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find series sol^n of differential Eq^n

$$(x-x^2)\frac{d^2y}{dx^2} + (1-5x)\frac{dy}{dx} - 4y = 0$$

$P_0 = (x-x^2) \quad P_1 = (1-5x) \quad P_2 = -4$

at $x=0$

$P_0 = 0$

$x=0$ is called singular point

$$y = \sum_{k=0}^{\infty} a_k x^{m+k} \quad (1)$$

$$\frac{dy}{dx} = \sum_{k=0}^{\infty} a_k (m+k) x^{m+k-1} \quad (2)$$

$$\frac{d^2y}{dx^2} = \sum_{k=0}^{\infty} a_k (m+k)(m+k-1) x^{m+k-2} \quad (3)$$

Put $y, \frac{dy}{dx}, \frac{d^2y}{dx}$ from (1),(2),(3) in (A)

$$(x-x^2)\sum_{k=0}^{\infty} a_k(m+k)(m+k-1)x^{m+k-2} + (1-5x)\sum_{k=0}^{\infty} a_k(m+k)x^{m+k-1} - 4\sum_{k=0}^{\infty} a_k x^{m+k} = 0$$

$$\sum_{k=0}^{\infty} a_k(m+k)(m+k-1)x^{m+k-1} - \sum_{k=0}^{\infty} a_k(m+k)(m+k-1)x^{m+k}$$
$$+ \sum_{k=0}^{\infty} a_k(m+k)x^{m+k-1} - 5\sum_{k=0}^{\infty} a_k(m+k)x^{m+k} - 4\sum_{k=0}^{\infty} a_k x^{m+k} = 0$$

$$\sum_{k=0}^{\infty} a_k\{(m+k)(m+k-1) + (m+k)\}x^{m+k-1} - \sum_{k=0}^{\infty} a_k\{(m+k)(m+k-1) + 5(m+k) + 4\}x^{m+k} = 0$$

$$\sum_{k=0} a_k\{(m+k)^2 - (m+k) + (m+k)\}x^{m+k-1} - \sum_{k=0}^{\infty} a_k\{(m+k)^2 - (m+k) + 5(m+k) + 4\}x^{m+k} = 0 \quad (B)$$

$$\sum_{k=0}^{\infty} a_k \{(m+k)^2\} x^{m+k-1} - \sum_{k=0}^{\infty} a_k \{(m+k)^2+4(m+k)+4\} x^{m+k} = 0 \quad (4)$$

on equating Lowest degree term i.e x^{m-1} which is obtained by putting $k=0$ in I^{st} Σ only

$a_0 m^2 = 0$ but $a_0 \neq 0 \Rightarrow m = 0, 0$

on Equating Coefficient of x^m which is obtained by Putting $k=1$ in I^{st} Σ and $k=0$ in II^{nd} Σ

$$\Rightarrow a_1 (m+1)^2 - a_0 \{m^2+4m+4\} = 0$$

$$a_1 = \frac{a_0 (m+2)^2}{(m+1)^2}$$

on Equating Coefficient of x^{m+1} which is obtained by Putting $k=2$ in I^{st} Σ and $k=1$ in II^{nd} Σ

$$a_2 (m+2)^2 - a_1 \{(m+1)^2 + 4(m+1) + 4\} = 0$$

$$a_2 = \frac{a_1 \{(m+1)^2 + 4(m+1) + 4\}}{(m+2)^2}$$

$$a_2 = \frac{\{(m+1)^2 + 4(m+1) + 4\}}{(m+2)^2} \frac{a_0 (m+2)^2}{(m+1)^2}$$

$$a_2 = (4m+8) a_0$$

on Equating coefficient of x^{m+2} which is obtained by Putting $k=3$ in I^{st} Σ and $k=2$ in II^{nd} Σ

$$a_3 (m+3)^2 - a_2 \{(m+2)^2 + 4(m+2) + 4\} = 0$$

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$$a_3 = \frac{a_2 \{(m+2)^2 + 4(m+2) + 4\}}{(m+3)^2}$$

$$a_3 = \frac{\{(m+2)^2 + 4(m+2) + 4\}(4m+8)a_0}{(m+3)^2}$$ ---

Put a_0, a_1, a_2, a_3 in

$$y = \sum_{k=0}^{\infty} a_k x^{m+k} \quad \text{--- (1)}$$

$$y = a_0 x^m + a_1 x^{m+1} + a_2 x^{m+2} + a_3 x^{m+3} + \cdots$$

$$y = x^m \left\{ a_0 + \left\{\frac{a_0 (m+2)^2}{(m+1)^2}\right\} x + \{(4m+8)a_0\} x^2 + \left\{\frac{(m+2)^2 + 4(m+2) + 4}{(m+3)^2}\{4m+8\}a_0\right\} x^3 \right\}$$

Ans