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Duckworth–Lewis and Twenty20 cricket

R Bhattacharya[1], PS Gill[2] and TB Swartz[1]*

[1]Simon Fraser University, Burnaby, British Columbia, Canada; and [2]University of British Columbia, Okanagan, Kelowna, British Columbia, Canada

Originally designed for 1-day cricket, this paper considers the use of the Duckworth–Lewis method as an approach to resetting targets in interrupted Twenty20 cricket matches. The Duckworth–Lewis table is reviewed and an alternative resource table is presented. The alternative table is constructed using observed scoring rates from international Twenty20 matches. A desideratum of a resource table is monotonicity in both the rows and columns corresponding to wickets and overs respectively. Consequently, a Gibbs sampling scheme related to isotonic regression is applied to the observed scoring rates to provide a non-parametric resource table. Taking into account the more aggressive batting style of Twenty20 compared to 1-day cricket, the resultant resource table is seen to possess sensible features. A discussion is provided concerning the use of the Duckworth–Lewis method applied to Twenty20.

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1. Introduction

When considering the substantial amount of research that has been directed toward the sporting world from a mathematical, statistical and operational research perspective, the Duckworth–Lewis method (Duckworth and Lewis, 1998, 2004) perhaps stands alone as the most significant contribution to sport. Prior to the adoption of the Duckworth–Lewis method, the resetting of targets in interrupted 1-day cricket matches was carried out via run rates. The difficulty with run rates is that targets are determined by taking the remaining overs into account, while ignoring the number of lost wickets. As is well known, batsmen tend to bat less aggressively and score fewer runs when more wickets have been taken. The Duckworth–Lewis method was utilized and gained prominence during the 1999 World Cup of Cricket, and since that time, it has been adopted by every major cricketing board and competition. In 1-day cricket, the Duckworth–Lewis method is based on the recognition that at the beginning of a match, each side has resources available (typically 50 overs and 10 wickets). When the match is shortened, the resources of one or both teams are reduced and the two teams usually have different resources for their innings. In this case, in an attempt to be fair, a revised target for the team batting second is set. The determination of the target using resources is known as the Duckworth–Lewis method. What makes the adoption of the Duckworth–Lewis method remarkable is that the method is widely perceived by the public as a black box procedure. Generally, people do not understand how the targets are set but they do agree that the targets are sensible or at least preferable to the approach based on run rates.

Historically, we note that although the Duckworth–Lewis method has gained worldwide prominence, there have been other attempts at establishing targets for interrupted 1-day matches. For example, for a brief period including the 1992 World Cup, the team batting second had its target reduced from the first innings total by n runs. The quantity n was determined as the number of runs scored in the first innings in the corresponding number of lost overs that had the least number of runs scored. This approach was immediately recognized as unfair and advantageous to the team batting first. Another short-lived approach was based on a modification of the previous system by further reducing the target by 0.5% for each over lost. It too was generally seen as advantageous to the team batting first.

There have been other proposals that have never been implemented. For example, Clarke (1988) developed a dynamic programming model where a target could be set such that the probability of winning prior to the interruption is equal to the probability of winning after the interruption. Christos (1998) proposed an alternative method based on run rates but where the number of wickets available is reduced proportional to the number of overs made available.

Correspondence: TB Swartz, Department of Statistics and Actuarial Science, Simon Fraser University, 8888 University Drive, Burnaby, BC V5A1S6, Canada.

E-mail: tim@stat.sfu.ca

Although the Duckworth–Lewis method was designed for 1-day cricket, it has also been applied to Twenty20 cricket. Twenty20 is a relatively new version of limited overs cricket with only 20 overs per side. In contrast to the 1-day game and first-class cricket (which can take up to 5 days to complete), Twenty20 matches have completion times that are comparable to other popular team sports. With the introduction of the biennial World Twenty20 tournament in 2007 and the Indian Premier League in 2008, Twenty20 cricket has gained widespread popularity.

Although Twenty20 cricket is similar to 1-day cricket, there exist subtle variations in the rules (eg fielding restrictions, limits on bowling, etc) between the two versions of cricket. The variations in the rules, and most importantly, the reduction of overs from 50 to 20 suggest that scoring patterns in Twenty20 may differ from the 1-day game. In particular, Twenty20 is seen as a more explosive game where the ability to score 4's and 6's is more highly valued than in 1-day cricket. Since the Duckworth–Lewis method (and its associated resource table) is based on the scoring patterns in 1-day cricket, it is therefore reasonable to ask whether the Duckworth Lewis method is appropriate for Twenty20. This is the focus of our paper.

With the rise of Twenty20, an investigation of the Duckworth–Lewis method applied to Twenty20 is timely. Up until this point in time, such an investigation might not have been possible due to the dearth of Twenty20 match results. We now have at our disposal nearly 100 international matches, and through the use of efficient estimation procedures, the question may be at least partially addressed. Also, since Twenty20 matches have a shorter duration, to date, very few matches have been interrupted and resumed according to Duckworth–Lewis. Consequently, if there is a problem with Duckworth–Lewis applied to Twenty20, it may not have yet manifested itself. Until a controversial outcome occurs, there may not be sufficient motivation to adjust the table.

In Section 2, we review the construction of the Duckworth–Lewis resource table and scale the table so that it is easily interpretable for Twenty20. Some comments are provided on aspects of the table. In Section 3, an alternative Twenty20 resource table is obtained using a non-parametric approach based on Gibbs sampling. The data used in the construction of the new table consist of all international Twenty20 matches to date involving nations from the International Cricket Council (ICC). We conclude with a short discussion in Section 4. A heat map is provided to facilitate comparisons between the two tables.

2. The Duckworth–Lewis resource table

In Table 1, we provide an abbreviated version of the Duckworth–Lewis resource table (Standard Edition) taken from the 2008–2009 ICC Playing Handbook found at www.icc-cricket.com. Note that in a full innings of 1-day cricket, a team begins batting with 100% of its resources available corresponding to 50 overs and zero wickets taken. As a simple example of the use of the Duckworth–Lewis resource table, consider a 1-day match where the team batting first scores 256 runs upon completion of its innings. It then rains prior to the resumption of the match. Due to the lost time, suppose that the team batting second receives only 30 overs for its innings. According to the resource table, the team batting second has only 75.1% of its resources available, and therefore its target for winning the match is set at $256(0.751) = 94$ runs. We contrast the Duckworth–Lewis target with the unreasonably low target of $256(30/50) = 154$ runs based on run rates.

Citing reasons of commerical confidentiality, Duckworth and Lewis (1998) provide only partial information concerning the construction of the resource table. However, they do disclose that the table entries are based on the estimation of the 20 parameters $Z_0(w)$ and $b(w)$, $w = 0, \ldots, 9$ corresponding to the function

$$Z(u, w) = Z_0(w)[1 - \exp\{-b(w)u\}] \tag{1}$$

Table 1 Abbreviated version of the Duckworth–Lewis resource table (Standard Edition)

Overs available	*Wickets lost*									
	0	1	2	3	4	5	6	7	8	9
50	100.0	93.4	85.1	74.9	62.7	49.0	34.9	22.0	11.9	4.7
40	89.3	84.2	77.8	69.6	59.5	47.6	34.6	22.0	11.9	4.7
30	75.1	71.8	67.3	61.6	54.1	44.7	33.6	21.8	11.9	4.7
25	66.5	63.9	60.5	56.0	50.0	42.2	32.6	21.6	11.9	4.7
20	56.6	54.8	52.4	49.1	44.6	38.6	30.8	21.2	11.9	4.7
10	32.1	31.6	30.8	29.8	28.3	26.1	22.8	17.9	11.4	4.7
5	17.2	17.0	16.8	16.5	16.1	15.4	14.3	12.5	9.4	4.6
1	3.6	3.6	3.6	3.6	3.6	3.5	3.5	3.4	3.2	2.5
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

The table entries indicate the percentage of resources remaining in a match with the specified number of wickets lost and overs available.

Table 2 The Duckworth–Lewis resource table (Standard Edition) scaled for Twenty20

Overs available	*Wickets lost*									
	0	1	2	3	4	5	6	7	8	9
20	100.0	96.8	92.6	86.7	78.8	68.2	54.4	37.5	21.3	8.3
19	96.1	93.3	89.2	83.9	76.7	66.6	53.5	37.3	21.0	8.3
18	92.2	89.6	85.9	81.1	74.2	65.0	52.7	36.9	21.0	8.3
17	88.2	85.7	82.5	77.9	71.7	63.3	51.6	36.6	21.0	8.3
16	84.1	81.8	79.0	74.7	69.1	61.3	50.4	36.2	20.8	8.3
15	79.9	77.9	75.3	71.6	66.4	59.2	49.1	35.7	20.8	8.3
14	75.4	73.7	71.4	68.0	63.4	56.9	47.7	35.2	20.8	8.3
13	71.0	69.4	67.3	64.5	60.4	54.4	46.1	34.5	20.7	8.3
12	66.4	65.0	63.3	60.6	57.1	51.9	44.3	33.6	20.5	8.3
11	61.7	60.4	59.0	56.7	53.7	49.1	42.4	32.7	20.3	8.3
10	56.7	55.8	54.4	52.7	50.0	46.1	40.3	31.6	20.1	8.3
9	51.8	51.1	49.8	48.4	46.1	42.8	37.8	30.2	19.8	8.3
8	46.6	45.9	45.1	43.8	42.0	39.4	35.2	28.6	19.3	8.3
7	41.3	40.8	40.1	39.2	37.8	35.5	32.2	26.9	18.6	8.3
6	35.9	35.5	35.0	34.3	33.2	31.4	29.0	24.6	17.8	8.1
5	30.4	30.0	29.7	29.2	28.4	27.2	25.3	22.1	16.6	8.1
4	24.6	24.4	24.2	23.9	23.3	22.4	21.2	18.9	14.8	8.0
3	18.7	18.6	18.4	18.2	18.0	17.5	16.8	15.4	12.7	7.4
2	12.7	12.5	12.5	12.4	12.4	12.0	11.7	11.0	9.7	6.5
1	6.4	6.4	6.4	6.4	6.4	6.2	6.2	6.0	5.7	4.4

The table entries indicate the percentage of resources remaining in a match with the specified number of wickets lost and overs available.

where $Z(u,w)$ is the average total score obtained in u overs in an unlimited overs match where w wickets have been taken. Although we readily admit the utility of the Duckworth–Lewis table in 1-day cricket, a number of questions arise based on (1) and the estimates found in Table 1:

- There are many parametrics curves that could be fit. Is (1) the best curve? Is there any advantage to a non-parametric fit?
- The function (1) is based on 1-day rules but pertains to unlimited overs cricket. Since 1-day cricket is limited overs cricket, is there an advantage in taking the structure of the 1-day game into account?
- How are the parameters estimated? If the 10 curves corresponding to $w = 0, \ldots, 9$ are fit separately, there are little data available beyond $u = 30$ for fitting the curve with $w = 9$. Also, the asymptotes for the curves with $w = 0, 1, 2$ (see Figure 1 of Duckworth and Lewis (1998)) fall beyond the range of the data.
- In Table 1, the last two columns have many identical entries going down the columns. Although very few matches occur under these conditions, is it really sensible for resources to remain constant as the available overs decrease? This is a consequence of the asymptote imposed by (1).

For ease of discussion, we find it convenient to convert the Duckworth–Lewis resource table to the context of Twenty20. Specifically, we truncate the resource table to 20 overs and we scale the entries so that an innings beginning with 20 overs and zero wickets corresponds to 100% resources. Table 2 gives the full Duckworth–Lewis resource table (Standard Edition) for Twenty20 where the entries are obtained by dividing the corresponding entry in Table 1 by 0.566 (the resources remaining in a 1-day match where 20 overs are available and zero wickets taken).

3. A new resource table for twenty20

In the construction of a resource table for Twenty20, it is important to consider the scoring patterns specific to Twenty20. For that reason, we consider all international Twenty20 matches involving ICC teams that have taken place from 17 February 2005 through 9 November 2009. There are 85 such matches and details concerning these matches are available from www.cricinfo.com. Note that we have excluded the four shortened matches where the Duckworth–Lewis method was applied. We have also excluded Twenty20 matches involving non-ICC nations as we prefer to consider matches of a consistently high standard.

In the study of Twenty20 scoring patterns, we consider first innings data only, as scoring patterns in the second innings are influenced by the number of runs scored in the first innings. In the development of a 1-day cricket simulator, Swartz *et al* (2009) consider batting behaviour in the second innings. Although match summary results are

Table 3 The matrix $R = (r_{ow})$ of estimated resources for Twenty20

Overs available	*Wickets lost*									
	0	1	2	3	4	5	6	7	8	9
20	100.0									
19	93.6	83.0	110.2							
18	90.4	85.8	78.3							
17	86.7	80.5	82.8	53.7						
16	81.7	74.5	81.9	70.7	32.8					
15	76.5	71.4	71.5	65.9	59.9					
14	68.3	69.1	67.6	66.2	58.4					
13	63.8	68.2	62.4	62.9	59.0	24.3				
12	62.1	62.3	60.6	57.3	58.8	44.1				
11	60.5	56.3	57.0	53.6	61.0	39.7				
10	57.6	49.6	52.1	52.8	48.1	38.6	41.0	35.2		
9	54.9	52.1	43.6	49.0	44.1	33.8	35.0	29.7		
8	51.0	46.4	41.7	42.2	41.2	36.7	27.5	28.7		
7	48.6	45.8	38.9	35.9	39.1	34.8	24.1	25.5		
6	54.0	37.9	36.6	30.3	36.2	31.3	20.9	21.4	26.7	
5		44.0	32.5	25.4	28.7	29.4	23.9	17.1	14.9	
4			28.2	23.4	22.5	22.2	20.9	14.3	10.6	6.7
3			20.6	19.9	16.9	17.8	15.8	12.4	7.6	1.2
2			21.2	17.6	11.9	13.4	10.6	11.0	7.2	1.4
1				8.7	5.2	7.3	6.0	5.5	6.0	2.6

Missing entries correspond to match situations where data are unavailable.

readily available from the Cricinfo website, our investigation requires ball-by-ball data, and for this, we have coded a Java script to parse the associated commentary log for each match. The script extracts the relevant details on a ball-by-ball basis, and stores the data in a tabular form for easy access.

For each match, define $x(u, w(u))$ as the runs scored from the stage in the first innings where u overs are available and $w(u)$ wickets have been taken until the end of the first innings. We calculate $x(u, w(u))$ for all values of u that occur in the first innings for each match beginning with $u = 20$ and $w(u) = w(20) = 0$.

We then calculate the matrix $R = (r_{uw})$ where r_{uw} is the estimated percentage of resources remaining when u overs are available and w wickets have been taken. We calculate (100%) r_{uw} by averaging $x(u, w(u))$ over all matches where $w(u) = w$ and dividing by the average of $x(20, 0)$ over all matches. The denominator is simply the average number of first inning runs over all matches. In the case of $u = 0$, we set $r_{uw} = r_{0w} = 0.0\%$. The matrix R is therefore a first attempt at a resource table for Twenty20 and is given in Table 3. Note that $r_{20,0} = 100\%$ as desired. Although R is a non-parametric estimate of resources and makes no assumptions concerning the scoring patterns in Twenty20, it is less than ideal. First, there are many table entries where there are missing data for the given situation. In addition, Table 3 does not exhibit the monotonicity that we expect. Logically, we require a resource table that is non-decreasing as we go from left to right along rows and we require a resource table that is non-decreasing as we go down columns. We also observe some conspicuous entries in Table 3, particularly the entry of 110.2% resources corresponding to 19 overs available and two wickets taken. This entry is clearly misleading and should be less than 100%. It arises due to the small sample size (two matches) corresponding to the given situation. For our non-parametric resource table (upcoming), we have found that the estimation procedure is robust to observations based on small sample sizes as the surrounding observations based on larger sample sizes have greater influence in the determination of the table. We have therefore retained conspicuous observations such as 110.2%. We view our investigation of Duckworth/Lewis in Twenty20 as one of discovery rather than an attempt to replace the Duckworth/Lewis table.

To impose the monotonicity constraints in the rows and columns, we refer to the general problem of isotonic regression. For our purposes, we consider the minimization of

$$F = \sum q_{uw}(r_{uw} - y_{uw})^2 \tag{2}$$

with respect to the matrix $Y = (y_{uw})$ where the double summation corresponds to $u = 1, \ldots, 20$ and $w = 0, \ldots, 9$, the q_{uw} are weights and the minimization is subject to the constraints $y_{uw} \geqslant y_{u,w+1}$ and $y_{u,w} \geqslant y_{u-1,w}$. In addition, we impose $y_{20,0} = 100$, $y_{0,w} = 0$ for $w = 0, \ldots, 9$ and $y_{u,10} = 0$ for $u = 1, \ldots, 20$.

Although the fitting of Y is completely non-parametric, there are some arbitrary choices that have been made in the minimization of (2). First, not only was the choice of 'squared error' discrepancy in (2) convenient for computation, minimization of the function F with squared error discrepancy corresponds to the method of constrained maximum likelihood estimation where the data r_{uw} are independently normally distributed with means y_{uw} and variances $1/q_{uw}$. Second, we choose to consider a matrix Y: 20×10 based on overs. Alternatively, we might have considered a larger matrix Y: 120×10 based on balls. We prefer the overs formulation as it involves less missing data and leads to a less computationally intensive optimization. With a matrix Y based on overs, it is possible to interpolate on a ball-by-ball basis if required. Third, we have made a simple choice with respect to the weights q_{uw}. We set $1/q_{uw}$ equal to the sample variance used in the calculation of r_{uw} divided by the sample size. The rationale is that when r_{uw} is less variable, there is stronger belief that y_{uw} should be close to r_{uw}.

In Table 4, we give a non-parametric resource table based on the minimization of (2). An algorithm for isotonic regression in two variables was first introduced by Dykstra and Robertson (1982). Fortran code was subsequently developed by Bril *et al* (1984). We have used an R code implementation that is available from the Iso package on the Cran website (www.cran.r-project.org). The programme requires 27 iterations to achieve convergence. What is unsatisfactory about Table 4 is that it suffers from the same criticism that was directed at the Duckworth–Lewis resource table. There is a considerable number of adjacent entries in Table 4 that have the same value. Again, it is not sensible for resources to remain constant as available overs decrease or wickets increase. The problem is that in the minimization of (2), various fitted y's occur on the boundaries imposed by the monotonicity constraints. Table 4 is also unsatisfactory from the point of view that it is incomplete; missing values correspond to match situations where data are unavailable.

To address the above criticisms, we now take a slightly different approach to estimation. As previously mentioned, we recognize that (2) arises from the normal likelihood

$$\exp\left\{-\frac{1}{2}\sum q_{uw}(r_{uw}-y_{uw})^2\right\}. \qquad (3)$$

We therefore consider a Bayesian model where the unknown parameters in (3) are the y's. A flat default prior is assigned to the y's subject to the monotonicity constraints. It follows that the posterior density takes the form (3) and that Gibbs sampling can be carried out via sampling from the full conditional distributions

$$[\,y_{uw}|\cdot\,] \sim \text{Normal}\,(r_{uw}, 1/q_{uw}) \qquad (4)$$

subject to the local constraints on y_{uw} in the given iteration of the algorithm. Sampling from (4) is easily carried out using a normal generator and rejection sampling according to the constraints. Although in statistical terminology, (3) takes a parametric form, we refer to the approach as non-parametric since no functional relationship is imposed on the y's.

Table 4 A non-parametric resource table for Twenty20 based on isotonic regression

Overs available	*Wicket lost*									
	0	1	2	3	4	5	6	7	8	9
20	100.0									
19	93.6	85.5	85.5							
18	90.4	85.5	80.8							
17	86.7	80.8	80.8	64.7						
16	81.7	77.4	77.4	64.7	55.9					
15	76.5	71.5	71.5	64.7	55.9					
14	68.8	68.8	67.6	64.7	55.9					
13	66.6	66.6	62.6	62.6	55.9	38.4				
12	62.2	62.2	60.6	57.3	55.9	38.4				
11	60.5	56.8	56.8	54.8	54.8	38.4				
10	57.6	52.1	52.1	52.1	48.1	38.4	34.1	29.3		
9	54.9	52.1	46.5	46.5	44.1	36.3	34.1	29.3		
8	51.0	46.4	42.0	42.0	41.2	36.3	28.6	28.6		
7	48.6	45.8	38.9	37.3	37.3	34.8	25.3	25.3		
6	39.7	39.7	36.6	32.8	32.8	31.3	23.0	21.4	21.4	
5		39.7	32.5	28.0	28.0	28.0	23.0	17.1	15.5	
4			27.9	23.4	22.5	22.2	20.9	14.3	10.7	10.7
3			20.7	19.9	17.4	17.4	15.8	12.4	7.7	7.7
2			20.7	17.6	12.5	12.5	10.8	10.8	7.2	1.8
1				8.7	6.6	6.6	6.0	5.7	5.7	1.8

The table entries indicate the percentage of resources remaining in a match with the specified number of wickets lost and overs available. Missing entries correspond to match situations where data are unavailable.

In Table 5, the estimated posterior means of the y's obtained through Gibbs sampling are given, and these provide an alternative resource table for Twenty20. The computations pose no difficulties and the estimates stabilize after 50 000 iterations. For cases of missing data, we impute the missing r's with the Duckworth-Lewis table entries. The imputation is in the spirit of a Bayesian approach where prior information is utilized. Unlike Table 4, note that we have a complete table. Also, we no longer have adjacent table entries with identical values and this is due to the sampling approach. Finally, we remark that the methodology allows the input of expert opinion. For example, suppose that there is expert consensus that a table entry y_{ij} ought to be tied down to a particular value a. To force this table entry, all that is required is to set $r_{ij} = a$ and assign a sufficiently small standard deviation $1/\sqrt{q_{ij}}$.

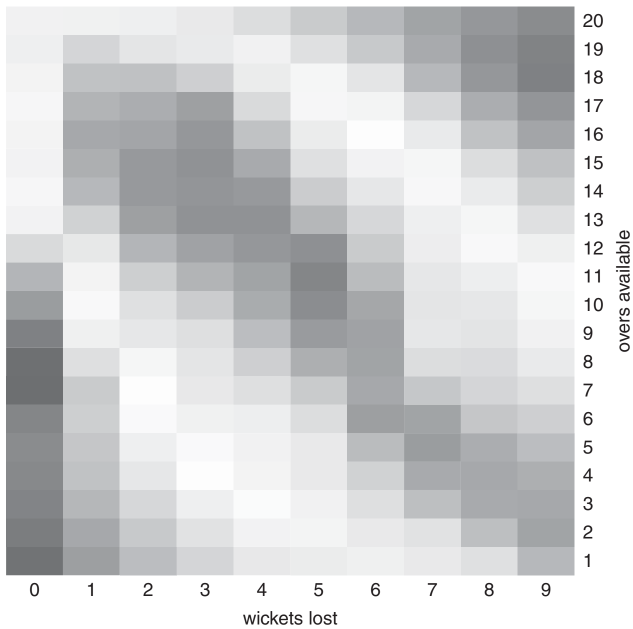

Figure 1 Heat map of the absolute differences between the Duckworth-Lewis resource table (Table 2) and the non-parametric resource table based on Gibbs sampling (Table 5). Darker shades indicate larger differences.

4. Discussion

Our intention now is to compare the scaled Duckworth–Lewis resource table originally designed for 1-day cricket (Table 2) with the alternative non-parametric resource table based on Twenty20 matches (Table 5). To facilitate the comparison, we take the absolute values of the differences between the two tables, and produce a heat map as given in Figure 1. The darker shades of the heat map indicate the greatest disagreement between the two tables. We investigate these areas of disagreement.

From Figure 1, we observe that the greatest absolute differences occur in three regions. First, large differences occur in the top-right hand corner and bottom-left hand corner of the table. These are precisely the regions where

Table 5 A non-parametric resource table for Twenty20 based on Gibbs sampling

Overs available	*Wickets lost*									
	0	1	2	3	4	5	6	7	8	9
20	100.0	96.9	93.0	87.9	81.3	72.2	59.9	44.8	29.7	17.6
19	95.6	90.9	87.7	83.0	76.9	68.3	56.5	42.0	27.2	15.3
18	91.7	86.7	82.9	78.7	73.2	65.4	54.2	40.2	25.7	13.9
17	87.7	82.3	78.9	73.8	69.7	62.8	52.2	38.7	24.6	12.8
16	83.5	78.2	75.3	70.5	66.4	60.2	50.3	37.4	23.5	12.0
15	79.2	74.3	70.9	66.9	62.6	57.4	48.4	36.2	22.7	11.2
14	75.1	70.7	67.3	63.7	59.3	54.6	46.4	35.0	21.8	10.5
13	71.5	67.4	63.6	60.3	56.2	51.5	44.3	33.8	21.0	9.8
12	68.3	63.7	60.2	56.8	52.9	47.5	41.9	32.6	20.2	9.1
11	65.0	59.9	56.6	53.3	49.7	43.9	39.3	31.3	19.4	8.5
10	61.3	56.0	52.6	50.1	46.0	40.8	36.1	30.0	18.6	7.9
9	57.9	52.3	47.9	46.1	42.5	37.8	33.1	28.3	17.7	7.2
8	54.0	48.3	44.3	41.7	38.9	34.9	30.2	26.1	16.7	6.6
7	49.3	44.2	40.2	37.4	35.4	32.1	27.2	23.4	15.7	5.9
6	41.7	38.5	35.7	33.0	31.7	29.0	24.2	20.0	14.5	5.2
5	36.2	33.4	31.0	28.6	27.3	25.5	21.5	17.0	12.2	4.4
4	30.8	28.0	26.1	24.1	22.4	20.7	18.3	14.2	10.0	3.5
3	25.4	22.8	21.1	19.4	17.7	16.5	14.4	11.6	7.9	2.5
2	19.7	17.2	15.5	14.1	12.7	11.9	10.6	9.3	6.2	1.6
1	13.7	11.3	9.7	8.5	7.3	6.7	6.0	5.2	4.2	0.9

The table entries indicate the percentage of resources remaining in a match with the specified number of wickets lost and overs available.

very little or no data are available. We do not view these regions as too important as the resetting of targets would rarely use these entries. It is interesting however that the non-parametric approach (Table 5) provides more resources in these regions than the Duckworth/Lewis approach. Consider a Twenty20 match with a single over remaining and two wickets lost. In a match that averages 150 runs, Duckworth/Lewis suggests that $(0.064)150 = 9.6$ runs is expected in the final over for an average of $9.6/6 = 1.6$ runs per ball. On the other hand, Table 5 suggests $(0.097)150 = 14.6$ runs in the final over for an average of $14.6/6 = 2.4$ runs per ball. According to our intuition, in this situation with an extremely aggressive and talented batsmen, 2.4 runs per ball may be reasonable.

The more interesting discrepancy occurs in the 'middle' of an innings (8–13 overs available with 3–6 wickets lost). In this stage of an innings, the non-parametric approach based on Gibbs sampling (Table 5) suggests that there is up to 5% fewer resources remaining than provided by the Duckworth–Lewis method. This coincides with our intuition as we believe that up to this stage in an innings, batting is more aggressive in Twenty20 than in 1-day cricket. Recall that in 1-day cricket, a team needs to protect its wickets over a longer period of overs. Consequently, up until the middle stage, more resources are conserved in the 1-day game than in Twenty20. We remark that a difference of 5% resources may be very meaningful as a target of 240 runs diminished by 5% gives 228 runs.

We revisit one of the four applications of Duckworth–Lewis in international Twenty20, which occurred in a match between England and the West Indies during the 2009 World Cup. This was the crunch game of the tournament for a place in the semi-finals. England scored 161 run at the expense of six wickets in the full first innings. The second innings was shortened to 9 overs with a target of 80 runs for the West Indies. West Indies scored 82 runs in 8.2 overs to eliminate England from the tournament. The English fans were upset and the Guardian claimed that the Duckworth–Lewis system will be reviewed to take into account Twenty20 matches (http://www.cricinfo.com/ci-icc/content/story/409482.html). In this match, we note that the Professional Edition of Duckworth-Lewis (which is not available from the 2008–2009 ICC Playing Handbook) was used to set the target at 80. For comparison, we refer to Table 2 (based on the Standard Edition) that sets a target of $(0.518)161 = 84$ runs. The non-parametric resource table (Table 5) gives an even higher target of $(0.579)161 = 94$ runs.

As a second example of the implementation of Duckworth–Lewis in Twenty20, we consider another controversial match that occurred after our data collection phase, and again involved England and the West Indies. On 3 May, in the 2010 World Cup, England batted first, scoring an impressive 191 runs at the expense of five wickets. England's captain Paul Collingwood commented 'Ninety-five percent of the time when you get 191 runs on the board you are going to win the game'. In this match, rain interrupted the second innings after 2.2 overs. When the match resumed, the Professional Edition of Duckworth-Lewis provided a target of 60 runs from 6 overs. The West Indies reached the target in 5.5 overs and won the match amidst great complaints (http://www.cricinfo.com/world-twenty20-2010/content/current/story/458375.html). Had our Table 5 been used, there would have been 41.7% resources available from 6 overs and this leads to a higher and more reasonable target of $(0.417)191 = 80$ runs.

It is important to emphasize that we do not endorse our non-parametric resource table (Table 5) as a replacement for the Duckworth–Lewis resource table in Twenty20. Our resource table is based on only 85 matches, too small a sample to provide confident table entries. However, we believe that our table does suggest that there may be some meaningful differences between the scoring rates in 1-day cricket and Twenty20 cricket. As more Twenty20 matches become available, we endorse a review of the use of Duckworth–Lewis in Twenty20 and the estimation techniques used in the construction of the associated resource table.

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