2-1

Differⁿ

Order of D.E ⇒ The order of a diffⁿ is the order of highest ordered derivative occuring in the diffⁿ.

Degree of DE ⇒ The degree of a diffⁿ eqⁿ is the degree of highest ordered derivative occuring in it when the derivatives are the free form fractional powers.

Q $$\left(1+\frac{d^2y}{dx^2}\right)^{4/3} = \frac{dy}{dx}$$

order degree

$$\left(1+\frac{d^2y}{dx^2}\right)^4 = \left(\frac{dy}{dx}\right)^3$$

Q $$\left(\frac{d^3y}{dx^3}\right)^{3/2} = \left(\frac{d^3y}{dx^3}\right)^{2/3}$$

$$\frac{\left(\frac{d^3y}{dx^3}\right)^{3/2}}{\left(\frac{d^3y}{dx^3}\right)^{2/3}} = 1$$

$$\left(\frac{d^3y}{dx^3}\right)^{\frac{3}{2}-\frac{2}{3}} = 1$$

$$\left(\frac{d^3y}{dx^3}\right)^{5} = 1$$

degree = 5

order = 3

Formation of a D.E ⇒

1. Write the eq^n of given family of curves.

2. Differentiate as no. of times as no. of arbitary constant occuring in it.

3. Eliminate the arbitary constant from the given eq^n by using the eq^n obtained in step (1) and step (2)

Q. Find the DE of the family of circle of the form $x^2+y^2+2gx=0$

given eq^n is $x^2+y^2+2gx=0$ — (1)

$$2x+2y\frac{dy}{dx}+2g=0 \Rightarrow g=-x-y\frac{dy}{dx} \quad \text{— (2)}$$

Sub (2) in (1)

$$x^2+y^2+2x\left(-x-y\frac{dy}{dx}\right)=0$$

$$x^2+y^2-2x^2-2xy\frac{dy}{dx}=0$$

$$\boxed{y^2-x^2-2xy\frac{dy}{dx}=0}$$ required DE.

find the DE of family of curve of the form $y = A\cos x + B\sin x$

The given eq^n is $y = A\cos x + B\sin x$ ①

$$\frac{dy}{dx} = -A\sin x + B\cos x \quad ②$$

$$\frac{d^2y}{dx^2} = -A\cos x - B\sin x \quad ③$$

① + ③

$$\frac{d^2y}{dx^2} + y = 0$$

find the D.E of family of curve of the form $y = Ax + Bx^2$

[when function are Algebric and expo^n]

$$DE = \begin{vmatrix} y & x & x^2 \\ y' & 1 & 2x \\ y'' & 0 & 2 \end{vmatrix}$$

$$y(2) - x(2y' - 2xy'') + x^2(-y'')$$

$$= x^2y'' - 2xy' + 2y = 0$$

Variable separable :→

$$\frac{dy}{dx} = f(x) \cdot f(y)$$

$$\Rightarrow \frac{dy}{dx} = e^{x-y} + x^2e^{-y}$$

$$\frac{dy}{dx} = e^xe^{-y} + x^2e^{-y}$$

$$e^y\,dy = (e^x + x^2)\,dx$$

$$\int e^y\,dy = \int (e^x + x^2)\,dx$$

$$e^y = e^x + \frac{x^3}{3} + c$$

Q $$\frac{dy}{dx} = 1 + x + y + xy$$

$$\frac{dy}{dx} = (1+x) + y(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y)$$

$$\frac{dy}{(1+y)} = (1+x)\,dx$$

$$\ln(1+y) = x + \frac{x^2}{2} + c$$

→ Reducible to variable-separable

$$\frac{dy}{dx} = f(x,y)$$

$$\frac{dy}{dx} = \sin(x+y) \quad -(1)$$

Let $x + y = t \quad -(2)$

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{dt}{dx} - 1 \quad -(3)$$

sub (2) + sub (3)

$$\frac{dt}{dx} - 1 = \sin t$$

$$\frac{dt}{dx} = 1 + \sin t$$

$$\frac{dt}{1+\sin t} = dx$$

$$= \frac{(1-\sin t)}{(1-\sin^2 t)} dt = \int dx$$

$$\int \frac{(1-\sin t)}{\cos^2 t} dt = \int dx$$

$$= \int \sec^2 t - \int \sec t \tan t = \int dx$$

$$= \tan t - \sec t = x + c$$

$$= \tan(x+y) - \sec(x+y) = x + c$$

Q
$$\frac{dy}{dx} = (4x+y+1)^2 \quad \text{--- (1)}$$

let $4x+y+1 = t$ --- (2)

$$4 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{dt}{dx} - 4 \quad \text{--- (3)}$$

sub (2) + su (3) in (1)

$$\frac{dt}{dx} - 4 = t^2$$

$$\frac{dt}{dx} = t^2 + 4$$

$$\frac{dt}{t^2+4} = dx$$

$$\frac{1}{2}\tan^{-1}(t/2) = x + c$$

$$\frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right) = x + c$$

$$\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)}$$ where

f_1 and f_2 are homogenous of same degree, put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Q $$\frac{dy}{dx} = \frac{x^2 y}{x^3+y^3} \quad -(1)$$

Put $y = vx$ (2)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad -(3)$$

Sub (2) and (3) in eqn (1)

$$v + x\frac{dv}{dx} = \frac{x^2 \cdot vx}{x^3+v^3x^3}$$

$$v + x\frac{dv}{dx} = \frac{v}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{v}{1+v^3} - v$$

$$x\frac{dv}{dx} = \frac{v - v - v^4}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$$

$$\left(\frac{1+v^3}{v^4}\right)dv = \frac{-dx}{x}$$

$$= (v^{-4}) + \frac{1}{v}\,dv = -\int \frac{1}{x}\,dx$$

$$= \frac{v^{-3}}{-3} + \ln v = -\ln x + c$$

$$\frac{-1}{3v^3} + \ln v + \ln x = c$$

$$\frac{-1}{3v^3} + \ln(vx) = c$$

since $v = y/x$

$$\boxed{\frac{-1}{3}\frac{x^3}{y^3} + \ln y = c}$$

Q $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ —(1)

Put $y = vx$ —(2)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad —(3)$$

$\Rightarrow$ $v + x\frac{dv}{dx} = v + \tan v$

$$x\frac{dv}{dx} = \tan v$$

$$\frac{dv}{\tan v} = \frac{dx}{x}$$

$$\int \cot v \, dv = \int \frac{dx}{x}$$

$$\ln \sin v = \ln x + \ln c$$

$$\ln \sin v = \ln xc$$

$$\sin v = xc$$

$$\boxed{\sin(y/x) = xc}$$

Linear diff^n equation :— A diff^n eq^n is said to be linear diff^n eq^n if the degree of dependent variable and its diff^n coefficient is one and they are not multiplied together.

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Ex. $\frac{dy}{dx} + xy = x^3$ is linear

$\frac{dy}{dx} + y^2 x + y^2 = x^2$ is non-linear

The standard form of L.D.E is

$$\frac{dy}{dx} + Py = Q$$ where P and Q are function of x

$$I.F = e^{\int P dx}$$

Solution is

$$y(I.F) = \int Q(I.F)\,dx$$

Q $\frac{dy}{dx} + 2xy = e^{-x^2}$; $y(0)=1$

$$\frac{dy}{dx} + Py = Q$$

$P = 2x$, $Q = e^{-x^2}$

$$e^{\int P dx} = e^{\int 2x\,dx} = e^{x^2}$$

$$y \times e^{x^2} = \int e^{x^2} \cdot e^{-x^2}\,dx$$

$$\boxed{y e^{x^2} = x + c}$$

$y \to 1$, $x \to 0$

$1 = c$

$$y e^{x^2} = x + 1$$

$$\boxed{y = (x+1) e^{-x^2}}$$

Q $\quad x^2\frac{dy}{dx} + 2xy = \frac{2\ln x}{x}$; $y(1) = 0$

$y(e) = \underline{\ ?\ }$

$$\frac{dy}{dx} + \frac{2}{x}y = \frac{2\ln x}{x^3} \quad -(1)$$

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{2}{x}, \quad Q = \frac{2\ln x}{x^3}$$

$$I.F = e^{\int P\,dx} = e^{\int \frac{2}{x}dx} = x^2$$

Solution $\quad y(x)^2 = \int \frac{2\ln x}{x^3} \times x^2\,dx$

$$yx^2 = \frac{2(\ln x)^2}{2} + c$$

$$yx^2 = (\ln x)^2 + c$$

$y(1) = 0 \quad y \to 0, \quad x \to 1$

$$0 = 0 + c$$

$$\boxed{yx^2 = (\ln x)^2 + 0}$$

$y(e) \to$

$= ye^2 = 1 \qquad \boxed{y = 1/e^2}$

Q If $y' - x \neq 0$ then the solⁿ of

$y'(y'+y) = x(x+y)$ with $y(0) = 2$ is

$$(y')^2 + yy' = x^2 + xy$$

$$(y')^2 + yy' - xy - x^2 = 0$$

$$(y')^2 - x^2 + yy' - xy = 0$$

$$(y'+x)(y'-x) + (y'-x)y = 0$$

$(y'-x)(y'+x+y) = 0$

$y'-x \neq 0 \quad \therefore \quad y'+x+y = 0$

$\Rightarrow \quad \frac{dy}{dx} + y = -x \quad -(1)$

$P = 1, \quad Q = -x$

$IF = e^x$

$y(e^x) = \int e^x(-x)\,dx$

$y e^x = -(xe^x - e^x) + c \quad \Big| \; \begin{matrix} +x & -1 & +0 \\ e^x & e^x & e^x \end{matrix}$

$\boxed{y = -(x-1) + ce^{-x}}$

$y(0) = 2 \qquad y \to 2, \quad x \to 0$

$2 = 1 + c \Rightarrow (c = 1)$

$\boxed{y = -x + 1 + e^{-x}}$

Labneze: $\rightarrow \frac{dy}{dx} - xy = x^3 y^2 \quad -(1)$

$\frac{1}{y^2}\frac{dy}{dx} - \frac{x}{y} = x^3 \quad -(2)$

let $-\frac{1}{y} = z \quad -(3)$

$\frac{1}{y^2}\frac{dy}{dx} = \frac{dz}{dx} \quad -(4)$

$\Rightarrow \quad \frac{dz}{dx} + xz = x^3$

which is LDE in z

$$\frac{dZ}{dx} + PZ = Q$$

$$\Rightarrow P = x, \quad Q = x^3$$

$$IF = e^{\int x\,dx} = e^{x^2/2}$$

Soln $\Rightarrow$ $$\cancel{Z}\, e^{x^2/2} = \int Q \times IF\, dx$$

$$= -\frac{1}{y} e^{x^2/2} = \int x^3 \cdot e^{x^2/2} dx$$

$$= -\frac{1}{y} e^{\frac{x^2}{2}} = \int x \cdot x^2 e^{x^2/2} dx$$

$$= \text{let } \frac{x^2}{2} = t$$

$$(x\,dx = dt)$$

$$\Rightarrow \quad = \int 2t e^t dt$$

$$= 2(te^t - e^t) + c$$

$$= -\frac{1}{y} e^{x^2/2} = 2\left(\frac{x^2}{2} e^{x^2/2} - e^{x^2/2}\right) + c$$

$$= -\frac{1}{y} = 2\left(\frac{x^2}{2} - 1\right) + c e^{-x^2/2}$$

Q $$\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$$

$$\frac{1}{\cos^2 y} \cdot \frac{dy}{dx} + \frac{x \cdot 2 \sin y \cos y}{\cos^2 y} = x^3$$

$$\sec^2 y \frac{dy}{dx} + 2x \tan y = x^3 \quad \text{—(2)}$$

$$\text{let } \tan y = Z \quad \text{—(3)}$$

$$\sec^2 y \frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow \frac{dz}{dx} + 2x z = x^3$$

$$\frac{dz}{dx} + Pz = Q$$

$$P = 2x, \quad Q = x^3$$

$$IF = e^{x^2}$$

$$\Rightarrow z(IF) = \int Q(IF)\, dx$$

$$\tan y \, e^{x^2} = \int x^3 e^{x^2} dx$$

$$\text{L.'} \quad x^2 = t \Longrightarrow 2x\,dx = dt$$

$$\tan y \, e^t = \int t \cdot e^t \frac{dt}{2}$$

$$\tan y \, e^t = \frac{e^t}{2}(t-1)$$

$$\tan y \, e^{x^2} = \frac{e^{x^2}}{2}(x^2-1) + c$$

$$\tan y = \frac{\cancel{e^{x^2}}}{2}(x^2-1) + c e^{-x^2}$$

Exact diff^n :→ The diff eq^n $M\,dx + N\,dy = 0$ is said to be an exact diff^n if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

the sol^n

$$\int M\,dx + \int (\text{term in } N \text{ free from } x)\, dy = c$$

y const.

Q $(xy^2+x)\,dx + (x^2y-y)\,dy = 0$

$M = xy^2 + x$, $N = x^2y - y$

$\frac{\partial M}{\partial y} = 2yx$, $\frac{\partial N}{\partial x} = 2xy$

Sol^n $= \int M\,dx + \int (\text{term in N free from } x)\,dy = c$

$= \int (xy^2+x)\,dx + \int -y\,dy = c$

$= \frac{y^2 \times x^2}{2} + \frac{x^2}{2} - \frac{y^2}{2} = c$

$x^2y^2 + x^2 - y^2 = 2c$

Q $(e^y+1)\cos x\,dx + e^y \sin x\,dy = 0$

$M = (e^y+1)\cos x$, $N = e^y \sin x$

$\frac{\partial M}{\partial y} = \cos x(e^y)$, $\frac{\partial N}{\partial x} = e^y \cos x$

$= \int M\,dx + \int \text{terms in N free from } x\,dy = c$

$\int (e^y+1)\cos x\,dx + \int 0 = c$

$= e^{(y+1)}\sin x = c$

Reduced to exact :-

* If M and N are homogenous funⁿ of same degree then $IF = \dfrac{1}{Mx+Ny}$

* If M and N are not homogenous but
$M = y f_1(x,y)$, $N = x f_2(x,y)$
then $IF = \dfrac{1}{Mx-Ny}$ (x or y) terms into में हो।

* If $\dfrac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N} = f(x)$ or constant
then $I.F = e^{\int f(x)\,dx}$

* If $\dfrac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M} = f(y)$ or constant then
$IF = e^{\int f(y)\,dy}$

Q $(y-xy^2)\,dx-(x+x^2y)\,dy=0$ —(1)

$M = y-xy^2$, $N = -x-x^2y$

$\dfrac{\partial M}{\partial y} = 1-2xy$, $\dfrac{\partial N}{\partial x} = -1-2xy$

$IF = \dfrac{1}{Mx-Ny} = \dfrac{1}{xy-x^2y^2+xy+x^2y^2} = \dfrac{1}{2xy}$

(1) $\times IF = \left(\dfrac{y-xy^2}{2xy}\right)dx - \left(\dfrac{x+x^2y}{2xy}\right)dy = 0$

$$\left(\frac{1}{2x}-\frac{y}{2}\right)dx - \left(\frac{1}{2y}+\frac{x}{2}\right)dy = 0 \quad -(2)$$

$$M_1 = \frac{1}{2x}-\frac{y}{2}, \quad M_2 = -\left(\frac{1}{2y}+\frac{x}{2}\right)$$

$$\frac{\partial M_1}{\partial y} = -1/2, \quad \frac{\partial M_2}{\partial x} = -1/2$$

the soln

$$\int M_1\,dx + \int (\text{terms in } N \text{ free from } x)\,dy = c$$

$$\int\left(\frac{1}{2x}-\frac{y}{2}\right)dx + \int\left(\frac{-1}{2y}\right)dy = c$$

$$= \frac{1}{2}\ln x - \frac{y}{2}x - \frac{1}{2}\ln y = c$$

$$\ln x - xy - \ln y = 2c$$

$$\boxed{\ln(x/y) - xy = 2c}$$

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Q $(x^2+y^2+2x)\,dx + 2y\,dy = 0 \quad -(1)$

$$M = x^2+y^2+2x \qquad N = 2y$$

$$\frac{\partial M}{\partial y} = 2y, \quad \frac{\partial N}{\partial x} = 0$$

$$\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N} = \frac{2y}{2y} = 1 \qquad IF = e^{\int 1\,dx} = e^x$$

$$\Rightarrow (1)\,e^x \Rightarrow e^x(x^2+y^2+2x)\,dx + e^x\,2y\,dy = 0$$

$$M_1 = e^x(x^2+y^2+2x), \quad N_1 = e^x\,2y$$

$$\frac{\partial M_1}{\partial y} = e^x(2y) \quad , \quad \frac{\partial N_1}{\partial x} = 2ye^x$$

Solution is

$$\int M_1 dx + \int \text{terms of } N \text{ w/o } x \cdot dy = c$$

$$\int e^x(x^2+y^2+2x)dx + 0 = c$$

$$\int \underline{e^x x^2} + \int e^x y^2 + \int e^x 2x = c$$

$$= x^2e^x - \cancel{2xe^x} + \cancel{2e^x} + y^2e^x + \cancel{2xe^x} - \cancel{2e^x} = c$$

$$= x^2e^x + y^2e^x = c$$

$$(x^2+y^2)e^x = c$$

Orthogonal Trajectories :→
(Path)

1. find the DE of given family of curves
2. Replace $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ to get DE of orthogonal.
   (Replace $\frac{1}{r}\frac{dr}{d\theta}$ by $-r\frac{d\theta}{dr}$ if it is in polarform.
3. Solve the resultant diff eqn to get O.T.

Q find the O.T of family of curves of the forms $y = k(x-1)$

$\longrightarrow$ $y = k(x-1)$

$$\frac{dy}{dx} = k$$

$$y = \frac{dy}{dx}(x-1)$$

replace $\frac{dy}{dx} \to -\frac{dx}{dy}$

so $y = \frac{-dx}{dy}(x-1)$

$= y\,dy = -dx(x-1)$

$$\int y\,dy = \int -x\,dx + \int dx$$

$$\frac{y^2}{2} = \frac{-x^2}{2} + x + c$$

$$= \frac{x^2}{2} + \frac{y^2}{2} = x + c$$

$$\boxed{x^2 + y^2 - 2x = 2c}$$ required O.T

Q find the O.T of family of curve of the form $r = a(1+\cos\theta)$

The given eq^n is $r = a(1+\cos\theta)$ — (1)

$$\frac{dr}{d\theta} = a(-\sin\theta) \quad \text{— (2)}$$

$$\boxed{\frac{1}{r}\frac{dr}{d\theta} = \frac{-\sin\theta}{1+\cos\theta}} \quad \text{--- (3)}$$

replace $\frac{1}{r}\frac{dr}{d\theta} = -r\frac{d\theta}{dr}$

$$\cancel{-}r\frac{d\theta}{dr} = \cancel{-}\frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{1+\cos\theta\, d\theta}{\sin\theta} = \frac{dr}{r}$$

$$= \frac{2\cos^2\theta/2\, d\theta}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \frac{dr}{r}$$

$$= \int \cot\theta/2\, d\theta = \int \frac{1}{r}dr$$

$$\frac{\ln\sin\frac{\theta}{2}}{1/2} = \ln r + \ln c$$

$$2\ln\sin\frac{\theta}{2} = \ln rc$$

$$\sin^2\theta/2 = rc$$

$$\left(\frac{1-\cos^2\theta}{2}\right) = rc$$

$$\boxed{r = \frac{1}{2c}(1-\cos\theta)}$$

Higher order linear diff^n with constant coefficients :-

$$\frac{d^n y}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + \text{------} a_n y = f(x)$$

$$(D^n + a_1 D^{n-1} + \text{-----} a_n)\,y = f(x)$$

If $f(x) = 0$ then it is homogeneous L.D.E.

general solution $\boxed{G.S = C.F}$

If $f(x) \neq 0$ then it is nonhomogenous LDE $\boxed{G.S = CF + PI}$

⇒ roots of auxillary equation		corresponding complementary function
real and distinct	m_1, m_2, m_3	$c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$
real and equal	m, m, m	$(c_1 + c_2 x + c_3 x^2) e^{mx}$
imaginary	$\alpha \pm i\beta$	$e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$
	$\alpha \pm i\beta, \alpha \pm i\beta$	$e^{\alpha x}[(c_1 + c_2 x)\cos\beta x + (c_3 + c_4 x)\sin\beta x]$

Q $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

$(D^2 - 5D + 6)y = 0$

AE is $m^2 - 5m + 6 = 0$

$m = 2, 3$

$C.F = c_1 e^{2x} + c_2 e^{3x}$

$y = c_1 e^{2x} + c_2 e^{3x}$

(c) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

$(D^2 - 4D + 4)y = 0$

AE is $m^2 - 4m + 4 = 0$

$m = 2, 2$

Solution

$y = (c_1 + c_2 x) e^{2x}$

(b) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

$(D^2 - 2D + 2)y = 0$

AE is $m^2 - 2m + 2 = 0$

$m = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2} = (1 \pm i)$

Solution is

$y = e^{x} \cancel{\cos}(c_1 \cos x + c_2 \sin x)$

(Q) If e^{2x} and e^{3x} are two independen solution of a second order diff.^{n} then the DE is

Given that $y = c_1 e^{2x} + c_2 e^{3x}$

roots are 2, 3

$\Rightarrow$ DE is $[(D-2)(D+3)]y = 0$

$= (D^2 - 5D + 6)y = 0$

(R) In complementary solution the coefficien of arbitary constant is called independent solution.

I- (1) $e^{\alpha x}$ [non homogenous]

Replace D by α in $f(D)$

(a) $(D^2-6D+8)y = e^{6x}$

$AE = m^2-6m+8$

$m = 4, 2$

$\Rightarrow \quad y = c_1e^{4x}+c_2e^{2x}$

$PI = \frac{1}{D^2-6D+8}e^{6x} = \frac{e^{6x}}{36-36+8}$

$= \frac{e^{6x}}{8}$

$y = c_1e^{4x}+c_2e^{2x}+\frac{e^{6x}}{8}$

(b) $(D^2-6D+9)y = e^{3x}$

m^2-6m+9

$\boxed{m = 3, 3}$

$y = (c_1+c_2x)e^{3x}$

$PI = \frac{1}{D^2-6D+9}xe^{3x} = \frac{1}{0}$ ← case fail

$= \frac{xe^{3x}}{2D-6}$ case fail

$= \frac{x^2e^{3x}}{2}$

$\Rightarrow y = (c_1+c_2x)e^{3x}+\frac{x^2e^{3x}}{2}$

(f) $(D^2+9)y = 2^x$

$AE = m^2+9 = 0$

$m = \pm 3i$

$y = c_1 \cos 3x + c_2 \sin 3x$

$PI = \frac{1}{D^2+9} 2^x = \frac{1}{D^2+9} e^{\ln 2^x}$

$= \frac{1}{D^2+9} e^{x \ln 2}$

$= \frac{1}{(\ln 2)^2+9} e^{\ln 2 x}$

$= \frac{2^x}{9.48}$

$\boxed{GS = CI + PI}$

(g) $(D^2+4)y = 16$

$AE = m^2+4$

$(m = \pm 2i)$

$y = c_1 \cos 2x + c_2 \sin 2x$

$PI = \frac{16}{D^2+4} = \frac{e^0 . 16}{D^2+4}$

$= \frac{16}{0+4} = 4$

$(GS = C.F + PI)$

TYPE II

$\cos \alpha x$ or $\sin \alpha x$

Hint : Replace D^2 by $-(\alpha^2)$

Q $(D^2+9)y = \sin 2x$

$AE = m^2+9 \Rightarrow m = \pm 3i$

$y = c_1 \cos 3x + c_2 \sin 3x$

$PI = \frac{1}{D^2+9} \sin 2x = \frac{\sin 2x}{-4+9}$

$= \frac{\sin 2x}{5}$

$GS = CF + PE$

Q $(D^2+4)y = \cos 2x$

$y = c_1 \cos 2x + c_2 \sin 2x$

$PI = \frac{\cos 2x}{D^2+4}$ Case fail

$= \frac{x \times (\cos 2x)}{2D} = x \frac{1}{2D} \cos 2x$

$= \frac{x}{2} \times \frac{1}{D} \cos 2x = \frac{x}{2} \times \frac{\sin 2x}{2}$

$= \frac{x}{4} \sin 2x$

$GS = CF + PI$

Q $(D^2-4D+3)y = \cos x$

$CF = c_1 e^x + c_2 e^{3x}$

$$= \frac{1}{D^2+4D+3}\cos x$$

$D^2 = -1$

$$= \frac{1}{-1-4D+3}\cos x = \frac{1}{2-4D}\cos x$$

$$= \boxed{\frac{(2+4D)}{4-16D^2}\cos x}$$

$(D^2=-1)$

$$= \frac{2\cos x - 4\sin x}{4+16} = \frac{2\cos x - 4\sin x}{20}$$

T-3

(4)

x^k

write $f(D)$ in the form $(1+t)^{-1}$ or $(1-t)^{-1}$.

Q $(D^2-3D+2)y = x^2$

$CF = y = c_1 e^x + c_2 e^{2x}$

$$PI = \frac{1}{2-3D+D^2}x^2 = \frac{1}{(1-D)(2-D)}x^2$$

$$= \left[\frac{1}{1-D} - \frac{1}{2-D}\right]x^2$$

$(1+x)^n$

$1+nx+n(n-1)\frac{x^2}{2!}+$

$n(n-1)(n-2)\frac{x^3}{3!}$

$$= \frac{x^2}{1-D} - \frac{x^2}{2(1-D/2)}$$

$$= (1-D)^{-1}x^2 - \frac{1}{2}\left(1-\frac{D}{2}\right)^{-1}x^2$$

$$\boxed{\left(1+D+D^2+D^3+\dots\right)}x^2 - \frac{1}{2}\left(1+\frac{D}{2}+\frac{D^2}{4}+\frac{D^3}{8}+\dots\right)$$

$D(x^2) = 2x, \quad D^2(x^2) = 2$

$D'''(x^2) = 0$

$$\Rightarrow (1+D+D^2)x^2 - \frac{1}{2}\left(1+\frac{D}{2}+\frac{D^2}{4}\right)x^2$$

$$= x^2+2x+2 - \frac{x^2}{2} - \frac{x}{2} - \frac{1}{2}$$

$$PI = \frac{x^2}{2} + \frac{3x}{2} + \frac{4}{2}$$

Q. $(D^2-4D+4)y = x^2$

$$CF = (c_1 + c_2 x)e^{2x}$$

$$PI = \frac{1 \cdot x^2}{4-4D+D^2} = \frac{1}{(2-D)^2}x^2$$

$$= \frac{1}{4\left(1-\frac{D}{2}\right)^2}x^2$$

$$= \frac{1}{4}\left(1-\frac{D}{2}\right)^{-2}x^2$$

$\because (1-t)^{-2} = 1+2t+3t^2+4t^3+\dots$

$$= \frac{1}{4}\left[1 + D + 3\frac{D^2}{4} + 4 \cdot \frac{D^3}{8} + \dots\right] \quad \uparrow \rightarrow \text{negl.}$$

$$= \frac{1}{4}\left[1+D+\frac{3D^2}{4}\right]x^2$$

$$= \frac{1}{4}\left[x^2+2x+\frac{3}{2}\right]$$

TYPE -IV $e^{\alpha x}\cos\beta x$ or $e^{\alpha x}\sin\beta x$

or $e^{\alpha x}\, x^{k}$

Hint $\Rightarrow$

$$\frac{1}{f(D)} e^{\alpha x}.V = e^{\alpha x}\left\{\frac{1}{f(D+\alpha)} V\right\}$$

where $V = \sin\beta x$ or $\cos\beta x$ or x^k

Q $(D^2-5D+6)\,y = e^{3x}\cos x$

$$CF = c_1 e^{2x} + c_2 e^{3x}$$

$$PI = \frac{1}{D^2-5D+6} e^{3x}\cos x$$

$$= e^{3x}\left\{\frac{1}{(D+3)^2-5(D+3)+6}\cos x\right\}$$

$$= e^{3x}\left\{\frac{1}{D^2+D}\cos x\right\}$$

$D^2 = -1$

$$= e^{3x}\left\{\frac{1}{-1+D}\cos x\right\}$$

$$= e^{3x}\left\{\frac{D+1}{(D-1)(D+1)}\cos x\right\}$$

$$= e^{3x}\left\{\frac{(D+1)\cos x}{D^2-1}\right\}$$

$(D^2 = -1)$

$$\frac{e^{-3x}}{-2}\left\{-\sin x+\cos x\right\}$$

$GS = CF + PI$

Q $(D^2 - 6D + 9)y = e^{3x}x^2$

$CF = (c_1 + c_2 x)e^{3x}$

$$PI = \frac{e^{3x} \cdot x^2}{D^2 - 6D + 9}$$

$$= e^{3x} \times \frac{1}{(D+3)^2 - 6(D+3) + 9} x^2$$

$$= e^{3x} \times \frac{1}{D^2} x^2$$

$$= e^{3x} \times \frac{1}{D} \times \frac{1}{D} \times x^2$$

$$= e^{3x} \times \frac{1}{D} \times \frac{x^3}{3}$$

$$= \boxed{e^{3x} \times \frac{x^4}{12}}$$

TYPE-V $x \sin \alpha x$ or $x \cos \alpha x$

$$\frac{1}{f(D)} x.V = x\frac{1}{f(D)}V - \frac{f'(D)}{[f(D)]^2}V$$

where $V = \sin \alpha x$ or $\cos \alpha x$

$$(D^2+9)y = x\sin 2x$$ (Q-1)

$$CF = c_1\cos 3x + c_2 \sin 3x$$

$$PI = \frac{1}{(D^2+9)} x\sin 2x$$

$$= x\cdot\frac{1}{(D^2+9)}\sin 2x - \frac{2D(\sin 2x)}{(D^2+9)^2}$$

$$= x \times \frac{1}{-4+9}\sin 2x - \frac{2\times\cos 2x\times 2}{(-4+9)^2}$$

$$= \frac{x}{5}\sin 2x - \frac{4}{25}\cos 2x$$

Q $(D^2+4)y = x\cos 2x$

$$CF = c_1\cos 2x + c_2\sin 2x$$

$$PI = \frac{1}{(D^2+4)} x\cos 2x$$

$$= x\frac{1}{(D^2+4)}\cos 2x - \frac{2D\cos 2x}{(D^2+4)^2}$$

$(D^2=-4)$ case fail

$$= \frac{x\times x\cdot 1}{2D}\cos 2x - x\frac{2D}{2(D^2+4)\cancel{2D}}\cos 2x$$

differentiate

$$= \frac{x^2}{2}\times\frac{1}{D}\cos 2x - \frac{x}{2}\frac{\cos 2x}{(D^2+4)}$$ $D^2=-4$ fail

$$= \frac{x^2\sin 2x}{2\cdot 2} - \frac{x^2}{2\times 2D}\cos 2x$$