

Q3 $\frac{d^2y}{dx^2} + 2x \frac{dy}{dx} + (x^2+1)y = (x^3+3x)$

Let,
 $y = u.v$

$P = 2x \quad | \quad Q = (x^2+1) \quad | \quad R(x^3+3x)$
 $u = e^{-\frac{1}{2} \int P dx} = e^{-\frac{1}{2} \int 2x dx} = e^{-\frac{1}{2} \cdot 2x^2/2} = e^{-\frac{1}{2}x^2}$

$$\left\{ \begin{aligned} \frac{d^2v}{dx^2} + IV &= R/u \quad \text{--- (1)} \\ I &= Q - \frac{1}{2} \frac{dP}{dx} - \frac{1}{4} P^2 \end{aligned} \right.$$

$$\frac{dP}{dx} = 2$$

$$I = (x^2+1) - \frac{1}{2}(2) - \frac{1}{4}(2x)^2$$

$$I = x^2 + 1 - 1 - x^2$$

$$I = 0$$

Put in (1)

$$\frac{d^2v}{dx^2} + 0v = \frac{(x^3+3x)}{e^{-\frac{1}{2}x^2}}$$

$$\frac{d^2v}{dx^2} = (x^3+3x) \cdot e^{\frac{1}{2}x^2}$$

$$\frac{dv}{dx} = \frac{(x^3+3x) e^{\frac{1}{2}x^2}}{\frac{1}{2}}$$

$$\frac{dv}{dx} = \int x^3 e^{\frac{1}{2}x^2} dx + \int 3x e^{\frac{1}{2}x^2} dx + C_1$$

$$\frac{dv}{dx} = \int x^2 \cdot x e^{\frac{1}{2}x^2} dx + 3 \int x e^{\frac{1}{2}x^2} dx + C_1$$

$$\frac{dv}{dx} = \int 2$$

Put $\frac{1}{2}x^2 = t \Rightarrow x^2 = 2t$
 on diff ~~case~~
 $\frac{1}{2} 2x dx = dt$
 $x dx = dt$

$$\frac{dv}{dx} = \int 2t e^t dt + 3 \int e^t dt + C_1$$

$$\frac{dv}{dx} = 2 \int t e^t dt + 3 \int e^t dt + C_1$$

$$\int x e^{\frac{1}{2}x^2} dx$$

Put

$$\frac{1}{2}x^2 = t$$

$$\frac{1}{2} 2x dx = dt$$

$$x dx = dt$$

$$\int e^t dt$$

$$+ \frac{e^t}{\frac{1}{2}}$$

$$= 2e^{\frac{1}{2}x^2}$$

$$\frac{dv}{dx} = 2(t e^t - e^t) + 3e^t + C_1$$

$$\frac{dv}{dx} = 2 \left(\frac{1}{2} x^2 e^{\frac{1}{2}x^2} - e^{\frac{1}{2}x^2} \right) + 3e^{\frac{1}{2}x^2} + C_1$$

$$\frac{dv}{dx} = x^2 e^{\frac{1}{2}x^2} - 2e^{\frac{1}{2}x^2} + 3e^{\frac{1}{2}x^2} + C_1$$

$$\frac{dv}{dx} = x^2 e^{\frac{1}{2}x^2} + e^{\frac{1}{2}x^2} + C_1$$

$$v = \int \left(x^2 e^{\frac{1}{2}x^2} \right) dx + \int e^{\frac{1}{2}x^2} dx + \int C_1 dx + C_2$$

$$v = x \int (x e^{\frac{1}{2}x^2}) - \int \left[\frac{d}{dx} (x) \int x e^{\frac{1}{2}x^2} dx \right] dx + \int e^{\frac{1}{2}x^2} dx$$

$$v = x e^{\frac{1}{2}x^2} - \int 1 \cdot e^{\frac{1}{2}x^2} dx + \int e^{\frac{1}{2}x^2} dx + C_1 x + C_2$$

$$v = x e^{\frac{1}{2}x^2} + C_1 x + C_2 \quad \text{--- (2)}$$

$$y = u \cdot v$$

$$y = e^{-\frac{1}{2}x^2} \left(x e^{\frac{1}{2}x^2} + C_1 x + C_2 \right)$$

$$y = x e^{-\frac{1}{2}x^2 + \frac{1}{2}x^2} + C_1 x e^{-\frac{1}{2}x^2} + C_2 e^{-\frac{1}{2}x^2}$$

$$y = x + (C_1 x + C_2) e^{-\frac{1}{2}x^2}$$