Date: / / Page no:

10/MAR/25 MONDAY

"SYLLABUS" MATHEMATICS – II

UNIT-I ORDINARY DIFFERENTIAL EQ^n – (I)

Topics
⇒ Differential Eq^n of 1st order & 1st Degree
1.1 Linear Diff. Eq^n. 1.2 Bernoullis Eq^n
1.3 Exact Diff. Eq^n
⇒ Differential Eq^n with constant Coefficient
⇒ Homogeneous linear Diff^n Eq^n
⇒ Simultaneous linear Diff. Eq^n.
⇒ Eq^n Solvable for P, X, and Y

UNIT-II ORDINARY DIFFERENTIAL Eq^n – (II)

Topics ⇒ 2nd order linear Diff. Eq^n with variable Coefficient
⇒ Bessel's function.
⇒ Legendres Polynomial
⇒ Power Series
⇒ Frobenius Method of Series Sol^n
⇒ Variation of Parameters.

UNIT-III PARTIAL DIFFERENTIAL Eq^n

Topics ⇒ Formation of Partial Differential Eq^n
1.1 By eliminating the ~~arbi~~ orbitary Constant.
1.2 By eliminating orbitary function

Methods
1. Lagrange's Methods
2. Non-Linear P.D. Eq^n
3. Charpits Method.
4. Homogeneous linear P.D. Eq^n
5. Non-Homogeneous linear P.D. Eq^n
6. Reducable to P.D Eq^n

UNIT-IV (DIFFERENTIATION)

Topics
1. Funct^n of Complex variables
2. Cauchy's theorem
3. Residue theorem

UNIT-V (VECTOR CALCULAS)

Part 1 (Differentiation)
Part 2 (Integration)

UNIT-I ordinary differential Eq^n — (I)

Topic → linear Diff. Eq^n of 1^{st} order

11/MAR/25

order = 1
Degree/Power = 1

$$\frac{dy}{dx} + 2x = e^x$$

LINEAR EQ^n

order = 2
Degree/Power = 1
Linear Eq^n

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin x$$

$$\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 2y = \cos x$$

L. Eq^n
order = 3
Degree/Power = 1

$$\left(\frac{d^2y}{dx^2}\right)^3 + 4\left(\frac{dy}{dx}\right) + 2y = \sin x$$

Non-L. Eq^n order = 2
D/P = 3

Higher order linear Differential Eq^n with Constant Co-efficient

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin x$$

$$\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 2y = \cos x$$

x - linear
$x = CF + PI$
t - linear
$t = CF + PI$

Its Solution is $y = CF + PI$

where, CF is Complimentry function
& PI is called Particular Intigrel.

Standard form

For x - linear

$$\boxed{\frac{dx}{dy} + Px + Q}$$

for y - linear

$$\boxed{\frac{dy}{dx} + Py = Q}$$

I.f = {Integration factor}

$$I.f = e^{\int P dy}$$

$$I.f = e^{\int P dx}$$

Complete Solution

$$\boxed{x\, I.f = \int (Q . I.f)\, dy + C}$$

$$\boxed{y . I.f = \int (Q . I.f)\, dx + C}$$

11/MAR/25

Date: / / Page no:

Q.1 $\Rightarrow \frac{dy}{dx} + 2y = e^x$

$\frac{dy}{dx} + Py = Q$

$\{P = 2 \;, Q = e^x\}$

$I.f = e^{\int P dx}$

$I.f = e^{\int 2 dx} = e^{2x}$

$y . I.f = \int (Q . I.f) dx + c$

$y . e^{2x} = \int (e^x . e^{2x}) dx + c$

$y . e^{2x} = \int e^{3x} dx + c$

$y . e^{2x} = \frac{e^{3x}}{3} + c$

$y = \frac{e^{3x}}{3 . e^{2x}} + c$

$y = \frac{e^x}{3} + c$ Ans

Q.2 $\Rightarrow \frac{dy}{dx} + xy = \sin x$

$\frac{dy}{dx} + Py = Q$

$\{P = x \;, Q = \sin x\}$

$I.f = e^{\int P dx}$

$I.f = e^{\int x dx} = e^{x^2/2}$

$y . I.f = \int (Q . I.f) dx + c$

$y . e^{x^2/2} = \int (\underset{II}{\sin x} \; \underset{I}{e^{x^2/2}}) dx + c$

$y . e^{x^2/2} = -e^{x^2/2} \cos x + \int e^{x^2/2} \frac{2x}{2} \cos x \, dx + c$

$y . e^{x^2/2} = -e^{x^2/2} \cos x + \int x e^{x^2/2} \cos x \, dx + c$

$y = \frac{-e^{x^2/2} \cos x}{e^{x^2/2}} + \frac{\int x e^{x^2/2} \cos x \, dx + c}{e^{x^2/2}}$

$y = -\cos x + e^{-x^2/2} \int x e^{x^2/2} \cos x \, dx + c$

Q

Date: / / Page no:

12/MAR/25

Q1 ⇒ $x\frac{dy}{dx} - 2y = x^2$

Divide x on Both side

$\frac{dy}{dx} - \frac{2y}{x} = \frac{x^2}{x}$

$\frac{dy}{dx} + Py = Q$

$\left\{ P = -\frac{2}{x},\ Q = x \right\}$

$I.f = e^{\int -\frac{2}{x} dx} = e^{-2\int \frac{1}{x} dx}$

$I.f = e^{-2\log x} = e^{\log x^{-2}}$

$e^{\log A} = A$

$I.f = x^{-2} = \frac{1}{x^2}$

$y.\ I.f = \int (Q.\ I.f)\, dx + c$

$y.\frac{1}{x^2} = \int \left(x \frac{1}{x^2}\right) dx + c$

$y\frac{1}{x^2} = \int \frac{1}{x}\, dx + c$

$y.\frac{1}{x^2} = \log x + c$

$y = x^2 \log x + cx^2$ Ans

Q2 $(1+y^2)dx = (\tan^{-1}y - x)\,dy$

$\frac{dx}{dy} = \frac{(\tan^{-1}y - x)}{(1+y^2)}$

$\frac{dx}{dy} = \frac{\tan^{-1}y}{(1+y^2)} - \frac{x}{(1+y^2)}$

$\frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{\tan^{-1}y}{(1+y^2)}$

$\frac{dx}{dy} + Px = Q$

$P = \frac{1}{(1+y^2)},\ Q = \frac{\tan^{-1}y}{1+y^2}$

$I.f. = e^{\int \frac{1}{1+y^2} dy}$

$I.f. = e^{\tan^{-1}y}$

$x.\ I.f = \int (Q.\ I.f.)\, dy + c$

$x.e^{\tan^{-1}y} = \int \left(\frac{\tan^{-1}y}{(1+y^2)} e^{\tan^{-1}y} \right) dy + c$

put ⇒ $\tan^{-1}y = t$

$\frac{1}{1+y^2} dy = dt$

$x.e^{\tan^{-1}y} = \int \left(\underset{u}{t}\ \underset{v}{e^t} \right) dt + c$

$xe^t = \left[te^t - \int 1 e^t dt \right] + c$

$x.e^t = \left[te^t - e^t \right] + c$

$x.\cancel{e^t} = \cancel{e^t}(t-1) + c$

$x = (t-1) + ce^{-t}$

$x = \tan^{-1}y - 1 + ce^{-\tan^{-1}y}$

12/MAR/25

Q.3 $(1+y^2)+(x-e^{-\tan^{-1}y})\frac{dy}{dx}=0$

$(x-e^{-\tan^{-1}y})\frac{dy}{dx}=-(1+y^2)$

$(x-e^{-\tan^{-1}y})=-(1+y^2)\frac{dx}{dy}$

$\frac{dx}{dy}=\frac{(x-e^{-\tan^{-1}y})}{-(1+y^2)}$

$\frac{dx}{dy}=\frac{x}{-(1+y^2)}+\frac{e^{-\tan^{-1}y}}{(1+y^2)}$

$\frac{dx}{dy}+\frac{1}{(1+y^2)}x=\frac{e^{-\tan^{-1}y}}{(1+y^2)}$

$\frac{dx}{dy}+Px=Q$

$P=\frac{1}{1+y^2}$, $Q=\frac{e^{-\tan^{-1}y}}{(1+y^2)}$

$I.f=e^{\int\frac{1}{1+y^2}dy}$

$I.f=e^{\tan^{-1}y}$

for complete Soln

$x.I.f=\int(Q.I.f)dy+c$

$x.e^{\tan^{-1}y}=\int\left(\frac{e^{-\tan^{-1}y}}{(1+y^2)}e^{\tan^{-1}y}\right)dy+c$

$xe^{\tan^{-1}y}=\int\frac{1}{(1+y^2)}dy+c$

$\boxed{x.e^{\tan^{-1}y}=\tan^{-1}y+c}$

Q.4 $\frac{dy}{dx}-y=e^{3x}$

$\frac{dy}{dx}+Py=Q$

$\{P=-1,\ Q=e^{3x}\}$

$I.f.=e^{\int Pdx}$

$I.f.=e^{\int-1dx}$

$I.f=e^{-\int dx}$

$I.f=e^{-x}=\frac{1}{e^x}$

$y.I.f.=\int(Q.I.f)dx+c$

$y.\frac{1}{e^x}=\int\left(e^{3x}\frac{1}{e^x}\right)dx+c$

$y.\frac{1}{e^x}=\int e^{2x}dx+c$

$y\frac{1}{e^x}=\frac{e^{2x}}{2}+c$

$\boxed{y=\frac{e^{3x}}{2}+ce^x}$ Ans

Q.5

Lebnitz's L.D. Eqn

$e \quad \int \frac{f'(x)}{f(x)} = \log f(x) + C$

12|MAR|25

Date: / / Page no:

Q.5 ⇒ $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$

Divide $(1+x^2)$ on both side

$$\frac{dy}{dx} + \frac{2xy}{(1+x^2)} = \frac{4x^2}{(1+x^2)}$$

In the form

$$\frac{dy}{dx} + Py = Q$$

$$\left\{ P = \frac{2x}{(1+x^2)}, \quad Q = \frac{4x^2}{(1+x^2)} \right\}$$

$$I.f = e^{\int P dx} = e^{\int \frac{2x}{(1+x^2)}}$$

$$I.f = e^{\log(1+x^2)}$$

$$(e^{\log A} = A)$$

$$I.f. = (1+x^2)$$

$$y.\, I.f = \int (Q.\, I.f)\, dx + C$$

$$y.\,(1+x^2) = \int \left(\frac{4x^2}{\cancel{(1+x^2)}} \cancel{(1+x^2)} \right) dx + C$$

$$y\,(1+x^2) = \int 4x^2\, dx + C$$

$$y\,(1+x^2) = \left(\frac{4x^3}{3} + C \right)$$

$$\boxed{y\,(1+x^2) = \frac{4x^3}{3} + C} \quad \underline{\text{Ans}}$$

Q.6 ⇒ $(1+x^2)\frac{dy}{dx} + 2xy = \cos x$

Divide $(1+x^2)$ on both side

$$\frac{dy}{dx} + \frac{2xy}{(1+x^2)} = \frac{\cos x}{(1+x^2)}$$

$$\frac{dy}{dx} + Py = Q$$

$$\left\{ P = \frac{2x}{(1+x^2)}, \quad Q = \frac{\cos x}{(1+x^2)} \right\}$$

$$I.f = e^{\int P dx} = e^{\int \frac{2x}{1+x^2} dx}$$

$$\int \frac{f'(x)}{f(x)} = \log f(x) + C$$

$$I.f = e^{\log(1+x^2)}$$

$$e^{\log A} = A$$

$$I.f = (1+x^2)$$

$$y.\, I.F = \int (Q.\, I.f)\, dx + C$$

$$y\,(1+x^2) = \int \frac{\cos x}{\cancel{(1+x^2)}} \cancel{(1+x^2)}\, dx + C$$

$$y\,(1+x^2) = \int \cos x\, dx + C$$

$$\boxed{y\,(1+x^2) = \sin x + C} \quad \underline{\text{Ans}}$$

21|Mar|25

Date: / / Page no:

Bernoullie's Equation

$\frac{dy}{dx} + Py = Qy^n$ (It is Bernoullie linear differential Eqn in Y - linear.)

for Solution Divide by (y^n)

$\frac{1}{y^n}\frac{dy}{dx} + \frac{P}{y^n}y = Q$

$\frac{1}{y^n}\frac{dy}{dx} + \frac{P}{y^{n-1}} = Q$

Put $\frac{1}{y^{n-1}} = t$ on diff. w.r.t. 'x'

$-y^{(n-1)-1} \frac{d}{}$

$-(n-1)\, y^{-(n-1)-1} \frac{dy}{dx} = \frac{dt}{dx}$

$-(n-1)\, y^{\{n+1-1\}} \frac{dy}{dx} = \frac{dt}{dx}$

$-(n-1)\, y^{-n} \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{1}{y^n} \cancel{y^{n}} \frac{dy}{dx} = \frac{1}{-(n-1)} \frac{dt}{dx}$

$\Rightarrow \frac{-1}{(n-1)}\frac{dt}{dx} + tP = Q$

multiply by $-(n-1)$

$\frac{dt}{dx} \underbrace{\boxed{-P(n-1)}}_{P'}\, t = \underbrace{\boxed{-Q(n-1)}}_{Q'}$ — (1)

Sol^n of (1) is

$t \times If = \int Q' \times If\, dx + C$

where $If = e^{\int P' dx}$

then Replace t by $\frac{1}{y^{n-1}}$

Q.1

$\frac{\log x}{x^2}$ ILATE $\frac{\log x}{(\log x)^2}$ A $\frac{\log x}{x^2}$

Date: / / Page no:

Q.1 :-

$$x\frac{dy}{dx} + y = y^2 \log_e x$$

Divide

$$\frac{dy}{dx} + \frac{1}{x}y = \frac{y^2 \log x}{x} \quad \text{—(1)}$$

Divide Eqn by y^2

$$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{x}\cdot\frac{1}{y} = \frac{\log_e x}{x} \quad \text{—(1)}$$

Put $\frac{1}{y} = t \Rightarrow \frac{-1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

$$\frac{1}{y^2}\frac{dy}{dx} = -\frac{dt}{dx} \quad \text{Put in (1)}$$

$$-\frac{dt}{dx} + \frac{1}{x}t = \frac{\log_e x}{x}$$

$$\frac{dt}{dx} - \frac{1}{x}t = \frac{-\log_e x}{x} \quad \text{—(2)}$$

Linear in t

$P = \frac{-1}{x}$ / $Q = \frac{-\log_e x}{x}$

$$I.f = e^{\int P\,dx} = e^{\int \frac{-1}{x}dx} = e^{-\log x} = e^{\log x^{-1}}$$

$$= x^{-1} = \frac{1}{x}$$

Sol^n of (2)

$$t \times I.f. = \int Q \cdot I.f + C$$

$$t \times \frac{1}{x} = \int \frac{1}{x}\cdot\frac{-\log_e x}{x}dx + C$$

$$t \times \frac{1}{x} = -\int \frac{\log x}{x^2}dx + C$$

$$\frac{t}{x} = -\int \Big(\underset{I}{\log x} \times \underset{II}{\frac{1}{x^2}}\Big)dx + C$$

$$\frac{t}{x} = -\left\{\log_e x \times \frac{x^{-2+1}}{-2+1} - \int \frac{1}{x}\frac{x^{-2+1}}{-2+1}dx\right\} + C$$

$$\frac{t}{x} = -\left\{\frac{\log_e x}{-x} - \int \frac{1}{x^2}dx\right\} + C$$

$$\frac{t}{x} = -\left\{\frac{\log_e x}{-x} + \int \frac{1}{x^2}dx\right\} + C$$

$$\frac{t}{x} = -\left\{\frac{\log_e x}{-x} + \frac{x^{-2+1}}{-2+1}\right\} + C$$

$$\frac{t}{x} = -\left\{\frac{\log_e x}{-x} + \frac{1}{-x}\right\} + C$$

$$\frac{t}{x} = \frac{\log_e x}{x} + \frac{1}{x} + C$$

$$\frac{1}{y} \times \frac{1}{x} = \frac{\log_e x}{x} + \frac{1}{x} + C$$

21/Mar/25

Date: / / Page no:

Q.1 $\frac{dy}{dx}(x^2y^3+xy)=1$

Sol^n ⟹

$\frac{dy}{dx}=\frac{1}{x^2y^3+xy}$

$\frac{dx}{dy}=x^2y^3+xy$

$\frac{dx}{dy}-xy=x^2y^3$ —(1)

Divide eqn (i) by x^2

$\frac{1}{x^2}\frac{dx}{dy}-\frac{1}{x}y=y^3$ —(2)

Put $\frac{1}{x}=t \Rightarrow \frac{-1}{x^2}\frac{dx}{dy}=\frac{dt}{dy}$

$=\frac{1}{x^2}\frac{dx}{dy}=-\frac{dt}{dy}$

Put in (ii)

$-\frac{dt}{dy}-ty=-y^3$

$\frac{dt}{dy}+ty=-y^3$

$\{P=y\ ,\ Q=-y^3\}$

$I.f.=e^{\int Pdy}=e^{\int y\,dy}=e^{y^2/2}$

Complete sol^n ⟹ for t-linear

$t\times I.f.=\int(Q.I.f.)dy+c$

$t\times e^{y^2/2}=\int(-y^3e^{y^2/2})dy+c$

$t.e^{y^2/2}=-\int(y^3e^{y^2/2})dy+c$

let, $\frac{y^2}{2}=u$

$y^2=2u$ $\quad$ $y\,dy=du$

$t.e^{y^2/2}=-\int(y.y^2e^{y^2/2})dy+c$

$t\,e^{u}=-\int \underset{I}{2u}\,\underset{II}{e^{u}}\,du+c$

$t\,e^{u}=-2\{ue^{u}-e^{u}\}+c$

$t\,e^{y^2/2}=-2\left\{\frac{y^2}{2}e^{y^2/2}-e^{y^2/2}\right\}+c$

$t\,e^{y^2/2}=-y^2e^{y^2/2}+2e^{y^2/2}+c$

$t=-y^2+2+ce^{-y^2/2}$

$\frac{1}{x}=-y^2+2+ce^{-y^2/2}$

$x=-y^2+2+ce^{-y^2/2}$

21/Mar/25

Q1 $\frac{dy}{dx}(x^2y^3+xy)=1$.

Sol^n → $\frac{dy}{dx}=\frac{1}{x^2y^3+xy}$

$\frac{dx}{dy}=x^2y^3+xy$

$\frac{dx}{dy}-xy=x^2y^3$ — (1)

Divide Eqn (1) by x^2

$\frac{1}{x^2}\frac{dx}{dy}-\frac{1}{x}y=y^3$ — (2)

Put $\frac{1}{x}=t \Rightarrow -\frac{1}{x^2}\frac{dx}{dy}=\frac{dt}{dy}$

Put in (2)

$-\frac{dt}{dy}-ty=y^3$

$\frac{dt}{dy}+ty=-y^3$.

$\{P=y,\ Q=-y^3\}$ I.f.

I.f. $=e^{\int P dy}=e^{\int y dy}=e^{y^2/2}$

Complete Sol^n for t- linear

$t.\text{I.f.}=\int(Q.\text{If})dy+C$

$t.e^{y^2/2}=\int(-y^3e^{y^2/2})dy+C$

$te^{y^2/2}=-\int(y^3e^{y^2/2})dy+C$

let, $\frac{y^2}{2}=u \Rightarrow y^2=2u$

$y\,dy=du$

$t.e^{y^2/2}=-\int(y.y^2e^{y^2/2})dy+C$

$t.e^u=-\int \underset{\text{I}}{2u}\,\underset{\text{II}}{e^u}\,du+C$

$t\,e^u=-2\{ue^u-e^u\}+C$

$te^{y^2/2}=-2\{\frac{y^2}{2}e^{y^2/2}-e^{y^2/2}\}+C$

$te^{y^2/2}=-y^2e^{y^2/2}+2e^{y^2/2}+C$

$t=-y^2+2+Ce^{-y^2/2}$

Replace the value of t

$\frac{1}{x}=-y^2+2+Ce^{-y^2/2}$

$x=\frac{1}{-y^2+2+Ce^{-y^2/2}}$ Ans

21/Mar/25

Date: / / Page no:

Q.2 $\Rightarrow \frac{dy}{dx} + (2x\tan^{-1}y - x^3)(1+y^2) = 0$

Divide By $(1+y^2)$

$$\frac{1}{1+y^2}\frac{dy}{dx} + (2x\tan^{-1}y - x^3) = 0$$

$$\frac{1}{1+y^2}\frac{dy}{dx} + 2x\tan^{-1}y = x^3 \quad (1)$$

Put $\tan^{-1}y = t$

on diff. w.r.t. x

$$\frac{1}{1+y^2}\frac{dy}{dx} = \frac{dt}{dx}$$

Put in (1)

$$\frac{dt}{dx} + 2xt = x^3 \qquad \text{I.f.}$$

$$\text{I.f.} = e^{\int P\,dx} = e^{\int 2x\,dx} = e^{x^2}$$

Complete Sol^n for t - linear

$$t.\text{I.f.} = \int (Q.\text{I.f.})\,dx + C$$

$$t\,e^{x^2} = \int (x^3 e^{x^2})\,dx + c$$

$$t\,e^{x^2} = \int (x.x^2 e^{x^2})\,dx + c$$

let $x^2 = u$

$2x\,dx = du$

$x\,dx = \frac{du}{2}$

$$t\,e^{x^2} = \frac{1}{2}\int u e^u\,du + c$$

$$t\,e^{x^2} = \frac{1}{2}\{ue^u - e^u\} + C$$

$$t\,e^{x^2} = \frac{1}{2}\{x^2 e^{x^2} - e^{x^2}\} + c$$

$$t\,e^{x^2} = \frac{1}{2}\{$$

$$t\,e^{x^2} = \left\{\frac{1}{2}x^2 e^{x^2} - \frac{1}{2}e^{x^2}\right\} + c$$

$$t = \frac{1}{2}x^2 - \frac{1}{2} + ce^{-x^2}$$

$$t = \frac{1}{2}\{x^2 - 1\} + ce^{-x^2}$$

$$\tan^{-1}y = \frac{1}{2}\{x^2 - 1\} + ce^{-x^2}$$

Ans

०१/मार्च/२५

Q.3 → $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$

$\sec y = \frac{1}{\cos y}$

$\tan y = \frac{\sin y}{\cos y}$

Multiply both side by $\cos y$

$\cos y \frac{dy}{dx} - \frac{\sin y}{1+x} = (1+x)e^x$

Let $\sin y = t$ on diff. w.r.t x

$\cos y \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - \frac{1}{1+x} t = (1+x)e^x$

$\frac{dt}{dx} + Pt = Q$

$\left\{ P = -\frac{1}{1+x}, \ Q = (1+x)e^x \right\}$

$I.f. = e^{\int P dx} = e^{\int -\frac{1}{1+x} dx} = e^{-\int \frac{1}{1+x} dx}$

$I.f. = e^{-\log(1+x)} = e^{\log(1+x)^{-1}}$

$e^{\log A} = A$

$I.f = (1+x)^{-1} = \frac{1}{(1+x)}$

Complete Solⁿ →

$t \times I.f. = \int (Q.I.f) dx + C$

$t \times \frac{1}{(1+x)} = \int \cancel{(1+x)} e^x \frac{1}{\cancel{(1+x)}} dx + C$

$\frac{t}{(1+x)} = \int e^x dx + C$

$\frac{t}{(1+x)} = e^x + C$

$t = (1+x)e^x + (1+x)C$

$\sin y = (1+x)e^x + (1+x)C$

Ans

21/Mar/25

Date: / / Page no:

Q.4 $\frac{dy}{dx} = 2y\tan x + y^2\tan^2 x$

Sol^n =>

$\frac{dy}{dx} - 2y\tan x = y^2\tan^2 x$

Divide Both side by y^2

$\cancel{\frac{1}{y^2}\frac{dy}{dx} - 2\tan x\frac{1}{y}}$

$\frac{1}{y^2}\frac{dy}{dx} - 2\tan x\frac{1}{y} = -\tan^2 x$ — (1)

Put $\frac{1}{y} = t$ on diff. w.r.t. x

$-\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{1}{y^2}\frac{dy}{dx} = -\frac{dt}{dx}$

Put in (1)

$-\frac{dt}{dx} - 2\tan x\, t = \tan^2 x$

$\frac{dt}{dx} + 2\tan x\, t = -\tan^2 x$

$\frac{dt}{dx} + Pt = Q$

$\{P = 2\tan x,\ Q = -\tan^2 x\}$

$I.f. = e^{\int 2\tan x\,dx} = e^{2\int\tan x}$

$I.f. = e^{2\log\sec x} = e^{\log\sec^2 x}$

$(e^{\log A} = A)$

$I.f. = \sec^2 x$

$t.I.f = \int (Q.I.f)\,dx + C$

$t\sec^2 x = \int -\tan^2 x\sec^2 x\,dx + C$

let $\tan x = u$

$\sec^2 x\,dx = du$

$t\sec^2 x = \int -u^2\,du + C$

$t\sec^2 x = -\frac{u^3}{3} + C$

$t\sec^2 x = -\frac{\tan^3 x}{3} + C$

$\frac{\sec^2 x}{y} = -\frac{\tan^3 x}{3} + C$

Ans

(2) Homogeneous (M3) (7-14 Marks)

Date: / / Page no:

25/Mar/25

⇒ Higher order linear differential eq^n with Constant Co-efficient

$\frac{dy}{dx^2} \Big| \frac{d^3y}{dx^3} \Big| \frac{d^4y}{dx}$

Rules:

$$\left(\frac{d^2y}{dx^2}\right)^1 + 3\left(\frac{dy}{dx}\right)^1 - 2(y)^1 = Q \rightarrow f^n \text{ of } (x)$$

first order $\frac{dy}{dx}$

(1) Higher order
(2) Linear (degree 1)
(3) constant coefficient

$$\boxed{y = Cf + PI}$$

$$\left(\frac{d^3y}{dx^3}\right)^1 + 4\left(\frac{dy}{dx}\right)^1 + 2(y)^1 = Q$$

$$\boxed{y = Cf + PI}$$

Cf = Complimentary function
PI = Particular Integral

y-linear $\left(\frac{d^3y}{dx^3}\right)^1 + 5\left(\frac{dy}{dx}\right)^1 + 2(y)^1 = Q$

$$y = Cf + PI$$

x-linear $\left(\frac{d^2x}{dt^2}\right)^1 + 3\left(\frac{dx}{dt}\right)^1 + 2(x)^1 = t^3 + 4t^2$

$$x = Cf + PI$$

Rules to find Cf

$$\frac{d^2y}{dx^2} + \frac{3dy}{dx} + 2y = Q \rightarrow f^n \text{ of } (x)$$

$$[D^2y + 3Dy + 2y] = Q, \quad D = \frac{d}{dx}, \quad D^2 = \frac{d^2}{dx^2} \quad \Big| \quad D^3 = \frac{d^3}{dx^3}$$

$$[D^2 + 3D + 2]y = Q$$

Replace

D = M

$\rightarrow m^2 + 3M + 2 = Q \rightarrow$ A.E. (Auxillary Eq^n)

$\rightarrow m = -1, 2.$

Eg. ⇒

Date: / / Page no:

$m = m_1, m_2$ e.g. $= m = -1, 2$

(1) $m_1 \neq m_2$ (Real

Sol^n $Cf = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

(2) $m_1 = m_2$ (Real) Eg. $= m = -3, -3$

Sol^n → $cf = (c_1 + c_2 x) e^{m_1 x}$

(3) ~~m = x~~ $m = \alpha \pm i\beta$ (complex)

Sol^n ⇒ $cf = \{c_1 \cos\beta x + c_2 \sin\beta x\} e^{\alpha x}$

4 E.g. ⇒ $m = 5 \pm 3i$

$cf = \{c_1 \cos 3x + c_2 \sin 3x\} e^{5x}$

26 Marks

$$\frac{d^3y}{dx^3} + \frac{3d^2y}{dx^2} - \frac{2dy}{dx} + 4y = Q$$

$(D^3 + 3D^2 - 2D + 4)y = Q$

Auxillary Equation

→ $M^3 + 3m^2 - 2M + 4 = 0$

$m = m_1, m_2, m_3$

(1) $m_1 \neq m_2 \neq m_3$ (Real) E.g. $\{3, 5, -6\}$

$C.f. = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$

(2) $m_1 = m_2 \neq m_3$ (Real) E.g. $\{3, 3, 5\}$

$C.f. = (c_1 + c_2 x) e^{m_1 x} + c_3 e^{m_3 x}$

(3) $m_1 = m_2 = m_3$ {Real} E.g. $\{3, 3, 3\}$

$C.f. = \{c_1 + c_2 x + c_3 x^2\} e^{m_1 x}$

(4) $m = \alpha \pm i\beta$, m_3 → Real

E.g. $\{5 \pm 3i, -2\}$

$Cf = \{c_1 \cos\beta x + c_2 \sin\beta x\} e^{\alpha x} + c_3 e^{m_3 x}$

$Cf = \{c_1 \cos 3x + c_2 \sin 3x\} e^{5x} + c_3 e^{-2x}$

26/Mar/25

Date: / / Page no:

$$\frac{d^4y}{dx^4}+\frac{3d^3y}{dx^3}+\frac{2dy}{dx}-5y=Q$$

$(D^4+3D^3+2D-5)y=Q$

Auxiliary eqn

$M^4+3M^3+2M-5=0$

$m=m_1, m_2, m_3, m_4$

(1) $m_1 \neq m_2 \neq m_3 \neq m_4$ (Real)

$Cf = C_1e^{m_1x}+C_2e^{m_2x}+C_3e^{m_3x}+C_4e^{m_4x}$

(2) $m_1 = m_2 \neq m_3 \neq m_4$ (Real)

$Cf = (C_1+C_2x)e^{m_1x}+C_3e^{m_3x}+C_4e^{m_4x}$

(3) $m_1=m_2=m_3 \neq m_4$ (Real)

$C.f. = \{C_1+C_2x+C_3x^2\}e^{m_1x}+C_4e^{m_4x}$

(4) $m_1=m_2=m_3=m_4$ (Real)

$C.f = \{C_1+C_2x+C_3x^2+C_4x^3\}e^{m_1x}$

(5) $m=\alpha \pm i\beta$, $\{\underline{m_3 \neq m_4}\} \to$ Real

$C.f. = \{C_1\cos\beta x+C_2\sin\beta x\}e^{\alpha x}+C_3e^{m_3x}+C_4e^{m_4x}$

(6) $m=\alpha \pm i\beta$, $\{m_3=m_4\}$ (Real)

$C.f = \{C_1\cos\beta x+C_2\sin\beta x\}e^{\alpha x}+\{C_3+C_4x\}e^{m_3x}$

(7) $m=\alpha_1 \pm i\beta_1$, $\alpha_2 \pm i\beta_2$

$C.f. = \{C_1\cos\beta_1 x+C_2\sin\beta_1 x\}e^{\alpha_1 x}+\{C_3\cos\beta_2 x+C_4\sin\beta_2 x\}e^{\alpha_2 x}$

(8) $m=\alpha \pm i\beta$, $\alpha \pm i\beta$

$C.f = \{(C_1+C_2x)\cos\beta x+(C_3+C_4x)\sin\beta x\}e^{\alpha x}$

26/03/25

Date: / / Page no:

Rull's to find Particular Integral (P.I.)

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=Q$$

$$(D^2+3D+2)y=Q$$

$$f(D)y=Q$$

$$PI=\frac{1}{f(D)}Q \Rightarrow PI=\frac{1}{D^2+3D+2}(Q)$$

for e.g.

(i) $Q=e^{ax} \Rightarrow e^{3x}, e^{-x}$ (2)

$$P.I.=\frac{1}{f(D)}e^{ax}, \quad D\equiv a \quad \text{Replace } (\leftrightarrow)$$

$$P.I.=\frac{1}{f(a)}e^{ax}$$

$$P.I.=\frac{1}{D^2+3D+2}e^{5x}$$

$$P.I.=\frac{1}{5^2+3\times5+2}e^{5x}$$

$$\boxed{P.I.=\frac{1}{42}e^{5x}}$$

$$P.I.=\frac{1}{f(D)}e^{ax}=\frac{1}{f(a)}e^{ax}$$

But if $f(a)=0$ case fail

$$P.I.=x\frac{1}{f'(D)}e^{ax} \quad \text{Now } D\equiv a$$

$$P.I.=x\frac{1}{f'(a)}e^{ax}$$

But if again $f'(a)=0$ again case fail

$$P.I.=x\frac{1}{f'(D)}e^{ax}$$

$$P.I=x^2\frac{1}{f''(D)}e^{ax} \quad D\equiv a$$

$$\boxed{P.I=x^2\frac{1}{f''(a)}e^{ax}}$$

27/MAR/25

Date: / / Page no:

Case ② Rules for $\phi =$ $\sin bx$ (or) $\cos bx$

If $\phi = \sin bx$ or $\cos bx$

$$P.I = \frac{1}{f(D)}\{\sin bx \text{ or } \cos bx\}$$

$$D^2 \equiv -(b^2)$$

$$P.I. = \frac{1}{D^2+3D+5}\sin 2x$$

$$D^2 \equiv -b^2 = -(2^2) = -4$$

$$P.I. = \frac{1}{-4+3D+5}\sin 2x$$

$$P.I. = \frac{1}{3D+1}\sin 2x$$

$$P.I. = \frac{(3D-1)}{(3D+1)(3D-1)}\sin 2x$$

$$P.I. = \frac{(3D-1)\sin 2x}{9D^2-1}$$

$$P.I. = \frac{3D\sin 2x - \sin 2x}{9(-4)-1}$$

$$P.I = \frac{3D\sin 2x - \sin 2x}{-36-1}$$

$$P.I = \frac{3D\sin 2x - \sin 2x}{-37}$$

~~P.I = 3x cos 2x(2)~~

$$P.I. = \frac{3(\cos 2x)2 - \sin 2x}{-37}$$

If

$$P.I = \frac{1}{D^2+4}\cos 2x$$

$$D^2 \equiv -(b^2) = -(2^2) = -4$$

$$P.I. = \frac{1}{-4+4}\cos 2x$$

Case fail

$$P.I. = x\frac{1}{2D}\cos 2x$$

$$P.I. = x\frac{D\cos 2x}{2D^2}$$

$$P.I = \frac{-x\sin 2x \cdot 2}{-8}$$

$$P.I = \frac{x\sin 2x}{4}$$

27/March/25

Date: / / Page no:

CASE - III If Q = Polynomial or algebraic

x^3+3x^2-2

$x+3$

x^2+4

x^2+2x-1

{Apply Binomial}

$$P.I. = \frac{1}{f(D)} Q \Rightarrow \text{Polynomial}$$

$$P.I. = \frac{1}{[1 \pm \phi(D)]}[Q]$$

$$P.I. = [1 \pm \phi(D)]^{-1} Q$$

Binomial $\Rightarrow [1+P]^n = 1 + nP + \frac{n(n-1)}{\lfloor 2} P^2 + \frac{n(n-1)(n-2)}{\lfloor 3} P^3 + \dots$

CASE - IV

If $Q = e^{ax} \cdot \phi(x)$

$$P.I. = \frac{1}{f(D)} e^{ax} \cdot \phi(x)$$

$$P.I. = e^{ax} \frac{1}{f(D+a)} \phi(x)$$

28/March/25

Case - V :

If $Q = x^m V$

$$P.I. = \frac{1}{f(D)} Q$$

$$P.I. = \frac{1}{f(D)} \{x^m \cdot V\}$$

$$P.I. = x^m \frac{1}{f(D)} V + m x^{m-1} \left\{\frac{d}{dD}\left(\frac{1}{f(D)}\right)\right\} V + \frac{m(m-1)}{\lfloor 2} \left\{\frac{d^2}{dD^2}\left(\frac{1}{f(D)}\right)\right\} V + \dots$$

28/Mar/25

Date: / / Page no: __

Q.1 $\frac{d^2y}{dx^2} + \frac{3dy}{dx} + 2y = e^{2x}$

$D^2y + 3Dy + 2y = Q e^{2x}$

$(D^2 + 3D + 2)y = Q e^{2x}$

$D = d/dx$

Auxiliary Eqn

$m^2 + 3m + 2 = 0$

$\Rightarrow m = -2, -1$ ($m_1 \neq m_2$, Real)

$C.f. = [C_1 e^{-2x} + C_2 e^{-x}]$

$P.I = \frac{1}{D^2+3D+2} e^{2x}$

$D = a = 2$

$P.I = \frac{1}{2^2+3D+2} e^{2x}$

$P.I = \frac{1}{12} e^{2x}$

Complete Soln

$y = Cf + PI$

$y = [C_1 e^{-2x} + C_2 e^{-x}] + \frac{1 e^{2x}}{12}$

Q.2 $\frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y = \sin 2x$

$(D^2 - 3D + 2)y = \sin 2x$

$D = d/dx$

Auxiliary Eqn

~~$(m^2 - 3m + 2)y$~~

$m^2 - 3m + 2 = 0$

$m = 1, 2$

$C.f = C_1 e^{x} + C_2 e^{2x}$

$P.I = \frac{1}{D^2 - 3D + 2} \sin 2x$

$D^2 = -(b^2) = -2^2 = -4$

$P.I = \frac{1}{-4 - 3D + 2} \sin 2x$

$P.I. = \frac{1}{-3D - 2} \sin 2x$

$P.I. = \frac{-1}{(3D+2)} \sin 2x$

$P.I = \frac{-1(3D-2)}{(3D+2)(3D-2)} \sin 2x$

$P.I = \frac{-(3D \sin 2x - 2\sin 2x)}{9D^2 - 4}$

$P.I = \frac{-(3D \sin 2x - 2\sin 2x)}{9(-4) - 4}$

$P.I = -\frac{(3D \sin 2x - 2\sin 2x)}{-36 - 4}$

$P.I = \frac{+(3D\cos 2x \cdot 2 - 2\sin 2x)}{+40}$

$PI = \frac{3D\cos 2x - \sin 2x}{20}$

~~$y = C.f + P.I$~~

$y = [C_1 e^{x} + C_2 e^{2x}] + \frac{3D\cos 2x - \sin 2x}{20}$

Q.3

22/Mar/25

$$\begin{array}{c|c} 2 & 4 \\ 2 & 2 \\ & 1 \end{array}$$

Date: / / Page no:

Q.3 $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-2x}$

$(D^2 + 4D + 4)y = e^{-2x}$

Auxiliary Eq^n

$M^2 + 4M + 4 = 0$

$m = -2, -2$

$(m_1 = m_2, \text{Real})$

$C.f. = \{C_1 + C_2x\}e^{-2x}$

$P.I. = \frac{1}{D^2+4D+4} e^{-2x}$

$\cancel{D^2 \equiv -(b^2) = -2^2 = -4}$

$\cancel{P.I = \frac{1}{4+}}$ $D = b = -2$

$P.I = \frac{1}{(-2)^2 + 4(-2) + 4} e^{-2x}$

$P.I = \frac{1}{4-8+4} e^{-2x}$

$P.I = \frac{1}{0} e^{-2x}$

case fail

$P.I = x \frac{1}{f'(D)} e^{2x}$

$P.I = x \frac{1}{2D+4} e^{-2x}$

$P.I = x \frac{1}{2(-2)+4} e^{-2x}$

$P.I = x \frac{1}{-4+4} e^{-2x}$ 0

case fail

$P.I = x^2 \frac{1}{2} e^{-2x}$

$y = C.f. + P.I$ $\quad y = \{C_1 + C_2x\}e^{-2x} + x^2 \frac{1}{2} e^{-2x}$ Ans

Q.4 $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \cos 3x$

Sol^n

$(D^2 + 6D + 9)y = \cos 3x$

Auxiliary Eq^n

$M^2 + 6M + 9 = 0$

$M = -3, -3$

$m_1 \neq m_2$

$C.f = \{C_1 + C_2x\}e^{-3x}$

$P.I = \frac{1}{D^2+6D+9} \cos 3x$

$D^2 \equiv -(b^2) = -(3^2) = -9$

$P.I = \frac{1}{-9+6D+9} \cos 3x$

$P.I = \frac{1}{6D} \cos 3x$

$P.I. = \frac{D \cos 3x}{6D^2}$

$P.I = \frac{3(-\sin 3x)}{6(-9)}$

$P.I = \frac{3(-\sin 3x)}{-54}$

$P.I = \frac{\sin 3x}{18}$

$y = C.f + P.I$

$y = \{C_1 + C_2x\}e^{-3x} + \frac{\sin 3x}{18}$

$P.I = \frac{1}{6} \int \cos 3x$

$P.I = \frac{1}{6} \frac{\sin 3x}{3}$

$P.I = \frac{\sin 3x}{18}$

01/04/25

Date: / / Page no:

Hyperbolic

Q1 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \cosh 3x$

$(D^2 - 6D + 9)y = \cosh 3x$

$m^2 - 6m + 9 = 0$

$Cf = (C_1 + C_2 x)e^{3x}$

$P.I. = \frac{1}{D^2 - 6D + 9} \cosh 3x$

$P.I. = \frac{1}{D^2 - 6D + 9} \left\{ \frac{e^{3x} + e^{-3x}}{2} \right\}$

$P.I. = \frac{1}{2}\left[\frac{e^{3x}}{D^2 - 6D + 9} + \frac{1}{D^2 - 6D + 9} e^{-3x} \right\}$

① $D = a = 3$

② $D = a = -3$

$P.I. = \frac{1}{2}\left[\frac{1}{(3)^2 - 6(3) + 9} e^{3x} + \frac{1}{(-3)^2 - 6(-3) + 9} e^{-3x} \right]$

$P.I. = \frac{1}{2}\left[\frac{1}{9 - 18 + 9} e^{3x} + \frac{1}{9 + 18 + 9} e^{-3x} \right]$

→ 0 case fail

$P.I. = \frac{1}{2}\left[\frac{1}{D^2 - 6D + 9} e^{3x} + \frac{1}{36} e^{-3x} \right]$

$P.I. = \frac{1}{2}\left[x \frac{1}{2D - 6} e^{3x} + \frac{1}{36} e^{-3x} \right]$

$P.I. = \frac{1}{2}\left[x \frac{1}{2(3) - 6} e^{3x} + \frac{1}{36} e^{-3x} \right]$

$P.I. = \frac{1}{2}\left[\frac{x \cdot 1}{0} e^{3x} + \frac{1}{36} e^{-3x} \right]$

$P.I. = \frac{1}{2}\left[x^2 \frac{1}{2} e^{3x} + \frac{1}{36} e^{-3x} \right]$

$\cosh ax = \frac{e^{ax} + e^{-ax}}{2}$

$\sinh ax = \frac{e^{ax} - e^{-ax}}{2}$

$D\{\cosh ax\} = a \sinh ax$

$D\{\sinh ax\} = a \cosh ax$

$\int \sinh ax = \frac{\cosh ax}{a}$

$\int \cosh ax = \frac{\sinh ax}{a}$

$\tanh ax = \frac{\sinh ax}{\cosh ax}$

$\tanh ax = \frac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}$

Complete sol^n

$y = C.f. + P.I.$

$y = (C_1 + C_2 x)e^{3x} + \frac{1}{2}\left[x^2 \frac{1}{2} e^{3x} + \frac{1}{36} e^{-3x} \right]$

0 1

Q.2

Auxiali

P.I

P.I. =

P.I.

P.

P.

P.

01/04/25

$\frac{1}{2}$ $-\frac{5}{2}$ $\frac{1+2}{2}$ $\frac{-1-2}{2}$ $-\frac{3}{2}$

Date: / / Page no:

Q.2 $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sinh 2x + \sin 2x + \cos 3x + \cosh 3x$

$(D^2+3D+2)y = \sinh 2x + \sin 2x + \cos 3x + \cosh 3x$

Auxiliary Eqn

$(m^2+3m+2) = 0$

$m = -2, -1$

$C.f. = c_1 e^{-2x} + c_2 e^{-x}$

$P.I. = \frac{1}{D^2+3D+2}\left\{\sinh 2x + \sin 2x + \cos 3x + \cosh 3x\right\}$

$P.I. = \frac{1}{D^2+3D+2}\left\{\frac{e^{2x}-e^{-2x}}{2} + \sin 2x + \cos 3x + \frac{e^{3x}+e^{-3x}}{2}\right\}$

$P.I. = \frac{1}{2}\left[\frac{1}{D^2+3D+2}e^{2x} - \frac{1}{D^2+3D+2}e^{-2x}\right] + \frac{1}{D^2+3D+2}\sin 2x + \frac{1}{D^2+3D+2}\cos 3x + \frac{1}{2}\left[\frac{1}{D^2+3D+2}e^{3x} + \frac{1}{D^2+3D+2}e^{-3x}\right]$

$P.I. = \frac{1}{2}\left[\frac{1}{2^2+3(2)+2}e^{2x} - \frac{1}{(-2)^2+3(-2)+2}e^{-2x}\right] + \frac{1}{-4+3D+2}\sin 2x + \frac{1}{-9+3D+2}\cos 3x + \frac{1}{2}\left[\frac{1}{3^2+3(3)+2}e^{3x} + \frac{1}{(-3)^2+3(-3)+2}e^{-3x}\right]$

$P.I. = \frac{1}{2}\left[\frac{1}{12}e^{2x} - \frac{1}{0}e^{-2x}\right] + \frac{1}{3D-2}\sin 2x + \frac{1}{3D-7}\cos 3x + \frac{1}{2}\left[\frac{1}{20}e^{3x} + \frac{1}{2}e^{-3x}\right]$

$P.I. = \frac{1}{2}\left[\frac{1}{12}e^{2x} - x\frac{1}{2D+3}e^{-2x}\right] + \frac{(3D+2)}{9D^2-4}\sin 2x + \frac{(3D+7)}{9D^2-49}\cos 3x + \frac{1}{2}\left[\frac{1}{20}e^{3x} + \frac{1}{2}e^{-3x}\right]$

$P.I. = \frac{1}{2}\left[\frac{1}{12}e^{2x} - x\frac{1}{-4+3}e^{-2x}\right] + \frac{3D\sin 2x + 2\sin 2x}{9(-4)-4} + \frac{3D\cos 3x + 7\cos 3x}{9(-9)-49} + \frac{1}{2}\left[\frac{1}{20}e^{3x} + \frac{1}{2}e^{-3x}\right]$

$P.I. = \frac{1}{2}\left[\frac{1}{12}e^{2x} + x\,e^{-2x}\right] + \frac{6\cos 2x + 2\sin 2x}{-40} + \left(\frac{-9\sin 3x + 7\cos 3x}{-130}\right) + \frac{1}{2}\left[\frac{1}{20}e^{3x} + \frac{1}{2}e^{-3x}\right]$

$P.I. = \frac{1}{2}\left[\frac{1}{12}e^{2x} + xe^{-2x}\right] - \left(\frac{6\cos 2x + 2\sin 2x}{40}\right) + \left(\frac{9\sin 3x - 7\cos 3x}{130}\right) + \frac{1}{2}\left[\frac{1}{20}e^{3x} + \frac{1}{2}e^{-3x}\right]$

$y = C.f. + P.I.$

$y = c_1 e^{-2x} + c_2 e^{-x} + \frac{1}{2}\left[\frac{1}{12}e^{2x} + xe^{-2x}\right] - \left(\frac{6\cos 2x + 2\sin 2x}{40}\right) + \left(\frac{9\sin 3x - 7\cos 3x}{130}\right) + \frac{1}{2}\left[\frac{1}{20}e^{3x} + \frac{1}{2}e^{-3x}\right]$

01/04/25

Date: / / Page no:

Q.3 $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=\cosh 2x+\sin 2x+\sinh 3x+\cos 3x$

$(D^2+5D+6)y=\cosh 2x+\sin 2x+\sinh 3x+\cos 3x$

Auxiliary Eqn

$M^2+5M+6=0$

$m=-2,-3$

$m_1\neq m_2$

So C.f. $=(C_1e^{-2x}+C_2e^{-3x})$

$P.I.=\frac{1}{D^2+5D+6}\{\cosh 2x+\sin 2x+\sinh 3x+\cos 3x\}$

$P.I.=\frac{1}{D^2+5D+6}\left\{\left(\frac{e^{2x}+e^{-2x}}{2}\right)+\sin 2x+\left(\frac{e^{3x}-e^{-3x}}{2}\right)+\cos 3x\right\}$

$P.I.=\frac{1}{2}\left[\frac{1}{D^2+5D+6}e^{2x}+\frac{1}{D^2+5D+6}e^{-2x}\right]+\frac{1}{D^2+5D+6}\sin 2x+\frac{1}{2}\left[\frac{1}{D^2+5D+6}e^{3x}-\frac{1}{D^2+5D+6}e^{-3x}\right]$

$D\equiv a=2$ $D^2=-(b)^2=-(2^2)=-4$ $+\frac{1}{D^2+5D+6}\cos 3x$

$P.I.=\frac{1}{2}\left[\frac{1}{2^2+5(2)+6}e^{2x}+\frac{1}{(-2)^2+5(-2)+6}e^{-2x}\right]+\frac{1}{-4+5D+6}\sin 2x+\frac{1}{2}\left[\frac{1}{3^2+5(3)+6}e^{3x}-\frac{1}{(-3)^2+5(-3)+6}e^{-3x}\right]$

$+\frac{1}{-9+5D+6}\cos 3x$

$P.I.=\frac{1}{2}\left[\frac{1}{20}e^{2x}+\frac{1}{0}e^{-2x}\right]+\frac{1}{5D+2}\sin 2x+\frac{1}{2}\left[\frac{1}{30}e^{3x}-\frac{1}{0}e^{-3x}\right]+\frac{1}{5D-3}\cos 3x$

$P.I.=\frac{1}{2}\left[\frac{1}{20}e^{2x}+x\frac{1}{2D+5}e^{-2x}\right]+\frac{(5D-2)\sin 2x}{25D^2-4}+\frac{1}{2}\left[\frac{1}{30}e^{3x}-x\frac{e^{-3x}}{2D+5}\right]+\frac{(5D+3)\cos 3x}{25D^2-9}$

$P.I.=\frac{1}{2}\left[\frac{1}{20}e^{2x}+x\frac{1}{2(-2)+5}e^{-2x}\right]+\frac{5D\sin 2x-2\sin 2x}{25(-4)-4}+\frac{1}{2}\left[\frac{1}{30}e^{3x}-x\frac{e^{-3x}}{2(-3)+5}\right]$

$+\frac{5D\cos 3x+3\cos 3x}{25(-9)-9}$

$P.I.=\frac{1}{2}\left[\frac{1}{20}e^{2x}+xe^{-2x}\right]+\left(\frac{10\cos 2x-2\sin 2x}{-104}\right)+\frac{1}{2}\left[\frac{1}{30}e^{3x}+xe^{-3x}\right]$

$+\left(\frac{-15\sin 3x+3\cos 3x}{-234}\right)$

$P.I.=\frac{1}{2}\left[\frac{1}{20}e^{2x}+xe^{-2x}\right]+\left(\frac{-10\cos 2x+2\sin 2x}{104}\right)+\frac{1}{2}\left[\frac{1}{30}e^{3x}+xe^{-3x}\right]$

$+\left(\frac{15\sin 3x-3\cos 3x}{234}\right)$

$-225-9$
-234

$= \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$

Date: / / Page no:

$y = C.F. + P.I.$

$$y = C_1 e^{-2x} + C_2 e^{-3x} + \frac{1}{2}\left[\frac{1}{20} e^{2x} + x e^{-2x}\right] + \left(\frac{-10\cos 3x + 25\sin 2x}{104}\right) + \frac{1}{2}\left[\frac{1}{30} e^{3x} + x e^{-3x}\right] + \left(\frac{15\sin 3x - 3\cos 3x}{234}\right)$$

Ans

02/09/25

Q1 $\frac{d^4y}{dx^4} - a^4 y = e^{ax}$

$(D^4 - a^4) y = e^{ax}$

$M^4 - a^2 = 0$
$M = 1$, $1 - a^2 = 0$
$a^2 = 1$
a

Auxiliary Eqn
$M^4 - (a^2)^2 = 0$
$M^4 = (a^2)^2$
$M^2 = \sqrt{(a^2)^2}$
$M^2 = \pm a$
$M = a, -a$

$\cancel{M^4 - a^2 = 0}$
$M^4 = a^2$
$(M^2)^2 = (a^2)^2$
$M^2 = \pm a$
$M = \sqrt{\pm a}$
$\sqrt{-a}, \sqrt{+a}$
$\sqrt{a}\,i, \sqrt{a}$

$P.I = \frac{1}{D^4 - a^4} e^{ax}$
$D = a$
$P.I = \frac{1}{a^4 - a^4} e^{ax}$
$= 0$ case fail
$P.I = x \frac{1}{4D^3} e^{ax}$
$P.I = x \frac{1}{4a^3} e^{ax}$

$M^4 - a^4 = 0$
$(m^2)^2 - (a^2)^2 = 0$
$(m^2 - a^2)(m^2 + a^2) = 0$

$m^2 - a^2 = 0$	$m^2 + a^2 = 0$
$m^2 = a^2$	$m^2 = -a^2$
$m = \pm a$	$m = \pm a i = \alpha \pm i\beta$ $\alpha = 0$, $\beta = a$

$m_1 \neq m_2$
(Real)

$C.f. = C_1 e^{-ax} + C_2 e^{ax} + \{C_3 \cos ax + C_4 \sin ax\} e^{0x}$

$y = C.f. + P.I$
$y = C_1 e^{-ax} + C_2 e^{ax} + \{C_3 \cos ax + C_4 \sin ax\} + \frac{x}{4} \times \frac{1}{a^3} e^{ax}$

Ans

$y = C_1 e^{-ax} + C_2 e^{ax} + \{C_3 \cos ax + C_4 \sin ax\} + \frac{x}{4} \times \frac{1}{a^3} e^{ax}$

02/09/25 BO

om 1-1

4

Date: / / Page no:

Q2 $(D^2+1)^3(D^2+D+1)^2 y = e^x$

$(m^2+1)^3(m^2+m+1)^2=0$

$(m^2+1)^3=0$ | $(m^2+m+1)^2=0$

$(m^2+m+1)(m^2+m+1)(\cancel{m^2+m+1})=0$

$(m^2+1)(m^2+1)(m^2+1)=0$

$m=\pm i$, $m=\pm i \pm i$

$\alpha \pm i\beta \Rightarrow \alpha=0, \beta=1$

$m^2+m+1=0$

$m=\dfrac{-1\pm\sqrt{3}i}{2}$

$m=\dfrac{-1}{2}\pm\dfrac{\sqrt{3}}{2}i$

$m^2+m+1=0$

$m=\dfrac{-1\pm\sqrt{3}i}{2}$

$m=\dfrac{-1}{2}\pm\dfrac{\sqrt{3}i}{2}$

$\alpha \pm i\beta$

$\alpha=\dfrac{-1}{2}, \beta=\sqrt{3}/2$

$$C.f.=\{(c_1+c_2x+c_3x^2)\cos x+(c_4+c_5x+c_6x^2)\sin x\}e^{0x}$$
$$+\{(c_7+c_8x)\cos\tfrac{\sqrt{3}}{2}x+(c_9+c_{10}x)\sin\tfrac{\sqrt{3}}{2}x\}e^{-\frac{1}{2}x}$$

$$P.I.=\frac{1}{(D^2+1)(D^2+1)(D^2+1)}e^x \times \frac{1}{(D^2+D+1)(D^2+D+1)}e^x$$

$$P.I.=\frac{1}{2\times2\times2}e^x \times \frac{1}{3\times3}e^x$$

$$P.I.=\frac{1}{8}e^x \times \frac{1}{9}e^x$$

$$P.I.=\frac{1}{72}e^x$$

$y=C.f.+P.I.$

$$y=\{(c_1+c_2x+c_3x^2)\cos x+(c_4+c_5x+c_6x^2)\sin x\}e$$
$$+\{(c_7+c_8x)\cos\tfrac{\sqrt{3}}{2}x+(c_9+c_{10}x)\sin\tfrac{\sqrt{3}}{2}x\}e^{-\frac{1}{2}x}+\frac{1}{72}e^x$$

Engineering,

02/04/25

Date: / / Page no:

Q.5 $\frac{d^4y}{dx^4} - 7\frac{d^3y}{dx^3} + 18\frac{d^2y}{dx^2} - 20\frac{dy}{dx} + 8y = (e^x+1)^2$

Solⁿ ⇒ $(D^4 - 7D^3 + 18D^2 - 20D + 8)y = (e^x+1)^2$

∴ Auxiliary Eqⁿ

$M^4 - 7M^3 + 18M^2 - 20M + 8 = 0$

$$\begin{array}{c|ccccc} & 1 & -7 & 18 & -20 & 8 \\ m=1 & - & 1 & -6 & 12 & -8 \\ \hline & 1 & -6 & 12 & -8 & 0 \end{array}$$

$m^3 - 6m^2 + 12m - 8 = 0$

$$\begin{array}{c|cccc} & 1 & -6 & 12 & -8 \\ m=2 & - & 2 & -8 & 8 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

$M^2 - 4M + 4 = 0$

$M = 2, 2$

$M = 1, 2, 2, 2 \Rightarrow$ Real

$C.F. = C_1e^x + (C_2 + C_3x + C_4x^2)e^{2x}$

for P.I.

$P.I. = \frac{1}{D^4 - 7D^3 + 18D^2 - 20D + 8}(e^x+1)^2$

$P.I. = \frac{1}{D^4 - 7D^3 + 18D^2 - 20D + 8}(e^{2x} + 1 + 2e^x)$

$P.I. = \frac{1}{D^4 - 7D^3 + 18D^2 - 20D + 8}e^{2x} + \frac{1}{D^4 - 7D^3 + 18D^2 - 20D + 8}e^{0x} + \frac{2}{D^4 - 7D^3 + 18D^2 - 20D + 8}e^x$

$P.I. = \frac{1}{2^4 - 7(2)^3 + 18(2)^2 - 20(2) + 8}e^{2x} + \frac{1}{8}e^{0x} + \frac{2}{1^4 - 7(1)^3 + 18(1)^2 - 20(1) + 8}e^x$

$16-56+72-40+8=0$

$32-84+72-20$

$-104+104=0$

$48-84+36$

$84-84$

$1-7+18-20+8$

$-27+27=0$

$4-21+36-20$

$-41+40$

Date: / / Page no:

$$P.I. = \frac{1}{0}e^{2x} + \frac{1}{8}e^{0x} + \frac{2}{0}e^{x}$$

case fail   case fail

So

$$P.I = \frac{1}{D^4-7D^3+18D^2-20D+8}e^{2x} + \frac{1}{8}e^{0x} + \frac{2}{D^4-7D^3+18D^2-20D+8}e^{x}$$

$$P.I. = \frac{x\,e^{2x}}{4D^3-21D^2+36D-20} + \frac{1}{8}e^{0x} + \frac{x\,2}{4D^3-21D^2+36D-20}e^{x}$$

$$P.I. = \frac{x\,e^{2x}}{4(2)^3-21(2)^2+36(2)-20} + \frac{1}{8}e^{0x} + \frac{2x\,e^{x}}{4(1)^3-21(1)^2+36(1)-20}$$

$$P.I. = \frac{x\,e^{2x}}{0} + \frac{1}{8} + \frac{2x\,e^{x}}{-1}$$

case fail

$$P.I. = \frac{x.x\,e^{2x}}{12D^2-42D+36} + \frac{1}{8} - 2x\,e^{x}$$

$$P.I. = \frac{x^2e^{2x}}{12(2)^2-42(2)+36} + \frac{1}{8} - 2x\,e^{x}$$

$$P.I. = \frac{x^2e^{2x}}{0} + \frac{1}{8} - 2x\,e^{x}$$

case fail

$$\boxed{P.I = \cancel{\frac{x\,x^2e^{2x}}{24D-42}}}$$

$$P.I. = \frac{x\,x^2e^{2x}}{24D-42} + \frac{1}{8} - 2x\,e^{x}$$

$$P.I. = \frac{x^3e^{2x}}{6} + \frac{1}{8} - 2x\,e^{x}$$

$$y = C.f. + P.I$$

$$y = C_1e^{x} + (C_2+C_3x+C_4x^2)e^{2x} + \frac{x^3e^{2x}}{6} + \frac{1}{8} - 2x\,e^{x}$$

Ans

08/04/25

Date: / / Page no: 8

$1+3-1$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$4 \times 1 \times -2$

Q.2

$(D^3+3D^2-2D)y = x^2$

Solve:-

Auxiliary Eqn

$M^3+3M^2-2M=0$

$M(M^2+3m-2)=0$

$m=0 \mid m^2+3m-2=0$

$m = \frac{-3 \pm \sqrt{9-4\times1(-2)}}{2\times1}$

$\frac{-3 \pm \sqrt{9+8}}{2}$

$\frac{-3 \pm \sqrt{17}}{2}$

$m = \frac{-3 \pm \sqrt{17}}{2} \to \frac{-3+\sqrt{17}}{2}, \frac{-3-\sqrt{17}}{2}$

$C.f. = C_1 e^{0x} + C_2 e^{\frac{-3+\sqrt{17}}{2}x} + C_3 e^{\frac{-3-\sqrt{17}}{2}x}$

$P.I = \frac{1}{D^3+3D^2-2D}(x^2)$

$P.I = \frac{1}{-2D\left\{1 - \frac{D^3+3D^2}{2D}\right\}}(x^2)$

$P.I. = \frac{1}{-2D\left\{1-\frac{D^2+3D}{2}\right\}}x^2$

$P.I. = \frac{1}{-2D}\left\{1-\frac{D^2+3D}{2}\right\}^{-1}(x^2)$

Apply Binomial

$(1+P)^n = 1+nP+\frac{n(n-1)}{\lfloor 2}P^2+\frac{n(n-1)(n-2)}{\lfloor 3}P^3+ \ldots$

$P.I. = \frac{1}{-2D}\left\{1+(-1)\left(-\frac{D^2+3D}{2}\right)+\frac{(-1)(-1-1)}{\lfloor 2}\left(-\frac{D^2+3D}{2}\right)^2+ \ldots\right)x^2$

$P.I = \frac{1}{-2D}\left\{x^2+\frac{D^2(x^2)+3Dx^2}{2}+\frac{D^4x^2+9D^2x^2+6D^3x^2}{4}+\right\}$

$P.I = \frac{1}{-2D}\left\{x^2+\frac{2+3\times2x}{2}+\frac{0+9\times2+6\times0}{4}+0\right\}$

$P.I = \frac{1}{-2D}\left\{x^2+1+3x+\frac{9}{2}\right\}$

$P.I = \frac{1}{-2}\int\left\{x^2+1+3x+\frac{9}{2}\right\}dx$

$= -\frac{1}{2}\left\{\frac{x^3}{3}+x+\frac{3x^2}{2}+\frac{9}{2}x\right\}$

P.I =

$y = C.f. + P.I.$

$y = C_1e^{0x}+C_2e^{\frac{-3+\sqrt{17}}{2}x}+C_3e^{\frac{-3-\sqrt{17}}{2}x} - \frac{1}{2}\left\{\frac{x^3}{3}+x+\frac{3x^2}{2}+\frac{9x}{2}\right\}$

Date: / / Page no:

$n^2 = \cancel{1}-1$

$n = \pm\sqrt{-1}$

$n = \pm i$

Q.2 $(D^2+1)y = \cosh x \cos x$

Auxiliary Eqⁿ

$M^2+1=0$

$m^2 = -1$ $\quad m = \pm i$ $\quad \alpha = 0 \quad \beta = 1$

$\cancel{m = \pm 1}$

$\cancel{m = 1, -1}$ $\quad C.f. = (C_1 \cos x + C_2 \sin x)e^{0x}$

$\cancel{m_1 \neq m_2}$

$\cancel{C.f. = C_1 e^{x} + C_2 e^{-x}}$

$$P.I. = \frac{1}{(D^2+1)} \cosh x \cos x$$

$$P.I. = \frac{1}{(D^2+1)} \frac{(e^{x}+e^{-x})}{2} \cos x$$

$$P.I. = \frac{1}{(D^2+1)} \left(\frac{e^{x}\cos x}{2} + \frac{e^{-x}\cos x}{2} \right)$$

$$P.I. = \frac{1}{2}\left\{ \frac{1}{(D^2+1)} e^{x}\cos x + \frac{1}{(D^2+1)} e^{-x}\cos x \right\}$$

$$P.I. = \frac{1}{2}\left\{ e^{x} \frac{1}{(D+1)^2+1} \cos x + e^{-x} \frac{1}{(D-1)^2+1} \cos x \right\}$$

$$P.I. = \frac{1}{2}\left\{ e^{x} \frac{1}{D^2+2D+2} \cos x + e^{-x} \frac{1}{D^2-2D+2} \cos x \right\}$$

$D^2 = -(b^2) = -(1^2) = -1$

$$P.I. = \frac{1}{2}\left\{ e^{x} \frac{1}{-1+2D+2} \cos x + e^{-x} \frac{1}{-1-2D+2} \cos x \right\}$$

$$P.I. = \frac{1}{2}\left\{ e^{x} \frac{1}{2D+1} \cos x + e^{-x} \frac{1}{1-2D} \cos x \right\}$$

$$P.I. = \frac{1}{2}\left[e^{x} \frac{(2D-1)}{(2D+1)(2D-1)} \cos x + e^{-x} \frac{(1+2D)}{(1-2D)(1+2D)} \cos x \right]$$

$$P.I. = \frac{1}{2}\left\{ e^{x} \frac{2D\cos x - \cos x}{4D(-1)-1} + e^{-x} \frac{\cos x + 2D\cos x}{1-4(-1)} \right.$$

$$P.I. = \frac{1}{2}\left[\frac{e^{x}(2(-\sin x) - \cos x)}{-5} + e^{-x}\frac{(\cos x + 2(-\sin x))}{5} \right]$$

$y = C.f. + P.I.$

$$y = (C_1 \cos x + C_2 \sin x)e^{0x} + \frac{1}{2}\left\{ \frac{e^{x}(2(-\sin x) - \cos x)}{-5} + e^{-x}\frac{(\cos x + 2(-\sin x))}{5} \right\}$$

Date: / / Page no:

$(D^2+1)y = xe^x$

Auxiliary

$M^2+1=0$

$m = \pm i \Rightarrow C.f = (c_1\cos x + c_2\sin x)$

$m = \alpha \pm i\beta$

$\alpha = 0, \beta = 1$

$P.I. = \frac{1}{(D^2+1)} xe^x$

$= e^x \frac{1}{(D+1)^2+1} x$

$= e^x \frac{1}{D^2+2D+1+1} x$

$= e^x \frac{1}{D^2+2D+2} x$

$= e^x \frac{1}{2\{1+\frac{D^2+2D}{2}\}} (x)$

$= e^x \frac{1}{2} \{1 + \frac{D^2+2D}{2}\}^{-1} (x)$ Binomial

$= e^x \frac{1}{2} \left[1 + (-1)\left(\frac{D^2+2D}{2}\right) + \cdots\right] x$

$= e^x \frac{1}{2} \left[x - \frac{D^2(x) + 2Dx}{2} + \cdots\right.$

$= e^x \frac{1}{2} \left[x - \frac{0+2}{2}\right]$

$= \frac{e^x}{2}\left[x-1\right] = P.I$

$y = C.f + P.I$

$y = (c_1\cos x + c_2\sin x) + \frac{e^x}{2}\left[x-1\right]$

$D^2 + 1 + 2D - 5D - 5 + 6$

$D^2 - 3D + 2$

Date: / / Page no:

$(D^2 - 5D + 6)y = (x^2+1)e^x$

$D = m$

$m^2 - 5m + 6 = 0$

$m^2 - 2m - 3m + 6 = 0$

$m(m-2) - 3(m-2) = 0$

$m = 2, 3$

$m_1 \neq m_2$

$C.f. = \{C_1 e^{2x} + C_2 e^{3x}\}$

$P.I = \frac{1}{f(D)}(x^2+1)e^x$

$P.I. = \frac{1}{D^2 - 5D + 6}(x^2+1)e^x \quad \{a = 1\}$

$P.I = e^x \frac{1}{(D+1)^2 - 5(D+1) + 6}(x^2+1)$

$P.I = e^x \frac{1}{D^2 - 3D + 2}(x^2+1)$

$P.I = e^x \frac{1}{2\left\{1 + \frac{D^2 - 3D}{2}\right\}}(x^2+1)$

$P.I = e^x \frac{1}{2}\left\{1 + \frac{D^2 - 3D}{2}\right\}^{-1}(x^2+1)$

$(1+P)^n = 1 + nP + \frac{n(n-1)P^2}{\underline{|2}} + \ldots$

$P.I = \frac{e^x}{2}\left\{1 + (-1)\left(\frac{D^2 - 3D}{2}\right) + \frac{(-1)(-1-1)}{\underline{|2}}\left(\frac{D^2 - 3D}{2}\right)^2 + \ldots\right)(x^2+1)$

$P.I. = \frac{e^x}{2}\left\{1 - \cancel{(D^2 x^2 + 1} - 3Dx\right.$

$P.I = \frac{e^x}{2}\left\{1 + (-1)\left\{\frac{D^2(x^2+1) - 3D(x^2+1)}{2}\right.\right.$

$\left.+ \left(\frac{D^4 + 9D^2 - 6D^3}{4}\right)(x^2+1) + \ldots\right)$

$P.I = \frac{e^x}{2}\left\{1 - \left\{\frac{2 - 6x}{2}\right\} + \frac{0 + 9\times 2 - 0}{4}\right\}$

$P.I. = \frac{e^x}{2}\left\{1 - \{2 - 6\}\right.$

$P.I. = \frac{e^x}{2}\left\{1 - \{1 - 3x\} + \frac{9}{2}\right\}$

complete sol^n

$y = C.f. + P.I$

$y = \{C_1 e^{2x} + C_2 e^{3x}\} + \left\{\frac{e^x}{2}\left\{1 - (1 - 3x) + \frac{9}{2}\right\}\right\}$

Ans

$-2, -1 = 2$
-3
2

$3 | 3$
$| 1$

Date: / / Page no.

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2^x$$

$$(D^2 - 3D + 2)y = 2^x$$

$$M^2 - 3D + 2 = 0$$

$$M = 2, 1$$

$$M_1 \neq M_2$$

So

$$C.F. = C_1 e^{2x} + C_2 e^{x}$$

$$P.I. = \frac{1}{D^2 - 3D + 2} 2^x$$

$$P.I. = \frac{1}{D^2 - 3D + 2} e^{x\log 2}$$

$$P.I. = \frac{1}{(\log 2)^2 - 3(\log 2) + 2}$$

$a^2+b^2=(a+b)(a+b)$

$a^2-b^2=(a-b)(a+b)$

07/04/25

Date: / / Page no:

Q $\dfrac{d^2y}{dx^2}+a^2y=\tan ax$

$e^{i\theta}=\cos\theta+i\sin\theta$

$e^{-i\theta}=\cos\theta-i\sin\theta$

$(D^2+a^2)y=\tan ax$

$m^2+a^2=0$

$m=\pm ai=\alpha\pm i\beta$

$a^2+b^2=(a+b)(a+b)$

$\cancel{D^2+a^2=P}$

$C.F.=\{c_1\cos ax+c_2\sin ax\}e^{0x}$

$P.I.=\dfrac{1}{(D^2+a^2)}(\tan ax)$

$P.I.=\dfrac{1}{(D-ai)(D+ai)}\tan ax$

$P.I.\ \dfrac{1}{(D-ai)(D+ai)}=\dfrac{A}{(D-ai)}+\dfrac{B}{(D+ai)}$; $A=\dfrac{1}{2ai}$, $B=-\dfrac{1}{2ai}$

$D-ai\to 0$, $D=ai$; $D+ai\to 0$, $D=-ai$

$P.I.=\left\{\dfrac{1/2ai}{(D-ai)}+\dfrac{(-1/2ai)}{(D+ai)}\right\}\tan ax$

$P.I=\dfrac{1}{2ai}\left\{\dfrac{1}{(D-ai)}\tan ax-\dfrac{1}{(D+ai)}\tan ax\right\}$ —— (i)

for

$\dfrac{1}{D-ai}\tan ax=\dfrac{1}{(D-ai)}1\times\tan ax=\dfrac{1}{(D-ai)}e^{aix-aix}\tan ax$

$=\dfrac{1}{(D-ai)}e^{aix}\cdot e^{-aix}\tan ax$ $\left[\dfrac{1}{f(D)}e^{ax}\phi(x)=e^{ax}\dfrac{1}{f(D+a)}\phi(x)\right]$

$=e^{aix}\dfrac{1}{(D+ai-ai)}e^{-aix}\tan ax=e^{aix}\dfrac{1}{D}\int e^{-aix}\tan ax\,dx$

$=e^{aix}\int e^{-aix}\tan(ax)\,dx$ $\{\because e^{-i\theta}=\cos\theta-i\sin\theta\}$

$=e^{aix}\int\{\cos ax-i\sin ax\}\tan(ax)\,dx$

$=e^{aix}\int\{\cos ax-i\sin ax\}\dfrac{\sin ax}{\cos ax}\,dx$

Date: / / Page no:

$= e^{aix}\int\left\{\sin ax - \frac{i\sin^2 ax}{\cos ax}\right\}dx$

$= e^{aix}\int\left\{\sin ax - \frac{i(1-\cos^2 ax)}{\cos ax}\right\}dx$

$= e^{aix}\int\{\sin ax - i(\sec ax - \cos ax)dx\}$

$= e^{aix}\left[-\frac{\cos ax}{a} - i\left\{\frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a}\right\}\right]$ — (ii)

For $\frac{1}{D+ai}\tan ax$

$= \frac{1}{D+ai} 1 \times \tan(ax)$

$= \frac{1}{D+ai} e^{aix - aix} \tan(ax)$

$= \frac{1}{D+ai} e^{aix} \cdot e^{-aix} \tan(ax)$

$= e^{-aix}\frac{1}{(D - ai + ai)} e^{aix} \tan(ax)$

$= e^{-aix} \frac{1}{D} e^{aix} \tan(ax)$

$= e^{-aix}\int e^{aix}\tan(ax)\,dx$

$= e^{-aix}\int\{\cos ax + i\sin ax\}\tan(ax)\,dx \quad \{\because e^{i\theta} = \cos\theta + i\sin\theta\}$

$= e^{-aix}\int\{\cos ax + i\sin ax\}\frac{\sin ax}{\cos ax}\,dx$

$= e^{-aix}\int\left\{\sin ax + \frac{i\sin^2 ax}{\cos ax}\right\}dx$

$= e^{-aix}\int\left\{\sin ax + \frac{i(1-\cos^2 ax)}{\cos ax}\right\}dx$

$$= e^{-aix}\int \{\sin ax + i(\sec ax - \cos ax)\}dx$$

$$= e^{-aix}\left[-\frac{\cos ax}{a} + i\left\{\frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a}\right\}\right] \text{ --- (iii)}$$

Put (ii) & (iii) in Eqn (i)

$$P.I. = \frac{1}{2ai}\left\{\left(e^{aix}\left[-\frac{\cos ax}{a} - i\left\{\frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a}\right\}\right]\right) - \left(e^{-aix}\left[-\frac{\cos ax}{a} + i\left\{\frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a}\right\}\right]\right)\right\}$$

Complete solution $y = C.F. + P.I$

$$y = \{c_1\cos ax + c_2\sin ax\}e^{0x} + \frac{1}{2ai}\left[\left(e^{aix}\left[-\frac{\cos ax}{a} - i\left\{\frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a}\right\}\right]\right) - \left(e^{-aix}\left[-\frac{\cos ax}{a} + i\left\{\frac{\log(\sec ax + \tan ax)}{a} - \frac{\sin ax}{a}\right\}\right]\right)\right]$$

Ans

$e^{\log A} = A$

$e^{\log x} = e^{z}$

7/07/25

Cauchy

Higher order homogenous linear differential equation

$$x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 4y = Q$$

$$[x^3D^3 + 3x^2D^2 + 2xD + 4]y = Q \quad , D = d/dx$$

put $x = e^z \Rightarrow \log x = z$

$x^3D^3 \cancel{x = e^z} = D'(D'-1)(D'-2)$

$x^2D^2 = D'(D'-1)$

$xD = D'$

$D' = d/dx$

$$\Rightarrow [D'(D'-1)(D'-2) + 3D'(D'-1) + 2D' + 4]y = Q$$

07/04/25

Date: / / Page no:

Q.1 $\frac{d^2y}{dx^2} + a^2y = \sec ax$

$(D^2+a^2)y = \sec ax$

$\hookrightarrow$ Auxiliary Eqⁿ $D=M$

$M^2+a^2=0$

$M^2=-a^2 \Rightarrow M=\pm ai = \alpha \pm i\beta$

$C.F. = \{c_1\cos ax + c_2\sin ax\}e^{0x}$

$P.I. = \frac{1}{D^2+a^2}\sec ax$

$P.I. = \frac{1}{(D-ai)(D+ai)}\sec ax$

$\cancel{P.I.} = \frac{1}{(D-ai)(D+ai)} = \frac{A}{(D-ai)} + \frac{B}{(D+ai)}$

$A = \frac{1}{2ai}$, $B = \frac{-1}{2ai}$

$D-ai = 0 \Rightarrow D=ai$, $D+ai=0 \Rightarrow D=-ai$

$P.I. = \left\{\frac{1/2ai}{(D-ai)} + \frac{(-1/2ai)}{(D+ai)}\right\}\sec ax$

$P.I. = \frac{1}{2ai}\left\{\frac{1}{(D-ai)}\sec ax - \frac{1}{(D+ai)}\sec ax\right\}$ — (i)

for $\cancel{\frac{1}{D-ai}}$ [redacted]

$= \frac{1}{D-ai}\sec ax = \frac{1}{(D-ai)}\times 1\times \sec ax = \frac{1}{(D-ai)}e^{aix-aix}\times \sec ax$

$= \frac{1}{D-ai}e^{aix}\cdot e^{-aix}\sec ax \quad \left[\frac{1}{f(D)}e^{ax}\phi(x) = e^{ax}\frac{1}{f(D+a)}\phi(x)\right]$

$= e^{aix}\frac{1}{(D+ai-ai)}e^{-aix}\sec ax = e^{aix}\int e^{-aix}\sec(ax)\,dx$

$= e^{aix}\int\{\cos ax - i\sin ax\}\sec(ax)\,dx \quad \{\because e^{-i\theta} = \cos\theta - i\sin\theta\}$

$$= e^{aix}\int \{\cos ax - i\sin ax\}\frac{1}{\cos ax}dx$$

$$= e^{aix}\int\{1 - i\tan ax\}dx$$

$$= e^{aix}\left[x - \frac{i\log\sec ax}{a}\right] \quad \text{--- (ii)}$$

for $\frac{1}{(D+ai)}\sec ax = \frac{1}{(D+ai)}1\times\sec ax = \frac{1}{(D+ai)}e^{aix-aix}\sec ax$

$$= \frac{1}{D+ai}e^{aix}\cdot e^{-aix}\sec ax$$

$$= e^{-aix}\frac{1}{D-ai+ai}e^{aix}\sec ax$$

$$= e^{-aix}\frac{1}{D}e^{aix}\sec ax = e^{-aix}\int e^{aix}\sec ax$$

$\{\because e^{i\theta} = \cos\theta + i\sin\theta\}$

$$= e^{-aix}\int(\cos ax + i\sin ax)\sec ax$$

$$= e^{-aix}\int(1 + i\tan ax)$$

$$= e^{-aix}\left[x + \frac{i\log\sec ax}{a}\right] \quad \text{--- (iii)}$$

Put (ii) & (iii) in Eqn (i)

$$P.I. = \frac{1}{2ai}\left\{\left(e^{aix}\left[x - \frac{i\log\sec ax}{a}\right]\right) - \left(e^{-aix}\left[x + \frac{i\log\sec ax}{a}\right]\right)\right\}$$

Complete Soln

$$y = C.F. + P.I.$$

$$y = \{C_1\cos ax + C_2\sin ax\}e^{0x} + \frac{1}{2ai}\left[\left(e^{aix}\left[x - \frac{i\log\sec ax}{a}\right]\right) - \left(e^{-aix}\left[x + \frac{i\log\sec ax}{a}\right]\right)\right]$$

Ans

07/04/25

Date: / / Page no:

Q.2 $\frac{d^2y}{dx^2} + 4y = \tan 2x$

$(D^2+4)y = \tan 2x$

→ Auxiliary Eqn

$M^2 + 4 = 0$

$M^2 = -4 \Rightarrow M = \pm 2i = \alpha \pm \beta$ $\alpha = 0$, $\beta = 2$

$C.F. = \{C_1 \cos 2x + C_2 \sin 2x\} e^{0x}$

$P.I. = \frac{1}{D^2+4} \tan 2x = \frac{1}{D^2+4} 1 \times \tan 2x$ $\cancel{= \frac{1}{D^2+4} e^{2ix-2ix} \tan 2x}$

$P = \frac{1}{D^2+4} \tan 2x$

$= \frac{1}{(D-2i)(D+2i)} = \frac{A}{(D-2i)} + \frac{B}{(D+2i)}$

$A = \frac{1}{4i}$, $B = -\frac{1}{4i}$

$D-2i = 0$, $D = 2i$; $D+2i = 0$, $D = -2i$

$P.I. = \left\{ \frac{\frac{1}{4i}}{(D-2i)} + \frac{\left(-\frac{1}{4i}\right)}{(D+2i)} \right\} \tan 2x$

$P.I. = \frac{1}{4i} \left\{ \frac{1}{(D-2i)} \tan 2x - \frac{1}{(D+2i)} \tan 2x \right\}$ — (i)

for $\frac{1}{(D-2i)} \tan 2x = \frac{1}{(D-2i)} e^{2ix-2ix} \tan 2x = \frac{1}{(D-2i)} e^{2ix} \cdot e^{-2ix} \tan 2x$

$= e^{2ix} \frac{1}{(D+2i-2i)} e^{-2ix} \tan 2x = e^{2ix} \frac{1}{D} e^{-2ix} \tan 2x$

$= e^{2ix} \int e^{-2ix} \tan 2x \, dx$

$= e^{2ix} \int [\cos 2x - i \sin 2x] \tan 2x \, dx$ $\{ e^{-i\theta} = \cos\theta - i\sin\theta \}$

Date: / / Page no:

$$= e^{2ix}\int\left[\cos 2x - i\sin 2x\right]\frac{\sin 2x}{\cos 2x}\,dx$$

$$= e^{2ix}\int\left[\sin 2x - i\frac{(\sin^2 2x)}{\cos 2x}\right]dx$$

$$= e^{2ix}\int\left[\sin 2x - i\frac{(1-\cos^2 2x)}{\cos 2x}\right]dx$$

$$= e^{2ix}\int\left[\sin 2x - i(\sec 2x - \cos 2x)\right]dx$$

$$= e^{2ix}\left[-\frac{\cos 2x}{2} - i\left(\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2}\right)\right] \quad \text{--- (ii)}$$

for $\frac{1}{(D+2i)}\tan 2x = \frac{1}{(D+2i)} e^{2ix-2ix}\tan 2x = \frac{1}{(D+2i)} e^{2ix}\cdot e^{-2ix}\tan 2x$

$$= e^{-2ix}\frac{1}{(D-2i+2i)} e^{2ix}\tan 2x = e^{-2ix}\frac{1}{D} e^{2ix}\tan 2x$$

$$= e^{-2ix}\int e^{2ix}\tan 2x\,dx$$

$$= e^{-2ix}\int\left[\cos 2x + i\sin 2x\right]\tan 2x\,dx$$

$$= e^{-2ix}\int\left[\cos 2x + i\sin 2x\right]\frac{\sin 2x}{\cos 2x}\,dx$$

$$= e^{-2ix}\int\left[\sin 2x + i\frac{(\sin^2 2x)}{\cos 2x}\right]dx$$

$$= e^{-2ix}\int\left[\sin 2x + i\frac{(1-\cos^2 2x)}{\cos 2x}\right]dx$$

$$= e^{-2ix}\int\left[\sin 2x + i(\sec 2x - \cos 2x)\right]dx$$

$$= e^{-2ix}\left[\frac{-\cos 2x}{2} + i\left(\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2}\right)\right] \quad \text{--- (iii)}$$

Put this value (ii) & (iii) in Eqn (i)

$$P.I. = \frac{1}{4i}\left\{\left(e^{2ix}\left[-\frac{\cos 2x}{2} - i\left(\frac{\log(\sec 2x+\tan 2x)}{2} - \frac{\sin 2x}{2}\right)\right]\right) - \left(e^{-2ix}\left[-\frac{\cos 2x}{2} + i\left(\frac{\log(\sec 2x+\tan 2x)}{2} - \frac{\sin 2x}{2}\right)\right]\right)\right\}$$

Complete Sol^n $y = C.F. + P.I.$

$$y = \{c_1\cos 2x + c_2\sin 2x\}e^{x} + \frac{1}{4i}\left\{\left(e^{2ix}\left[-\frac{\cos 2x}{2} - i\left(\frac{\log(\sec 2x+\tan 2x)}{2} - \frac{\sin 2x}{2}\right)\right]\right) - \left(e^{-2ix}\left[-\frac{\cos 2x}{2} + i\left(\frac{\log(\sec 2x+\tan 2x)}{2} - \frac{\sin 2x}{2}\right)\right]\right)\right\}$$

Ans

$(D-2i)(D+2i) = D^2-(2i)^2$
$= D^2 - 2i^2$
Date: / / Page no.
$= D^2+4$

Q.3: $\frac{d^2y}{dx^2}+4y = \sec 2x$

Solⁿ → $(D^2+4)y = \sec 2x$

↳ Auxiliary eqⁿ

$M^2+4=0$

$M^2=-4 \Rightarrow M = \pm 2i = \alpha \pm \beta i$; $\alpha = 0$, $\beta = 2$

$C.F. = \{C_1\cos 2x + C_2 \sin 2x\}e^{0x}$

$P.I. = \frac{1}{D^2+4}\sec 2x$

$P.I. = \frac{1}{(D-2i)(D+2i)} = \frac{A}{(D-2i)} + \frac{B}{(D+2i)}$; $A = \frac{1}{4i}$, $B = \frac{-1}{4i}$

$D-2i=0$, $D=2i$; $D+2i=0$, $D=-2i$

$P.I. = \left\{\frac{\frac{1}{4i}}{(D-2i)} + \frac{\frac{-1}{4i}}{(D+2i)}\right\}\sec 2x$

$P.I. = \frac{1}{4i}\left\{\frac{1}{(D-2i)}\sec 2x - \frac{1}{(D+2i)}\sec 2x\right\}$ — (i)

for $\frac{1}{D-2i}\sec 2x = \frac{1}{D-2i}e^{2ix-2ix}\sec 2x = \frac{1}{D-2i}e^{2ix}\cdot e^{-2ix}\sec 2x$

$= e^{2ix}\frac{1}{D+2i-2i}e^{-2ix}\sec 2x = e^{2ix}\frac{1}{D}e^{-2ix}\sec 2x$

$= e^{2ix}\int e^{-2ix}\sec 2x\, dx$ $\quad \{\because e^{-i\theta} = (\cos\theta - i\sin\theta)\}$

$= e^{2ix}\int[\cos 2x - i\sin 2x]\sec 2x\, dx$ $\quad \sec 2x = \frac{1}{\cos 2x}$

$= e^{2ix}\int[1-i\tan 2x]dx$

$= e^{2ix}\left[x - i\frac{(\log\sec 2x)}{2}\right]$ — (ii)

Date: / / Page no:

for $\frac{1}{(D+2i)} \sec 2x = \frac{1}{(D+2i)} e^{2ix-2ix} \sec 2x$

$= \frac{1}{(D+2i)} e^{2ix} \cdot e^{-2ix} \sec 2x$

$= e^{-2ix} \frac{1}{(D-2i+2i)} e^{2ix} \sec 2x$

$= e^{-2ix} \frac{1}{D} e^{2ix} \sec 2x$

$= e^{-2ix} \int e^{2ix} \sec 2x \, dx$ $\quad \left\{ \because e^{i\theta} = (\cos\theta + i\sin\theta) \right\}$

$= e^{-2ix} \int [\cos 2x + i \sin 2x] \sec 2x \, dx$

$= e^{-2ix} \int [1 + i \tan 2x] dx$

$= e^{-2ix} \left[x + i \frac{(\log \sec 2x)}{2} \right]$ — (iii)

Put (ii) & (iii) in Eqn (i)

~~Complete Sol^n $y = C.F. + P.I.$~~

~~$y = \{c_1 \cos 2x + c_2 \sin 2x\} e^{0x} +$~~

$P.I = \frac{1}{4i} \left\{ \left(e^{2ix} \left[x - i \frac{(\log \sec 2x)}{2} \right] \right) - \left(e^{-2ix} \left[x + i \frac{(\log \sec 2x)}{2} \right] \right) \right\}$

Complete Sol^n $y = C.f. + P.I.$

$$y = \{c_1 \cos 2x + c_2 \sin 2x\} e^{0x} + \frac{1}{4i} \left\{ \left(e^{2ix} \left[x - i \frac{(\log \sec 2x)}{2} \right] \right) - \left(e^{2ix} \left[x + i \frac{(\log \sec 2x)}{2} \right] \right) \right\}$$

Ans

Date: / / Page no:

Q4 $\frac{d^2y}{dx^2} + a^2y = \operatorname{cosec}(ax)$

Sol^n $\Rightarrow$ $(D^2 + a^2)y = \operatorname{cosec}(ax)$

$\hookrightarrow$ Auxiliary Eqn

$M^2 + a^2 = 0$ $\quad \alpha = 0$, $\beta = a$

$M^2 = -a^2 \Rightarrow M = \pm ai = \alpha \pm \beta i$

$\boxed{C.f. = \{C_1 \cos ax + C_2 \sin ax\} e^{0x}}$

$P.I. = \frac{1}{D^2 + a^2} \operatorname{cosec} ax$

$P.I. = \frac{1}{(D-ai)(D+ai)} \operatorname{cosec} ax$

$= \frac{1}{(D-ai)(D+ai)} = \frac{A}{(D-ai)} + \frac{B}{(D+ai)}$ $\quad A \to \frac{1}{2ai}$, $B \to \frac{-1}{2ai}$

$\hookrightarrow = 0 \quad D = ai \qquad \hookrightarrow = 0 \quad D = -ai$

$= \left\{ \frac{\frac{1}{2ai}}{(D-ai)} + \frac{\left(\frac{-1}{2ai}\right)}{(D+ai)} \right\} \operatorname{cosec} ax$

$= \frac{1}{2ai} \left\{ \frac{1}{(D-ai)} - \frac{1}{(D+ai)} \right\} \operatorname{cosec} ax$

$P.I. = \frac{1}{2ai} \left\{ \frac{1}{(D-ai)} \operatorname{cosec} ax - \frac{1}{(D+ai)} \operatorname{cosec} ax \right\} \quad - (i)$

for $\frac{1}{(D-ai)} \operatorname{cosec} ax = \frac{1}{(D-ai)} e^{aix - aix} \operatorname{cosec} ax = \frac{1}{(D-ai)} e^{aix} \cdot e^{-aix} \operatorname{cosec} ax$

$= e^{aix} \frac{1}{(D + \cancel{ai} - \cancel{ai})} e^{-aix} \operatorname{cosec} ax = e^{aix} \frac{1}{D} e^{-aix} \operatorname{cosec} ax$

$= e^{aix}\int e^{-aix} \operatorname{cosec} ax \, dx$ $\left\{ \because e^{-i\theta} = (\cos\theta - i\sin\theta) \right\}$

$\operatorname{cosec} ax = \frac{1}{\sin ax}$

$= e^{aix}\int [\cos ax - i\sin ax] \operatorname{cosec} ax \, dx$

$= e^{aix}\int [\cot ax - i(1)] \, dx$

$= e^{aix}\left[\frac{\log(\sin ax)}{a} - ix\right]$. — (ii)

for $\frac{1}{(D+ai)} \operatorname{cosec} ax = \frac{1}{(D+ai)} e^{aix-aix} \operatorname{cosec} ax = \frac{1}{(D+ai)} e^{aix} \cdot e^{-aix} \operatorname{cosec} ax$

$= e^{-aix} \frac{1}{(D-ai+ai)} e^{aix} \operatorname{cosec} ax = e^{-aix} \frac{1}{D} e^{aix} \operatorname{cosec} ax$

$= e^{-aix}\int e^{aix} \operatorname{cosec} ax \, dx$ $\left\{ \because e^{i\theta} = \cos\theta + i\sin\theta \right\}$

$= e^{-aix}\int [\cos ax + i\sin ax] \operatorname{cosec} ax \, dx$

$= e^{-aix}\int [\cot ax + i(1)] \, dx$

$= e^{-aix}\left[\frac{\log(\sin ax)}{a} + ix\right]$ — (iii)

Put (ii) & (iii) in eqn (i)

$P.I. = \frac{1}{2ai}\left\{ \left(e^{aix}\left[\frac{\log(\sin ax)}{a} - ix\right]\right) - \left(e^{-aix}\left[\frac{\log(\sin ax)}{a} + ix\right]\right) \right\}$

~~Complete~~ Complete . Sol^n $y = C.F. + P.I$

$y = \{C_1 \cos ax + C_2 \sin ax\} e^{0x} + \frac{1}{2ai}\left\{ \left(e^{aix}\left[\frac{\log(\sin ax)}{a} - ix\right]\right) - \left(e^{-ax}\left[\frac{\log(\sin ax)}{a} + ix\right]\right) \right\}$ Ans

Date: / / Page no:

$$= e^{aix} \int e^{-aix} \operatorname{cosec} ax \, dx \qquad \left\{ \because e^{-i\theta} = (\cos\theta - i\sin\theta) \right\}$$

$$\operatorname{cosec} ax = \frac{1}{\sin ax}$$

$$= e^{aix} \int [\cos ax - i\sin ax] \operatorname{cosec} ax \, dx$$

$$= e^{aix} \int [\cot ax - i(1)] \, dx$$

$$= e^{aix} \left[\frac{\log(\sin ax)}{a} - ix \right] \quad \text{— (ii)}$$

for $\frac{1}{(D+ai)} \operatorname{cosec} ax = \frac{1}{(D+ai)} e^{aix-aix} \operatorname{cosec} ax = \frac{1}{(D+ai)} e^{aix} \cdot e^{-aix} \operatorname{cosec} ax$

$$= e^{-aix} \frac{1}{(D-ai+ai)} e^{aix} \operatorname{cosec} ax = e^{-aix} \frac{1}{D} e^{aix} \operatorname{cosec} ax$$

$$= e^{-aix} \int e^{aix} \operatorname{cosec} ax \, dx \qquad \left\{ \because e^{i\theta} = \cos\theta + i\sin\theta \right\}$$

$$= e^{-aix} \int [\cos ax + i\sin ax] \operatorname{cosec} ax \, dx$$

$$= e^{-aix} \int [\cot ax + i(1)] \, dx$$

$$= e^{-aix} \left[\frac{\log(\sin ax)}{a} + ix \right] \quad \text{— (iii)}$$

Put (ii) & (iii) in Eqn (i)

$$P.I. = \frac{1}{2ai} \left\{ \left(e^{aix} \left[\frac{\log(\sin ax)}{a} - ix \right] \right) - \left(e^{-aix} \left[\frac{\log(\sin ax)}{a} + ix \right] \right) \right\}$$

~~Comple~~ complete Sol^n $y = C.F. + P.I$

$$y = \{C_1 \cos ax + C_2 \sin ax\} e^{0x} + \frac{1}{2ai} \left\{ \left(e^{aix} \left[\frac{\log(\sin ax)}{a} - ix \right] \right) - \left(e^{-ax} \left[\frac{\log(\sin ax)}{a} + ix \right] \right) \right\}$$

Ans

07/04/25

Date: / / Page no:

Q.5 $\Rightarrow \dfrac{d^2y}{dx^2} + 4y = \operatorname{Cosec}(2x)$

$(D^2+4)y = \operatorname{Cosec} 2x$

$\rightarrow$ Auxiliary Eqn

$M^2 + 4 = 0$

$M^2 = -4 \Rightarrow M = \pm 2i = \alpha \pm \beta i$; $\alpha = 0$, $\beta = 2$

$C.f. = \{C_1 \cos 2x + C_2 \sin 2x\} e^{0x}$

$P.I. = \dfrac{1}{D^2+4} \operatorname{Cosec} 2x$

$= \dfrac{1}{(D-2i)(D+2i)} = \dfrac{A}{(D-2i)} + \dfrac{B}{(D+2i)}$; $A \to \dfrac{1}{4i}$, $B \to \dfrac{-1}{4i}$

$(D-2i) \to = 0$, $D = 2i$; $(D+2i) \to = 0$, $D = -2i$

$P.I. = \left\{ \dfrac{1/4i}{(D-2i)} + \dfrac{(-1/4i)}{(D+2i)} \right\} \operatorname{Cosec}(2x)$

$P.I. = \dfrac{1}{4i} \left\{ \dfrac{1}{(D-2i)} - \dfrac{1}{(D+2i)} \right\} \operatorname{Cosec}(2x)$

$P.I. = \dfrac{1}{4i} \left\{ \dfrac{1}{(D-2i)} \operatorname{Cosec} 2x - \dfrac{1}{(D+2i)} \operatorname{Cosec} 2x \right\}$ — (i)

for $= \dfrac{1}{(D-2i)} \operatorname{Cosec} 2x = \dfrac{1}{(D-2i)} e^{2ix-2ix} \operatorname{Cosec} 2x = \dfrac{1}{(D-2i)} e^{2ix} \cdot e^{-2ix} \operatorname{Cosec} 2x$

$= e^{2ix} \dfrac{1}{(D+2i-2i)} e^{-2ix} \operatorname{Cosec} 2x - e^{2ix} \dfrac{1}{D} e^{-2ix} \operatorname{Cosec} 2x$

$= e^{2ix} \int e^{-2ix} \operatorname{Cosec} 2x \, dx$ $\quad \{\because e^{-i\theta} = \cos\theta - i\sin\theta\}$

$= e^{2ix} \int [\cos 2x - i \sin 2x] \operatorname{Cosec} 2x \, dx$

$= e^{2ix} \int [\cot 2x - i(1)] \, dx$

$= e^{2ix} \left[\dfrac{\log \sin 2x}{2} - ix \right]$ — (ii)

$y = (\cdot$

Date: / / Page no:

for $= \frac{1}{(D+2i)} \operatorname{Cosec} 2x = \frac{1}{(D+2i)} e^{2ix-2ix} \operatorname{Cosec} 2x = \frac{1}{(D+2i)} e^{2ix} \cdot e^{-2ix} \operatorname{cosec}$

$= e^{-2ix} \frac{1}{(D-2i+2i)} e^{2ix} \operatorname{Cosec} 2x$

$= e^{-2ix} \frac{1}{D} e^{2ix} \operatorname{Cosec} 2x$

$= e^{-2ix} \int e^{2ix} \operatorname{Cosec} 2x \, dx \qquad \left\{ \because e^{i\theta} = \cos\theta + i \sin\theta \right\}$

$= e^{-2ix} \int [\cos 2x + i \sin 2x] \operatorname{Cosec} 2x \, dx$

$= e^{-2ix} \int [\cot 2x + i(1)] \, dx$

$= e^{-2ix} \left[\frac{\log \sin 2x}{2} + ix \right] \quad \text{—} \; (iii)$

Put (ii) & (iii) in Eqn (i)

$P.I. = \frac{1}{4i} \left\{ \left(e^{2ix} \left[\frac{\log \sin 2x}{2} - ix \right] \right) - \left(e^{-2ix} \left[\frac{\log \sin 2x}{2} + ix \right] \right) \right\}$

Complete Sol^n $y = C.f. + P.I.$

$y = \left\{ C_1 \cos 2x + C_2 \sin 2x \right\} e^{0x} + \left\{ \frac{1}{4i} \left[\left(e^{2ix} \left[\frac{\log \sin 2x}{2} - ix \right] \right) - \left(e^{-2ix} \left[\frac{\log \sin 2x}{2} + ix \right] \right) \right] \right\}$ Ans

$1 \quad -1 + 2$

$(-1) - 1 + 2$

08/04/25

$m^2 - 2m + 2$

-2×1

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2}$

$\frac{2 \pm 2i}{2}$

Printed

Solve the diff. Eqn $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = \left(x + \frac{1}{x}\right)$

$\left[x^3D^3 + 2x^2D^2 + 2\right]y = \left(x + \frac{1}{x}\right)$ $\quad m = -1$

$D = d/dx$

$C_1 e^{m_1 x} + (C_2\cos x + C_3\sin x)e^{x}$

Put $x = e^z \Rightarrow \log x = z$

By this substitution

$C_1 e^{-z} + (C_2\cos z + C_3\sin z)e^{z}$

$x^3D^3 = D'(D'-1)(D'-2)$

$x^2D^2 = D'(D'-1)$ $\quad \Big\}\ D' = d/dz$

$xD = D'$

$\left\{D'(D'-1)(D'-2) + 2D'(D'-1) + 2\right]y = e^z + \frac{1}{e^z}$

$\left\{D'^3 - 3D'^2 + \cancel{2D'} + 2D'^2 - \cancel{2D'} + 2\right\}y = e^z + e^{-z}$

$\left\{D'^3 - D'^2 + 2\right\}y = e^z + e^{-z}$

$M^3 - m^2 + 2 = 0$

$m = -1$

$$\begin{array}{c|cccc} & 1 & -1 & 0 & 2 \\ m = -1 & - & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$m^2 - 2m + 2 = 0$

$m = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times 2}}{2 \times 1}$

$m = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2}$

$m = 1 \pm i = \alpha \pm i\beta$

$\alpha = 1,\ \beta = 1$

$m = -1,\ 1 \pm i$

$C.F. = C_1 e^{-z} + \{C_2\cos z + C_3 \sin z\}e^{z}$

$P.I. = \frac{1}{(D'^3 - D'^2 + 2)}(e^z + e^{-z})$

$P.I. = \frac{e^z}{(D'^3 - D'^2 + 2)} + \frac{e^{-z}}{D'^3 - D'^2 + 2}$

$P.I. = \frac{e^z}{1^3 - 1^2 + 2} + \frac{e^{-z}}{(-1)^3 - (-1)^2 + 2}$

$P.I. = \frac{e^z}{2} + \frac{e^{-z}}{0}$ $\quad -1-1+2$ case

$P.I. = \frac{e^z}{2} + z\frac{e^{-z}}{3D'^2 - 2D'}$

$P.I. = \frac{e^z}{2} + z\frac{e^{-z}}{3(-1)^2 - 2(-1)}$

$P.I. = \frac{e^z}{2} + z\frac{e^{-z}}{5}$ Ans

$y = C.F. + P.I.$

$y = C_1 e^{-z} + \{C_2\cos z + C_3 \sin z\}e^{z} + \frac{e^z}{2} + z\frac{e^{-z}}{5}$ Ans

Date: / / Page no:

$y = C_1\frac{1}{x} + \{C_2\cos(\log x) + C_3\sin(\log x)\}x + \frac{1}{2}(x) + \frac{\log x}{5}\frac{1}{x}$

$\log x^2$

09/04/25

m^2+m-12

$C_1e^{3z}+C_2e^{-4z}$

$$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = x^3\log x$$

$[x^2D^2+2xD-12]y = x^3\log x,\ D = d/dx$

$x = e^z \Rightarrow \log x = z$

$x^2D^2 = D'(D'-1)$, $xD = D'$, $D = d/dz$

$\{D'(D'-1)+2D'-12\}y = (e^z)^3 z$

$\{D'^2+D'-12\}y = e^{3z}z$

$m^2+m-12=0$

$m = 3,-4$

$C.f. = C_1e^{3z}+C_2e^{-4z}$

$P.I. = \frac{1}{D'^2+D'-12}e^{3z}z$

$P.I. = e^{3z}\frac{1}{(D'+3)^2+(D'+3)-12}(z)$

$P.I. = e^{3z}\frac{1}{D'^2+6D'+9+D'+3-12}(z)$

$P.I. = e^{3z}\frac{1}{D'^2+7D'}(z)$

$P.I. = e^{3z}\frac{1}{7D'\left(1+\frac{D'}{7}\right)}(z)$

$P.I. = \frac{1}{7D'}\left\{1+\frac{D'}{7}\right)^{-1}(z)$

$P.I. = (1+p)^n = 1+np+\frac{n(n-1)}{\lfloor 2}p^2+\dots$

$= \frac{1}{7D}\left\{1+(-1)\left(\frac{D'}{7}\right)+\dots\right\}(z)$

$= \frac{1}{7D}\left\{1+(-1)\left(\frac{D'}{7}\right)\right.$

$= \frac{1}{7D}\left\{z+(-1)\left(\frac{D'(z)}{7}\right)+\dots\right\}$

$= \frac{1}{7D}\left\{z-\frac{1}{7}\right\} = \frac{1}{7}\int\left(z-\frac{1}{7}\right) =$

$= \frac{1}{7}\left\{\frac{z^2}{2}-\frac{1}{7}z\right\} = P.I.$

$y = C.f. + P.I. = \{C_1e^{3z}+C_2e^{-4z}\} + \frac{1}{7}\left\{\frac{z^2}{2}-\frac{1}{7}z\right\}$

$y = \left\{C_1x^3+\frac{C_2}{x^4}\right\}+\frac{1}{7}\left\{\frac{(\log x)^2}{2}-\frac{1}{7}\log x\right\}$

$z\sin z$ $\quad M^2+1=0$, $M^2=-1$, $M=\pm i$ $\quad$ 08/04/25

Date: / / Page no:

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{y}{x}=\frac{\log x\sin(\log x)}{x}$$

Multiply By x

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\log x\sin(\log x)$$

$$(x^2D^2+xD+1)y=\log x\sin(\log x)$$

$x^2D^2=D'(D'-1)$ $\quad$ put $x=e^z$, $\log x=z$

$xD=D'$

$$(D'(D'-1)+D'+1)y=z\sin(z)$$

$$\{D'^2+1\}y=z\sin z$$

$M^2+1=0$

$m=\pm i=\{c_1\cos z+c_2\sin z\}e^{0z}$

$$P.I.=\frac{1}{D^2+1}z\sin z$$

$$P.I.=\frac{1}{f(D)}\left(x^mV=x^m\frac{1}{f(D)}V+mx^{m-1}\frac{d}{dx}\left(\frac{1}{f(D)}\right)V+\cdots\right.$$

$$P.I.=\frac{1}{(D'^2+1)}(z\sin z)=z\frac{1}{(D'^2+1)}\sin z+1\cdot z^{1-1}\frac{d}{dD'}\left\{\frac{1}{D'^2+1}\right\}\sin z$$

$$P.I.=z\frac{1}{-1+1}\sin z+(-1)(D'^2+1)^{-2}(2D')(\sin z)$$

$$P.I.=z\frac{1}{D'^2+1}\sin z-\frac{2D'}{(D'^2+1)^2}(\sin z)\quad\to D'^4+2D'^2+1$$

$$\cancel{P.I.=z^2\frac{1}{2D'}\sin z-z\frac{2D'}{D'}(-2D')}$$

$$P.I.=z^2\frac{1}{2D'}\sin z-z\frac{2D'}{4D'^3+4D'}\sin z$$

$$P.I.=\frac{z^2}{2}\frac{D'\sin z}{D'^2}-z\frac{2D'}{}$$