Spanning Properties of Emanation Graphs with Three Rays

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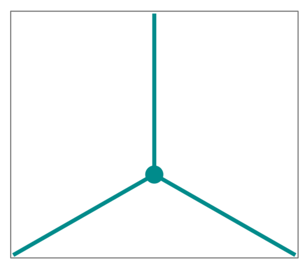
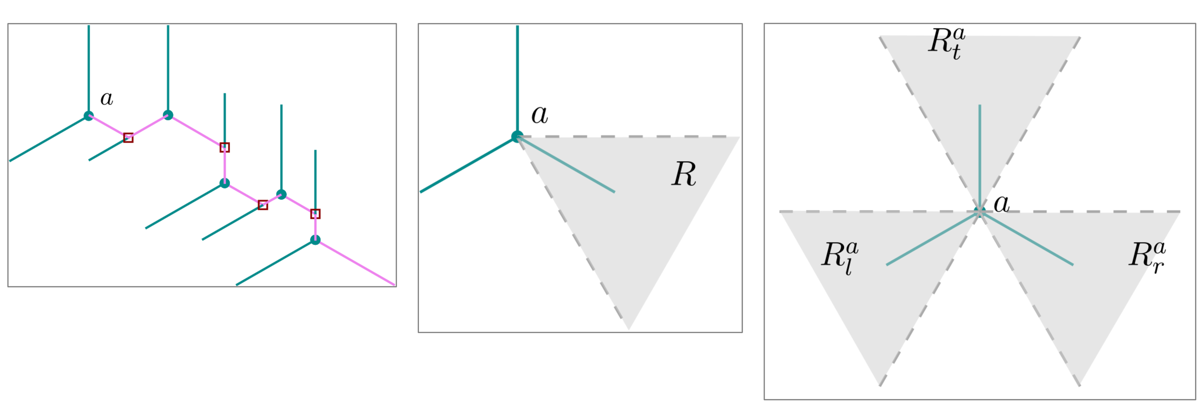
What is E3?

- An E3 graph of a given point set is constructed as follows:
  - We shoot three rays from each point such that the rays emanating from a point are equally apart.
  - If two rays intersect, the shorter ray stops the longer ray (breaking ties arbitrarily) and continues until it hits the bounding box of the graph.
  - At each intersection, we create a new Steiner point.

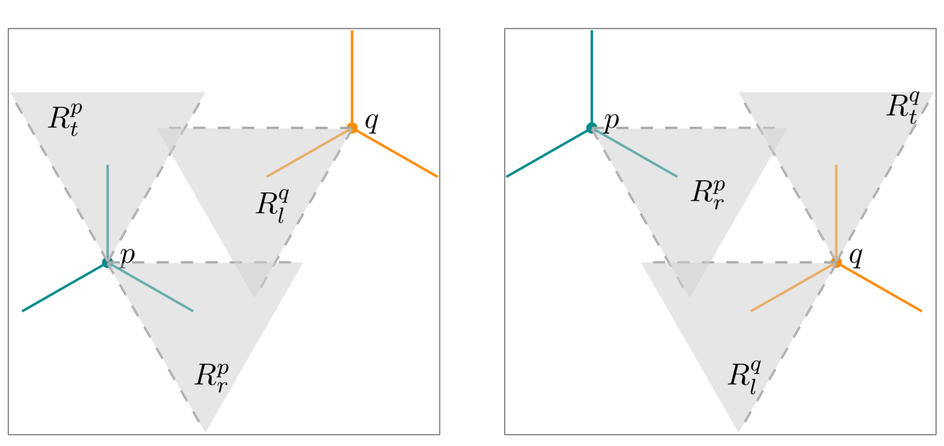


E3 is Connected

- We construct a greedy right paths from the point a by repeatedly following a right ray and the ray that stops it.

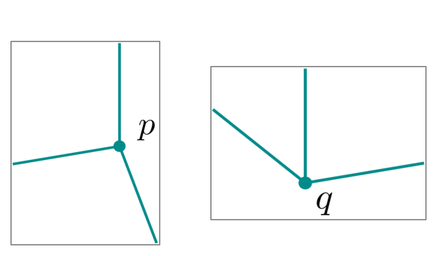


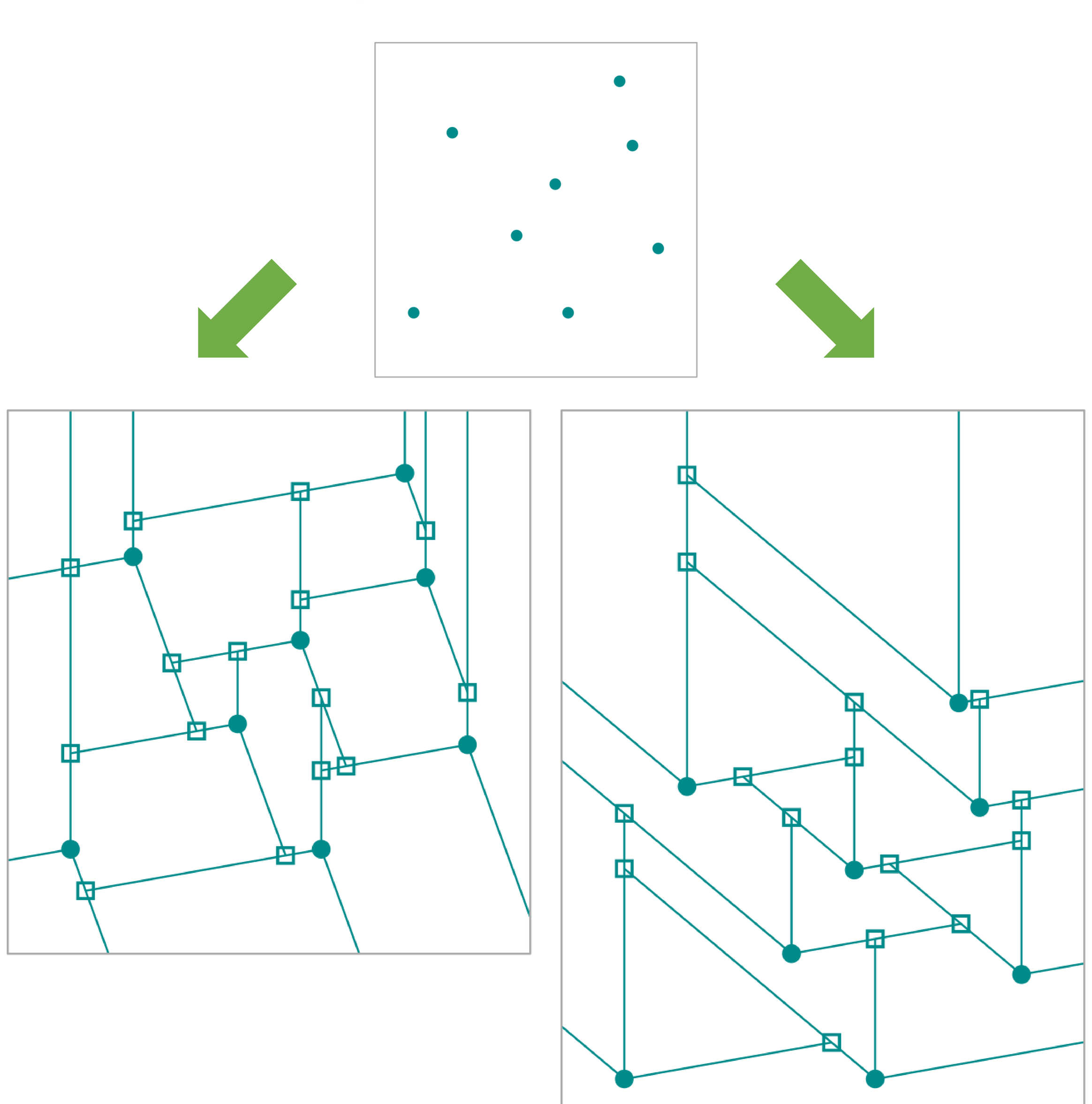
- We show that the greedy paths are bounded by 60 degree cones such as the region R shown above. We can similarly construct a top and left cone for the point a that bound its top and left greedy path.
- For two arbitrary points p and q in an E3 graph, we use their cone intersections and the planarity of E3 to show that there is always a path connecting them.



Generalized E3 is Connected

- We consider E3 when the rays are NOT equally apart, there are two categories:
  - At least 2 of the ray angles are larger than 90 degrees (p)
  - One of the ray angles is at least 180 degrees (q)





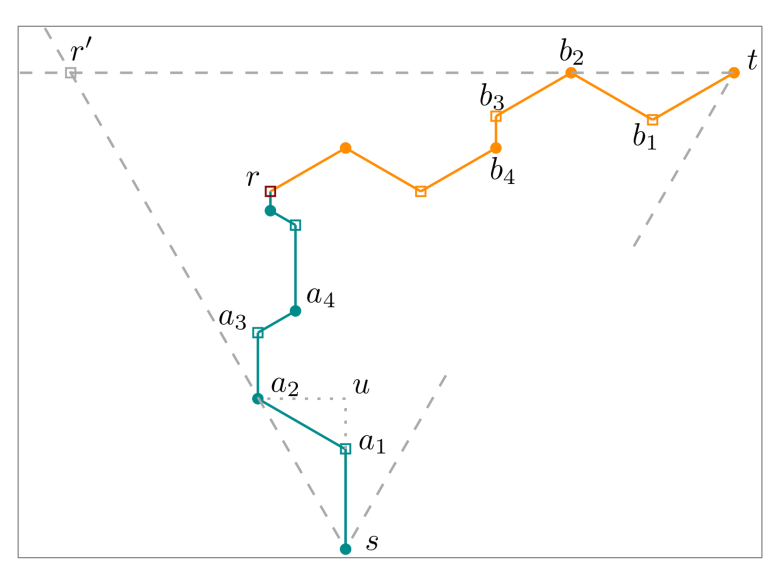
- We generalize the proof for E3 to show that Generalized E3 is connected.

Results

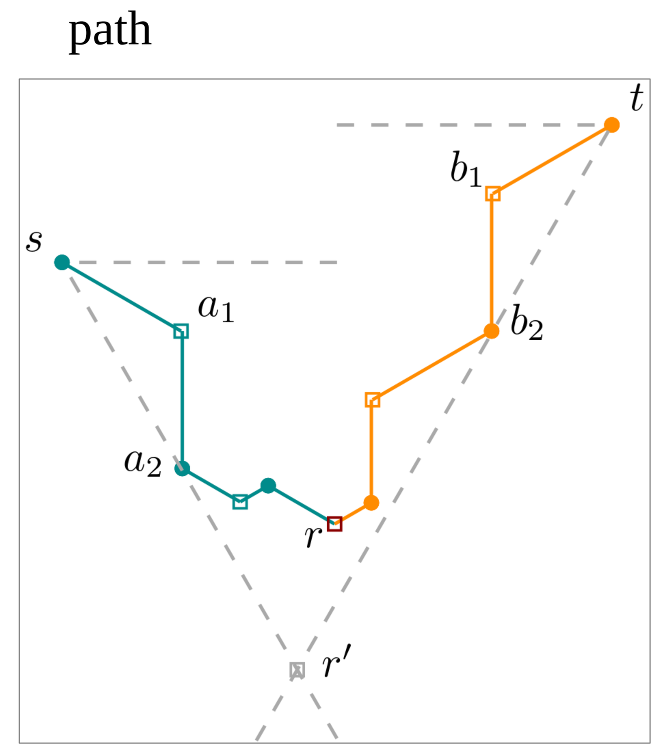
- E3 is connected, a spanner, and improves a previously known spanner E4
- E'3 is planar
- Generalized E3 is connected

E3 is a Spanner

- The spanning ratio is the maximum ratio of the path distance over Euclidean distance over all pairs of vertices in a geometric graph. If the spanning ratio is a constant t, then the graph is called a t-spanner.
- To calculate the spanning ratio of E3, we consider two types of path constructions:
  - A top path intersecting a left (right) path



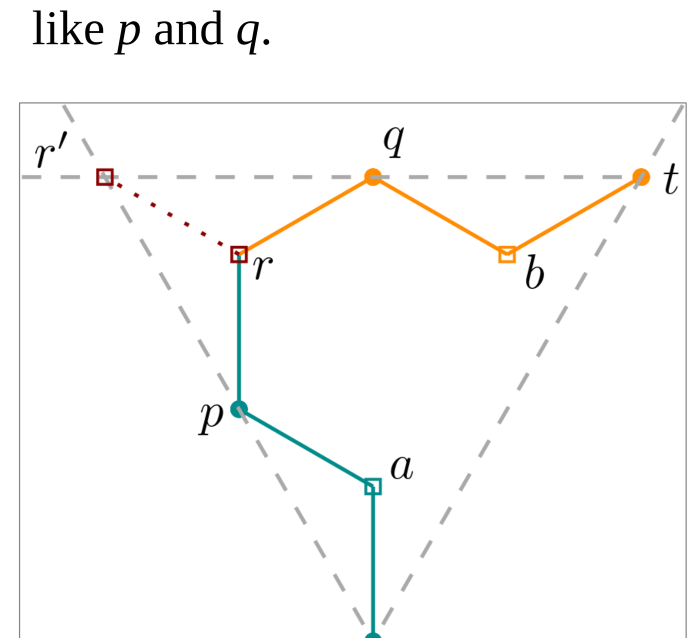
  - A left path intersecting a right path



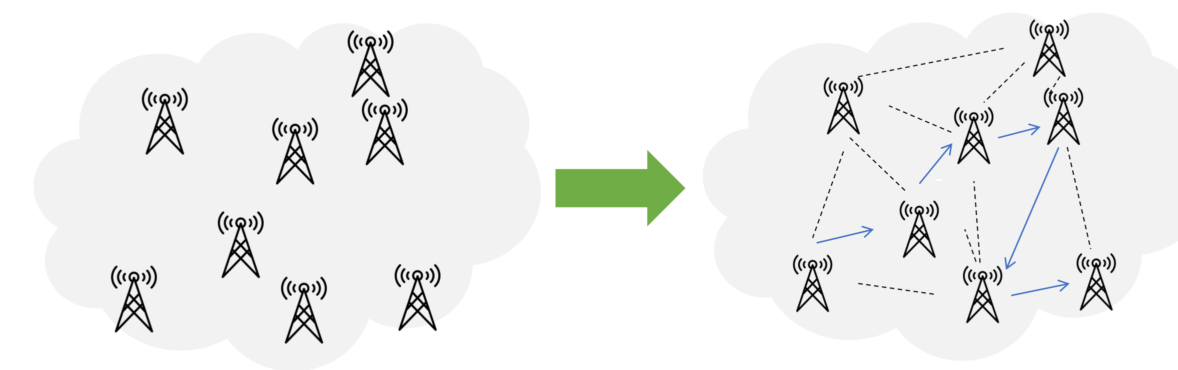
  - Both types yield the same spanning ratio of $\frac{4}{\sqrt{3}} \approx 2.309$.
  - This is an improvement from the spanning ratio of 3.162 of E4.

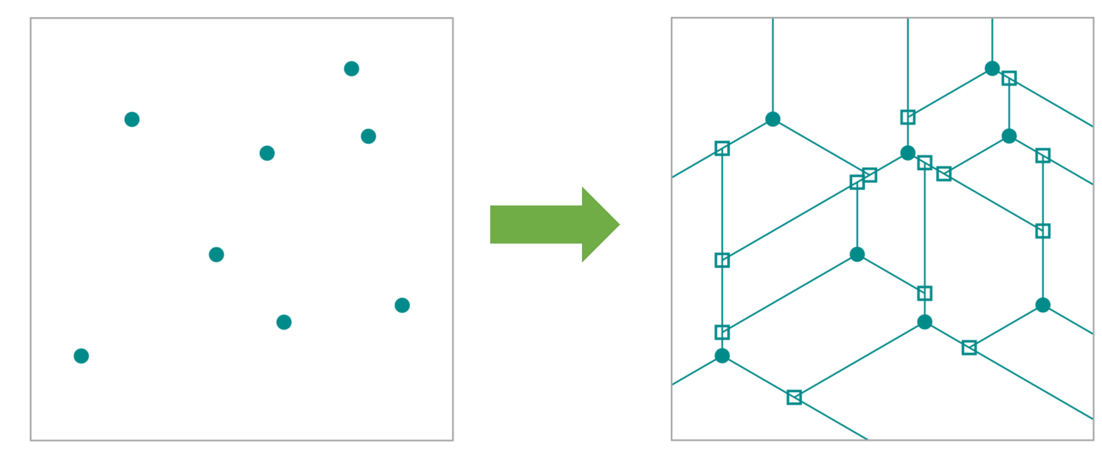
2.309 is the Best Possible Spanning Ratio for E3

- The spanning ratio of the path from s to t indefinitely approaches $\frac{4}{\sqrt{3}}$ as $|rr'|$ is minimized by subdividing sr' and tr' and adding more points like p and q.



Motivation

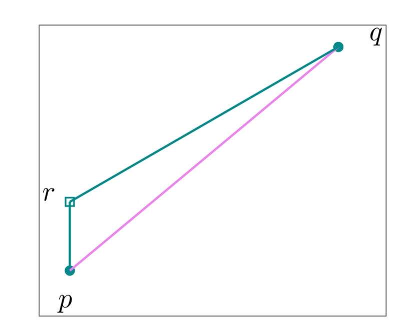


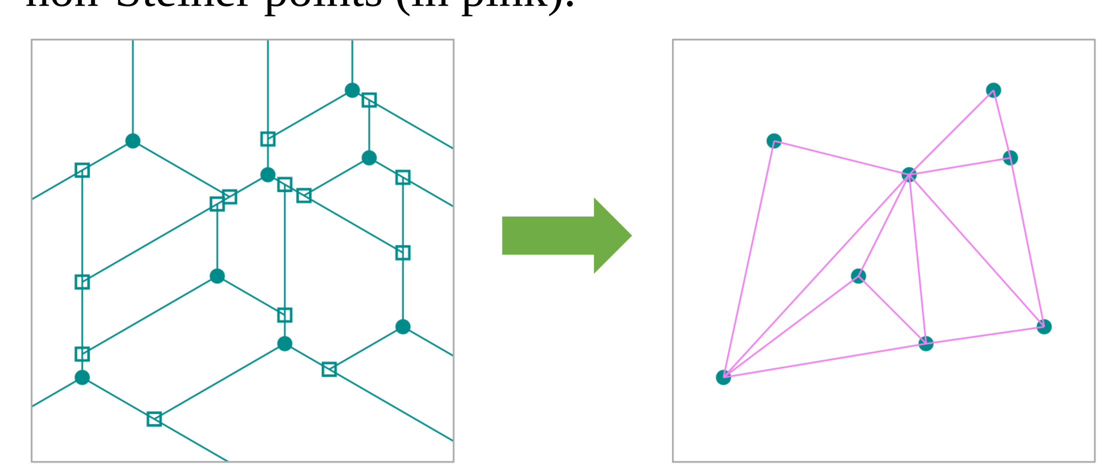


We study geometric graphs with good spanning properties to better model large networks used for communication and visualization.

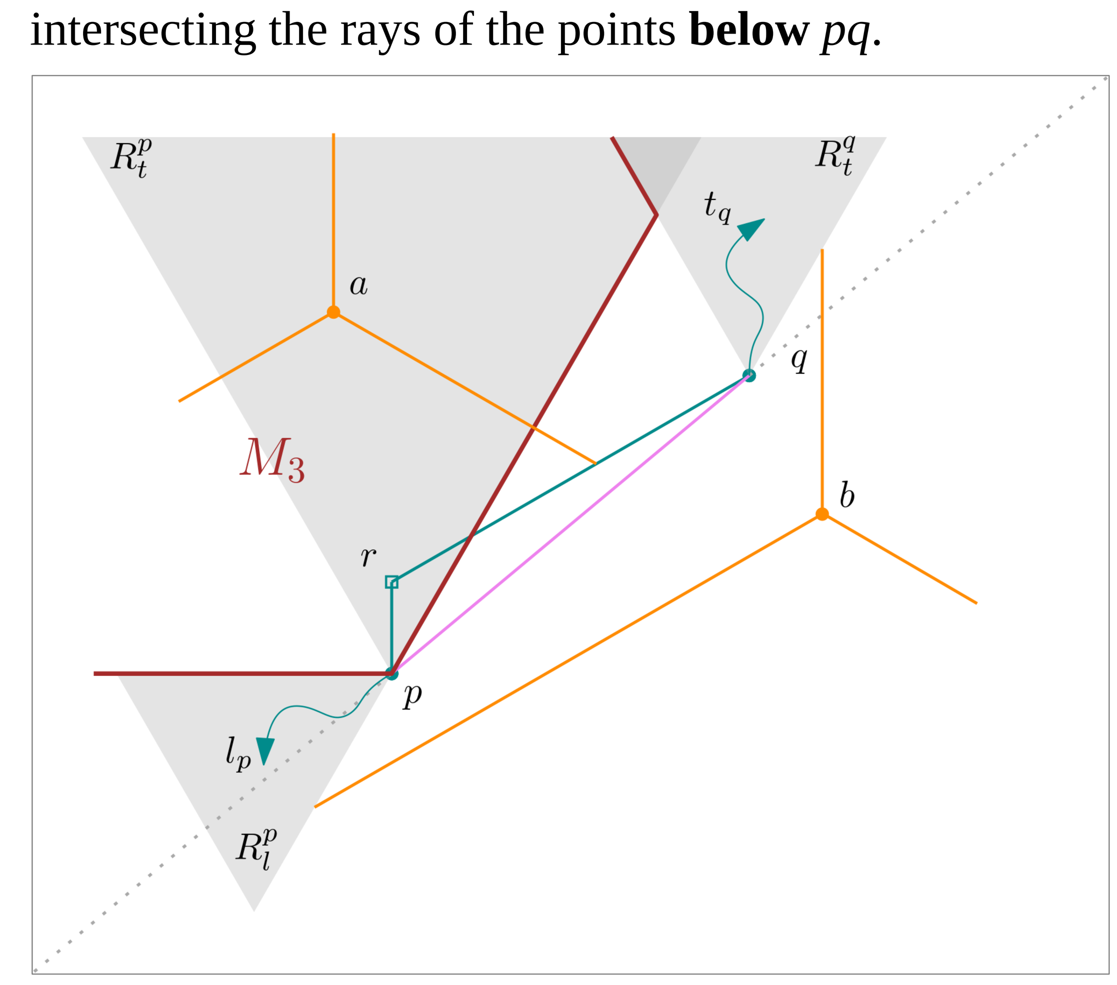
E'3 is Planar

- We create E'3 graphs from E3 graphs by replacing the intersecting rays and the resulting Steiner points (in cyan) with edges connecting the non-Steiner points (in pink).





- Assume pq is an edge in E'3. We show no other edge in E'3 can cross pq. We divide the half plane above pq into 5 regions. For each region, we show the rays of the points **above** pq are blocked by the paths of p and q from intersecting the rays of the points **below** pq.



- The rays of the point a in region $M3$ cannot intersect with the rays of b due to the left and top paths of p and q.

Directions for Future Research

- The spanning ratio of E'3
- The spanning ratio of Generalized E3
- The planarity of Generalized E'3

Acknowledgement

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