

Collateral and bank screening as complements: A spillover effect

Sonny Biswas*

Abstract

I analyze a novel spillover effect from collateralized to uncollateralized loans. High-type borrowers have good projects while low-type borrowers do not know their project quality. High-type borrowers post collateral, and a monopolist bank screens only low-type borrowers' projects. Different from existing models, equilibrium collateral requirements are stricter than the minimum necessary to achieve separation, despite costly collateral. When high-type borrowers post more collateral, the bank charges a higher interest rate to low-type borrowers. This enhances the bank's incentives to screen the low-types' projects, thereby improving the average quality of uncollateralized loans.

JEL Classification: D82, G21

Keywords: Collateral, Bank screening, Adverse Selection, Countervailing incentives.

*University of Bristol Business School. 15-19 Tyndall's Park Road, Bristol, BS8 1PQ. s.biswas@bristol.ac.uk.
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1 Introduction

Collateral in debt contracts serve a multitude of functions. One function of collateral is to protect lenders' interests in defaults (e.g., Tirole 2006). Modelling this particular function of collateral, Asriyan et al. (2022) show in a dynamic general equilibrium setting that credit booms associated with higher collateral values can crowd out bank screening leading to protracted economic crises (see also, Manove et al. 2001 and Gorton and Ordoñez 2014). Instead, I consider the screening function of collateral, which is that it solves an adverse selection problem. My analysis implies that, aside from being substitutes (as in existing models), collateral and direct screening can be complements since collateral availability enhances banks' direct screening incentives through a spillover effect. The model provides a new explanation for the empirical observation that banks screen riskier borrowers more diligently in weaker economic conditions.

In traditional adverse selection models of collateral (e.g., Bester 1985, Besanko and Thakor 1987a), collateral sorts borrowers into risk-types, with no residual uncertainty regarding borrowers' riskiness, which makes direct screening by banks redundant. Posting collateral entails a deadweight loss, and hence, the minimum amount of collateral that achieves separation between types is used, and no more. I present a setting in which the use of collateral eliminates information asymmetry but does not eliminate all uncertainties. Direct bank screening (e.g., pre-lending information acquisition) plays a role despite the use of collateral. In equilibrium, collateral requirements can be higher than the minimum that is necessary to separate borrower-types. This result allows me to derive a spillover effect which has not been considered previously.

I consider a model in which there are two types of borrowers who apply for credit from a monopolistic bank: a high-type borrower has a good project, while a low-type borrower may have a good or a bad project. Each borrower has personal assets which they may pledge as collateral to secure a loan; these assets are in addition to what is normally available to the lender in the case of default (as in Chan and Kanatas 1985). High-type borrowers post collateral which separates

them from low-type borrowers, and hence, the bank does not screen the high-types' projects; this is the substitution relationship between collateral and bank screening which is studied in existing models (e.g., Manove et al. 2001). The novelty in my model is as follows: conditional on separation between borrower-types, more collateral posted by the high-type allows the bank to charge a higher interest rate to the low-type, which provides improved incentives to the bank to screen the low-types' projects (i.e., screening and collateral are also complements). Hence, the average quality of uncollateralized loans approved by the bank increases in the availability of collateral, i.e., there is a positive spillover effect from collateralized to uncollateralized loans. This effect arises when the level of collateral availability is intermediate and posting collateral is not too costly.

There are two distinguishing features of my model: First, even after the resolution of information asymmetry between the monopolist lender and borrowers, the lender cannot extract the full surplus from screening unless collateral availability is sufficiently high. What drives this result in my model is that the demand for collateralized and uncollateralized loans depends on the contract terms, beyond separation. As a result of this, effort provision by the bank may be inefficiently low since it does not fully internalize the benefits of screening. Second, in contrast to existing screening models, equilibrium collateral requirements can be higher than the minimum necessary for separation. By increasing the collateral requirements for high-type borrowers, the bank makes it less attractive for low-type borrowers to mimic; this allows the bank to set a higher interest rate for the low-type. Since the bank retains more of the surplus from screening, the bank screens more diligently. The model delivers new empirical implications:

First, banks screen the riskier uncollateralized loans more diligently when economic conditions deteriorate; this prediction is consistent with findings in Howes and Weitzner (2023). Second, an increase in collateral availability leads to a higher average quality of uncollateralized loans. Third, a higher collateral availability leads to more (resp. less) uncollateralized credit in the economy, if the low-type have a high (resp. low) fraction of good projects.

The purpose of the model is to capture a credit market characterized by severe information asymmetry. I have in mind small, private firms seeking to raise financing from banks. These borrowers often have access to limited sources of funds, and hence, the banks providing financing to them have pricing power, reflecting my assumption that the bank is a monopolist. Unless banks can perform screening, the severity of information asymmetry may result in no firms obtaining financing. If the bank does not lend to them, borrowers may borrow from personal networks, which is an extremely common practice among smaller firms (see e.g., Lee and Persson 2016 and Zaccaria 2023). Since friends and family would arguably observe their type, high-type borrowers obtain more funds. This feature is reflected in the model by assuming that high-type borrowers have higher outside options. Finally, small firms often borrow against personal assets (see e.g., Bahaj et al. 2020), which is how I model collateral here.

Related literature. In one set of theories collateral alleviates borrower moral hazard concerns and (observably) riskier borrowers pledge collateral (e.g., Boot et al. 1991 and Boot and Thakor 1994). In a signalling model, Ordonez et al. (2019) show that privately informed borrowers can use secured loans to signal their type when collateral values are uncertain. In dynamic models (e.g., Rajan and Winton 1995, Donaldson et al. 2020b and Donaldson et al. 2020a), collateral assigns priority to a loan over unsecured loans.

My model belongs to a complementary set of theories in which collateral sorts borrowers into (unobservable) risk-classes (e.g., Bester 1985, Chan and Kanatas 1985, Besanko and Thakor 1987a, Besanko and Thakor 1987b): these make up the screening or adverse selection theories of collateral. My model differs from existing screening models on two grounds: first, equilibrium collateral requirements may be higher than necessary for separation, and second, the monopolist bank cannot extract the full surplus from screening despite full resolution of information asymmetry, unless collateral availability is sufficiently high.

In Manove et al. (2001), collateral sorts borrowers into risk-types and reduces costly screening

by banks, even when it is socially optimal to screen (see also Hainz et al. 2013, Goel et al. 2014, Gorton and Ordoñez 2014, Gorton and Ordoñez 2020, Degryse et al. 2021, and Asriyan et al. 2022 who model collateral and bank screening as substitutes). Similar to these studies, in my model the bank does not screen borrowers who post collateral. However, different from them, in my model the greater use of collateral leads to more efficient screening of uncollateralized loans, i.e., collateral complements direct screening through a spillover effect from collateralized to uncollateralized loans. This effect arises when the level of collateral availability is intermediate and posting collateral is not too costly.

In my model, starting from a pooling equilibrium, the use of collateral alters the pool of projects being screened (the screening pool), which affects screening intensity. The importance of the quality of the screening pool in my model is reminiscent of several existing papers, e.g., Broecker (1990), Shaffer (1998), Marquez (2002), Vanasco (2017), and Hu (2022). Different from these papers, my main results are derived holding constant the screening pool: beyond achieving separation, a higher availability of collateral does not affect the screening pool, but affects screening incentives by diverting the surplus from low-type borrowers to the lender.

2 Model set-up

I consider a three-date economy, $t = 0, 1, 2$. There is a continuum of entrepreneurs (borrowers), each with access to a project. The borrowers do not have corporate funds and seek financing for their projects from a monopolistic bank. All agents are risk-neutral and protected by limited liability. The risk-free rate is normalized to 0, so there is no discounting. The project is of fixed scale and requires an investment, normalized to 1, at $t = 1$. A borrower is either high-type, with probability q_1 , or low-type, with probability $(1 - q_1)$. A borrower's type is determined by nature, and it is the borrower's private information. If the borrower is high-type, the project is good with certainty. If the borrower is low-type, the project is good with probability, q_2 , and bad

with the complementary probability, $(1 - q_2)$. A low-type borrower does not know if her project is of good or bad quality. A good project either succeeds with probability, p , and produces X , or fails with probability, $(1 - p)$, and produces 0, at $t = 2$. A bad project produces 0 with certainty. The payoff structure is illustrated in Figure 1. The bank can directly screen borrowers' projects (e.g., through pre-lending information acquisition) to gauge the project quality. I make two parametric assumptions regarding the profitability of projects:

A1: $pX - 1 > 0$

A2: $q_p pX - 1 < 0$, where $q_p \equiv q_1 + (1 - q_1)q_2$

Assumption *A1* indicates that good projects are profitable. If types are separated, a high-type borrower's project is not screened as it is known that she has a good project. Therefore, the screening pool is made up of either only low-type borrowers' projects (separation) or both low- and high-type borrowers' projects (pooling). q_2 is the fraction of good projects in a screening pool containing only low-type borrowers' projects, while q_p is the fraction of good projects in a screening pool containing both borrower-types' projects. *A2* implies that it is unprofitable for the bank to lend without screening unless it is known that the borrower is high-type. This assumption ties in well with the objective of the model to describe a credit market characterized by severe information asymmetry.

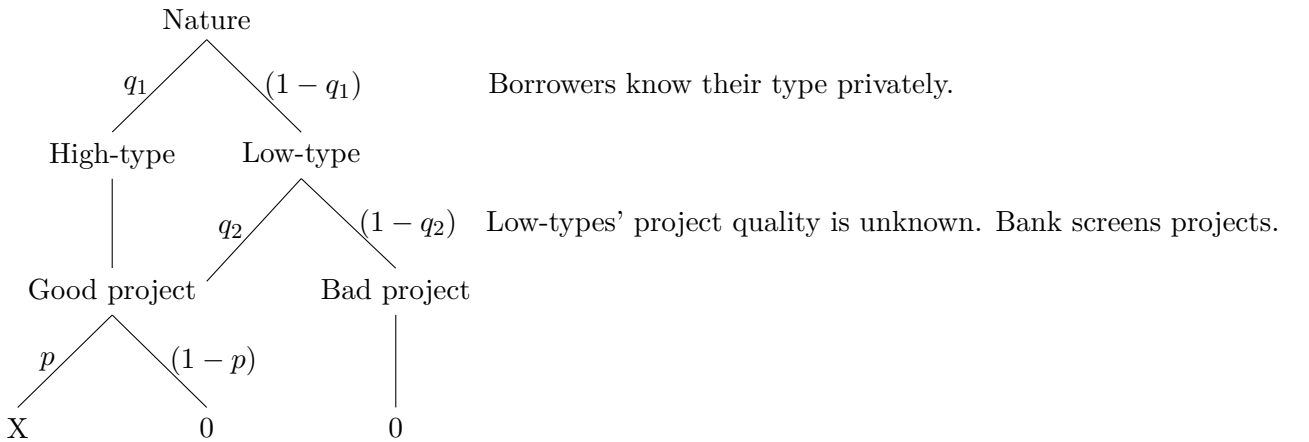


Figure 1: Payoff tree

The bank offers debt contracts (loans) at $t = 0$.¹ A borrower applies for the loan, which may or may not reveal her type. Observing a borrower's type does not necessarily reveal the project quality. Bank screening produces a signal, s_g (project is good) or s_b (project is bad), and the signal is informative but noisy²:

$$Pr(s = s_g | \text{project} = \text{good}) = Pr(s = s_b | \text{project} = \text{bad}) = \beta \geq \frac{1}{2} \quad (1)$$

β is the precision of the signal, which increases in screening intensity, F , as follows:

$$\beta = \frac{1}{2} + F \quad (2)$$

where $F \in (0, \frac{1}{2})$. If $F = 0$, the signal is pure noise and $\beta = \frac{1}{2}$. For $F = \frac{1}{2}$, $\beta = 1$ and screening perfectly reveals the project's quality. The cost of screening is given by $\frac{\tau}{2}F^2$, with $\tau > 0$ and sufficiently large, in a sense made precise later. The functional form implies that the screening cost is increasing and convex in screening intensity. Convexity reflects increasing difficulty for the bank to find out more and more about a project (see e.g., Song and Thakor 2010). Having incurred a non-zero screening cost such that $F > 0$, the bank must only agree to lend if the signal is good, $s = s_g$; otherwise, the bank might as well not have incurred the cost in the first place. Given an observed signal, the posterior probability of the project quality depends on the composition of the screening pool:

$$Pr(\text{project} = \text{good} | s_g, q_A) = \frac{\beta q_A}{[\beta q_A + (1 - \beta)(1 - q_A)]} \quad (3)$$

$q_A \in \{q_2, q_p\}$ is the unconditional probability of a good project in the screening pool. $q_A = q_2$ if the screening pool contains low-type borrowers only, while $q_A = q_p$ if the pool contains both

¹The assumption that only debt contracts are offered is without loss of generality since, given the structure of returns in our model, all contracts (sharing rules) are equivalent.

²Since the bank is a monopolist, whether the signal is observable/contractible or not does not affect the analysis.

types. For the pure noise signal, when $F = 0$ and $\beta = \frac{1}{2}$, the conditional probability that the project is good becomes equal to the prior, which is given by the fraction of the good projects in the screening pool, q_A .

If screening, the bank extends credit if the signal is good, $s = s_g$. Given the quality of the screening pool, q_A , the probability of the good signal is $Pr(s = s_g|q_A)$, which is given by the denominator in Equation (3). If the signal is positive, the project quality is good with probability $Pr(\text{project} = \text{good}|s_g, q_A)$ (Equation (3)). If the project is good, it succeeds with probability, p , and the bank receives a repayment of R . If the good project fails, or the project is bad, the repayment is 0. The bank's expected payoff from an uncollateralized loan (i.e., a contract featuring only interest rates) is:

$$\begin{aligned} & Pr(s = s_g|q_A)[Pr(\text{project} = \text{good}|s_g, q_A)pR - 1] - \frac{\tau}{2}F^2 \\ & = \beta q_A p R - \beta(2q_A - 1) - (1 - q_A) - \frac{\tau}{2}F^2 \end{aligned} \quad (4)$$

A3: The cost of screening, τ , is sufficiently high:

$$\frac{\tau}{2} > q_p(pX - 2) + 1 \quad (5)$$

Assumption A3 ensures that the solution of the bank's problem is interior, $F < \frac{1}{2}$.

Although borrowers do not have any corporate funds to invest in the project, they have personal assets, W , which they can pledge as collateral. In collateralized loans, the bank can seize the collateral if the borrower fails to deliver the promised repayments. These assets are in addition to what is normally available to the lender in the case of a default (see Chan and Kanatas 1985). Posting collateral may be costly with the cost given by $k \geq 0$ (see Parlatore (2019) for a microfoundation for this cost), and the cost is assumed to be proportional to the amount of collateral posted. Pledging collateral can be costly due to the disparity between the

valuation of the borrower and the lender. The disparity can arise since the borrower may be the best user of the asset, while the lender lacks the expertise to use the asset.

As I consider monopolistic banks, collateral would not play a role if high- and low-types have identical outside options (see page 677 in Besanko and Thakor 1987a, also Lengwiler and Rishabh 2017).³ Therefore, I assume that a high-type borrower has a higher outside option than a low-type borrower (see also, Sengupta 2007, Sengupta 2014, and Freixas and Rochet 2008).

A4: *High-type borrowers have outside option, H , with $H \in (0, pX - 1)$; while low-type borrowers' outside option is normalized to 0.*

The higher outside option of high-type borrowers can be understood as follows: while borrower-type is hidden from the bank, borrowers' friends and family can observe their type. Then, they can fund the project, but at a smaller scale. In this case, the high-type will invest and produce H , while the low-type will not invest. These frictions are of particular relevance in the credit markets since new entrepreneurs often find it difficult to acquire bank loans but can raise seed funding from personal networks (see e.g., Lee and Persson 2016 and Zaccaria 2023). That high-type borrowers have a higher outside option than low-type borrowers gives rise to countervailing incentives which implies that low-type borrowers potentially mimic high-type borrowers and makes collateral relevant in my model.⁴

Timeline. At $t = 0$, a bank offers a loan contract which consists of the interest rate and collateral requirement. Contracts are complete. Each borrower applies for a contract. The bank decides whether or not to screen the borrower's project at $t = 1$. If screening, the bank further decides the intensity of screening. Depending on the outcome of screening, the bank

³With symmetric outside options, the bank's objective is to deter the high-type from mimicking the low-type, while extracting the surplus. Since it is more attractive for high-type borrowers to post collateral than low-type borrowers, increasing collateral requirements is not effective in deterring the high-type.

⁴Laffont and Martimort (2002) (pages 101-115) discuss the impact of type-dependent reservation utilities on the principal-agent problem and give an overview of several applications that have appeared in the broader contract theory literature.

then approves or rejects the borrower's application. If the loan is approved, the bank funds the project. If the application is rejected, the borrower makes her type-specific outside option. The payoffs are realized at $t = 2$, when all agents consume the output. The timing is illustrated in Figure 2.

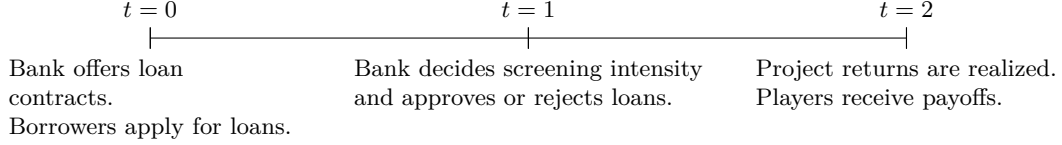


Figure 2: Timeline

Equilibrium definition. Since the uninformed player (the bank) moves first, this is a screening game. I look for the pure strategy subgame-perfect Nash equilibrium, i.e., I solve the game backwards and assume sequential rationality.⁵ The bank maximizes its expected payoff subject to the individual rationality and incentive compatibility constraints of each borrower-type. Additionally, the loan repayment rates must satisfy the borrowers' limited liability constraints and the non-negativity constraints (the feasibility constraints). We know from Rothschild and Stiglitz (1976) that a competitive equilibrium may not exist in a screening game due to banks undercutting one another. However, the non-existence problem does not arise in my setting since there is no threat of undercutting with a monopolistic bank.

3 Analysis

3.1 Symmetric information benchmark

In the symmetric information benchmark, a borrower's type is observable, but in the case of a low-type borrower, neither the bank nor the borrower knows the project quality. The bank offers type-contingent contracts, (R_i, C_i) , $i \in \{l, h\}$. R_i is the repayment rate and C_i is the

⁵Despite the presence of private information, the appropriate solution concept is a non-Bayesian equilibrium concept since the uninformed player moves first by offering contracts. Therefore, there are no Bayesian inferences to be made when the contracts are offered.

collateral requirement. Since it is assumed that collateral is costly, in the symmetric information benchmark, the bank sets $C_i = 0$ for each type.

If the borrower is high-type, the bank does not screen and always approves the loan application. If, on the other hand, the borrower is low-type, the bank incurs positive screening cost and extends credit only if the signal is positive, i.e., $s = s_g$. The bank solves:

$$\begin{aligned}
& \text{Max}_F && \beta q_2 p R_l - \beta(2q_2 - 1) - (1 - q_2) - \frac{\tau}{2} F^2 \\
& \text{subject to} && \\
& \text{(IRH)} && p(X - R_h) \geq H \\
& \text{(IRL)} && \beta q_2 p(X - R_l) \geq 0 \\
& \text{(FCs)} && 0 \leq R_i \leq X \ \forall i \in \{l, h\}
\end{aligned} \tag{6}$$

The bank maximizes its expected payoff with respect to the choice of screening intensity, F . Since only low-type borrowers' projects are screened, the screening pool consists of q_2 good projects (substitute $q_A = q_2$ in Equation (4) to derive the objective function). The bank sets the repayment rates such that borrowers' Individual Rationality (or IR) constraints are satisfied. These are IRH and IRL for high- and low-type borrowers, respectively. The left-hand side (LHS) of an IR constraint is the expected payoff of the corresponding borrower-type and the right-hand side (RHS) is their outside option. The FCs are the feasibility constraints which are the non-negativity and limited liability constraints. In the symmetric information benchmark, the monopolist lender extracts the full surplus, and hence, the screening decision is efficient. I summarize this case in the following proposition:

Proposition 1 (Symmetric information) *In the symmetric information benchmark, high-type borrowers always receive credit. The bank screens low-type borrowers' projects and grants credit only if the signal is positive. The repayments, R_i , and the screening intensity, F^{SI} , are:*

$$R_h = X - \frac{H}{p} \quad (7)$$

$$R_l = X \quad (8)$$

$$F^{SI} = \frac{1}{\tau}(q_2(pX - 2) + 1) \quad (9)$$

Proof. The proof is in the appendix. ■

3.2 Equilibrium

In this section, I derive the equilibrium for the asymmetric information case. Before characterizing the equilibrium, I state the following Lemma:

Lemma 1 *There does not exist a separating equilibrium in which only low-type borrowers obtain financing, while the high-type do not obtain financing.*

Proof. The proof is in the appendix. ■

Given Lemma 1, the possibilities are that the equilibrium is either pooling, with or without financing, or separating, in which high-type firms obtain financing without being screened. I consider these possibilities below.

By the revelation principle of Myerson (1979), the bank can induce truth-telling from borrowers by offering two incentive compatible contracts.⁶ Suppose that the bank offers type-contingent contracts, (R_i, C_i) , $i \in \{l, h\}$. Since high-type borrowers are less likely to default, it is relatively less costly for them to post collateral. Thus, the high-type posts more collateral than the low-type, i.e., $C_h > C_l$. Without loss of generality, the offered contracts are $(R_l, 0)$ and (R_h, C_h) .

⁶The revelation principle may be invoked due to the assumptions of sequential rationality and complete contracts. I discuss the renegotiations case in Section 6.3.

In the separating equilibrium, the bank screens only low-type borrowers' projects and solves:

$$\begin{aligned}
& \underset{F}{\text{Max}} && \beta q_2 p R_l - \beta(2q_2 - 1) - (1 - q_2) - \frac{\tau}{2} F^2 \\
& \text{subject to} && \\
& (\text{IRH}') && p(X - R_h) - (1 - p)C_h \geq H \\
& (\text{IRL}) && \beta q_2 p(X - R_l) \geq 0 \\
& (\text{ICH}) && p(X - R_h) - (1 - p)C_h \geq \beta p(X - R_l) + (1 - \beta)H \\
& (\text{ICL}) && \beta q_2 p(X - R_l) \geq q_2(p(X - R_h) - (1 - p)C_h) - (1 - q_2)C_h \\
& (\text{FCs}) && 0 \leq R_i \leq X \quad \forall i \in \{l, h\}
\end{aligned} \tag{10}$$

The LHS of high-type borrowers' new IR constraint reflects that when the project fails, she loses the posted collateral. The LHS of the IC constraint for borrower-type i is her expected payoff when she tells the truth, while the RHS is her expected payoff from mimicking the other type. If mimicking low-type borrowers, a high-type borrower's project is screened. Due to noisy screening, the high-type borrower is denied credit with probability $(1 - \beta)$, in which case she makes her outside option, H . If mimicking high-type borrowers, a low-type borrower is always granted credit, and with probability q_2 , the project is good.

First, consider the case that $W = 0$. In this case, borrowers cannot pledge collateral to secure loans and the equilibrium is pooling in which high- and low-type borrowers apply for the identical contract, i.e., the promised repayment is $R_h = R_l = \hat{R}$.⁷ In a pooling equilibrium, the low-type extract informational rents, which are increasing in the outside option of the high-type, H . Hence, bank profits in a pooling equilibrium are falling in H . There exists a threshold, H_1 , such that for $H = H_1$, bank profits in a pooling equilibrium are 0. I state these results in the following proposition:

⁷In competitive credit markets, the high-type can potentially break the pooling equilibrium by accepting a worse interest rate which makes it unattractive for the low-type to participate (see e.g., Bernhardt et al. 2022). However, since I consider a monopolist bank, each borrower-type is already on their participation constraint implying that separation through participation constraints cannot take place in this setting.

Proposition 2 (No collateral) *When neither borrower-type posts collateral, the equilibrium is pooling with financing if $H \leq H_1$ and there is no financing if $H > H_1$. In a pooling equilibrium, the bank screens both high- and low-type borrowers' projects and grants credit if screening produces a good signal. The repayment, \hat{R} , and the screening intensity, F^P , are:*

$$\hat{R} = X - \frac{H}{p} \quad (11)$$

$$F^P = \frac{1}{\tau}(q_p(pX - H - 2) + 1) \quad (12)$$

Proof. The proof is in the appendix. ■

Next, consider the case that borrowers may post collateral. From the constraints in (10), we derive the feasible bounds on collateral requirements:

$$\underbrace{\frac{q_2 H (1 - \beta^C)}{(1 - q_2)}}_{\equiv \hat{C}} \leq C_h \leq \underbrace{\frac{q_2 H}{(1 - q_2)}}_{\equiv \bar{C}} \quad (13)$$

The lower bound, \hat{C} , comes from violating the low-types' IC constraint and the upper bound, \bar{C} , comes from the feasibility constraint that the interest rate cannot be higher than X . Below the lower bound, there is no feasible interest rate for which the low-type do not mimic the high-type. At the upper bound, the surplus extracted by the low-type goes to 0, as in the symmetric information benchmark. Note that when $H = 0$, both the lower and upper thresholds of C_h , \hat{C} and \bar{C} , become equal to 0, and given Proposition 2, collateral cannot effectively separate borrower-types.

Suppose that $W = \gamma \bar{C} + (1 - \gamma) \hat{C}$, with $\gamma \in [0, 1]$. γ equals 0 if borrowers have just enough assets to achieve separation, but no more. A higher γ allows a borrower to post more collateral. Therefore, an increase in γ can be interpreted as an increase in the availability of collateral. Having introduced the parameter, γ , we are ready to fully characterize the equilibrium in

Proposition 3.

Proposition 3 (Costless collateral) *Separation arises if $W \geq \hat{C}^*$, where \hat{C}^* is:*

$$\hat{C}^* = \frac{q_2 H}{1 - q_2} \left[\frac{1}{2} - \frac{1}{\tau} (q_2 (pX - H - 2) + 1) \right] \quad (14)$$

High-type borrowers post all pledgeable assets as collateral, $C_h = W$, and obtain credit. The bank screens low-type borrowers' projects and grants credit only if the signal is positive. The repayments, R_i , and the screening intensity, F^C , are:

$$R_h = X - \frac{1}{p} (H + (1 - p)W) \quad (15)$$

$$R_l = X - \frac{H}{p} + \frac{\gamma H}{p} \quad (16)$$

$$F^C = \frac{1}{\tau} (q_2 (pX - H - 2) + 1 + \gamma q_2 H) \quad (17)$$

Proof. The proof is in the Appendix. ■

For $\gamma = 0$, the low-type extract a part of the surplus since $R_l = \hat{R} < X$. An increase in γ implies higher collateral requirements for the high-type, which allows the bank to increase the repayment that the low-type is charged. A higher R_l , in turn, improves the bank's incentive to screen the low-types' projects, F^C . In this sense, collateral and bank screening are complements due to a spillover from collateralized to uncollateralized loans.

Given the positive spillover effect of collateral, if collateral is costless, collateral requirements are as high as possible. Thus, different from existing screening models of collateral, the equilibrium level of collateral is uniquely pinned down in my model when collateral is costless. $F^{SI} \geq F^C$ if $(1 - \gamma) \geq 0$: For $\gamma = 1$, the equilibrium becomes identical to the symmetric information benchmark in terms of the intensity with which the uncollateralized loans are screened. For $\gamma < 1$, the screening intensity differs from the symmetric information benchmark since low-

type borrowers extract a fraction of the surplus. Since low-type borrowers extract a fraction of the surplus when the availability of collateral is limited, the bank's screening incentives are lower.⁸

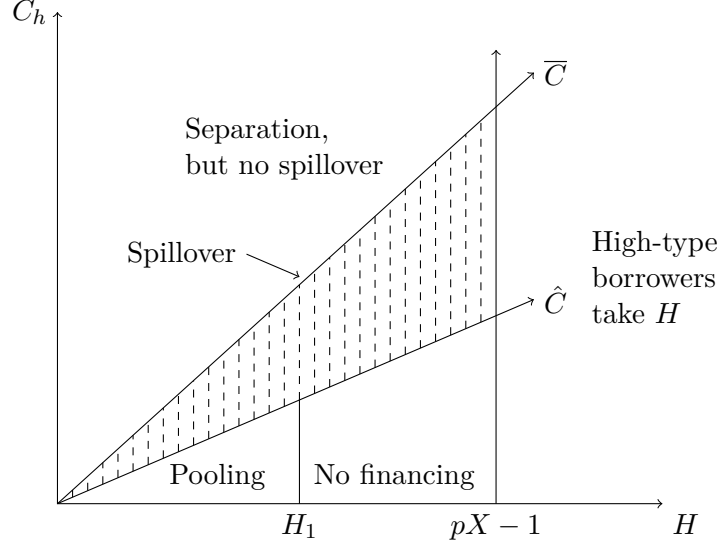


Figure 3: Equilibrium regions

In Figure 3, I identify which equilibrium arises in each parameter region. I plot the bounds on C_h from Equation (13) and the upper bound on H from Assumption A4 in the (C_h, H) space. Both the bounds on C_h start at the origin and are upward sloping and linear in this space, \bar{C} being steeper than \hat{C} . The upper bound on H is represented by the vertical line, $H = pX - 1$. To the right of this vertical line, the high-type takes their outside option, and the bank screens the low-types' projects with intensity, $F = F^{SI}$, and provides funds to them at $R_l = X$ if screening produces a positive signal.

The parameters of interest lie to the left of the vertical line, $H < pX - 1$. Below the shaded region, collateral requirements are insufficient to foster separation: there is a pooling equilibrium with financing to the left of H_1 , while there is no financing in equilibrium to the right of H_1 (see Proposition 2; H_1 may or may not lie in the permissible parameter range, $H \in (0, pX - 1)$).

The spillover effect arises in the shaded region. Along the lower contour of the shaded region,

⁸Note that given the constraints of the economy, a regulator who maximizes net social surplus cannot improve upon this outcome. The regulator faces the same set of constraints as the bank and since the constraints bind to determine the contract terms, the regulator's solution coincides with that of the bank.

collateral requirements are just enough to separate borrower-types as long as $C_h > 0$, but the corresponding R_l from the ICL is $R_l = \hat{R} < X$. For any H , an increase in C_h allows the bank to charge a higher interest rate to the low-type. At the upper contour of the shaded region, the corresponding R_l from ICL equals X , i.e., the bank extracts all the surplus from screening. Above the shaded region, higher collateral requirements does not affect the equilibrium, since R_l cannot be increased beyond X .

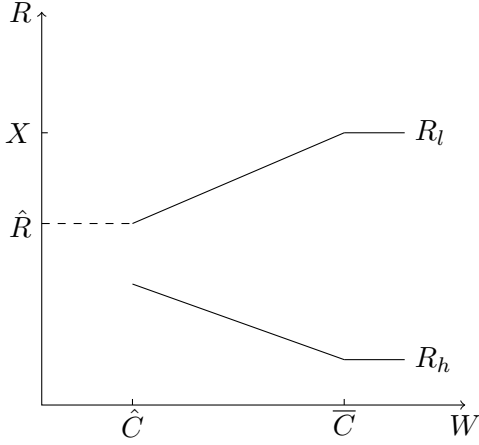


Figure 4: Repayment

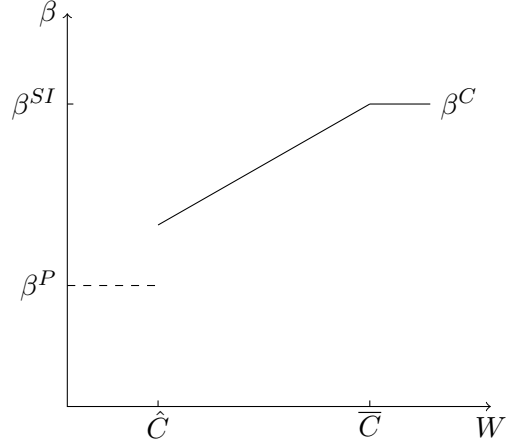


Figure 5: Effort

In Figure 4, I plot the repayment in each equilibrium against collateral availability, W . For $W < \hat{C}$, the equilibrium is pooling (assuming $H < H_1$), and the repayment is \hat{R} (dashed line). At $W = \hat{C}$ there is separation; the low-type are charged $R_l = \hat{R}$, while the high-type are charged $R_h = \hat{R} - \frac{1-p}{p}\hat{C} < \hat{R}$. As collateral availability increases beyond \hat{C} , the high-types' repayment falls in order to ensure that the IRH constraint is not violated when collateral requirements increase. At the same time, the bank charges a higher repayment to the low-type till $W = \bar{C}$ at which point $R_l = X$.

In Figure 5, I plot the screening precision in each equilibrium against collateral availability, W . For $W < \hat{C}$, the equilibrium is pooling (assuming $H < H_1$), and precision is β^P (dashed line). At $W = \hat{C}$ there is separation; only the low-type are screened, and there is a discrete increase in screening precision. As collateral availability increases beyond \hat{C} , the repayment charged to the low-type is higher which leads to better screening incentives and higher precision. At

$W = \bar{C}$, equilibrium screening precision is the same as in the symmetric equilibrium benchmark, $\beta^C = \beta^{SI}$. Beyond $W = \bar{C}$ this effect no longer exists since $R_l = X$ and the low-type cannot be charged a higher interest rate.

Next, I consider the case that pledging collateral is costly, i.e., $k > 0$. The lender faces the following trade-off. On the one hand, $R_l(C_h)$ is increasing in C_h , which implies that as the high-type posts a higher level of collateral, the bank extracts more of the surplus from lending to the low-type. On the other hand, collateral entails a deadweight loss (and the cost is borne by the monopolist lender). Due to this trade-off, there exists a threshold, \bar{k} , such that for $k = \bar{k}$, bank profits in a separating equilibrium are the same whether collateral requirements are low, $C_h = \hat{C}$, or high, $C_h = \bar{C}$ (and profits are lower for $C_h = \bar{C}$ when k is higher).

Lemma 2 *Suppose $k > 0$ and $C_h \geq \bar{C}$. In a separating equilibrium, collateral requirements are high, $C_h = \bar{C}$, if $k \leq \bar{k}$, and low, $C_h = \hat{C}$, otherwise.*

Proof. The proof is in the appendix. ■

In existing screening models of collateral, the equilibrium features the minimum level of collateral required to separate, whenever collateral entails a non-zero cost. My setting yields a different result for the following reason: as high-type borrowers post a higher amount of collateral, the bank extracts more of the surplus from lending to low-type borrowers. This, in turn, improves the bank's incentives to screen the low-type and leads to higher bank profitability. That is, there is a positive spillover effect from collateralized to uncollateralized loans.

For given collateral requirements, bank profits are falling in a separating equilibrium when the cost of posting collateral is higher, while profits in a pooling equilibrium are unaffected. There exist thresholds k_γ^s , such that for $k = k_\gamma^s$, bank profits in a separating equilibrium with $\gamma \in \{0, 1\}$ are 0, and profits are negative for a higher k . There exist thresholds, k_γ^p , such that for $k = k_\gamma^p$, bank profits in a separating equilibrium with $\gamma \in \{0, 1\}$ equal bank profits in the pooling equilibrium, and profits are higher in the pooling equilibrium for a higher k . I characterize the

equilibrium in the proposition below:

Proposition 4 (Costly collateral) *Suppose $W \geq \bar{C}$. The equilibrium is separating with $C_h = \bar{C}$ for a low cost of collateral, $k \leq \min(\bar{k}, k_1^s, k_1^p)$, separating with $C_h = \hat{C}^*$ for an intermediate cost of collateral, $\bar{k} < k \leq \min(k_0^s, k_0^p)$. For a high cost of collateral, $k > \max(k_0^p, k_1^p)$, the equilibrium is pooling if $H \leq H_1$, and there is no financing if $H > H_1$ and $k > \max(k_0^s, k_1^s)$.*

Proof. The proof is in the appendix. ■

The spillover effect arises when the level of collateral availability is intermediate, $\hat{C}^* \leq W < \bar{C}$, and the cost of posting collateral is sufficiently low, $k \leq \min(\bar{k}, k_1^s, k_1^p)$. For these parameters, an increase in collateral availability leads to higher collateral requirements, which has knock-on effects on the terms of uncollateralized loans and the bank's incentives to screen these projects.

I combine an ex-ante screening device (i.e., collateral) and an ex-post screening device (i.e., direct screening by the bank) to deliver the spillover effect result. The two screening devices play distinct roles in my model since the ex-ante device resolves information asymmetry and the ex-post device solves (noisily) residual uncertainty. To illustrate the role of each, it is useful to consider benchmarks by shutting down the other screening device:

1. Assume that $q_2 = 0$. In this case, there is no scope for direct screening by the bank since conditional on separation, there is no residual uncertainty. High-type borrowers post collateral and obtain financing. Low-type borrowers do not obtain financing. Assuming non-zero q_2 , but setting $\tau = \infty$ also produces a similar outcome. In this case, $F \rightarrow 0$, and given Assumption A2, low-type borrowers do not obtain financing. Since collateral is costly and there are no benefits of collateral beyond separating borrower-types, the minimum amount of collateral necessary for separation is used, and no more (identical to standard screening models like Besanko and Thakor (1987a) and Bester 1985).
2. Assume that $W = 0$ or $k = \infty$, i.e., posting collateral is infeasible. This implies that

there is pooling of high- and low-type borrowers if $H \leq H_1$ and no bank financing in equilibrium if $H > H_1$ (see Proposition 2). In either case, since there is no separation of borrower-types, a spillover effect cannot arise.

Therefore, the spillover effect does not arise if either of the screening devices is shut down.

3.3 Extension: More than two types

In the baseline, I consider two types of borrowers. In Section 6.2, I consider an extension by introducing intermediate-type borrowers. Intermediate borrowers have a good project with probability $q_m \in (q_2, 1)$; they are more likely to have good projects than low-type borrowers, but may possess bad projects with some positive probability. I show that there exists a partial pooling equilibrium in which the higher types pool on the same contract and separate from the low-type. Suppose that collateral availability $W = \hat{C}$; then, high-type and intermediate borrowers post collateral to separate from low-type borrowers. With that as the starting point, an increase in collateral availability does not affect the screening pool but allows the bank to extract more of the surplus from low-type borrowers, which, in turn, positively affects the bank's screening incentives. Thus, as in the baseline model, there arises a positive spillover effect from collateralized to uncollateralized loans.

4 Empirical relevance and implications

4.1 Empirical relevance

In this section, I discuss how realistic two of the key assumptions of my model are.

Bank market power: I have considered a monopolist bank. The results go through qualitatively if banks retain positive market power in the uncollateralized loan market. As long as banks have positive market power, they increase their surplus by increasing collateral requirements for

high-type borrowers since it allows them to charge higher interest rates to low-type borrowers. Thus, with positive bank market power, I can perform the comparative static analyses with respect to the availability of collateral, which deliver the key results of my model.⁹

Conceptually, one could assume that direct screening by banks is a scarce skill, which would allow banks to extract surplus in the uncollateralized loan market. In a sample of 48 countries between 1995 and 2007, Forssbaeck and Shehzad (2015) estimate that the country-level loan market Lerner index, a measure of market power, has a mean close to 50% (see also Delis et al. 2016 and Beck et al. 2013). The estimates in these studies indicate that banks enjoy substantial market power, which gives empirical relevance to my model.

Role of collateral. In the model, high-type borrowers use assets unconnected to the firm (e.g., personal assets or third-party guarantees) as collateral to separate themselves from low-type borrowers. The empirical relevance of my model hinges on whether personal assets or third-party guarantees are indeed used as corporate collateral and, if so, how widespread the phenomenon is. Bahaj et al. (2020) present compelling evidence that the use of personal assets as corporate collateral can mitigate financing frictions and that an increase in the value of directors' personal assets leads to higher firm-level investment (see also Anderson et al. 2022 and Beyhaghi 2022).¹⁰

4.2 Empirical implications

My model shares with other screening models the prediction that safer borrowers post collateral to separate from riskier borrowers; this prediction has considerable empirical support – Berger et al. (2011) provide evidence that unobserved risk is negatively correlated with collateral; see

⁹The interesting results in my model arise from the interaction between the low-types' incentive compatibility constraint and the high-types' binding participation constraint. If the degree of competition is very high such that the high-types' participation constraint becomes slack, the analysis changes fundamentally, and becomes similar to models with competitive lenders such as Besanko and Thakor (1987a).

¹⁰Most of the evidence on directors pledging personal assets to secure corporate loans relates to privately held companies, but there is anecdotal evidence that this practice is also prevalent in public companies. E.g., in 2020, Richard Branson pledged as collateral his private Caribbean island (Necker Island) to raise funds from the UK government to prevent the collapse of his Virgin Group empire.

also, Ioannidou et al. (2022) and Godlewski and Weill (2011).

From Equation (36), the spillover effect from collateralized to uncollateralized borrowers arise through two parameters – high-types’ outside option, H , and the availability of collateral, γ . Comparative statics with respect to these variables generate new predictions which other related models cannot deliver. In deriving these predictions, I consider the case that the cost of pledging collateral is small (i.e., $k \leq \min(\bar{k}, k_1^s, k_1^p)$) and the borrowers have intermediate levels of personal wealth, $\hat{C}^* \leq W < \bar{C}$ (these parameters correspond to the shaded region in Figure 3).

Prediction 1. *An increase in high-type borrowers’ outside options leads to a higher intensity with which banks screen uncollateralized loans.*

From Equation (17), equilibrium screening intensity, F^C , is falling in high-type borrowers’ outside option, H .¹¹ The intuition is as follows: as high-type borrowers’ outside options increase, it becomes more attractive for low-type borrowers to mimic the high-type, which lowers the interest rate that the low-type can be charged. In turn, this reduces the bank’s incentive to screen low-type borrowers diligently since they retain less of the surplus.

It seems reasonable to interpret high-type borrowers’ outside option as reflecting the strength of local economic conditions: in weaker economic conditions (e.g., high local unemployment rate), neither borrower-type obtains financing from personal networks since friends and family are themselves under-employed and financially constrained, while when economic conditions improve and financial constraints of friends and family ease, high-type borrowers have better outside options since they can obtain alternate financing more easily. With this interpretation in mind, the prediction is consistent with the findings in Howes and Weitzner (2023) that the quality of information produced by banks at loan origination improves as local economic conditions deteriorate, i.e., the local unemployment rate increases (see also, Lisowsky et al.

¹¹ $\frac{\partial F^C}{\partial H} = -\frac{1}{\tau} q_2 (1 - \gamma) < 0$

2017).¹² Further, also consistent with my model, Howes and Weitzner (2023) find that the cyclical sensitivity of information quality is driven by loans which have larger loss given defaults (in my model these are the loans which are not backed by collateral). The findings in Howes and Weitzner (2023) can also be explained by Dang et al. (2012). My model offers an alternative explanation to Dang et al. (2012); in their model, information production is triggered by a fall in the value of collateral, while in my model, information production is triggered by a fall in borrowers' outside options (which corresponds to higher local unemployment rate in the empirical setting of Howes and Weitzner 2023). As a direct implication of higher screening intensity in economic downturns, the model predicts that lending standards are countercyclical. Asea and Blomberg (1998), Dell'Ariccia et al. (2012), Rodano et al. (2018), and Cao et al. (2022) present empirical evidence in support of this prediction.

Prediction 2. *An increase in collateral availability leads to an improved average quality of uncollateralized loans issued.*

As high-type borrowers post more collateral, low-type borrowers are charged higher interest rates and, hence, their projects are screened more diligently. More diligent screening by the bank reduces errors, i.e., fewer good projects are rejected and more bad projects are rejected, leading to a higher average quality of the uncollateralized loans. To the best of my knowledge, this prediction has not been tested.¹³

Prediction 3. *Uncollateralized lending increases in collateral availability if low-type borrowers have many good projects, and falls in collateral availability, otherwise.*

The total amount of uncollateralized lending is $L^S = (1 - q_1)(q_2\beta^C + (1 - q_2)(1 - \beta^C))$. Differ-

¹²The sample in Howes and Weitzner (2023) contain many small and non-public firms which is the market segment that I model in this paper; in the same database, Beyhaghi (2022) finds that around 46% of loans carry third-party guarantees, consistent with the role of collateral considered here.

¹³Ioannidou et al. (2022) consider the spillover effect from secured to unsecured loans for a given borrower, which is not the case in my model.

entiating L^S with respect to γ :

$$\frac{\partial L^S}{\partial \gamma} = \frac{q_2 H}{\tau} (1 - q_1)(2q_2 - 1) \quad (18)$$

The derivative is positive (resp. negative) and uncollateralized lending increases (resp. falls) in the availability of collateral if low-type borrowers have a high (resp. low) fraction of good projects, $q_2 > 0.5$ (resp. $q_2 < 0.5$). The intuition is as follows: As the availability of collateral increases, the bank exerts a higher screening effort and identifies project quality more accurately. Therefore, if there are many (resp. few) good projects in the pool of uncollateralized loan applications, the bank increases (resp. reduces) its supply of uncollateralized credit. Thus, the impact of an increase in collateral availability will vary in the cross-section; industries or countries with high (resp. low) q_2 will experience an increase (a fall) in the levels of bank credit.

5 Concluding remarks

I present a model of a credit market characterized by severe information asymmetry. The market segment consists of small, private firms with limited data and limited assets; often, these firms obtain financing by pledging personal assets of directors. As in existing screening models, high-type borrowers post collateral to separate from low-type borrowers; since the high-type post collateral, the bank does not screen their projects (this is the well-known substitution relationship between collateral and bank screening, as in Manove et al. 2001). The novel implication is that, when posting collateral is not too costly and the level of collateral availability is intermediate, increasing collateral requirements for high-type borrowers allows banks to charge a higher interest rate to the low-type, which improves their incentives to screen uncollateralized loan applications from the low-type. Thus, the model uncovers a new mechanism of how an increase in collateral availability affects information production.

6 Appendix

6.1 Omitted proofs

Proof of Lemma 1. The proof is by contradiction. Suppose that the bank commits to serve low-type borrowers only. At $t = 0$ the bank offers an $R = X$ contract. This contract violates the high-types' participation constraint, so they stay out. Only low-type borrowers apply for this contract. The bank screens the low-types' projects and extends credit whenever the signal is positive. At the same time, at $t = 1$, the bank knows that borrowers who are not served are the high-type (because they turned down the $R = X$ contract at $t = 0$). Then, the bank offers a contract to the high-type with interest rate, $R = X - \frac{H}{p}$, which satisfies the high-types' participation constraint. Moreover, anticipating this outcome the low-type will not apply for the $R = X$ contract at $t = 0$. Therefore, since the bank cannot credibly commit to not serve high-type borrowers at $t = 1$, the conjectured equilibrium unravels.

Proof of Proposition 1. I solve for the equilibrium by backward induction and begin with the bank's screening decision at $t = 2$. An interior solution to the bank's problem is given by the first order condition with respect to F :

$$q_2(pR_l - 2) + 1 - \tau F = 0 \tag{19}$$

At $t = 0$, the bank sets repayment rates such that the IR constraints of borrowers are satisfied. The IR constraints bind for both types. To see why this is the case, consider a candidate equilibrium in which the IR constraint for type i is not binding. Then, the bank can increase its profits by increasing R_i a little, without violating the other constraints. Hence the candidate equilibrium with non-binding IR constraint for either type is not stable. From the relevant IR constraints, the repayment rates, R_h and R_l , are derived (Equations (7) and (8)). The non-negativity constraints are satisfied if $pX > H$, which holds due to Assumption A4, and the

limited liability constraints are always satisfied, binding for low-type borrowers and slack for high-type borrowers. Substitute R_l in the first order condition to derive the screening intensity, F^{SI} (Equation (9)). Finally, I check that the solution is interior, i.e., $0 < F^{SI} < \frac{1}{2}$. Take the extreme value, $pX \rightarrow 1$ which does not violate any of the assumptions: F^{SI} becomes $\frac{1}{\tau}(1 - q_2)$, which is strictly positive since $q_2 < 1$. Increasing pX leads to a *higher* F , therefore, $F^{SI} > 0$ holds for all parameters. Assumption $A3$ ensures $F^{SI} < \frac{1}{2}$.

Proof of Proposition 2. Suppose that $C_h = 0$ and there is a separating equilibrium with $R_h \neq R_l$. The IC constraints are jointly satisfied only if $(1 - \beta) \leq 0$. However, this condition is never satisfied for any $\beta < 1$. For $\beta = 1$, it is satisfied for $R_h = R_l$, which violates the starting assumption of $R_h \neq R_l$. Further, as shown in Lemma 1, time inconsistency rules out a separating equilibrium in which only low-type borrowers obtain financing. Thus, separation does not arise with $C_h = 0$. There are two possibilities – either there is no bank financing in equilibrium or there is a pooling equilibrium in which both borrowers-types apply for the identical contract, i.e., the promised repayment is $R_h = R_l = \hat{R}$.

In a pooling equilibrium, the bank solves the following problem:

$$\begin{aligned}
& \text{Max}_F \quad \Pi_P = \beta q_p p \hat{R} - \beta(2q_p - 1) - (1 - q_p) - \frac{\tau}{2} F^2 \\
& \text{subject to} \\
& \text{(IRH)} \quad p(X - \hat{R}) \geq H \\
& \text{(IRL)} \quad \beta q_2 p(X - \hat{R}) \geq 0 \\
& \text{(FCs)} \quad 0 \leq \hat{R} \leq X
\end{aligned} \tag{20}$$

I solve for the equilibrium by backward induction and begin with the bank's screening decision at $t = 2$. An interior solution to the bank's problem is given by the first order condition:

$$q_p(p\hat{R} - 2) + 1 - \tau F = 0 \tag{21}$$

At $t = 0$, the bank sets the repayment rates such that the IR constraints of the borrowers are satisfied. In order to make sure that the high-type participate, her individual rationality constraint must be satisfied. This automatically satisfies low-type borrowers' individual rationality constraint. From the IRH constraint, derive the interest rate in the pooling equilibrium, \hat{R} (Equation (11)). The condition is satisfied with equality. If it is slack, the bank can increase its profits by increasing \hat{R} a little, without violating any other relevant constraints. Substitute \hat{R} in the first order condition to derive the screening intensity in the pooling equilibrium, F^P (Equation (12)). Finally, I check that the solution is interior, i.e., $0 < F^P < \frac{1}{2}$. Take the extreme values, $pX \rightarrow 1$ and $H \rightarrow pX - 1$: F^P becomes $\frac{1}{\tau}(1 - q_p pX)$, which is strictly positive given Assumption A2. Increasing pX and/or reducing H leads to a *higher* F , therefore $F^P > 0$ holds for all parameters. Assumption A3 ensures $F^P < \frac{1}{2}$.

Using \hat{R} , bank profits in a pooling equilibrium becomes:

$$\Pi_P = \beta^P(q_p(pX - H - 2) + 1) - (1 - q_p) - \frac{\tau}{2}F^{P2} \quad (22)$$

Derivating Π_P with respect to H :

$$\frac{\partial \Pi_P}{\partial H} = -q_p \underbrace{\left[\frac{1}{2} + \frac{1}{\tau}(q_p(pX - H - 2) + 1) \right]}_{\beta^P} < 0 \quad (23)$$

Π_P is falling in H . There exists a threshold, H_1 such that for $H = H_1$, $\Pi_P = 0$. H_1 is given by:

$$H_1 = \frac{\tau - 4q_2 - 4q_1 \pm \tau((\tau - 8q_2 - 8q_1 + 8q_1q_2 + 8)/\tau)^{0.5} + 4q_1q_2 + 2pXq_1 + 2pXq_2 - 2pXq_1q_2 + 2}{2(q_1 + q_2 - q_1q_2)} \quad (24)$$

There are two roots of H_1 . The relevant root must be in the permissible range for H which is $(0, pX - 1)$, from Assumption A4. If the smaller root is greater than $pX - 1$, then there is no pooling equilibrium with financing. If the bigger root is smaller than 0, then there is always

financing in a pooling equilibrium. If $H \leq H_1$, bank profit is positive (weakly, if $H = H_1$), and there exists a pooling equilibrium in which both borrower-types obtain financing. If $H > H_1$, bank profit is strictly negative, and there is no bank financing in equilibrium.

Proof of Proposition 3. To solve the problem I initially assume that the ICH constraint is satisfied. After solving the modified problem, I verify that a solution exists which does not violate the starting assumption. In the relaxed problem, the IRH' constraint must bind; if not binding, the bank can increase R_h a little to increase its profits without violating the other constraints. Next, note that either the IRL or ICL constraint must bind. If neither constraint is binding the bank can increase R_l a little to increase its profits without violating the other constraints. It is the ICL constraint which binds, and not the IRL constraint, if the RHS of the ICL constraint is greater than 0. Using the IRH' constraint, the ICL constraint binds if:

$$C_h \leq \frac{q_2 H}{(1 - q_2)} \equiv \bar{C} \quad (25)$$

Equation (25) represents an upper bound on the amount of collateral used. If this condition is violated, then the IRL constraint binds which implies $R_l = X$.

From the IRH' constraint, high-type borrowers' repayment rate when the loan is collateralized becomes:

$$R_h(C_h) = X - \frac{1}{p}(H + (1 - p)C_h) \quad (26)$$

The interest rate is falling in the level of collateral, to allow the high-type to achieve their outside option, in expectation. The limited liability constraint ($R_h < X$) is always satisfied. The non-negativity constraint for a high-type borrower's repayment rate is satisfied, $R_h \geq 0$, if:

$$H \leq pX - (1 - p)C_h \quad (27)$$

Notice that the RHS is falling in C_h , implying that a higher C_h makes the condition more binding. That is, if the condition is satisfied for the upper bound of C_h , it will be satisfied for smaller values of C_h . Substituting $C_h = \bar{C}$ and simplifying:

$$H \leq \frac{pX(1 - q_2)}{1 - pq_2} \quad (28)$$

Equation (27) is satisfied if the RHS above is greater than $pX - 1$ which is the upper bound on H from Assumption A4:

$$\frac{pX(1 - q_2)}{1 - pq_2} \geq pX - 1 \quad (29)$$

$$\implies pq_2(pX - 1) \geq q_2pX - 1 \quad (30)$$

The above condition is always satisfied since the LHS is positive (given Assumption A1) while the RHS is negative (given Assumption A2 and $q_2 \leq q_p$). Thus, Equation (27) is always satisfied.

Supposing that Equation (25) is satisfied, i.e., the ICL constraint binds, and using $R_h(C_h)$, the repayment rate charged to low-type borrowers, $R_l(C_h)$, is derived from ICL:

$$R_l(C_h) = X - \frac{q_2H - (1 - q_2)C_h}{\beta^C q_2 p} \quad (31)$$

The upper bound, $C_h \leq \bar{C}$, ensures that the limited liability constraint is satisfied, i.e., $R_l \leq X$.

The non-negativity constraint is satisfied if $R_l \geq 0$, which gives a lower bound as follows:

$$C_h \geq \frac{q_2(H - \beta^C pX)}{(1 - q_2)} \equiv \underline{C} \quad (32)$$

Finally, I verify that bounds on C_h do not violate the starting assumption that the ICH con-

straint is satisfied. Substituting $R_h(C_h)$ and $R_l(C_h)$ in the ICH constraint:

$$C_h \geq \frac{q_2 H(1 - \beta^C)}{(1 - q_2)} \equiv \hat{C}(\beta^C) \quad (33)$$

The ICH constraint is satisfied only if collateral is sufficiently large, $C_h \geq \hat{C}$. Combining with the feasibility constraints of low-type borrowers' repayment rate, the collateral that high-type borrowers need to post, C_h , in order to achieve separation lies in the range, $\max(\underline{C}, \hat{C}) \leq C_h \leq \bar{C}$. It is easily verified that $\hat{C} < \bar{C}$ is always satisfied for any $\beta > 0$, and $\hat{C} > \underline{C}$ is satisfied as long as $pX > H$, which holds due to Assumption, A4. Therefore, the feasible range of collateral requirements for which separation is achieved is given by $C_h \in [\hat{C}, \bar{C}]$.

Consider the case that for a given γ , $C_h = \lambda \bar{C} + (1 - \lambda) \hat{C}$ with $\lambda \in [0, \gamma]$; λ is to be determined in equilibrium. Substituting R_h , R_l , and C_h in the bank's objective function and taking the first order condition gives the equilibrium screening intensity:

$$F^C = \frac{1}{\tau} (q_2(pX - H - 2) + 1 + \lambda q_2 H) \quad (34)$$

Substituting F^C into C_h , the equilibrium level of collateral is:

$$C_h = \frac{q_2 H}{1 - q_2} \left[1 - (1 - \lambda) \left[\frac{1}{2} + \frac{1}{\tau} (q_2(pX - H - 2) + 1 + \lambda q_2 H) \right] \right] \quad (35)$$

By setting $\lambda = 0$ in C_h , we derive the minimum level of collateral requirements which is necessary to achieve separation between borrower-types, \hat{C}^* (Equation (14)). To fully characterize the equilibrium, substitute F^C and C_h into R_h and R_l . High-type borrowers' repayment rate, R_h is falling in the equilibrium amount of collateral (such that the high-type are indifferent with regards to the level of collateral requirement), while low-type borrowers' repayment is increasing

in the amount of collateral, as follows:

$$R_l = X - \frac{H}{p} + \frac{\lambda H}{p} \quad (36)$$

For $\lambda = 0$, $R_l = \hat{R}$. For $\lambda = 1$, a low-type borrower's repayment becomes $R_l = X$. Therefore, by setting $\lambda = \gamma$, the bank extracts the maximum surplus from lending to the low-type borrower (if $\gamma = 1$, the bank extracts the full surplus).

Proof of Lemma 2. Consider the case that for a given γ , $C_h = \lambda \bar{C} + (1 - \lambda)\hat{C}$ with $\lambda \in [0, \gamma]$; λ is to be determined in equilibrium. The lemma states that λ is always given by a corner solution, i.e., either $\lambda = 0$ if posting collateral is sufficiently costly or $\lambda = \gamma$ if posting collateral is cheap. The bank's objective function in the separating equilibrium, including the cost of collateral, is:

$$\begin{aligned} \Pi_S = & q_1(pR_h + (1 - p)C_h - 1 - kC_h) \\ & + (1 - q_1) \left(\beta^C(q_2(pR_l - 2) + 1) - (1 - q_2) - \frac{\tau}{2}F^{C^2} \right) \end{aligned} \quad (37)$$

With probability q_1 the bank makes collateralized loans to high-type borrowers (the top line). The collateral cost is incurred whether or not the project fails (e.g., to transfer and store the collateral when the loan is approved); it could be adapted to the case that the cost of collateral is only incurred on the failure of the project without qualitatively affecting the results. With probability $(1 - q_1)$ the bank makes uncollateralized loans to the low-type (the second line). Derivating Π_S with respect to λ and k ,

$$\frac{\partial \Pi_S^2}{\partial \lambda \partial k} = -q_1 \frac{\partial C_h}{\partial \lambda} \quad (38)$$

$$= - \underbrace{\frac{q_1 q_2 H}{1 - q_2}}_{>0} \left[\frac{1}{2} + \frac{1}{\tau} (q_2(pX - H - 2) + 1 + \lambda q_2 H - (1 - \lambda)q_2 H) \right] \quad (39)$$

The cross derivative is negative if the second term above is positive:

$$\frac{1}{2} + \frac{1}{\tau}(q_2(pX - H - 2) + 1 + \lambda q_2 H) > \frac{1}{\tau}(1 - \lambda)q_2 H \quad (40)$$

$$\implies \frac{1}{2} + \frac{1}{\tau}(q_2(pX - 2) + 1) > \frac{2}{\tau}(1 - \lambda)q_2 H \quad (41)$$

The RHS is increasing in λ and falling in H . Thus, the condition becomes most binding for the smallest λ and highest H . Taking the extreme values, $\lambda = 0$ and $H = pX - 1$, Equation (41) becomes:

$$\tau > 2(q_2 pX - 1) \quad (42)$$

Equation (42) is always satisfied since $\tau > 0$ and the RHS is negative due to Assumption A2 and $q_2 < q_p$. Given that Equation (42) is satisfied for the extreme values considered above, Equation (41) will be satisfied for any feasible values of λ and H . Thus, the cross derivative, $\frac{\partial \Pi_S^2}{\partial \lambda \partial k}$, is negative for all feasible parameters.

Suppose that collateral is costless, i.e., $k = 0$. In this case, Π_S is increasing in λ , since conditional on separation, the higher use of collateral allows the bank to extract the full surplus from lending to the low-type, which leads to higher profits. Since $\frac{\partial \Pi_S^2}{\partial \lambda \partial k} < 0$, as k increases, $\frac{\partial \Pi_S}{\partial \lambda}$ is falling and it becomes negative for some k which is sufficiently high. When the derivative is (weakly) positive, i.e., bank profit is (weakly) increasing in λ , we set λ as high as possible (i.e., $\lambda = \gamma$); otherwise, we set $\lambda = 0$. Suppose $\gamma = 1$. There exists a threshold, \bar{k} , such that for $k = \bar{k}$, profits in the separating equilibrium with $\lambda = 1$ equals the profits in the separating equilibrium with $\lambda = 0$. The expression for \bar{k} is:

$$\bar{k} = \frac{(q_1 - 1)(q_2 - 1)(\tau - 4q_2 - q_2 H + 2pq_2 X + 2)}{q_1(\tau - 4q_2 - 2q_2 H + 2pq_2 X + 2)} \quad (43)$$

The bank sets $\lambda = 1$ for $k \leq \bar{k}$ and sets $\lambda = 0$ for $k > \bar{k}$.

Proof of Proposition 4. There are four possibilities: separating with $C_h = \bar{C}$ (i.e. $\gamma = 1$), separating with $C_h = \hat{C}^*$ (i.e. $\gamma = 0$), pooling in which no collateral is used, and no financing for either borrower-type. Which equilibrium emerges depends on exogenous parameters.

Bank profits in a separating equilibrium are falling in the cost of collateral, k . The exist thresholds k_γ^s such that when $k = k_\gamma^s$, $\Pi_S(\gamma) = 0$ for $\gamma \in \{0, 1\}$. k_γ^s are given by:

$$k_0^s = \frac{-(2\tau((q_1 - 1)((2q_2 + q_2H - q_2pX - 1)^2/(2\tau) - q_2 + (2q_2 + q_2H - q_2pX - 1)(\tau - 4q_2 - 2q_2H + 2q_2pX + 2))/(2\tau) + 1) - q_1(H - pX + 1)(q_2 - 1))}{q_1q_2H(4q_2 + \tau + 2q_2H - 2q_2pX - 2)} \quad (44)$$

$$k_1^s = \frac{(q_1(H - pX + 1) + ((q_1 - 1)(4q_2^2 - \tau - 4q_2 + 2pXq_2 + p^2X^2q_2^2 - 4pXq_2^2 + pXq_2\tau + 1))/(2\tau))(q_2 - 1)}{Hq_1q_2} \quad (45)$$

While bank profits in a separating equilibrium are falling in the cost of collateral, k , bank profits in a pooling equilibrium, Π_P , are invariant in k . The exists thresholds k_γ^p such that when $k = k_\gamma^p$, $\Pi_S(\gamma) = \Pi_P$ for $\gamma \in \{0, 1\}$, and profits are higher in the pooling equilibrium for higher k . k_γ^p are given by:

$$k_0^p = \frac{-(2\tau(q_2 - 1)((q_1 - 1)((2q_2 + q_2H - pq_2X - 1)^2/(2\tau) - q_2 + ((2q_2 + q_2H - pq_2X - 1)(\tau - 4q_2 - 2q_2H + 2pq_2X + 2))/(2\tau) + 1) - ((q_1 + q_2 - q_1q_2)(H - pX + 2) - 1)((q_1 + q_2 - q_1q_2)(H - pX + 2) - 1)/\tau - 1/2) - q_1 - q_1(H - pX + 1) + q_2(q_1 - 1) + ((q_1 + q_2 - q_1q_2)(H - pX + 2) - 1)^2/(2\tau) + 1))}{q_1q_2H(4q_2 + \tau + 2q_2H - 2pq_2X - 2)} \quad (46)$$

$$k_1^p = \frac{(q_2 - 1)(q_1 + ((q_1 + q_2 - q_1q_2)(H - pX + 2) - 1)((q_1 + q_2 - q_1q_2)(H - pX + 2) - 1)/\tau - 1/2) + q_1(H - pX + 1) - q_2(q_1 - 1) - ((q_1 + q_2 - q_1q_2)(H - pX + 2) - 1)^2/(2\tau) + ((q_1 - 1)(4q_2^2 - \tau - 4q_2 + 2pXq_2 + X^2p^2q_2^2 - 4pXq_2^2 + pXq_2\tau + 1))/(2\tau) - 1)}{Hq_1q_2} \quad (47)$$

For $k = 0$, $\Pi_S(\gamma = 1) > \Pi_S(\gamma = 0) > \max(0, \Pi_P)$. $\Pi_S(\gamma)$ are falling in k with a constant slope, $-q_1C_h(\gamma)$. Π_P is invariant in k ; it is positive when $H < H_1$. For any given set of parameters, it is necessary to order \bar{k} , k_0^s , k_1^s , k_0^p , and k_1^p to determine the equilibrium.

Suppose that $k \leq \min(\bar{k}, k_1^s, k_1^p)$. Then, $\Pi_S(\gamma = 1) > \max[\Pi_S(\gamma = 0), \Pi_P, 0]$, and collateral requirements are high, $C_h = \bar{C}$.

Suppose that $\bar{k} < k \leq \min(k_0^s, k_0^p)$. Then, $\Pi_S(\gamma = 0) > \max[\Pi_S(\gamma = 1), \Pi_P, 0]$, and collateral requirements are low, $C_h = \hat{C}$.

Suppose that $k > \max(k_0^p, k_1^p)$ and $H \leq H_1$. Then, $\Pi_P > \max[\Pi_S(\gamma = 1), \Pi_S(\gamma = 0), 0]$, and there is a pooling equilibrium with financing.

Suppose that $k > \max(k_0^s, k_1^s)$ and $H > H_1$. Then, $\max[\Pi_S(\gamma = 1), \Pi_S(\gamma = 0), \Pi_P] < 0$, and there is no financing in equilibrium.

6.2 More than two types of borrowers

In this extension, I consider the case that there are three types of borrowers: high-type, intermediate, and low-type. An intermediate borrower has a good project with probability, $q_m \in (q_2, 1)$, i.e., intermediate borrowers may have bad projects, but they have good projects with a higher probability than low-type borrowers. Intermediate borrowers have a lower outside option than high-type borrowers, $H_m < H$. I consider the costless collateral case, $k = 0$. To simplify exposition, I assume that $q_m = 1 - \epsilon$ where ϵ is positive but arbitrarily small; this assumption implies that the bank would lend to intermediate borrowers without screening their project.¹⁴

Proposition 5 (Three-type case) *There exists a partial pooling equilibrium for intermediate levels of collateral availability, $\hat{C} \leq W < C'$, in which the higher types post collateral to separate from the low-type. C' is given by:*

$$C' = \frac{q_m H - H_m}{(1 - q_m)} \quad (48)$$

Proof. Suppose that the bank offers a contract intended for intermediate borrowers, (R_m, C_m) ,

¹⁴The cost of screening exceeds the gains from screening if $q_m p X - 1 > \beta q_m p X - \beta(2q_m - 1) - (1 - q_m) - \frac{\tau}{2} F^2$, where the LHS (resp. RHS) is the NPV of lending to intermediate borrowers without (resp. with) screening. Using $q_m = 1 - \epsilon$, this condition simplifies to $\epsilon < \frac{(1-\beta)(pX-1) + \frac{\tau}{2} F^2}{(1-\beta)(pX-1) + \beta}$. Since the RHS is positive, it is always possible to find an ϵ sufficiently small such that this condition holds.

with $0 < C_m < C_h$. An intermediate borrower's IR constraint is as follows:

$$q_m(p(X - R_m) - (1 - p)C_m) - (1 - q_m)C_m \geq H_m \quad (49)$$

Intermediate borrowers mimic the high-type if they are better off mimicking the high-type than truthfully revealing their type:

$$\overbrace{q_m(p(X - R_m) - (1 - p)C_m) - (1 - q_m)C_m}^{H_m} > \quad (50)$$

$$\begin{aligned} & q_m \underbrace{(p(X - R_h) - (1 - p)C_h)}_H - (1 - q_m)C_h \\ \implies & C_h > \frac{q_m H - H_m}{(1 - q_m)} = C' \end{aligned} \quad (51)$$

Comparing C' and Equations (25), $\bar{C} < C'$ if:

$$H_m < \frac{q_m H(1 - q_2) - q_2 H(1 - q_m)}{1 - q_2} \quad (52)$$

With $q_m \rightarrow 1$, the above condition becomes $H > H_m$, which is always satisfied (i.e. $\bar{C} < C'$). Thus, for $\hat{C} \leq W < C'$, the higher types pool on the (R_h, W) contract, while the low-type picks the $(R_l, 0)$ contract. ■

Consider the parameters, $\hat{C} \leq W < \bar{C}$. Low-type borrowers do not mimic high-type borrowers since their IC constraint is violated, but intermediate borrowers do. Given that it is the high-types' IR constraint which must bind in this partial pooling equilibrium, the contract terms, (R_h, C_h) , is identical to the baseline. For $W = \hat{C}$, the higher types separate from the low-type, but the low-type extract a fraction of the surplus since $R_l < X$. As the availability of collateral increases, the bank increases collateral requirements for the higher types, which allows the bank to charge a higher interest rate to the low-type and positively affects the bank's incentive to screen the low-types' projects. Thus, as in the baseline model, the positive spillover

effect from collateralized to uncollateralized loans arises. The analysis generalizes to any number of borrower-types, but it becomes more complicated without additional qualitative insights.

6.3 Renegotiation case

In this extension, I consider the case that renegotiations are allowed, even if one party is worse off as a result of it. Suppose that contracts separate borrower-types and low-type borrowers apply for uncollateralized loans. In the absence of renegotiations, the bank screens applicants' projects, and if screening produces a good signal, $s = s_g$, the bank offers a loan at the initial promised interest rate, $R_l(C_h)$. In this case, low-type borrowers earn informational rents if the availability of collateral is limited, $R_l(C_h) < X$ for $C_h < \bar{C}$.

However, when we allow for renegotiations, the lender can withdraw the initial offer of interest rate, making a new take-it-or-leave-it offer of $R_l = X$ at $t = 1$. Since separation has already taken place at the earlier date due to the use of collateral, low-type borrowers do not hold an informational advantage at $t = 1$, and the bank extracts the full surplus. At $t = 0$, borrowers optimally anticipate this renegotiation at the interim date, and the separating equilibrium collapses for any $C_h < \bar{C}$. This is an inefficient outcome since pooling leads to waste of resources and may also lead to the market breakdown equilibrium in which no borrowers obtain finance.

In this scenario, a benevolent social planner whose objective is to maximize net social surplus will step in to improve the allocation. One solution would be to announce that renegotiated contracts will be made void unless both parties are (weakly) better off under the updated terms compared to the terms in the original contract. This announcement will effectively prevent the unravelling of the separating equilibrium. Thus, when the planner implements optimal policies, all the results in the baseline model go through unchanged.

References

- Anderson, G., Bahaj, S., Chavaz, M., Foulis, A., and Pinter, G. (2022). Lending relationships and the collateral channel. *Review of Finance*.
- Asea, P. and Blomberg, B. (1998). Lending cycles. *Journal of Econometrics*, 83(1).
- Asriyan, V., Laeven, L., and Martin, A. (2022). Collateral booms and information depletion. *Review of Economic Studies*, 89(2):517–555.
- Bahaj, S., Foulis, A., and Pinter, G. (2020). Home values and firm behaviour. *American Economic Review*, 110(7):2225–2270.
- Beck, T., De Jonghe, O., and Schepens, G. (2013). Bank competition and stability: cross-country heterogeneity. *Journal of Financial Intermediation*, 22(2):218–244.
- Berger, A., Frame, W. S., and Ioannidou, V. (2011). Tests of ex ante versus ex post theories of collateral using private and public information. *Journal of Financial Economics*, 100(1):85 – 97.
- Bernhardt, D., Koufopoulos, K., and Trigilia, G. (2022). Profiting from the poor in competitive lending markets with adverse selection. *Working paper*.
- Besanko, D. and Thakor, A. (1987a). Collateral and rationing: Sorting equilibria in monopolistic and competitive credit markets. *International Economic Review*, 28(3):671–89.
- Besanko, D. and Thakor, A. V. (1987b). Competitive equilibrium in the credit market under asymmetric information. *Journal of Economic Theory*, 42(1):167 – 182.
- Bester, H. (1985). Screening vs. rationing in credit markets with imperfect information. *American Economic Review*, 75(4):850–55.
- Beyhaghi, M. (2022). Third-party credit guarantees and the cost of debt: Evidence from corporate loans. *Review of Finance*, 26(2):87–317.

- Boot, A., Thakor, A., and Udell, G. (1991). Secured lending and default risk: Equilibrium analysis, policy implications and empirical results. *Economic Journal*, 101(406):458–72.
- Boot, A. W. A. and Thakor, A. V. (1994). Moral hazard and secured lending in an infinitely repeated credit market game. *International Economic Review*, 35(4):899–920.
- Broecker, T. (1990). Credit-worthiness tests and interbank competition. *Econometrica*, 58(2):429–452.
- Cao, Q., Pietro, M. D., Kokas, S., and Minetti, R. (2022). Liquidity and discipline. bank due diligence over the business cycle. *Journal of the European Economic Association*, 20(5):2136–2180.
- Chan, Y.-S. and Kanatas, G. (1985). Asymmetric valuations and the role of collateral in loan agreements. *Journal of Money, Credit and Banking*, 17(1):84–95.
- Dang, T. V., Gorton, G., and Holmstrom, B. (2012). Ignorance, debt and financial crises. *Working paper*.
- Degryse, H., Karapetyan, A., and Karmakar, S. (2021). To ask or not to ask? collateral and screening in lending relationships. *Journal of Financial Economics*, 142(1):239–260.
- Delis, M. D., Kokas, S., and Ongena, S. (2016). Foreign ownership and market power in banking: Evidence from a world sample. *Journal of Money, Credit and Banking*, 48(2-3):449–483.
- Dell’Ariccia, G., Igan, D., and Laeven, L. (2012). Credit booms and lending standards: Evidence from the subprime mortgage market. *Journal of Money, Credit and Banking*, 44(367-384):367–384.
- Donaldson, J., Gromb, D., and Piacentino, G. (2020a). Conflicting priorities: A theory of covenants and collateral. *Working paper*.

- Donaldson, J., Gromb, D., and Piacentino, G. (2020b). The paradox of pledgeability. *Journal of Financial Economics*, 137(3):591–605.
- Forssbaeck, J. and Shehzad, C. T. (2015). The conditional effects of market power on bank risk - cross country evidence. *Review of Finance*, 19:1997–2038.
- Freixas, X. and Rochet, J.-C. (2008). *Microeconomics of Banking*. MIT Press, London, England.
- Godlewski, C. J. and Weill, L. (2011). Does collateral help mitigate adverse selection? a cross-country analysis. *Journal of Financial Services Research*, 40:49–78.
- Goel, A. M., Song, F., and Thakor, A. V. (2014). Correlated leverage and its ramifications. *Journal of Financial Intermediation*, 23(4):471 – 503.
- Gorton, G. and Ordoñez, G. (2014). Collateral crises. *American Economic Review*, 104(2):343–78.
- Gorton, G. and Ordoñez, G. (2020). Good booms, bad booms. *Journal of the European Economic Association*, 18(2):618–665.
- Hainz, C., Weill, L., and Godlewski, C. J. (2013). Bank competition and collateral: Theory and evidence. *Journal of Financial Services Research*, 44:131–148.
- Howes, C. and Weitzner, G. (2023). Bank information production over the business cycle. *Working paper*.
- Hu, Y. (2022). A dynamic theory of bank lending, firm entry, and investment fluctuations. *Journal of Economic Theory*, 204:105515.
- Ioannidou, V., Pavanini, N., and Peng, Y. (2022). Collateral requirements and adverse selection in lending markets. *Journal of Financial Economics*, 144(1):93–121.
- Laffont, J.-J. and Martimort, D. (2002). *The theory of incentives*. Princeton University Press, Princeton, NJ.

- Lee, S. and Persson, P. (2016). Financing from family and friends. *Review of Financial Studies*, 29(9):2341–2386.
- Lengwiler, Y. and Rishabh, K. (2017). Credit from the monopoly bank. *Working paper*.
- Lisowsky, P., Minnis, M., and Sutherland, A. (2017). Economic growth and financial statement verification. *Journal of Accounting Research*, 55:745–794.
- Manove, M., Padilla, A. J., and Pagano, M. (2001). Collateral versus project screening: A model of lazy banks. *RAND Journal of Economics*, 32(4):726–744.
- Marquez, R. (2002). Competition, adverse selection, and information in the banking industry. *Review of Financial Studies*, 15(3):901–926.
- Myerson, R. B. (1979). Incentive compatibility and the bargaining problem. *Econometrica*, 47(1):61–73.
- Ordóñez, G., Perez-Reyna, D., and Yogo, M. (2019). Leverage dynamics and credit quality. *Journal of Economic Theory*, 183:183–212.
- Parlato, C. (2019). Collateralizing liquidity. *Journal of Financial Economics*, 131(2):299–322.
- Rajan, R. and Winton, A. (1995). Covenants and collateral as incentives to monitor. *Journal of Finance*, 50(4):1113–1146.
- Rodano, G., Serrano-Velarde, N., and Tarantino, E. (2018). Lending standards over the credit cycle. *Review of Financial Studies*, 31:2943–2982.
- Rothschild, M. and Stiglitz, J. (1976). Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. *Quarterly Journal of Economics*, 90(4):629–649.
- Sengupta, R. (2007). Foreign entry and bank competition. *Journal of Financial Economics*, 84(2):502–528.

- Sengupta, R. (2014). Lending to uncreditworthy borrowers. *Journal of Financial Intermediation*, 23(1):101–128.
- Shaffer, S. (1998). The winner’s curse in banking. *Journal of Financial Intermediation*, 7(4):359 – 392.
- Song, F. and Thakor, A. (2010). Financial system architecture and the co-evolution of banks and capital markets. *Economic Journal*, 120(547):1021–1055.
- Tirole, J. (2006). *The theory of corporate finance*. Princeton University Press, Princeton, NJ.
- Vanasco, V. (2017). The downside of asset screening for market liquidity. *Journal of Finance*, 72(5):1937–1982.
- Zaccaria, L. (2023). Are family and friends the wrong investors? evidence from U.S. start-ups. *Journal of Corporate Finance*, 79:102368.