The spillover effect of collateral

Sonny Biswas*

Abstract

I analyze a novel spillover effect from secured to unsecured lending. A high type borrower has a good project while the low type does not know her project's quality. The high type posts collateral and the bank screens only the low type's project. Different from existing models, equilibrium collateral requirements are stricter than the minimum necessary to achieve separation, despite costly liquidation of collateral. When the high type posts more collateral, the lender charges a higher interest rate to the low type, which enhances its incentives to screen the low type's project, thereby improving the average quality of unsecured loans.

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*School of Accounting and Finance, University of Bristol, s.biswas@bristol.ac.uk.

1 Introduction

Collateral in debt contracts serve a multitude of functions. One function of collateral is that it protects lenders' interests in defaults (e.g., Tirole 2006, Donaldson et al. 2020b). Modelling this particular function of collateral, Asriyan et al. (2021) show in a dynamic general equilibrium setting that credit booms associated with higher collateral values can crowd out bank screening which leads to protracted economic crises. Instead, I consider another widely studied function of collateral which is that it solves an adverse selection problem, and show that higher collateral values can improve the equilibrium outcome. In my model, higher collateral values in secured lending lead to improved incentives to screen the unsecured loans, and hence, a higher average quality of unsecured loans, i.e., there is a positive spillover effect from secured to unsecured lending.

In the traditional adverse selection models of collateral (e.g., Bester 1985, Besanko and Thakor 1987a), collateral sorts borrowers into risk types, with no residual uncertainty regarding the borrowers' riskiness which makes direct screening by banks (e.g., through information acquisition) redundant. Liquidating collateral entails a deadweight loss, and hence, the minimum amount of collateral that achieves separation between types is used, and no more. In my setting, there are two important differences: first, direct screening supplements the use of collateral, and second, equilibrium collateral requirements can be higher than what is necessary to separate borrower types. These features together allow me to derive a new spillover effect which has not been considered in the literature. My model reconciles mixed evidence in the literature on the impact of higher availability of collateral on overall lending in the economy. I provide a new explanation for the findings in Acharya and Subramanian (2009) that innovative firms lower their borrowing when creditor rights are higher.

I present a model in which there are two types of borrowers who apply for credit from a monopolistic bank: a high type borrower has a good project with certainty, while a low type borrower may have a good or a bad project. Borrowers privately know their type, but the low type borrower's project quality is unknown to everyone. The high type borrower has a higher reservation utility (outside option) which gives rise to countervailing incentives, i.e., the low type borrower potentially mimics the high type borrower. The type-dependent outside option may arise if borrowers can raise funds from familiar sources (i.e., the financier can observe the borrowers' type) and invest in the project, but at a reduced scale.

On receiving a loan application, the bank may exert costly effort to directly screen the project to gauge whether it is good or bad; notably, the bank screens the project, not the borrower. Screening is noisy and the precision of the signal produced by screening increases in the effort exerted by the bank (the screening intensity). Zero effort produces a pure noise signal, such that the probability of detecting a good project becomes equal to the prior, which is given by the fraction of good projects in the pool of borrowers being screened (the screening pool).

In the symmetric information benchmark borrower types are observable. Project quality may be inferred from the borrower type, but is not directly observed. A high type borrower's project is not screened since it is known with certainty that she has a good project. The bank screens a low type borrower's project and extends credit only if the signal is good. The monopolistic bank sets type-contingent interest rates which satisfy the respective participation constraints, and extracts the full surplus from lending.

In the incomplete information case, borrower type is private information and interest rates alone cannot separate borrower types; the low type borrower mimics the high type borrower since the high type's intended contract specifies a smaller interest rate. Two distortions arise compared to the symmetric information benchmark: first, the screening pool contains a higher fraction of good projects since both high and low type borrowers' projects are screened, and second, the low type borrower extracts informational rents. The rents extracted by the low type borrower are increasing in the high type borrower's outside option: the interest rate charged by the bank in the pooling equilibrium needs to be low enough to satisfy the high type borrower's participation constraint, which is the more binding out of the two types' participation constraints due to the high type borrower's higher outside option.

Enlarging the contracting space from just interest rates to include both interest rates and collateral allows the separation of borrowers into high and low types. It should be noted that the contract terms only partially resolve uncertainty since while the borrower types are separated, without direct screening it remains uncertain whether the low type borrower has a good or a bad quality project; this information cannot be elicited through contracts since the low type borrower herself does not know her project's quality. The high type borrower applies for secured loans, while the low type borrower applies for unsecured loans; only the low type borrower's project is screened, same as in the symmetric information benchmark. Jimenez et al. (2006) provide evidence in support of the view that when the unobserved component of risk is considered (and the observed component of risk is controlled for), the high credit quality borrowers are more likely to pledge collateral (see also Berger et al. 2011a, Berger et al. 2011b and Godlewski and Weill 2011).

Suppose that sufficient collateral is available which allows separation and that liquidating collateral is not too costly (it could be costless). Given separation, a higher amount of collateral posted by the high type makes the high type's (secured) contract less attractive for the low type to mimic and allows the bank to charge a higher interest rate to the low type. The high type's interest rate is adjusted with respect to the collateral pledged, such that the high type's participation constraint always binds and the high type borrower is indifferent between posting less or more collateral.

If the availability of collateral is limited, the low type cannot be charged a high interest rate, and hence, the low type extracts informational rents (the low type's participation constraint is slack). So, the first of the two distortions with respect to the symmetric information benchmark is eliminated since the screening pool is identical to the full information case. However, since the low type borrower extracts informational rents, the second distortion with respect to the symmetric information benchmark is not eliminated. In particular, the screening intensity is lower compared to the symmetric information benchmark. If the availability of collateral is plentiful, the bank sets high collateral requirements, which makes the secured contract unattractive for the low type borrower. This allows the bank to increase the interest rate offered to the low type borrowers all the way till the low type's participation constraint binds. Then the entire surplus from lending accrues to the bank. Therefore, with plentiful availability of collateral, both distortions with respect to the full information benchmark are eliminated and the efficient outcome is achieved.

In contrast to the existing adverse selection models (e.g., Bester 1985), the level of collateral is uniquely pinned down in my model if liquidation of collateral is costless. If liquidating collateral entails a cost, in the existing models, the minimum amount necessary to separate is used; beyond this point, an increase in the availability of collateral does not impact the equilibrium outcome. In my model, the equilibrium collateral requirements can be higher than the minimum necessary for separation since when the high type borrower posts more collateral, the bank extracts more of the surplus from screening the unsecured loans (and screens more diligently, as a result). This result allows me to derive new empirical implications:

First, an increase in the availability of collateral leads to a higher average quality of unsecured loans (unambiguously); this is the spillover effect of secured lending and I am unaware of existing empirical tests of this prediction. Second, higher collateral availability may lead to an increase or a fall in the overall credit in the economy, depending on the distribution of projects. Thus, my model reconciles mixed empirical evidence on the impact of higher creditor rights on the overall level of credit in the economy. Specifically, the prediction is that if the low type have a high (resp. low) fraction of good projects, then higher collateral stock leads to improved screening which, in turn, leads to higher (resp. lower) approvals of unsecured loan applications.

The prediction is consistent with evidence in Acharya and Subramanian (2009) that innovative firms (presumably, with a lower fraction of good projects relative to traditional firms with tried-and-tested technologies) borrow less when creditor rights are higher.

In the baseline model, I assume that outcome of screening is contractible. In this case, the equilibrium is identical to the full information benchmark when the availability of collateral is plentiful and the bank can set sufficiently high collateral requirements. If the availability of collateral is limited, there is distortion compared to the full information benchmark since the low type borrower extracts informational rents. However, taking the degree of collateral availability as exogenously given, the regulator cannot improve on the equilibrium outcome. Consider, instead, that the outcome of screening is not contractible. If the availability of collateral is plentiful and the bank can set sufficiently high collateral requirements, then whether or not the outcome of screening is contractible is inconsequential. If the availability of collateral is limited, the bank threatens to hold up the low type borrower and the separating equilibrium unravels. This gives rise to a scope for policy intervention in the model. The regulator can step in and impose a ceiling on the interest rate that can be charged on unsecured loans. The interest rate ceiling resolves the hold-up problem and restores the separating equilibrium.

Related literature. In one set of theories collateral alleviates borrower moral hazard concerns and the (observably) riskier borrowers are more likely to pledge collateral (e.g., Boot et al. 1991 and Boot and Thakor 1994). My model belongs to a complementary set of theories which show that collateral sorts borrowers into (unobservable) risk classes (e.g., Bester 1985, Chan and Kanatas 1985, Besanko and Thakor 1987a, Besanko and Thakor 1987b): these make up the adverse selection theories of collateral. In these theories, collateral requirements solve a problem of pre-lending information asymmetry between the borrower and the lender and improve equilibrium credit allocation. The two novel features of my model discussed above distinguishes it from the adverse selection models and generates new implications. Specifically, I can perform comparative static analyses with respect to the degree of collateral availability, which is not possible in the traditional models.

In the traditional adverse selection models (e.g., Bester 1985), the use of collateral keeps the bad borrowers out and leads to lower lending, while in the moral hazard theories (e.g., Boot et al. 1991), more collateral availability leads to higher lending. In contrast, my model predicts that a higher availability of collateral may lead to higher or lower overall credit provision, depending on the distribution of projects. Donaldson et al. (2020b) show that collateralized borrowing by a borrower encumbers her assets and can lead to lower borrowing by the same borrower in the future. In contrast, in my setting, collateral pledged by some borrowers affects the amount of credit available to borrowers who do not pledge collateral.

Asriyan et al. (2021) show that when borrowers posts more collateral, the bank has a smaller incentive to expend costly resources to acquire information. This leads to a lower stock of information in the economy which can then lead to protracted crises, i.e., a credit boom due to a higher availability of collateral can lead to crises. In Manove et al. (2001), collateral requirements lead to the sorting of borrowers into risk types and reduce costly screening by banks, even when it is socially optimal to screen (see also Hainz et al. 2013, Karapetyan and Stacescu 2017 and Goel et al. 2014 who model collateral and bank screening as substitutes). Similar to these studies, in my model, the bank does not screen the borrower who posts collateral. However, different to them, the bank's incentives to screen the unsecured loan applications are affected in my setting. Indeed, I uncover a new complementarity relationship which is that when the secured borrowers post more collateral, the bank screens the unsecured loan applications more diligently (the positive spillover effect of secured lending).

Ioannidou et al. (2021) provide empirical evidence that, for the borrowers which take out both secured and unsecured loans simultaneously, the spillover effect from the secured to unsecured loans can be substantial. Donaldson et al. 2021 provide further evidence on spillovers across different classes of debt issued by the same borrower (the sovereign, in their case). I present a first theoretical analysis of the spillover effect from secured borrowers to unsecured borrowers (i.e., from borrowers who pledge collateral to borrowers who do not).

Rajan and Winton (1995) consider a dynamic model with multiple creditors and show that a bank's ability to acquire additional collateral when borrower's prospects worsen gives it effective priority over the other creditors at the liquidation stage, and enhances the bank's ex-ante monitoring incentives. While I too investigate the interaction between contract design and the bank's incentives, I consider a static model with a single creditor. My model is static in the sense that no new information becomes available between the bank granting a loan to the borrower and the repayment date. Further, since there is a single creditor, there is no conflict among creditors to resolve. Donaldson et al. (2020b) and Donaldson et al. (2020a) also consider dynamic models in which there are conflicts among multiple creditors, which can be resolved by assigning priority to secured debt. Again, since there is only one period and a single creditor, collateral plays a different role in my model. In particular, alternate contractual arrangements such as the use of covenants or debt maturity cannot substitute (or complement) the use of collateral in my model.

Inderst and Mueller (2007) provide a theory of bank entry in which the use of collateral improves the uninformed entrant bank's ex-ante credit decision by enhancing its incentives to finance the marginally profitable projects. Different from my setting, their theory relies on competition among local (or informed) and transaction (or foreign) lenders and does not feature borrower-side frictions such as adverse selection or moral hazard. Sengupta (2007) and Sengupta (2014) model how the use of collateral interacts with the entry of uninformed (foreign) lenders in the presence of informed incumbent lenders. They show that the uninformed lenders may use collateral to cream-skim the high quality borrowers. In contrast, in my setting there is a monopolistic lender who competes for both the high and low quality borrowers and uses both collateral and direct screening.

In my model, the use of collateral alters the screening pool, which has implications for the screening intensity. The central importance of the quality of the screening pool in my model is reminiscent of the game in Broecker (1990). In his model, the bank which offers the lower loan interest rate gets the higher quality pool of applicants, while the bank which offers the higher loan interest rate inherits the leftover applicants, who have been previously rejected by the first bank. As competition intensifies, the applicant pool becomes worse and worse since the rejected loan applicants can apply at other banks (see also Marquez 2002 and Shaffer 1998). While Broecker (1990) and the related articles interact the quality of the pool with bank competition, I interact it with the use of collateral.

2 Model

2.1 Set-up

I consider a four-date economy, $t = 0, 1, 2, 3$. There is a continuum of entrepreneurs (borrowers) with access to a project. The borrowers are penniless and seek financing for their projects from a monopolistic bank. All agents are risk-neutral and protected by limited liability. The risk-free rate is normalized to 0, so there is no discounting. The project is of fixed scale and requires an investment, normalized to 1, at $t = 2$. A borrower is either high type, with probability q_1, or low type, with probability, $(1-q_1)$. Borrower type is determined by nature, and it is the borrower's private information. If the borrower is low type, the project is good with some probability, q_2, and bad with the complementary probability, $(1 - q_2)$. The low type borrower does not know if her project is good or bad quality. If the borrower is high type, the project is good with certainty. A good project either succeeds with probability, p, and produces X, or fails with probability, $(1 - p)$, and produces 0, at $t = 3$. A bad project produces 0 with certainty. The payoff structure for each borrower is illustrated in Figure 1. I make two parametric assumptions regarding the profitability of projects:

A1: $pX - 1 > 0$

A2: $q_p pX - 1 < 0$*, where* $q_p \equiv q_1 + (1 - q_1)q_2$

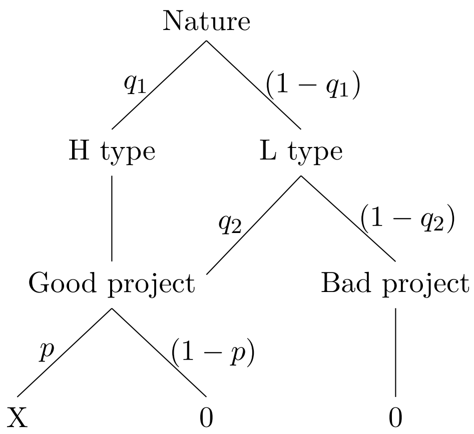

Figure 1: Payoff tree

Assumption *A1* indicates that the good projects are profitable. If types are separated, the high type borrower's project is not screened as the high type borrower has a good project with certainty. Therefore, the screening pool is made up of either only low type borrowers' projects or both low and high type borrowers' projects (a mixed pool). q_2 is the fraction of good project in an screening pool containing only low type borrowers' projects, while q_p is the fraction of good projects in an screening pool containing both high and low type borrowers' projects. Assumption *A2* implies that it is not profitable to lend without screening unless it is known with certainty that the borrower is high type.

The bank offers debt contracts at $t = 0$.[1] A borrower applies for a debt contract, which may or may not reveal her type. The borrower type does not directly reveal the project quality. The bank can screen the borrower's project in order to identify the quality of the project. screening produces a signal s_g (project is good) or s_b (project is bad), and the signal is informative but

[1]The assumption that only debt contracts are offered is without loss of generality since, given the structure of returns in our model, all contracts (sharing rules) are equivalent.

noisy:

$$Pr(s = s_g|\text{project} = good) = Pr(s = s_b|\text{project} = bad) = \beta \geq \frac{1}{2} \tag{1}$$

β is the precision of the signal, which increases in screening intensity, F, as follows:

$$\beta = \frac{1}{2} + F \tag{2}$$

where $F \in (0, \frac{1}{2})$. If $F = 0$, the signal is pure noise and $\beta = \frac{1}{2}$. For $F = \frac{1}{2}$, $\beta = 1$ and screening perfectly reveals the project's quality. The cost of screening is given by $\frac{\tau}{2}F^2$, with $\tau > 0$ and sufficiently large, in a sense made precise later. The functional form implies that the screening cost is increasing and convex in screening intensity. Convexity reflects increasing difficulty for the bank to find out more and more about a project (see e.g., Song and Thakor 2010). Having incurred a non-zero screening cost such that $F > 0$, the bank must only agree to lend if the signal is good, $s = s_g$; otherwise, the bank might as well not have incurred the cost in the first place. Given an observed signal, the posterior probability of the project quality depends on the composition of the screening pool. The posterior probabilities are given as follows:

$$Pr(\text{project} = \text{good}|s_g, q_A) = \frac{\beta q_A}{[\beta q_A + (1-\beta)(1-q_A)]} \tag{3}$$

$$Pr(\text{project} = \text{good}|s_b, q_A) = \frac{\beta(1-q_A)}{[\beta(1-q_A) + (1-\beta)q_A]} \tag{4}$$

$q_A \in \{q_2, q_p\}$ is the unconditional probability of a good project in the screening pool. $q_A = q_2$ if the screening pool contains low type borrowers only, while $q_A = q_p$ if the pool contains both types. The conditional probability of a bad project is $Pr(\text{project} = \text{bad}|s) = 1 - Pr(\text{project} = \text{good}|s)\ \forall\ s \in \{s_g, s_b\}$. For the pure noise signal, when $F = 0$ and $\beta = \frac{1}{2}$, the conditional probability that the project is good becomes equal to the prior, which is given by the fraction of the good projects in the screening pool, q_A. Therefore, the pure noise signal detects a good project with a lower probability when the screening pool is made up of low type borrowers only, compared to the case of the mixed pool (since $q_2 < q_p$).

If screening, the bank extends credit if the signal is good, $s = s_g$. Given the quality of the screening pool, q_A, the probability of the good signal is, $Pr(s = s_g|q_A)$, which is given by the denominator in Equation (3). If the signal is positive the project quality is good with probability, $Pr(\text{project} = \text{good}|s_g, q_A)$ (Equation (3)). If the project is good it succeeds with probability, p, and the bank receives a repayment of R. If the good project fails, or the project is bad, the repayment is 0. The bank's expected payoff from an unsecured loan (i.e., a contract featuring only interest rates) is:

$$Pr(s = s_g|q_A)[Pr(\text{project} = \text{good}|s_g, q_A)pR - 1] - \frac{\tau}{2}F^2 \tag{5}$$

A3: The cost of screening τ is sufficiently high:

$$\frac{\tau}{2} > q_p(pX - 2) + 1 \tag{6}$$

Assumption *A3* ensures that the solution of the bank's problem satisfies $F < \frac{1}{2}$.

A4: *The high type borrower has an outside option, H, with $H \in (0, pX - 1)$; while the low type borrower's outside option is normalized to* 0*.*

The higher outside option of the high type borrower can be understood as follows: while the borrower type is hidden from the bank, the borrowers' family members can observe their type. Then, the family members can fund the project, but at a smaller scale. In this case, the high type will invest, and produce H, while the low type will not invest. These frictions are of particular relevance in the credit markets since new entrepreneurs often find it difficult to acquire bank loans but can raise seed funding from family members.

As I consider monopolistic banks, collateral does not play a role if high and low types have identical outside options (see page 677 in Besanko and Thakor 1987a). That the high type borrower has a higher outside option than the low type borrower gives rise to countervailing incentives which implies that the low type borrower potentially mimics the high type borrower.[2] The countervailing incentives makes collateral relevant in my model.

Timeline. At $t = 0$, a bank offers a loan contract ζ which consists of the interest rate and collateral requirement. Each borrower applies for a contract at $t = 1$. This action may or may not reveal the borrower's type. At $t = 2$, the bank decides whether or not to screen the borrower's project. If screening, the bank further decides the intensity of screening. Since the bank's screening decision is perfectly anticipated at $t = 0$, it can be thought of as part of the offered contracts, i.e., the contract ζ implicitly consists of the bank's screening effort. Depending on the outcome of screening, the bank then approves or rejects the borrower's application. If the loan is approved, the bank funds the project. If the application is rejected, the borrower makes her type-specific outside option. The payoffs are realized at $t = 3$, when all agents consume the output. Note that between loan approval at $t = 2$ and the final date, no new information arrives or is generated, which makes this a static model with no scope to explore debts of different maturities. The timing is illustrated in Figure 2.

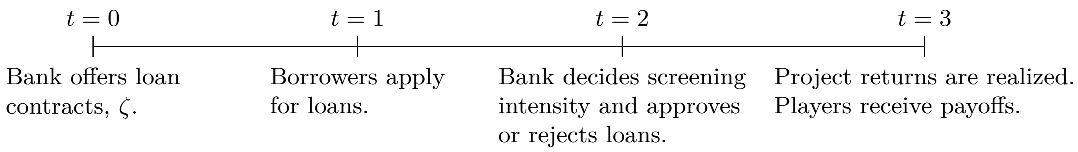

Figure 2: Timeline

Equilibrium definition. Since the uninformed player (the bank) moves first, this is a screening game. I look for the pure strategy subgame-perfect Nash equilibrium.[3] The bank maximises its

[2] Laffont and Martimort (2002) (pages 101-115) discuss the impact of type-dependent reservation utilities on the principal-agent problem and give an overview of several applications that have appeared in the broader contract theory literature.

[3] Despite the presence of private information, the appropriate solution concept is a non-Bayesian equilibrium concept. The reason that this is a screening game in which the uninformed player moves first by offering contracts. Therefore, there are no inferences to be made once the contracts are offered (since the offers are made by the uninformed players to the informed players). So, upon observing the contracts (or at any other point in the game) there is no Bayesian updating of beliefs by any player. The choice of equilibrium concept is consistent

expected payoff subject to the individual rationality and incentive compatibility constraints of each borrower type. Additionally, the loan repayment rates must satisfy the borrowers' limited liability constraints and the non-negativity constraints (the feasibility constraints). We know from Rothschild and Stiglitz (1976) that a competitive equilibrium may not exist in a screening game due to banks undercutting one another. However, the non-existence problem does not arise in my setting, since there is no threat of undercutting with a monopolistic bank.

2.2 Full information benchmark

In the full information benchmark, a borrower's type is observable, but in the case of a low type borrower, neither the bank nor the borrower knows the project quality. The bank offers a menu of type-contingent contracts, $(R_t, C_t),\ t \in \{l, h\}$. R_t is the repayment rate and C_t is the collateral requirement; the bank can seize the collateral from the borrower in the event of no repayment. It is assumed that collateral is costly, the cost may be minimal or substantial (discussion is provided later). In the full information benchmark, the bank sets $C_t = 0$ for each type to minimize deadweight loss.

If the borrower is high type, the bank does not screen and always approves the loan application. If, on the other hand, the borrower is low type, the bank incurs positive screening cost, say F^{fi}, and extends credit only if the signal is positive, i.e., $s = s_g$. The bank solves the following problem:

$$\begin{aligned}
&\underset{F}{\text{Max}} && \beta q_2 p R_l - \beta(2q_2 - 1) - (1 - q_2) - \frac{\tau}{2}F^2 \\
&\text{subject to} && \\
&\text{(IRH)} && p(X - R_h) \geq H \qquad (7)\\
&\text{(IRL)} && \beta q_2 p(X - R_l) \geq 0 \\
&\text{(FCs)} && 0 \leq R_t \leq X\ \forall t \in \{l, h\}
\end{aligned}$$

with other models featuring screening games such as Rothschild and Stiglitz (1976) and Besanko and Thakor (1987a).

The bank maximizes its expected payoff with respect to the choice of screening intensity, F. Since only the low type borrower's project is screened, the screening pool consists of q_2 (substitute $q_A = q_2$ in Equation (5) to derive the objective function). The bank sets the repayment rates such that the Individual Rationality (or IR) constraints of the borrowers are satisfied, IRH and IRL, for the high and low type borrowers, respectively. The left hand side (LHS) of an IR constraint is the expected payoff of the corresponding borrower type and the right hand side (RHS) is her outside option. The FCs are the feasibility constraints which are the non-negativity and limited liability constraints.

Proposition 1 (Full information). *In the full information benchmark, the high type borrower always receives credit. The bank screens the low type borrower and grants credit only if the signal is positive. The equilibrium is characterized as follows:.*

$$R_h = X - \frac{H}{p} \tag{8}$$

$$R_l = X \tag{9}$$

$$F^{fi} = \frac{1}{\tau}(q_2(pX - 2) + 1) \tag{10}$$

Proof. The proof is in the appendix. ■

The bank grants credit to all high type borrowers (no errors). A low type borrower with a good project receives credit with probability, $\beta^{fi} = \frac{1}{2} + F^{fi}$. Due to noisy screening, some bad projects are financed, and some good projects are denied credit. With probability, $(1-\beta^{fi})$, the bank incorrectly rejects a low type borrower with a good project and incorrectly grants credit to a low type borrower with a bad project.

2.3 Equilibrium without collateral

In this section, I consider the incomplete information problem, with the restriction that the credit policy only involves setting the interest rate, R_t. In a separating equilibrium the contracts should ensure that neither type is better off mimicking the other (the Incentive Compatibility (or IC) constraints, ICH and ICL, corresponding to the high and low type borrowers, respectively, are satisfied). The IC constraints are as follows:

$$\text{(ICH)} \quad p(X - R_h) \geq \beta p(X - R_l) + (1 - \beta)H \tag{11}$$

$$\text{(ICL)} \quad \beta q_2 p(X - R_l) \geq q_2 p(X - R_h) \tag{12}$$

The LHS of the IC constraint for borrower type t is her expected payoff when she tells the truth, while the RHS is her expected payoff from mimicking the other type. If mimicking the low type borrower, the high type borrower's projects is screened. Due to noisy screening, the high type borrower is denied credit with probability $(1 - \beta)$, in which case she makes her outside option, H. If mimicking the high type borrower, the low type borrower is always granted credit, and with probability q_2, the project is good. The IC constraints are jointly satisfied only if:

$$(1 - \beta) \leq 0 \tag{13}$$

However, this condition is never satisfied as long as the solution is interior, i.e., $\beta < 1$.

Lemma 1: *Contracts featuring interest rates only are not incentive compatible.*

Next, I look for a pooling equilibrium in which the high and low type borrowers apply for the identical contract (repayment, $R_h = R_l = \hat{R}$). In a pooling equilibrium both high and low type borrowers' projects are screened, and a borrower is granted credit if screening produces a good signal, i.e., $s = s_g$. The bank solves the following problem:

$$\begin{aligned}
&\underset{F}{\text{Max}} && \beta q_p p\hat{R} - \beta(2q_p - 1) - (1 - q_p) - \frac{\tau}{2}F^2 \\
&\text{subject to} && \\
&\text{(IRH)} && p(X - \hat{R}) \geq H \\
&\text{(IRL)} && \beta q_2 p(X - \hat{R}) \geq 0 \\
&\text{(FCs)} && 0 \leq \hat{R} \leq X
\end{aligned} \tag{14}$$

When both high and low type borrowers' projects are screened, the screening pool consists of q_p good projects (substitute $q_A = q_p$ in Equation (5) to derive the objective function).

Proposition 2 (No-collateral equilibrium). *When the contracting space is restricted to interest rates only, the equilibrium is pooling. Both high and low type borrowers' projects are screened, and a borrower is granted credit if screening produces a good signal. The equilibrium is characterized as follows:*

$$\hat{R} = X - \frac{H}{p} \tag{15}$$

$$F^P = \frac{1}{\tau}(q_p(pX - H - 2) + 1) \tag{16}$$

Proof. The proof is in the appendix. ■

Note that in the no-collateral equilibrium the low type borrower extracts informational rents, $\hat{R} < R_l$. The bank screens both borrower types' projects. In contrast to the full information benchmark, some high type borrowers do not receive credit, due to screening being noisy. The high type borrower and a low type borrower with a good project receive credit with probability, $\beta^P = \frac{1}{2} + F^P$, and is rejected with probability, $(1 - \beta^P)$. A low type borrower with a bad project receives credit with probability, $(1 - \beta^P)$.

Comparison with the full information benchmark. The screening intensity in the pooling equilibrium, F^P, differs from the screening intensity in the full information benchmark, F^{fi}, due to two distortions: the low type borrower extracts informational rents, $\hat{R} < R_l$, and the screening pool contains a higher fraction of good projects in the pooling equilibrium, since the high type borrower's project is also screened, $q_p > q_2$.

In the full information benchmark, the bank keeps the entire surplus by perfectly price-discriminating across borrower types. In the pooling equilibrium, the low type borrower extracts informational rents since the low type borrower pools with the high type borrower who has a higher outside option, which negatively affects the bank's screening incentives since some of the surplus from screening is appropriated by the low type borrower. Higher the high type borrower's outside option, H, greater the (negative) distortion in the screening incentives in the pooling equilibrium compared to the full information benchmark. The reason is that higher H implies that the low type borrower extracts a larger fraction of the surplus, which negatively impacts the bank's screening incentives.

The second difference relates to the composition of the screening pool. In the full information benchmark, the pool is of a lower quality since only the low type borrower's project is screened, $q_2 < q_p$. The effect of the pool composition on screening incentives may be positive or negative. Comparing (10) and (16), for low (resp. high) profitability projects, the screening intensity is lower (resp. higher) in the pooling equilibrium, compared to the full information benchmark.

2.4 Equilibrium with collateral

In this section, I introduce collateral into the contracting space. Assume that the borrowers have assets-in-place, W, which they can pledge as collateral. In addition to the interest rate, the bank specifies collateral requirements, $C \leq W$. In secured loans, if the borrower fails the bank can seize the collateral and recoup some of its investment. By the revelation principle of Myerson (1979), the bank can induce truth telling from the borrowers by offering two incentive compatible contracts. Suppose that the bank offers a menu of contracts, $(R_t, C_t),\ t \in \{l, h\}$.

Pledged assets may be seized by the bank if the borrower defaults. Since the high type borrower is less likely to default, it is relatively less costly for the high type to post collateral. Thus, the high type posts more collateral than the low type, i.e., $C_h > C_l$. Without loss of generality, the offered contracts are $(R_l, 0)$ and (R_h, C_h): since the purpose of collateral is purely to separate borrower types, asking both types to post collateral will only make constraints more binding. To start with, I assume that pledging collateral is costless. In the separating equilibrium, the bank screens only the low type borrower's project and solves the following problem:

$$
\begin{aligned}
&\underset{F}{\text{Max}} && \beta q_2 p R_l - \beta(2q_2 - 1) - (1 - q_2) - \frac{\tau}{2}F^2 \\
&\text{subject to} && \\
&\text{(IRH')} && p(X - R_h) - (1-p)C_h \geq H \\
&\text{(IRL)} && \beta q_2 p(X - R_l) \geq 0 \\
&\text{(ICH')} && p(X - R_h) - (1-p)C_h \geq \beta p(X - R_l) + (1-\beta)H \\
&\text{(ICL')} && \beta q_2 p(X - R_l) \geq q_2(p(X - R_h) - (1-p)C_h) - (1-q_2)C_h \\
&\text{(FCs)} && 0 \leq R_t \leq X\ \forall t \in \{l, h\}
\end{aligned}
\tag{17}
$$

The RHS of the high type borrower's new IR and IC constraints reflect that when the project fails, she loses the posted collateral. The IRL and the LHS of the ICL are the same as before since the low type borrower's intended contract is not secured by collateral, $C_l = 0$. However, the RHS of the ICL is affected since if she mimics the high type borrower she can lose the posted collateral if she has a good project which fails nonetheless or if she has a bad project.

To solve the problem I initially assume that the ICH' constraint is satisfied. After solving the modified problem, I verify that a solution exists which does not violate the starting assumption. In the relaxed problem, the IRH' constraint must bind; if not binding, the bank can increase

R_h a little to increase its profits without violating the other constraints. Next, note that either the IRL or ICL' constraint must bind. If neither constraint is binding the bank can increase R_l a little to increase its profits without violating the other constraints. It is the ICL' constraint which binds, and not the IRL constraint, if the RHS of the ICL' constraint is greater than 0. Using the IRH' constraint, the ICL' constraint binds if:

$$C_h \leq \frac{q_2 H}{(1-q_2)} \equiv \overline{C} \tag{18}$$

Equation (18) represents an upper bound on the amount of collateral used. If this condition is violated, then the IRL constraint binds which implies $R_l = X$.

From the IRH' constraint, the high type borrower's repayment rate when the loan is secured becomes:

$$R_h(C_h) = X - \frac{1}{p}(H + (1-p)C_h) \tag{19}$$

The interest rate is falling in the level of collateral, to allow the high type borrower to achieve her outside option, in expectation. The limited liability constraint ($R_h < X$) is always satisfied. The non-negativity constraint for the high type borrower's repayment rate is satisfied, $R_h \geq 0$, if:

$$H \leq pX - (1-p)C_h \tag{20}$$

Note that the RHS is falling in C_h, implying that a higher C_h makes the condition more binding. If the condition is satisfied for the upper bound on C_h, then it will be satisfied for any other value of C_h. Using $C_h = \overline{C}$, Equation (20) becomes $-H(1+pq_2) \leq pX(1-q_2)$. Since the LHS is negative and the RHS is positive, the condition is always satisfied.

Supposing that Equation (18) is satisfied, i.e., the ICL' constraint binds and using $R_h(C_h)$, the repayment rate charged to the low type borrower, $R_l(C_h)$, becomes:

$$R_l(C_h) = X - \frac{q_2 H - (1-q_2)C_h}{\beta(C_h)q_2 p} \tag{21}$$

The upper bound, $C_h \leq \overline{C}$, ensures that the limited liability constraint is satisfied, i.e., $R_l \leq X$. The non-negativity constraint is satisfied if $R_l \geq 0$, which gives a lower bound as follows:

$$C_h \geq \frac{q_2(H - \beta(C_h)pX)}{(1-q_2)} \equiv \underline{C} \tag{22}$$

Finally, I verify that bounds on C_h do not violate the starting assumption that the ICH' constraint is satisfied. Substituting $R_h(C_h)$ and $R_l(C_h)$ in the ICH' constraint:

$$C_h \geq \frac{q_2 H(1 - \beta^S(C_h))}{(1-q_2)} \equiv \hat{C}(\beta^S) \tag{23}$$

The ICH' constraint is satisfied only if collateral is sufficiently large, $C_h \geq \hat{C}$. Combining with the feasibility constraints of the low type borrower's repayment rate, the collateral, C_h, that the high type borrower needs to post in order to achieve the separation lies in the range, $\max(\underline{C},\ \hat{C}) < C_h < \overline{C}$. It is easily verified that $\hat{C} < \overline{C}$ is always satisfied for any $\beta > 0$, and $\hat{C} > \underline{C}$ is satisfied as long as $pX > H$, which holds due to Assumption, *A4*. Therefore, the feasible range of collateral requirements for which separation is achieved is given by $C_h \in [\hat{C}, \overline{C}]$.

Consider the case that $C_h = \gamma\overline{C} + (1-\gamma)\hat{C}(\beta^S) \equiv C^\gamma(\beta^S)$ with $\gamma \in [0,1]$. $\gamma = 0$ if borrowers have just enough assets to achieve separation, but no more. γ is increasing in the amount of collateral that the borrowers can post, and $\gamma = 1$ if the borrower is unconstrained, i.e., $W \geq \overline{C}$. Therefore, an increase in γ can be interpreted as an increase in the availability of collateral.

Substituting R_h, R_l, and C^γ in the bank's objective function and taking the first order condition gives the equilibrium screening intensity:

$$F^S = \frac{1}{\tau}(q_2(pX - H - 2) + 1 + \gamma q_2 H) \tag{24}$$

Substituting F^S into C^γ, the equilibrium level of collateral is:

$$C^\gamma = \frac{q_2 H}{1 - q_2}(1 - (1 - \gamma)\beta^S) \tag{25}$$

For any γ, collateral is relevant only for $H > 0$. For an arbitrarily small H, a positive amount of collateral achieves separation between types.

To fully characterize the equilibrium, substitute F^S and C^γ into R_h and R_l. The high type borrower's repayment rate, $R_h(C^\gamma)$ is falling in the equilibrium amount of collateral, while the low type borrower's repayment is increasing in the level of collateral, as follows:

$$R_l(C^\gamma) = \underbrace{X - \frac{H}{p}}_{\hat{R}} + \frac{\gamma H}{p} \tag{26}$$

For $\gamma = 0$, which corresponds to the minimum collateral requirement which achieves separation, $R_l(C^\gamma) = \hat{R}$. For $\gamma = 1$, the low type borrower's repayment becomes $R_l(C^\gamma) = X$. Therefore, by setting $\gamma = 1$, the bank extracts the full surplus from lending to the low type borrower. As long as collateral is costless, the bank increases the requirement to the full extent, $C_h = \overline{C}$, whenever the borrower has sufficient pledgeable wealth, $W \geq \overline{C}$. Thus, γ takes a value less than 1 only if the borrower is wealth-constrained. Different to existing models, the equilibrium level of collateral is uniquely pinned down even if collateral is costless. Lemma 2 and Proposition 3 fully characterize the equilibrium.

Lemma 2. *Suppose that $\hat{C} \leq W \leq \overline{C}$ and collateral is costless. The high type borrower posts the maximum amount of collateral available to her upto $\overline{C}$, i.e., $C_h = \min(W, \overline{C})$.*

Proposition 3 (Collateral Equilibrium). *The high type borrower applies for the secured loan,* $(R_h(C^\gamma), C^\gamma)$*, and always receives credit. The low type borrower applies for the unsecured loan,* $(R_l(C^\gamma), 0)$*; the bank screens the low type borrower's project with an intensity,* $F^S(C^\gamma)$*, and grants credit only if the signal is positive.*

Comparison with the full information benchmark. For $W \geq \overline{C}$, the equilibrium becomes identical to the full information benchmark in terms of the screening pool and the intensity with which the unsecured loans are screened. For $W < \overline{C}$, the collateral equilibrium bears some similarities with both the full information benchmark and the pooling equilibrium. The bank screens only the low type borrower's projects, as in the full information benchmark. The screening intensity, F^S, differs from the full information benchmark, F^{fi}, since the low type borrower extracts informational rents, as in the pooling equilibrium. $F^{fi} \geq F^S$ if:

$$(1 - \gamma) \geq 0 \tag{27}$$

This condition is always satisfied for any γ (with equality for $\gamma = 1$). Since the low type borrower extracts rents when the availability of collateral is limited, the screening incentives are negatively distorted. When the availability of collateral is not limited, $F^{fi} = F^S$.

So far I have assumed that collateral is costless to pledge. Suppose that there is a cost of posting collateral, k. Pledging collateral can be costly due to the disparity between the valuation of the borrower and the lender. The disparity can arise since the borrower may be the best user of the asset, while the lender (or whoever the lender can sell the asset to) lacks the expertise to use the asset. The disparity can also arise since the lender has to transport and store the asset on seizure and there may be additional legal costs. Parlatore (2019) provides a microfoundation for this cost: borrowers with persistent investment opportunities value access to collateral more than lenders do since these assets allow the borrowing firms to take advantage of their investment opportunities. In the reduced form, the cost can be thought of as a deadweight loss. The cost is relevant when borrowers pledge discrete assets as collateral (which is how collateral is modelled here), rather than borrowing against the going concern value of the business.

The lender faces the following trade-off. On the one hand, $R_l(C_h)$ is increasing in C_h, which implies that as the high type borrower posts a higher level of collateral, the bank extracts more of the surplus from lending to the low type borrower. On the other hand, collateral entails a deadweight loss (and the cost is borne the monopolist lender). Indeed, if collateral is prohibitively costly, the bank entirely foregoes the use of collateral and offer the same contract to all borrowers featuring only the interest rate. From this trade off, I present the following result:

Proposition 4 (Costly collateral). *Suppose that the availability of collateral is plentiful. The collateral requirements are low, $C_h = \hat{C}$, if collateral is moderately costly, $\bar{k} < k < k^P$, and high, $C_h = \overline{C}$, if the cost of collateral is small, $k < \bar{k}$. If collateral is very costly, $k > k^P$, collateral is not used and the equilibrium is pooling.*

Proof. The proof is in the appendix, where I also define and derive $\bar{k}$ and k^P. ■

In existing models of collateral (e.g., Bester 1985), the equilibrium features the minimum level of collateral required to separate, whenever collateral entails a non-zero cost. My setting yields a different result for the following reason: $R_l(C_h)$ is increasing in C_h. This implies that as the high type borrower posts a higher level of collateral, the bank extracts more of the surplus from lending to the low type borrower. This, in turn, improves the bank's screening incentives, and pushes the equilibrium towards the full information benchmark.

A numerical example illustrates that the various outcomes described above are feasible. The benchmark parameter values are: $q_1 = 0.1$, $q_2 = 0.25$, $p = 0.7$, $X = 4$, $H = 0.6$, $\tau = 3$. These parameters satisfy all the assumptions, *A1-A4*. In this example, $\bar{k} = 6.95$ and $k^P = 12.11$. When the cost of collateral is small, $k < 6.95$, the equilibrium is separating and $C_h = 0.2$. For moderately costly collateral, $6.95 < k < 12.11$, the equilibrium is separating and $C_h = 0.03$.

In this case, the low type borrower extracts informational rents. For very costly collateral, $k > 12.11$, collateral is not used and the equilibrium is pooling.

The analysis so far assumes that the bank serves both high and low type borrowers. By committing to serve the high type, the bank is giving up rents to the low type. If what the bank makes from the high type is smaller than what the bank loses to the low type, it could be the case that the bank chooses to not serve the high type, while extracting full surplus from the low type. However, such an equilibrium is not time-consistent since the bank cannot commit to not serve the high type at a later date after types are revealed at the initial date.

Lemma 3. *A separating equilibrium in which the bank only serves the low type is time-inconsistent.*

Proof. The proof is in the appendix. ■

Thus, the only equilibrium possible is the one described in Proposition 4.

Remark. In the existing adverse selection models of collateral with price-setting monopolistic banks, collateral is not used in loan contracts unless it makes the loan riskless (e.g., Besanko and Thakor 1987a and Lengwiler and Rishabh 2017). In these models (with symmetric outside options), the monopolistic bank's objective is to deter the high type from mimicking the low type while extracting the full surplus from lending. Since it is more attractive for the high type borrower to post collateral than the low type borrower, increasing collateral requirements is not effective in deterring the high type borrower. In contrast to these models, in my model there are countervailing incentives due to type-dependent outside options (same as Sengupta 2014). This implies that the bank's problem is to deter the low type borrower from mimicking the high type borrower and using collateral requirements achieves this objective.

3 Empirical Implications

In this section, I describe the empirical predictions arising out of the model. In deriving the predictions, I consider the case that the cost of collateral is small, $k < \bar{k}$. Borrower wealth is $\hat{C} \leq W < \overline{C}$ which suggests that the availability of collateral is sufficient to induce separation between borrower types, but it is not unlimited.

Prediction 1. *A higher availability of collateral leads to an improved average quality of unsecured loans.*

This prediction embodies the spillover effect from secured to unsecured loans. As the secured borrowers post more collateral, the unsecured borrowers are charged higher interest rates, and hence, screened more diligently. More diligent screening by the bank reduces errors, i.e., fewer good projects are rejected and more bad projects are rejected, leading to a higher average quality of the unsecured loans. To the best of my knowledge, this spillover effect from secured to unsecured lending has not been tested.

To test the prediction, we need to identify settings with an exogenous change in the pledgeable wealth of the borrowers. One example is the setting of Bolivia, studied by Ioannidou et al. (2015). The banking sector in Bolivia used to be largely dollarized up until mid-2000s. Hence, monetary policy changes were transmitted exogenously from the US, even though the business cycles in US and Bolivia were not synchronous. This positively impacted the amount of collateral available to the Bolivian borrowers, unrelated to the macroeconomic conditions in the country. Haselmann et al. (2010), Aretz et al. (2020), and Vig (2013) study legal reforms which (arguably) exogenously affect the availability of collateral.

Prediction 2. *The overall lending increases (resp. falls) as the availability of collateral increases, if the low type borrowers have many (resp. few) good projects.*

The bank extends credit to all good borrowers and to the bad borrowers whose projects yield a positive signal when screened. The total amount of lending is L^S:

$$L^S(C_h) = q_1 + (1 - q_1)(\beta^S(C_h)q_2 + (1 - \beta^S(C_h))(1 - q_2)) \tag{28}$$

Differentiating L^S with respect to the parameter reflecting the availability of collateral, γ:

$$\frac{\partial L^S}{\partial \gamma} = \frac{q_2 H}{\tau}(1 - q_1)(2q_2 - 1) \lesseqgtr 0 \tag{29}$$

The sign of the derivative is not clear, which indicates that as the availability of collateral increases, the probability of lending may increase or decrease, depending on parameters. Specifically, the derivative is positive when $q_2 > 0.5$, i.e., the probability of lending increases in the availability of collateral if the low type borrowers have a sufficiently high fraction of good projects. For $q_2 < 0.5$, the probability of lending falls in the availability of collateral. The intuition for the result is as follows. As the availability of collateral increases, the bank exerts a higher screening effort and identifies project quality more accurately. Therefore, if there are many good projects in the pool of unsecured loan applications, i.e., $q_2 > 0.5$, the bank increases its supply of credit to the economy, and if there are fewer good projects in the pool of unsecured loan applications, the bank tightens the overall credit supply.

Several studies (e.g., Haselmann et al. 2010 and Aretz et al. 2020) document that increasing the availability of collateral leads to an increase in credit in the economy, while others (e.g., Acharya et al. 2011, Vig 2013, and Cho et al. 2014) find that firms reduce leverage (i.e., bank lending falls) when creditor rights in bankruptcy increase (e.g., due to greater supply of collateral). The prediction in my model is consistent with the conflicting empirical evidence that an increase in creditor rights in bankruptcy may have a positive or a negative impact on the overall credit provision in the economy.

In contrast to existing models, my model generates specific conditions under which bank credit increases or falls following an increase in the availability of collateral. Specifically, my model predicts that credit provision falls as collateral availability increases when q_2 is low. The empirical evidence in Acharya and Subramanian (2009) can be interpreted as supportive. Acharya and Subramanian (2009) find that higher creditor rights in bankruptcy, including the protection of secured creditors' rights in bankruptcy, leads to lower borrowing by innovative firms. One can interpret q_2 to be falling in firm innovativeness; in particular, innovative projects, by their very nature, are less likely to be successful compared to tried-and-tested technologies. Thus, my theory provides a new explanation for the empirical findings in Acharya and Subramanian (2009).

4 Policy implications

So far I have assumed that the outcome of screening is contractible. In this section, I consider the case that the outcome of screening is privately observed and hence cannot be contracted upon. In particular, contracts are not always enforceable, and there is a possibility that a party reneges on a contract in an attempt to renegotiate the initially agreed terms.

Suppose that contracts separate borrower types and the low type borrower applies for the unsecured loans. The monopolistic bank can pick from many unsecured loan applications which borrower's project to screen at $t = 2$. If contracts are enforceable, the bank screens a randomly drawn applicant's project, and if screening produces a good signal, $s = s_g$, the bank offers the loan at the initial promised interest rate, $R_l(C^\gamma)$. Note that in this case, the low type borrower earns informational rents if the availability of collateral is limited, $R_l(C^\gamma) < X \ \forall \gamma < 1$.

However, if contracts are not enforceable, the monopolistic lender will refuse to screen a borrower's project unless she agrees to renegotiate the repayment rate; the borrower cannot commit to not renegotiate since the bank can pick another borrower's project to screen (borrowers are competing with one another for funds). The lender withdraws the initial offer of interest rate, making a take-it-or-leave-it offer of $R_l = X$ at $t = 2$. Since separation has already taken place at the earlier date due to collateral, the low type borrower does not hold an informational advantage at $t = 2$, and the bank extracts the full surplus. The borrowers optimally anticipate this renegotiation at $t = 0$, and the separation takes place only if the high type borrower posts sufficient collateral such that the low type borrower does not mimic for $R_l = X$, i.e., $C_h = \overline{C}$. For $W < \overline{C}$ the separating equilibrium unravels.

Proposition 5 (Screening not contractible). *Suppose that the cost of collateral is small, $k < \bar{k}$.*

1. *Consider borrowers with high pledgeable wealth, $W \geq \overline{C}$. The high type borrower applies for the secured loan, $(R_h(\overline{C}), \overline{C})$, and the low type borrower applies for the unsecured loan.*

2. *For borrowers with limited pledgeable wealth, $\hat{C} \leq W < \overline{C}$, the equilibrium is pooling, even though separation would be feasible with contractible screening.*

The amount of screening and lending in the collateral equilibrium are identical to the full information benchmark case if the availability of collateral is not limited, i.e., $W \geq \overline{C}$. However, in the pooling equilibrium several distortions arise. Therefore, to the extent that collateral is not too expensive, the regulator prefers separation to pooling. Suppose that collateral is almost costless. With enforceable contracts, the equilibrium outcome coincides with the full information outcome if the availability of collateral is not limited. If the availability of collateral is limited, $\hat{C} \leq W < \overline{C}$, separation still takes place, but there is distortion from the full information case since the low type borrower extracts informational rents. That the low type borrower extracts rents is simply a transfer, and in itself, is not undesirable from the regulator's perspective. However, when the low type borrower extracts rents, it reduces the monopolistic lender's screening incentives, which is undesirable from the regulator's perspective. Then, the regulator can boost the availability of collateral to improve the outcome. However, if the availability of collateral is taken as exogenously given, then the regulator cannot improve upon the equilibrium outcome.

Policy. *With the outcome of screening not contractible and limited collateral availability, the regulator sets an interest rate ceiling, $R \leq R_l(C^\gamma)$, on the unsecured loans.*

Suppose that $\hat{C} \leq W < \overline{C}$ and contracts are not enforceable due to the screening outcome not being contractible. The separating equilibrium unravels due to a hold-up problem. In this scenario, a benevolent regulator may step in to improve the allocation. The regulator sets a cap on the interest rate that a low type borrower may be charged as follows: $R \leq R_l(C^\gamma)$. By capping the interest rate, the hold-up problem is effectively eliminated. At $t = 0$, the bank is able to commit to not renegotiate a higher interest rate with the low type borrower, as it is ruled out by the regulator. The unravelling of the separating equilibrium is prevented. The policy can be implemented by only capping the interest rates on the unsecured loans.

Note that the commitment problem becomes relevant only if the borrower has enough pledgeable asset to separate, but the availability of collateral is limited, i.e., $\hat{C} \leq W < \overline{C}$, where both $\hat{C}$ and $\overline{C}$ are increasing in the high type borrower's outside option, H. Suppose that H is such that the borrower is unconstrained, i.e., $W \geq \overline{C}$. In this case, the possibility of renegotiation does not affect the structure of contracts at $t = 0$; the high type borrower pledges $\overline{C}$ and there is separation. Suppose that there is an increase in H which pushes $\overline{C}$ up such that $\hat{C} \leq W < \overline{C}$. In this case, separation is still feasible if contracts are complete, but separation unravels if contracts are incomplete. Therefore, for a higher H, the limited commitment problem is more likely to become relevant.

5 Concluding remarks

When the contracting space does not include collateral, the high type borrower cannot indicate her type, and the equilibrium is pooling: both high and low type borrowers' projects are screened. The expansion of the contracting space to include collateral allows the separation of borrower types. The high type borrower applies for secured loans and is always granted credit, while the low type borrower applies for unsecured loans, and the bank screens only the low type borrower's project. In contrast to existing models, the lender sets higher collateral requirements in equilibrium than what is necessary for separation. This allows me to perform novel comparative static analyses. My model suggests that an increase in the availability of collateral impacts screening incentives positively and the overall credit provision by banks in nuanced ways. In terms of policy, the efficient outcome obtains when the outcome of screening is contractible. If there is scope for renegotiation after contracts are written due to the outcome of screening not being contractible, and additionally, the availability of collateral is limited, then the regulator can intervene to prevent a hold-up problem and deliver the efficient equilibrium.

6 Appendix

Proof of Proposition 1. I solve for the equilibrium by backward induction and begin with the bank's screening decision at $t = 2$. An interior solution to the bank's problem is given by the first order condition:

$$q_2(pR_l - 2) + 1 - \tau F = 0 \tag{30}$$

At $t = 0$, the bank sets the repayment rates such that the IR constraints of the borrowers are satisfied. The IR constraints bind for both types. To see why this this the case, consider a candidate equilibrium in which the IR constraint for type t is not binding. Then, the bank can increase its profits by increasing R_t a little, without violating the other constraint. Hence the candidate equilibrium with non-binding IR constraint for either type is not stable. From the relevant IR constraints, the repayment rates, R_h and R_l, are derived (Equations (8) and (9)). The non-negativity constraints are satisfied if $pX > H$, which holds due to Assumption *A4*, and the limited liability constraints are always satisfied, binding for the low type borrower and slack for the high type borrower. Substitute R_l in the first order condition to derive the screening intensity, F^{fi} (Equation (10)). Finally, I check that the solution is interior, i.e., $0 < F^{fi} < \frac{1}{2}$. Take the extreme values, $pX \to 1$ and, which satisfy Assumptions *A1-A4*: F^{fi} becomes $\frac{1}{\tau}(1 - q_2)$, which is strictly positive. Increasing pX leads to a *higher* F, therefore, $F^{fi} > 0$ always holds. Assumption *A3* ensures $F^{fi} < \frac{1}{2}$.

Proof of Proposition 2. I solve for the equilibrium by backward induction and begin with the bank's screening decision at $t = 2$. An interior solution to the bank's problem is given by the first order condition:

$$q_p(p\hat{R} - 2) + 1 - \tau F = 0 \tag{31}$$

At $t = 0$, the bank sets the repayment rates such that the IR constraints of the borrowers are satisfied. In order to make sure that the high type borrower participates, her individual rationality constraint must be satisfied. This automatically satisfies the low type borrower's individual rationality constraint. From the IRH constraint, derive the interest rate in the pooling equilibrium, $\hat{R}$ (Equation (15)). The condition is satisfied with equality. If it is slack, the bank can increase its profits by increasing $\hat{R}$ a little, without violating any other relevant constraints. Substitute $\hat{R}$ in the first order condition to derive the screening intensity in the pooling equilibrium, F^P (Equation (16)). Finally, I check that the solution is interior, i.e., $0 < F^P < \frac{1}{2}$. Take the extreme values, $pX \to 1$ and $H \to pX - 1$, which satisfy Assumptions *A1-A4*: F^P becomes $\frac{1}{\tau}(1 - q_p pX)$, which is strictly positive given Assumption *A2*. Increasing pX and/or reducing H leads to a *higher* F, therefore, $F^P > 0$ always holds. Assumption *A3* ensures $F^P < \frac{1}{2}$.

Proof of Proposition 4. Following the discussion in the text, in a separating equilibrium the collateral requirements lies in the range $C_h \in (\hat{C},\ \overline{C})$. Re-write C_h as a convex combination of the two corner values, $C_h = (1 - \gamma)\hat{C} + \gamma\overline{C}$, with $\gamma \in [0, 1]$. The first part of the proposition states that γ is always given by a corner solution, i.e., either $\gamma = 0$ or $\gamma = 1$. The bank's objective function in the separating equilibrium, including the cost of collateral is:

$$\begin{aligned}\Pi_S =& q_1(pR_h + (1-p)C_h - 1 - kC_h) \\ &+ (1-q_1)\left(\beta^S(q_2(pR_l - 2) + 1) - (1-q_2) - \frac{\tau}{2}F^{S2}\right)\end{aligned} \tag{32}$$

With probability q_1 the bank makes secured loans to the high type borrower (the top line). The collateral cost is incurred whether or not the project fails (e.g., to transfer and store the collateral when the loan is approved); it could be adapted to the case that the cost of collateral is only incurred on the failure of the project without qualitatively affecting the results. With probability $(1 - q_1)$ the bank makes unsecured loans to the low type. Derivating with respect to γ and k,

$$\frac{\partial \Pi_S^2}{\partial\gamma\partial k} = -\frac{\partial C_h}{\partial\gamma} = \hat{C} - \overline{C} < 0 \tag{33}$$

Suppose that collateral is costless, i.e., $k = 0$. Then, Π_S is increasing in γ, since conditional on separation, the higher use of collateral allows the bank to extract the full surplus from lending to the low type borrower, which leads to higher profits. From Equation (33), as k increases, $\frac{\partial \Pi_S}{\partial\gamma}$ is falling. There must exist $k = \bar{k}$ such that:

$$\frac{\partial \Pi_S}{\partial\gamma}\bigg|_{k=\bar{k}} = 0 \tag{34}$$

When $k = \bar{k}$, the bank is indifferent in the choice of γ. For $k < \bar{k}$ (resp. $k > \bar{k}$), the derivative is positive (resp. negative), i.e., bank profit is increasing (resp. falling) in γ. Hence, we set γ as high (resp. low) as possible, i.e., $\gamma = 1$ (resp. $\gamma = 0$).

The profit in the separating equilibrium is falling in k, while the profit in the pooling equilibrium, Π_P, is unaffected in k:

$$\Pi_P = \beta^P(q_p(pX - H - 2) + 1) - (1 - q_2) - \frac{\tau}{2}F^{P2} \tag{35}$$

k_γ^P is the value of k which makes the bank indifferent between the pooling equilibrium and the separating equilibrium, given γ:

$$\Pi_P = \Pi_S|_{\gamma\in\{0,1\}, k=k_\gamma^P} \tag{36}$$

$k^P = \max(k_0^P,\ k_1^P)$. For $k > k^P$ it is profitable for the bank to forego the separating in favour of the pooling equilibrium. The expressions for k_γ^P and $\bar{k}$ are as follows:

$$k_0^P = \frac{\begin{gathered}(q_2-1)(q_1+4q_2-4q_p+2Hq_2-2Hq_p-4q_1q_2+q_1\tau-4Hq_2^2+4Hq_p^2+4q_1q_2^2-4q_2^2\\ +4q_p^2-h^2q_2^2+h^2q_p^2+4Hq_1q_2^2+4pq_2^2X-4pq_p^2X+h^2q_1q_2^2-2Hq_1q_2+2Hq_1\tau+Hq_2\tau\\ -Hq_p\tau-2pq_2X+2pq_pX-p^2q_2^2X^2+p^2q_p^2X^2-Hq_1q_2\tau+2pq_1q_2X-2pq_1\tau X-pq_2\tau X+pq_p\tau X\\ +p^2q_1q_2^2X^2+2Hpq_2^2X-2Hpq_p^2X-4pq_1q_2^2X+pq_1q_2\tau X-2Hpq_1q_2^2X)\end{gathered}}{Hq_1q_2(4q_2+\tau+2Hq_2-2pq_2X-2)} \tag{37}$$

$$k_1^P = \frac{\begin{gathered}(q_2-1)(q_1+4q_2-4q_p-2Hq_p-4q_1q_2+q_1\tau+4Hq_p^2+4q_1q_2^2-4q_2^2+4q_p^2+h^2q_p^2\\ +4pq_2^2X-4pq_p^2X+2Hq_1\tau-Hq_p\tau-2pq_2X+2pq_pX-p^2q_2^2X^2+p^2q_p^2X^2\\ +2pq_1q_2X-2pq_1\tau X-pq_2\tau X+pq_p\tau X+p^2q_1q_2^2X^2-2Hpq_p^2X-4pq_1q_2^2X+pq_1q_2\tau X)\end{gathered}}{2Hq_1q_2\tau} \tag{38}$$

$$\bar{k} = \frac{(q_1-1)(q_2-1)(\tau-4q_2-q_2H+2pq_2X+2)}{q_1(\tau-4q_2-2q_2H+2pq_2X+2)} \tag{39}$$

Proof of Lemma 3. Suppose that at $t = 0$, the bank offers an $R = X$ contract. This contract violates the high types' participation constraint, so they stay out. Only the low type borrowers apply for this contract at $t = 1$. At $t = 2$, the bank screens the projects of the low type and extends credit whenever the signal is positive. At the same time, at $t = 2$, the bank knows that the borrowers who are not served are the high type (because they turned down the $R = X$ contract at $t = 0$). Then, the bank offers a contract to the high type with interest rate,

$R = X - \frac{H}{p}$. Moreover, anticipating this outcome the low type will not apply for the $R = X$ contract at $t = 1$. Therefore, since the bank cannot credibly commit to not serve the high type borrower at $t = 2$ (even if parameters are such that it would like to be able to commit), the equilibrium unravels.

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