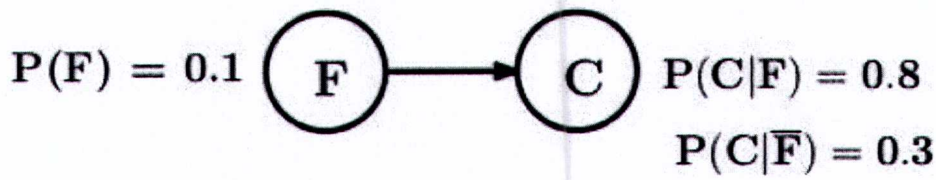
Week 4 Assignment

Topic: Probability and Bayes learning

1. Consider the following Bayesian network, where F stands for Flu and C stands for Coughing. Find P(C).



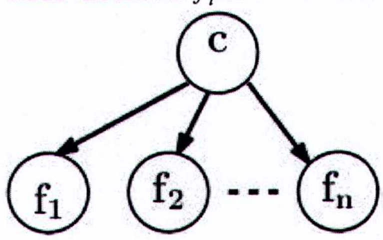
A) **0.35**
B) 0.77
C) 0.24
D) 0.5

$$P(C) = P(C|\bar{F}) \cdot P(\bar{F}) + P(C|F) \cdot P(F).$$

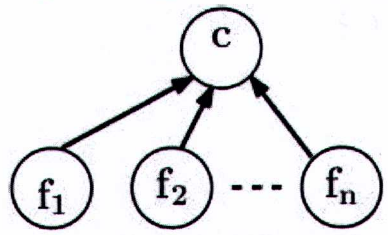
$$= 0.3 \times 0.9 + 0.8 \times 0.1$$

$$= 0.27 + 0.08 = 0.35$$

2. Which of the following graphical models capture the Naive Bayes assumption, where c represents the class label and f_i are the features.



✓A)



B)

C) It cannot be captured by a graphical model

$$P(c|f_1, f_2, .. f_n) = \frac{P(f_1, f_2, .. f_n | c) \cdot P(c)}{P(f_1, f_2, .. f_n)}.$$

Naive Bayes assumption, assumes independence among attributes f_i when class is given.

$$P(f_1, f_2, .. f_n | c) = P(f_1|c) \cdot P(f_2|c) .. P(f_n|c).$$

So, <u>option A</u> is the correct answer.

3. Bayesian Network is a graphical model that efficiently encodes the joint probability distribution for a large set of variables.

A) True ✓

B) False

Please refer to the lecture notes.

4. A fair coin is tossed three times and a T (for tails) or H (for heads) is recorded, giving us a list of length 3. Let X be the random variable which is zero if no T has another T adjacent to it, and is one otherwise. Let Y denote the random variable that counts the number of T's in the three tosses. Find P(X=1, Y=2).

A) ⅛

B) 2/8 ✓

C) ⅝

D) ⅞

The possibilities are as follows:

	X	Y
TTT	1	3
$\boxed{\text{TTH}}$	1	2
THT	0	2
THH	0	1
$\boxed{\text{HTT}}$	1	2
HTH	0	1
HHT	0	1
HHH	0	0

$\therefore P(x=1, y=2) = \frac{2}{8}$.

B is the correct answer.

5. Two cards are drawn at random from a deck of 52 cards without replacement. What is the probability of drawing a 2 and an Ace in that order?

A) 4/51

B) 1/13

C) 4/256

D) 4/663

$$\frac{4}{52} \cdot \frac{4}{51} = \frac{4}{663}.$$

6. A and B throw alternately a pair of dice. A wins if he throws 6 before B throws 7 and B wins if she throws 7 before A throws 6. If A begins, his chance of winning would be:

A) 30/61

B) 31/61

C) 1/2

D) 6/7

probability of A's throwing 6 with 2 dices $= \frac{5}{36}$.

probability of B's throwing 7 with 2 dices $= \frac{1}{6}$.

A can win if he throws 6 in the $1^{st}, 3^{rd}, 5^{th}, 7^{th} \ldots$ throws.

Probability of A's winning $= \frac{5}{36} + \frac{31}{36} \cdot \frac{5}{6} \cdot \frac{5}{36} + \frac{31}{36} \cdot \frac{5}{6} \cdot \frac{31}{36} \cdot \frac{5}{6} \cdot \frac{5}{36} + \cdots$

$= \frac{30}{31}$.

7. Diabetic Retinopathy is a disease that affects 80% people who have diabetes for more than 10 years. 5% of the Indian population has been suffering from diabetes for more than 10 years. Answer the following questions. What is the joint probability of finding an Indian suffering from Diabetes for more than 10 years and also has Diabetic Retinopathy?

A) 0.024

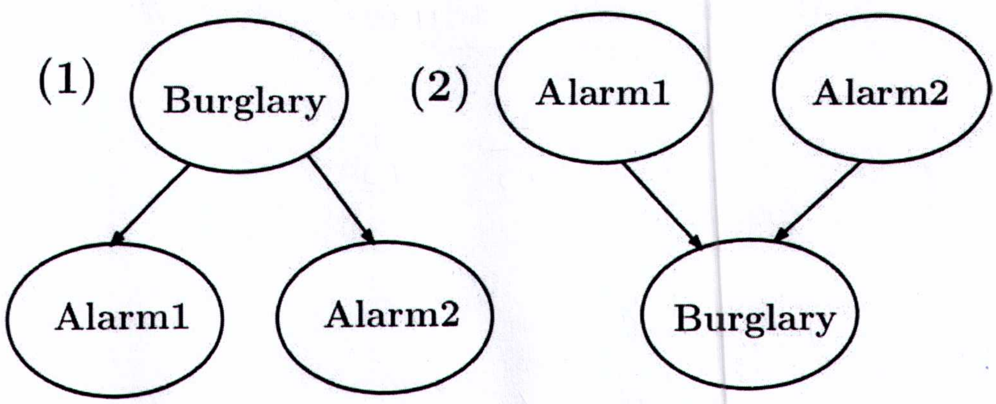
B) 0.040

C) 0.076

D) 0.005

Joint probability = $0.8 \times 0.05 = 0.04$.

8. To safeguard your house, you recently installed two different burglary alarm systems by two different reputable manufacturers that use completely different sensors for their alarm systems. Alarm1 means that the first alarm system rings, Alarm2 means that the second alarm system rings, and Burglary means that a burglary is in progress.
Which one of the two Bayesian networks given below makes independence assumptions that are **not true**?



A) 1

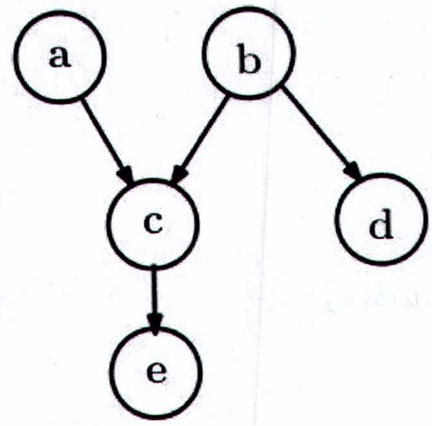
B) 2 ✓

C) Both

D) None of these

The second one falsely assumes that Alarm1 and Alarm2 are independent if value of Burglary is unknown. However, if the alarms are working as intended they should not be independent.

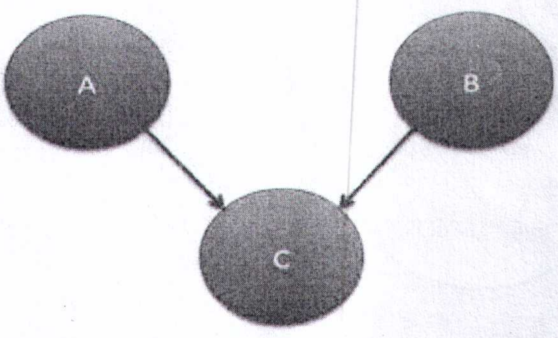
9. Consider the following graphical model. Mark which of the following pair of random variables is independent given no evidence?



A) a, b
B) c, d
C) e, d
D) c, e

Refer to the lecture video.

10. In the following Bayesian network A, B and C are Boolean random variables taking values in {True, False}. Which of the following statements is true?



A) The value of C is not given. If the value of B changes from True to False, the conditional probability of A, P(A|B) changes.
B) The value of C is given to be True. If the value of B changes from True to False, the conditional probability of A, P(A|B) changes.
C) Neither A nor B
D) Both A and B

Please refer to the lecture video.