Variation of parameters :-

$$f(D)y = R(x)$$

$$C.F = c_1 U(x) + c_2 V(x)$$

$$PI = A\cdot U(x) + B\,V(x) \text{ where}$$

$$A = -\int \frac{V(x)\,R(x)\,dx}{U\frac{dV}{dx} - V\frac{dU}{dx}}$$

$$\begin{vmatrix} U & V \\ \frac{dU}{dx} & \frac{dV}{dx} \end{vmatrix}$$

$$B = \int \frac{U(x)\,R(x)\,dx}{U\frac{dV}{dx} - V\frac{dU}{dx}}$$

Q $(D^2+9)y = \sec 3x$

AE is $m^2+9=0$

$$C.F = c_1 \cos 3x + c_2 \sin 3x$$

$$U = \cos 3x, \quad V = \sin 3x$$

$$\frac{dU}{dx} = -\sin 3x \cdot 3, \quad \frac{dV}{dx} = 3\cos 3x$$

$$U\frac{dV}{dx} - V\frac{dU}{dx} = 3\cos^2 3x + 3\sin^2 3x = 3$$

$$A = -\int \frac{V(x)\,R(x)\,dx}{U\frac{dV}{dx} - V\frac{dU}{dx}} = -\int \frac{\sin 3x \sec 3x}{3}$$

$$= -\frac{1}{3}\int \tan 3x \, dx = +\frac{1}{9}\ln(\cos 3x)$$

$$B = \int \frac{U(x)\,R(x)\,dx}{U\frac{dV}{dx} - V\frac{dU}{dx}} = \int \frac{\cos 3x \sec 3x}{3}$$

$$= \frac{x}{3}$$

$$PI = A\,U(x) + B\,V(x)$$

$$= \frac{\ln\cos 3x}{9}\cos 3x + \frac{x}{3}\sin 3x$$

(c) $(D^2+1)y = \operatorname{cosec} x$

$$CF = c_1\cos x + c_2\sin x$$

$$U = \cos x, \quad V = \sin x$$

$$\frac{dU}{dx} = -\sin x, \quad \frac{dV}{dx} = \cos x$$

$$U\frac{dV}{dx} - V\frac{dU}{dx} = 1$$

$$A = -\int \frac{V(x)\,R(x)\,dx}{U\frac{dV}{dx} - V\frac{dU}{dx}} = -\int \frac{\sin x \cdot \operatorname{cosec} x \, dx}{1}$$

$$= -x$$

$$B = \int \frac{U(x)\,R(x)\,dx}{U\frac{dV}{dx} - V\frac{dU}{dx}} = \int \cos x \cdot \operatorname{cosec} x \, dx$$

$$= \int \cot x \, dx$$

$$= \ln\sin x$$

$$PI = -x\cos x + \sin x \ln(\sin x)$$

Cauchy - Eulers diff^n eq^n of → D

(Diff eq^n with variable coefficients)

$$x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_n y = f(x)$$

$$(x^n D^n + a_1 x^{n-1} D^{n-1} + \dots + a_n) y = f(x)$$

① $x = e^z \Leftrightarrow z = \ln x$

② $x\frac{d}{dx} = xD = \theta = \frac{d}{dz}$

$x^2 D^2 = \theta(\theta - 1)$; $x^3 D^3 = \theta(\theta-1)(\theta-2)$

Q $(x^2 D^2 + xD + 1) y = x^2$

$\Rightarrow [\theta(\theta-1) + \theta + 1] y = (e^z)^2$

$(\theta^2 - \theta + \theta + 1) y = e^{2z}$

$(\theta^2 + 1) y = e^{2z}$

AE is $m^2 + 1 = 0$

$\boxed{m = \pm i}$

$C.F = c_1 \cos z + c_2 \sin z$

$PI = \frac{1}{\theta^2 + 1} e^{2z} = \frac{e^{2z}}{4+1} = \frac{e^{2z}}{5}$

$\theta = 2$

$G.S = PI + CF$

$= c_1 \cos z + c_2 \sin z + \frac{e^{2z}}{5}$

$= c_1 \cos \ln x + c_2 \sin \ln x + \frac{x^2}{5}$

$\Rightarrow (x^3D^3 + 2x^2D^2 + 2)y = 0$

$(\theta(\theta-1)(\theta-2) + 2(\theta)(\theta-1) + 2)y = 0$

$(\theta^3 - 2\theta^2 - \theta^2 + 2\theta + 2\theta^2 - 2\theta + 2)y = 0$

$(\theta^3 - \theta^2 + 2)y = 0$

AE is $m^3 - m^2 + 2 = 0$

$(m+1)(m^2 - 2m + 2) = 0$

$m = -1, \quad m = \frac{2 \pm \sqrt{4-8}}{2} = 1 \pm i$

$CF = c_1 e^{-z} + e^{z}(c_2 \cos z + c_3 \sin z)$

$= c_1 x^{-1} + x[c_2 \cos \ln x + c_3 \sin \ln x)]$

Partial diff eq^n :-

$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} = F$

(1) if $B^2 - 4AC = 0$ then the P.D.E is parabola.

(2) If $B^2 - 4AC < 0$ then the P.D.E is Elliptic.

(3) If $B^2 - 4AC > 0$ then the P.D.E is hyperbolic.

(1) 1-D wave eqn

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial t^2} - a^2 \frac{\partial^2 u}{\partial x^2} = 0$$

$A = 1, B = 0, C = -a^2$

$B^2 - 4AC = 0 - 4(1)(-a^2) > 0$

hyperbolic

(2) 1-D heat eqn

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial u}{\partial t} - a^2 \frac{\partial^2 u}{\partial x^2} = 0$$

$A = 0, B = 0, C = -a^2$

$B^2 - 4AC = 0 - 0 = 0$

parabolic

(3) laplas eqn (2-D heat eqn)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$A = 1, B = 0, C = 1$

$B^2 - 4AC = 0 - 4 < 0$

elliptic

Solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, $u(0,y) = 8e^{-3y}$

by method of separation of variable

$$\frac{\partial U}{\partial x} = 4\frac{\partial U}{\partial y} \quad \text{—(1)}$$

$$U = XY \quad \text{—(2)}$$

$$\frac{\partial U}{\partial x} = X'Y \quad (3), \quad \frac{\partial U}{\partial y} = XY' \quad (4)$$

Sub (2)(3)(4) in (1)

$$X'Y = 4XY'$$

$$\frac{X'}{X} = \frac{4Y'}{Y} = k \text{ (say)}$$

$X' = kX$

$X' - kX = 0$

$m - k = 0$

$(m - k)$

$$\frac{X'}{X} = k, \quad \frac{Y'}{Y} = \frac{k}{4} \quad \text{—(6)}$$

root is k (5) root is $k/4$

$$X = c_1 e^{kx} \quad \text{—(7)}, \quad Y = c_2 e^{ky/4} \quad \text{—(8)}$$

Sub (7) and (8) in (2)

$$U(x,y) = c_1 e^{kx} \cdot c_2 e^{\frac{k}{4}y}$$

$$U(x,y) = c e^{kx} e^{\frac{y}{4}y} \quad \text{where } c_1 c_2 = c$$

now $U(0,y) = c e^{\frac{k}{4}y}$

given $U(0,y) = 8e^{-3y}$

$$(c = 8), \quad (k = -12)$$

Sol^n. → U(x,... $-12x$ ...

Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$; $u(x,0) = 6e^{-3x}$

Q2 by the method of separation of variables

$\rightarrow$ $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$

$\boxed{U = XT}$; $\frac{\partial u}{\partial x} = X'T$, $\frac{\partial u}{\partial t} = XT'$

$\Rightarrow$ $X'T = 2XT' + XT$

$X'T - XT = 2XT'$

$T(X' - X) = 2XT'$

$\left(\frac{X' - X}{X}\right) = \frac{2T'}{T} = k$ (say)

$\frac{X' - X}{X} = k$, $\frac{2T'}{T} = k$

$\frac{X'}{X} = k+1$, $T' = \frac{T}{2}k$

$X' = (k+1)X$; $T' - \frac{T}{2}k = 0$

$X' - (k+1)X = 0$; root $= k/2$

$\boxed{X = c_2 e^{(k+1)x}}$ $\boxed{T = c_1 e^{k/2\, t}}$

$\Rightarrow$ $U = c_1 e^{k/2\, t} c_2 e^{(k+1)x}$

$\boxed{U = c_1 c_2 e^{\frac{k}{2}t} e^{(k+1)x}}$

$\Rightarrow$ $U(x,t) = c_1 c_2 e^{\frac{k}{2}t} e^{(k+1)x}$ $\boxed{c = c_1 c_2}$

$U(x,0) = c\, e^{0} e^{(k+1)x}$

$6e^{-3x} = c\, e^{(k+1)x}$

$\Rightarrow$ $k+1 = -3$ $c = 6$

$\boxed{k = -4}$, $\boxed{c_1 c_2 = 6}$

$\boxed{U = 6e^{-2t}e^{-3x}}$

Series solution

Solution of Diffrential Equations by Power Series Solution

Some Definations :-

Ordinary Point and Singular Point

Consider the Diffrential equation

$$P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2(x)y = 0 \quad \text{---} (1)$$

Where $P_0(x)$, $P_1(x)$ and $P_2(x)$ are polynomials in x.

Case - I

Ordinary Points :- If $P_0(x) \neq 0$ at $x = a$, then the point $x = a$ is called an ordinary point of the differential equation ..(1).

Case - II

Singular Points :- If $P_0(x) = 0$ at $x = a$, then the point $x = a$ is called a Singular point of Diffrential equation (1)

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Example :- (I) ~~$(1+x^2)$~~

(1) $(1+x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = 0$

$P_0(x) = 1+x^2$ Put $x=0$

So $P_0(x) = 1+0 = 1$

$P_0(x) \neq 0$ at Point $x=0$

So This is ordinary point.

(2) $\dfrac{d^2y}{dx^2} + xy = 0$

$\Rightarrow P_0(x) = 1$ Put $x=0$

$P_0(x) = 1 \neq 0$

$\Rightarrow P_0(x) \neq 0$ at Point $x=0$

So this is ordinary Point.

(3) $x^2\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + 3y = 0$

$\Rightarrow P_0(x) = x^2$ Put $x=0$

So $P_0(x) = 0$ at Point $x=0$

So this is Regular Singular Point or Singular Point.

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Solution by first case Ordinary Point.

Series Solution of the Differential Equation.

Working Rule when $x=0$ is an ordinary Point.

Step-01
Given Differential equation is

$$P_0(x)\frac{d^2y}{dx^2}+P_1(x)\frac{dy}{dx}+P_2(x)\,y=0 \quad (1)$$

Find the value of $P_0(x)$, $P_1(x)$, and $P_2(x)$

Step-02 :- ~~Let series Solution of (1)~~ be

Check ordinary and Singular Points.

Step-03 :- Let series Solution of eq. (1) be

$$y=A_0+A_1x+A_2x^2+A_3x^3+\cdots+A_nx^n+\cdots\infty \quad (2)$$

Step-04 :- Find value $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ from equation (2)

Substitute the value of

y, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in eq. (1) and get an equation called identity Equation.

Step-05 :- Equate to zero the coeficients x^0, x^1, x^2, x^3 ---- etc.

Solve there relations [called recurrence Relations) and get the values of A_2, A_3 A_4, A_5 ----- etc in terms of A_0 and A_1.

Step-06 :- Substitute the value of A_2, A_3, A_4, A_5 ---- etc in eq. (2). Thus we get the required series solution of eq. (1) in orbitrary constaints A_0 and A_1.

Q.1 :- Solve the Diffrential equation by Series Solution.

$$\frac{d^2y}{dx^2} + xy = 0$$

Solution: Given Diffrential equation

$$\frac{d^2y}{dx^2} + xy = 0 \quad \text{-----} (1)$$

we know that

$$P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2(x)y = 0 \quad \text{----} (2)$$

Compair eq. (1) and (2)

$$P_0(x) = 1, \quad P_1(x) = 0, \quad P_2(x) = x$$

Then $P_0(x) = 1$. put $x = 0$

then $P_0(x) = 1$

so $P_0(x) \neq 0$

Then this is ordinary point at Point $x = 0$.

Then complete solution of eq. (1)

$$y = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + A_4 x^4 + \text{-----} A_n x^n + \text{-----} \infty \quad (3)$$

$\therefore$ Now eq. 3 D. w. r. to x

$$\frac{dy}{dx} = A_1 + 2A_2 x + 3A_3 x^2 + 4A_4 x^3 + \\ + \text{-----} + n A_n x^{n-1} + \text{---} \infty$$

Again D. w. r. to x

$$\frac{d^2y}{dx^2} = 2A_2 + 6A_3 x + 12 A_4 x^2 + \text{----} \\ + \text{---} n(n-1) A_n x^{n-2} + \text{---} \infty$$

Now Put value y, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$
~~Now~~ in eq. 1

$$\frac{d^2y}{dx^2} + x \cdot y = 0$$

$$\Rightarrow \left\{ 2A_2 + 6A_3 x + 12 A_4 x^2 + \text{----} + n(n-1) A_n x^{n-2} + \text{---} \infty \right\}$$

$$+ x(A_0 + A_1 x + A_2 x^2 + A_3 x^3 + \text{----} \\ + A_n x^n + \text{--} \infty) = 0$$

$$\Rightarrow (2A_2 + 6A_3 x + 12 A_4 x^2 + \text{----} + n(n-1) A_n x^{n-2} + \text{----} \infty)$$

$$+ (A_0 x + A_1 x^2 + A_2 x^3 + A_3 x^4 + \text{-----} + A_n x^{n+1} + \text{--} \infty) = 0$$

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$(n+2)(n+2-1)A_{n+2}x^{n+2-2}$ (7)

$0x^0 + 0.x^1 + 0x^i$

x^n

$\Rightarrow \left(2A_2 + 6A_3x + 12A_4x^2 + \cdots\right.$

$\left.\cdots + n(n-1)A_nx^{n-2} + \cdots\infty\right)$

$+ \left(A_0x + A_1x^2 + A_2x^3 + A_3x^4 + \cdots\right.$ $= 0$

$\left.\cdots + A_nx^{n+1} + \cdots\infty\right)$ ---- (4)

So eq. (4) is called identity Equation

Now Equate to coeficient of $x^0 = 1$

$= 2A_2 = 0$

$\boxed{A_2 = 0}$

Now Equate to coeficient of x^1

$= 6A_3 + A_0 = 0$

$6A_3 = -A_0$

$\boxed{A_3 = \frac{-A_0}{6}}$

Now Equate to coeficient of x^2

$= 12A_4 + A_1 = 0$

$= 12A_4 = -A_1$

$\boxed{A_4 = \frac{-A_1}{12}}$

Now Equate the coeficient of x^n

$$(n+1)(n+2)A_{n+2} + A_{n-1} = 0$$

$$\boxed{A_{n+2} = -\frac{A_{n-1}}{(n+1)(n+2)}}$$

This is called Recurrence Relation.

Now Put $n=0$

$$A_{0+2} = -\frac{A_{0-1}}{(0+1)(0+2)}$$

$$A_2 = -\frac{A_{-1}}{2} \quad \boxed{A_{-1}=0}$$

$$\boxed{A_2 = 0}$$

Now Put $n=1$

$$A_{1+2} = -\frac{A_{1-1}}{(1+1)(1+2)}$$

$$\boxed{A_3 = -\frac{A_0}{6}}$$

Now Put $n=2$

$$A_{2+2} = -\frac{A_{2-1}}{(2+1)(2+2)}$$

$$\boxed{A_4 = -\frac{A_1}{12}}$$

Now Put $n=3$

$$A_{3+2} = -\frac{A_{3-1}}{(3+1)(3+2)}$$

$$A_5 = -\frac{A_2}{20} = 0$$

$$\boxed{A_5 = 0}$$

Now Put $n=4$

$$A_{4+2} = -\frac{A_{4-1}}{(4+1)(4+2)}$$

$$= -\frac{A_3}{30} \quad \frac{A_0}{6 \times 30}$$

$$\boxed{A_6 = \frac{A_0}{180}}$$

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Now Put $n=5$

$$A_{5+2} = \frac{-A_{5-1}}{(5+1)(5+2)}$$

$$A_7 = \frac{-A_4}{42} = \frac{A_1}{12\times 42}$$

$$\boxed{A_7 = \frac{A_1}{504}}$$

Now $A_2, A_3, A_4, A_5, A_6, A_7$ --- value in eq. (3).

$$y = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + A_4 x^4 + A_5 x^5 + A_6 x^6 + A_7 x^7 + \cdots \infty$$

$$y = A_0 + A_1 x + 0\cdot x^2 + \left(\frac{-A_0}{6}\right)x^3 + \left(\frac{-A_1}{12}\right)x^4 + 0\cdot x^5 + \left(\frac{A_0}{180}\right)x^6 + \left(\frac{A_1}{504}\right)x^7 + \cdots \infty$$

$$y = A_0 + A_1 x - \frac{A_0}{6}x^3 - \frac{A_1}{12}x^4 + \frac{A_0}{180}x^6 + \frac{A_1}{504}x^7 + \cdots \infty$$

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$$y = \left[1 - \frac{x^3}{6} + \frac{x^6}{180} + \cdots \infty\right] A_0 + \left[x - \frac{x^4}{12} + \frac{x^7}{504} + \cdots \infty\right] A_1$$

Hence the complete solution of eq.(1).

Q-2 Solve the Differential Equation by Series solution.

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$$

Solution:- Given the D. E.

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0 \quad \text{----} (1)$$

we know that

$$P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2(x)\,y = 0 \quad \text{----} (2)$$

compair eq. (1) and eq. (2)

$P_0(x) = 1 - x^2$, $P_1(x) = -x$, $P_2(x) = 4$

Now . $P_0(x) = 1 - x^2$ Put $x = 0$

$$P_0(x) = 1 - 0 = 1$$

$$P_0(x) \neq 0 \text{ at Point } x = 0$$

So
This is ordinary point at point $x = 0$.

Then complete solution of eq. (1)

$$y = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + \dots + A_n x^n + \dots \quad (3)$$

Now D. w. r. to x

$$\frac{dy}{dx} = A_1 + 2A_2 x + 3A_3 x^2 + \dots + nA_n x^{n-1} + \dots$$

Now Again D. w. r. to x

$$\frac{d^2y}{dx^2} = 2A_2 + 6A_3 x + \dots + n(n-1)A_n x^{n-2} + \dots \infty$$

Now . Put values y, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in eq. (1).

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Now eq. 1

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} - x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$$

$\Rightarrow$ Put values y, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

$$\Rightarrow \Big[\{2A_2 + 6A_3x + ---- + n(n-1)A_nx^{n-2} + ---\infty\} -$$

$$-x^2\{2A_2 + 6A_3x + ---- + n(n-1)A_nx^{n-2} + ---\} -$$

$$-x\{A_1 + 2A_2x + 3A_3x^2 + --- + nA_nx^{n-1} --\} +$$

$$+4\{A_0 + A_1x + A_2x^2 + A_3x^3 + --- + A_nx^n + --\}\Big] = 0$$

$$\Rightarrow \Big[\{2A_2 + 6A_3x + ---- + n(n-1)A_nx^{n-2} ----\} +$$

$$+\{-2A_2x^2 - 6A_3x^3 - ----- - n(n-1)A_nx^n --\} +$$

$$+\{-A_1x - 2A_2x^2 - 3A_3x^3 - ----- - nA_nx^n ---\} +$$

$$+\{4A_0 + 4A_1x + 4A_2x^2 + 4A_3x^3 + --- ---4A_nx^n + ---\} = 0$$

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This is called identity equation.

Now Equate the coeficients of x^0

$$2A_2 + 4A_0 = 0$$

$$2A_2 = -4A_0$$

$$\boxed{A_2 = -2A_0}$$

Now Equate the coeficients of x^1.

$$6A_3 - A_1 + 4A_1 = 0$$

$$6A_3 + 3A_1 = 0$$

$$6A_3 = -3A_1$$

$$\boxed{A_3 = -\frac{A_1}{2}}$$

finaly Now Equate the coeficient of x^n.

$$(n+1)(n+2)A_{n+2} - n(n-1)A_n - nA_n + 4A_n = 0$$

$$(n+1)(n+2)A_{n+2} = n(n-1)A_n + nA_n - 4A_n$$

$$(n+1)(n+2)A_{n+2} = \{n(n-1) + n - 4\}A_n$$

$$(n+1)(n+2)A_{n+2} = \{n^2 - \cancel{n} + \cancel{n} - 4\}A_n$$

$$(n+1)(n+2)A_{n+2} = \{n^2-4\}A_n$$

$$A_{n+2} = \frac{\cancel{(n+2)}(n-2)A_n}{(n+1)\cancel{(n+2)}}$$

$$\boxed{A_{n+2} = \frac{(n-2)}{(n+1)}A_n} \quad \text{---} \quad (4)$$

This is called Recurrence Relation.

Now Put value $n=0$

$$A_{0+2} = \frac{(0-2)}{(0+1)}A_0$$

$$\boxed{A_2 = -2A_0}$$

Now Put value $n=1$

$$A_{1+2} = \frac{1-2}{1+2}A_1$$

$$\boxed{A_3 = -\frac{A_1}{2}}$$

Now Put values $n=2$

$$A_{2+2} = \frac{2-2}{2+1}A_2$$

$$\boxed{A_4 = 0}$$

Now Put value $n=3$

$$A_{3+2} = \frac{3-2}{3+1}A_3$$

$$A_5 = \frac{A_3}{4} = \frac{-A_1}{4\times 2}$$

$$\boxed{A_5 = -\frac{A_1}{8}}$$

Now Put value $n=4$

$$A_{4+2} = \frac{4-2}{4+1} A_4$$

$$A_6 = \frac{2A_4}{5} = \frac{2\times 0}{5}$$

$$\boxed{A_6 = 0}$$

Now Put value $n=5$

$$A_{5+2} = \frac{5-2}{5+1} A_5$$

$$A_7 = \frac{3}{6} A_5 = \frac{\cancel{3}}{\cancel{6}_2}\left(\frac{-A_1}{8}\right)$$

$$\boxed{A_7 = -\frac{A_1}{16}}$$

Now Put values A_2, A_3, A_4, A_5, A_6 and A_7 -- etc in eq. (3)

$$y = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + A_4 x^4 + A_5 x^5 + A_6 x^6 + A_7 x^7 + \text{-----}$$

$$y = A_0 + A_1 x + (-2A_0)x^2 + \left(-\frac{A_1}{2}\right)x^3 + 0.x^4 + \left(-\frac{A_1}{8}\right)x^5 + 0.x^6 + \left(-\frac{A_1}{16}\right)x^7 + \text{---}$$

So then

$$y = A_0 + A_1 x - 2A_0 x^2 - \frac{A_1}{2}x^3 - \frac{A_1}{8}x^5 - \frac{A_1}{16}x^7 + \text{-----}$$

$$\boxed{y = (1-2x^2) A_0 + \left\{ x - \frac{x^3}{2} - \frac{x^5}{8} - \frac{x^7}{16} - \cdots \right\} A_1}$$

Hence
This is complete solution of eq. (1)

Rough Work

$$\cdots + n \cdot (n-1) A_n x^{n-2} + \cdots$$

$$n = n+2$$

$$(n+2)(n+2-1) A_{n+2} x^{(n+2)-2}$$

$$(n+2)(n+1) A_{n+2} x^n$$

$$y = \cdots + A_n x^n + A_{n+1} x^{n+1} + A_{n+2} x^{n+2} \cdots$$

$$\frac{dy}{dx} = n A_n x^{n-1} + (n+1) A_{n+1} x^n + (n+2) A_{n+2} x^{n+1}$$

$$\frac{d^2y}{dx^2} = n(n-1) A_n x^{n-2} + n(n+1) A_{n+1} x^{n-1} + (n+1)(n+2) A_{n+2} x^n$$

Rough Work

$$--nA_n x^{n+1} --$$

$n = n-1$

$$(n-1) A_{n-1} x^{n-\cancel{1}+\cancel{1}}$$

$$(n-1) A_{n-1} \cdot x^n$$

$$n(n+1) A_n x^{n+1}$$

$$A_n x^{n+1}$$

$n = n-1$

$$A_{n-1} x^{n-\cancel{1}+\cancel{1}}$$

$$A_{n-1} x^n$$

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Q.3 $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$

Solve by series solution method.

Q.4 $(2-x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 0$

Solve by series solution method.

Q.5 $\frac{d^2y}{dx^2} + x^2 y = 0$

Q.6 $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 4y = 0$

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