Efficient on-balance sheet financing with non-exclusive contracts

Sonny Biswas*

Kostas Koufopoulos[†]

Xingzhu Li[‡]

Abstract

A bank seeks financing for two unrelated projects but cannot commit to exclusively

deal with a single investor. Despite this restriction, on-balance sheet financing with cross-

subsidization across projects arises as the unique equilibrium and the outcome is efficient

(second-best). Given the informational frictions considered, our solution strictly dominates

off-balance sheet financing with voluntary support for all parameters, implying that a ra-

tional profit-maximizing bank will never choose the latter. Different from existing models,

the cross-subsidization benefits are independent of the correlation structure across project

returns, which generates new empirical implications.

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*University of Bristol Business School. s.biswas@bristol.ac.uk.

[†]Department of Economics and Related Studies, University of York. kostas.koufopoulos@york.ac.uk.

[‡]Department of Economics and Related Studies, University of York. xingzhu.li@york.ac.uk.

1 Introduction

A bank has the choice to finance lending either on- or off- balance sheet. From the bank's perspective, off-balance sheet financing is advantageous since it allows the bank to lay off credit risk: if a loan is off the books, the bank's assets are protected from losses coming from it. Yet, banks frequently provide voluntary support to their sponsored entities (see e.g., Acharya et al. 2013 and Kacperczyk and Schnabl 2013). Segura and Zeng (2020) present a model to resolve this puzzle – voluntary support under off-balance sheet financing can signal a bank's type and resolve an adverse selection problem. We reconsider their model and show that on-balance sheet financing (even when contracts are non-exclusive) dominates the off-balance sheet financing solution for all parameter values. Thus, which combination of frictions rationalizes the provision of voluntary support in off-balance sheet financing remains an open question.

In the model, there are two investment opportunities for a bank – the first suffers from an effort moral hazard problem, while the second suffers from an adverse selection problem. Regarding the project associated with adverse selection, we show that a fully separating equilibrium always exists in which only good banks invest. Separation arises through the violation of the bad bank's participation constraint rather than through the standard incentive compatibility constraint. The bank sells off the expected net present value of the adverse selection project and uses these funds to reduce the face value of debt which is a claim on the cash flow generated by the moral hazard project. The lower promised repayment relaxes the bank's effort moral hazard constraint and boosts effort provision in the first project. We show that this equilibrium allocation coincides with the planner's solution (i.e., the equilibrium allocation is second-best).

In Segura and Zeng (2020), off-balance sheet financing allows a good bank to signal the quality of the second project to a greater extent through the provision of voluntary support but it reduces effort provision in the first project. Thus, while the provision of voluntary support relaxes the adverse selection constraint in the second project, it makes the moral hazard friction

in the first project more binding. In contrast, in our solution, the funds raised from the pre-sale of the expected NPV of the adverse selection project are directly invested in the moral hazard project which relaxes the moral hazard constraint. The separating equilibrium is costlessly achieved through the violation of the participation constraint of the bad banks. Thus, while in off-balance sheet financing mode, the solution of the adverse selection friction has a perverse impact on the moral hazard constraint, in our case, the solution of the adverse selection friction relaxes the moral hazard constraint.

In summary, the off-balance sheet solution of Segura and Zeng (2020) is inefficient for three reasons: First, they do not allow for direct investment by the bank in the second project which leads to either an inefficient partial pooling equilibrium or a complete market breakdown equilibrium. Second, in their solution, the bank uses all cash flows generated by assets-in-place for signaling purposes (money-burning) which makes the effort moral hazard problem more binding and leads to lower effort provision. In contrast, signaling in our solution is not socially costly, and we show that there exists a unique equilibrium which is always separating where only good banks invest. Third, our solution increases effort provision even further by transferring the expected NPV of the second project to reduce the promised repayment for the first project. Hence, given the informational frictions in this model, the off-balance sheet financing mode is inefficient and does not maximize bank profit. Therefore, a rational profit-maximizing bank will never choose it.

It should be noted that we restrict attention to non-exclusive contracts only, which means that the bank cannot commit to exclusively deal with a single investor. This implies that the contracts used to raise the funds for investment in the two projects are zero-profit on a contract-by-contract basis. Of course, the second-best outcome could also be implemented using exclusive contract menus which would potentially allow for cross-subsidization across contracts within the menu.

Our theory is consistent with evidence on the existence of cross-subsidization across unrelated activities in financial institutions (see e.g., Drucker and Puri 2005, Griffin et al. 2007, Santikian 2014, and Jenkinson et al. 2018). Drucker and Puri 2005 find evidence of price discounts in the context of repeated lending to borrowers. We model the financing decision of the bank, and hence, our implications relate to the bank's relationship with its investors, rather than the relationship with its borrowers, but the principle of cross-subsidization is the same in both cases. We generate additional testable empirical predictions: First, the spread on debt is lower and bank monitoring intensity is higher for banks with better growth prospects. Second, the spread on debt falls in the bank's growth opportunities if these are positively correlated with its core activities, while the direction of the effect is not clear if there is a negative correlation.

Relation to the wider literature.

Our theory provides a new rationale for the why financial intermediation exists. In most existing theories, financial intermediaries have special skills in acquiring or processing information which facilitates financing. For example, Diamond (1984) and Holmstrom and Tirole (1997) emphasize the role of banks as monitors – in Diamond (1984) banks mitigate an ex post moral hazard problem through monitoring, while in Holmstrom and Tirole (1997) they mitigate an ex ante moral hazard problem. Ramakrishnan and Thakor (1984) and Allen (1990) focus on the screening role of banks in which banks can distinguish between good and bad borrowers. In these models, banks have a monitoring or screening cost advantage over non-intermediated outcomes (depending on the model, this advantage is either assumed, or it arises as an equilibrium outcome). In contrast, banks in our model do not have superior skills in resolving any informational frictions, yet they generate value through two novel channels:

First, in our model an efficient separating equilibrium arises through socially costless signaling, whereas, in existing banking theories, separation is achieved through costly information

¹There is a separate strand of the literature which focuses on the liquidity creation/provision role of banks (see e.g., Diamond and Dybvig 1983, Gorton and Penacchi 1990, Diamond and Rajan 2001, and Donaldson et al. 2018).

production. Second, in our model there is cross-subsidization across projects which is absent in existing models: the expected NPV of the adverse selection problem is transferred to the first project to relax the moral hazard constraint. Coval and Thakor (2005) also do not assume that banks have special information processing skills; rational banks form a beliefs-bridge between optimistic entrepreneurs and pessimistic investors. In our theory, all agents are rational, so we explore a different role of banks than the one considered by Coval and Thakor (2005).

Improved effort incentives due to cross-subsidization across projects is not a new idea. In Tirole (2006), cross-pledging across imperfectly correlated projects subject to effort moral hazard leads to greater effort provision by the manager (see also Diamond 1984, Cerasi and Daltung 2000, Laux 2001, Axelson et al. 2009, and Maurin et al. 2023) – the optimal contract entails that the bank is only compensated when all projects succeed; this relaxes the limited liability constraint of the agent and induces the efficient level of effort provision. The cross-pledging mechanism relies on exploiting the diversification effect resulting from imperfectly correlated projects and it is ineffective if projects are perfectly correlated. Our mechanism works differently – There exists a unique separating equilibrium in which only good banks invest and they effectively pledge the expected cash flows from the second project to the first, which reduces the cost of external financing, thereby boosting effort provision. Thus, in contrast to existing models, the correlation structure across projects does not play a role which implies that the cross-subsidization channel in our model is distinct from the well-known diversification effect.

Our results may be interpreted as providing a rationale for tying in universal banks. Suppose that the commercial banking division serves borrowers who need monitoring, while the investment banking division identifies good firms and underwrites their issues. Then, should the two divisions operate separately or should the two divisions be integrated? Existing models in this literature emphasize increased cost efficiency in universal banking due to economies of scope (e.g., Kanatas and Qi 1998 and Kanatas and Qi 2003) and improved credit allocation (e.g., Loranth and Morrison 2012). In contrast to these models, we present a model that considers

the bank's financing decision and demonstrate how the bank's ability to mobilize funds within its unrelated operations can generate efficiency gains by transferring resources to where they are the most beneficial.

2 Model

2.1 Set-up

There are four dates $t \in \{0, 1, 2, 3\}$. There is a bank which has no funds of its own at t = 0 and obtains a pay-off from assets-in-place at t = 1. The bank has two investment opportunities, the first at t = 0 and the second at t = 2; each project is of scale 1 unit and generates a payoff in the period following the investment. The funds are raised from competitive external investors. All agents are risk-neutral and the discount rate is 0.

The first project is always good (g), while the second project may be either good or bad (b). In either case, a project produces R per unit of investment if it succeeds and zero if it fails. A project succeeds with probability p_i . We assume that only good projects are profitable:

A1:
$$p_g R > 1 > p_b R$$

The first project to be undertaken at t=0 is subject to effort moral hazard. The project succeeds with probability p_g and fails with probability $1-p_g$. The bank can exert unobservable effort $e \in [0,1]$ at a cost c(e) to reduce the probability of failure. Taking into account the bank's effort provision, the first project's success probability is $p_g + me$ and its failure probability is $1-p_g - me$, with $m \le 1-p_g$ to ensure non-negative probability of failure. m is a constant which is interpreted as the marginal value of effort. The cost of effort c(e) satisfies:

A2: i.
$$c(0) = 0$$
, ii. $c'(0) = 0$ and $c'(1) > mR$ and iii. $c''(e) > \left(\frac{m}{p_g}\right)^2$

The bank obtains pay-off Y at t=1 from its assets-in-place. We assume that Y is not so large such that the t=0 debt becomes riskless, as otherwise the first-best effort level could be

implementable:

A3:
$$Y < 1 - \alpha(p_q R - 1)$$

At t=1, the bank privately learns whether the second project is good or bad. The second project to be undertaken at t=2 is of type g with probability $\alpha \in (0,1)$. The bank privately observes the project's type. We assume that α is small such that the average NPV of the pool is negative:

A4:
$$\alpha < \frac{1-p_b}{p_g R - p_b R}$$

The timeline is illustrated in Figure 1.

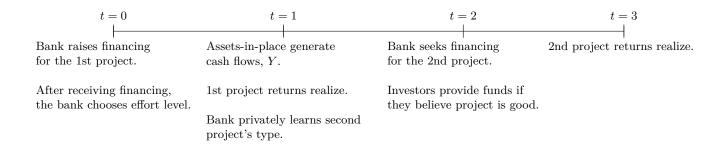


Figure 1: Timeline

2.2 The Game

We consider a two-stage sequential game, the second stage of which itself is a standard two-stage signaling game.

• Stage 1: At t = 0 (when the bank is unaware of its own type), competitive investors propose the following contract: $(D_1, D_{0,3}, K, Z)$. For a price, Z, the investor obtains debt with face value, $D_{0,3}$, which is a claim on the cash flow generated by the adverse selection project, if undertaken, and this debt is senior to any security which may be subsequently issued. The bank may use these funds to directly invest in the moral hazard project or consume it. If the bank chooses to invest it in the project (resp. consume it), it needs to

raise an amount, 1 - Z units (resp. 1 unit), which the investor provides subject to the promised repayment, D_1 , being sufficient for the investor to break even. The investor gets D_1 if the first project succeeds and Y - K if it fails.² In the next stage, the bank will try to raise financing from a potentially different investor playing a two-stage signaling game which follows.

- Stage 2a (which occurs at t = 2): the bank, now privately aware of its type, seeks to raise funds for the second project from competitive investors (who may be different from the one who provided the funds at t = 0). The bank offers a contract which specifies whether K will be invested directly in the second project or not, and offers to investors debt with face value, $D_{2,3}$, which is junior to the debt issued at t = 0, in exchange for the remaining amount of funds necessary for undertaking the project.
- Stage 2b: given the contract offered by the bank in Stage 2a, investors will form beliefs about the second project's type. Given these beliefs, investors will decide whether they will accept the contract and provide the funds necessary to undertake the project or not.

We look for the pure strategy Perfect Bayesian equilibria of this game that satisfy the Intuitive Criterion of Cho and Kreps (1987).³ Note that we do not impose exogenously whether the bank pre-sells a fraction of the cash flows from the second project at t = 0 or not, i.e., K, Z, and $D_{0,3}$ are chosen optimally in equilibrium.

2.3 First-best

In the first-best, the effort level exerted by the bank is observable and verifiable, and the type of the second project is publicly known. In this case, the bank always undertakes the first project

²We obtain identical results if there are two separate contracts at this stage of the game offered by two different investors, one relating to the first project and another relating to the second project.

 $^{^{3}}$ The appropriate solution concept is Perfect Bayesian since in the second stage of the game banks may signal about their type through the use of K, which implies that inferences are made.

and, given Assumption 2, the level of effort is:

$$c'(e^{FB}) = mR \tag{1}$$

The bank invests in the second project only if it is good.

2.4 Equilibrium

The two key dates of strategic interaction between the bank and investors are t = 0 and t = 2 when the bank raises funds for the projects. We solve for the equilibrium using backward induction.

2.4.1 Financing at t=2

Below, we list all the candidate equilibria of the subgame which starts at t = 2 (Stage 2a), given the terms of the contract signed in Stage 1 of the game:

- 1. A candidate separating equilibrium in which only bad banks obtain financing.
- 2. A candidate pooling equilibrium in which both firm types obtain financing.
- 3. A candidate pooling equilibrium in which neither firm type obtains financing (market breakdown).
- 4. A candidate separating equilibrium in which both good and bad banks obtain financing, but at different terms.
- 5. A candidate separating equilibrium in which only good banks obtain financing.

Below we consider each candidate equilibrium.

Lemma 1 There cannot exist a separating equilibrium in which only bad banks obtain financing or both good and bad banks obtain financing but at different terms.

Proof. The proof is in the Appendix.

Suppose that the contract in the first stage specifies a pair, $(K, D_{0,3})$. Bank *i*'s profit is denoted Π_i and its participation constraint is given by:

$$\Pi_i = p_i(R - (D_{0,3} + D_{2,3})) - K \ge 0 \tag{2}$$

We derive the slope of $\Pi_i = 0$ in the $(K, D_{2,3})$ space by totally differentiating Equation (2) with respect to $D_{2,3}$ and K:

$$\frac{dD_{2,3}}{dK}\bigg|_{\Pi_{i=0}} = -\frac{1}{p_i} < 0 \tag{3}$$

Investors at t=2 facing a type i bank make an expected profit which is denoted γ_i , and they provide the necessary funds for investment if $\gamma_i \geq 0$:

$$\gamma_i = p_i \min(D_{2,3}, (R - D_{0,3})) - (1 - K) \ge 0 \tag{4}$$

In the main text, we consider the case that $D_{2,3} \leq R - D_{0,3}$ holds as this is the relevant case for the results.⁴ We derive the slope of $\gamma_i = 0$ in the $(K, D_{2,3})$ space by totally differentiating Equation (4) with respect to $D_{2,3}$ and K:

$$\left. \frac{dD_{2,3}}{dK} \right|_{C=0} = -\frac{1}{p_i} < 0 \tag{5}$$

implies that $D_{2,3} = \frac{1}{p_i}$ for K = 0 and $D_{2,3} = 0$ for K = 1. A bank of type i is willing to undertake the project in the region below its participation constraint, while an investor facing a bank of type i is willing to provide funds in the region above the corresponding participation constraint. Additionally, we plot the pooling participation constraint (blue dashed line) – this is the average of γ_b and γ_g , and on the y-axis it lies above R since the average NPV of the pool is negative (Assumption A4).

From Equation (4), we derive the minimum promised repayment (face value of debt, $D_{2,3}$) for which investors at t = 2 will provide funds:

$$D_{2,3} \ge \frac{1-K}{p_i} \tag{6}$$

At this lower bound, the intersections of $\Pi_i = 0$ and $\gamma_g = 0$ with the vertical axis coincide, and since $\Pi_g = 0$ and $\gamma_g = 0$ have the same slope, these two lines coincide (the red and blue line in Figure 2b). In this case, the shaded area in Figure 2a shrinks to the $\Pi_g = \gamma_g = 0$ line in Figure 2b.

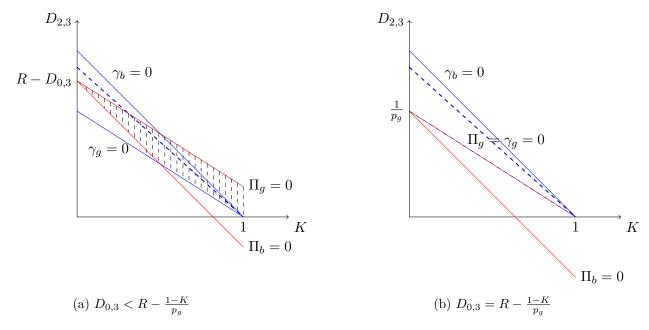


Figure 2: Feasible equilibria

Lemma 2 There cannot exist a pooling equilibrium in which both good and bad banks obtain

financing.

Proof. The proof is in the Appendix.

Lemma 3 Depending on the terms of the contract offered in Stage 1 of the game, there either exists a pooling equilibrium in which neither bank obtains financing (market breakdown) or a separating equilibrium in which only good banks obtain financing.

Proof. We present a graphical proof and refer to Figure 2.

There exists a financing equilibrium in the shaded region in Figure 2a: here, investors' participation constraints are slack and they make positive profits when they face good banks (i.e., $\gamma_g > 0$), a good bank's participation constraint is satisfied (i.e., $\Pi_g \geq 0$), while a bad bank's participation constraint is (weakly) violated (i.e., $\Pi_b \leq 0$). Financing is infeasible above the shaded region since the good bank's participation constraint is violated. Below the shaded region, the market breaks down since either the bad bank's participation constraint is satisfied and/or investors' participation constraint is violated.

Both the breakdown equilibrium and the separating equilibrium with only good banks participating are feasible along the bad bank's participation constraint bordering the shaded region up to $\left(K=0,D_{2,3}=\frac{1}{p_g}\right)$. For these parameters, a bad bank is indifferent between participating or not which implies that the breakdown equilibrium cannot be ruled out for these parameters in this subgame. Note in Figure 2b, when the lower bound on $D_{2,3}$ binds (see Equation (6)), the participation constraint of the bad bank lies entirely below the $\gamma_g=0$ line and the coexistence of the breakdown equilibrium and the separating equilibrium is possible only at the vertical intercept, $\left(K=0,D_{2,3}=\frac{1}{p_g}\right)$.

The proposition below summarizes the results from the Lemmas 1-3, and characterizes the equilibria that may arise in the two-stage signaling game played at t = 2:

Proposition 1 In the two-stage signaling game played at t = 2, there can exist two potential

equilibria:

- 1. a separating equilibrium in which only good banks invest and
- 2. a pooling equilibrium in which no banks invest (market breakdown).

Below, we show that in the whole game the only equilibrium that survives is the separating equilibrium in which only good banks invest.

2.4.2 Financing at t = 0

At t = 0, investors offer a contract to the bank which specifies the terms and conditions under which they will purchase a fraction of the cash flow from the second project, $D_{0,3}$, at a price, Z, and also, provide the bank with the additional funds required for investment in the first project, 1 - Z. The lower bound on $D_{2,3}$ that we derived in the previous section (Equation (6)), puts an upper bound on how much of the cash flow from the second project can be sold to the t = 0investors, and for a given K, this upper bound is as follows:

$$D_{0,3} \le R - \frac{1 - K}{p_q} \tag{7}$$

If this upper bound is violated, there will be too little cash flow left over at t = 3 for the t = 2 investors, who will then refuse to provide the funds required to undertake the second project. If the first project succeeds, investors obtain the promised repayment, D_1 . In the case of failure, investors obtain Y - K. The bank retains K whether or not the first project succeeds. At t = 2, the bank privately knows the second project's type to be good or bad. For any pair, $(K, D_{0,3})$, which satisfies Equation (7), the t = 2 investors will offer the funds necessary for investment in the second project with face value of debt, $D_{2,3}$, conditional on the bank's decision of whether to invest K directly in the second project or to consume it. That is, conditional on the bank investing K directly, the pairs $(K, D_{2,3})$ that violate the bad bank's participation constraint

while satisfying the good bank's one, and ensuring that investors make weakly positive profits (i.e., $\gamma_g \geq 0$) are those that belong to the shaded area in Figure 2. Given the contract offered, investors' zero profit condition at t = 0 is as follows:

$$\underbrace{(p_g + me)D_1 + (1 - p_g - me)Y - K - (1 - Z)}_{\text{Investors' expected profit from 1st project}} + \underbrace{\alpha p_g D_{0,3} - Z}_{\text{2nd project}} = 0$$
 (8)

From the zero profit condition, we derive the repayment, D_1 . The bank's expected profits is:

$$\Pi_1 = \underbrace{(p_g + me)(R - D_1 + Y) - c(e) - Z}_{\text{Bank's expected profit from 1st project}} + \underbrace{\alpha p_g(R - (D_{0,3} + D_{2,3})) + (1 - \alpha)K + Z}_{\text{2nd project}}$$
(9)

The first part is the expected profit from the first project, net of the cost of effort. The second part is the expected profit from the second project depending on the bank's use of K. Given the contract chosen by the bank, it maximizes its expected profit from the first project by choosing the level of effort. Taking the first order condition with respect to the effort level, solving the investor's zero profit condition (Equation (8)) for D_1 , and substituting it into the first order condition, we obtain the equilibrium effort level exerted by the bank:

$$c'(e^*) = m(R - D_1^* + Y)$$

$$= m\left(R - \frac{1 - Y}{p_g + me} - \frac{K - \alpha p_g D_{0,3}}{p_g + me}\right)$$
(10)

Because c(e) is strictly convex in e, c'(e) is strictly increasing in e. Since the right hand side of Equation (10) is falling in K and increasing in $D_{0,3}$, a lower K and a higher $D_{0,3}$ implies a higher effort level. The intuition is as follows: The higher the face value of debt, $D_{0,3}$, the higher the amount of funds that the bank raises from pre-selling expected cash flows from the second project, which can then be invested in the first project. Hence, the remaining amount of funds to be raised to invest in the first project falls, and so the face value of debt issued at t = 0, D_1 , which is a claim on the cash flow generated by the first project falls. This, in turn,

implies that the cash flow that the bank retains in the case that the first project succeeds is higher, which relaxes the effort moral hazard constraint, and therefore, the bank exerts more effort.

Below, we present two lemmas which allows us to characterize the equilibrium in Proposition 2.

Lemma 4 From the t = 0 perspective, there cannot exist an equilibrium in which a good project at t = 2 does not obtain financing.

Proof. The proof is in the Appendix.

Starting from a non-financing equilibrium, an investor can profitably deviate by offering a new pair, $(K, D_{0,3})$, which would allow the t = 2 investors to offer a $D_{2,3}$ in the shaded region in Figure 2a which satisfies the good bank's and investors' participation constraints and violates the bad bank's participation constraint at t = 2.

Lemma 5 There cannot exist an equilibrium in which the contract specifies K > 0. In any equilibrium, the contract must specify K = 0 and $D_{0,3} = R - \frac{1}{p_g}$.

Proof. The proof is in the Appendix.

Bank profit is maximized for the smallest possible K and the largest possible $D_{0,3}$ (from Equation (10)). Competition among investors drives K to 0 and $D_{0,3}$ to its upper bound, $R - \frac{1}{p_g}$. For any K > 0, there exists a deviation in which the t = 0 investors offer a smaller K and purchase a higher fraction of the cash flows from the second project, which, in turn, relaxes the bank's effort moral hazard constraint by reducing D_1 , and increases bank value. Effectively, a positive K implies that some resources are left unused at t = 0, which leads to an inefficiency. Note from Lemma 3, K = 0 can potentially lead to two equilibria in the second stage subgame: one is the separating one considered above and the other is the market

breakdown equilibrium. But from the perspective of t = 0, the market breakdown equilibrium cannot exist due to the possibility of profitable deviations. Thus, the separating equilibrium with $\left(K = 0, \ D_{0,3} = R - \frac{1}{p_g}\right)$ is the unique equilibrium of the full game. While K equals 0 on the equilibrium path, it is the off-path threat of positive K that sustains the equilibrium.

Substituting the equilibrium values, $\left(K=0,\ D_{0,3}^*=R-\frac{1}{p_g}\right)$ in Equation (8) we derive the promised repayment in the first project, D_1^* :

$$D_1^* = \frac{1 - (1 - p_g - me)Y}{p_g + me} - \frac{\alpha(p_g R - 1)}{p_g + me}$$
(11)

Lemmas 4 and 5 rule out all candidate equilibria, but one: $\left(D_1 = D_1^*, K = 0, D_{0,3}^* = R - \frac{1}{p_g}\right)$. Now we need to show that there is no profitable deviation from this equilibrium. To see why this is the case, note that any deviation would have to increase bank profits by relaxing the moral hazard constraint further. To do so, D_1 must be lower than D_1^* . However, since the full expected surplus from the second project is already used to minimize D_1 and investors are on their participation constraints, $D_1 < D_1^*$ is not feasible. Thus, this candidate equilibrium exists. Using $D_1 = D_1^*$, K = 0, and $D_{0,3}^* = R - \frac{1}{p_g}$ in Equation (10), the equilibrium effort level is given by:

$$c'(e^*) = m \left(R - \frac{1 - Y}{p_g + me} + \frac{\alpha(p_g R - 1)}{p_g + me} \right)$$
 (12)

Given Assumption A3, $D_1^* > Y$, implying that the debt is risky. In turn, this prevents equilibrium level of effort from reaching the first-best level, i.e., $e^* < e^{FB}$. This is the reason why optimality requires that we set K = 0 (its lowest possible value) and $D_{0,3} = R - \frac{1}{p_g}$ (its highest value) in order to achieve the maximum possible effort level (second best, see Section 2.5). We characterize the equilibrium in Proposition 2.

Proposition 2 There exists a unique equilibrium in which the contract specifies $(D_1 = D_1^*, K = D_1^*)$

0, $D_{0,3}^* = R - \frac{1}{p_g}$) at t = 0. There is full separation in the second project and only good banks participate. Effort provision in the first project is given by $c'(e^*)$.

The contract in Proposition 2 does not include provision for financing the second project and competitive investors make zero profits in expectation, as in Segura and Zeng (2020). Still, our solution differs since the bank pre-sells the NPV of the second project at t=0 to reduce the amount it needs to borrow for the first project. We do not impose how much of the cash flow from the second project the bank pre-sells, and allow for all possibilities, including no pre-selling (which corresponds to the solution in Segura and Zeng (2020)). We show that not pre-selling is off the equilibrium path. That is, a rational profit-maximizing bank will never choose to not pre-sell.

2.5 The planner's solution

The objective of the planner is to maximize the net social surplus (the bank value). The net social surplus consists of two parts: the value created by the project associated with adverse selection and the value created by the project associated with effort moral hazard. With regards to the project which is subject to adverse selection, maximization of the net social surplus requires that only good projects are undertaken. The planner can achieve this separation by giving K to the bank and allow banks to play a two-stage signaling game similar to the one we consider above (stages 2a and 2b) where the role of the investors is played by the planner. Furthermore, in order to maximize the aggregate net social surplus, the planner will extract the full NPV of the second project and transfer it to the first project to relax the effort moral hazard constraint as much as possible. Hence, at the optimum, he will set K = 0. Formally,

the planner's problem reduces to:

$$\max_{\tau_1, \tau_2, K} (p_g + me)(R - \tau_1 + Y) - c(e)$$

subject to

(IC)
$$c'(e^*) = m(R - \tau_1 - Y)$$
 (13)

(PC)
$$p_q(R - \tau_3) - K \ge 0 \ge p_b(R - \tau_3) - K$$

(FC)
$$(p_g + me)\tau_1 + (1 - p_g - me)Y - K + \alpha(p_g\tau_3 - (1 - K)) \ge 1$$

(LL)
$$au_3 \leq R$$

The planner provides the required funds and sets transfers from the bank to himself as τ_1 which is the repayment at t=1 and τ_3 which is the repayment at t=3. The planner maximizes the objective function with respect to the effort exerted by the bank subject to four constraints. The first constraint is the effort moral hazard constraint (IC). The second constraint is related to truth-telling about the second project. The third constraint is the planner's feasibility constraint (FC). The final constraint is the limited liability constraint.

The planner's objective is to maximize the net social surplus. Given that full separation can be achieved on the second project and the planner can extract the full surplus (NPV) on this project, the planner's problem reduces to maximizing the bank's effort in the first project. From the IC, this can be done by reducing τ_1 to the maximum extent possible (given Assumption A3, the first-best cannot be reached). To minimize τ_1 , the FC must bind. The choice variables are τ_3 and K. From the FC the aggregate effect of K is $-(1-\alpha)K$, implying that a strictly positive K diverts resources which could be used to reduce τ_1 and increase effort. Thus, at the optimum, the planner will set K = 0. Also, from the second line of the FC, higher the τ_3 , lower is the τ_1 consistent with the planner's FC being satisfied. Thus, the planner will set the transfer to the maximum possible amount consistent with the limited liability constraint, i.e., $\tau_3 = R$. $\tau_3 = R$ and K = 0 satisfies the truth-telling constraint, and hence, is consistent with

a separating equilibrium which allows the planner to extract the full surplus.⁵ The planner's allocation becomes $(\tau_1 = D_1^*, K = 0, j = 1, \tau_3^* = R)$, which coincides with the competitive equilibrium allocation, and hence, the equilibrium allocation is optimal (second-best).

Proposition 3 The equilibrium allocation of our game coincides with the planner's solution, and hence, it is efficient (second-best).

3 Benchmarks

In this section, we contrast our solution with two benchmarks (market financing and disintermediation).

3.1 Market versus intermediated financing

Instead of a bank seeking joint financing for the two projects, suppose that there are two separate firms, each managing one of the projects, which obtain financing directly from the market. We assume that the firm managing the first (resp. second) project has assets-in-place which produce $Y_1 > 0$ (resp. $Y_2 > 0$), with $Y_1 + Y_2 = Y$.

With regards to the second project, good firms achieve separation by investing an amount, K, in the project. The good firm offers a pair (K, D_3) along the lower contour of the shaded region in Figure 2a. The offer is on the bad bank's participation constraint for $K \in [0, \bar{K}]$ and on investors' participation constraint (when facing good borrowers) for $K \in (\bar{K}, 1]$, where \bar{K} is the intersection point of Π_b and γ_g . It is assumed that bad banks stay out in case of indifference between investing or not. Despite perfect competition, market investors obtain positive profits in expectation and the expected profits are smaller as direct investment by the firm increases (for $K > \bar{K}$, investors' expected profits become 0). In this case, the firm invests all available

⁵By an argument similar to the one developed in Lemma 4 we can show that this is indeed the unique equilibrium in the game played by the planner and the investors.

cash flows, $K = Y_2$, into the project to minimize sharing profits with investors. Regardless of the split of the surplus, the outcome is efficient since the equilibrium is separating and only good projects obtain financing for any $Y_2 > 0$.

With regards to the first project, the zero profit condition of investors is given as follows:

$$(p_g + me)D_1 + (1 - p_g - me)Y_1 - 1 = 0 (14)$$

From the zero profit condition, we derive the repayment, D_1^D . The firm maximizes its expected profit from the first project by choosing the level of effort:

$$\Pi_1^D = (p_q + me)(R - D_1^D + Y_1) - c(e)$$
(15)

Taking the first order condition, and substituting the investor's zero profit condition (Equation (14)), we obtain the equilibrium effort exerted:

$$D_1^D = \frac{1 - (1 - p_g - me)Y_1}{p_g + me} \tag{16}$$

$$c'(e^D) = m(R - D_1^D + Y_1) (17)$$

Proposition 4 Intermediated financing strictly dominates market financing for all parameter values.

Proof. The proof is in the Appendix.

 $e^D < e^*$ for all parameter values. The reason is that with direct financing, the two projects are separately financed and the borrower cannot bring forward the expected profits to investors in the second project to reduce the repayment for the first project, i.e., $D_1^D < D_1^*$ for any $Y_1 \leq Y$. Since the provider of effort retains more of the surplus from effort provision in the case

of intermediated financing, effort is higher in this case compared to the market financing case.

Intermediated and direct financing deliver equivalent outcomes (in terms of efficiency) in the case of the second project, while intermediation delivers a strictly more efficient outcome in the case of the first project. The result is driven by cross-subsidization across projects in the intermediation case.

3.2 On- versus Off-balance sheet

Segura and Zeng (2020) consider a very similar setting and consider the case that the first project may be funded on- or off-balance sheet. First, we recap their analysis in brief, and then we provide a comparison.

Under the on-balance sheet financing mode, investors in the first project have unlimited recourse to cash flows from the assets-in-place. In the absence of cross-subsidization across projects, the promised repayment to investors in the first project, D_1^{on} , and effort provision, e^{on} , are given by:

$$D_1^{on} = \frac{1 - (1 - p_g - me)Y}{p_g + me} \tag{18}$$

$$c'(e^{on}) = m(R - D_1^{on} + Y) (19)$$

Different from us, Segura and Zeng (2020) assume that investors make zero profits on each project. For this reason, there is a partial pooling equilibrium in which all good banks and a fraction of bad banks invest in the second project.

Under the off-balance sheet financing mode, in the event of failure of the first project, there is no obligation for the bank to make repayments using cash flows arising from the bank's assetsin-place. However, the good bank can signal its second project's type by voluntarily using cash flows from the assets-in-place to repay the investors in the first project. The promised repayment to the investors in the first project, D_1^{off} , and effort provision, e^{off} , are given by:

$$D_1^{off} = \frac{1 - \alpha Y}{p_g + me} \tag{20}$$

$$c'(e^{off}) = m(R - D_1^{off}) (21)$$

The ratio of marginal cost of providing voluntary support to the marginal benefit is smaller for good banks compared to bad banks, implying that voluntary support can be an effective signalling device. There is a partial pooling equilibrium in which all good banks and a fraction of bad banks invest; due to the voluntary support, the fraction of bad banks investing in the off-balance sheet case is always smaller than in the on-balance sheet case.⁶

The trade-off between on-balance sheet financing (without cross-subsidization across projects) and off-balance sheet financing is the following: under on-balance sheet financing, the effort provision in the first project is higher, while under off-balance sheet financing signalling in the second project is stronger. The optimal financing mode is determined from this trade-off. Our solution differs since we allow (but do not impose) for the two projects to be jointly financed. We do not assume that investors make zero profits on a project-by-project basis; instead, investors must make weakly positive profits on aggregate. To facilitate comparisons, we present the following Lemma:

Lemma 6 The face value of debt and the corresponding effort levels under different financing modes are as follows: $D_1^* < D_1^{on} < D_1^{off}$ and $e^* > e^{on} > e^{off}$.

Proof. The proof is in the Appendix.

Our solution, (D_1^*, e^*) , strictly dominates both the on-balance sheet solution without cross-subsidization, (D_1^{on}, e^{on}) , and the off-balance sheet solution, (D_1^{off}, e^{off}) .

1. $e^* > e^{on} > e^{off}$ (Lemma 6). Effort provision in our solution dominates the effort provision

in either case above for all parameter values since $D_1^* < D_1^{on} < D_1^{off}$. Intuitively, the first-period debt is priced by rationally anticipating that investors will make positive expected profits in the second project; this reduces the promised repayment D_1 and, since the bank retains more of the surplus from exerting effort, effort provision is higher.

2. In relation to the second project, Segura and Zeng (2020) obtain a partial pooling equilibrium in which all good banks and a fraction of bad banks invest if cash flows from the assets-in-place are sufficiently high (inefficiency); otherwise, there is a market breakdown equilibrium with no financing at all (inefficiency). In contrast, our equilibrium is always separating in which only good banks invest (efficiency) even if the cash flows generated by the assets-in-place is 0.

Proposition 5 For all parameter values, on-balance sheet financing with cross-subsidization across projects strictly dominates on-balance sheet financing without cross-subsidization and off-balance sheet financing.

In the choice of the financing mode, Segura and Zeng (2020) trade-off the efficiency gains from mitigating the adverse selection friction with the efficiency gains from relaxing the moral hazard friction; mitigating one makes the other friction more binding. In our case, the solution of the adverse selection friction relaxes the moral hazard friction; indeed, as we show in Section 2.5, our solution implements the second-best outcome. Hence, given the informational frictions in this model, the off-balance sheet financing mode is inefficient and does not maximize bank profit. Therefore, a rational profit-maximizing bank will never choose it.

4 Empirical implications

In this section, we present the key implications arising out of our analysis. First, we present a lemma which forms the basis the empirical predictions below:

Lemma 7 The increase in bank profits is higher than the direct increase in profits due to an increase in its expected growth opportunities, α .

The intuition is as follows: At t = 0, a higher α implies higher expected returns on the second project which are fully captured by investors. This has two effects: a direct one and an indirect one. The increase in the expected profits made by investors directly reduces the face value of debt, D_1 . This fall in D_1 relaxes the effort moral hazard constraint in the first project and leads to higher effort provision which, in turn, reduces the bank's default probability, and hence, the spread on debt falls further.

We state the testable empirical predictions below:

- 1. The spread on debt is lower and bank monitoring intensity is higher for banks with higher expected growth opportunities.
 - Banks which have stronger growth prospects, i.e., a higher α , can use the future profits to reduce the spread on debt due to the two effects discussed above. A key feature of this prediction is that the lower spread is, in part, driven by higher effort provision by the bank. This is potentially testable. One can proxy bank monitoring with borrower site visits, hiring third party appraisers, or the frequency with which banks demand loan-specific information (see Gustafson et al. 2021).
- 2. The spread on debt falls in the bank's growth opportunities if these are positively correlated with its core activities. If they are negatively correlated, the effect may reverse.
 - If there is a positive correlation between the bank's growth opportunities, α , and the return on its assets-in-place, Y, then an increase in one is accompanied by an increase in the other. The effect of both changes are in the same direction, which is that the spread on debt falls. If on the other hand, Y and α are negatively correlated, then an increase in one is accompanied by a fall in the other. As a result, these two changes have opposite effects on the spread and which effect dominates depends on the parameters.

5 Conclusion

We present a model in which there are two investment opportunities for a bank; the first suffers from an effort moral hazard problem, while the second suffers from an adverse selection problem. We show that on-balance sheet financing with cross-subsidization across projects strictly dominates direct financing, on-balance sheet financing without cross-subsidization and off-balance sheet financing for all parameter values, and delivers the second-best outcome. In our solution, expected profits from projects which suffer from adverse selection are used to induce higher effort provision in projects subject to effort moral hazard. Griffin et al. 2007, Santikian 2014, and Jenkinson et al. 2018 provide empirical evidence consistent with cross-subsidization across unrelated activities in financial institutions. Our analysis implies that, given the informational frictions in the present setting, the optimal solution requires intermediation rather than off-balance sheet financing. Hence, in order to provide a rationale for disintermediation (off-balance sheet financing), we need to identify a different combination of frictions.

6 Appendix

6.1 Proofs

Proof of Lemma 1:

Proof. Due to limited liability, $D_{2,3} \leq R - D_{0,3}$. In a separating equilibrium in which bad banks obtain financing without pooling with good banks, investors' participation constraint is violated since for any $D_{2,3}$ satisfying the limited liability constraint (even for $D_{0,3} = 0$, and hence, for all feasible $D_{0,3}$), $p_bD_{2,3} < 0$ (see Assumption A1). Therefore, there cannot exist a separating equilibrium in which bad banks obtain financing.

Proof of Lemma 2:

Proof. Suppose that the equilibrium is pooling. In Figure 2a, the blue dashed line denotes the

combination of K and $D_{2,3}$ such that investors break even in a pooling equilibrium. Investors provide funds in a pooling equilibrium for any combination of K and $D_{2,3}$ which lies above the blue dashed line. Note that the blue dashed line lies entirely above the zero profit line of bad banks, $\Pi_b = 0$ – bad banks participate on or below this line. This implies that for any K, the repayment that investors require to break even in a pooling equilibrium is such that it delivers negative profits for bad banks, which violates the rationality assumption. Thus, the conjectured equilibrium cannot exist.

Proof of Lemma 4:

Proof. Suppose that there exists a market breakdown equilibrium at t=2. Given the beliefs associated with the breakdown equilibrium, the expected payoff that the t=0 investors obtain from the second project is 0, which implies that the price that they are willing to pay for this cash flow is Z=0. In this case, the bank needs to raise the full 1 unit in order to invest in the moral hazard project. Thus, the face value of debt in the first project in this case is higher than in the case when it is expected that there will be financing for the good-type second project. From Lemma 3, for any K > 0 and $D_{0,3} \leq R - \frac{1}{p_g}$, the t = 2 investors can achieve separation in the shaded region in Figure 2a, such that they can offer financing in the good-type second project. Given this offer, good banks at t=2 will invest since their participation constraint is satisfied, while bad banks will not invest since their participation constraint is violated in this region. By the Intuitive Criterion, investors assign a probability 1 to the event that only good banks will invest. Thus, there exists a profitable deviation from the K=0 market breakdown equilibrium in which investors offer K > 0 which ensures that good projects obtain financing at t=2. Anticipating that good banks will invest, the t=0 investors are willing to purchase a portion of the second project for Z>0, which reduces the bank's financing requirement for the first project and, in turn, reduces the promised repayment for the first project, D_1 . This lower D_1 will induce higher effort provision and lead to higher profits which the bank will (at least partly) capture. Therefore, this deviation is strictly profitable both for the bank and the investors, and hence, the market breakdown equilibrium cannot exist.

Proof of Lemma 5:

Proof. Suppose that K > 0 and $D_{0,3} < R - \frac{1}{p_g}$. It is clear from Equation (10) that bank profits are maximized when K takes its smallest value and $D_{0,3}$ takes its largest possible value. Hence, for any K > 0 and $D_{0,3} < R - \frac{1}{p_g}$, a new investor will enter the market to offer a smaller K and a higher $D_{0,3}$, and attract the bank profitably. The new contract will reduce the promised repayment for the first project, D_1 . A lower D_1 leads to higher effort in the first project and increases bank value, so the bank accepts the deviant offer. Thus, competition among investors sets $K^* = 0$ and $D_{0,3}^* = R - \frac{1}{p_g}$.

Proof of Proposition 4:

Proof. With regards to the adverse selection project, there is full separation between good and bad banks in both intermediated and market financing cases. Thus, to show that intermediated financing dominates market financing in terms of efficiency, we need to show that $e^* > e^D$ for all parameter values. First, we show that $D_1^* > D_1^D$. Using Equations 11 and 16:

$$D_1^* = \frac{1 - (1 - p_g - me)Y}{p_g + me} - \underbrace{\frac{\alpha(p_g R - 1)}{p_g + me}}_{>0} < \frac{1 - (1 - p_g - me)Y_1}{p_g + me} = D_1^D$$
 (22)

Using $D_1^* < D_1^D$ and Equations 12 and 17:

$$c'(e^*) = m(R - D_1^* + Y) > m(R - D_1^D + Y) = c'(e^D)$$
(23)

Since c'(e) is increasing in e, it holds that $e^* > e^D$.

Proof of Lemma 6:

Proof. First we show that $D_1^* < D_1^{on} < D_1^{off}$. Noting that $1 - p_g - \alpha > me$ since it is assumed

that $1 - p_g \ge me$ and using Equations 11, 18, and 20:

$$D_{1}^{*} = \frac{1 - (1 - p_{g} - me)Y}{p_{g} + me} - \underbrace{\frac{\alpha(p_{g}R - 1)}{p_{g} + me}}_{>0} < \underbrace{\frac{1 - (1 - p_{g} - me)Y}{p_{g} + me}}_{D_{1}^{on}}$$

$$= \underbrace{\frac{1 - (1 - p_{g} - \alpha - me)Y - \alpha Y}{p_{g} + me}}_{\text{add and subtract } \alpha Y} < \frac{1 - \alpha Y}{p_{g} + me} = D_{1}^{off}$$
(24)

Using $D_1^* < D_1^{on} < D_1^{off}$ and Equations 12, 19, and 21:

$$c'(e^*) = m(R - D_1^* + Y) > \underbrace{m(R - D_1^{on} + Y)}_{c'(e^{on})} > m(R - D_1^{off}) = c'(e^{off})$$
(25)

Since c'(e) is increasing in e, it holds that $e^* > e^{on} > e^{off}$.

6.2 Limited liability constraint at t = 2

In the main text, we consider the case the limited liability constraint of the bank at t = 2 is not binding. Below, we consider the case in which the limited liability constraint is binding (i.e., $D_{2,3} > R - D_{0,3}$). In this case, the participation constraint of an investor facing a type g bank becomes:

$$\gamma_q = p_q(R - D_{0.3}) - (1 - K) \ge 0 \tag{26}$$

When the participation constraint binds we get, $K \geq 1 - p_g(R - D_{0,3})$, which is the green vertical line in the $(K, D_{2,3})$ space in Figure 3. For an equilibrium to be feasible, it must lie on or to the right of this green vertical line. When the limited liability constraint is slack (see Figure 2a), this line is in the second quadrant and the green line intersects the horizontal axis at K < 0, and when it just binds (see Figure 2b), it is along the vertical axis. Thus, for $D_{0,3} \leq R - \frac{1-K}{p_g}$, this line does not affect the analysis. When the limited liability constraint is

binding, the vertical line appears in the first quadrant, ruling out some of the equilibria. The set of feasible financing equilibria is along the $\Pi_g = \gamma_g = 0$ line, but on or to the right of the green line.

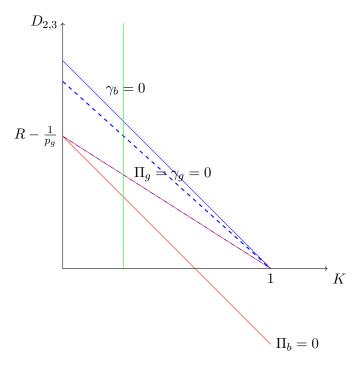


Figure 3: Feasible equilibria, $D_{0,3} > R - \frac{1-K}{p_g}$

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