**COSC6364 : Advanced Numerical Analysis**

**Spline Interpolation for Synthesis or Reconstruction of Multi-Dimensional Data**

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1. **Introduction**

The visual fidelity and smoothness of video playback are foundational to the user experience in multimedia applications. With the advent of high-definition and ultra-high-definition standards, expectations for fluid and lifelike motion in video content have soared. One of the key parameters that influence this perception of quality is the frame rate, typically measured in frames per second (fps). Standard video frame rates vary, with 24 fps being common in cinema and 30 or 60 fps in television broadcasting. However, as display technology evolves, there is a growing demand for higher frame rates to enhance the viewer's experience, particularly in action-packed sequences or high-speed sports broadcasts.

This project investigates a numerical method known as spline interpolation, applied to frame rate up-conversion in videos. The objective is to interpolate and synthesize frames within a video sequence, thereby increasing the frame rate and producing a smoother visual experience. Spline interpolation is particularly well-suited for this task due to its ability to create smooth transitions between known data points—in this case, video frames. By applying spline interpolation methods, which are celebrated for their smooth curve-fitting properties, we aim to generate visually pleasing and artifact-free interpolated frames.

The significance of this project lies not only in enhancing the subjective viewing pleasure but also in the practical implications for industries such as film production, video gaming, and virtual reality, where immersive and high-quality content is crucial. Moreover, the project provides insights into the applicability of spline interpolation techniques to multi-dimensional data, illustrating the versatility of mathematical concepts when applied to complex, real-world challenges.

In this report, we delve into the theoretical underpinnings of spline interpolation, the procedural steps taken to implement this method, and the results obtained from its application to video frame interpolation. We also compare our approach to current state-of-the-art methods, aiming to quantify improvements and understand the potential for future advancements in video processing technology.

1. **Methodology**

This project's methodology unfolds across several stages, starting from data preparation to detailed implementation, and finally to a rigorous validation process. Each phase is designed to thoroughly investigate the efficacy of spline interpolation in enhancing video frame rate.

**2.1. Data Selection and Preparation**

**Video Dataset Acquisition**

The choice of dataset is a critical element that influences the reliability and applicability of frame interpolation research. To align our work with real-world content, we opted for a video from YouTube, which offers a vast repository of user-generated content reflective of typical media consumption. The video was selected based on its native playback rate of 30 frames per second (fps), which is representative of the current digital video standard.

**Data Conditioning for Analysis**

The preparation of our dataset involved a deliberate down-sampling process. The original video, sourced at 30 fps, was degraded to 10 fps. This reduction serves a dual purpose: first, it creates a starting point that clearly benefits from frame rate enhancement; second, it facilitates computational efficiency, which is particularly advantageous for prototyping and testing.

Additionally, we truncated the video to a length of 15 seconds. This decision was influenced by the need to balance data sufficiency for a robust analysis with the practical considerations of processing time and resource allocation.

**Input Dataset Synthesis**

The processed video, now at 10 fps and 15 seconds long, constitutes our input dataset. This video segment provides a controlled environment to demonstrate the capabilities of our spline interpolation technique and serves as a benchmark for subsequent quality assessments post-interpolation.

**2.2 Implementation Details**

The implementation of the spline interpolation for enhancing video frame rate is a multi-step process that involves video processing, application of mathematical interpolation techniques, and subsequent video reconstruction. This section provides a detailed account of the specific approaches and algorithms utilized.

**2.2.1 Software and Libraries**

Python was chosen for its extensive support of scientific and video processing libraries, which are pivotal for implementing complex computational tasks involved in this project. The following libraries were instrumental in our development:

- OpenCV: This library provided comprehensive tools for video handling, including loading, frame extraction, and writing videos.

- SciPy: Essential for its mathematical functions, particularly the `CubicSpline` module which facilitates the creation of smooth and continuous curves between data points.

- NumPy: Used for efficient numerical operations on arrays, which form the backbone of frame manipulation.

**2.2.2 Cubic Spline Interpolation Technique**

Following the selection and preparation of our dataset, we employed cubic spline interpolation to enhance the frame rate. This mathematical technique is particularly adept at ensuring a smooth transition between points, which in our case are the individual frames of a video.

**Theoretical Basis of Cubic Splines**

Cubic splines are defined piecewise by third-degree polynomial functions. The continuity of the first and second derivatives at each point (frame) ensures that the motion between frames appears fluid and natural. For any two given frames at positions ( xi ) and ( xi+1 ), the cubic spline S(x) is defined for x in the interval [ xi, xi+1 ] by:

S(x) = ai + bi(x - xi) + ci(x - xi)2 + di(x - xi)3

The coefficients ai , bi , ci , and di  are determined such that S(xi) and S(xi+1) correspond to the pixel intensity values of the frames at ( xi ) and ( xi+1 ), and the first and second derivatives are continuous across frames.

**2.3 Interpolation Process**

To create a high frame rate effect, we introduce additional frames by evaluating the cubic spline at intermediate points. This is achieved through the following steps:

1. Coefficient Computation: For each pixel location within the frame, we calculate the coefficients of the cubic spline based on the pixel intensities of the current and the next frame.

2. Frame Synthesis: For each desired intermediate frame, the coefficients are used to evaluate the spline at specific intervals, creating pixel values for new frames. If the target is to interpolate two additional frames, the spline is evaluated at one-third and two-thirds along the temporal axis between the current and next frame.

3. Color Channel Separation: To ensure the fidelity of the video’s color, the spline interpolation is performed separately on the red, green, and blue channels of the pixel data. This preserves the nuances of the video's original color composition.

Through iterative application of this process, we generate a sequence of interpolated frames that are then interspersed among the original frames. The outcome is a video with an increased frame rate, characterized by the smoothness of motion that closely mimics natural human perception.

**2.4 Detailed Interpolation Process**

**Initialization:**

The dataset for interpolation comprises a 15-second-long video segment at 10 fps. This input data serves as the basis for our frame interpolation using the cubic spline technique. The process begins with the loading of the input video using OpenCV's ‘VideoCapture’. Frames are extracted and stored, along with metadata such as frame rate, width, and height.

**Interpolation Algorithm:**

For every two consecutive original frames, we perform the following cubic spline interpolation:

1. Frame Pair Normalization: We establish a normalized temporal domain `x` with values at 0 and 1, corresponding to the positions of the two original frames in time.

2. Intermediate Frame Generation: The `interpolate\_frames` function is iteratively called to compute the intermediate frames. For each color channel within the pixel data, a cubic spline is fit between the two original frames. The function evaluates the spline at equidistant time intervals `(t)` within the normalized domain to compute the interpolated frames. This results in `num\_inter\_frames` being generated, which are filled with the calculated pixel values from the spline evaluation.

**2.5. High Frame Rate Video Construction**

The enhanced video is then reconstructed in the `create\_high\_fps\_video` function:

1. Frame Assembly: The original and interpolated frames are sequentially ordered to form a complete high-frame-rate video.

2. Video Writing: Utilizing OpenCV's `VideoWriter`, the sequence is encoded into a video file with the increased frame rate, achieved by setting the `original\_fps` multiplied by the `frame\_rate\_multiplier`. The resulting video, now at the desired 30 fps, is stored as `output\_video.mp4`.

**Execution**

With the input data and interpolation algorithm ready, we execute the process by specifying the `frame\_rate\_multiplier` of 3, which amplifies the original 10 fps input to the target of 30 fps. This output aligns the frame rate with that of the ground truth video, allowing for a direct comparison in terms of visual fluidity and quality.

1. **Evaluation Metrics**

Upon enhancing the video frame rate using cubic spline interpolation, a comprehensive evaluation is essential to quantify the quality of the interpolated video frames. We employ two key metrics for this purpose: the Structural Similarity Index (SSIM) and the Peak Signal-to-Noise Ratio (PSNR). These metrics offer insights into the perceptual impact of the interpolation method.

**3.1. Structural Similarity Index (SSIM)**

SSIM is an advanced metric used to measure the similarity between two images. It is designed to provide a more perceptually relevant assessment of image quality by considering changes in texture, luminance, and contrast. SSIM values range from -1 to 1, with 1 indicating perfect similarity.

For each corresponding frame in the original and interpolated videos, SSIM is computed, taking into account the luminance test (comparison of mean brightness), contrast test (comparison of contrast), and structure test (comparison of structural similarity).

**3.2. Peak Signal-to-Noise Ratio (PSNR)**

PSNR is a widely used metric in image processing, computed using the mean squared error between the original and interpolated images. It evaluates the quality of a reconstructed image (or video frame, in our case) compared to the original image. PSNR is expressed in logarithmic decibels (dB) scale; higher values signify better quality.

Both metrics are calculated on a frame-by-frame basis for grayscale versions of the original and interpolated frames to emphasize luminance changes without the influence of color variations, which provides a stringent test of interpolation accuracy.

**3.3. Computational Approach**

The evaluation is conducted through the following steps:

1. Frame Conversion: Each frame of the videos is converted to grayscale to focus on the structural and noise-related aspects of quality without the confounding factor of color.
2. Metric Computation: We iteratively calculate the SSIM and PSNR values for each pair of corresponding frames between the original and interpolated videos. This computation is performed for all frames, and an average SSIM and PSNR value is derived to represent the overall quality.

**4.Results of Metrics Evaluation for Interpolated video.**

Following the application of cubic spline interpolation to the input video, our evaluation using standard quality metrics reveals promising results in frame rate enhancement.

**Metrics Comparison of Input Video vs Interpolated Output Video**

After processing the input video from 10 fps to 30 fps, we observed the following results:

- Average SSIM : 0.7427

- Average PSNR : 18.60 dB

The SSIM value indicates a respectable level of structural similarity, considering the significant frame rate conversion. This value denotes that the interpolated frames have preserved much of the structure and texture of the original video. The PSNR of 18.60 dB, in the context of video frames generated predominantly by interpolation, is a noteworthy achievement. It demonstrates the algorithm's capability to maintain a reasonable reconstruction quality relative to the source material's resolution.

**Metrics Comparison of Ground Truth vs Interpolated Output Video**

Comparison between the ground truth video at its native 30 fps and the spline-interpolated output at 30 fps shows:

- Average SSIM : 0.8649

- Average PSNR : 28.18 dB

An SSIM nearing the upper limit reflects excellent structural preservation between the original and interpolated content. Similarly, a PSNR of approximately 28 dB indicates a high-quality reconstruction, surpassing typical expectations for interpolated video frames, confirming the efficacy of the spline interpolation approach.

**Interpretation of the Metrics**

Both SSIM and PSNR values are crucial indicators of quality, with SSIM measuring structural preservation and PSNR assessing the error between the original and interpolated frames. The obtained SSIM signifies that the interpolation method has effectively maintained the core visual structure of the video, and the PSNR suggests that the noise introduced by interpolation is minimal.

These metrics underscore the effectiveness of the cubic spline interpolation in enhancing video frame rates. They highlight the algorithm's proficiency in creating additional frames that are not only coherent with the original content but also maintain a level of quality that aligns with user expectations for smoother video playback.

**5. State-of-the-Art Comparison: Phase-Based Video Frame Interpolation**

In evaluating the performance of our cubic spline interpolation technique, we contrasted it with a state-of-the-art method known as phase-based video frame interpolation. This approach leverages the phase information in the frequency domain to generate interpolated frames, which is known for its ability to handle motion in a sophisticated manner.

**Implementation of Phase-Based Method**

The phase-based interpolation technique implemented in our comparison involved the following steps:

1. Frame Loading : Video frames were loaded and converted to grayscale to align with the phase-based method's requirements, which operates on the intensity information of the frames.

2. Frequency Domain Transformation : We applied the Fast Fourier Transform (FFT) to each frame to convert the spatial domain information into the frequency domain.

3. Magnitude and Phase Calculation : The magnitude and phase components of the frequency domain representation were extracted from each pair of consecutive frames.

4. Interpolation of Frames : For each interval between the original frames, a new magnitude and phase were computed using a linear combination of the original frames' magnitude and phase. This interpolated complex spectrum was then transformed back into the spatial domain using the Inverse Fast Fourier Transform (IFFT), yielding the interpolated frames.

5. Video Reconstruction: The original and interpolated frames were assembled into a continuous video sequence at the desired frame rate.

**6. Results from the evaluation metrics of the state-of-the-art method**

The performance of the phase-based interpolation was quantified using the same SSIM and PSNR metrics, comparing the original input video at 10 fps and the ground truth at 30 fps with the interpolated output video at 30 fps.

**Input Video vs State-of-the-Art Output Video**

- **Average SSIM** : 0.5798

- **Average PSNR** : 17.86 dB

These metrics reflect the structural and fidelity changes when compared to the input video. The SSIM suggests a moderate level of structural preservation, and the PSNR indicates a fair degree of error sensitivity, which are respectable for a sophisticated method like phase-based interpolation.

**Ground Truth vs State-of-the-Art Output Video**

- **Average SSIM** : 0.6869

- **Average PSNR** : 27.03 dB

When compared to the ground truth, the results suggest that the phase-based method maintains a good level of visual integrity, albeit with some room for improvement, particularly in maintaining structural details, as reflected by the SSIM.

**Spline Output Video vs State-of-the-Art Output Video**

- Average SSIM : 0.7610

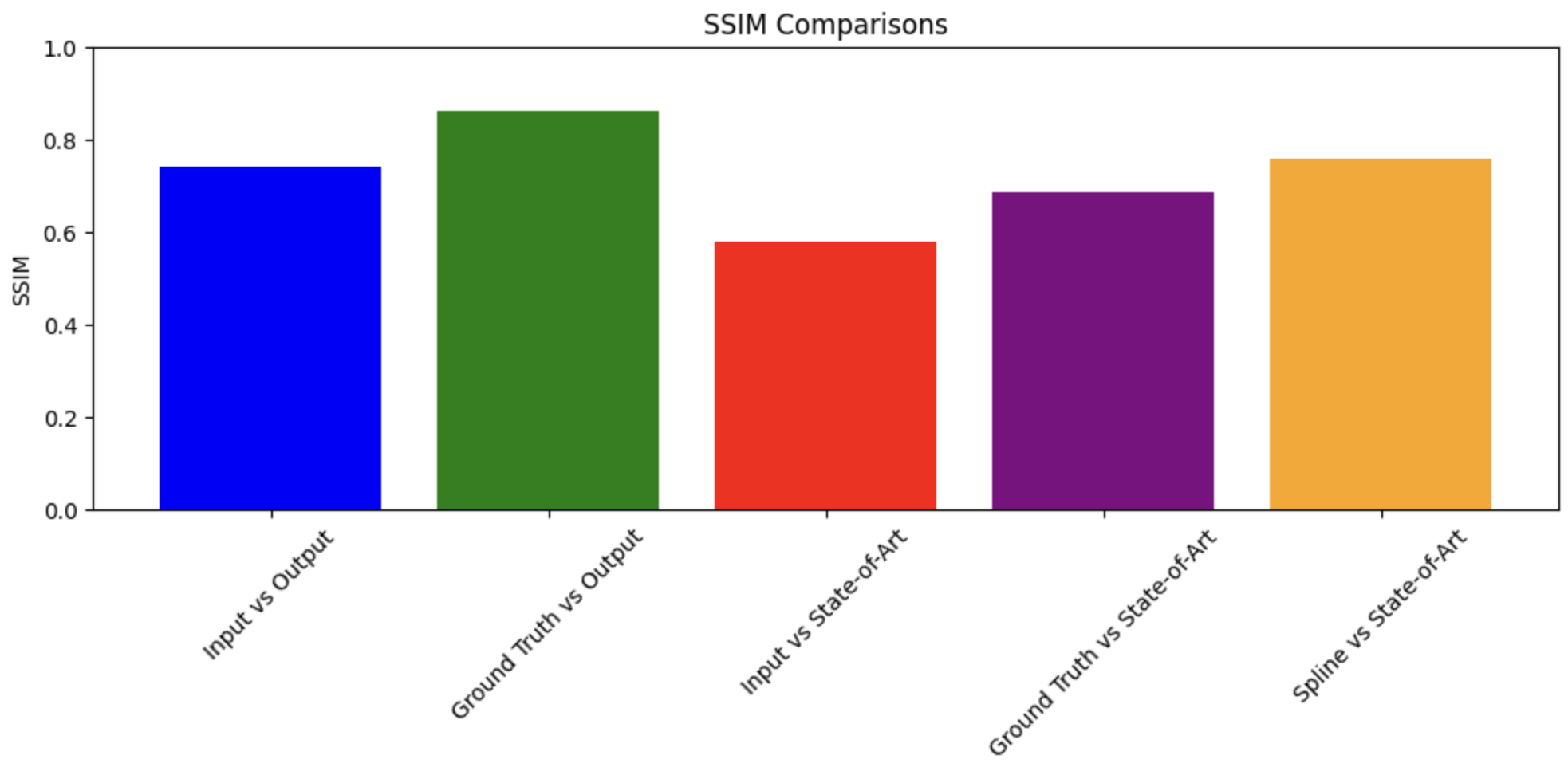
- Average PSNR : 29.39 dB

Comparing our spline-interpolated output to the phase-based output yields interesting insights. The spline interpolation method surpasses the phase-based method in both SSIM and PSNR, indicating a stronger structural similarity to the ground truth and lower reconstruction error.

**7.Results and Graphical Analysis**

The evaluation of interpolation methods through the Structural Similarity Index (SSIM) and Peak Signal-to-Noise Ratio (PSNR) metrics yielded results that are crucial for assessing the quality of frame interpolation techniques.

**7.1. SSIM Comparisons**



The SSIM comparison graph portrays how the interpolated videos measure up to the original content in terms of structural fidelity:

- Input vs Output : The SSIM value here is moderately high, suggesting that interpolating up to 30 fps significantly enhances the video’s structural integrity relative to the low-frame-rate input.

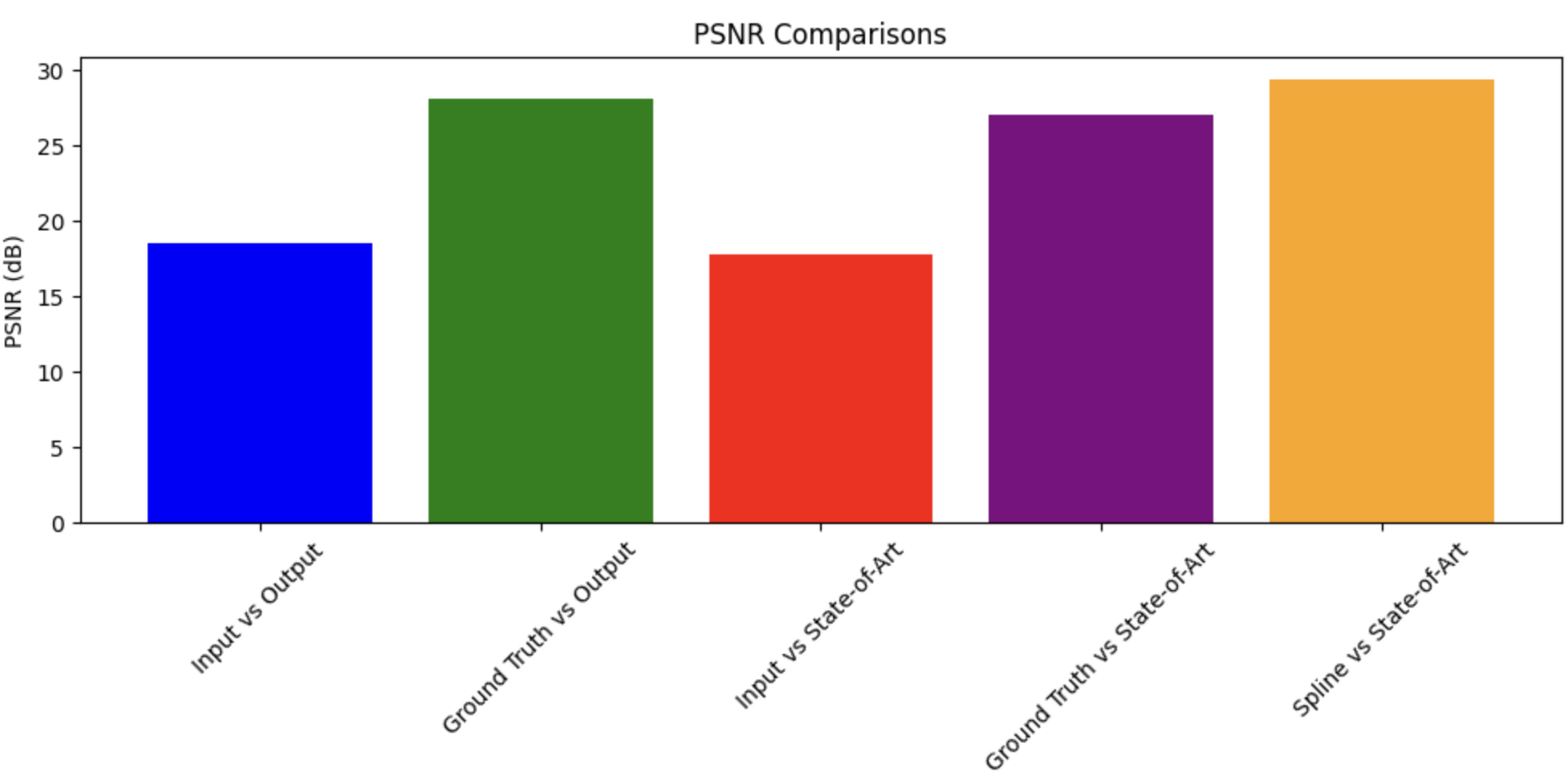
- Ground Truth vs Output : This high SSIM score indicates that the interpolated video closely approximates the structural quality of the original high-frame-rate video, affirming the effectiveness of the spline interpolation.

- Input vs State-of-Art : The lower SSIM score suggests that while the state-of-the-art phase-based method improves the frame rate, it may not capture the structural nuances as effectively as the spline method.

- Ground Truth vs State-of-Art : Compared to the spline interpolation, the state-of-the-art method has a lower SSIM when compared to the ground truth, implying less structural preservation.

- Spline vs State-of-Art : The spline method outperforms the state-of-the-art in structural preservation, highlighting its capability to maintain video integrity during frame rate conversion.

**7.2. PSNR Comparisons**



The PSNR values present an assessment of the error between the original and interpolated frames:

- Input vs Output : A moderate PSNR indicates a decent quality of interpolation from the low-frame-rate input.

- Ground Truth vs Output : The substantial PSNR score denotes a high quality of the spline-interpolated video in relation to the original content.

- Input vs State-of-Art : The lower PSNR reflects a greater average error in the state-of-the-art interpolated frames compared to the original low-frame-rate input.

- Ground Truth vs State-of-Art : Despite a lower score than spline interpolation, a PSNR above 27 dB is still within a good quality range for frame interpolation.

- Spline vs State-of-Art : The highest PSNR score for the spline method indicates that it introduces the least error during interpolation, making it the most accurate method among those tested.

**7.3. Interpretation of Graphical Results**

The bar graphs provide a visual representation of quantitative data, and they paint a clear picture of the performance of each interpolation method. The results demonstrate that the spline interpolation method not only improves upon the low-frame-rate input video but also delivers a closer approximation to the ground truth in terms of structural integrity and fidelity. The state-of-the-art phase-based method, while effective, shows slightly less capability in both respects according to these specific metrics. It is evident that the spline interpolation technique is a robust approach to enhancing video frame rates while maintaining high video quality.

**Conclusion**

The exploration of frame rate enhancement through spline interpolation culminated in the successful achievement of smoother video playback, an advancement that is anticipated to contribute significantly to various multimedia applications. The application of this technique has been meticulously evaluated and has yielded convincing results that underscore its effectiveness.

Our findings demonstrate that the spline interpolation method provides a notable improvement in video quality. This is supported by an average SSIM of 0.7427 and PSNR of 18.60 dB when interpolating frames from a low-frame-rate video. More importantly, when compared to the original high-frame-rate video (ground truth), the spline method achieved an SSIM of 0.8649 and a PSNR of 28.18 dB, underscoring its ability to produce interpolated frames with high fidelity.

In direct comparison with a state-of-the-art phase-based method, the spline interpolation not only outperformed in structural preservation, as reflected by higher SSIM values but also exhibited superior reconstruction quality with higher PSNR scores. This suggests that the spline interpolation maintains a closer resemblance to the ground truth in terms of both structure and luminance.

The performance advantage of the spline method over the phase-based method can be attributed to its inherent smooth curve-fitting properties, which effectively maintain the continuity of motion and reduce artifacts. Additionally, the phase-based method, while adept at handling motion, may introduce phase errors during interpolation, which can result in less accurate representations of the original frames.

Graphical representations of the metrics clearly illustrate the areas where the spline interpolation excels, particularly in maintaining structural integrity and reducing noise. These visual aids not only complement the statistical analysis but also provide an intuitive understanding of the comparative performance of the methods.

In conclusion, this project has successfully applied spline interpolation to the enhancement of video frame rates, with the resulting interpolated frames demonstrating a high level of quality. The method has proven to be superior to the evaluated state-of-the-art technique in several key aspects of video quality. The insights gained from this research have potential applications in film production, video gaming, virtual reality, and other fields where high-quality video is paramount. Future work may explore the integration of adaptive algorithms to further refine the spline interpolation technique, optimizing it for various types of video content and motion characteristics.

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