VECTOR

$\bar{a}\cdot\bar{b} = |\bar{a}|\,|\bar{b}|\cos\theta$

$\bar{a}\times\bar{b} = |\bar{a}|\,|\bar{b}|\sin\theta\,\hat{n}$

$\bar{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

tangent $\dfrac{d\bar{r}}{dt} = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j}$

∇

$$i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

~~(Scalar)~~ $\phi(x,y,z) \Rightarrow \nabla\phi$ gradient

Vector $\vec{F}$ → $\nabla\cdot\vec{F}$ scalar, div F

→ $\nabla\times F$, Curl F

$$i\frac{\partial}{\partial x}\phi + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

$$\phi(x,y,z) = c$$

$$\frac{\partial\phi}{\partial x}\frac{dx}{dt} + \frac{\partial\phi}{\partial y}\frac{dy}{dt} + \frac{\partial\phi}{\partial z}\frac{dz}{dt} = 0$$

$$\underbrace{\left(i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}\right)}_{\nabla\phi\ \to\ \text{normal}}\cdot\underbrace{\left(i\frac{dx}{dt} + j\frac{dy}{dt} + k\frac{dz}{dt}\right)}_{\frac{d\bar{r}}{dt}\ \to\ \text{tangent}} = 0$$

* The gradient of a scalar point function $\phi(x,y,z)$ is given by

$$\text{grad}\,\phi = \nabla\phi = \left(i\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\phi$$

$$= i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}$$

which is a vector point function.

* The unit normal vector to the surface $\phi(x, y, z)$ is given by

$$\hat{n} = \frac{\nabla\phi}{|\nabla\phi|}$$

* If θ is the angle b/w two surfaces $f(x,y,z) = c$ and $g(x,y,z) = t$ at point P then $\cos\theta = \frac{\nabla f_{at P} \cdot \nabla g_{at P}}{|\nabla f_{at P}| \, |\nabla g_{at P}|}$

i.e. angle b/w tangents is always equal to angle b/w their normal.

* If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = \sqrt{x^2+y^2+z^2}$ then

(i) $\nabla(f(r)) = \frac{f'(r)}{r}\bar{r}$ (ii) $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$

* The directional derivative of surface $\phi(x,y,z)$ at point P in the dirn of a vector $\bar{a}$ is given by $\left(\nabla\phi_{at P} \cdot \frac{\bar{a}}{|\bar{a}|}\right)$

* The max. value of directional derivative to the surface $\phi(x,y,z)$ at point P is given by $|\nabla\phi_{\text{at}\,P}|$

$$\nabla\phi \cdot \frac{\bar{a}}{|\bar{a}|} = |\nabla\phi| \left|\frac{\bar{a}}{|\bar{a}|}\right| \cos\theta$$

$$\left|\frac{\bar{a}}{|\bar{a}|}\right| \downarrow 1 \qquad \cos\theta \downarrow 1$$

* The divergence of a vector point function $\bar{F} = F_1\hat{i} + F_2\bar{j} + F_3\bar{k}$ is given by

$$\bar{F} = \nabla \cdot F = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \cdot (\bar{F}_1 i + F_2\bar{j} + F_3 k)$$

$$= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$ which is a scaler point function.

$\nabla \cdot \bar{F} = 0 \Longleftrightarrow \bar{F}$ is a solenoidal vector.

* Div (Curl F) = 0 i.e curl $\bar{F}$ is always solenoidal vector

* The curl of a vector point function $\bar{F} = F_1 i + F_2 \hat{j} + F_3 \hat{k}$ is given by

$$\text{curl}\,\bar{F} = \nabla \times \bar{F}$$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$= i\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right) - \bar{j}\left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right) + \bar{k}\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)$$

$\Rightarrow$ which is a vector point function

$\nabla \times \vec{F} = 0 \Leftrightarrow \bar{F}$ is irrotational vector

Curl (grad ϕ) $= 0$

$\nabla \times (\nabla \cdot \phi) = 0$

Q $\nabla (r^n) =$ ____

$\because f(r) = r^n, \quad f'(r) = n r^{n-1}$

$$\nabla (f r) = \frac{f'(r)}{r} \bar{r}$$

$$= \frac{n r^{n-1}}{r} \times \bar{r}$$

$$= n r^{n-2} \bar{r}$$

(2) $\nabla (\ln r)$. $f(r) = \ln r, \quad f'(r) = 1/r$

$$\nabla f(r) = \frac{f'(r)}{r} \bar{r} = \frac{1/r}{r} \bar{r}$$

$$= \frac{1}{r^2} \bar{r}$$

(3) $\text{div}(r^2 \nabla \ln r) = \_\_\_$

$$\text{div}\left[\cancel{r^2}\left(\frac{1}{\cancel{r^2}}\vec{r}\right)\right] = \nabla \cdot \vec{r}$$

$$\Rightarrow \left[\hat{i}\left(\frac{\partial}{\partial x}\right) + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] \cdot (x\hat{i} + y\hat{j} + z\hat{k})$$

$$= \frac{\partial}{\partial x}x + \frac{\partial}{\partial y}y + \frac{\partial}{\partial z}z$$

$$= 1 + 1 + 1 = 3$$

(4) $\text{curl}(r^2 \nabla \ln r)$

$$\nabla \times \left(\cancel{r^2}\frac{1}{\cancel{r^2}}\vec{r}\right)$$

$$= \nabla \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix}$$

$$\hat{i}(0) - \hat{j}(0) + \hat{k}(0)$$
$$= 0$$

Q $\nabla^2\left(\frac{1}{r}\right)$: $f(r) = \frac{1}{r}$, $f'(r) = \frac{-1}{r^2}$

$f''(r) = 2/r^3$

$$\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$$

$$\nabla^2\left(\frac{1}{r}\right) = \frac{2}{r^3} + \frac{2}{r} \times \frac{-1}{r^2}$$

$$= \frac{2}{r^3} - \frac{2}{r^3} = 0$$

(1) $\phi : x^3 - y^3 + x^2 z$ at $(1,1,-2)$

$$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

$$= i(3x^2+2xz) + j(-3y^2) + \hat{k}(x^2)$$

Unit Normal $\hat{n} = \dfrac{\nabla\phi}{|\nabla\phi|}$

$$\nabla\phi = i(3-4) + j(-3) + k(1) \text{ at } (1,1,-2)$$

$$= -\hat{i} - 3\hat{j} + \hat{k}$$

$$|\nabla\phi| = \sqrt{11}$$

$$\Rightarrow \hat{n} = \frac{-\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{11}}$$

WBB $\bar{F} = \boxed{r^n \bar{r}}$ is Solenoidal

$$\bar{F} = r^n(xi + yj + zk)$$

$$\bar{F} = (r^n x i + r^n y j + r^n z k)$$

$$\boxed{\nabla \cdot \bar{F} = 0}$$

$$\frac{\partial}{\partial x}(r^n x) + \frac{\partial}{\partial y}(r^n y) + \frac{\partial}{\partial z}(r^n z) = 0$$

$$r^n + x n r^{n-1}\frac{\partial r}{\partial x} + r^n + y n r^{n-1}\frac{\partial r}{\partial y} + r^n + z n r^{n-1}\frac{\partial r}{\partial z} = 0$$

$$3r^n + n r^{n-1}\left(x\frac{\partial r}{\partial x} + y\frac{\partial r}{\partial y} + z\frac{\partial r}{\partial z}\right) = 0$$

$$3r^n + n r^{n-1} r = 0$$

$$3r^n + n r^n = 0$$

$(3+n)r^n = 0$

$\boxed{n = -3}$

$\Rightarrow$ $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$r = |\bar{r}| = \sqrt{x^2+y^2+z^2}$ is homogenous of degree 1

$x\frac{\partial r}{\partial x} + y\frac{\partial r}{\partial y} + z\frac{\partial r}{\partial z} = nr = 1r$

Q $|\bar{F}| = r^n$ and $\nabla \cdot \bar{F} = 0$ then $n =$ ___

$\downarrow$

$\bar{F} = r^n \times \frac{\bar{r}}{r} = r^{n-1}\bar{r}$ and $\nabla \cdot \bar{F} = 0$

so from previous problem

$n - 1 = -3$

$\boxed{n = -2}$

Q If $\bar{F} = \frac{(x,y,z)}{(x^2+y^2+z^2)^{3/2}}$, $(x,y,z) \in R^3$ then $\nabla \cdot \bar{F}$ ___

$\bar{F} = \frac{(x\hat{i}+y\hat{j}+z\hat{k})}{(x^2+y^2+z^2)^{3/2}} = \frac{\bar{r}}{(r^2)^{3/2}} = \frac{\bar{r}}{r^3}$

$= r^{-3}\bar{r}$ we know the it is a solenidal $\nabla \cdot F = 0$

WB Q3

$\phi = x^2yz + 4xz^2$

$\nabla\phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$

$i(2xyz + 4z^2) + j(x^2 z) + k(x^2 y + 8xz)$

$\nabla\phi_{\text{at}(1,-2,1)} = i(4+4) + j(-1) + k = (-2-8)$

$\vec{a} = \sqrt{2i - j + 2k} = |a| = \sqrt{4+1+4}$
$= \sqrt{9} = 3$

$D.D = \nabla\phi_{\text{at } p}\dfrac{\vec{a}}{|a|}$

$= (8i - j - 10k)\left(\dfrac{2i - j - 2k}{3}\right)$

$= \dfrac{16+1+20}{3} = \dfrac{37}{3}$

(4) $\phi: x^{2/3} + y^{2/3}$

$\nabla\phi = i\dfrac{\partial\phi}{\partial x} + j\dfrac{\partial\phi}{\partial y}$

$= i\left(\frac{2}{3}\right)x^{-1/3} + j\left(\frac{2}{3}y^{-1/3}\right)$

$\nabla\phi_{(8,8)} = i\left(\frac{2}{3}\right)(8)^{-1/3} + j\left(\frac{2}{3}8^{-1/3}\right)$

$= \frac{1}{3}i + \frac{1}{3}j$

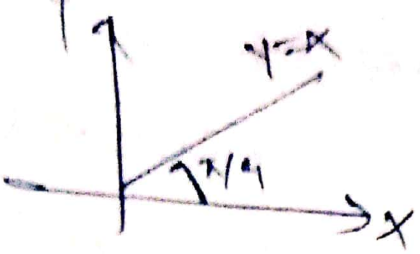
$\bar{r} = xi + yj$

$\vec{r} = r\cos\theta\, i + r\sin\theta\, j$

$\bar{r} = r(\cos\theta\, i + \sin\theta\, j)$

$\dfrac{\bar{r}}{r} = \cos\frac{\pi}{4}i + \sin\frac{\pi}{4}j$

$\dfrac{\bar{r}}{r} = \frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j$



$$D.D. = \nabla a \text{ at } p \, \frac{\bar{a}}{|a|}$$

$$= \nabla a \text{ at } p \, \frac{\bar{r}}{|\bar{r}|}$$

$$= \left(\frac{1}{3} i + \frac{1}{3} j\right) \cdot \left(\frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} j\right)$$

$$= \frac{1}{3\sqrt{2}} + \frac{1}{3\sqrt{2}}$$

$$= \frac{\sqrt{2}}{3}$$

Green theorem :⇒ If $F_1(x, y)$ and $F_2(x, y)$ are two differential function defined on a region R bounded by a single closed curve c then

$$\underbrace{\oint_C F_1 \, dx + F_2 \, dy}_{\text{work done}} = \iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\,dy$$

Stoke's theorem :⇒

If $F = F_1 i + F_2 \hat{j} + F_3 \hat{k}$ is a diff^n vector point function defined on an open surface S bounded by a simple closed curve c then

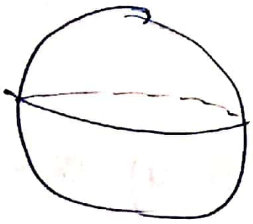
$$\underbrace{\oint_C \bar{F} \, d\bar{r}}_{\text{work done}} = \iint_S (\nabla \times F) \cdot \hat{n} \, ds$$

Gauss Divergence theorem :-

If $\vec{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$ is a differentiable vector point function defined on a closed surface S enclosing volume V then $\boxed{\oint\!\!\oint_S \vec{F}\cdot\hat{n}\,ds = \iiint_V \nabla\cdot\vec{F}\,dV}$

flux

$\hat{n} = \dfrac{\nabla\phi}{|\nabla\phi|}$

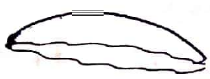


Remember

Surface (S)

closed surface $\longrightarrow$ Gauss divergence

open surface $\longrightarrow$ stoke's theorem



simple closed curve

$F_1 i + F_2 j \longrightarrow$ green theorem

$F_1 i + F_2 J + F_3 k \longrightarrow$ stokes theorem

(8) By gauss $\iint_S F\cdot\hat{n}\,dS = \iiint_V \nabla\cdot\bar{F}\,dV$

$F = xi + yj + zk$

$\nabla\cdot F = \frac{\partial}{\partial x}x + \frac{\partial}{\partial y}y + \frac{\partial}{\partial z}z$

$= 1+1+1 = 3$

$= \iiint_V 3\,dV = 3V$

where V is volume of $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

$V = \frac{4}{3}\pi abc$

$= \frac{3\times 4}{3}\pi abc$

$= 4\pi abc$

(6) By Gauss $\iint_S F\cdot\hat{n}\,dS = \iint \nabla\bar{F}\,dV$

$\iint \underbrace{x\sin y, \cos^2 x, 2z-2\sin y}_{F} \cdot \underbrace{(x,y,z)}_{\hat{n}}\,dS$

$\nabla\cdot F = \frac{\partial}{\partial x}(x\sin y) + \frac{\partial}{\partial y}(\cos^2 x) + \frac{\partial}{\partial z}(2z-2\sin y)$

$\sin y + 0 + 2 - \sin y$

$= 2$

$x^2+y^2+z^2=1$

$\nabla\phi = i(2x) + j(2y) + k(2z)$

$|\nabla\phi| = \sqrt{4x^2+4y^2+4z^2} = \sqrt{4} = 2$

$$n = \frac{\nabla\phi}{|\nabla\phi|} = \frac{2x\hat{i} + 2y\hat{j} + 2z\hat{k}}{2}$$

$$= x\hat{i} + y\hat{j} + z\hat{k}$$

$$= \iiint_V 2\,dv = 2V \text{ where } V \text{ is}$$

volume of $x^2+y^2+z^2=1$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1)^3$$

$$= \frac{4\pi}{3}$$

$$= 2 \times \frac{4\pi}{3}$$

$$= \frac{8\pi}{3}$$

(8) $\iint_S \left((2x^2+3x) - y^2 + 5z^2\right) ds$

$F \cdot \hat{n}$

$$\iint_S \left[(2x+3)\hat{i} - y\hat{j} + 5z\hat{k}\right] \cdot (x\hat{i} + y\hat{j} + z\hat{k})\, ds$$

$$\nabla \cdot F = \frac{\partial}{\partial x}(2x+3) + \frac{\partial}{\partial y}(-y) + \frac{\partial}{\partial z}(5z)$$

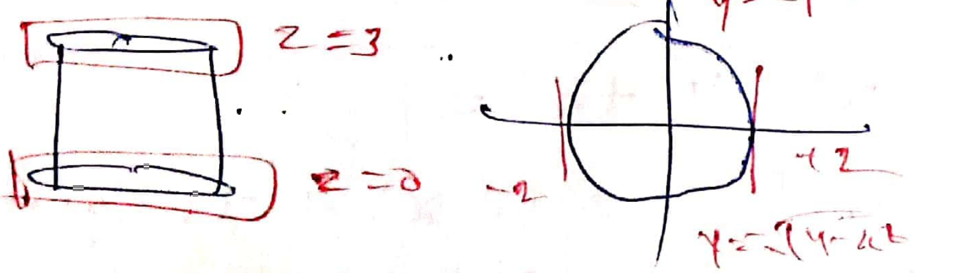
$$2 - 1 + 5 = 6$$

$$= \iiint_V 6\,dv = 6V = 6 \times \frac{4}{3}\pi r^3$$

$$= 6 \times \frac{4}{3} \times \pi \times 1$$

$$= 8\pi$$

(9) $x^2+y^2=4,\ z=0,\ z=3$



$F = 4x\,i - 2y^2\,j + z^2\,k$

$\nabla \cdot \overline{F} = \frac{\partial}{\partial x}(4x) + \frac{\partial}{\partial y}(-2y^2) + \frac{\partial}{\partial z}(z^2)$

$= 4 - 4y + 2z$

$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{3} (4 - 4y + 2z)\, dz\, dy\, dx$

put $x = r\cos\theta$, $y = r\sin\theta$

Method (2)

$\iiint dx\,dy\,dz = \int_0^{z}\int_0^{2\pi}\int_0 r\,dr\,d\theta\,dz$

$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left[(4-4y)z + \frac{2 \times z^2}{2}\right]_0^3 dy\, dx$

$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (12 - 12y + 9)\, dy\, dx$

$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 21\, dy\, dx - \underbrace{\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 12y\, dy\, dx}_{0}$

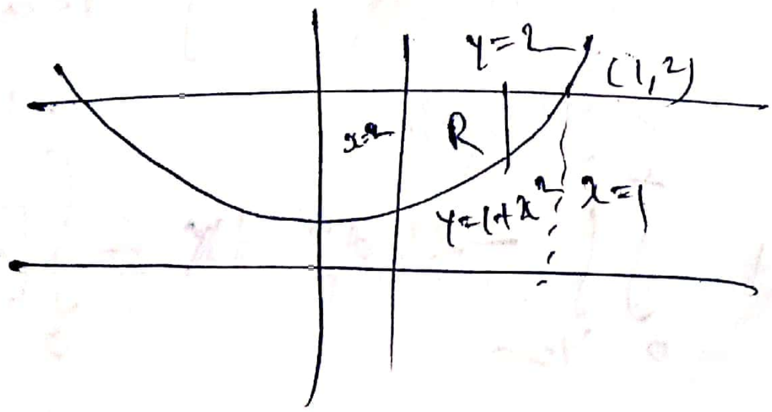
(odd fun)

$21(4\pi)$

$= \underline{84\pi}$

$\boxed{\iint dy\, dx = \text{area of } x^2 + y^2 = 4 = \pi(2)^2 = 4\pi}$

(9) by green



$$\int_C F_1\,dx + F_2\,dy = \iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\,dy$$

$F_1 = \frac{e^y}{x}$ $\qquad F_2 = e^y \ln x + x$

$\frac{\partial F_1}{\partial y} = \frac{e^y}{x}$, $\quad \frac{\partial F_2}{\partial x} = \frac{e^y}{x} + 1$

$$= \iint \left(\cancel{\frac{e^y}{x}} + 1 - \cancel{\frac{e^y}{x}}\right) dy\,dx$$

$$= \int_{x=1/2}^{1} \int_{1+x^2}^{2} dy\,dx$$

$$= \int_{1/2}^{1} [y]_{1+x^2}^{2}\,dx = \int_{1/2}^{1} (1-x^2)\,dx$$

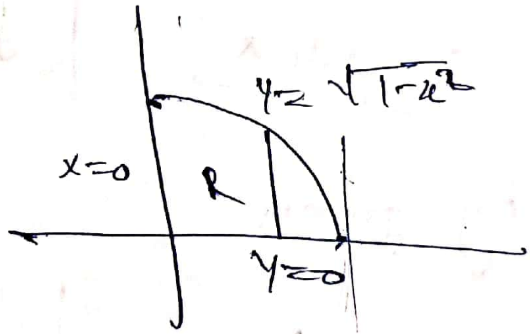
$$= \left(x - \frac{x^3}{3}\right)_{1/2}^{1} = \left(\frac{1}{1} - \frac{1}{3}\right) - \left(\frac{1}{2} - \frac{1}{24}\right)$$

$$= \frac{5}{24}$$

(10) $$\int_C F_1\,dx + F_2\,dy = \iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\,dy$$

$\Rightarrow F_1 = 2x^2 + y^2$

$\frac{\partial F_1}{\partial y} = 2y$



$F_2 = e^y$

$\frac{\partial F_2}{\partial x} = 0$

$$\Rightarrow \int_0^1 \int_{y=0}^{y=\sqrt{1-x^2}} -2y\,dy\,dx = \int_0^1 \left[-y^2\right]_0^{\sqrt{1-x^2}} dx$$

8/3/17 $\int_0^1 (-1+x^2)\,dx = \left[-x+\frac{x^3}{3}\right]_0^1$

$= \frac{1}{3}-1 = \frac{-2}{3}$

(18) $\int_C yz\,dx + xz\,dy + xy\,dz$

from $(1,1,0) \to (2,3,2)$

$\Rightarrow$ Note :- No theorem is applicable because here curve is line segment which is not closed.

$\Rightarrow \int_{(1,1,0)}^{(2,3,2)} d(xyz) = [xyz]_{(1,1,0)}^{(2,3,2)}$

$= 12$

(19) $(0,0,0),\ (1,1,1)$

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z_2-z_1}{z_2-z_1}$

$\frac{x}{1} = \frac{y}{1} = \frac{z}{1} = t$ (say)

$x=t, \quad y=t, \quad z=t$

$dx=dt, \quad dy=dt \quad dz=dt$

t varying from $0 \to 1$

work done $= \int_C \vec{F}\cdot d\vec{r}$

$= \int_C F_1\,dx + F_2\,dy + F_3\,dz$

$$= \int_0^1 5t^2\,dt + (t^2-t)\,dt + 3t\,dt$$

$$= \int_0^1 (6t^2+2t)\,dt = \frac{6}{3} + \frac{2}{2} = 3$$

W16 (3) by stoke's $\int_C \bar{F}\cdot d\bar{r} = \iint_S (\nabla\times F)\cdot\hat{n}\,ds$

$$= \iint_S (-\hat{i}-\hat{j}-\hat{k})\cdot\hat{k}\,ds$$

$$= \iint (-1)\,ds$$

$= (-1)S$ where S is the area of $x^2+y^2=1$

$S = (\pi)(1)^2 = \pi$ | $z=0$ surface xy

$= -\pi$

$$\nabla\times\bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & z & x \end{vmatrix} = -\hat{i}-\hat{j}-\hat{k}$$

$\hat{n} = \hat{k}$

[assume it in a ~~ato~~ anticlockwise direction] (right hand rule)

$x = \cos\theta$, $y = \sin\theta$ $\quad z=0$

$x^2+y^2=1$ $\quad$ xy plane

(12) Stokes $\int_C \bar{F}\cdot d\bar{r} = \iint_S (\nabla\times\bar{F})\cdot\hat{n}\, ds$

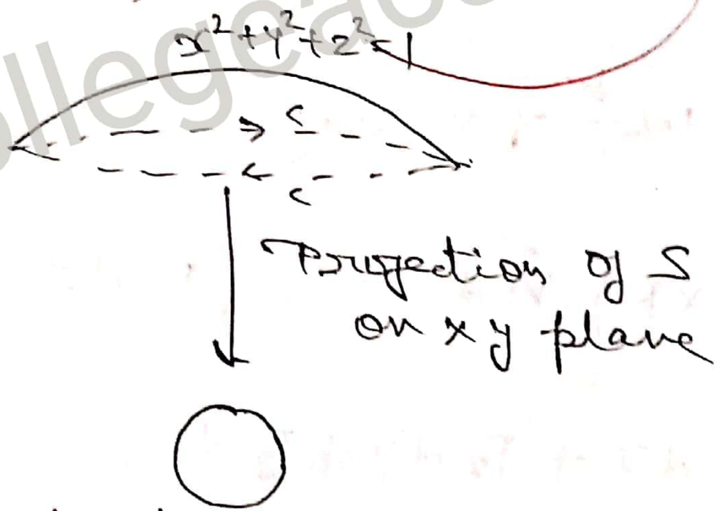
use for plain surface

$$\nabla\times\bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2x-y & -yz^2 & -y^2z \end{vmatrix}$$

$$= \hat{k}$$

$$\hat{n} = \frac{\nabla\phi}{|\nabla\phi|} = x\hat{i}+y\hat{j}+z\hat{k}$$

$\Rightarrow$



$$ds = \frac{dx\,dy}{|\hat{n}\cdot\hat{k}|} \quad \text{on } xy \text{ plane}$$

$$ds = \frac{dy\,dz}{|\hat{n}\cdot\hat{i}|} \quad \text{on } yz \text{ plane}$$

$$ds = \frac{dx\,dz}{|\hat{n}\cdot\hat{j}|} \quad \text{on } xz \text{ plane}$$

$$\Rightarrow \iint \hat{k}\cdot(x\hat{i}+y\hat{j}+z\hat{k})\frac{dx\,dy}{|\hat{n}\cdot\hat{k}|}$$

$$\iint z\,\frac{dx\,dy}{|(x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{k}|}$$

$$\iint z \frac{dx\,dy}{z} = \iint dx\,dy$$

$$= \text{Area of circle } x^2+y^2=1$$

$$\pi(1) = \pi$$

Here we use projection because double integration deals with two variable variation but any point on the surface which is in space (above the xy plain) have three variable variation

2nd method (use for open surface) line integr

$$\Rightarrow \iint_S (\nabla \times F)\cdot \hat{n}\, ds = \int_C \bar{F}\cdot d\bar{r}$$

$$= \int F_1\,dx + F_2\,dy + F_3\,dz$$

$$= \int_C (2x-y)\,dx - yz^2\,dy - y^2z\,dz$$

$$= \int_0^{2\pi} (2\cos\theta - \sin\theta)(-\sin\theta\, d\theta) + 0 + 0$$

$$= \int_0^{2\pi} -2\cos\theta\sin\theta + \sin^2\theta\, d\theta$$

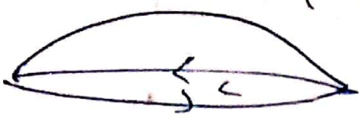
$$= \int_0^{2\pi} -\sin 2\theta + \int_0^{2\pi} \frac{1-\cos 2\theta}{2}\, d\theta$$

$$= \int_0^{2\pi} -\sin 2\theta \overset{0}{} + \int_0^{2\pi} \frac{1}{2}\, d\theta - \int_0^{2\pi} \frac{\cos 2\theta}{2}\, d\theta$$

π

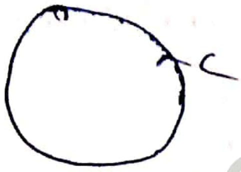
(1-4)

$x^2+y^2+z^2=1$



lies on xy plane $z=0$

$(x^2+y^2=1)$

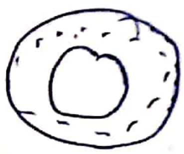


$r=1$, $x = \cos\theta$, $dx = \sin\theta \, d\theta$

$y = \sin\theta$, $dy = \cos\theta \, d\theta$

(18) $\omega = \{(x,y,z) \in R^3, \ 1 \le x^2+y^2+z^2 \le 4\}$

$$\iint_{\partial\omega} \bar{F}\cdot\hat{n}\, d\omega = \iiint_V \nabla\cdot\bar{F}\, dV$$



$$\Rightarrow \bar{F} = \frac{x,y,z}{(x^2+y^2+z^2)^{3/2}} = \frac{\bar{r}}{(r^2)^{3/2}} = \frac{\bar{r}}{r^3}$$

$$= r^{-3}\bar{r}$$

we know that

it is solenoid $\nabla\cdot\bar{F} = 0$

$$= \iiint_V \nabla\cdot\bar{F}\, dV = 0$$