Laws of Forces

2.1. FORCE

Force is some thing which *changes or tends to change the state of rest or of uniform motion of a body in a straight line*. Force is the direct or indirect action of one body on another. The bodies may be in direct contact with each other causing direct motion or separated by distance but subjected to gravitational effects.

There are different kinds of forces such as gravitational, frictional, magnetic, inertia or those cause by mass and acceleration. A static force is the one which is caused without relative acceleration of the bodies in question.

The force has a magnitude and direction, therefore, it is **vector**. While the directions of the force is measured in absolute terms of angle relative to a co-ordinate system, the magnitude is measured in different units depending on the situation.

When a force acts on a body, the following effects may be produced in that body : (*i*) *It may bring a change in the motion of the body i.e., the motion may be accelerated or retarded ; (ii) it may balance the forces already acting on the body thus bringing the body to a state of rest or of equilibrium, and (iii) it may change the size or shape of the body i.e., the body may be twisted, bent, stretched, compressed or otherwise distorted by the action of the force.*

2.2. UNITS OF FORCE

The two commonly used units of force are :

1. Absolute units
2. Gravitational units.

Absolute units. Because the mass and acceleration are measured differently in different systems of units, so the units of force are also different in the various systems as given below :

In the F.P.S. (Foot-Pound-Second) system the absolute unit of force is a *poundal* which is that much force as produces an acceleration of 1 ft/sec^2 in a mass of one pound.

In the C.G.S. (Centimetre-Gram-Second) system the absolute unit of force is *dyne* which is that much force as produces an acceleration of 1 cm/sec^2 in a mass of one gram.

In the M.K.S. (Metre-Kilogram-Second) system the absolute unit of force is a *newton* which is that much force as produces an acceleration of 1 m/sec^2 in a mass of the kilogram.

Obviously, 1 newton = 10^5 dynes.

Gravitational units of force. Gravitational units are the units which are used by engineers for all practical purposes. These units depend upon the weight of a body (*i.e.*, the force with which the body is attracted towards the centre of the earth). Now the weight of a body of mass m (*i.e.*, the quantity of matter contained in a body). $= mg$, where g is the acceleration due to gravity.

So the gravitational units of force in the three systems of units *i.e.*, F.P.S., C.G.S. and M.K.S. are Pound weight, Gram weight and kilogram weight.

The relationship of units of force is given as under :

1 lb wt. (or lbf) = g poundal = 32.2 poundals (app.)

1 gm wt. (or gmf) = g dynes = 981 dynes (app.)

1 kg wt. (or kgf) = g newtons = 9.81 newtons (app.)

which means

Gravitational unit of force = 'g' times the corresponding absolute units of force.

Usually, kg, wt (or kgf) is written simply as kg.

2.3. CHARACTERISTICS OF A FORCE

The characteristics or elements of the force are the quantities by which a force is fully represented. These are :

1. Magnitude (*i.e.*, 50 N, 100 N, etc.)
2. Direction or line of action (angle relative to a co-ordinate system).
3. Sense or nature (push or pull).
4. Point of application.

2.4. REPRESENTATION OF FORCES

Forces may be represented in the following two ways :

1. Vector representation
2. Bow's notation.

Vector representation. A force can be represented graphically by a vector as shown in Figs. 1.2 and 1.3.

Bow's notation. It is a method of designating a force by writing two capital letters one on either side of the force a shown in Fig. 2.1, where force P_1 (200 N) is represented by AB and force P_2 (100 N) by CD.

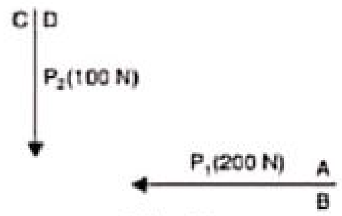

Fig. 2.1

2.5. CLASSIFICATION OF FORCES

There are several ways in which forces can be classified. Some of the important classifications are given as under :

1. According to the effect produced by the force :

(*i*) **External force.** When a force is applied external to a body it is called **external force.**

(*ii*) **Internal force.** The resistance to deformation, or change of shape, exerted by the material of a body is called an **internal force.**

(*iii*) **Active force.** An **active force** is one which causes a body to move or change its shape.

(*iv*) **Passive force.** A force which prevents the motion, deformation of a body is called a **passive force.**

2. According to nature of the force :

(i) **Action and reaction.** Whenever there are two bodies in contact, each exerts a force on the other. Out of these forces one is called action and other is called reaction. *Action and reaction are equal and opposite.*

(ii) **Attraction and repulsion.** These are actually non-contacting forces exerted by one body or another without any visible medium transmission such as magnetic forces.

(iii) **Tension and thrust.** When a body is dragged with a string the force communicated to the body by the string is called the *tension* while, if we push the body with a rod, the force exerted on the body is called a *thrust*.

3. According to whether the force acts at a point or is distributed over a large area.

(i) **Concentrated force.** The force whose point of application is so small that it may be considered as a point is called a concentrated force.

(ii) **Distributed force.** A **distributed force** is one whose place of application is area.

4. According to whether the force acts at a distance or by contact.

(i) **Non-contacting forces or forces at a distance.** Magnetic, electrical and gravitational forces are examples of non-contacting forces or forces at a distance.

(ii) **Contacting forces or forces by contact.** The pressure of steam in a cylinder and that of the wheels of a locomotive on the supporting rails are examples of contacting forces.

2.6. FORCE SYSTEMS

A *force system* is a collection of forces acting on a body in one or more planes.

According to the relative positions of the lines of action of the forces, the forces may be classified as follows :

1. Coplanar concurrent collinear force system. It is the simplest force system and includes those forces whose vectors lie along the same straight line (refer Fig. 2.2.).

Fig. 2.2

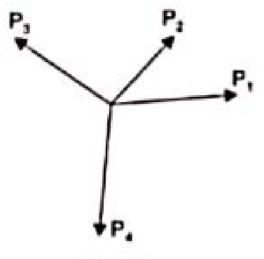

Fig. 2.3

2. Coplanar concurrent non-parallel force system. Forces whose lines of action pass through a common point are called **concurrent forces.** In this system lines of action of all the forces meet at a point but have different directions in the same plane as shown in Fig. 2.3.

3. Coplanar non-concurrent parallel force system. In this system, the lines of action of all the forces lie in the same plane and are parallel to each other but may not have same direction as shown in Fig. 2.4.

4. Coplanar non-concurrent non-parallel force system. Such a system exists where the lines of action of all forces lie in the same plane but do not pass through a common point. Fig. 2.5 shows such a force system.

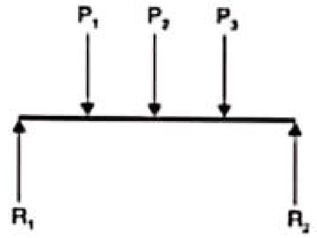

Fig. 2.4

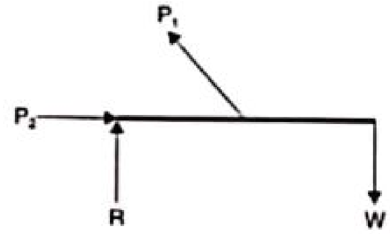

Fig. 2.5

5. Non-coplanar concurrent force system. This system is evident where the lines of action of all forces do not lie in the same plane but do pass through a common point. An example of this force system is the forces in the legs of tripod support for camera (Fig. 2.6).

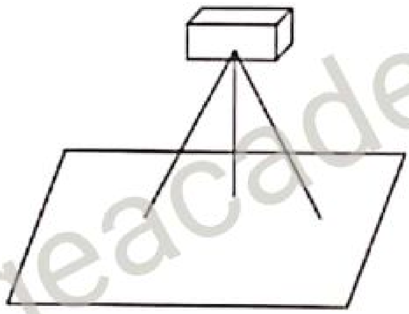

Fig. 2.6

6. Non-coplanar non-concurrent force system. Where the lines of action of all forces do not lie in the same plane and do not pass through a common point, a non-coplanar non-concurrent system is present.

2.7. FREE BODY DIAGRAMS

A body may consist of more than one element and supports. Each element or support can be isolated from the rest of the system by incorporating the net effect of the remaining system through a set of forces. This *diagram of the isolated element or a portion of the body along with the net effects of the system on it is called a 'free-body diagram'.* Free-body diagrams are useful in solving the forces and deformations of the system.

In case of a body in Fig. 2.7, we remove the supporting springs and replace it by the reactive force R equal to W in magnitude. The Fig. 2.7 (*c*) in which the body is completely isolated from its support and in which all forces acting on it are shown by vectors is called a *free body diagram.*

Let us consider another case of a beam shown in Fig. 2.8 (*a*). The beam is supported on a hinge at the left end and on a roller at the right end. The hinge offers vertical and horizontal reaction whereas the roller offers vertical reaction. The beam can be isolated from the supports by setting equivalent forces of the supports. Fig. 2.8 (*b*) illustrates the free body diagram of the beam in which R_1 and R_2 are reactions of the hinge support and R_3 the reaction of the roller support. Similarly, the free body diagrams of hinge and roller supports are shown in Figs. 2.8 (*c*) and 2.8 (*d*) respectively.

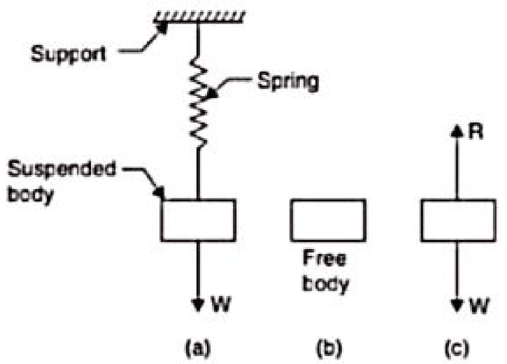

Fig. 2.7

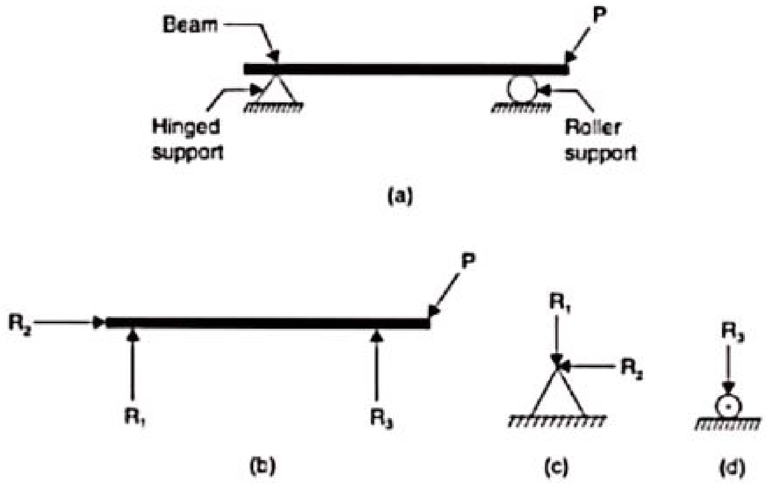

Fig. 2.8

2.8. TRANSMISSIBILITY OF A FORCE

The principle of transmissibility of forces states that *when a force acts upon a body, its effect is the same whatever point in its line of action is taken as the point of the application provided that the point is connected with the rest of the body in the same invariable manner.*

A force may be considered as acting at any point on its line of action so long as the direction and magnitude are not changed. Suppose a body (Fig. 2.9) is to be moved by a horizontal force P applied by hooking a rope to some point on the body. The force P will have the same effect if it is applied at 1, 2, 3 (Fig. 2.10) or any point on its line of action. This property of force is called *transmissibility*.

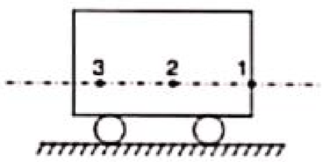

Fig. 2.9

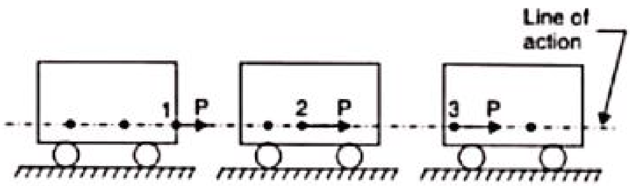

Fig. 2.10

2.9. PARTICLE

A body whose dimensions are practically negligible is called a *particle*. In any problem of mechanics, when the applied forces have no tendency to rotate the body on which they act, the body may be considered as a particle. Forces acting on the particle are concurrent, the point through which they pass being the point representing the particle.

2.10. RESULTANT FORCE

A *resultant force is a single force which can replace two or more forces and produce the same effect on the body as the forces.* It is fundamental principle of mechanics, demonstrated by experiment, that when a force acts on a body which is free to move, the motion of the body is in the direction of the force, and the distance travelled in a unit time depends on the magnitude of the force. Then, for a system of concurrent forces acting on a body, the body will move in the direction of the resultant of that system, and the distance travelled in a unit time will depend on the magnitude of the *resultant*.

2.11. COMPONENT OF A FORCE

As two forces acting simultaneously on a particle acting along directions inclined to each other can be replaced by a single force which produces the same effect as the given force, similarly, a single force can be replaced by two forces acting in directions which will produce the same effect as the given force. This breaking up of a force into two parts is called the *resolution of a force.* The force which is broken into two parts is called the *resolved force* and the parts are called *component forces* or the *resolutes.*

Generally, a force is resolved into the following two types of components :

1. Mutually perpendicular components
2. Non-perpendicular components.

1. Mutually perpendicular components. Let the force P to be resolved is represented in magnitude and direction by oc in Fig. 2.11. Let P_x is the component of force P in the direction oa making an angle α with the direction oc of the force. Complete the rectangle $oacb$. Then the other component P_y at right angle to P_x will be represented by ob which is also equal to ac.

From the right-angled triangle oac

$$P_x = oa = P \cos \alpha$$
$$P_y = ac = P \sin \alpha.$$

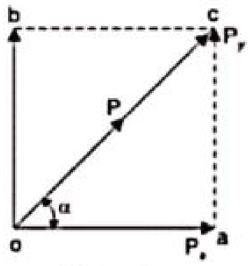

Fig. 2.11

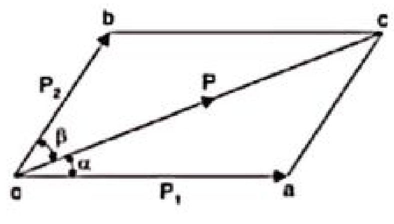

Fig. 2.12

2. Non-perpendicular components. Refer Fig. 2.12. Let oc represents the given force P in magnitude and direction to some scale. Draw oa and ob making angle α and β with oc. Through c draw ca parallel to ob and cb parallel to oa to complete the parallelogram $oacb$. Then the vectors oa and ob represent in magnitude and direction (to the same scale) the components P_1 and P_2 respectively.

Now from the triangle oac, by applying sine rule,

$$\frac{oa}{\sin\beta} = \frac{oc}{\sin[180 - (\alpha+\beta)]} = \frac{ac}{\sin\alpha}$$

or
$$\frac{P_1}{\sin\beta} = \frac{P}{\sin(\alpha+\beta)} = \frac{P_2}{\sin\alpha}$$

$\therefore$
$$P_1 = P \cdot \frac{\sin\beta}{\sin(\alpha+\beta)} \qquad ...(2.1)$$

and
$$P_2 = P \cdot \frac{\sin\alpha}{\sin(\alpha+\beta)} \qquad ...(2.2)$$

2.12. PRINCIPLE OF RESOLVED PARTS

The principle of resolved parts states : "*The sum of the resolved parts of two forces acting at a point in any given direction is equal to the resolved parts of their resultant in that direction.*

Refer Fig. 2.13. Let the two forces P and Q be represented by the sides oa and ob of the parallelogram $oacb$ and the resultant R of these two forces is given by the diagonal oc in magnitude and direction. Let ox is the given direction. Draw bf, ae, cd and ag perpendicular to cd.

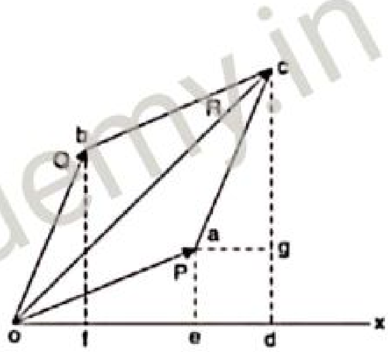

Fig. 2.13

Now from the two triangles obf and acg which are same in all respects, we get

$$of = ag = ed$$

$\therefore$
$$od = oe + ed = oe + of$$

But oe, of and od represent the resolved components or parts of the forces P, Q and R respectively in the direction of ox.

It may be noted that this principle *holds good for any number of forces.*

2.13. LAWS OF FORCES

The method of determination of the resultant of some forces acting simultaneously on a particle is called *composition of forces.* The various laws used for the composition of forces are given as under :

1. Parallelogram law of forces
2. Triangle law of forces
3. Polygon law of forces.

1. Parallelogram law of forces. It states as under :

"*If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram then their resultant may be represented in magnitude and direction by the diagonal of the parallelogram which passes through their point of intersection.*"

Refer Fig. 2.14. Let two forces P and Q acting simultaneously on a particle be represented in magnitude and direction by the adjacent sides oa and ob of a parallelogram $oacb$ drawn from a point o, their resultant R will be represented in magnitude and direction by the diagonal oc of the parallelogram.

The value of R can be determined either graphically or analytically as explained below :

Graphical method. Draw vectors oa and ob to represent to some convenient scale the forces P and Q in magnitude and direction. Complete the parallelogram $oacb$ by drawing ac parallel to ob and bc parallel to oa. The vector oc measured to the same scale will represent the resultant force R.

Analytical method. As shown in Fig. 2.14, in the parallelogram $oacb$, from c drop a perpendicular cd to oa at d when produced. Now from the geometry of the figure.

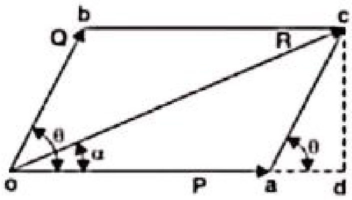

Fig. 2.14

$$\angle cad = \theta,\ ac = Q$$

$$\therefore \quad cd = Q \sin \theta$$

and

$$ad = Q \cos \theta$$

From right-angled triangle, odc

$$oc = \sqrt{(od)^2 + (cd)^2}$$

$$= \sqrt{(oa + ad)^2 + (cd)^2}$$

or

$$R = \sqrt{(P + Q\cos\theta)^2 + (Q \sin \theta)^2}$$

$$= \sqrt{P^2 + Q^2 \cos^2 \theta + 2PQ \cos \theta + Q^2 \sin^2 \theta}$$

$$= \sqrt{P^2 + Q^2 (\sin^2 \theta + \cos^2 \theta) + 2PQ \cos \theta}$$

$$= \sqrt{P^2 + Q^2 + 2PQ \cos \theta} \qquad (\because \ \sin^2 \theta + \cos^2 \theta = 1)$$

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta} \qquad ...(2.3)$$

Let the resultant makes an angle α with P as shown in figure.

Then

$$\tan \alpha = \frac{cd}{od} = \frac{cd}{oa + ad}$$

$$= \frac{Q \sin \theta}{P + Q \cos \theta} \qquad ...(2.4)$$

Case 1. If $\theta = 0°$, *i.e.*, when the forces P and Q act along the same straight line then equation (2.3) reduces to

$$R = P + Q \qquad (\because \ \cos 0° = 1)$$

Case 2. If $\theta = 90°$, *i.e.*, when the forces P and Q act at right angles to each other, then

$$R = \sqrt{P^2 + Q^2} \qquad (\because \cos 90° = 0)$$

Case 3. If $\theta = 180°$, *i.e.*, the forces P and Q act along the same straight line but in opposite directions, then

$$R = P - Q \qquad (\because \cos 180° = -1)$$

The resultant will act in the direction of the greater force.

2. Triangle law of forces. It states as under :

"If two forces acting simultaneously on a body are represented in magnitude and direction by the two sides of triangle taken in order then their resultant may be represented in magnitude and direction by the third side taken in opposite order."

Let P and Q be the two coplanar concurrent forces. The resultant force R in this case can be obtained with the help of the triangle law of forces both graphically and analytically as given below :

Graphical method. Refer Fig. 2.15. Draw vectors *oa* and *ac* to represent the forces *P* and *Q* to some convenient scale in magnitude and direction. Join *oc* which will represent the resultant force *R* in magnitude and direction to the same scale.

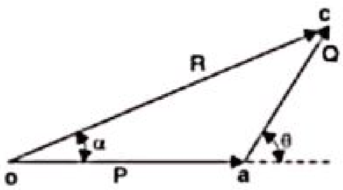

Fig. 2.15

Analytical method. From the geometry of triangle *oac* (Fig. 2.15).

$$\angle coa = \alpha, \angle oca = \theta - \alpha, \angle cao = 180° - \theta$$

$$\therefore \quad \frac{oa}{\sin(\theta - \alpha)} = \frac{ac}{\sin \alpha} = \frac{oc}{\sin(180° - \theta)}$$

or
$$\frac{P}{\sin(\theta - \alpha)} = \frac{Q}{\sin \alpha} = \frac{R}{\sin(180° - \theta)}$$

or
$$\frac{P}{\sin(\theta - \alpha)} = \frac{Q}{\sin \alpha} = \frac{R}{\sin \theta} \quad \text{...(2.5)}$$

3. Polygon law of forces. It states as under :

"*If a number of coplanar concurrent forces, acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon taken in order, then their resultant may be represented in magnitude and direction by the closing side of a polygon, taken in the opposite order*".

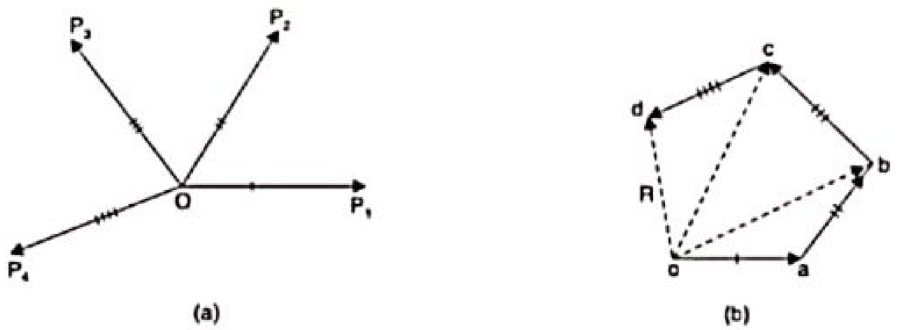

Fig. 2.16

If the forces P_1, P_2, P_3, and P_4 acting simultaneously on a particle be represented in magnitude and direction by the sides *oa, ab, bc* and *cd* of a polygon respectively, their resultant is represented by the closing side *do* in the opposite direction as shown in Fig. 2.16 (*b*).

The law is actually an extension of triangle law of forces. This is so because *ob* is the resultant of *oa* and *ab* and therefore *oc* which is resultant of *ob* and *bc* is also the resultant of *oa, ab* and *bc*. Similarly, *od* is the resultant of *oc* and *cd* and therefore of *ob, bc* and *cd* and finally of *oa, ab, bc* and *cd*.

2.14. RESULTANT OF SEVERAL COPLANAR CONCURRENT FORCES

To determine the resultant of a number of coplanar concurrent forces any of the following two methods may be used :

1. Graphical method (Polygon law of forces)
2. Analytical method (Principle of resolved parts).

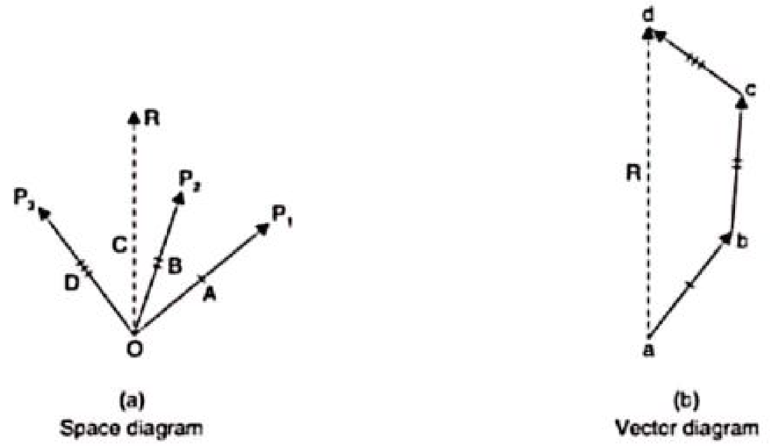

Fig. 2.17

Resultant by graphical method. Fig. 2.17 (*a*) shows the forces P_1, P_2 and P_3 simultaneously acting at a particle O.

Draw a vector *ab* equal to force P_1 to some suitable scale and parallel to the line of action of P_1.

From '*b*' draw vector *bc* to represent force P_3 in magnitude and direction.

Now from '*c*' draw vector *cd* equal and parallel to force P_3. Join *ad* which gives the required resultant in magnitude and direction, the direction being *a* to *d* as shown in the vector diagram.

Resultant by analytical method. Refer Fig. 2.18.

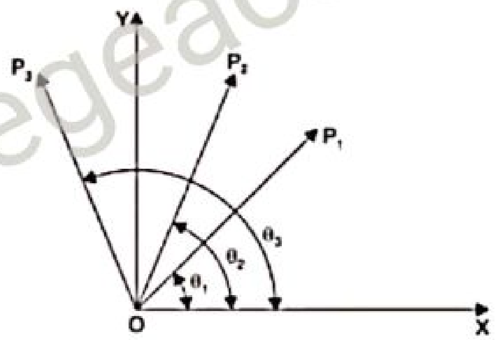

Fig. 2.18

The resolved parts in the direction OX and OY of

P_1 are $P_1 \cos \theta_1$ and $P_1 \sin \theta_1$, respectively,

P_2 are $P_2 \cos \theta_2$ and $P_2 \sin \theta_2$ respectively

and P_3 and $P_3 \cos \theta_3$ and $P_3 \sin \theta_3$ respectively.

If the resultant R makes an angle θ with OX then by the principle of resolved parts :

$$R \cos \theta = P_1 \cos \theta_1 + P_2 \cos \theta_2 + P_3 \cos \theta_3$$
$$= \Sigma H \quad \text{...(i)}$$

and

$$R \sin \theta = P_1 \sin \theta_1 + P_2 \sin \theta_2 + P_3 \sin \theta_3$$
$$= \Sigma V \quad \text{...(ii)}$$

Now, by squaring and adding eqns. (*i*) and (*ii*), we get

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} \quad ...(2.6)$$

and by dividing eqn. (*ii*) by eqn. (*i*), we get

$$\frac{R \sin \theta}{R \cos \theta} = \tan \theta = \frac{\Sigma V}{\Sigma H}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) \quad ...(2.7)$$

It may be noted that while solving problems proper care must be taken about the signs (+ve or –ve) of the resolved parts. Following sign conventions may be kept in view :

Vertical components

Upward direction ↑ Positive (+)
Downward direction ↓ Negative (–)

Horizontal components

Directions : From left to right → Positive (+)
: From right to left ← Negative (–)

2.15. EQUILIBRIUM CONDITIONS FOR COPLANAR CONCURRENT FORCES

When several forces act on a particle, the particle is said to be in *equilibrium* if there *is no unbalanced* forces acting on it, *i.e.*, the resultant of all the forces acting on the particle is *zero.*

Analytical and graphical conditions of equilibrium of coplanar concurrent forces are given as under :

Analytical conditions :

1. The algebraic sum of components of all the forces in any direction which may be taken as horizontal, in their plane must be zero. Mathematically, $\Sigma H = 0$.

2. The algebraic sum of components of all the forces in a direction perpendicular to the first direction, which may be taken as *vertical*, in their plane, must be zero. Mathematically, $\Sigma V = 0$.

Graphical conditions. The force polygon, *i.e.*, force or vector diagram *must close.*

2.16. LAMI'S THEOREM

It states as under :

"If three coplanar forces acting on a point in a body keep it in equilibrium, then each force is proportional to the sine of the angle between the other two forces."

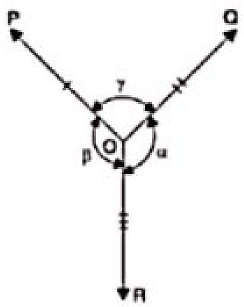

Fig. 2.28

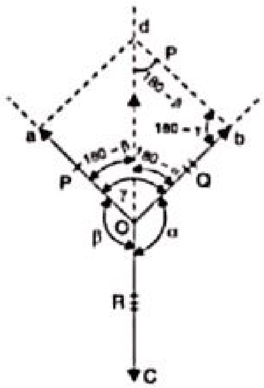

Fig. 2.29

Fig. 2.28 shows three forces P, Q and R acting at a point O. Let the angle between P and Q be γ, between Q and R be α and between R and P be β. If these forces are in equilibrium then according to Lami's theorem :

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma} \qquad ...(2.8)$$

Proof. Let us first consider the two forces P and Q which are represented by the two sides oa and ob of a parallelogram $oadb$ as shown in Fig. 2.29. Then the resultant of these two *forces* will be given by od (the diagonal of the parallelogram) in magnitude and direction. This means od should be equal to R in magnitude but opposite in direction to oc as P, Q and R are in equilibrium.

From geometry of parallelogram $oadb$, we find

$$bd = P \text{ and } ad = Q$$

$$\angle bod = (180 - \alpha)$$

and

$$\angle bdo = \angle aod = \angle(180 - \beta)$$

Now, from triangle *obd*,

$$\angle obd = 180^\circ - \angle bod - \angle bdo$$

($\because$ Sum of all the angles of triangle = 180°)

$$= 180 - (180 - \alpha) - (180 - \beta) = \alpha + \beta - 180^\circ$$

But $\alpha + \beta + \gamma = 360^\circ$ ($\because$ Sum of all the angles at a point = 360°)

Subtracting 180° from both the sides, we get

$$\alpha + \beta + \gamma - 180^\circ = 360^\circ - 180^\circ$$

$$(\alpha + \beta - 180^\circ) + \gamma = 180^\circ$$

$$\angle obd + \gamma = 180^\circ$$

$$\angle obd = 180 - \gamma$$

Now, applying sine formula to triangle *obd*,

$$\frac{bd}{\sin(180 - \alpha)} = \frac{ob}{\sin(180 - \beta)} = \frac{od}{\sin(180 - \gamma)}$$

$$\frac{P}{\sin(180 - \alpha)} = \frac{Q}{\sin(180 - \beta)} = \frac{R}{\sin(180 - \gamma)}$$

Hence $$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}.$$

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Framed Structure / Truss.

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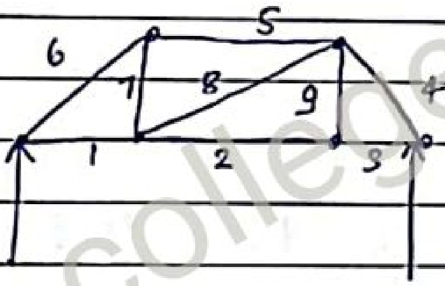
Truss or framed str. - It's composed of several bars or rods joined together in a particular fashion as per the requirement, these bars are called the members of the structure. A members under tension, is called tie & a member under compression strut

Types of Truss.

1. Perfect or efficient truss

$\boxed{m = 2j - 3}$, where. m = No. of member
j = No. of joints



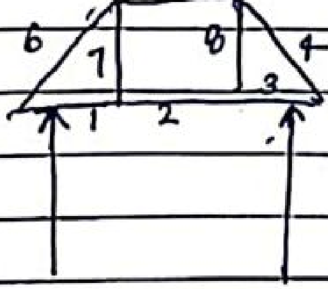
$m = 9$,
$j = 6$.

$m = 2j - 3$.
$9 = 9$ ✓.

$\therefore$ Perfect Truss.

2. Imperfect or deficient truss

• if $m \neq 2j - 3$
$<$



$m = 8$
$j = 6$

$8 = 2j - 3$
$8 \neq 9$
$8 \neq 9$.
$<$

$\therefore$ Truss is imperfect or Def Truss.

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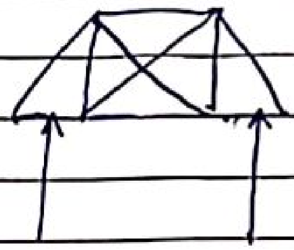
3. Imperfect / Redundant Truss

- If $m \neq 2j-3$.
  $>$



$m = \cancel{10}\,12$
$j = 6$

$m = 2j-3$
$12 \neq 11$
$>$ $\therefore$ Imperfect or Redundant Truss.

Analysis of framed str.:

assumptions - -

1. All the members are pin jointed.
2. The frame is loaded only at joints.
3. The frame is considered as a perfect frame or perfect truss.
4. The self weight of the members is neglected.

Methods

1) Method of joints.

In this method, each & every joint is treated separately as a free body in eq^m. The unknown forces are then determine by eq^m eq^n

$\Sigma H = 0$ & $\Sigma V = 0$

A joint is taken for analysis only when there are not more than 2 unknown forces acting at that point.

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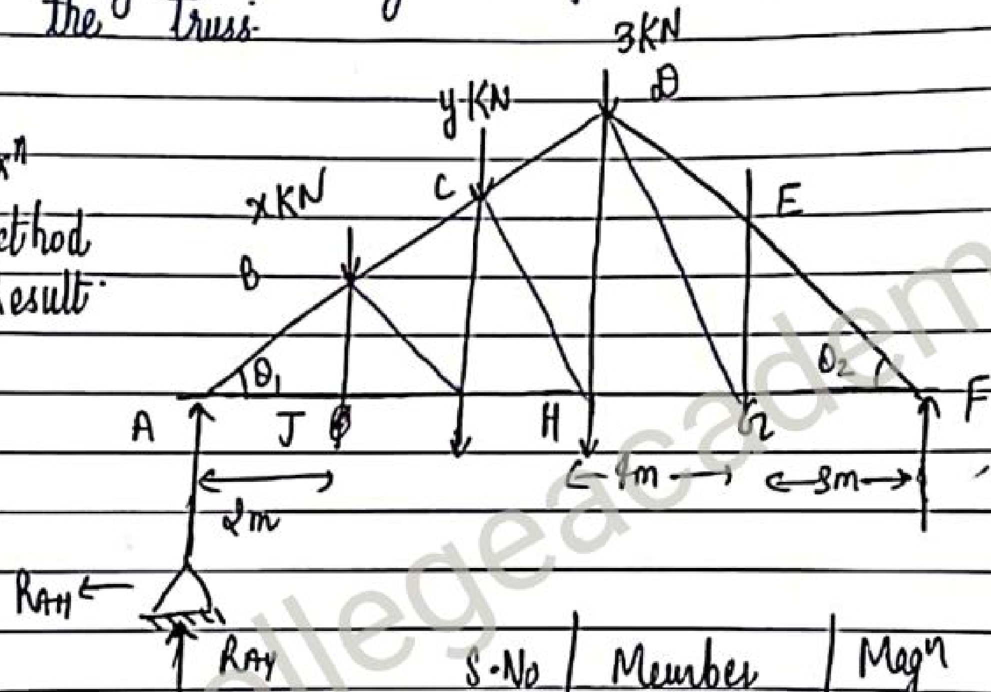
To start with a joint is taken at which there are not more than 2 unknown forces. After calculating all forces acting at that joint, next joint is taken up at which there are not more than 2 unknown forces. The process is continued untill all the joints are considered thereby, calculating the forces in all the members of the truss.

- Rx^n
- Method
- Result.



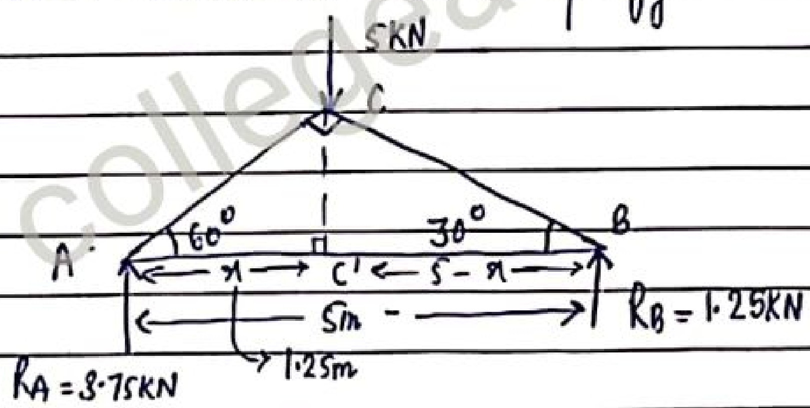
S.No	Member	Mag^n	Nature
1.	AB	— —	Tensile
2.	BC	— —	compr.
|	|	|	
|	|	|	

2) √ Method of section / Moment. This method is particularly convenient when the forces in few members are req^r to be find out. In this method a section line is passed through the members & in which forces are req^r to be determine. A part of

the structure on any side of the section line is then treated as a free body in eq^m under the action of all external forces. The unknown forces are then determined by applic^n of eq^m $\Sigma M = 0$ with proper sign convention. It may be noted that while drawing a section line, care should be taken not to cut more than 3 ~~forces~~ members in which the forces are unknown.

Q.

Q^M A truss is loaded as shown, specify



Reactions :-

Let R_A & R_B be the rx^{n}s at A & B respectively

Taking moment about A.

$$\Sigma M_A = 0. \quad [\circlearrowleft = +ve \quad \circlearrowright = -ve]$$

$$\boxed{R_B \cdot 5 = 5 \cdot \cancel{4} \cdot R_C}$$

$$\sqrt{3} = \frac{CC'}{x} \qquad \frac{1}{\sqrt{3}} = \frac{CC'}{5-x}$$

$$\sqrt{3}x = \frac{5-x}{\sqrt{3}} \qquad x = 5/4 \, m$$

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$\boxed{R_C = 4R_B}$ $\boxed{R_B = 1.25\ KN}$

Considering the vertical eq^m

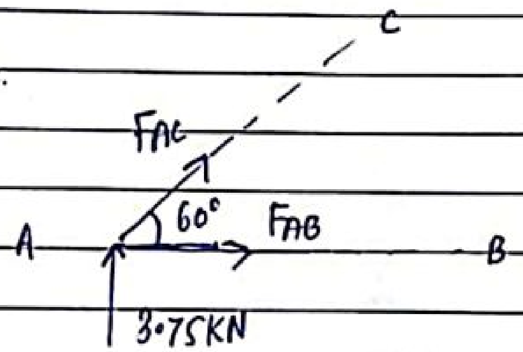
$$\therefore \quad \Sigma V = 0 \quad [\uparrow = +ve,\ \downarrow = -ve]$$

$$R_A + R_B - 5 = 0.$$

$$R_A = 5 - 1.25 \qquad \boxed{R_A = 3.75\ KN}$$

Method of joints

Joint (A)



Considering vertical eq^m $\Sigma V = 0.$

$$F_{AC} \sin 60 + 3.75 = 0.$$

$$F_{AC} = -\frac{3.75}{\sin 60}$$

$$\boxed{F_{AC} = -4.33\ KN}$$

Considering Hz eq^m $\Sigma H = 0.$

$$F_{AC} \cos 60° + F_{AB} = 0$$

$$F_{AB} = -\frac{1}{2} \times (-4.33)$$

$$\boxed{F_{AB} = 2.165\ KN}$$

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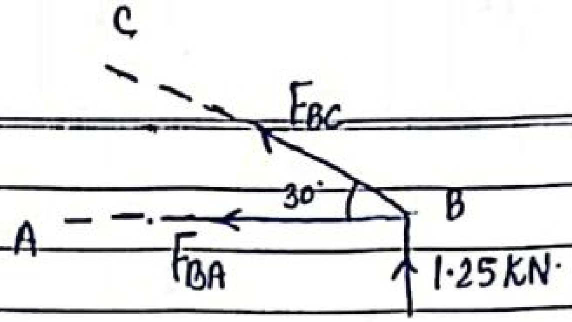
Joint (B)



Considering vertical eq^m $\Sigma V = 0$

$$1.25 + F_{BC} \cdot \sin 30 = 0$$

$$\boxed{F_{BC} = -2.5\,KN}$$

consider Hz eq^m $\Sigma H = 0$.

$$-F_{BA} - F_{BC} \cdot \cos 30 = 0.$$

$$F_{BA} = -F_{BC} \cdot \frac{\sqrt{3}}{2}$$

$$F_{BA} = -(-2.5) \times \sqrt{3}/2$$

$$\boxed{F_{BA} = 2.164\,KN}$$

Result

S.No	Member	Force	Nature
1.	AB	2.165 KN	Tensile
2.	BC	2.5 KN	Compressive
3.	AC.	4.33 KN	Compressive

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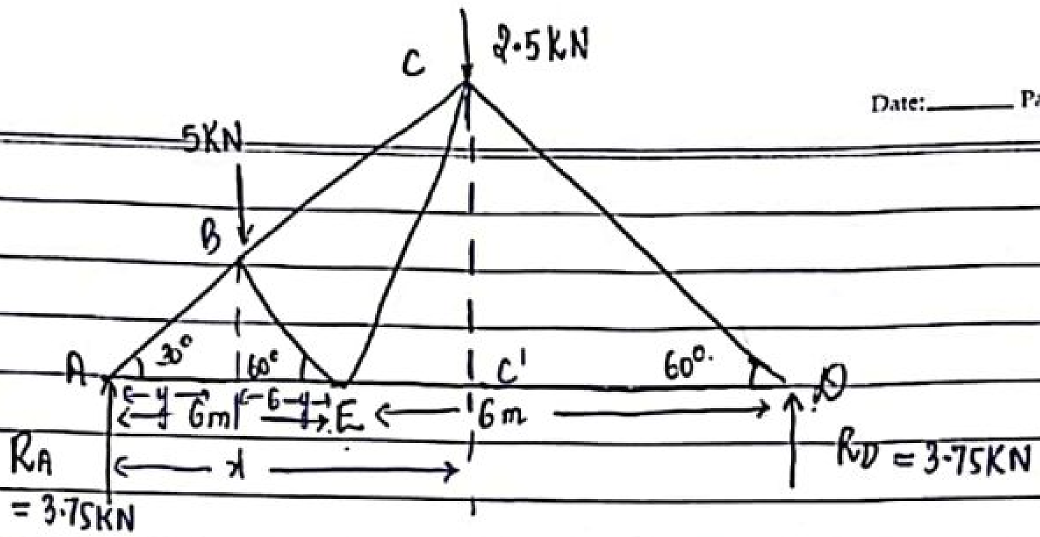
$\underline{Q^n}$



$\underline{Rx^n}$

Let R_A & R_D be the rx^ns at A & D - -

Now taking Moment about A $\sum M_A = 0$

~~$R_D \cdot 12$~~.

from Δs ACC' & $CC'D$.

$\tan 30° = \frac{CC'}{x}$ & $\tan 60° = CC'/(12-x)$

$x = 9m$

from Δ's ABB' & BEB'

$\frac{1}{\sqrt{3}} = \frac{BB'}{y}$ $\sqrt{3} = \frac{BB'}{6-y}$.

$y = 4.5m$

$\sum M_A = 0.$

$R_D \cdot 12 - 5 \times 4.5 - 2.5 \times 9 = 0.$

$\boxed{R_D = 3.75KN}$

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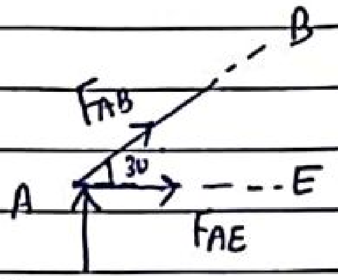
$\Sigma V = 0.$ & $R_A + R_D - 5 - 2.5 = 0$

$\boxed{R_A = 3.75\,KN}$

Method of joints

Joint (A)



vertical eq^m $\Sigma V_A = 0.$

$F_{AB} \cdot \frac{1}{2} + R_A = 0.$

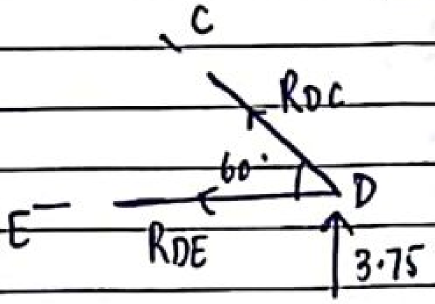
$F_{AB} = -2 \cdot 3.75$

$\boxed{F_{AB} = -7.50\,KN}$

$\Sigma H = 0.$ $F_{AB} \cdot \frac{\sqrt{3}}{2} + F_{AE} = 0.$

$\boxed{F_{AE} = 6.495\,KN}$

Joint (D)



$\Sigma V = 0$

$R_{DC} \cdot \frac{\sqrt{3}}{2} + 3.75 = 0.$

$\boxed{\cancel{R_{DC} = -7.50\,KN}}$ $\boxed{R_{DC} = -4.33\,KN}$

$\Sigma H = 0$

$R_{DE} + R_{DC} \cdot \frac{1}{2} = 0$ $\boxed{R_{DE} = 2.165\,KN}$

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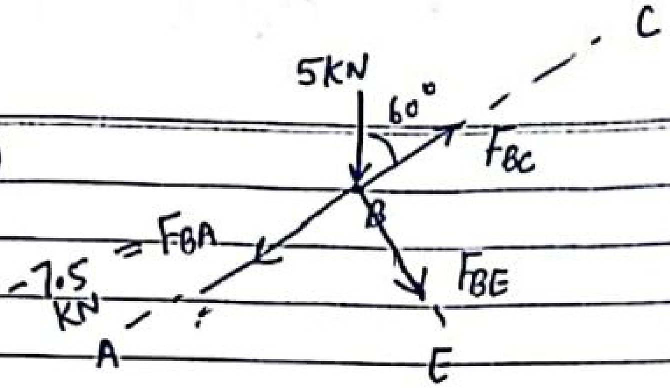
Joint (B)



resolving forces along line ABC.

$$-(-7.5) - 5\cdot\frac{1}{2} + F_{BC} = 0.$$

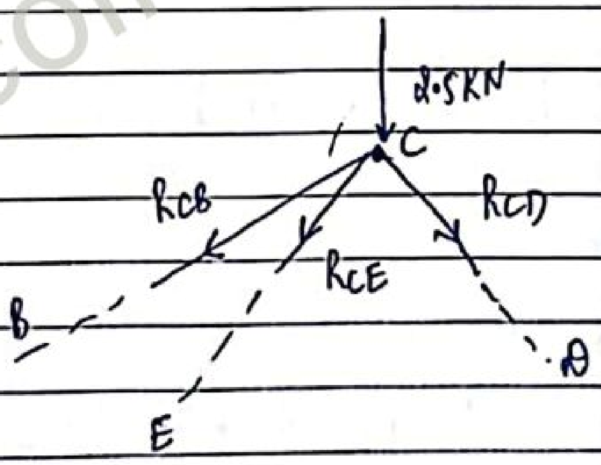
$\boxed{F_{BC} = -5KN}$ $\boxed{\cancel{F_{BC} = 10KN}}$

selor resolving forces along ~~line BE~~ ⊥ar to ABC

$$-5\cdot\frac{\sqrt{3}}{2} - F_{BE} = 0$$

$\boxed{F_{BE} = -4.33KN}$

Joint (C)



~~GA~~ 2.5KN

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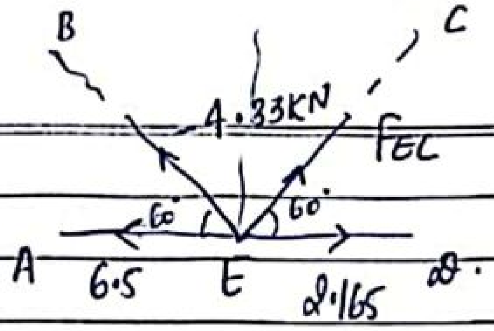
Joint (E)



$\Sigma V = 0$ $\qquad F_{EC} \cdot \frac{\sqrt{3}}{2} - 4.33 \cdot \frac{\sqrt{3}}{2} = 0$

$$\boxed{F_{EC} = 4.33 KN}$$

Result :-

S.No	Member	Force	Nature
1.	AB		
2.	AE		
3.	DC		
4.	BE		
5.	BC		
6.	EC		
7.			

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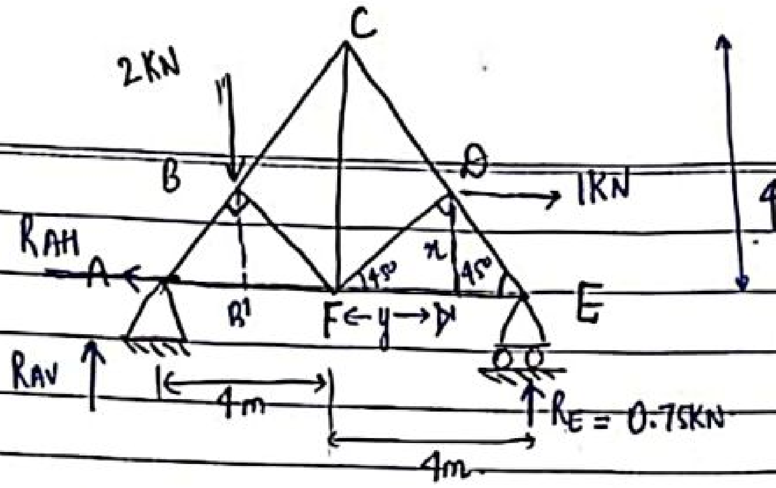
Q) 1



$\underline{R_{x^n}}$ Let: R_{AV} & R_{AH} be the rx^n at A
R_E be rx^n at E.

Taking Moment about A, $\Sigma M_A = 0$.

In Δ ~~CFE~~ & $DD'E$ & FDD'

$$\frac{\cancel{x}}{y} = \frac{\cancel{x}}{4-y} \qquad y = 2$$

similarly $BB' = 2m$.

$\Sigma M_A = 0$

$$R_E \times 8 - 1 \cdot 2 - 2 \times 2 = 0$$

$$\boxed{R_E = 0.75KN}$$

Considering vertical eq^n $\Sigma V = 0$.

$$R_{AV} + R_E - 2 = 0.$$

$$R_{AV} = 2 - R_E$$

$$\boxed{R_{AV} = 1.25KN}$$

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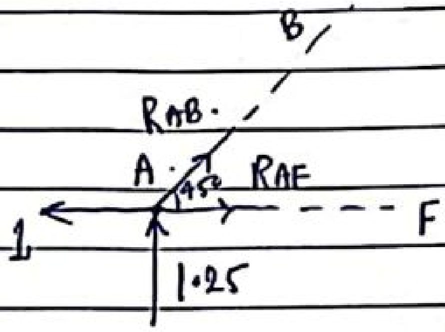
$\Sigma H = 0$ $\quad 1 - R_{AH} = 0$

$$\boxed{R_{AH} = 1 KN}$$

Method of joints.

Joint (A)



$\Sigma H = 0$ $\quad R_{AB} \cdot \frac{1}{\sqrt{2}} + R_{AF} = 1.$

$R_{AF} = 1 + \frac{1}{\sqrt{2}} \times 1.768.$

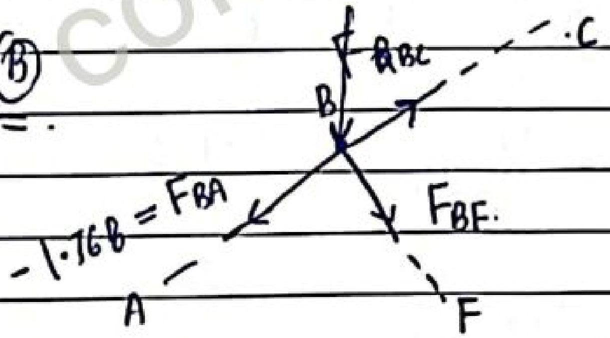
$$\boxed{R_{AF} = 2.25 KN}$$

$\Sigma V = 0.$

$R_{AB} \cdot \frac{1}{\sqrt{2}} + 1.25 = 0.$

$$\boxed{R_{AB} = -1.768 KN}$$

Joint (B)



resolving forces along line ABC.

$-(1.768) - \frac{2 \cdot 1}{\sqrt{2}} + F_{BC} = 0.$

$$\boxed{F_{BC} = -0.36 KN}$$

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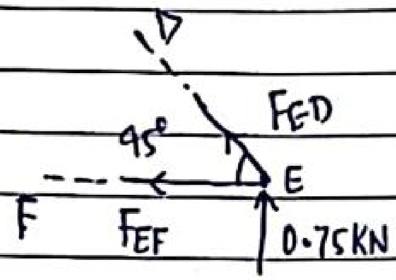
resolving forces ⊥ to line ABC.

$$-2\cdot\frac{1}{\sqrt{2}} - F_{BF} = 0.$$

$$\boxed{F_{BF} = -\sqrt{2}\ KN}$$

Joint (E)



~~ΣH=0.~~
ΣV=0

$$F_{ED}\cdot\frac{1}{\sqrt{2}} + 0.75 = 0$$

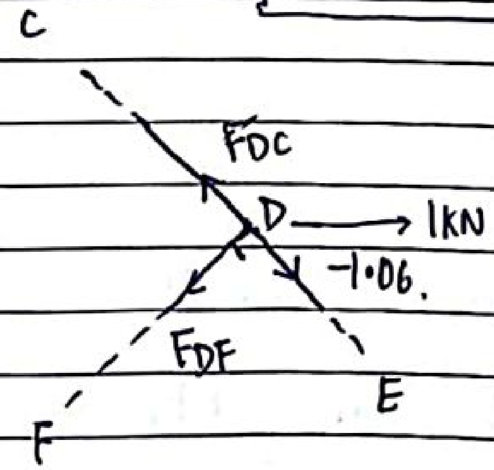
$$\boxed{F_{ED} = -1.06\ KN}$$

ΣH=0

$$-F_{ED}\cdot\frac{1}{\sqrt{2}} - F_{EF} = 0.$$

$$\boxed{F_{EF} = 0.75\ KN}$$

Joint (D)



resolving along line CDE.

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$$1 \cdot \frac{1}{\sqrt{2}} - 1.06 - F_{DC} = 0.$$

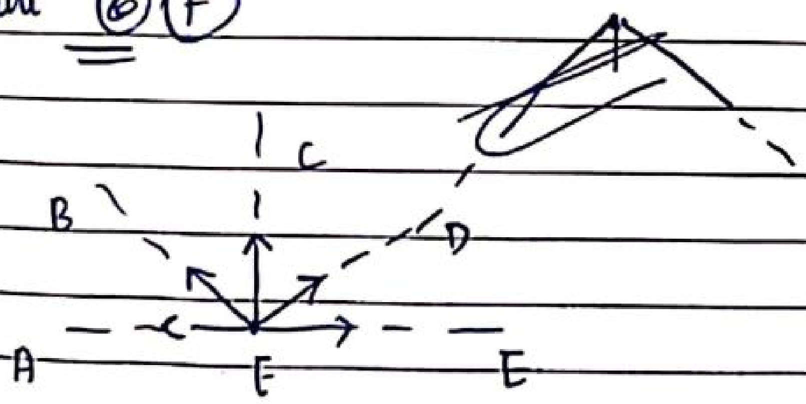
$$\boxed{F_{DC} = -0.35\,KN}$$

⊥ar to line

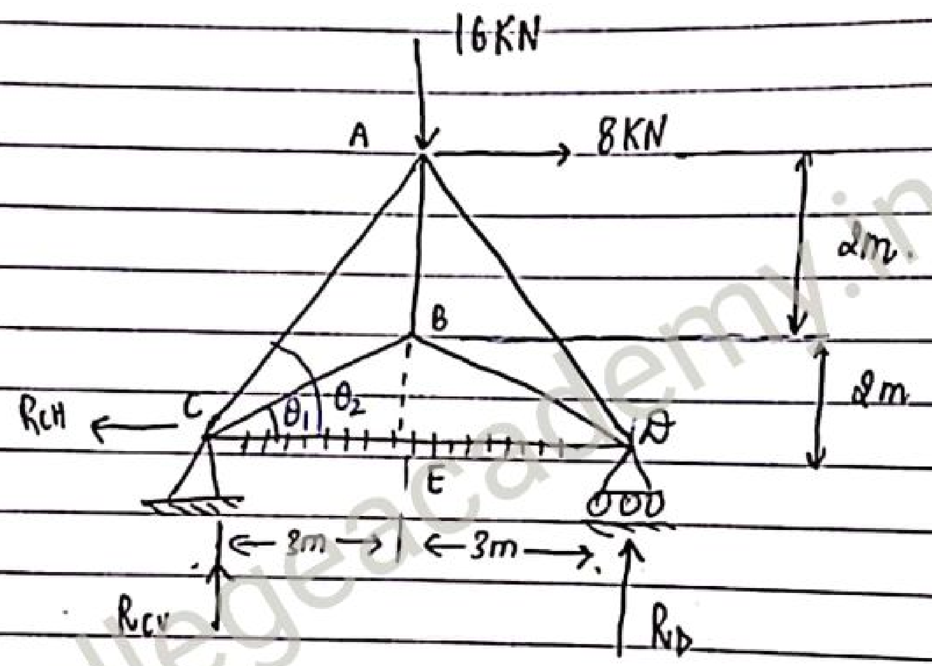
$$1 \cdot \frac{1}{\sqrt{2}} - F_{DF} = 0$$

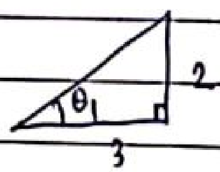
$$\boxed{F_{DF} = 0.707\,KN}$$

Joint (E) (F)



Q^n





$\tan^{-1} 2/3 = \theta_1$

$\boxed{\theta_1 = 33.7^\circ}$

$\tan^{-1} \frac{4}{3} = \theta_2$

$\boxed{\theta_2 = 53.13^\circ}$

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Rxn Take Moment about C.

$$16 \cdot 3 \quad 8 \cdot 4 \quad R_D \cdot 6 = 0$$

$$R_D \cdot 6 = 80$$

$$\boxed{R_D = 13.33 KN}$$

$\Sigma V = 0$ $\quad R_D + R_{CV} - 16 = 0.$

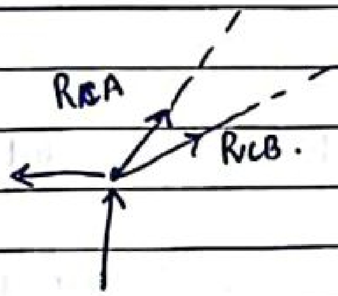
$$R_{CV} = 16 - 13.33$$

$$\boxed{R_{CV} = +2.66 KN}$$

$\Sigma H = 0$ $\quad \boxed{R_{CH} = 8KN}$

Method of joints

Joint C



$\Sigma H = 0$

$\Sigma V = 0$ $\quad$ ~~$R_{CA} \cdot \cos\theta_2$~~ $R_{CA} \sin\theta_2 + R_{CB} \sin\theta_1 + 2.66 = 0.$

$R_{CA} \sin\theta_2$

$$0.8 R_{CA} + R_{CB} \cdot 0.55 = -2.66.$$

$$R_{CB} \times 0.832 + R_{CA} \cdot 0.6 = 8$$

$\boxed{R_{CA} = -19.7}$ $\uparrow -20$ $\quad \boxed{R_{CB} = 23.82 KN}$ $\uparrow 24.02$

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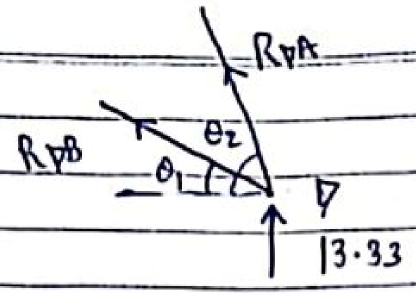
Joint (D)

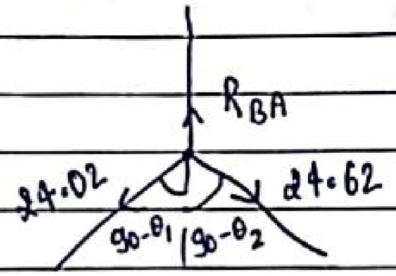


$$-13.33 = R_{DA}\ 0.79 + R_{DB}\ 0.554$$

$$R_{DA} \times 0.6 + 0.832\ R_{DB} = 0$$

$\boxed{R_{DA} = -33.337\ kN}$ $\boxed{R_{DB} = 24.02\ kN}$

Joint (B)



$$\cancel{R_{BA}}\quad R_{BA} = 24.02 \sin\theta_2 + 24.02 \sin\theta_1$$

$$= 19.696 +$$

$$\boxed{R_{BA} = 26.66\ kN}$$

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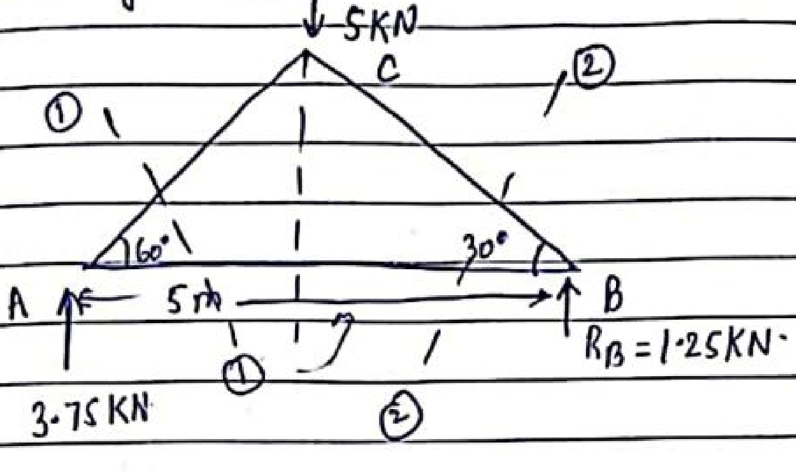
Q. Method of section / Moment.

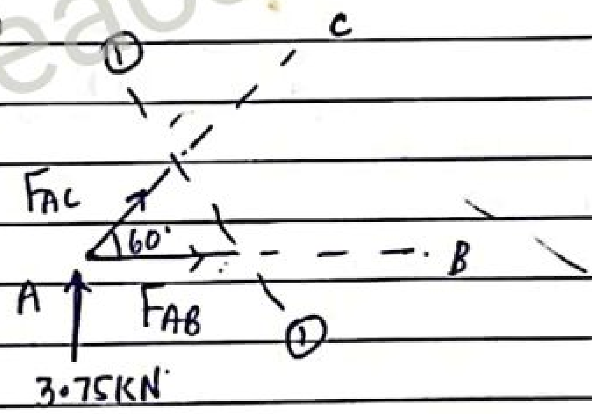


R_x^n

Method of section

Section line (1)-(1)

Left part.



Now taking Moment about B. [↺ = +ve ↻ = -ve]

$$\Sigma M_B = 0.$$

$$-3.75 \times 5 - F_{AC} \cdot \sin 60° \cdot 5 = 0.$$

$$\boxed{F_{AC} = -4.33\,KN}$$

$$\Sigma M_C = 0.$$

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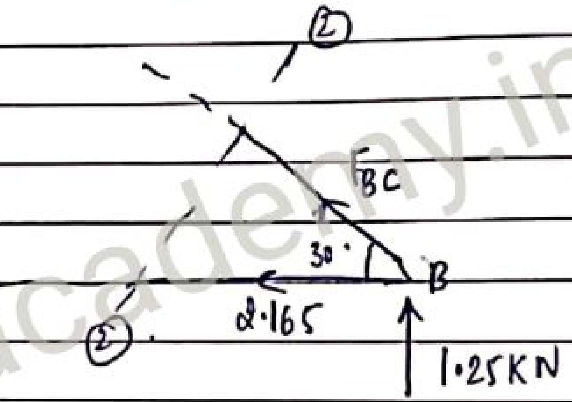
$$-3.75 \times 1.25 + F_{AB} \times 2.165 = 0$$

$$\boxed{F_{AB} = 2.165 KN}$$

Section line ② - ②
Right part



$$\Sigma M_A = 0.$$

$$1.25 \times 5 + F_{BC} \cdot \frac{1}{2} \cdot 5 = 0.$$

$$\boxed{F_B = -2.5 KN}$$

$$\Sigma M_C = 0.$$

$$1.25 \times 3.75 - 2.165 \times F_{AB} = 0$$

$$\boxed{F_{AB} = 2.165 KN}$$