CENTROID & MOMENT OF INERTIA, PLANE SURFACE

It is fixed for a given body.

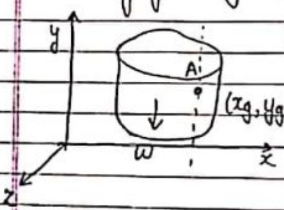
* Centre of mass (CM)
* Centre of gravity (CG) - Point where weight is assumed to be conc
* Centroid (C)

Different at different places due to difference in gravity. It can change from place to place

(For plane surfaces only) Centroid - Point where entire area is assumed to be concentrated

geometrical centre of surfaces having area and no mass.

Centre of Gravity:



$W \Rightarrow$ Magnitude

$\Rightarrow \Sigma dw_i$

$(x_g, y_g, z_g) \Rightarrow$ Location (x_g, y_g, z_g)

$\downarrow$

Varignon's theorem can be used to find location of CG.

$dW_1 \times x_1 + dW_2 \times x_2 + \cdots + dW_n \times x_n = W \times x_g$

$$= \int (dW) \times x = W \times x_g$$

$\left.\begin{matrix} x_g \times W \\ y_g \times W \\ z_g \times W \end{matrix}\right\}$ First moment of weight

$\because$ the power of $(x_g, y_g, z_g) = 1$

But if power $= 2$, then it would be called second moment of weight

$$\boxed{\begin{matrix} X_g = \dfrac{\int x\,dW}{W} \\ Y_g = \dfrac{\int y\,dW}{W} \\ Z_g = \dfrac{\int z\,dW}{W} \end{matrix}}$$

$W = Mg$ constant

$dW = \cancel{m\,dg} + dm \cdot g$

$dW = g\,dm$

$$x = \frac{\int x g\,dm}{mg}$$

$x_{CM} = \dfrac{\int x\,dm}{m}$	$y_{cm} = \dfrac{\int y\,dm}{m}$	$z_{cm} = \dfrac{\int z\,dm}{m}$

$\left.\begin{matrix} x_{CM} \times M \\ y_{CM} \times M \\ z_{CM} \times M \end{matrix}\right\}$ = First moment of mass

$\boxed{F = m a_{CM}}$

CENTROID (C)

$\rho = \dfrac{m}{V}$

$m = V\rho$

If "g" is constant, then CG, CM coincide

$dm = \rho dV + V d\rho$

$\rho \rightarrow$ constant

$\boxed{dm = \rho dV}$

$$\bar{x} = \frac{\int x \rho \, dm}{\rho V}$$

$$\boxed{\bar{x} = \frac{\int x \, dV}{V}}$$

$\bar{x}, \bar{y}, \bar{z}$ = geometrical centre of the volume of the object → First moment of volume

Volume = Area × height

$V = At$

$t \rightarrow$ constant

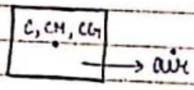
$dV = t\,dA$

$$\bar{x} = \frac{\int x t \, dA}{At}$$

$$\boxed{\bar{x} = \frac{\int x \, dA}{A}}$$

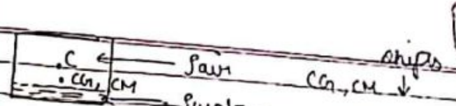
$\bar{x} \times A \rightarrow$ First moment of area

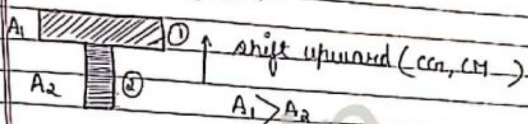
$\bar{x}, \bar{y}, \bar{z} \Rightarrow$ Geometrical centres of area

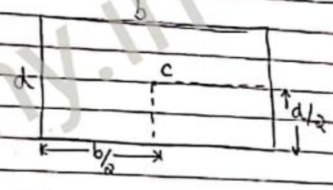


In following questions C, CM, CG will coincide.

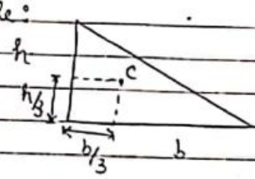
CM, CG shifts towards more mass & density

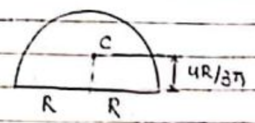
Ans.


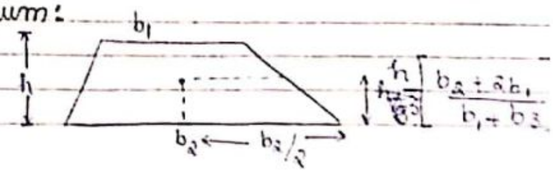
Ans.


1) Rectangle:


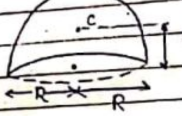
2) Circle:


3) Triangle:


4) Semi circle:


5) Trapezium:


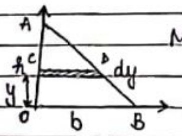
6) Hemisphere :



$\left.\begin{array}{l}\bar{x} \times A \\ \bar{y} \times A\end{array}\right\}$ First moment of area "A"

$\left.\begin{array}{l}x \times dA \\ y \times dA\end{array}\right\}$ First moment of area of strip having area "A"

$\left.\begin{array}{l}\int x\,dA \\ \int y\,dA\end{array}\right\}$ sum of first moment of Area of all strips having area "A".



Moment of Area of strip CD about "O" $= dA \times y$

$= CD \times dy \times y$

$\Delta ACD \sim \Delta AOB$

$$\frac{AC}{AO} = \frac{CD}{OB}$$

$$\frac{h-y}{h} = \frac{CD}{b}$$

Moment of Area (MOA) of strip CD $= \left(\frac{h-y}{h}\right) b \cdot y \cdot dy$

Total moment of area of all strips $= \int y\,dA$

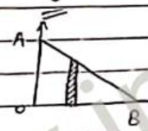
$$= \int_0^h \left(\frac{h-y}{h}\right) b\,y\,dy$$

$$= \frac{b}{h}\left[\frac{h^3}{2} - \frac{h^3}{3}\right] = \frac{bh^2}{6}$$

$$\bar{y} = \frac{\int y\,dA}{A}$$

$$= \frac{(bh^2/6)}{(bh/2)} = \frac{h}{3}$$

$$\bar{x} = \frac{b}{3}$$



Find CG of this figure

MOA $= dA \times y$

$= CD \times dy \times y$

$x = \frac{x^2}{4} \quad x = 4$

$$\text{Area} = \int_0^4 x\,dx - \int_0^4 \frac{x^2}{4}\,dx$$

$$\text{Area} = \frac{x^2}{2} - \frac{x^3}{12} = \frac{\cancel{2}x}{2} - \frac{\cancel{x}x^2}{\cancel{12}\,4} \qquad \frac{16}{2} - \frac{\cancel{64}\,16}{\cancel{12}\,3}$$

$$x - \frac{x^2}{4} \qquad \frac{16}{6}$$

$$\int_0^4 \left(x - \frac{x^2}{4}\right) x \cdot dx$$

$$\int_0^4 \left(x^2 - \frac{x^3}{4}\right) dx = \left[\frac{x^3}{3} - \frac{x^4}{12}\right]_0^4$$

$$\frac{64}{3} - \frac{64 \times \cancel{4}}{\cancel{12}\,3}$$

$$= \frac{64}{12} \quad \frac{16}{3}$$

$$= \frac{16}{3} \Big/ \frac{16}{6}$$

/6

$\bar{y} = 4$

MOA of strip $= dA \times x$

$= x \times dx \times (y_1 - y_2)$

Total MOA of all strips $= \int_0^4 (y_1 - y_2)\, x\, dx$

For $x = x$; $y_1 = x$

For $x = x$; $y_2 = \frac{x^2}{4}$

$$= \int_0^4 \left(x - \frac{x^2}{4}\right) x\, dx$$

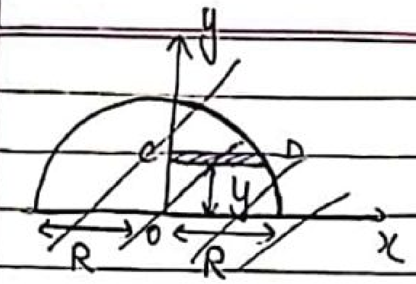
$$= \int_0^4 \left(x^2 - \frac{x^3}{4}\right) dx = 5.33 \text{ units}$$

$$\bar{x} = \frac{\int x\, dA}{A} = \frac{\int x\, dA}{\int dA}$$

$dA = (y_1 - y_2)\, dx$

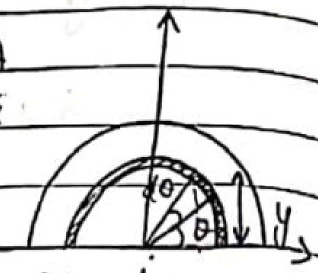
$$A = \int_0^4 \left(x - \frac{x^2}{4}\right) dx = 2.87 \text{ units}$$

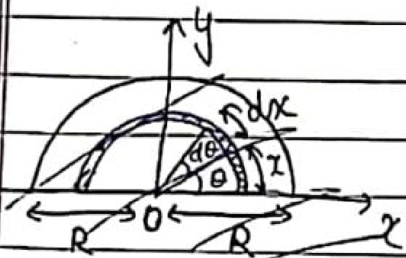
Ans.



$MOA = y \times dA$

$= y \times dy \, x$





$dx = r\,d\theta \qquad dA = \pi r\,dr$

$\frac{y}{r} = \sin\theta$

$y = \sin\theta \, r$

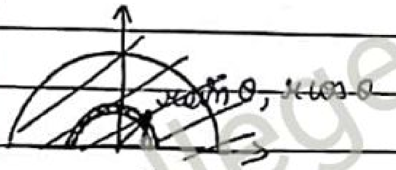
$\boxed{\sin[\theta r \pi r\,dr]}$

$d\theta \quad dr = r\,d\theta$

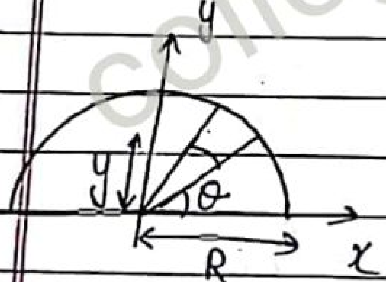
$\bar{x} = 0$ because symmetrical about origin

MOA of strip $d\theta = r \sin\theta \, \pi r\,dr$

$= r^2 \sin\theta \, \pi \, dr$



$r \sin\theta$



MOA of strip $= y\,dA$

$= y\,dr\,r\,d\theta$

$= r\sin\theta \, r\,dr\,d\theta$

Total MOA $= \int_0^{\pi} \int_0^{R} r^2 \sin\theta \, dr\,d\theta$

$= \frac{2R^3}{3}$

$y = r\sin\theta$

$\bar{y} = \frac{\int y\,dA}{A}$

$= \left(\frac{2R^3}{3}\right) \Big/ \left(\frac{\pi R^2}{2}\right) \quad = \frac{4R}{3\pi}$

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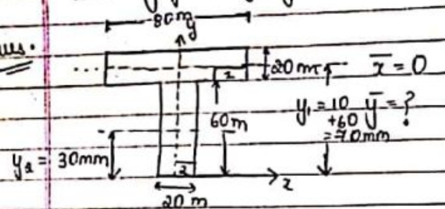
Centre of gravity of composite bodies:

Ans.



$\bar{x} = 0$ (∵ symmetrical about y-axis)

$y_1 = 10 + 60 = 70mm$ $\quad \bar{y} = ?$

$A_1 = 80 \times 20 = 1600mm^2$

$A_2 = 20 \times 60 = 1200mm^2$

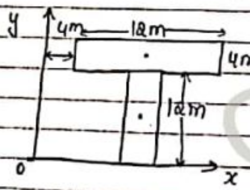
$$\bar{Y} = \frac{\int y\,dA}{A}$$

$$\bar{y} = \frac{\sum y_i \cdot A_i}{A}$$

$$\bar{y} = \frac{[(70) \times (80 \times 20)] + [30 \times (20 \times 60)]}{(80 \times 20) + (20 \times 60)}$$

$\bar{Y}_1 = 52.85mm > Y_2$

Ans.



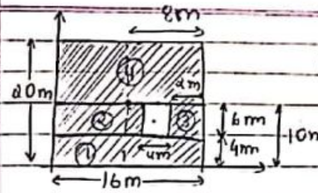
$\bar{x} = ?$

$\bar{y} = ?$

$$\bar{y} = \frac{6 \times 12 \times 4 + 14 \times 12 \times 4}{12 \times 4 + 12 \times 4} = \frac{6 + 14}{2} = 10$$

$$\bar{x} = \frac{10 \times 12 \times 4 + 10 \times 12 \times 4}{12 \times 4 + 12 \times 4} = 10$$

Ans.



$CG_{total} = CG_{shaded} - CG_{hole}$

$\bar{x}_{unshaded} = 12m$, $\bar{y}_{unshaded} = 7m$

$\bar{x}_{uns} = 12m$ $\quad \bar{y}_{uns} = 7m$

~~16×4×8+~~

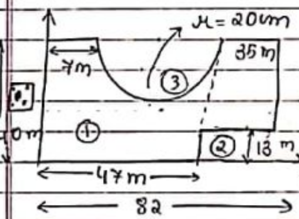
$$\bar{y} = \frac{3200 - 168}{320 - 24} = \frac{3032}{296} = 10.24$$

$$\bar{x}_{shaded} = \bar{x} = \frac{8 \times 16 \times 20 - 12 \times 6 \times 4}{16 \times 20 - 24}$$

$$= \frac{2560 - 288}{296}$$

$\bar{x} = 7.67$

$47 + 35 = 82$



$y_1 = 20 \quad x_1 = 41$

$x_2 = 64.5 \quad y_2 = 6.5$

$x_3 = 27 \quad y_3 = 31.5$

$47 + 17.5 = 64.5$

$$\frac{(41 \times 40 \times 82) - (64.5 \times 13 \times 35) - \left(27 \times \frac{3.14 \times 20 \times 20}{2}\right)}{40 \times 82 - 13 \times 35 - \frac{3.14 \times 20 \times 20}{2}}$$

$$\frac{88176.5}{2197}$$

$= 40.13$

$$\bar{y} = \frac{(20\times40\times82) - (6.5\times13\times35) - \left(31.5\times\frac{3.14\times20\times20}{2}\right)}{2197}$$

$$= \frac{42860.5}{2197}$$

$$= 19.5086$$

MOMENT OF INERTIA

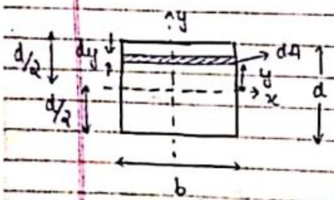
$I_{xx} = \int y^2 dA$ } Second moment of area. can never be -ve as square is present.

$I_{yy} = \int x^2 dA$

Axis of moment of inertia

$\bar{x} = \frac{\int x\,dA}{A}$ } First moment of Area

$\bar{y} = \frac{\int y\,dA}{A}$



$I_{xx} = \int y^2 b\,dy$

$= b\int_{-d/2}^{d/2} y^2 dy$

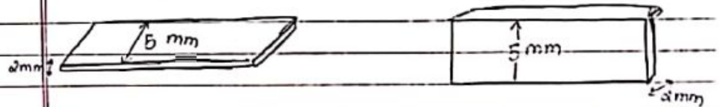
$b\left(\frac{y^3}{3}\right)_{-d/2}^{d/2}$

$\frac{b}{3}\left[\left(\frac{d}{2}\right)^3 - \left(\frac{-d}{2}\right)^3\right]$

$I_{xx} = \frac{bd^3}{12}$

Short trick of MI of rect. } when finding out M.I. about x-axis, the ~~side ⊥r to x-axis~~ cube of side ⊥r to x-axis is taken. ~~and~~ For finding out M.I. about y-axis, the cube of side ⊥r to y-axis is taken.

$I_{yy} = \frac{db^3}{12}$



$I_{xx} = \frac{5\times2^3}{12}$ $\quad I_{xx_2} = \frac{2\times5^3}{12}$

$\frac{I_{xx_1}}{I_{xx_2}} = \frac{5\times2^3}{2\times5^3} = \frac{4}{25}$

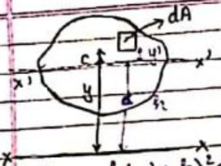
$I_{xx_2} = 6.25\, I_{xx}$

$I_{xx} = \int y^2 dA$

$I_{yy} = \int x^2 dA$

Greater the value of M.I., greater is the bending resistance.

Date:
Page No:



$y = y' + h$

$I_{xx} = \int (y' + h)^2 dA$

$I_{xx} = \int y^2 dA$

$= \int (y')^2 dA + \int 2hy' dA + \int h^2 dA$

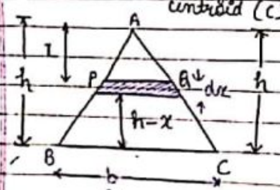
$I_{xx} = I_{x'x'} + 2h \int y' dA + h^2 A$

$I_{xx} = I_{x'x'} + h^2 A$

$I_{yy} = I_{y'y'} + Ah^2$ → normal distance between $y'y'$ & yy axes

This axis must pass through CG

$I_{\text{about any axis}} = I$ about a // axis passing through centroid (C) $+ Ah^2$ ← normal distance b/w both the bodies



Moment of inertia of strip PQ about BC $= \cancel{\ldots}$

$= (h-x)^2 dA$

$= (h-x)^2 \frac{bx}{h} dx$

$\frac{PQ}{BC} = \frac{x}{h}$

$PQ = \frac{x}{h} \times b$

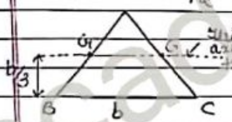
Total Moment of Inertia about base BC:

Date:
Page No:

$= \int_0^h (h-x)^2 \frac{bx}{h} dx$

$= \frac{b}{h} \int_0^1 x (h^2 + x^2 - 2xh) dx$

$= \frac{bh^3}{12}$



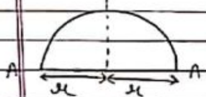
G.G. axis passes through centroid

$I_{BC} = I_{CG} + Ah^2$

$\frac{bh^3}{12} = I_{CG} + \frac{1}{2} bh \left(\frac{h}{3}\right)^2$

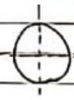
$I_{CG} = \frac{bh^3}{36}$

Parallel axis theorem: $I = I_{CG} + Ah^2$

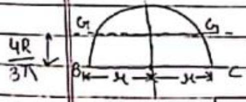


$I_{\text{semicircle about AA}} = \frac{\pi}{128} d^4$

(yahan par diameter ke about liya hai)



$I_{\text{full circle}} = \frac{\pi}{64} d^4$



$I_{\text{about BC}} = I_{CG} + Ah^2$

$\frac{\pi}{8} r^4 = I_{CG} + \frac{\pi r^2}{2} \left(\frac{4r}{3\pi}\right)^2$

$\boxed{I_{CG} = 0.11 r^4}$

aur yahan radius ke about

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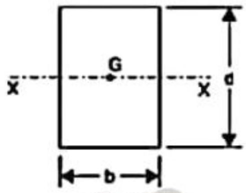
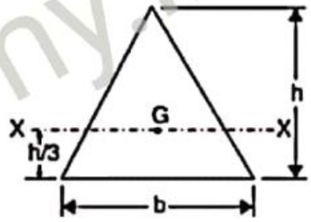
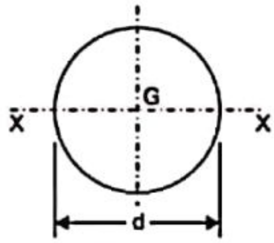
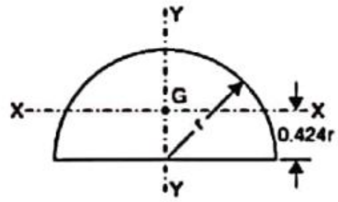
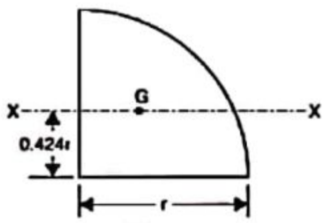
Table 5.1. Moments of Inertia for Simple Areas

Shape	*Moment of inertia*	*Simple areas*
Rectangle	$I_{xx} = \dfrac{bd^3}{12}$	 Fig. 5.2
Triangle	$I_{xx} = \dfrac{bh^3}{36}$	 Fig. 5.3
Circle	$I_{xx} = \dfrac{\pi d^4}{64}$	 Fig. 5.4
Semi-circle	$I_{xx} = 0.11r^4$ $I_{yy} = \dfrac{\pi d^4}{128}$	 Fig. 5.5
Quadrant	$I_{xx} = 0.055r^4$	 Fig. 5.6

5.4. RADIUS OF GYRATION OF THE SECTION

One of the properties of cross-section which influence the structural behaviour of the members is *radius of gyration*.

$$k_i = \sqrt{\frac{I_i}{A}} \qquad \ldots(5.7) \; (\because \; I_i = Aki^2)$$

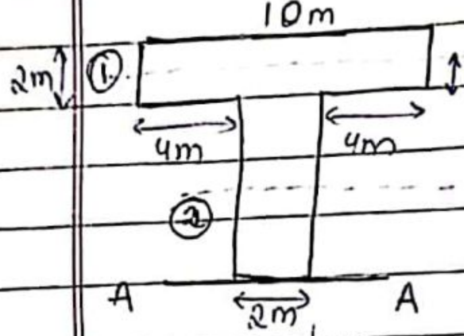
where I_i = moment of inertia about ith axis ; and

k_i = radius of gyration of area about ith axis.

Members, when subjected to axial forces tend to buckle. The load at which members will buckle is proportional to the square of the radius of the gyration. The radius of gyration is usually referred to *with respect to centroidal axes system of the reaction.*

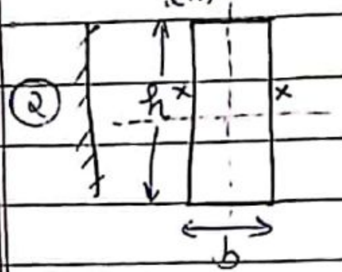
Date:
Page No:

I semi-circle about CC parallel to y-axis $= I_{base} = \frac{\pi}{8} r^4$



$I = I_{CG} + Ah^2$

$I_A = I_{A_1} + I_{A_2}$



$I_{xx} = \frac{bh^3}{12}$

$I_{yy} = \frac{hb^3}{12}$

$I_{A_2} = \frac{2 \times 8^3}{12} + (2 \times 8) \times 4^2$

~~$I_{A_1} =$~~

$(8+1)^2$

$I_{A_1} = \frac{10 \times 2^3}{12} + (10 \times 2) \times \left(8 + \frac{2}{2}\right)^2$

$= \frac{10 \times 3^3}{12} + (10 \times 2) \times 9^2$

$I = I_{A_1} + I_{A_2}$

$I_{CG} = I_{CG_1} + I_{CG_2}$

$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

$= \frac{(10 \times 2) \times \frac{2}{2} + (2 \times 8) \times \left(2 + \frac{8}{2}\right)}{20+16}$

$= 3.22$

~~I_{CG_1}~~ $= I_{G_1} = \frac{10 \times 2^3}{12} + (10 \times 2) \times (y_1)^2$

$$= \frac{10 \times 2^3}{12} + (10 \times 2) \times \left(3.22 - \frac{2}{2}\right)^2$$

$$I_{C_2} = \frac{2 \times 8^3}{12} + (2 \times 8) \times (10 - \bar{y})^2$$

$$= \frac{2 \times 8^3}{12} + 16 \times (6.78)^2$$