PHY 206: Physics Through Computational Thinking

Assignment 2

Question 1: Graph the potential for the system

$$\dot{x} = x - x^3,$$

and identify all equilibrium points.

Question 2: For each of the following vector fields, plot the potential function $V(x)$ and identify all the equilibrium points and their stability.

- $\dot{x} = x(1 - x)$
- $\dot{x} = r - x - x^3$
- $\dot{x} = x(x - 1)(x - 2)$
- $\dot{x} = x^2(6 - x)$
- $\dot{x} = \tan(x)$
- $\dot{x} = \ln x$

Question 3: The growth of cancerous tumors can be modelled by the Gompertz law $\dot{N} = -aNln(bN)$, where $N(t)$ is proportional to the number of cells in the tumor and $a, b > 0$ are parameters. Sketch the vector field, what are the equilibrium points and then graph N(t) for various initial values.

Question 4: Find the divergence and curl of the following vector fields. Also plot them in Mathematica and explain the result.

- $\vec{A}(\vec{r}) = (xy)\hat{i} + (yz)\hat{j}$

- $\vec{V}(\vec{r}) = (x^2 + y^2)\hat{i} + (x^2 - y^2)\hat{j}$

Question 5:
The magnetic field of a magnetic dipole is given by:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi r^3}\left[2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right].$$

Plot the magnetic field lines in the x-z plane as a vector plot and a stream plot.

Question 6: Draw the following functions first by hand, analyzing them for their zeros, divergences, extrema, and asymptotes. Then cross-check your sketch by plotting the function in Mathematica.

1. $\coth(x)$
2. $\frac{\ln x}{x}$
3. $e^{-x}\cos x$
4. x^x
5. $\frac{1}{x^{12}} - \frac{1}{x^6}$