = -e^x$

$\int dB = -\int e^x dx$

$B = -e^x$

from (1)

$y = xe^{2x} + (-e^x)(e^x)$

$y = [xe^{2x} - e^{2x}]$

$y = e^{2x}[x-1]$ Ans

~~Note~~ 3 Non-zero term 02/05/25

order - 2
~~Sere~~ Constant = 2

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Series Sol^n of differential Eq^n

$$\frac{d^2y}{dx^2} - y = 0$$

$$P_0\frac{d^2y}{dx^2} + P_1\frac{dy}{dx} + P_2 y = 0$$

where P_0, P_1 & P_2 are func^n of x

$$(2x - x^2)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 2y = 3$$

$P_0 = 2x - x^2$

$x = 0$ $P_0 = 0$

$x = 0$ is called Singular point

$$y = a_0x^m + a_1x^{m+1} + a_2x^{m+2} + a_3x^{m+3} + \ldots$$

$$y = x^m\{a_0 + a_1x + a_2x^2 + \ldots\}$$

$$y = x^m\sum_{K=0}^{\infty} a_K x^K$$

$$y = \sum_{K=0}^{\infty} a_K x^{m+K}$$

$$(x^2+1)\frac{d^2y}{dx^2} + 3x^2\frac{dy}{dx} + y = 0$$

$P_0 = x^2 + 1$

$x = 0$

$P_0 = 1$

~~let~~ $x = 0$ is called ordinary point

let Sol^n is

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \ldots$$

$$y = \sum_{K=0}^{\infty} a_K x^K$$

$a_0 \neq 0$

नहीं होना है

$\frac{dA}{dx} = \frac{-vB}{\omega}$ $\frac{dB}{dx} = \frac{uR}{\omega}$

Printed

find Series Soln of ($1-x^2$)

$$(1-x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = 0 \quad \text{— (A)}$$

$P_0 = 1 - x^2$

$x = 0$ $(P_0 = 1)$

$x = 0$ is ordinary

Point let Soln of (A) is

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots \quad (1)$$

$$\frac{dy}{dx} = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots \quad (2)$$

$$\frac{d^2y}{dx^2} = 2a_2 + 3.2a_3 x + 4\,3a_4 x^2 + \cdots \quad (3)$$

Put (1), (2), (3) in (A)

$$(1-x^2)\{2a_2 + 3.2a_3 x + 4.3a_4 x^2 + \cdots\} + 2x\{a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots\} + \{a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots\} = 0x + 0x^2 + 0x^3 + 0x^4$$

on equating Constant :

$$2a_2 + a_0 = 0 \Rightarrow a_2 = \frac{-a_0}{2}$$

on equating Coefficient of 'x'

$3.2a_3 + 2a_1 + a_1 = 0$

$3\,2a_3 + 3a_1 = 0$

$$a_3 = \frac{-a_1}{2}$$

on equating Coefficient of x^2

$4.3a_4 - 2a_2 + 2.2a_2 + a_2 = 0$

$4.3a_4 - 3a_2 = 0$

$a_4 = \frac{a_2}{4}$

$$a_4 = \frac{-a_0}{2.4}$$

Put in (1)

$$y = a_0 + a_1 x - \frac{a_0}{2}x^2 - \frac{a_1}{2}x^3 - \frac{a_0}{2.4}x^4 + \cdots$$

$C_1 \to C_1 x + C_2$

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Printed

find Series Sol^n $\frac{d^2y}{dx^2} - y = 0$

$\frac{d^2y}{dx^2} - y = 0$ — A

$P_0 = 1$, at $x=0$, $P_0 = 1 \neq 0$

$x=0$ is called ordinary point

Let Sol^n of (A) is

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots \quad (1)$$

$$\frac{dy}{dx} = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots \quad (2)$$

$$\frac{d^2y}{dx^2} = 2a_2 + 3.2a_3 x + 4.3a_4 x^2 + \cdots \quad (3)$$

Put $y, \frac{dy}{dx}, \frac{d^2y}{dx^2}$ from (1), (2), (3) in (A)

$$\{2a_2 + 3.2a_3 x + 4.3a_4 x^2 + \cdots\} - \{a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots\} = 0$$

on equating Constant

$$2a_2 - a_0 = 0$$

$$a_2 = \frac{a_0}{2}$$

on equating coefficient of 'x'

$$3.2a_3 - a_1 = 0 \Rightarrow a_3 = \frac{a_1}{3.2}$$

on equating Coefficient of 'x^2'

$$4.3a_4 - a_2 = 0 \Rightarrow a_4 = \frac{a_2}{4.3} \Rightarrow a_4 = \frac{a_0}{2.3.4}$$

on Equating Coefficient of 'x^3'

$$5.4a_5 - a_3 = 0 \quad a_5 = \frac{a_3}{5.4} \Rightarrow a_5 = \frac{a_1}{2.3.4.5}$$

put in (1)

$$y = a_0 + a_1 x + \frac{a_0}{2}x^2 + \frac{a_1}{3.2}x^3 + \frac{a_0}{4.3.2}x^4 + \frac{a_1}{2.3.4.5}x^5 + \cdots$$

Ans

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find series sol^n of

$$2x^2\frac{d^2y}{dx^2}+(2x^2-x)\frac{dy}{dx}+y=0$$

$P_0=2x^2$, $P_1=(2x^2-x)$, $P_2=1$

at $x=0$, $P_0=0$

$x=0$ is singular point

let sol^n is

$$y=\sum_{k=0}^{\infty}a_k x^{m+k} \quad (1)$$

$$\frac{dy}{dx}=\sum_{k=0}^{\infty}(m+k)a_k x^{m+k-1} \quad (2)$$

$$\frac{d^2y}{dx^2}=\sum_{k=0}^{\infty}(m+k)(m+k-1)a_k x^{m+k-2} \quad (3)$$

Put $y, \frac{dy}{dx}, \frac{d^2y}{dx^2}$ from (1) (2) (3) in (A)

$$\cancel{2x^2\sum_{k=0}^{\infty}(m+k)a_k(m+k-1)x^{m+}}$$

$$2x^2\sum_{k=0}^{\infty}(m+k)(m+k-1)a_k x^{m+k-2}+(2x^2-x)\sum_{k=0}^{\infty}(m+k)a_k x^{m+k-1}$$
$$+\sum_{k=0}^{\infty}a_k x^{m+k}=0$$

$$2\sum_{k=0}^{\infty}(m+k)(m+k-1)a_k x^{m+k}+2\sum_{k=0}^{\infty}(m+k)a_k x^{m+k+1}$$
$$-\sum_{k=0}^{\infty}(m+k)a_k x^{m+k}$$
$$+\sum_{k=0}^{\infty}a_k x^{m+k}=0$$

$$\sum_{k=0}^{\infty}a_k\{2(m+k)(m+k-1)-(m+k)+1\}x^{m+k}$$
$$+2\sum_{k=0}^{\infty}(m+k)a_k x^{m+k+1}$$

whole No differ x^n $\left(\frac{d}{dx}x^n\right)$ flew so m.

$$\sum_{K=0}^{\infty} a_K\{2(m+K)^2-2(m+K)-(m+K)+1\}x^{m+K} + 2\sum_{K=0}^{\infty} a_K(m+K)x^{m+K+1}=0$$

$$\sum_{K=0}^{\infty} a_K\{2(m+K)^2-3(m+K)+1\}x^{m+K} + 2\sum_{K=0}^{\infty} a_K(m+K)x^{m+K+1}=0 \quad \text{(4)}$$

on equating Coefficient of lowest degree term i.e x^m which is obtained by Putting $K=0$ in $I^{st}\ \Sigma$ only

$$a_0\{2m^2-3m+1\}=0$$

$\Rightarrow$ $a_0 \neq 0$ or $2m^2-3m+1=0 \rightarrow$ (indicial eq^n)

$m=\frac{1}{2}, 1$

on equating Coefficient x^{m+1} which obtained by putting $K=1$ in $I^{st}\ \Sigma$ and $K=0$ in $II^{st}\ \Sigma$

$$a_1\{2(m+1)^2-3(m+1)+1\}+2a_0 m=0$$

$$a_1=\frac{-2a_0 m}{2(m+1)^2-3(m+1)+1}$$

on Equating Coefficien of x^{m+2} which is obtained by putting $K=2$ in $I^{st}\ \Sigma$ and $K=1$ in $II^{st}\ \Sigma$

$$a_2\{2(m+2)^2-3(m+2)+1\}+2a_1(m+1)=0$$

$$a_2=\frac{-2a_1(m+1)}{2(m+2)^2-3(m+2)+1}$$

$$a_2 = \frac{-2(m+1)\ (-2a_0 m)}{\{2(m+2)^2-3(m+2)+1\}\{2(m+1)^2-3(m+1)+1\}}$$

$$a_2 = \frac{2.2m(m+1)a_0}{\{2(m+2)^2-3(m+2)+1\}\{2(m+1)^2-3(m+1)+1\}}$$

Put a_0, a_1, a_2 - - ——

In $y = \sum_{k=0}^{\infty} a_k x^{m+k}$ —(1)

$$y = a_0 x^m + a_1 x^{m+1} + a_2 x^{m+2} + - - ——$$

$$y = x^m\left\{a_0 \frac{-2a_0 m}{\{2(m+1)^2-3(m+1)+1\}} x + \frac{2.2m(m+1)a_0}{\{2(m+2)^2-3(m+2)+1\}\{2(m+1)^2-3(m+1)+1\}} x^2 + \right.$$

$m = \frac{1}{2}, 1$

$$y = x\left\{a_0 - \frac{2a_0}{\{2.4-3(2)+1\}} x + \frac{2.2(2)a_0}{\{2.9-3.3+1\}\{2.4-3(2)+1\}} x^2 + \cdots\right\}$$

$$+ x^{1/2}\left\{a_0 \frac{-2a_0 1/2}{\{2(\frac{1}{2}+1)^2-3(\frac{1}{2}+1)+1\}} x + \frac{2.2(\frac{1}{2})(1/2+1)a_0}{\{2(1/2+2)^2-3(\frac{1}{2}+2)+1\}\{2(\frac{1}{2}+1)^2-3(\frac{1}{2}+1)+1\}} x^2 + - -\right.$$

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find series sol^n of differential eq^n

$(x-x^2)\frac{d^2y}{dx^2} + (1-5x)\frac{dy}{dx} - 4y = 0$

$P_0 = (x-x^2) \quad P_1 = (1-5x) \quad P_2 = -4$

at $x=0$

$P_0 = 0$

$x=0$ is called singular point

$$y = \sum_{K=0}^{\infty} a_K x^{m+K} \quad (1)$$

$$\frac{dy}{dx} = \sum_{K=0}^{\infty} a_K (m+K) x^{m+K-1} \quad (2)$$

$$\frac{d^2y}{dx^2} = \sum_{K=0}^{\infty} a_K (m+K)(m+K-1) x^{m+K-2} \quad (3)$$

Put $y, \frac{dy}{dx}, \frac{d^2y}{dx}$ from (1),(2),(3) in (A)

$$(x-x^2)\sum_{K=0}^{\infty} a_K (m+K)(m+K-1) x^{m+K-2} + (1-5x)\sum_{K=0}^{\infty} a_K (m+K) x^{m+K-1} - 4\sum_{K=0}^{\infty} a_K x^{m+K} = 0$$

$$\sum_{K=0}^{\infty} a_K (m+K)(m+K-1) x^{m+K-1} - \sum_{K=0}^{\infty} a_K (m+K)(m+K-1) x^{m+K}$$

$$+ \sum_{K=0}^{\infty} a_K (m+K) x^{m+K-1} - 5\sum_{K=0}^{\infty} a_K (m+K) x^{m+K} - 4\sum_{K=0}^{\infty} a_K x^{m+K} = 0$$

$$\sum_{K=0}^{\infty} a_K \{(m+K)(m+K-1) + (m+K)\} x^{m+K-1} - \sum_{K=0}^{\infty} a_K \{(m+K)(m+K-1) + 5(m+K) + 4\} x^{m+K} = 0$$

$$\sum_{K=0} a_K \{(m+K)^2 - (m+K) + (m+K)\} x^{m+K-1} - \sum_{K=0}^{\infty} a_K \{(m+K)^2 - (m+K) + 5(m+K) + 4\} x^{m+K} = 0$$

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$$\sum_{K=0}^{\infty} a_K \{(m+K)^2\} x^{m+K-1} - \sum_{K=0}^{\infty} a_K \{(m+K)^2 + 4(m+K) + 4\} x^{m+K} = 0 \quad \text{---(4)}$$

on equating Lowest degree term i.e x^{m-1} which is obtained by putting $K=0$ in I^{st} Σ only

$a_0 m^2 = 0$ but $a_0 \neq 0 \Rightarrow m = 0, 0$

on Equating coefficient of x^m which is obtained by Putting $K=1$ in I^{st} Σ and $K=0$ in II^{nd} Σ

$\Rightarrow a_1(m+1)^2 - a_0\{m^2 + 4m + 4\} = 0$

$$a_1 = \frac{a_0 (m+2)^2}{(m+1)^2}$$

on Equating coefficient of x^{m+1} which is obtained by Putting $K=2$ in I^{st} Σ and $K=1$ in II^{nd} Σ

$$\cancel{a_2}\; a_2(m+2)^2 - a_1\{(m+1)^2+4(m+1)+4\} = 0$$

$$a_2 = \frac{a_1\{(m+1)^2+4(m+1)+4\}}{(m+2)^2}$$

$$a_2 = \frac{\{(m+1)^2+4(m+1)+4\}}{\cancel{(m+2)^2}}\,\frac{a_0\,\cancel{(m+2)^2}}{(m+1)^2}$$

$$a_2 = \frac{\{(m+1)^2+4(m+1)+4\}a_0}{(m+1)^2}$$

put a_0, a_1, a_2 ~~&~~ + - - -

in $y = \sum_{K=0}^{\infty} a_K x^{m+K}$ — (1)

$$y = a_0x^m + a_1x^{m+1} + a_2x^{m+2} + - - -$$

$m = 0$

$$y = x^m\left\{a_0 + \left\{\frac{a_0(m+2)^2}{(m+1)^2}\right\}x + \left\{\frac{(m+1)^2+4(m+1)+4}{(m+1)^2}\right\}a_0x^2 + - -\right.$$

$$y_{m=0} = x^0\left\{a_0 + \left\{\frac{a_0(0+2)^2}{(0+1)^2}\right\}x + \left\{\frac{(0+1)^2+4(0+1)+4}{(0+1)^2}\right\}a_0x^2 + - -\right.$$

$$y_{m=0} = \{a_0 + 4a_0x\} + \{1+4+4\}a_0x^2 + - \}$$

$$y_{m=0} = [a_0\{1+4x\} + 9a_0x^2 + - \}$$

$$y_{m=0} = \{a_0\{1+4x+9x^2\} + - \}$$

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Bessel's differential Eqn & its Series Solⁿ

Pointed Solⁿ

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0 \quad (A)$$ it is called Bessel's diffe. Eqn of n^{th} order

$P_0 = x^2$,
at,

$P_0 = x^2$ at $x=0 \Rightarrow P_0=0$ $x=0$ is singular point

let Solⁿ of A is

$$y=\sum_{k=0}^{\infty} a_k x^{m+k} \quad -(1)$$

$$\frac{dy}{dx}=\sum_{k=0}^{\infty} a_k (m+k) x^{m+k-1} \quad -(2)$$

$$\frac{d^2y}{dx^2}=\sum_{k=0}^{\infty} a_k (m+k)(m+k-1) x^{m+k-2} \quad -(3)$$

Put (1), (2), (3) in (A)

$$x^2\sum_{k=0}^{\infty} a_k (m+k)(m+k-1)x^{m+k-2}+x\sum_{k=0}^{\infty} a_k (m+k) x^{m+k-1}$$
$$+(x^2-n^2)\sum_{k=0}^{\infty} a_k x^{m+k}=0$$

$$\sum_{k=0}^{\infty} a_k (m+k)(m+k-1)x^{m+k}+\sum_{k=0}^{\infty} a_k (m+k) x^{m+k}$$
$$+\sum_{k=0}^{\infty} a_k x^{m+k+2}-n^2\sum_{k=0}^{\infty} a_k x^{m+k}=0$$

$$\sum_{k=0}^{\infty} a_k \{(m+k)(m+k-1)+(m+k)-n^2\} x^{m+k}+\sum_{k=0}^{\infty} a_k x^{m+k+2}=0$$

$$\sum_{K=0}^{\infty} a_K\left\{(m+K)^2-(m+K)+(m+K)-n^2\right\}x^{m+K}+\sum_{K=0}^{\infty} a_K x^{m+K+2}=0$$

$$\sum_{K=0}^{\infty} a_K\left\{(m+K)^2-n^2\right\}x^{m+K}+\sum_{K=0}^{\infty} a_K x^{m+K+2}=0 \quad \text{--- (4)}$$

$\sum_{K=0}^{\infty} a_K(m+1)$' on Equating lowest degree term i.e. x^m which is obtained by Putting $K=0$ in $I^{st}\Sigma$ only

$$a_0\left\{m^2-n^2\right\}=0 \Rightarrow \text{ but } a_0\neq 0,\ m^2-n^2=0$$
$$m=\pm n$$

Now Equating Coefficient of x^{m+1} put $K=1$ in $I^{st}\Sigma$ only,

$$a_1\left\{\underbrace{(m+1)^2-n^2}_{\neq 0}\right\}=0$$
$$\Rightarrow a_1=0$$

Now Equating Coefficient of x^{m+2} put $K=2$ in $I^{st}\Sigma$ and $K=0$ in $2^{nd}\Sigma$

$$a_2\left\{(m+2)^2-n^2\right\}+a_0=0$$
$$a_2=\frac{-a_0}{\left\{(m+2)^2-n^2\right\}}$$

on Equating Coefficient of x^{m+3} $K=3$ in $I^{st}\Sigma$ and $K=1$ in $2^{nd}\Sigma$

$$a_3\left\{(m+3)^2-n^2\right\}+a_1=0$$
$$a_3=\frac{-a_1}{\left\{(m+3)^2-n^2\right\}}=a_3=0 \quad \{\because a_1=0\}$$

On Equating coefficient of x^{m+4} $K=2$ in II^{nd} Σ and $K=4$ in I^{st} Σ

$$a_4\{(m+4)^2-n^2\}+a_2=0$$

$$a_4=\frac{-a_2}{(m+4)^2-n^2} \Rightarrow a_4=\frac{a_0}{\{(m+2)^2-n^2\}\{(m+4)^2-n^2\}}$$

Put in $y=\sum_{K=0}^{\infty} a_K x^{m+K}$ — (1)

$$y=\{a_0x^m+a_1x^{m+1}+a_2x^{m+2}+a_3x^{m+3}+a_4x^{m+4}+\cdots\}$$

$$y=x^m\{a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+\cdots\}$$

$$y=x^m\left\{a_0+0x-\frac{a_0}{(m+2)^2-n^2}x^2+0x^3+\frac{a_0}{\{(m+2)^2-n^2\}\{(m+4)^2-n^2\}}x^4+\cdots\right\}$$

$m=n$
$m=-n$

$$\hookrightarrow y=x^n\left\{a_0-\frac{a_0}{(n+2)^2-n^2}x^2+\frac{a_0}{\{(n+2)^2-n^2\}\{(n+4)^2-n^2\}}x^4+\cdots\right\}$$

$$x^{-n}\left\{a_0-\frac{a_0}{(-n+2)^2-n^2}x^2+\frac{a_0}{\{(-n+2)^2-n^2\}\{(-n+4)^2-n^2\}}x^4+\cdots\right\}$$

$$C_1\, y_{m=n}=x^n\left[a_0-\frac{a_0}{((n+2)-n)(n+2+n)}x^2+\frac{a_0}{[((n+2)-n)(n+2+n)][(n+4-n)(n+4+n)]}x^4+\cdots\right]$$

$$C_2\, y_{m=-n}=x^{-n}\left[a_0-\frac{a_0}{(2-n-n)(2-n+n)}x^2+\frac{a_0}{[(2-n-n)(2-n+n)][(4-n-n)(4-n+n)]}x^4+\cdots\right]$$

$$C_1\, y_{m=n}=x^n\left[a_0-\frac{a_0}{2\,2(n+1)}x^2+\frac{a_0}{[2\cdot 2(n+1)][4\cdot 2(n+2)]}x^4+\cdots\right].$$

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$$C_2 y_{m=-n} = x^{-n}\left[a_0 - \frac{a_0}{2.2(1-n)}x^2 + \frac{a_0}{[2.2(1-n)][4.2(2-n)]}x^4 + \cdots\right]$$

$$\therefore C_1 y_{m=n} + C_2 y_{m=-n} = y$$

$$y = C_1\left[x^n\left\{a_0 - \frac{a_0}{4(n+1)}x^2 + \frac{a_0}{4(n+1)\,8(n+2)}x^4 + \cdots\right\}\right.$$

$$\left. + C_2\left[x^{-n}\left\{a_0 - \frac{a_0}{4(1-n)}x^2 + \frac{a_0}{[4(1-n)\,8(2-n)}x^4 + \cdots\right]\right]\right.$$

Ans

$+\cdots\}$

$\}$

$]$

x^{m-K} x^{m-1K} 08/05/25

$P_0 = 1$

$P_0 = 0$ होगा

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legendre

legender differential EQn

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0 \quad \text{—(A)}$$

It is called n^{th} order of legender differential Eqn.

let Soln of (A) is

$$y = \sum_{K=0}^{\infty} a_K x^{m-K} \quad \text{—(1)}$$

$$\frac{dy}{dx} = \sum_{K=0}^{\infty} a_K(m-K)x^{m-K-1} \quad \text{—(2)}$$

$$\frac{d^2y}{dx^2} = \sum_{K=0}^{\infty} a_K(m-K)(m-K-1)x^{m-K-2} \quad \text{—(3)}$$

Put $y, \frac{dy}{dx}, \frac{d^2y}{dx^2}$ from (1) (2) (3) in (A)

$$(1-x^2)\left\{\sum_{K=0}^{\infty} a_K(m-K)(m-K-1)x^{m-K-2}\right\} - 2x\left\{\sum_{K=0}^{\infty} a_K(m-K)x^{m-K-1}\right.$$

$$+ n(n+1)y\left\{\sum_{K=0}^{\infty} a_K x^{m-K}\right\}.$$

$$\sum_{K=0}^{\infty} a_K(m-K)(m-K-1)x^{m-K-2} - \sum_{K=0}^{\infty} a_K(m-K)(m-K-1)x^{m-K}$$

$$-2\left\{\sum_{K=0}^{\infty} a_K(m-K)x^{m-K}\right. + n(n+1)\left\{\sum_{K=0}^{\infty} a_K x^{m-K}\right\}$$

$$\sum_{K=0}^{\infty} a_K\{m-K\}\{m-K-1\}x^{m-K-2}$$

$$-\sum_{K=0}^{\infty} a_K\left\{(m-K)(m-K-1) + 2(m-K) - n(n+1)\right\}x^{m-K}$$

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$$\sum_{k=0}^{\infty} a_k \{(m-k)^2-(m-k)\} x^{m-k-2}$$

$$- \sum_{k=0}^{\infty} a_k \{(m-k)^2-(m-k)+2(m-k)-n(n+1)\} x^{m-k}$$

$$\sum_{k=0}^{\infty} a_k \{(m-k)^2-(m-k)\} x^{m-k-2} - \sum_{k=0}^{\infty} a_k \{(m-k)^2+(m-k)-n(n+1)\} x^{m-k}$$

on Equating coefficient of highest degree term i.e. x^m which is obtained by putting $k=0$ in II^{nd} Σ only

$$a_0\{\cancel{(m-k)^2} m^2+m-n^2-n\}=0$$

but $a_0 \neq 0$

So

$$m^2+m-n^2-n=0$$

$$(m^2-n^2)+(m-n)=0$$

$$(m+n)(m-n)+(m-n)=0$$

$$(m+n+1)(m-n)=0$$

$$m=-(n+1)$$

$$m=n$$

on Equating coefficient of x^{m-1} which is obtained by putting $k=1$ in $\mathrm{II}\ \Sigma$ only

$$-a_1\{(m-1)^2+(m-1)-n(n+1)\}=0$$

$$\Rightarrow a_1=0$$

on Equating coefficient of x^{m-2} which is obtained by putting $k=0$ in $\mathrm{I}^{st}\ \Sigma$ and $k=2$

$$a_0\{m^2-m\}-a_2\{(m-2)^2+(m-2)-n(n+1)\}=0$$

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$$\boxed{a_2 = \frac{a_0 m(m-1)}{(m-2)^2+(m-2)-n(n+1)}}$$

on Equating coefficient of x^{m-3} which is obtained by Putting $K=1$ in $I^{st}\ \Sigma$ and $K=3$ in $II^{nd}\ \Sigma$

$$a_1\{(m-1)(m-2)\} - a_3\{(m-3)^2+(m-3)-n(n+1)\}=0$$

$$a_3 = \frac{a_1(m-1)(m-2)}{(m-3)^2+(m-3)-n(n+1)} = 0 \quad (\because a_1 = 0)$$

$$\boxed{a_3 = 0}$$

on Equating coefficient of x^{m-4} which is obtained by Putting $K=2$ in $I^{st}\ \Sigma$ and $K=4$ in $II^{nd}\ \Sigma$

$$a_2\{(m-2)(m-2-1)\} - a_4\{(m-4)^2+(m-4)-n(n+1)\}=0$$

$$a_4 = \frac{a_2(m-2)(m-3)}{\{(m-4)^2+(m-4)-n(n+1)}$$

$$a_4 = \frac{a_0 m(m-1)(m-2)(m-3)}{\{(m-2)^2+(m-2)-n(n+1)\}\{(m-4)^2+(m-4)-n(n+1)\}}$$

Put in $y = \sum_{K=0}^{\infty} a_K x^{m-K}$

$$y = a_0 x^m + a_1 x^{m-1} + a_2 x^{m-2} + a_3 x^{m-3} + a_4 x^{m-4} + \dots$$

$$y = x^m\{a_0 + a_1 x^{-1} + a_2 x^{-2} + a_3 x^{-3} + a_4 x^{-4} + \dots\}$$

$$C_1 y_{m=n} + C_2 y_{m=-(n+1)} = y$$ final answer

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$$y = x^m \left[a_0 + 0x^{-1} + \frac{a_0 m(m-1)}{(m-2)^2 + (m-2) - n(n+1)} x^{-2} + 0x^{-3} + \frac{a_0 m(m-1)(m-2)(m-3)}{\{(m-2)^2 + (m-2) - n(n+1)\}\{(m-4)^2 + (m-4) - n(n+1)\}} x^{-4} + \right]$$

$$y = x^m \left[a_0 + \frac{a_0 m(m-1)}{(m-2)^2 + (m-2) - n(n+1)} x^{-2} + \frac{a_0 m(m-1)(m-2)(m-3)}{\{(m-2)^2 + (m-2) - n(n+1)\}\{(m-4)^2 + (m-4) - n(n+1)\}} x^{-4} \right]$$

$m = -(n+1)$

$m = n$

$$y_{m=n} = x^n \left[a_0 + \frac{a_0 n(n-1)}{(n-2)^2 + (n-2) - n^2 - n} x^{-2} + \frac{a_0 n(n-1)(n-2)(n-3)}{\{(n-2)^2 + (n-2) - n^2 - n\}\{(n-4)^2 + (n-4) - n^2 - n\}} x^{-4} \right]$$

$$y_{m=n} = x^n \left[a_0 + \frac{a_0 (n^2 - n)}{n^2 + 4 - 4n + n - 2 - n^2 - n} x^{-2} + \frac{a_0 (n^2 - n)(n-2)(n-3)}{\{n^2 + 4 - 4n + n - 2 - n^2 - n\}\{n^2 + 16 - 8n + n - 4 - n^2 - n\}} x^{-4} \right]$$

$$y_{m=n} = x^n \left[a_0 + \frac{a_0 (n^2 - n)}{-4n + 2} x^{-2} + \frac{a_0 (n^2 - n)(n-2)(n-3)}{\{-4n + 2\}\{-8n + 12\}} x^{-4} \right]$$

$$\cancel{y_{m=n} = x^n a_0 [\;a_0 + a_0}$$

$$y_{m=n} = x^n a_0 \left[1 + \frac{(n^2 - n)}{2 - 4n} x^{-2} + \frac{(n^2 - n)(n-2)(n-3)}{(2 - 4n)(12 - 8n)} x^{-4} + \right]$$

$$y_{m=-(n+1)} = x^{-(n+1)} a_0 \left[1 + \frac{-(n+1)(-(n+1) - 1)}{-((n+1)+2)^2 + (-(n+1) - 2) - n(n+1)} x^{-2} + \frac{-(n+1)(-(n+1) - 1)(-(n+1) - 2)(-(n+1) - 3)}{\{-(n+1+2)^2 + (-(n+1) - 2) - n(n+1)\}\{(-(n+1) - 4)^2 + (-(n+1) - 4) - n(n+1)} x^{-4} \right]$$

$$y_{m=-(n+1)} = x^{-(n+1)} a_0 \left[1 + \frac{(-n-1)(-n-1-1)}{-(n+3)^2 - (n+3) - n^2 - n} x^{-2} + \frac{-(n+1) \cdot -(n+2) \cdot -(n+3) \cdot -(n+4)}{[-(n+3)^2 - (n+3) - n^2 - n][-(n+5)^2 - (n+5) - n^2 - n]} x^{-4} \right]$$

$$y_{m=-(n+1)} = x^{-(n+1)} a_0 \left[1 + \frac{(n+1)(n+2)}{-(n+3)^2 - (n+3) - n(n+1)} x^{-2} + \frac{(n+1)(n+2)(n+3)(n+4)}{[-(n+3)^2 - (n+3) - n(n+1)][-(n+5)^2 - (n+5) - n(n+1)]} x^{-4} \right]$$