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Innovative Applications of O.R.

A modified Duckworth–Lewis method for adjusting targets in interrupted limited overs cricket

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ABSTRACT

In this paper we present a modified Duckworth/Lewis method. The key modification is an improved functional form for the model describing the runs to be scored in an innings. In the course of our work we compare several alternative methods for resetting targets in limited overs cricket that have been proposed in the literature and conclude that the Duckworth/Lewis method is the most viable. Our analysis also suggests that it is reasonable to use a single method for both the 50-over and 20-over formats of the game.

1. Introduction

Cricket is a hugely popular sport around the world. An estimated three billion people are cricket fans, a figure that is larger only for soccer, which has an estimated 3.5 billion fans (www.digalist.com). In recent years cricket's governing body, the International Cricket Council (ICC), has sought to make cricket even more popular. In order to achieve this, one strategy the ICC has adopted is to introduce Twenty20 (T20), a shorter format of the game, with the intention of making cricket a faster, more exciting spectacle that might attract a new audience.

Broadly speaking cricket can be played in two formats: limited overs and non-limited overs games. Non-limited overs matches at the professional level typically last for several days. For example, in the case of international games between major cricket playing countries, a 'test match' lasts for five days. Limited overs matches on the other hand, are designed to start and finish on the same day. For example, One Day Internationals (ODI) are limited to 50 overs per side, whilst T20 matches are limited to 20 overs per side and are the shortest format of international cricket, with matches typically lasting for three hours, bringing the game closer to the time span of other popular spectator sports.

In comparison to other sports, limited overs cricket is particularly vulnerable to inclement weather – when it rains, or becomes too dark, cricket becomes too dangerous to play. As a consequence, when a ODI or T20 match is interrupted by rain or bad light, either or both of the competing teams can often not complete their allotted overs. Incomplete games are unsatisfactory for the players and fans alike and, to some extent negate the purpose of the shorter formats since an abandoned match offers minimal levels of excitement. Furthermore, to enable knockout tournament play, such as the ODI and T20 World Cups, games must reach a conclusion. Therefore, the cricket authorities have adopted quantitative methods to adjust scores and reset targets in matches when one or both sides cannot complete their allotted overs in order to ensure interrupted matches are concluded and a definite result is obtained.

Since the first limited overs match was played in 1962, cricket analysts have searched for a fair method to reset targets in interrupted matches. The issue was elevated to higher importance following the introduction of the ODI World Cup in 1975. Several methods have been tried by the ICC. The current method, the Duckworth-Lewis (D/L) method (Duckworth and Lewis, 1998) is now widely accepted as the fairest method available and has been in operation since 1997.

Several academic papers have appeared attempting to improve upon the D/L method and these can be split into two categories: resources based methods and probability-preserving based methods. Possibly the highest profile alternative is the VJD method of Jayadevan (2002) which can be interpreted in terms of resources. Stern (2009) proposes changing the resources table of the D/L method in the second innings to better reflect how teams batting second are able to adopt a different strategy from the team batting first. Bhattacharya et al. (2011) present an alternative resources table for the D/L method based on a non-parametric approach for T20 cricket. Preston and Thomas (2002) were the first authors to present a method for adjusting targets that preserves the probability of

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victory for each team as it stood before the interruption took place. Carter and Guthrie (2004) follow a similar ethos and present algorithms to preserve the probability of victory for several interruptions during a game. We do not consider a probability-conservation method as this methodology was found to give contradictory results in Duckworth and Lewis (2005). Further, it has never been adopted by the ICC and so we choose to seek improvements to the current methodology.

In this paper we first present a method for estimating the D/L model parameters and then show that the D/L method is superior to other proposed resources-based methods for resetting targets in cricket. We then examine the scoring patterns in T20 Internationals (T20Is) and the last 20 over of ODIs and conclude that, as Duckworth and Lewis do, it is reasonable to use a single model for both ODI and T20I formats. Next, we present a modified D/L method which provides a superior fit to data and proves to better reflect the run scoring pattern observed in limited overs cricket. Finally we present an empirical justification for the adjustment of our modified D/L model in the case of high scoring matches. In Section 2 we present the current D/L method and a method to estimate the parameters of the model. In Section 3 we describe our data and compare the runs scoring pattern of the two formats, ODI and T20I. Comparison of the D/L method with other resources-based methods for resetting targets is presented in Section 4. Section 5 describes our modified D/L model and we conclude with some closing remarks in Section 6.

2. The D/L method

The D/L method has been through two incarnations. The first was adopted by the ICC in 1997 and is described in Duckworth and Lewis (1998). The second version, known as the Professional Edition, was introduced in 2003 (see Duckworth and Lewis, 2004) so that the method produced fairer adjusted targets in high scoring interrupted games.

The fundamental idea behind the D/L method is to estimate the resources available, R, to each team. In an uninterrupted match each team will have 100% of its resources available and no target adjustment is necessary. However, if there is an interruption and the resources of team 1, R_1, are not equal to team 2's resources, R_2, then the target for team 2 must be adjusted. Let S be the total runs scored by team 1 (the team batting first), then the D/L method states that the par score for team 2 (the team batting second) T, is given by

$$\begin{aligned} T &= SR_2/R_1 && \text{if } R_2 \leqslant R_1, \\ T &= S + G(N)(R_2 - R_1) && \text{if } R_2 \geqslant R_1, \end{aligned} \tag{1}$$

where $G(N)$ is the average first innings total number of runs in an N-over match (N is typically either 50 or 20). The target for team 2 is then the next integer above T.

2.1. The D/L model

To estimate the resources available to a team, the D/L method uses a model of the average runs remaining to be scored, Z. The D/L model for the average runs in the remaining u overs and with w wickets lost is given by

$$Z(u,w) = Z_0F(w)[1 - \exp\{-bu/F(w)\}], \tag{2}$$

where Z_0 is the asymptotic average runs with no wickets lost in an infinite number of overs. $F(w)$ is a positive decreasing step function with $F(0) = 1$ and is interpreted as the proportion of runs that are scored with w wickets lost compared with that of no wickets lost and, hypothetically, infinitely many overs available. That is, $F(w) = \lim_{u\to\infty} Z(u,w)/Z(u,0)$. The ratio

$$P_N(u,w) = Z(u,w)/Z(N,0) \tag{3}$$

gives the average proportion of runs still to be scored in an innings with u overs remaining and with w wickets lost, which Duckworth and Lewis present as the proportion of remaining resources. For brevity, we refer to this as remaining resources, although strictly speaking it is a proportion.

Duckworth and Lewis (2004) modified the original 1998 model for high scoring matches. The idea being that the resources remaining, for a given number of wickets lost, decrease linearly when a team is chasing a well above average target. In other words, each over has equal value and so the distribution of runs scored per over tends to be uniform, provided that the number of wickets lost remains the same. For this purpose they include an extra parameter which they call the match factor and is denoted by λ. In matches with well above average targets, the parameter λ scales down the rate parameter b. As a result Z tends to linearity with respect to u. The D/L upgraded model is given by

$$Z(u,w|\lambda) = Z_0F(w)\lambda^{n(w)+1}[1 - \exp\{-bu/\lambda^{n(w)}F(w)\}], \tag{4}$$

where $n(w)$ is a positive decreasing function with $n(0) = 5$. Strictly speaking, we should not be conditioning only on λ, but to distinguish Z in Eq. (4) from Z in Eq. (2) we follow this notation of Duckworth and Lewis and continue with it throughout the paper. In innings i ($i = 1, 2$), following n_i interruptions (the jth interruption stops play when u_{1j} overs remain and w_j wickets have been lost and play is resumed when u_{2j} overs remain), the resources available is given by

$$R_i = 1 - \sum_{j=1}^{n_i}(P_N(u_{1\,j}, w_j|\lambda) - P_N(u_{2\,j}, w_j|\lambda)). \tag{5}$$

2.2. Estimation of the D/L model

The parameters to be estimated are Z_0, b, $F(w)$, $n(w)$ and λ, and estimation can be done in two stages. In the first stage Z_0, b, and $F(w)$ are estimated from Eq. (2). In the second stage we first need a functional form for $n(w)$. Duckworth and Lewis (2004) does not reveal the functional form of $n(w)$. However, our experimentation with Tables 1 and 2 in Duckworth and Lewis (2004) revealed that $n(w) = \alpha + \beta F(w)$, where $\alpha = 2$ and $\beta = 3$. This functional form was confirmed to be that used in the D/L method in correspondence with Tony Lewis and Frank Duckworth. $F(w)$ is estimated non-parametrically with two constraints: $F(w) \geqslant F(w+1) > 0$ and $F(0) = 1$. Finally, λ is estimated on a match-by-match basis, after the first innings has been completed in a game.

The estimation methodology used by Duckworth and Lewis remains unknown, so we now describe the approach adopted here to fit both the D/L model and our modified D/L model. We first describe how we estimate Z_0, b and $F(w)$ and then describe estimation of λ.

2.2.1. Estimation of Z_0, b and $F(w)$

In correspondence with Tony Lewis and Frank Duckworth, it was revealed that $F(w)$ was first estimated from data, and then subjectively smoothed to produce more intuitive behaviour of $Z(u,w)$. Following advice from Duckworth and Lewis, we keep these values, as this is how the model is adopted in practice (these values of $F(w)$ are given later in Table 2 below).

Let $x_i(u,w)$ be the observed runs scored in the remaining u overs of the first innings of match i when w wickets have been lost. Similarly, let $\bar{x}(u,w)$ be the observed mean runs scored in the remaining first innings. First innings, and not second innings, data is used here (as in Duckworth and Lewis) because the scoring pattern in the second innings will be affected by the target set in the first innings. The first innings run scoring pattern represents the true scoring pattern of a team trying to maximise its runs total, rather than a team trying to score enough runs to meet a target and win a game. To estimate Z_0 and b in Eq. (2), we minimise a weighted sum of squared errors, *WSSE*, given by

$$\begin{aligned} WSSE &= \sum_u \sum_w k(u,w) e^2(u,w) \\ &= \sum_u \sum_w k(u,w) (Z(u,w) - \bar{x}(u,w))^2, \end{aligned} \tag{6}$$

where $k(u,w)$ is a weighting function that is intended to account for two characteristics of $\bar{x}(u,w)$: first, there is heteroskedasticity present, in that the standard deviation of the mean runs remaining to be scored becomes smaller as the number of overs remaining decreases, and second, some of the observations are more likely to occur when a very strong team plays a weak team. For example, the situation when there are forty overs remaining and five wickets lost happens more frequently when a poor team is batting against a strong bowling attack. Thus, weighting this observation equally with more usual match situations is likely to result in biased parameter estimates.

We propose to weight the observations using a weighting function $k(u,w)$, given by

$$k(u,w) = \frac{\sqrt{n(u,w)}}{s(u,w)}, \tag{7}$$

where n is the number of data points and s is the standard deviation of the remaining runs in the innings. Further, for k to be finite and $\bar{x}(u,w)$ reliable, we discarded means calculated using fewer than five observations.

2.2.2. Estimating λ

Duckworth and Lewis (2004) estimate λ when team 1 scores well above average runs. For below average team 1 scores, $\lambda = 1$. λ depends only on team 1's score, S, the number of overs allotted before team 1 starts its innings, N, and α and β. In a match in which team 1's innings is uninterrupted, λ is estimated such that,

$$g(\lambda) = |Z(N,0|\lambda) - S| = 0. \tag{8}$$

However, if team 1 faces m interruptions then λ is optimised by minimising the following function

$$g(\lambda) = \left| Z(N,0|\lambda) - \sum_{i=1}^{m} \Delta S_i - S \right|, \tag{9}$$

where ΔS_i is the expected runs lost in the ith interruption and can be defined as

$$\Delta S_i = Z(u_{1i}, w_i|\lambda) - Z(u_{2i}, w_i|\lambda). \tag{10}$$

Although Duckworth and Lewis (2004) modifies the method and provides an algorithm for estimating the λ, it does not provide any statistical evidence to justify that this D/L *Professional Edition* is an improved version. We examine the issue of whether adjusting the model for high scoring matching improves the model fit in Section 5.3. Further, a computer programme CODA, only available to cricketing authorities, is required for the λ estimation in a given match. We developed R code (R Core Team, 2012) for optimising λ for any given type of interrupted match.

3. Data: ODI and T20

Estimation of the parameters was facilitated by collecting over-by-over data on 463 ODI uninterrupted matches from January 2008 to October 2011, and 198 uninterrupted T20I matches from the start of these games in February 2005 to September 2011. The data were obtained from the ESPN cricinfo website (www.cricinfo.com). Purpose written code was used to estimate all parameters of the model using standard optimisation routines in R (R Core Team, 2012).

Table 1 gives an extract of the average runs remaining to be scored with u overs remaining and when w wicket have been lost, denoted by $\bar{x}(u,w)$, for T20Is (left panel) and for ODIs (right panel). Some matches in our original data set were reduced to shorter matches before the first innings started. We include these matches in our estimation sample as the match was not interrupted during play. As such, the sample sizes for the start of the innings given in Table 1 are 458 (not 463) for ODI and 191 (not 198) for T20I.

Table 1
For each given u and w, the table entries give observed means, standard deviations and number of cases in the remaining innings for T20 Internationals (left panel) and ODIs (right panel).

u	w					u	w				
	0	1	3	5	7		0	1	3	5	7
T20I (February 2005–September 2011)						*ODI (January 2008–October 2011)*					
20	151.79	★	★	★	★	50	245.43	★	★	★	★
	34.01	★	★	★	★		63.09	★	★	★	★
	191	0	0	0	0		458	0	0	0	0
15	128.45	115.24	106.33	67.00	★	40	236.26	202.89	147.09	130.67	★
	27.98	28.25	32.32	★	★		38.39	49.99	52.98	24.91	★
	47	79	15	1	0		109	178	44	3	0
10	90.38	88.50	82.16	45.00	58.00	30	189.64	184.95	154.19	99.40	67.25
	17.85	21.84	20.02	21.13	★		26.01	40.18	40.82	40.42	45.10
	13	30	49	12	1		25	81	100	30	4
5	46.50	57.83	47.76	45.81	29.42	20	145.29	143.38	121.84	96.76	65.09
	17.68	14.74	17.17	11.55	11.09		24.34	30.49	29.32	34.19	31.11
	2	6	41	48	12		7	32	114	62	23
2	★	34.00	33.16	27.52	23.65	10	91.00	87.00	85.81	68.18	46.93
	★	9.64	11.92	9.39	11.33		4.24	10.47	19.39	19.82	24.43
	0	3	25	48	20		2	6	64	104	44
3	★	24.00	23.00	21.26	17.58	5	★	45.00	51.77	44.67	33.38
	★	2.83	9.89	7.52	7.63		★	2.83	11.71	15.17	16.26
	0	2	16	46	31		0	2	26	98	61
1	★	8.00	10.00	10.75	9.39	1	★	★	12.14	10.02	10.39
	★	★	5.61	4.33	4.12		★	★	6.59	3.59	5.23
	0	1	8	36	36		0	0	7	50	75

We fit our model to the combined ODI and T20 data. To test for whether combining the data is reasonable for estimation purposes, we tested for equality in means in $\bar{x}_{ODI}(u,w)$ and $\bar{x}_{T20}(u,w)$. To do this, at each overs remaining (ranging from 20 to one), for each value of w (ranging from 0 to 9) we obtained 131 means for T20. Of these, we have data on 94 means for the corresponding ODI data. Performing 94 independent t-tests, produced just three statistically significant differences in means at the 5% level. To further justify combining the ODI and T20 data, we next made the Šidák correction to the significance level in order to take account of performing multiple independent tests on a data set and found that no cells were significantly different at an overall significance level of 0.05. We also performed the tests with the Bonferroni correction and again found no cells suggested a statistically significant difference in means at an overall significance level of 0.05.

We perform a second type of test to see whether it is reasonable to use one model for both ODI and T20 formats. We fit our model (see Section 5) separately to the T20I data and the corresponding data for the final 20 overs of ODIs. We calculate standard errors on the fitted parameters using bootstrapping and use a standard Z-test to test for a significant difference in the estimated parameters from the two models. Only one of the parameters proves to be significantly different in the two estimations and we take this as further evidence that it is reasonable to combine data from the two formats.

It seems there is little evidence of a difference between the scoring patterns in the two forms of the game and we believe the debate over whether the D/L method should be re-estimated for T20 is borne from the underlying model in Eq. (2) not being appropriate for both ODI and T20, rather than from some fundamental difference in the scoring patterns of the two formats.

In addition to the evidence provided by the statistical tests performed above, we believe it is more appropriate, in an idealistic sense, to have one model for resources in cricket, regardless of the format. For example, suppose a ODI match is reduced to 20 overs per side. If two models are in existence (one for ODI and one for T20), which model would best be suited? In this case, having one overall model for scoring patterns in cricket is more attractive than having separate models. Fig. 1 shows the $\bar{x}(u,w)$ curves for the combined data.

We estimate the D/L model parameters using our more recent data set. Some improvements are immediately gained by using

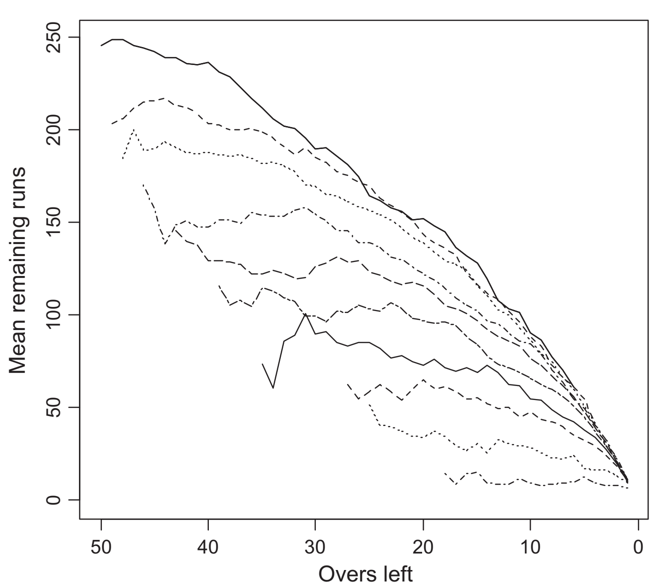

Fig. 1. $\bar{x}(u,w)$ for combined data. Top line is for no wickets lost, bottom line is for nine wickets lost.

these updated parameters. For example, the average runs scored in 50 over matches in our sample is approximately 245. Duckworth and Lewis state in their original paper (Duckworth and Lewis, 2004) that the average runs scored in the first innings, as implied by their model parameter estimates is 235 runs. However, refitting their original model to our updated data set we find the model implies the average runs to be 247 runs–closer to the observed average.

4. Comparing D/L method with previously proposed improvements

We now turn our attention to addressing the question of which resources-based method for resetting targets is best. Several methods have been proposed in the literature: the VJD method of Jayadevan (2002), Stern's (2009) adjusted D/L method and the D/L method for T20 presented in Bhattacharya et al. (2011), and we compare each of these to the D/L method.

Resources based methods for resetting targets in limited overs cricket are perhaps best represented graphically and we begin this section by presenting the D/L method as a series of three graphs. To simplify matters, we present the case when team 1's first innings total, $S = Z(N,0)$, i.e. team 1 scored the average number of runs in an N over innings. This implies $\lambda = 1$ and the revised target for team 2, T, would be reduced by $Z(u,w)$ if the second innings is ended with u overs remaining and w wickets lost. In other words, team 2 is compensated with $Z(u,w)$ runs for the loss of the remaining u overs. Fig. 2a shows $Z(u,w)$ as a function of u, which we denote as Z_u, for $w = 0, 1, \ldots, 9$. Fig. 2b shows $Z(u,w)$, as a function of w, which we denote as Z_w, for $u = 50, 45, \ldots, 5, 1$. Fig. 2c shows $\Delta Z_u = Z(u,w) - Z(u-1,w)$, which gives the number of runs the team will be compensated with if the team is deprived of the $(N-u)$th over for $w = 0, 1, \ldots, 9$.

In general, the behaviour of these curves is as one would expect. Indeed, on visual inspection, the form of Fig. 2a seems to resemble the form of Fig. 1. The other curves also appear to be intuitive: Fig. 2b shows that the number of runs a team will be compensated with, decreases as wickets lost increases, and Fig. 2c shows that for a given number of wickets lost, the number of runs a team will be compensated with, if the next over is lost, increases as the innings progresses. We note here that the D/L model implies that the value of the next over increases exponentially as the innings progresses (for fixed w) giving the shape of the curves in Fig. 2c. We revisit this point in Section 5.2.

We now contrast the D/L method as depicted in Fig. 2 with the VJD method of Jayadevan (2002), Stern's (2009) adjusted D/L method and the D/L method for T20 presented in Bhattacharya et al. (2011).

Fig. 3 gives the three resources curves for Jayadevan's VJD method, as calculated from the algorithm and tables given in his 2002 paper. It is clear that the behaviour of the method is not as one would expect. First, Z_u, in Fig. 3a, is flat for large regions of an innings. The consequences of this are unattractive. For example, suppose a team is chasing an average target of 245 and has lost six wickets and the innings is ended after 10 overs. This team would be compensated with the same number of runs as a team chasing the same target which had also lost six wickets but could not play the final 20 overs. This implies that overs 10 to 30 contribute zero resources to the team's innings. Somewhat contradictorily, if a team loses overs 10 to 30 and then play resumes, the VJD method compensates these overs with 26 runs. Further Fig. 3c shows how the VJD method is extremely erratic in respect of the runs awarded to a team for the loss of the $(N-u)$th over. It is clear from the curves displayed in Fig. 3, the VJD method produces contradictory reset targets and as such we believe it should not be considered as an alternative to the D/L method.

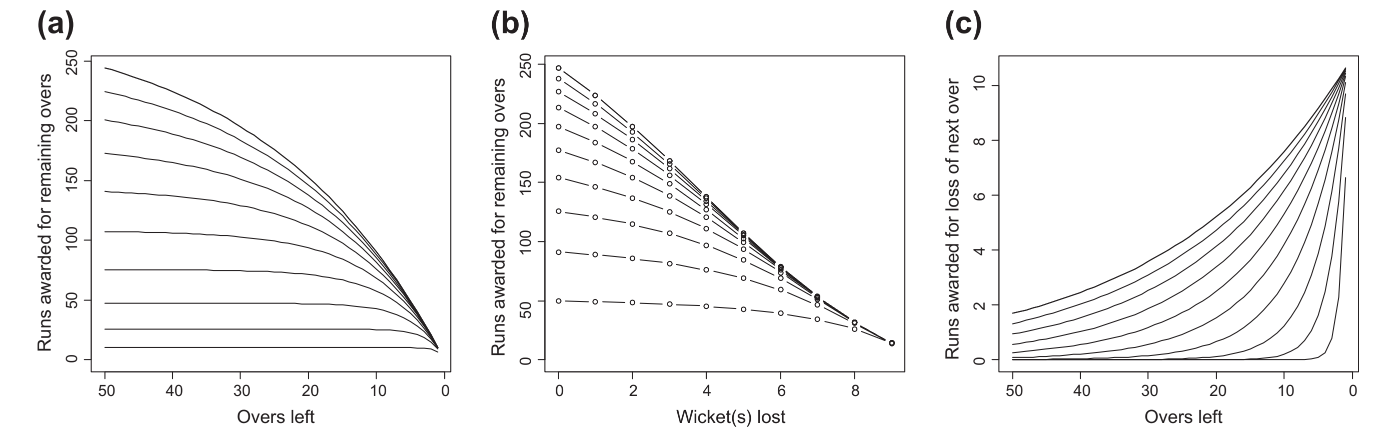

Fig. 2. D/L method presented graphically. (a) Z_u for $w = 0$ (top line), ..., 9 (bottom line), (b) Z_w for $u = 5$ (bottom line), ..., 50 (top line), (c) ΔZ_u for $w = 0$ (top line), ..., 9 (bottom line).

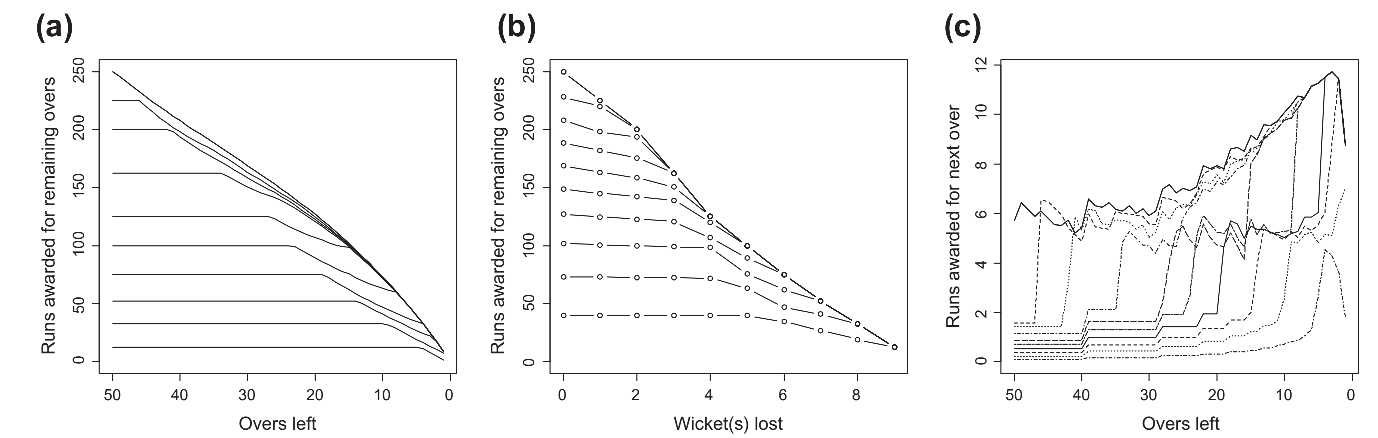

Fig. 3. Graphical representation of Jayadevan's VJD method for resetting targets in ODI cricket.

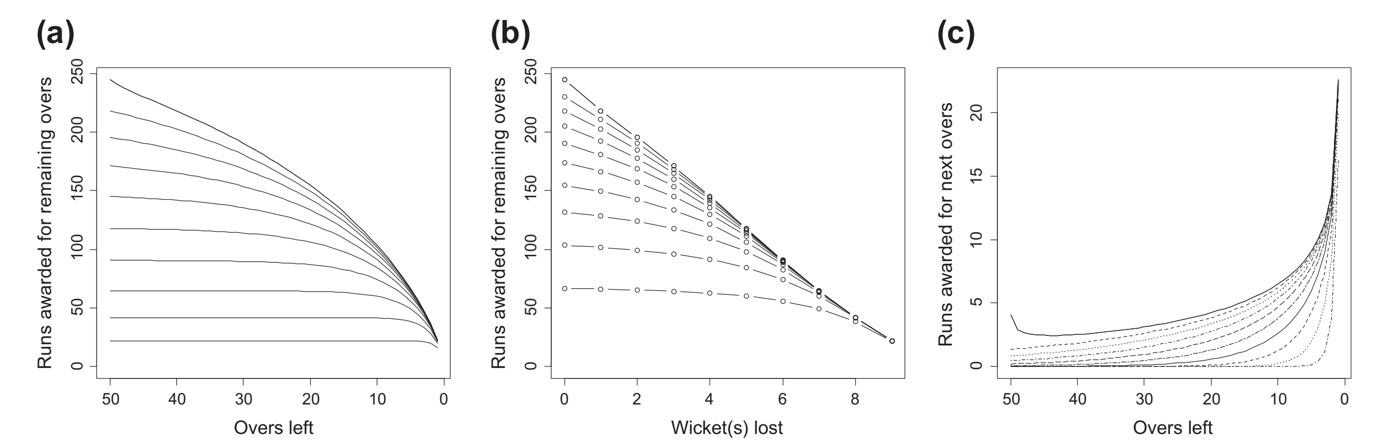

Fig. 4. Graphical representation of Stern's method for resetting targets in ODI cricket.

Fig. 4 depicts the method of Stern (2009) graphically as calculated using the algorithms given in his paper. The behaviours of Z_u and Z_w are as one would expect but Fig. 4c shows how the runs awarded for the loss of the $(N-u)$th over do not behave intuitively with respect to the over number. For example, the extremely rapid increase in the number of runs awarded for the loss of the $(N-u)$th over evident in the last few overs means that a team which has lost five wickets is compensated with 12.6 runs for the loss of the 49th over, and this rises to 22.4 runs for the loss of the final over. However, for five wickets lost, the observed average runs scored in the 49th and 50th overs are 10.6 and 10.3 respectively, based on 99 and 86 observations. We believe the D/L method better represents the scoring patterns evident in limited overs cricket.

Lastly, we show the three curves for the method of Bhattacharya et al. (2011) in Fig. 5, as calculated from the resources table given in their paper. This method does not recognise high or low scoring matches like the other three methods do. Moreover, the resources are estimated under two constraints: the resources must be non-decreasing with respect to overs remaining, and non-increasing with respect to wickets lost whilst no constraint is placed on the resources allocated to the loss of the $(N-u)$th over. As a consequence, the erratic behaviour shown in Fig. 5c results.

It is clear then that the D/L model behaves the more intuitively than these proposed alternatives.

5. A modified D/L model

Having demonstrated that the D/L method is superior to the VJD method, Stern's adjusted D/L method (Stern, 2009) and the semi-parametric D/L method of Bhattacharya et al. (2011), we now propose two improvements to the D/L model. First, we adopt an alternative estimation procedure for $F(w)$ and second we use a different model for $Z(u,w)$.

5.1. Estimation of F(w)

Duckworth and Lewis estimated $F(w)$ using nine parameters (one for each w, $w>0$) so that $F(w) \geqslant F(w+1) > 0$ and $F(0)=1$. However, this approach has unintuitive consequences on the value assigned to wicket partnerships, given by $\Delta Z_w = Z(u,w) - Z(u,w+1)$ for $u = 50, 45, \ldots, 5$. Fig. 6a shows ΔZ_w for the non-smoothed $F(w)$. It clearly results in an erratic pattern for the value of successive wickets. For example, until around the 45th over, the second wicket partnership is valued with fewer runs than the first and third wicket partnerships. Similarly, the fourth wicket partnership is valued with fewer runs than the third and fifth wicket partnerships. In communication with Duckworth and Lewis, they confirmed that their solution to this problem was to subjectively smooth $F(w)$. Our solution to this problem is to smooth $F(w)$ using

$$F(w) = \frac{\Phi(10;\mu_1,\theta_1) - \Phi(w;\mu_1,\theta_1)}{\Phi(10;\mu_1,\theta_1) - \Phi(0;\mu_1,\theta_1)} - \infty < \mu_1 < \infty, \quad \theta_1 > 0. \quad (11)$$

$\Phi$is the normal cumulative distribution function and μ_1, θ_1, and parameters to be estimated. Here $F(w)$ is a survival function based on a truncated normal distribution. We experimented with using several other functional forms based on the Cauchy, Gamma, Weibull and negative binomial distributions, however, (11) provided a superior fit to the data. We note that using a smoothed $F(w)$ does not improve the goodness-of-fit of the model, but of course, the main objective for using a smoothed $F(w)$ function (and the reason why Duckworth and Lewis subjectively smoothed $F(w)$) was not to improve the goodness-of-fit, but to produce a more intuitive and well-behaved ΔZ_w. This is evident from Fig. 6b which shows that as an innings progresses the runs value of each wicket partnership decreases. Further, unlike for the non-smoothed $F(w)$ in Fig. 6a, the relative importance of consecutive wicket partnerships changes smoothly and intuitively. Lastly, we note that using (11) means there is a reduction in the number of parameters to be estimated.

5.2. A model for Z(u,w)

Having identified an objective way to obtain a smooth $F(w)$ that produces a well-behaved function, we now propose an alternative functional form for $Z(u,w)$. We show that the new function provides an improved fit to the data, and show where this improvement is coming from.

Cumulative distribution functions provide a wide range of curves that can be used to model $Z(u,w)$, the runs remaining to be scored in an innings for a given number of wickets lost. This is because they are positive non-decreasing functions and the fundamental shape of Fig. 1 can be preserved. The curves need to be truncated at or above zero so that the domain is the positive real line. Before presenting our proposed model we first discuss a property of the runs scoring pattern implied by the exponential type curve of the current D/L method.

The decay towards the asymptotes, $Z_0F(w)$, is extremely rapid for (2). As a consequence, in some situations, particularly when a team has lost wickets in the early stage of the innings, losing overs provides very little (sometimes zero) compensation and hence the revised target will remain unchanged. For example, suppose a team is chasing a target of 250 and has lost six wickets after five overs. The existing D/L model provides no compensation (actually it provides 0.55 runs) to the team if it is deprived the next ten overs. With the advent of longer batting line-ups, it may be the case that this level of compensation is no longer reasonable.

We believe using an alternative to the exponential type function used for Z, specifically one that can allow for a slower decay towards the asymptotes will produce more acceptable reset targets. A slower decaying curve is tantamount to modelling Z using a probability distribution with a heavier tail.

We fitted many curves based on a wide range of distribution functions, including the Cauchy, the normal, the ex-Gaussian, the t-distribution, the gamma and the exponential (the current D/L model). The following model, based on the truncated-Cauchy distribution for Z (and the truncated-Normal distribution for $F(w)$) provided the best fit to the data.

We propose that the average number of runs scored in the remaining u overs once w wickets have been lost be given by

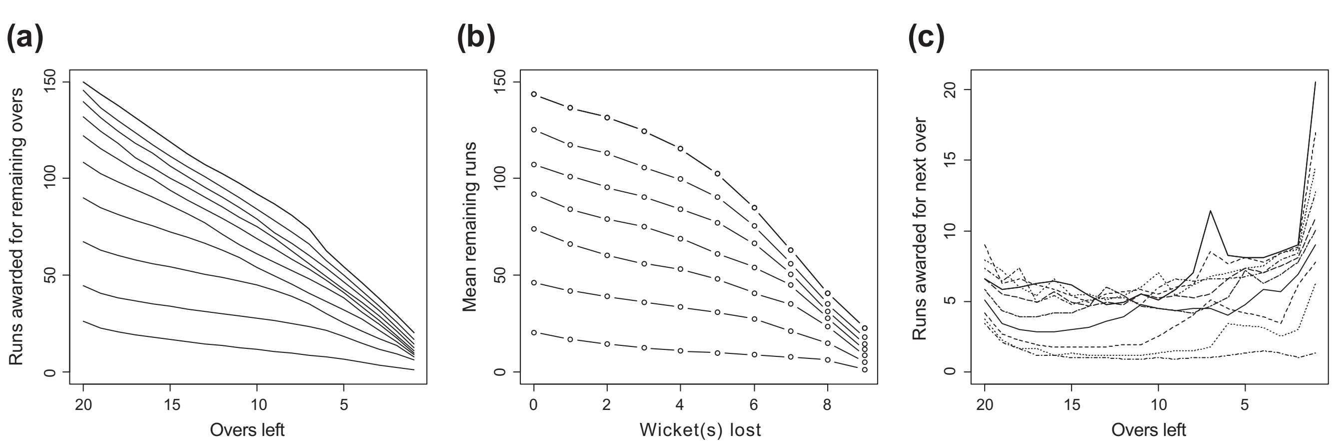

Fig. 5. Graphical representation of Bhattacharya et al. (2011) method for resetting targets in T20 cricket.

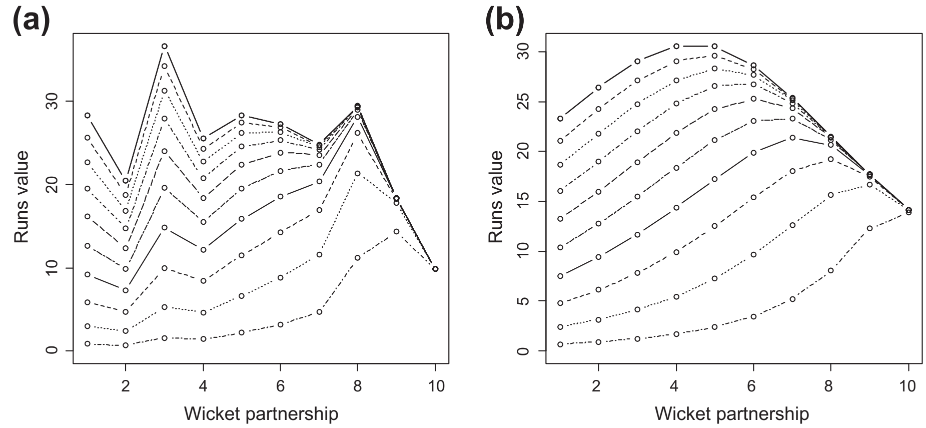

Fig. 6. Runs value allocated to wicket partnerships for a team batting second chasing an average target for (a) non-smoothed $F(w)$, and (b) smoothed $F(w)$. Top line is for 50 overs remaining and the bottom line is for five overs remaining.

Table 2
Fitted models and estimated parameters.

	D/L model, subjectively smoothed $F(w)$	D/L model, smoothed $F(w)$	Modified D/L model, smoothed $F(w)$
Z_0	288.6	291.9	340.0
θ_1		6.027	23.66
μ_1		0.896	−33.22
θ_0, (=1/b)	26.59	26.69	22.64
μ			−1.46
$F(1)$	0.884		
$F(2)$	0.760		
$F(3)$	0.630		
$F(4)$	0.500		
$F(5)$	0.375		
$F(6)$	0.262		
$F(7)$	0.164		
$F(8)$	0.089		
$F(9)$	0.035		
WSSE	2108.0	1735.9	1607.1

$$Z(u,w) = Z_0F(w)\left\{\frac{\tan^{-1}\left(\frac{u-\mu}{\theta_0F(w)}\right) - \tan^{-1}\left(\frac{-\mu}{\theta_0F(w)}\right)}{\frac{\pi}{2} - \tan^{-1}\left(\frac{-\mu}{\theta_0}\right)}\right\}, \tag{12}$$

where $F(w)$ is as defined in (11) above.

The D/L model in Eq. (2) has 11 parameters to be estimated (Z_0, b, and $F(1), \ldots, F(9)$). Our modified model has 5 parameters (Z_0, μ, θ_0, μ_1, and θ_1).

Table 2 shows the estimated parameters and the weighted sum of squared errors for three models: the D/L model with the subjectively smoothed values of $F(w)$, the D/L model with our smoothed $F(w)$, and our modified D/L model based on (12) with smoothed $F(w)$. In comparison with the original D/L model, our modified D/L model has a considerably lower weighted sum of squared errors. Interestingly, although the choice of the functional form of $F(w)$ changed the WSSE a great deal for the original D/L model, we found that it did not have such an effect on the WSSE for our modified D/L model.

The need for a heavier tailed distribution than the exponential can be seen in Fig. 7, which plots the observed mean remaining runs scored in an innings as the innings progresses, and the fitted lines according to the current D/L model and our modified D/L model. Although difficult to assess visually, there is some evidence that the exponential based model for $Z(u,w)$ is decaying too quickly, whereas the truncated-Cauchy distribution based model allows for slower decay for given w.

The rate of decay of the two models can be further analysed by examining the asymptotic behaviour of Eqs. (2) and (12) as the number of overs tends to infinity. For the D/L model fitted to our data, letting $u \to \infty$ produces an estimate of Z_0 = 291.9. This value represents the hypothetical score in an innings with unlimited overs (but restricted to the loss of 10 wickets, of course). By contrast, the corresponding value for the modified D/L method is 340. Thus, the modified D/L model implies an unlimited overs innings to have an average score 52 runs higher than the D/L model. These figures can be compared to the average first innings score

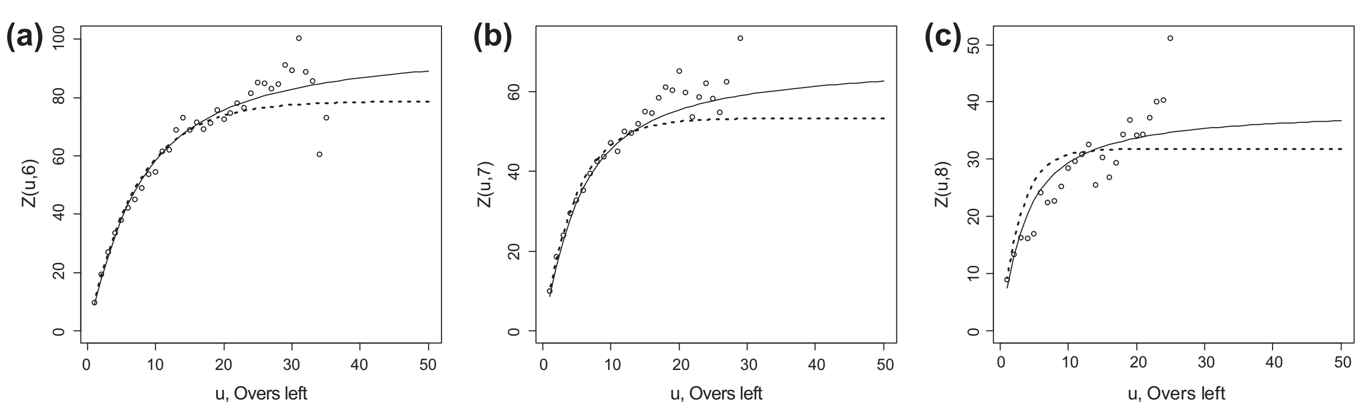

Fig. 7. $Z(u,w)$ with (a) w = 6, (b) w = 7 and (c) w = 8 for the exponential distribution based D/L model (dotted line) and the truncated Cauchy distribution based modified D/L model (solid line). The circles represent the observed values, denoted $\bar{x}(u,w)$ given in Table 1.

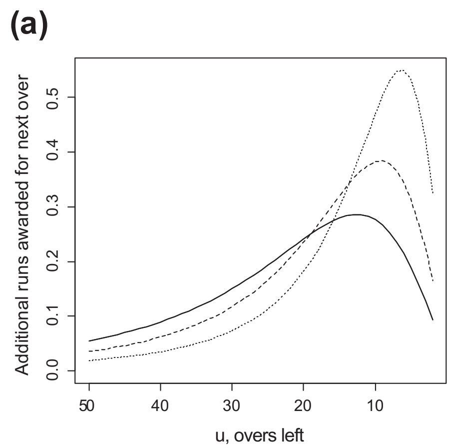

Fig. 8. $-\Delta^2 Z_u$ for (a) the modified D/L method and (b) the D/L method. The solid line is for $w = 0$, the dashed line is for $w = 2$ and the dotted line is for $w = 4$.

in Test matches, which, according to ESPNcricinfo.com, is 322 (all matches played between 1877 and 2007), a figure that is downwardly biased due to some innings being censored when the team declares. Of course, making this comparison is not completely appropriate for two reasons. First, a Test match is not absolutely unlimited (the game is restricted to a maximum of five days of play). Second, the conditions for play are not the same as for limited overs cricket. For example, wickets are prepared differently (to make batting conditions easier or more difficult), and the boundaries marking the edge of the field which defines the distance the ball must travel for the batter to score four or six runs off a single ball, are placed at different distances.

Fig. 8 shows $-\Delta^2 Z_u = \Delta Z_{u-1} - \Delta Z_u$, the extra number of runs a team is compensated with for the next over compared with the current over, for 0 (solid line), 2 (dashed line) and 4 (dotted line) wickets lost, for our modified model of $Z(u,w)$, Fig. 8a, and for the D/L model of $Z(u,w)$, Fig. 8b. We show Fig. 8 as the negative of $\Delta^2 Z_u$ so that the plot reflects the change in runs allocated to consecutive overs as the innings progresses.

As a consequence of the exponential type function in (2), $-\Delta^2 Z_u$ increases exponentially (Fig. 8b), regardless of the number of wickets lost and the number of overs remaining. This means that irrespective of the number of wickets lost, the batters are expected to score at an ever increasing run-rate. However, suppose a team has two overs remaining and has lost no wickets. The two batsmen are most likely already batting at maximum capacity and it seems unreasonable to expect them to score at an ever-increasing rate, which is what is implied by the current D/L model, if the resources allocated to overs is to increase at a constant rate. Some of the improved fit of (12) may be attributable to the rate of increase in the run-rate being able to slow down.

In contrast to Figs. 2–5 our modified D/L curves are presented in Fig. 9. The most notable difference between the D/L model (shown in Fig. 2) and our model is in Fig. 9c which shows how our model allows for the rate of increase of the single over resources to slow down for low wickets lost with few overs remaining as overs remaining decreases. Similarly, in the same figure, there is a slower decay that implies a heavier tail, which is especially evident when there are many wickets down in early stage of the innings.

5.3. Accounting for high and low scoring matches: estimation of a λ

Having developed a model which provides an improved fit to the data, and has more intuitive properties than the D/L model, we now incorporate an adjustment to the model that accounts for matches with well above average innings totals. Using the same notation of Duckworth and Lewis (2004), we introduce a parameter, λ, to our modified model.

As Duckworth and Lewis assume, in high scoring matches Z_u tends to become linear and hypothetically, the value of each wicket tends to zero. To understand this latter point, consider a match in which a team has 50 overs to bat and is chasing a target of 1800, so that six runs is required from each ball in the innings. This requirement is constant throughout the innings so that each ball has constant value to the batting team irrespective of the number of wickets lost.

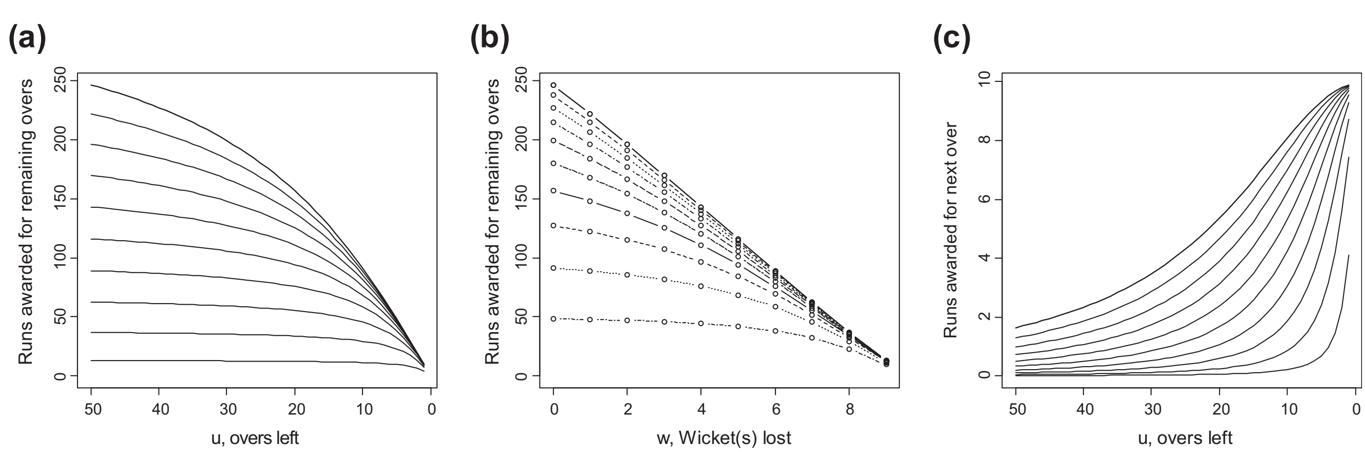

Fig. 9. Modified D/L method presented graphically. (a) Z_u for $w = 0, 1, \ldots, 9$, (b) Z_w for $u = 50, 45, \ldots, 5$, (c) ΔZ_u for $w = 0, 1, \ldots, 9$.

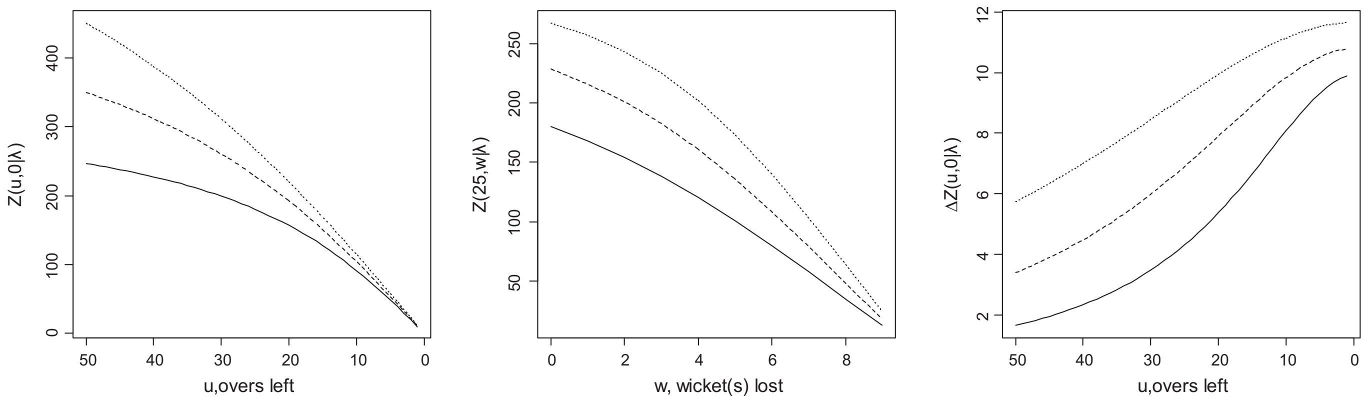

Fig. 10. (a) Z_u for $w = 0$, (b) Z_w for $u = 25$, (c) ΔZ_u for $w = 0$. Solid line is for $S = 246$ and $\lambda = 1$, the dashed line is for $S = 350$ and $\lambda = 1.087$ and the dotted line is for $S = 450$ and $\lambda = 1.172$.

In order to account for this effect, we scale up the parameters θ_0 and Z_0 in Eq. (9). This scaling allows each curve in Fig. 9a to become more linear accordingly. Hence, the model in Eq. (12) is altered to the following model given by

$$Z(u,w|\lambda) = Z_0\lambda^{n(w)+1}F(w)\left\{\frac{\tan^{-1}\left(\frac{u-\mu}{\theta_0\lambda^{n(w)}F(w)}\right) - \tan^{-1}\left(\frac{-\mu}{\theta_0\lambda^{n(w)}F(w)}\right)}{\frac{\pi}{2} - \tan^{-1}\left(\frac{-\mu}{\theta_0}\right)}\right\}. \tag{13}$$

Fig. 10 shows the visual demonstration of Z curves for different values of $\lambda(\geqslant 1)$.

The variation in the resources pattern with respect to S is intuitive, but Duckworth and Lewis do not give quantitative evidence that the addition of the λ parameter to their model results in an improvement to its performance. To show that the introduction of parameter λ significantly improves our modified D/L method, we do the following numerical experiment.

Consider a series of *ODI* matches, assume $Y_{i(u,w)}$ is team 1's runs scored when u overs are remaining and w wickets have been lost, where, $i = 1, 2, \ldots, n(u,w)$ and $n(u,w)$ is the number of data points we have (see Table 1). Suppose in the ith match, $\widehat{S}_i(u,w,\lambda)$ is the forecast runs, as predicted with u overs remaining and w wickets lost. We estimate λ at each $i(u,w)$ using the methodology described in Section 2.2.2. Using the modified D/L model, in Eq. (13), the projected runs is given by

$$\widehat{S}_i(u,w,\lambda_i) = Y_{i(u,w)}/\{1 - P_N(u,w|\lambda_i)\}, \tag{14}$$

where $P_N(u,w|\lambda_i)$ is the remaining resources defined in Eq. (3) but with Z calculated using Eq. (13). Now let S_i be the actual runs the team scored in the completed innings. For given u and w, the mean absolute error of this forecast is given by

$$MAE(u,w) = \sum_{i=1}^{n(u,w)} |\widehat{S}_i(u,w,\lambda_i) - S_i|/n(u,w). \tag{15}$$

To estimate the total error we have the weighted sum of mean absolute errors (*WSMAE*)

$$WSMAE = \sum_u \sum_w k(u,w)MAE(u,w), \tag{16}$$

Table 3
Goodness of fit measures for forecasted innings totals with and without λ.

Method	Modified D/L model without λ	Modified D/L model with λ	Number of matches	Number of forecasts
$WSMAE_{ODI}$	5311.1	3039.6	458	19,844
$WSMAE_{T20I}$	966.9	823.3	191	2844

where $k(u,w)$ is defined in Eq. (7). Table 3 gives *WSMAE* for ODI and T20I matches with forecasts based on the model with λ and without λ (which is equivalent to setting $\lambda = 1$).

The addition of the λ parameter clearly improves the forecasting power of the modified D/L model since the *WSMAE* is considerably lower for both ODI and T20I cricket.

Lastly, we note that in addition to the empirical evidence for using λ, such a modification corrects the shortcoming of revising a target by direct scaling as discussed in Duckworth and Lewis (1998). The example given by Duckworth and Lewis is of a team batting first scoring 80 runs for the loss of no wickets in 10 overs when play is interrupted and the match is reduced to ten overs per side. For this example, using Eq. (12), gives $R_1 = 0.0782$ and $R_2 = 0.3702$. Hence, the revised target $T = 80 \times 0.3702/0.0782 = 378.8$, an impossibly high target in just ten overs. As a result of this, Duckworth and Lewis (1998, 2004) suggest that in situations when team 2's available resources are greater than team 1's available resources, the revised target can be determined by $T = S + (R_2 - R_1)G(N)$. Since, in the given example, $R_2 > R_1$, therefore $T = 80 + (0.3702 - 0.0782) \times 245 = 151.4$.

We believe that the above shortcoming is not a consequence of using direct scaling, but rather, it is because no adjustment is made to the model to account for high scoring matches. For the above example our estimate of λ is 1.229553. Using this value in Eq. (13), gives $R_1 = 0.156, R_2 = 0.236$, and therefore, the revised target is $T = 80 \times 0.236/0.156 = 121.1$, which is a more acceptable revised target. Hence, given the addition of the λ parameter to take account of high scoring matches, there is no need to use the *ad hoc* scaling given in the second equation of Eq. 1. Thus, we use

$$T = SR_2/R_1 \quad \text{if } R_2 \leqslant R_1 \quad \text{or } R_2 \geqslant R_1, \tag{17}$$

to calculate par scores.

6. Conclusions

The Duckworth–Lewis method for adjusting targets in interrupted limited overs cricket matches is widely accepted as the fairest method available and is a great success story of Operational Research in practice. The D/L method is heavily scrutinised and academics continue to propose improvements and alternatives. We show here that the current D/L method has more attractive properties than the other proposed alternatives that rely on the concept of resources.

Next, we propose a modified D/L method which uses an alternative model for the runs remaining to be scored in an innings, $Z(u,w)$. We show that this model provides a superior fit to data and, in addition, has a possibly more intuitive behaviour in respect of the runs scoring pattern.

We provide statistical evidence for using a single model for resetting targets in both One Day International and Twenty20 cricket. Further, we show empirically that it is appropriate to take account of high scoring matches in the same way as Duckworth and Lewis do for their model, that is, the addition of the parameter λ.

We are happy to make our code available and invite interested readers to experiment with and test our model.

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