theorem

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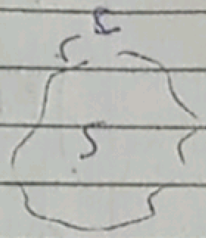
Stokes theorem

According to this theorem If surface (S) is bounded by closed curve (C) then By Stokes theorem $\int_C \vec{F}\cdot d\vec{r}$

$$\int_C \vec{F}\cdot d\vec{r} = \iint_S \text{curl}\,\vec{F}\cdot d\vec{S} = \iint_S \text{curl}\,\vec{F}\cdot\hat{n}\,dS$$

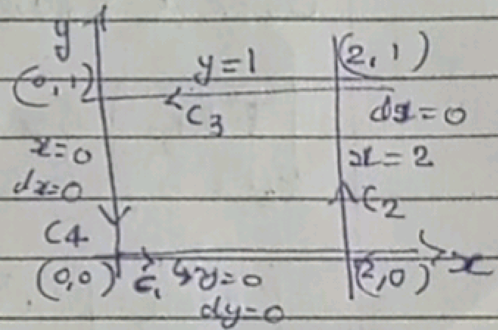


verify ~~Stookes~~ Stokes theorem for $\vec{F} = (x^2+y)\hat{i} + (y^2+z)\hat{j} + (z^2+x)\hat{k}$

when C is bounded by $C \to x=0, y=0, x=2, y=1$

$$\vec{F} = (x^2+y)\hat{i} + (y^2+z)\hat{j} + (z^2+x)\hat{k}$$

$C \to x=0, y=0, x=2, y=1$



$$LHS \Rightarrow \int_C \vec{F}\cdot d\vec{r} = \int_{C_1} \vec{F}\cdot d\vec{r}_1 + \int_{C_2} \vec{F}\cdot d\vec{r}_2 + \int_{C_3} \vec{F}\cdot d\vec{r}_3 + \int_{C_4} \vec{F}\cdot d\vec{r}_4$$

$z=0 \Rightarrow dz=0$

$C_3 \Rightarrow y=1,\ dy=0,\ z=0,\ dz=0$

$$\int_{C_3} \vec{F}\cdot d\vec{r}_3 = \int_{C_3} \{(x^2+y)\hat{i} + (y^2+z)\hat{j} + (z^2+x)\hat{k}\}\cdot\{\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz\}$$

$$\int_{C_3} \vec{F}\cdot d\vec{r}_3 = \int_{C_3} (x^2+y)\,dx + \int_{C_3} (y^2+z)\,dy + \int_{C_3} (z^2+x)\,dz = \int_{C_3} (x^2+1)\,dx + 0 + 0$$

$y=1 \quad dy=0 \quad dz=0$

$$C_3 \int_{x=2}^{0} (x^2+1)\,dx = \left\{\frac{x^3}{3} + x\right\}_{x=2}^{0} = \left(\frac{0^3}{3}+0\right) - \left(\frac{2^3}{3}+2\right)$$

$$= \frac{8}{3}+2 = \frac{\cancel{8+6}}{3}\ \frac{-14}{3}$$

$$\int_{C_3} \vec{F}\cdot d\vec{r}_3 = -\frac{14}{3}$$

$$= \frac{\cancel{8}}{3} \cancel{A}\ C_3$$

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$C_1 \Rightarrow y=0, dy=0, z=0, dz=0$

$$\int_{C_1} \vec{F}\,d\vec{r}_1 = \int_{C_1} \{(x^2+y)\hat{i} + (y^2+z)\hat{j} + (z^2+x)\hat{k}\}\{\hat{i}dx + \hat{j}dy + \hat{k}dz\}$$

$$\int_{C_1} \vec{F}\,d\vec{r}_1 = \int_{C_1} (x^2+y)dx + \int_{C_1} (y^2+z)dy + \int_{C_1} (z^2+x)dz$$

$y=0$, $dy=0$, $dz=0$

$$\int_{C_1} \vec{F}\,d\vec{r}_1 = \int_{C_1} x^2 dx = \int_0^2 x^2 dx = \left[\frac{x^3}{3}\right]_0^2 = \frac{8}{3}.$$

$C_2 \Rightarrow x=2, dx=0, z=0, dz=0$

$$\int_{C_2} \vec{F}\,d\vec{r}_2 = \int_{C_2} \{(x^2+y)\hat{i} + (y^2+z)\hat{j} + (z^2+x)\hat{k}\}\{\hat{i}dx + \hat{j}dy + \hat{k}dz\}$$

$$\int_{C_2} \vec{F}\,d\vec{r}_2 = \int_{C_2} (x^2+y)dx + \int_{C_2} (y^2+z)dy + \int_{C_2} (z^2+x)dz$$

$dx=0$, $z=0$, $dz=0$

$$\int_{C_2} y^2 dy = \int_0^1 y^2 dy = \left[\frac{y^3}{3}\right]_0^1 = \frac{1}{3}$$

$C_4 \Rightarrow x=0, dx=0, z=0, dz=0$

$$\int_{C_4} \vec{F}\,d\vec{r}_4 = \int_{C_4} \{(x^2+y)\hat{i} + (y^2+z)\hat{j} + (z^2+x)\hat{k}\}\{\hat{i}dx + \hat{j}dy + \hat{k}dz\}$$

$$\int_{C_4} \vec{F}\,d\vec{r}_4 = \int_{C_4} (x^2+y)dx + \int_{C_4} (y^2+z)dy + \int_{C_4} (z^2+x)dz$$

0, 0

$$\int_{C_4} y^2 dy = \int_1^0 y^2 dy = \left[\frac{y^3}{3}\right]_1^0 = -\frac{1}{3}$$

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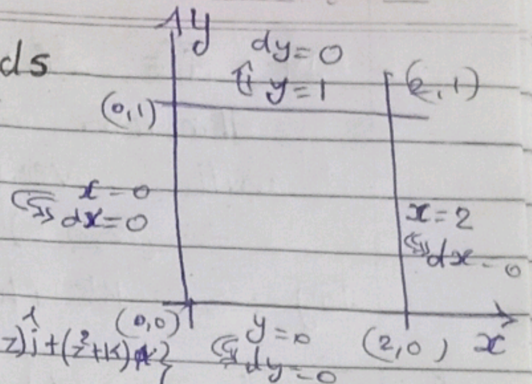
$$\int_c \vec{f}\cdot d\vec{r} = \iint_s \text{curl}\,\vec{f}\cdot d\vec{s} = \iint_s \text{curl}\,\vec{f}\cdot\hat{n}\,ds$$



R.H.S

$\Rightarrow \text{curl}\,\vec{f} = \vec{\nabla}\cdot\vec{f}$

$$\vec{\nabla}\times\vec{f} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\times\left((x^2+y)\hat{i} + (y^2+z)\hat{j} + (z^2+x)\hat{k}\right)$$

$z=0 \Rightarrow dz=0$

$$\text{curl}\,\vec{f} = \vec{\nabla}\times\vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x^2+y) & (y^2+z) & (z^2+x) \end{vmatrix} = \hat{i}\{0-1\} - \hat{j}\{1-0\} + \hat{k}\{0-1\}$$

$$= -\hat{i} - \hat{j} - \hat{k}$$

$$\hat{n} = \hat{k},\quad ds = \frac{dx\,dy}{|\hat{n}\cdot\hat{k}|} = \frac{dx\,dy}{|\hat{k}\cdot\hat{k}|} = \frac{dx\,dy}{|1|} = dx\,dy$$

$$\iint_s \text{curl}\,\vec{f}\cdot\hat{n}\,ds = \iint_s \{-\hat{i}-\hat{j}-\hat{k}\}\cdot\hat{k}\,dx\,dy$$

$$= -\iint_s dx\,dy = -\int_{x=0}^{2}\int_{y=0}^{1} dx\,dy = -\int_{x=0}^{2}(y)_{y=0}^{1}\,dx = -\int_{x=0}^{2} 1\,dx$$

$$= -(x)_{x=0}^{2}$$

$$= -2$$

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$$\int_C \vec{f}\,d\vec{r} = \int_{C_1} \vec{f}\,d\vec{r_1} + \int_{C_2} \vec{f}\,d\vec{r_2} + \int_{C_3} \vec{f}\,d\vec{r_3} + \int_{C_4} \vec{f}\,d\vec{r_4}$$

$$\int_C \vec{f}\,d\vec{r} = \frac{8}{3} + \frac{1}{3} - \frac{14}{3} - \frac{1}{3}$$

$$\int_C \vec{f}\,d\vec{r} = \frac{8+1-14-1}{3} = -\frac{6}{3} = -2$$

Know

$$\int_C \vec{f}\,d\vec{r} = \iint_S \text{curl}\,\vec{f}\,d\vec{s} = \iint_S \text{curl}\,\vec{f}\,\hat{n}\,ds = -2$$

Hence proved

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Q.1 $\vec{F} = x^2\hat{i} + y\hat{j} + z\hat{k}$ and S is bounded by $x=0$, $y=0$, $z=0$, $2x+3y+z=6$ then verify gauss divergence theorem

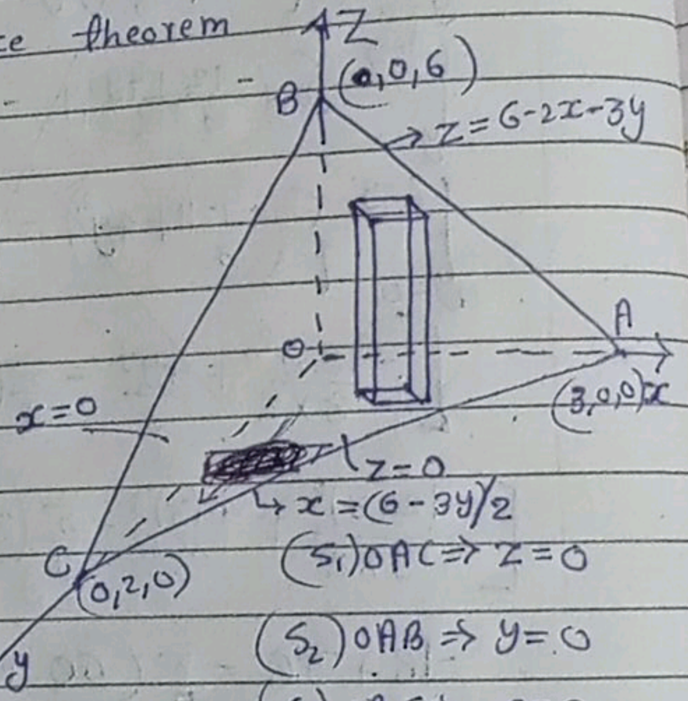
Sol^n Given, $\vec{f} = x^2\hat{i} + y\hat{j} + z\hat{k}$

$S \to x=0,\ y=0,\ z=0$

$2x+3y+z=6$

By gauss divergence theorem

$$\iint_S \vec{f}\,\hat{n}\,ds = \iiint_V div\,\vec{f}\,dv$$



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$$\iiint_V div\,\vec{f}\,dv$$

(S_1) OAC $\Rightarrow z=0$

(S_2) OAB $\Rightarrow y=0$

(S_3) OBC $\Rightarrow x=0$

(S_4) ABC $\Rightarrow 2x+3y+z=6$

$div\,\vec{f} = \vec{\nabla}\cdot\vec{f}$

$$\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(x^2\hat{i} + y\hat{j} + z\hat{k}\right)$$

$div\,\vec{f} = 2x+1+1 = (2x+2)$

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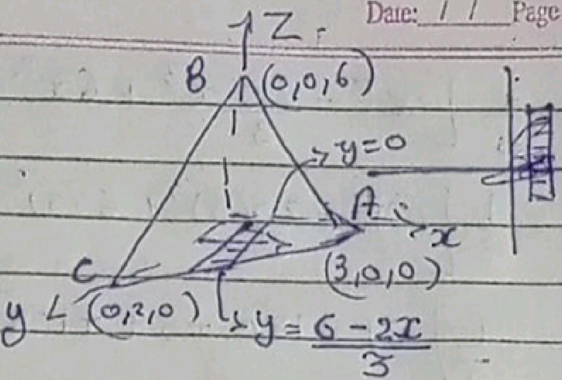
$$\int_{y=0}^{y=2}$$

$$\int_{x=0}^{3}\int_{y=0}^{y=6-2}$$



$$\int_{x=0}^{3}\int_{y=0}^{\frac{6-2x}{3}} (2x+2)(6-2x-3y)\,dx\,dy$$

$$\int_{x=0}^{3} (2x+2)\left[(6-2x)y - \frac{3y^2}{2}\right]_0^{\frac{6-2x}{3}} dx$$

$$\int_{x=0}^{3} (2x+2)\left[(6-2x)\left(\frac{6-2x}{3}\right) - \frac{3}{2}\left(\frac{6-2x}{3}\right)^2\right] dx$$

$$\int_{x=0}^{3} (2x+2)\left[\frac{(6-2x)^2}{3} - \frac{3}{2}\,\frac{(6-2x)^2}{9}\right] dx$$

$$\int_{x=0}^{3} (2x+2)\left[\frac{(6-2x)^2}{3} - \frac{(6-2x)^2}{6}\right] dx$$

$$\int_{x=0}^{3} (2x+2)\,\frac{(6-2x)^2}{6}\, dx$$

$$\int_{x=0}^{3} \frac{2}{6}(x+1)(6-2x)^2\, dx$$

$$\frac{1}{3}\int_{x=0}^{3} (x+1)(36+4x^2-24x)\,dx$$

$$\frac{1}{3}\int_{x=0}^{3}\left[36x+4x^3-24x^2+36+4x^2-24x\right]dx$$

$$\frac{1}{3}\int_{x=0}^{3}\left[4x^3-20x^2+12x+36\right]dx$$

$$\frac{1}{3}\left[\frac{4x^4}{4} - \frac{20x^3}{3} + \frac{12x^2}{2} + 36x\right]_{x=0}^{3}$$

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$$\frac{1}{3}\left[x^4 - \frac{20}{3}x^3 + 6x^2 + 36x\right]_{x=0}^{3}$$

$$\frac{1}{3}\left[(3)^4 - \frac{20}{3}(3)^3 + 6(3)^2 + 36(3)\right]$$

$$\frac{1}{3}\left[81 - 180 + 54 + 108\right]$$

$$\frac{1}{3}\left[63\right] = 21$$

$$\iint_S \vec{f}\cdot\vec{n}\,ds = \iiint_V \operatorname{div}\vec{f}\,dv = 21$$

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Vivo AI camera

17/08/25

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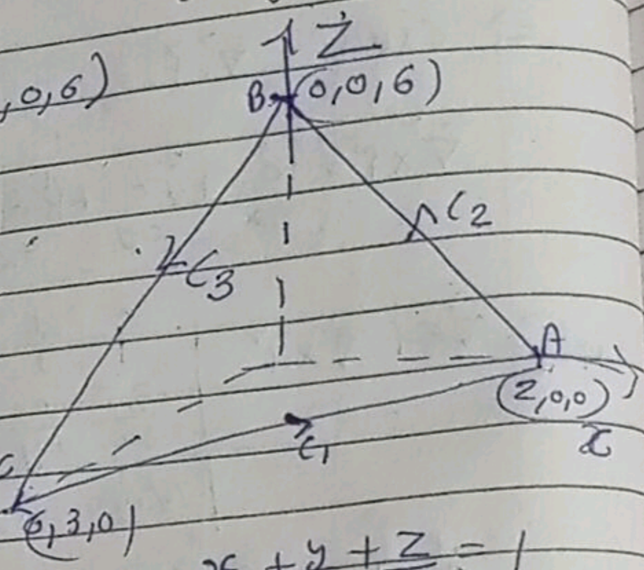
If $\vec{F} = z^2\hat{i} + x^2\hat{j} + y^2\hat{k}$ then verify ~~stock's~~ stokes theorem where C is boundary of ~~ft~~ triangle with vertax

$C \to (2,0,0), (0,3,0), (0,0,6)$

By stock's theorem



$$\int_C \vec{F}\cdot d\vec{r} = \iint_S \text{curl}\,\vec{F}\cdot\hat{n}\,ds \quad \text{--- (1)}$$

$$\text{L.H.S} \Rightarrow \oint_C \vec{F}\cdot d\vec{r} = \int_{C_1} \vec{F}\cdot d\vec{r}_1 + \int_{C_2} \vec{F}\cdot d\vec{r}_2 + \int_{C_3} \vec{F}\cdot d\vec{r}_3$$

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{6} = 1$$

$$\int_{C_2} \vec{F}\cdot d\vec{r}_2 = \int_{C_2} (z^2\hat{i} + x^2\hat{j} + y^2\hat{k})\cdot(\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz)$$

(S) $\Delta ABC \Rightarrow 3x + 2y + z = 6$

C_2 (lin AB, for this $y = 0$ }

$C_2 \Rightarrow 3x + z = 6,\ y = 0,\ dy = 0$

$C_1 \Rightarrow$ (lin CB) $\Rightarrow 3x + 2y = 6,\ z = 0,\ dz = 0$

C_3 (lin AC) $\Rightarrow 2y + z = 6,\ x = 0,\ dx = 0$

$$= \int_{C_2} z^2 dx + \underbrace{\int x^2 dy}_{C_2 \to 0} + \underbrace{\int y^2 dz}_{C_2 \to y=0}$$

$$= \int_{C_2 \to z = 6-3x} z^2 dx = \int_{C_2} (6-3x)^2 dx$$

$$= \int_{x=2}^{0} \{36 + 9x^2 - 36x\} dx$$

$$= \left\{36x + \frac{9x^3}{3} - \frac{36x^2}{2}\right\}_{x=2}^{0}$$

$$= -\{36\times 2 + 3(2)^3 - 18(2)^2\}$$

$$= -\{\cancel{72} + 24 - \cancel{72}\}$$

$$= -24$$

$C_1 \Rightarrow z=0,\ dz=0,$

$$\int_{C_1} \vec{f}\,d\vec{r}_1 = \int_{C_2} \{z^2\hat{i}+x^2\hat{j}+y^2\hat{k}\}\{\hat{i}dx+\hat{j}dy+\hat{k}dz\}$$

$$\int_{C_1} z^2 dx + \int_{C_1} x^2 dy + \int_{C_1} y^2 dz \qquad z=0,\ dz=0$$

$$= \int_{C_1} x^2 dy \qquad C_1 \to x = \frac{6-2y}{3}$$

$$= \int_{C_1} \left(\frac{6-2y}{3}\right)^2 dy$$

$$= \int_3^0 \left\{\frac{36+4y^2-24y}{9}\right\} dy$$

$$= \frac{1}{9}\left\{36y + \frac{4y^3}{3} - \frac{24y^2}{2}\right\}_{y=3}^{0}$$

$$= -\frac{1}{9}\left\{36\times 3 + \frac{4(3)^3}{3} - \frac{24(3)^2}{2}\right\}$$

$$= -\frac{1}{9}\{\cancel{108} + 36 - \cancel{108}\}$$

$$= -4$$

$$\int_C \vec{f}\,d\vec{r} = -4 - 24 - 18$$

$$\int_C \vec{f}\,d\vec{r} = -46$$

$C_3 \Rightarrow x=0,\ dx=0$

$$\int_{C_3} \vec{f}\,d\vec{r}_3 =$$

$$= \int_{C_3} \{z^2\hat{i}+x^2\hat{j}+y^2\hat{k}\}\{\hat{i}dx+\hat{j}dy+\hat{k}dz\}$$

$$= \int_{C_3} z^2 dx + \int_{C_3} x^2 dy + \int_{C_3} y^2 dz \qquad (0,\ 0)$$

$$= \int y^2 dz \qquad C_3 \to y = \frac{6-z}{2}$$

$$= \int_{C_3} \left(\frac{6-z}{2}\right)^2 dz$$

$$= \int_{C_3} \left(\frac{36+z^2-12z}{4}\right) dz$$

$$= \frac{1}{4}\int_{z=6}^{0} \{36+z^2-12z\} dz$$

$$= \frac{1}{4}\left\{36z + \frac{z^3}{3} - \frac{12z^2}{2}\right\}_{z=6}^{z=0}$$

$$= -\frac{1}{4}\left\{36\times 6 + \frac{(6)^3}{3} - \frac{12(6)^2}{2}\right\}$$

$$= -\frac{1}{4}\left\{\cancel{216} + \frac{216}{3} - \cancel{216}\right\}$$

$$= -\frac{1}{4}\times\frac{216}{3}$$

$$= -\frac{54}{3} = -18$$

R.H.S → ABC ⟹ $3x+2y+z=6$ (S)

$\Rightarrow \iint_S \text{curl}\vec{f}\cdot\hat{n}\,ds = ?$

$\text{curl}\vec{f} = \vec{\nabla}\times\vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z^2 & x^2 & y^2 \end{vmatrix} = \hat{i}\{2y-0\} - \hat{j}\{0-2z\} + \hat{k}\{2x-0\}$

$\text{curl}\vec{f} = 2y\hat{i} + 2z\hat{j} + 2x\hat{k}$

$\hat{n} = \frac{\text{grad}(s)}{|\text{grad}(s)|} = \frac{\vec{\nabla}(s)}{|\vec{\nabla}(s)|}$

$\vec{\nabla}(s) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(3x+2y+z-6) = 3\hat{i} + 2\hat{j} + K$

$\hat{n} = \frac{3\hat{i}+2\hat{j}+\hat{k}}{\sqrt{9+4+1}} = \frac{3\hat{i}+2\hat{j}+\hat{k}}{\sqrt{14}}$

$ds = \frac{dx\,dy}{|\hat{n}\cdot\hat{k}|} = \frac{dx\,dy}{\left|\left(\frac{3\hat{i}+2\hat{j}+\hat{k}}{\sqrt{14}}\right)\hat{k}\right|} = \frac{dx\,dy}{\frac{1}{\sqrt{14}}}$

$\iint_S \text{curl}\vec{f}\,\hat{n}\,ds = \iint_S (2y\hat{i}+2z\hat{j}+2x\hat{k})\left(\frac{3\hat{i}+2\hat{j}+K}{\sqrt{14}}\right)\frac{dx\,dy}{1/\sqrt{14}}$

$= \iint_S \{6y+4z+2x\}\,dx\,dy$

$z = (6-3x-2y)$

$= \iint_S \{6y+4(6-3x-2y)+2x\}\,dx\,dy$

$\cancel{= \iint_S \{4y-x+6\}\,dx\,dy}$

$= \iint_S \{6y+24-12x-8y+2x\}\,dx\,dy$

$= \iint_S \{-2y-10x+24\}\,dx\,dy$

$$\int_{x=0}^{2} 5 \int_{y=0}^{y=\frac{6-3x}{2}} \left\{24 - 10x - 2y\right\} dy\, dx$$

$$\int_{x=0}^{2} \left\{24y - 10xy - \frac{2y^2}{2}\right\}_{y=0}^{y=\frac{6-3x}{2}} dx$$

$$\int_{x=0}^{2} \left\{24\left(\frac{6-3x}{2}\right) - 10x\left(\frac{6-3x}{2}\right) - \left(\frac{6-3x}{2}\right)^2\right\} dx$$

$$\int_{x=0}^{2} \left\{72 - 36x - 30x + 15x^2 - \left(\frac{36+9x^2-36x}{4}\right)\right\} dx$$

$$\int_{x=0}^{2} \left\{72 - 66x + 15x^2 - 9 - \frac{9}{4}x^2 + 9x\right\} dx$$

$$\int_{x=0}^{2} \left\{63 - 57x + \frac{51x^2}{4}\right] dx$$

$$\left\{63x - \frac{57x^2}{2} + \frac{51}{4}\frac{x^3}{3}\right\}_{x=0}^{2}$$

$$= 63\times 2 - \frac{57(2)^2}{2} + \frac{17}{4}(2)^3$$

$$= 126 - 114 + 34 = 46$$

$$\int_C \vec{f}\, d\vec{r} = \iint_S \text{curl}\, \vec{f}\, \hat{n}\, ds = 46$$