17/06/25 analitic funcn

Contan diff

Conjugate

UNIT- IV

Date: / / Page no:

Complex variable

funcn

$i = \sqrt{-1}$

C Const

$a = 2 + 3i$

$Z = x + iy, \bar{Z} = x - iy$

$|Z| = \sqrt{x^2+y^2}$

$z = x + iy$ = Complex variable

$\boxed{R.P = x}$

$\boxed{I.P = y}$

$\boxed{f(z)} = z^2 + 3 = (x+iy)^2 + 3$

complex funcn $= x^2 + (iy)^2 + 2xyi + 3$

$f(z) = (x^2 - y^2 + 3) + (2xy)i$

$f(z) = u + iv$

$u = x^2 - y^2 + 3 \quad | \quad v = 2xy$

$fx \Rightarrow \boxed{f(x)} \Rightarrow x^2+2x+3$

cos x

$\lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$

$\lim_{\delta z \to} \frac{f(z+\delta z) - f(z)}{\delta z}$

$\lim_{\delta y \to} \frac{f(y+\delta y) - f(y)}{\delta y}$

Analytic funcn:

- → f(z) will be analytic → if it is single valued Evaluate
- → if it is differentiable in given domain Reimann
- → If f(z) satisfied Cauchy - Rimenn (C-R) Equation then f(z) called analytic funcn

→ If $f(z) = u + iv$ then (C-R) Eqn will be

$$C\text{-}R \Rightarrow \boxed{\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}} \text{ and } \boxed{\frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}}$$

Date: / / Page no: ____

$f(z) \Rightarrow z^2 + 3$

$f(z) = (x^2 - y^2 + 3) + 2xyi$

$u = x^2 - y^2 + 3 \mid v = 2xy$

$\dfrac{\partial u}{\partial x} = 2x \mid \dfrac{\partial u}{\partial y} = -2y$

$\dfrac{\partial v}{\partial x} = 2y \mid \dfrac{\partial v}{\partial y} = 2x$

$\Rightarrow$ C-R Eq^n

$\therefore f(z) \Rightarrow$ analytic

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \;\Big|\; \frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$$

Q.1 $\Rightarrow$ If $f(z) = z^2 - 4i$ wheather it is analytic or not

$f(z) = z^2 - 4i$

$f(z) = (x + iy)^2 - 4i$

$f(z) = x^2 - y^2 + 2xyi - 4i$

$f(z) = (x^2 - y^2) + (2xy - 4)i$

$u = x^2 - y^2 \mid v = 2xy - 4$

$\dfrac{\partial u}{\partial x} = 2x \mid \dfrac{\partial u}{\partial y} = -2y$

$\dfrac{\partial v}{\partial x} = 2y \mid \dfrac{\partial v}{\partial y} = 2x$

$\Rightarrow$ C-R Eq^n are

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \;\Big|\; \frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$$

$\cosh x = e^{x}$

$\cos(ix) = \cosh x$
$\sin(ix) = i \sinh x$

$f(z) = \cos(z) + 3\sin(z)$ wheat weather it is analytical or not

Solⁿ →

$$f(z) = \cos(z) + 3\sin(z)$$

$$f(z) = \cos(x+iy) + 3\sin(x+iy)$$

$$f(z) = \cos(x)\cos(iy) - \sin(x)\sin(iy) + 3\{\sin(x)\cos(iy) + \cos(x)\sin(iy)\}$$

$$f(z) = \cos x \cosh y - i \sin x \sinh y + 3 \sin x \cosh y + 3i \cos x \sinh y$$

$$f(z) = \{\cos x \cosh y + 3 \sin x \cosh y\} + i\{3\cos x \sinh y - \sin x \sinh y\}$$

$$u = \cos x \cosh y + 3 \sin x \cosh y$$

$$\frac{\partial u}{\partial x} = -\sin x \cosh y + 3\cos x \cosh y$$

$$\frac{\partial u}{\partial y} = -\cos x \sinh y - 3 \sin x \sinh y \quad \boxed{\text{①}}$$

$$V = 3\cos x \sinh y - \sin x \sinh y$$

$$\frac{\partial v}{\partial x} = -3\sin x \sinh y - \cos x \sinh y \quad \boxed{\text{①}}$$

$$\frac{\partial v}{\partial y} = 3\cos x \cosh y - \sin x \cosh y$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad / \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

→ C.R Eqⁿ

$f(z)$ is analytic

greens theorem
Printed

18/Jun/25

Date: / / Page no:

Method to Construct analytic function

Malney thomson mtd

we have to find $f(z) = u + iv$

Case 1st :- If u is given then we have to find $f(z)$

$\boxed{f(z) = \boxed{u} + iv}$

(1) $\frac{\partial u}{\partial x} = \phi_1(x, y)$ {let}

$\phi_1(z, 0) = ?$

(2) $\frac{\partial u}{\partial y} = \phi_2(x, y)$ {let}

$\Rightarrow \phi_2(z, 0) = ?$

$$f(z) = \int \phi_1(z, 0) - i \int \phi_2(z, 0)$$

Case 2nd If v is given and we have to find $f(z)$

$f(z) = ?$

$\frac{\partial v}{\partial y} = \psi_1(x, y)$ {let}

$\Rightarrow \psi_1(z, 0) = ?$

$\frac{\partial v}{\partial x} = \psi_2(x, y)$ {let}

$\psi_2(z, 0) = ?$

$$f(z) = \int \psi_1(z, 0) + i \int \psi_2(z, 0)$$

18/Jun/25

discovery of India

Date: / / Page no:

Q1 If $e^x \sin y + e^y \cos x$ is real part of analytic funcn then find that analytic funcn.

$$u = e^x \sin y + e^y \cos x$$

$$\frac{\partial u}{\partial x} = e^x \sin y - e^y \sin x = \phi_1(x,y)$$

$$\phi_1(z,0) = 0 - e^0 \sin z = -\sin z$$

$$\frac{\partial u}{\partial y} = e^x \cos y + e^y \cos x = \phi_2(x,y)$$

$$\phi_2(z,0) = e^z \cos(0) + e^0 \cos z$$

$$\phi_2(z,0) = e^z + \cos z$$

$$f(z) = \int \phi_1(z,0) - i \int \phi_2(z,0)$$

$$f(z) = \int (-\sin z) - i \int (e^z + \cos z)$$

$$f(z) = -(-\cos z) - i \{e^z + \sin z\}$$

$$f(z) = \cos z - i (e^z + \sin z \}$$

Q.2 ~~$xe^y + x^2 - y \sin x$~~ $xe^y + x^2 - y \sin x$ is Imaginary part of analytic funcn then find analytic func

$$V = xe^y + x^2 - y \sin x$$

$$\frac{\partial V}{\partial y} = xe^y + 0 - \sin x = \psi_1(x,y)$$

$$\Rightarrow \psi_1(z,0) = ze^0 - \sin z = z - \sin z$$

$$\frac{\partial V}{\partial x} = e^y + 2x - y \cos x = \psi_2(x,y)$$

$$\Rightarrow \psi_2(z,0) = e^0 + 2z - 0 \cos z = 1 + 2z$$

$$f(z) = \int \psi_1(z,0) + i \int \psi_2(z,0)$$

$$f(z) = \int (z - \sin z) + i \int (1 + 2z)$$

$$f(z) = \frac{z^2}{2} + \cos z + i \left(z + \frac{2z^2}{2}\right)$$

$$f(z) = \frac{z^2}{2} + \cos z + i (z + z^2)$$

18/JUN/25

Date: / / Page no:

Q.3 I.P $= e^x \sin y + \cos(x+y)$

$f(z) = ?$

R.P $= e^y \sin x - \sin(x-y)$

$f(z) = ?$

Q.3 $v = e^x \sin y + \cos(x+y)$

$v = e^x \sin y + \cos x \cos y - \sin x \sin y$

$\frac{\partial v}{\partial y} = e^x \cos y - \cos x \sin y - \sin x \cos y = \psi_1(x,y)$

$\Rightarrow \psi_1(z,0) = e^z \cos(0) - \cos z \sin(0) - \sin z \cos(0)$

$\psi_1(z,0) = e^z - 0 - \sin z = e^z - \sin z$

$\frac{\partial v}{\partial x} = e^x \sin y - \sin x \cos y - \cos x \sin y = \psi_2(x,y)$

$\Rightarrow \psi_2(z,0) = e^z \sin(0) - \sin(z)\cos(0) - \cos(z)\sin(0)$

$\psi_2(z,0) = 0 - \sin z - 0 = -\sin z$

$f(z) = \int \psi_1(z,0) + i \int \psi_2(z,0)$

$f(z) = \int (e^z - \sin z) + i \int (-\sin z)$

$f(z) = e^z + \cos z + i \cos z$

~~$f(z) = e^z + 2\cos z$~~

$f(z) = e^z + \cos z + i \cos z$

~~$f(z) =$~~

18|JUN|25

$$u = e^y \sin x - \sin(x-y) = e^y \sin x - [\sin x \cos y - \cos x \sin y]$$

$$f(z) = ?$$

$$\frac{\partial u}{\partial x} = e^y \cos x - \cos x \cos y - \sin x \sin y = \phi_1(x,y)$$

$$\phi_1(z,0) = e^0 \cos z - \cos z \cos 0 - \sin z \sin 0$$

$$\phi_1(z,0) = \cos z - \cos z = 0$$

$$\frac{\partial u}{\partial y} = \cancel{e^y \sin x + \sin x \sin y}$$

$$\frac{\partial u}{\partial y} = e^y \sin x + \sin x \sin y + \cos x \cos y = \phi_2(x,y)$$

$$\phi_2(z,0) = e^0 \sin z + \sin z \sin 0 + \cos z \cos 0$$

$$\phi_2(z,0) = \sin z + 0 + \cos z = \sin z + \cos z$$

$$f(z) = \int \phi_1(z,0) - i \int \phi_2(z,0)$$

$$f(z) = \int 0 - i \int (\sin z + \cos z)$$

$$f(z) = -i(-\cos z + \sin z)$$

$$f(z) = i(\cos z - \sin z)$$

18/JUN/25

$z = x+iy$

$|z| = \sqrt{x^2+y^2} = r \Rightarrow \boxed{|z| = r}$

$x^2+y^2=r^2$ -> Eqn of circle with centre (0,0)

$|z|=r$

$x = r\cos\theta$ | $y = r\sin\theta$

-> $z = x+iy = r\cos\theta + ir\sin\theta$

$z = r\{\cos\theta + i\sin\theta\}$

$\boxed{z = re^{i\theta}}$

(5,0) rad = r (a,0) centre, rad = r

$(x-a)^2+y^2=r^2$

$|z-a|=r$

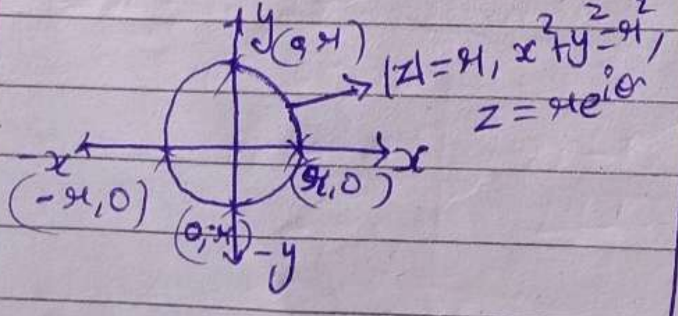
$|x+iy-a|=r$

$|(x-a)+iy|=r$

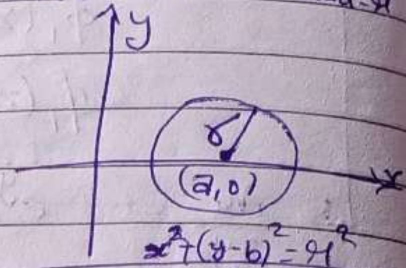
$\sqrt{(x-a)^2+y^2}=r$

$(x-a)^2+y^2=r^2$

$x^2+y^2=r^2$
$|z|=r$
$z=re^{i\theta}$
} circle Eqn centre (0,0) rad = r



$(x-a)^2+y^2=r^2$
$|z-a|=r$
$(z-a)=re^{i\theta}$
$z=re^{i\theta}+a$
} Eqn of circle centre (a,0) rad = r



$x^2+(y-b)^2=r^2$ (0,b) rad = r

$|z-ib|=r$

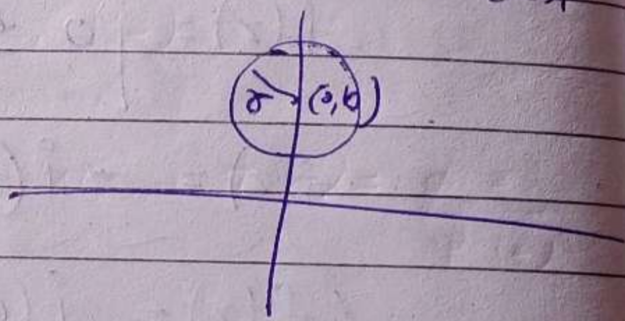
$|x+iy-ib|=r$

$|x+i(y-b)|=r$

$\sqrt{x^2+(y-b)^2}=r$

$x^2+(y-b)^2=r^2$

$x^2+(y-b)^2=r^2$
$|z-ib|=r$
$(z-ib)=re^{i\theta}$
$z=re^{i\theta}+ib$
} Eqn of circle with centre (0,b) rad = r



18|JUN|25

$(x-a)^2+(y-b)^2 = r^2$, centre (a,b)
rad = r

$|z-a-ib| = r$

$|x+iy-a-ib| = r$

$|(x-a)+i(y-b)| = r$

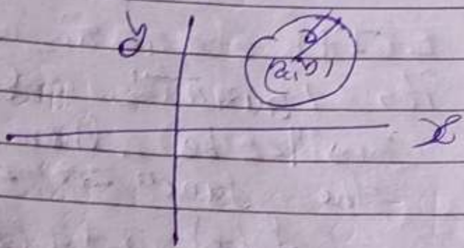
$\sqrt{(x-a)^2+(y-b)^2} = r$

$(x-a)^2+(y-b)^2 = r^2$

$(z-a-ib) = re^{i\theta}$

$z = a+ib+re^{i\theta}$

→ Eqn of circle with centre (a,b), rad = r



Pole of analytic funcn

If $z=a$ is point such that $f(z)$ vanishes it is called Pole of analytic funcn

$$f(z) = \frac{(z-1)}{(z-2)(z-5)}$$

$z=2, z=5$

Types of Poles

(1) Simple Pole :- A Pole of 1st order is called simple pole

(2) Pole of n^{th} order, when pole is repeated it is called pole of n^{th} order

$$f(z) = \frac{(z+3)}{(z+2)(z-5)^4}$$

$z=-2$ → single pole
$z=5,5,5,5$
↳ Pole of 4th order

18/06/25

Date: / / Page no:

zeroes $\boxed{z=a}$ is point such that $f(z)$ become zero then it is called zeroes of analytic funcⁿ

~~Residing~~ Residue.

If $z=a$ is pole then ~~at z=a~~ its Laurent's Expansion about $z=a$ will be then b_1 is called residue of the funcⁿ at $z=a$ that is in Laurent's Expansion

$$f(z) = a_0 + a_1(z-a) + a_2(z-a)^2 + \cdots$$
$$+ b_1(z-a)^{-1} + b_2(z-a)^{-2} + \cdots$$

Co-efficient of $z=a^{-1}$ that is value of b_1 is called residue

Mtd to find residue

(case 1) $\boxed{z=a} \to$ simple pole

(case a) $$Res(z=a) = \lim_{z \to a} (z-a) f(z)$$

(case b) $$Res(z=a) = \frac{\phi(a)}{\psi'(a)} \quad \text{where } f(z) = \frac{\phi(z)}{\psi(z)}$$

(Case 2) If $f=a$ is pole of n^{th} order then residue

$$Res(z=a) = \frac{1}{\lfloor n-1} \left\{ \frac{d^{n-1}}{dz^{n-1}} (z-a)^n f(z) \right\}_{z=a}$$

19/06/25

Date: / / Page no:

Q1 Find Residue and nature of pole

$$f(z) = \frac{z}{(z+1)(z-3)^2}$$

for pole $(z+1)(z-3)^2 = 0$

$z = -1, \; z = 3, 3 \Rightarrow z = -1$ is simple pole

$z = 3$ pole of 2^{nd} order

$$\text{Res}(z=a) = \lim_{z \to a} (z-a) f(z)$$

$$\text{Res}(z=-1) = \lim_{z \to -1} (z+1) \times \frac{z}{(z+1)(z-3)^2}$$

$$\text{Re}(z=-1) = \frac{-1}{\{(-1)-3\}^2} = \frac{-1}{16} = R_1$$

$$\text{Res}(z=a) = \frac{1}{\lfloor n-1} \left\{ \frac{d^{n-1}}{dz^{n-1}} (z-a)^n f(z) \right\}_{z=a}$$

$$\text{Res}(z=3) = \frac{1}{\lfloor 2-1} \left\{ \frac{d}{dz} \cancel{(z-3)^2} \times \frac{z}{(z+1)\cancel{(z-3)^2}} \right\}_{z=3}$$

$$\text{Res}(z=3) = \left\{ \frac{d}{dz} \left(\frac{z}{z+1} \right) \right\}_{z=3}$$

$$\text{Res}(z=3) = \left\{ \frac{(z+1).1 - z \times 1}{(z+1)^2} \right\}_{z=3} = \frac{1}{16} = R_2$$

19/06/25

Date: / / Page no:

Q2 If $f(z) = \cot z$ then find nature of pole and residue

Solve ⇒ $f(z) = \cot z = \frac{\cos z}{\sin z}$

for pole $\sin z = 0$

$z = n\pi = 0, \pi, 2\pi, 3\pi, \ldots, -\pi, -2\pi, -3\pi, \ldots$

$z = n\pi$ is simple pole

$$\text{Res}(z = n\pi) = \lim_{z \to n\pi} (z - n\pi) \frac{\cos z}{\sin z}$$ Not valid

So we have (case 2nd)

$$\boxed{\text{Res}(z=a) = \frac{\phi(a)}{\psi'(a)}}$$

$f(z) = \frac{\cos z}{\sin z} = \frac{\phi(z)}{\psi(z)}$ $\psi(z) = \sin z$, $\psi'(z) = \cos z$

$$\text{Res}(z = n\pi) = \frac{\phi(n\pi)}{\psi'(n\pi)} = \frac{\cos z}{\cos z} = 1 = R$$

Q.3 If $f(z) = \frac{(z+3)e^z}{z^2(z^2-3z+2)}$ find nature of pole & Residue

for Plole

$z^2(z^2-3z+2) = 0$

$z = 0, 0$, $z^2 - 3z + 2 = 0$

$z^2 - 2z - z + 2 = 0$

$(z-1)(z-2) = 0$

$z = 1, 2$

$z = 0, 0$ is pole of 2nd order

$z = 1, 2$ is simple pole

$$\text{Res}(z=0) = \frac{1}{\lfloor 2-1} \left\{ \frac{d}{dz} (z-0)^2 \frac{(z+3)e^z}{z^2(z^2-3z+2)} \right\}_{z=0}$$

19/06/25

$$= \left\{ \frac{(z^2-3z+2)e^z + e^z(z+3) - e^z(z+3)(2z-3)}{(z^2-3z+2)^2} \right\}_{z=0}$$

$$Res(z=0) = \frac{2(4)+9}{4} = 17/4 = R_1$$

$z=1$ is simple Pole

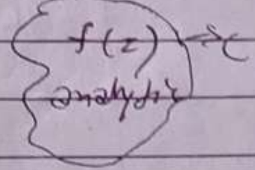
$$Res(z=1) = \lim_{z\to 1} (z-1) \times \frac{(z+3)e^z}{z^2(z-1)(z-2)} = \frac{4\times e}{-1} = R_2$$

$z=2$ is simple pole

$$Res(z=2) = \lim_{z\to 2} (z-2) \times \frac{(z+3)e^z}{z^2(z-1)(z-2)} = \frac{5e^2}{4} = R_3$$

Cauchy's Integral theorem

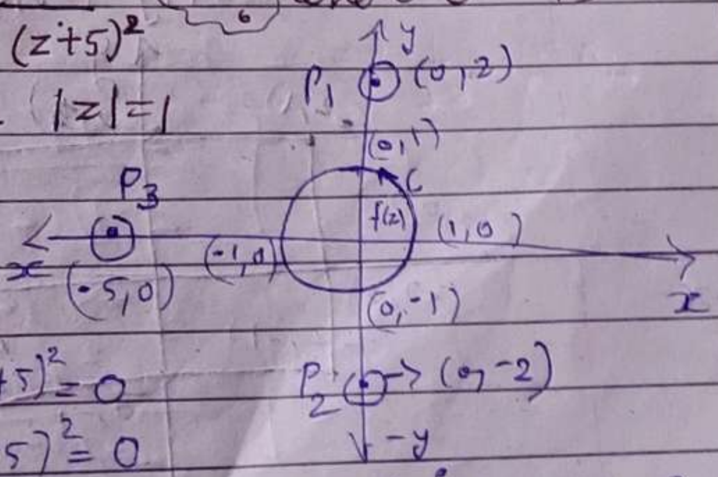
If $f(z)$ is analytic inside and on closed curve (C) then $\int_C f(z)\,dz = 0$



Q.1 Evaluate $\int_C \frac{z^2+3}{(z^2+4)(z+5)^2}\,dz$ where c is bounded by $c \to |z|=1$

$$\int_C \frac{z^2+3}{(z^2+4)(z+5)^2}$$

$c \to |z|=1$



for Pole $(z^2+4)(z+5)^2 = 0$

$z^2+4=0$ | $(z+5)^2=0$

$z = \pm 2i$ | $z=-5,-5$ $\Rightarrow z = x+iy = -5+0i$

$z = x+iy = \pm 2i = 0 \pm 2i$

19/06/25

$x = 2 \quad y = 1 \quad y = -1$

$x = -2 \quad y = -1 \quad y = 1$

$f(z) = \dfrac{z^2+3}{(z^2+4)(z+5)^2}$ is Analytic inside and on 'c'

because all pole out side 'c'

$\left\{ \therefore \text{By Cauchy theorem } \int_C \dfrac{z^2+3}{(z^2+4)(z+5)^2}\, dz = 0 \right\}$

Q.2 Evaluate $\int_C \dfrac{z^2}{(z^2-9)(z+5)}$ where 'c' is bounded by $c \Rightarrow z = 2 \pm i, -2 \pm i$

Solve → $C \Rightarrow z = 2 \pm i, -2 \pm i$

$$\boxed{\begin{aligned} z &= 2+i = x+iy \Rightarrow (2, 1) \\ z &= 2-i = x+iy \Rightarrow (2, -1) \\ z &= -2+i = x+iy \Rightarrow (-2, 1) \\ z &= -2-i = x+iy \Rightarrow (-2, -1) \end{aligned}}$$

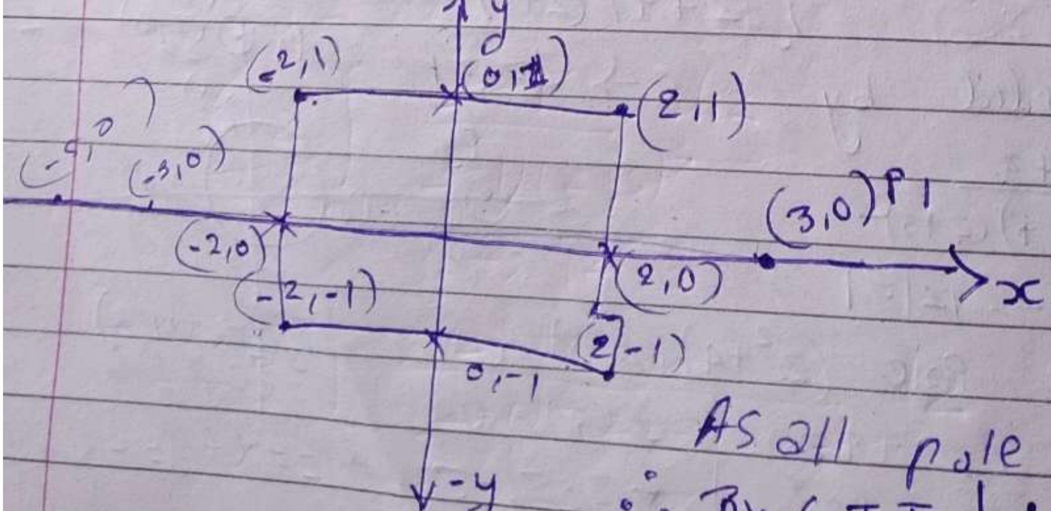
for Pole $(z^2-9)(z+5) = 0$

$z^2 - 9 = 0 \quad | \quad z+5 = 0$

$z = \pm 3 \quad | \quad z = -5 \quad (-5, 0)$

$z = 3 = x+iy \ (3, 0)$

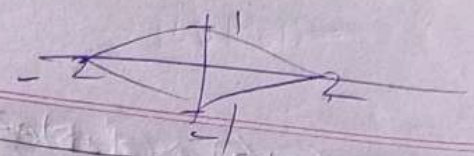
$z = -3 = x+iy \ (-3, 0)$



As all pole ~~inside~~ outside 'c'

$\therefore$ By C.I.T $\int f(z)\, dz = 0$

$$\int_C \frac{z^2}{(z^2-9)(z+5)}\, dz = 0$$



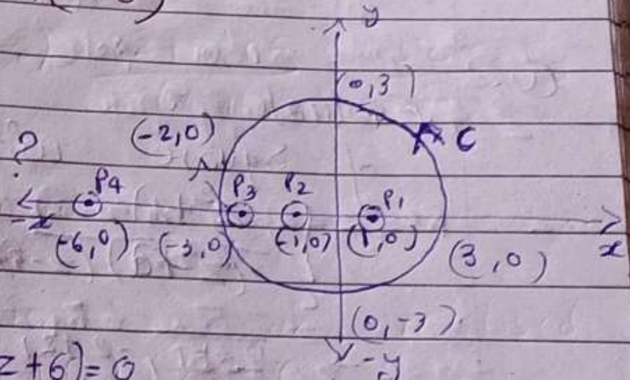
Cauchy residue theorem

If $f(z)$ is analytic inside and on closed curve 'c' except some finite pole then Cauchy residue theorem

$$\oint_c f(z)\,dz = 2\pi i \left\{ \text{sum of all residue for which pole in 'c'} \right\}$$

Q.1 Evaluate $\int_c \frac{(z^2+5)e^2}{(z^2-1)(z+2)(z+6)}\,dz = ?$ where c is bounded by $c \Rightarrow |z| = 3$

$$\oint_c \frac{(z^2+5)e^2}{(z^2-1)(z+2)(z+6)}\,dz = ?$$



$c \Rightarrow |z| = 3$

for pole $(z^2-1)(z+2)(z+6) = 0$

$z = \pm 1,\ z = -2,\ z = -6$

$(1,0)$ $z = 1 = x+iy$ | $z = -2 = x+iy$ $(-2,0)$

$(-1,0)$ $z = -1 = x+iy$ | $z = -6 = x+iy$ $(-6,0)$

$P_1 (1,0) \Rightarrow z = 1$

$P_2 (-1,0) \Rightarrow z = -1$

$P_3 (-2,0) \Rightarrow z = -2$

These pole lie in 'c'

All are single pole

$$\text{Res}(z=1) = \lim_{z \to 1} (z-1) \times \frac{(z^2+5)e^2}{(z-1)(z+1)(z+2)(z+6)} = \frac{6e}{42}$$

$$= \frac{e}{7} = R_1$$

$$\text{Res}(z=-1) = \lim_{z \to -1} (z+1) \times \frac{(z^2+5)e^2}{(z-1)(z+1)(z+2)(z+6)} = \frac{6e^{-1}}{-10}$$

$$= \frac{3}{-5} e^{-1} = R_2$$

Date: / / Page no:

$$\text{Res}(z=-2) = \lim_{z\to -2} \cancel{(z+2)} \times \frac{(z^2+5)e^z}{(z-1)(z+1)\cancel{(z+2)}(z+6)} = \frac{7e^{-2}}{12}$$

$$\int_c f(z)\,dz = 2\pi i\,(R_1+R_2+R_3) \Rightarrow \frac{e}{7} - \frac{3}{5}e^{-1} + \frac{7e^{-2}}{12}$$

$$\int_c \frac{(z^2+5)e^z}{(z^2-1)(z+2)(z+6)}\,dz = 2\pi i\left[\frac{e}{7} - \frac{3}{5}e^{-1} + \frac{7e^{-2}}{12}\right]$$

~~$f(z-2)=$~~

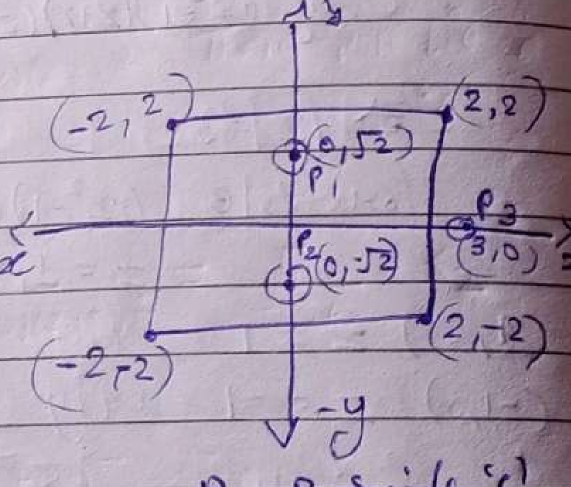
Q.2 Evaluate where 'c' is boundary of square with vertex

$$\int_c \frac{(z^2+1)}{(z^2+2)(z-3)^2}$$

$c \rightarrow \boxed{2\pm i,\ -2\pm 2i}$

(2,2), (2,−2)

(−2,2), (−2,−2)



P_1, P_2 inside 'c'

for pole

$(z^2+2)(z-3)^2$

Q.2 $$\int_c \frac{z^2+1}{(z^2-1)(z+5)^2}\,dz$$

$c \rightarrow |z|=6$

Q.3 $$\int_c \frac{(z^2+1)}{(z-1)^2(z+2)(z+4)}\,dz$$

$c \rightarrow |z|=3$

for pole

$(z^2+2)(z-3)^2=0$

$z^2+2=0 \quad (z-3)^2=0$

$z=\pm\sqrt{2}i \quad , z=3,3$ (2^{nd} order)

$z=-\sqrt{2}i$ } Simple pole

$z=\sqrt{2}i$ }

$(0,-\sqrt{2}),(0,\sqrt{2})$

$$\text{Res}(z=-\sqrt{2}i)=\lim_{z\to-\sqrt{2}i}\frac{\cancel{(z+\sqrt{2}i)}\times(z^2+1)}{(z-\sqrt{2}i)\cancel{(z+\sqrt{2})}(z-3)^2}$$

$$=\frac{-1}{-2\sqrt{2}i(-\sqrt{2}i-3)^2}=R_1$$

$$\text{Res}(z=\sqrt{2}i)=\lim_{z\to\sqrt{2}i}\frac{\cancel{(z-\sqrt{2}i)}(z^2+1)}{\cancel{(z-\sqrt{2}i)}(z+\sqrt{2}i)(z-3)^2}$$

$$=\frac{-1}{2\sqrt{2}i(\sqrt{2}i-3)^2}=R_2$$

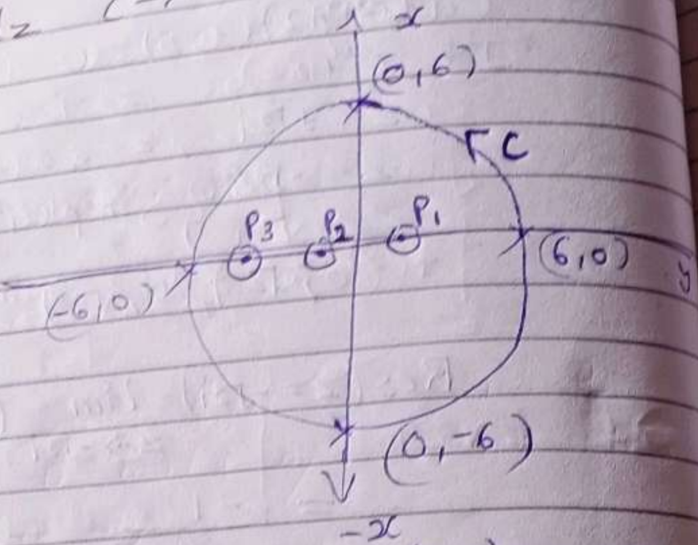
$$\int_C\frac{(z^2+1)}{(z^2+2)(z-3)^2}dz=2\pi i\{R_1+R_2\}$$

Ans

$$\int_C\frac{(z^2+1)}{(z^2+2)(z-3)^2}dz=2\pi i\left\{\frac{1}{2\sqrt{2}i(-\sqrt{2}i-3)^2}-\frac{1}{2\sqrt{2}i(\sqrt{2}i-3)^2}\right\}$$

Ans

Q.3 $\int_C \frac{z^2+1}{(z^2-4)(z+5)^2}\,dz \quad C \to |z|=6$



for pole

$(z^2-4)(z+5)^2=0$

$z^2-4=0 \quad (z+5)^2=0$

$z=\pm 2 \quad z=-5,-5$

$z=2 = x+iy \quad (2,0)\ P_1$

$z=-2 = x+iy \quad (-2,0)\ P_2$

$z=-5 = x+iy \quad (-5,0)\ P_3$

~~All are single~~

$(z+5)^2$ 2nd order pole

(z^2-4) is simple pole

$P_1 (2,0) \Rightarrow z=2$

$P_2 (-2,0) \Rightarrow z=-2$

$P_3 (-5,0) \Rightarrow z=-5$

These pole lie in 'C'

$Res(z=2) = \lim_{z\to 2} \cancel{(z-2)} \frac{z^2+1}{\cancel{(z-2)}(z+2)(z+5)^2}$

$\frac{5}{4\times 49} = \frac{5}{196} = R_1$

$Res(z=-2) = \lim_{z\to -2} (z+2) \frac{z^2+1}{(z-2)\cancel{(z+2)}(z+5)^2} = \frac{5}{-36} = R_2$

$Res(z=-5) = \frac{1}{\lfloor 2-1} \left\{ \frac{d^{2-1}}{dz^{2-1}} \cancel{(z+5)^2} \frac{z^2+1}{(z^2-4)\cancel{(z+5)^2}} \right\}$

$\Rightarrow \left\{ \frac{d}{dz} \frac{(z^2+1)}{(z^2-4)} \right\}_{z=-5}$

$\left\{ \frac{(z^2-4)(2z)-(z^2+1)(2z)}{(z^2-4)^2} \right\}$

$\left\{ \frac{\cancel{2z^3}-8z-\cancel{2z^3}-2z}{(z^2-4)^2} \right\}$

$\frac{-10(-5)}{((-5)^2-4)^2} = \boxed{\frac{50}{441} = R_3}$

$+\times 9$

~~0.025 − 0.138 + 0.113~~

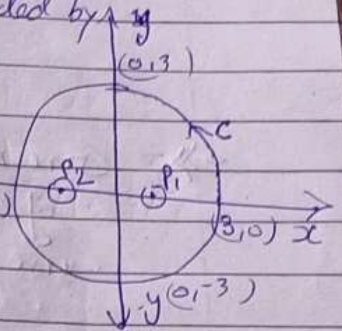
Date: / / Page no:

$$\int_C \frac{z^2+1}{(z^2+4)(z+5)^2}\,dz = 2\pi i\{R_1+R_2+R_3\}$$

$$\Rightarrow 2\pi i\left\{\frac{5}{196}-\frac{5}{36}+\frac{5}{441}\right\}$$

$$\Rightarrow 2\pi i \times 0 = 0 \quad \underline{\text{Ans}}$$

Q3 $\Rightarrow$ $\displaystyle\int_C \frac{(z^2+1)}{(z-1)^2(z+2)(z+4)}\,dz$ where c is bounded by $|z|=3$



Sol $\Rightarrow$ for pole —

$$(z-1)^2(z+2)(z+4)=0$$

$$z = 1,1, \quad z=-2, \quad z=-4$$

P_1 $z=1 \Rightarrow x+iy=(1,0)$ is Pole of 2nd order

P_2 $z=-2 \Rightarrow x+iy=(-2,0)$ } simple pole

P_3 $z=-4 \Rightarrow x+iy=(-4,0)$

$$\text{Res}(z=1) = \frac{1}{\lfloor 2-1}\left\{\frac{d^{2-1}}{dz^{2-1}}\cancel{(z-1)^2}\frac{(z^2+1)}{\cancel{(z-1)^2}(z+2)(z+4)}\right\}_{z=1}$$

$$=\left\{\frac{d}{dz}\frac{(z^2+1)}{(z+2)(z+4)}\right\}_{z=1}$$

$$=\left\{\frac{(z+2)(z+4)(2z)-(z^2+1)\{(z+2)+(z+4)\}}{\{(z+2)(z+4)\}^2}\right\}_{z=1}$$

$$=\left[\frac{(3\times5\times2)-2\{3+5\}}{(3\times5)^2}\right] = \frac{30-16}{225} = \frac{14}{225} = R_1$$

$$\text{Res}(z=-2) = \lim_{z\to -2}\cancel{(z+2)}\frac{(z^2+1)}{(z-1)^2\cancel{(z+2)}(z+4)}$$

$$=\frac{(-2)^2+1}{(-2-1)^2(-2+4)} = \frac{5}{18} = R_2$$

Date: / / Page no:

$$\int_C \frac{(z^2+1)}{(z-1)^2(z+2)(z+4)}\,dz = 2\pi i\left\{R_1+R_2\right\}$$

$$= 2\pi i\left\{\frac{14}{225}+\frac{5}{18}\right\}$$

$$= 2\pi i\left\{\frac{252+8625}{\cancel{4050}\;2025}\right\}$$

$$= \pi i\left\{\frac{8877}{2025}\right\}$$