PHY 206: Physics Through Computational Thinking

Assignment 3

Question 1: Visualizing the Electric Field of a Quadrupole

In electrodynamics, a quadrupole consists of four charges arranged symmetrically. Consider four point charges:

- $+Q$ at (a, a)
- $-Q$ at $(-a, a)$
- $+Q$ at $(-a, -a)$
- $-Q$ at $(a, -a)$

The total electric potential at a point (x, y) due to these charges is given by:

$$V(x, y) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{4} \frac{Q_i}{\sqrt{(x - x_i)^2 + (y - y_i)^2}}$$

where (x_i, y_i) are the positions of the charges and Q_i are their respective magnitudes. The electric field is given by:

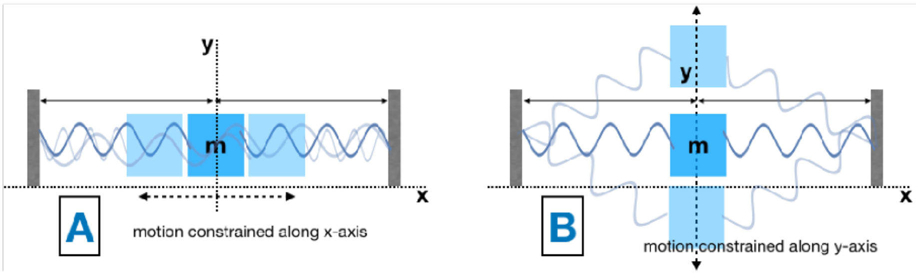
$$\mathbf{E} = -\nabla V = \left(-\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y} \right)$$

(a) **Vector Plot:** Generate a **vector field plot** of $\mathbf{E}$ in the xy-plane for the quadrupole.

(b) **Stream Plot:** Generate a **streamline plot** to visualize the continuous flow of the electric field lines.

(c) **Contour Plot:** Plot the **contours** of the electric potential $V(x, y)$.

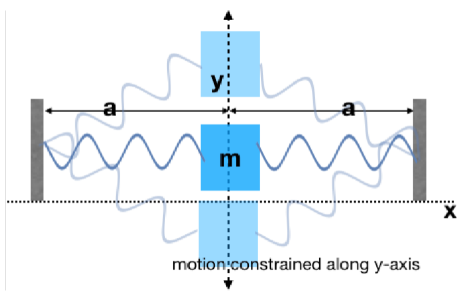
Question 2: Simple Harmonic Oscillator For the spring mass systems shown below, both the systems have mean position of the block of mass m in the center and is connected by ideal springs of spring constant k and ideal length a_0 on each side stretched to length $2a_0$ at mean position. System A is constrained to oscillate horizontally while system B is constrained to oscillate



vertically.

Ignoring effect of gravity, if frequency of small oscillation for system A is ω_A and in system B is ω_B, then find the ratio ω_A/ω_B.

Question 3: Simple Harmonic Oscillator Consider the oscillator shown below made up of block of mass m and ideal springs of natural length a_0 and spring constant k. Oscillator is constrained to move along the y?axis as shown in the figure



Small y expansion of the potential for $r = \frac{a}{a_0} > 1$ has the form: $V(y) = V_0 + \alpha y^2 + \beta y^4 +?$ where α and β are constants. Find the ratio α/β.

Question 4: Projectile
A projectile is launched with an initial velocity $v_0 = 20\,\mathrm{m/s}$ from the ground, neglecting air resistance. The parametric equations describing the motion of the projectile are:

$$x(t) = v_0 \cos(\theta)t, \quad y(t) = v_0 \sin(\theta)t - \frac{1}{2}gt^2,$$

where $g = 9.8\,\mathrm{m/s}^2$ is the acceleration due to gravity, and θ is the launch angle.

1. Derive the time of flight $t_{\text{flight}}(\theta)$ as a function of the launch angle θ.

2. Plot the trajectory of the projectile for a specific angle ($\theta = 45°$).

3. Create an interactive plot where the launch angle θ can be varied to visualize how the trajectory changes.

4. Plot the Range of the projectile for different launch angles. Hence determine the angle for which the range is maximum

5. Plot graphically the scenario where the ground has an inclination of 0.5 degrees. Can you visually guess the angle at which the range is maximum?

Question 5: Non-dimensionalization Consider the differential equation:

$$a\frac{dx}{dt} + bx = Af(t)$$

where x is the dependent variable and t is the independent variable. The parameters a, b, and A are constants, and $f(t)$ is a given function of time.

Perform a **non-dimensionalization** of this equation by introducing suitable dimensionless variables:

$$X = \frac{x}{x_0}, \quad T = \frac{t}{t_0}$$

where x_0 and t_0 are appropriate characteristic scales.

Express the resulting differential equation in terms of the new unitless variables X and T, and identify x_0 and t_0 in terms of the given parameters.

Question 6: Physical Examplary on Non-dimensionalization
Consider the equation governing Newton's law of cooling:

$$mc\frac{dT}{dt} = -hA(T - T_{\text{surrounding}})$$

where $T(t)$ is the temperature of the object, $T_{\text{surrounding}}$ is the temperature of the surrounding environment, m is the mass of the object, c is its specific heat capacity, h is the heat transfer coefficient, and A is the surface area of the object. Perform a **non-dimensionalization** of this equation by introducing suitable dimensionless variables:

- Let Θ represent the temperature,
- Let τ represent time.

Express the resulting differential equation in terms of the dimensionless variables Θ and τ.

Question 7: Derivatives Through Mathematica
Consider the following potential energy function:

$$V(x) = \frac{1}{4}ax^4 - \frac{1}{2}bx^2$$

Where:

- a and b are positive constants,
- x is the displacement from equilibrium.

Let's put aside our pens and papers for a moment and switch to Mathematica and

(a) **Find the equilibrium points** of the system by setting the force to zero.

(b) **Check the stability** of the equilibrium points by computing the second derivative of the potential energy function.[Hint: maxima lead to an unstable equilibrium point, and minima lead to a stable equilibrium point.]

(c) **Find the frequency of oscillation** for small oscillations around the stable equilibrium points. Given that the formula for frequency is:

$$\nu = \frac{1}{2\pi}\sqrt{\frac{1}{m}\left(\frac{d^2V}{dx^2}\right)_{\text{eq}}}$$

where $\left(\frac{d^2V}{dx^2}\right)_{\text{eq}}$ is the second derivative of the potential energy function evaluated at the equilibrium point and m is the mass, express the frequency ν in terms of the constants a, b, and the mass m.