Collateral and bank screening as complements: A spillover effect

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Abstract

I analyze a novel spillover effect from collateralized to uncollateralized loans. High-type borrowers have good projects while low-type borrowers do not know their project quality. High-type borrowers post collateral, and a monopolist bank screens only low-type borrowers' projects. Different from existing models, equilibrium collateral requirements are stricter than the minimum necessary to achieve separation, despite costly collateral. When high-type borrowers post more collateral, the bank charges a higher interest rate to low-type borrowers. This enhances the bank's incentives to screen the low-types' projects, thereby improving the average quality of uncollateralized loans issued, i.e., the use of collateral complements direct screening.

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Keywords: Collateral, Bank screening, Adverse Selection, Countervailing incentives.

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1 Introduction

Collateral in debt contracts serve a multitude of functions. One function of collateral is that it protects lenders' interests in defaults (e.g., Tirole 2006). Modelling this particular function of collateral, Asriyan et al. (2022) show in a dynamic general equilibrium setting that credit booms associated with higher collateral values can crowd out bank screening which leads to protracted economic crises (see also, Manove et al. 2001 and Gorton and Ordoñez 2014). Instead, I consider the screening function of collateral, which is that it solves an adverse selection problem, and show that higher collateral values can improve equilibrium screening incentives, i.e., collateral and direct screening are complements (not substitutes, as in existing models).

In the traditional adverse selection models of collateral (e.g., Bester 1985, Besanko and Thakor 1987a), collateral sorts borrowers into risk types, with no residual uncertainty regarding the borrowers' riskiness which makes direct screening by banks (e.g., through information acquisition) redundant. Posting collateral entails a deadweight loss and, hence, the minimum amount of collateral that achieves separation between types is used and no more. I present a setting in which the use of collateral eliminates information asymmetry but does not eliminate all uncertainties. Hence, direct screening supplements the use of collateral and, in equilibrium, collateral requirements can be higher than the minimum that is necessary to separate borrower-types. This result allows me to derive a new spillover effect which has not been considered previously.

As in existing models, in my model high-type borrowers post collateral to separate from low-type borrowers. The novelty in my model is as follows: conditional on separation between borrower-types, more collateral posted by the high-type allows the bank to charge a higher interest rate to the low-type, which provides improved incentives to the bank to screen the low-types' projects (i.e., screening and collateral are complements). Hence, the average quality of uncollateralized loans approved by the bank increases in the availability of collateral in the economy, i.e., there is a positive spillover effect from collateralized to uncollateralized loans.

Model Preview. There are two types of borrowers who apply for credit from a monopolistic bank: a high-type borrower has a good project, while a low-type borrower may have a good or a bad project. Borrowers privately know their type, but a low-type borrower's project quality is unknown to everyone. High-type borrowers have a higher reservation utility (outside option) which gives rise to countervailing incentives, i.e., low-type borrowers may mimic high-type borrowers. On receiving a loan application, the bank may exert costly effort to directly screen the project to gauge whether it is good or bad.

In the full information benchmark, the bank always lends to high-type borrowers, and extends credit to a low-type borrower only if screening her project yields a positive signal. In the incomplete information case, interest rates alone cannot separate borrower-types; low-type borrowers mimic high-type borrowers since the high-types' intended contract specifies a lower interest rate. Two distortions arise compared to the full information benchmark: first, the pool of projects being screened (the screening pool) contains a higher fraction of good projects since both high- and low-type borrowers' projects are screened and, second, low-type borrowers capture a fraction of the surplus.

Enlarging the contracting space from just interest rates to include both interest rates and collateral allows the separation of borrower-types. Without direct screening it remains uncertain whether a low-type borrower has a good or a bad project; this information cannot be elicited through contracts since low-type borrowers do not know their project quality. High-type borrowers post collateral, while low-type borrowers apply for uncollateralized loans; only the low-types' projects are screened, as is the case in the full information benchmark. If the availability of collateral is limited, collateral requirements are low and the bank cannot charge low-type borrowers a high interest rate and, hence, low-type borrowers extract a fraction of the surplus (the low-types' participation constraint is slack). Note that this is surprising because there is no residual information asymmetry once borrower-types are separated: the low-type may have a good or a bad project, but neither the bank nor the borrower know the project type. What drives this result in my model is the fact that the demand for the two types of debt is endogenous to the contract terms, beyond separation. Due to not being able to extract the full surplus from low-type borrowers, even though the screening pool becomes identical to the full information case, effort provision by the bank is inefficient.

If the availability of collateral is plentiful, the bank sets high collateral requirements for the high-type, which allows the bank to increase the interest rate offered to low-type borrowers all the way till the low-types' participation constraint binds. Then the entire surplus from lending accrues to the bank. Therefore, with plentiful availability of collateral, the bank exerts the efficient effort level since it internalizes the full benefit of screening. Thus, in contrast to existing screening models, equilibrium collateral requirements can be higher than the minimum necessary for separation. This result allows me to derive new empirical implications:

First, an increase in the availability of collateral leads to a higher average quality of uncollateralized loans (unambiguously); this is the spillover effect of collateralized lending. Second, higher collateral availability may lead to an increase or a fall in the uncollateralized credit in the economy, depending on the distribution of projects. Specifically, the prediction is that if the low-type have a high fraction of good projects, then higher collateral stock leads to higher approvals of uncollateralized loan applications.

Related literature. In one set of theories collateral alleviates borrower moral hazard concerns and the (observably) riskier borrowers are more likely to pledge collateral (e.g., Boot et al. 1991 and Boot and Thakor 1994). In a signalling model, Ordonez et al. (2019) show that privately informed borrowers can use secured loans to signal their type when collateral values are uncertain. In dynamic models (e.g., Rajan and Winton 1995, Donaldson et al. 2020b and Donaldson et al. 2020a), collateral assigns priority to a loan over unsecured loans.

My model belongs to a complementary set of theories which show that collateral sorts borrowers into (unobservable) risk classes (e.g., Bester 1985, Chan and Kanatas 1985, Besanko and

Thakor 1987a, Besanko and Thakor 1987b): these make up the adverse selection or screening theories of collateral.[1] My model differs from the existing screening models on two grounds: first, equilibrium collateral requirements may be higher than necessary for separation and, second, the monopolist bank cannot extract the full surplus despite full resolution of asymmetric information. These two novel features distinguishes mine from existing screening models of collateral and allows me to perform comparative static analyses with respect to the degree of collateral availability, which is not possible in the traditional models.

In Manove et al. (2001), collateral lead to the sorting of borrowers into risk types and reduce costly screening by banks, even when it is socially optimal to screen (see also Asriyan et al. 2022, Hainz et al. 2013, Karapetyan and Stacescu 2014, Degryse et al. 2021, and Goel et al. 2014 who model collateral and bank screening as substitutes). Similar to these studies, in my model the bank does not screen the borrower who posts collateral (this is the classic substitution result). However, different to them, the greater use of collateral leads to more efficient screening of uncollateralized loans.

Several recent articles connect the relationship between collateral values and information production to business cycle dynamics; a high availability of collateral leads to too little information production, and a subsequent negative shock to collateral values leads to deep crises (see e.g., Gorton and Ordoñez 2014, Gorton and Ordoñez 2020, and Asriyan et al. 2022). In my model, a high availability of collateral encourages more information production that, in turn, improves the equilibrium outcome.

In my model, starting from a pooling equilibrium, the use of collateral alters the screening pool, which affects the screening intensity. The importance of the quality of the screening pool in my model is reminiscent of several existing papers, e.g., Broecker (1990), Shaffer (1998), Marquez (2002), Vanasco (2017), Hu (2022), and Ahnert and Kuncl (2021). Different from these

[1] Consistent with these models, Berger et al. (2011) provide empirical evidence that unobserved risk is negatively correlated with pledging collateral; see also Ioannidou et al. (2022), Godlewski and Weill (2011).

papers, my main results are derived holding constant the screening pool: beyond achieving separation, a higher availability of collateral does not affect the screening pool, but affects screening incentives by diverting the surplus from low-type borrowers to the lender.

2 Model

2.1 Set-up

I consider a three-date economy, $t = 0, 1, 2$. There is a continuum of entrepreneurs (borrowers) with access to a project. The borrowers are penniless and seek financing for their projects from a monopolistic bank. All agents are risk-neutral and protected by limited liability. The risk-free rate is normalized to 0, so there is no discounting. The project is of fixed scale and requires an investment, normalized to 1, at $t = 1$. A borrower is either high-type, with probability q_1, or low-type, with probability $(1 - q_1)$. Borrower type is determined by nature, and it is the borrower's private information. If the borrower is low-type, the project is good with some probability, q_2, and bad with the complementary probability, $(1 - q_2)$. A low-type borrower does not know if her project is good or bad quality. If the borrower is high-type, the project is good with certainty. A good project either succeeds with probability, p, and produces X, or fails with probability, $(1-p)$, and produces 0, at $t = 2$. A bad project produces 0 with certainty. Unconditionally, the probability of a good projects is q_p, with $q_p \equiv q_1 + (1 - q_1)q_2$. The payoff structure for each borrower is illustrated in Figure 1. I make two parametric assumptions regarding the profitability of projects:

A1: $pX - 1 > 0$

A2: $q_p pX - 1 < 0$

Assumption *A1* indicates that the good projects are profitable. If types are separated, a high-type borrower's project is not screened as high-type borrowers have a good project with certainty. Therefore, the screening pool is made up of either only low-type borrowers' projects or both low- and high-type borrowers' projects (a mixed pool). q_2 is the fraction of good project in a screening pool containing only low-type borrowers' projects, while q_p is the fraction of good projects in a screening pool containing both high and low-type borrowers' projects. *A2* implies that it is not profitable to lend without screening unless it is known with certainty that the borrower is high-type.

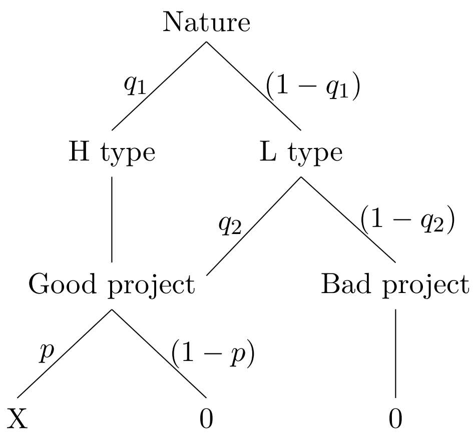

Figure 1: Payoff tree

The bank offers debt contracts at $t = 0$.[2] A borrower applies for a debt contract, which may or may not reveal her type. The borrower type does not directly reveal the project quality. The bank can screen the borrower's project to gauge the quality of the project. Screening produces a signal s_g (project is good) or s_b (project is bad), and the signal is informative but noisy:

$$Pr(s = s_g|\text{project} = \textit{good}) = Pr(s = s_b|\text{project} = \textit{bad}) = \beta \geq \frac{1}{2} \tag{1}$$

β is the precision of the signal, which increases in screening intensity, F, as follows:

$$\beta = \frac{1}{2} + F \tag{2}$$

[2] The assumption that only debt contracts are offered is without loss of generality since, given the structure of returns in our model, all contracts (sharing rules) are equivalent.

where $F \in (0, \frac{1}{2})$. If $F = 0$, the signal is pure noise and $\beta = \frac{1}{2}$. For $F = \frac{1}{2}$, $\beta = 1$ and screening perfectly reveals the project's quality. The cost of screening is given by $\frac{\tau}{2}F^2$, with $\tau > 0$ and sufficiently large, in a sense made precise later. The functional form implies that the screening cost is increasing and convex in screening intensity. Convexity reflects increasing difficulty for the bank to find out more and more about a project (see e.g., Song and Thakor 2010). Having incurred a non-zero screening cost such that $F > 0$, the bank must only agree to lend if the signal is good, $s = s_g$; otherwise, the bank might as well not have incurred the cost in the first place. Given an observed signal, the posterior probability of the project quality depends on the composition of the screening pool. The posterior probabilities are given as follows:

$$Pr(\text{project} = \text{good}|s_g, q_A) = \frac{\beta q_A}{[\beta q_A + (1-\beta)(1-q_A)]} \tag{3}$$

$$Pr(\text{project} = \text{good}|s_b, q_A) = \frac{(1-\beta) q_A}{[\beta(1-q_A) + (1-\beta) q_A]} \tag{4}$$

$q_A \in \{q_2, q_p\}$ is the unconditional probability of a good project in the screening pool. $q_A = q_2$ if the screening pool contains low-type borrowers only, while $q_A = q_p$ if the pool contains both types. The conditional probability of a bad project is $Pr(\text{project} = \text{bad}|s) = 1 - Pr(\text{project} = \text{good}|s)\ \forall\ s \in \{s_g, s_b\}$. For the pure noise signal, when $F = 0$ and $\beta = \frac{1}{2}$, the conditional probability that the project is good becomes equal to the prior, which is given by the fraction of the good projects in the screening pool, q_A. Therefore, the pure noise signal detects a good project with a lower probability when the screening pool is made up of low-type borrowers only, compared to the case of the mixed pool (since $q_2 < q_p$).

If screening, the bank extends credit if the signal is good, $s = s_g$. Given the quality of the screening pool, q_A, the probability of the good signal is $Pr(s = s_g|q_A)$, which is given by the denominator in Equation (3). If the signal is positive the project quality is good with probability $Pr(\text{project} = \text{good}|s_g, q_A)$ (Equation (3)). If the project is good it succeeds with probability, p, and the bank receives a repayment of R. If the good project fails, or the project is bad, the repayment is 0. The bank's expected payoff from an uncollateralized loan (i.e., a contract featuring only interest rates) is:

$$\begin{aligned} & Pr(s = s_g|q_A)[Pr(\text{project} = \text{good}|s_g, q_A)pR - 1] - \frac{\tau}{2}F^2 \\ \implies \ & \beta q_A pR_l - \beta(2q_A - 1) - (1 - q_A) - \frac{\tau}{2}F^2 \end{aligned} \tag{5}$$

A3: The cost of screening τ is sufficiently high:

$$\frac{\tau}{2} > q_A(pX - 2) + 1 \tag{6}$$

Assumption *A3* ensures that the solution of the bank's problem satisfies $F < \frac{1}{2}$.

As I consider monopolistic banks, collateral would not play a role if high- and low-types have identical outside options (see page 677 in Besanko and Thakor 1987a, also Lengwiler and Rishabh 2017).[3] Therefore, I assume that a high-type borrower has a higher outside option than a low-type borrower (similar to Sengupta 2014).

A4: *High-type borrowers have an outside option, H, with $H \in (0, pX - 1)$; while low-type borrowers' outside option is normalized to* 0*.*

The higher outside option of high-type borrowers can be understood as follows: while borrower-type is hidden from the bank, borrowers' family members can observe their type. Then, the family members can fund the project, but at a smaller scale. In this case, the high-type will invest and produce H, while the low-type will not invest. These frictions are of particular relevance in the credit markets since new entrepreneurs often find it difficult to acquire bank loans but can raise seed funding from family members. That high-type borrowers have a higher outside option than low-type borrowers gives rise to countervailing incentives which implies that

[3] With symmetric outside options, the bank's objective is to deter the high-type from mimicking the low-type while extracting the full surplus. Since it is more attractive for high-type borrowers to post collateral than low-type borrowers, increasing collateral requirements is not effective in deterring the high-type.

low-type borrowers potentially mimic high-type borrowers.[4] The countervailing incentives make collateral relevant in my model.

Timeline. At $t = 0$, a bank offers a loan contract ζ which consists of the interest rate and collateral requirement. Each borrower applies for a contract at $t = 1$. The bank decides whether or not to screen the borrower's project. If screening, the bank further decides the intensity of screening. Since the bank's screening decision is perfectly anticipated at $t = 0$, it can be thought of as part of the offered contracts, i.e., the contract ζ implicitly consists of the bank's screening effort. Depending on the outcome of screening, the bank then approves or rejects the borrower's application. If the loan is approved, the bank funds the project. If the application is rejected, the borrower makes her type-specific outside option. The payoffs are realized at $t = 2$, when all agents consume the output. The timing is illustrated in Figure 2.

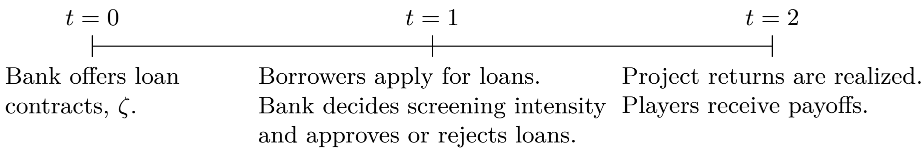

Figure 2: Timeline

Equilibrium definition. Since the uninformed player (the bank) moves first, this is a screening game. I look for the pure strategy subgame-perfect Nash equilibrium.[5] The bank maximizes its expected payoff subject to the individual rationality and incentive compatibility constraints of each borrower type. Additionally, the loan repayment rates must satisfy the borrowers' limited liability constraints and the non-negativity constraints (the feasibility constraints). We know from Rothschild and Stiglitz (1976) that a competitive equilibrium may not exist in a screening game due to banks undercutting one another. However, the non-existence problem does not

[4] Laffont and Martimort (2002) (pages 101-115) discuss the impact of type-dependent reservation utilities on the principal-agent problem and give an overview of several applications that have appeared in the broader contract theory literature.

[5] Despite the presence of private information, the appropriate solution concept is a non-Bayesian equilibrium concept since the uninformed player moves first by offering contracts. Therefore, there are no inferences to be made once the contracts are offered and upon observing the contracts there is no Bayesian updating of beliefs by any player. See also Rothschild and Stiglitz (1976) and Besanko and Thakor (1987a).

arise in my setting since there is no threat of undercutting with a monopolistic bank.

2.2 Full information benchmark

In the full information benchmark, a borrower's type is observable, but in the case of a low-type borrower, neither the bank nor the borrower knows the project quality. The bank offers a menu of type-contingent contracts, $(R_i, C_i),\ i \in \{l, h\}$. R_i is the repayment rate and C_i is the collateral requirement; the bank can seize the collateral from the borrower in the event of no repayment. It is assumed that collateral is costly, the cost may be minimal or substantial (discussion is provided later). In the full information benchmark, the bank sets $C_i = 0$ for each type to minimize deadweight loss.

If the borrower is high-type, the bank does not screen and always approves the loan application. If, on the other hand, the borrower is low-type, the bank incurs positive screening cost, say F^{fi}, and extends credit only if the signal is positive, i.e., $s = s_g$. The bank solves:

$$
\begin{aligned}
&\underset{F}{\text{Max}} && \beta q_2 p R_l - \beta(2q_2 - 1) - (1 - q_2) - \frac{\tau}{2}F^2 \\
&\text{subject to} && \\
&\text{(IRH)} && p(X - R_h) \geq H \\
&\text{(IRL)} && \beta q_2 p(X - R_l) \geq 0 \\
&\text{(FCs)} && 0 \leq R_i \leq X\ \forall\ i \in \{l, h\}
\end{aligned}
\tag{7}
$$

The bank maximizes its expected payoff with respect to the choice of screening intensity, F. Since only low-type borrowers' projects are screened, the screening pool consists of q_2 (substitute $q_A = q_2$ in Equation (5) to derive the objective function). The bank sets the repayment rates such that the Individual Rationality (or IR) constraints of the borrowers are satisfied. These are IRH and IRL for high- and low-type borrowers, respectively. The left-hand side (LHS) of an IR constraint is the expected payoff of the corresponding borrower type and the right-hand side (RHS) is their outside option. The FCs are the feasibility constraints which are the non-negativity and limited liability constraints.

Proposition 1 ***(Full information)*** *In the full information benchmark, high-type borrowers always receive credit. The bank screens low-type borrowers' projects and grants credit only if the signal is positive. The equilibrium is characterized as follows:*

$$R_h = X - \frac{H}{p} \tag{8}$$

$$R_l = X \tag{9}$$

$$F^{fi} = \frac{1}{\tau}(q_2(pX - 2) + 1) \tag{10}$$

Proof. The proof is in the appendix. ■

The bank grants credit to all high-type borrowers. A low-type borrower with a good project receives credit with probability, $\beta^{fi} = \frac{1}{2} + F^{fi}$. Due to noisy screening, some bad projects are financed, and some good projects are denied credit. With probability, $(1 - \beta^{fi})$, the bank incorrectly rejects a low-type borrower with a good project and incorrectly grants credit to a low-type borrower with a bad project.

2.3 Equilibrium without collateral

In this section, I consider the incomplete information problem, with the restriction that the credit policy only involves setting the interest rate, R_i. In a separating equilibrium, the contracts should ensure that neither type is better off mimicking the other, i.e., the Incentive Compatibility (or IC) constraints, ICH and ICL, corresponding to high- and low-type borrowers, respectively, are satisfied. The IC constraints are as follows:

$$\text{(ICH)} \quad p(X - R_h) \geq \beta p(X - R_l) + (1 - \beta)H \tag{11}$$

$$\text{(ICL)} \quad \beta q_2 p(X - R_l) \geq q_2 p(X - R_h) \tag{12}$$

The LHS of the IC constraint for borrower type i is her expected payoff when she tells the truth, while the RHS is her expected payoff from mimicking the other type. If mimicking low-type borrowers, a high-type borrower's project is screened. Due to noisy screening, the high-type borrower is denied credit with probability $(1 - \beta)$, in which case she makes her outside option, H. If mimicking high-type borrowers, a low-type borrower is always granted credit, and with probability q_2, the project is good. The IC constraints are jointly satisfied only if $(1 - \beta) \leq 0$. However, this condition is never satisfied as long as the solution is interior, i.e., $\beta < 1$.

Lemma 1 *Contracts featuring interest rates only cannot separate borrower-types.*

Proof. The proof is in the Appendix. ■

Next, I look for a pooling equilibrium in which high- and low-type borrowers apply for the identical contract (repayment, $R_h = R_l = \hat{R}$). In a pooling equilibrium both high- and low-type borrowers' projects are screened, and a borrower is granted credit if screening produces a good signal, i.e., $s = s_g$. The bank solves the following problem:

$$\begin{aligned}
&\underset{F}{\text{Max}} && \beta q_p p\hat{R} - \beta(2q_p - 1) - (1 - q_p) - \frac{\tau}{2}F^2 \\
&\text{subject to} && \\
&\text{(IRH)} && p(X - \hat{R}) \geq H \\
&\text{(IRL)} && \beta q_2 p(X - \hat{R}) \geq 0 \\
&\text{(FCs)} && 0 \leq \hat{R} \leq X
\end{aligned} \tag{13}$$

When both high- and low-type borrowers' projects are screened, the screening pool consists of

q_p good projects (substitute $q_A = q_p$ in Equation (5) to derive the objective function).

Proposition 2 ***(No-collateral equilibrium)*** *When the contracting space is restricted to interest rates only, the equilibrium is pooling. Both high- and low-type borrowers' projects are screened, and a borrower is granted credit if screening produces a good signal. The equilibrium is characterized as follows:*

$$\hat{R} = X - \frac{H}{p} \tag{14}$$

$$F^P = \frac{1}{\tau}(q_p(pX - H - 2) + 1) \tag{15}$$

Proof. The proof is in the appendix. ■

Note that in the no-collateral equilibrium, low-type borrowers extract a fraction of the surplus since $\hat{R} < R_l$. The bank screens both borrower-types' projects. In contrast to the full information benchmark, some high-type borrowers do not receive credit due to screening being noisy. High-type borrowers and low-type borrowers with good projects receive credit with probability, $\beta^P = \frac{1}{2} + F^P$, and is rejected with probability, $(1 - \beta^P)$. A low-type borrower with a bad project receives credit with probability, $(1 - \beta^P)$. The screening intensity in the pooling equilibrium, F^P, differs from the screening intensity in the full information benchmark, F^{fi}, due to two distortions: low-type borrowers extract a fraction of the surplus, $\hat{R} < R_l$, and the screening pool contains a higher fraction of good projects in the pooling equilibrium, since high-type borrowers' projects are also screened, $q_p > q_2$.

2.4 Equilibrium with collateral

In this section, I introduce collateral into the contracting space. Assume that the borrowers have assets-in-place, W, which they can pledge as collateral. In addition to the interest rate, the bank specifies collateral requirements, $C \leq W$. In collateralized loans, if the borrower fails the bank can seize the collateral and recoup some of its investment. These assets are in addition to what is normally available to the lender in the case of a default, i.e., these could be personal assets or third-party guarantees (see Chan and Kanatas 1985).[6]

By the revelation principle of Myerson (1979), the bank can induce truth-telling from the borrowers by offering two incentive compatible contracts. Suppose that the bank offers a menu of contracts, $(R_i, C_i),\ i \in \{l, h\}$. Since high-type borrowers are less likely to default, it is relatively less costly for the high-type to post collateral. Thus, the high-type posts more collateral than the low-type, i.e., $C_h > C_l$. Without loss of generality, the offered contracts are $(R_l, 0)$ and (R_h, C_h). To start with, I assume that pledging collateral is costless. In the separating equilibrium, the bank screens only low-type borrowers' projects and solves the following problem:

$$
\begin{aligned}
&\underset{F}{\text{Max}} && \beta q_2 p R_l - \beta(2q_2 - 1) - (1 - q_2) - \frac{\tau}{2} F^2 \\
&\text{subject to} && \\
&\text{(IRH')} && p(X - R_h) - (1 - p)C_h \geq H \\
&\text{(IRL)} && \beta q_2 p(X - R_l) \geq 0 \\
&\text{(ICH')} && p(X - R_h) - (1 - p)C_h \geq \beta p(X - R_l) + (1 - \beta)H \\
&\text{(ICL')} && \beta q_2 p(X - R_l) \geq q_2(p(X - R_h) - (1 - p)C_h) - (1 - q_2)C_h \\
&\text{(FCs)} && 0 \leq R_i \leq X \ \forall\ i \in \{l, h\}
\end{aligned}
\tag{16}
$$

The RHS of high-type borrowers' new IR and IC constraints reflect that when the project fails, she loses the posted collateral. The IRL and the LHS of the ICL are the same as before since a low-type borrower's intended contract is not collateralized, $C_l = 0$. However, the RHS of the ICL is affected since if she mimics high-type borrowers, she can lose the posted collateral whether she has a good project that fails nonetheless or a bad project.

[6]Although third-party guarantees are not strictly an asset that may be seized, lenders have collateral in the form of expected recovery value since guarantors would repay or the lender has access to their assets (see e.g., Ordonez et al. 2019).

From the constraints in (16), we derive the feasible bounds on collateral requirements, $C_h \in [\hat{C}, \overline{C}]$ (see Equations (24) and (29)); the lower-bound comes from violating the low-types' IC constraint and the upper-bound comes from the feasibility constraint that the interest rate cannot be higher than X. Consider the case that $C_h = \gamma\overline{C} + (1-\gamma)\hat{C} \equiv C^\gamma$ with $\gamma \in [0,1]$. $\gamma = 0$ if borrowers have just enough assets to achieve separation, but no more. γ is increasing in the amount of collateral that borrowers can post, and $\gamma = 1$ if the borrower is unconstrained, i.e., $W \geq \overline{C}$. Therefore, an increase in γ can be interpreted as an increase in the availability of collateral. Having introduced the parameter, γ, we are ready to fully characterize the equilibrium in Proposition 3.

Proposition 3 ***(Collateral Equilibrium)*** *Suppose $k = 0$. High-type borrowers post collateral, $C_h = \min(W, \overline{C})$ and receive credit. The bank screens low-type borrowers' projects and grants credit only if the signal is positive. The equilibrium is characterized as follows:*

$$R_h = X - \frac{1}{p}(H + (1-p)C_h) \tag{17}$$

$$R_l = X - \frac{H}{p} + \frac{\gamma H}{p} \tag{18}$$

$$F^S = \frac{1}{\tau}(q_2(pX - H - 2) + 1 + \gamma q_2 H) \tag{19}$$

Proof. The proof is in the Appendix. ■

For $\gamma = 0$, the low-type extracts a fraction of the surplus since $R_l = \hat{R} < X$. A higher γ leads to an increase in R_l, which in turn improves the bank's incentive to screen the low-type's project, F^S. For this reason, if collateral is costless, the collateral requirement is as high as possible. Thus, different to existing screening models of collateral, the equilibrium level of collateral is uniquely pinned down in my model, even when collateral is costless.

For $W \geq \overline{C}$, the equilibrium becomes identical to the full information benchmark in terms of the screening pool and the intensity with which the uncollateralized loans are screened.

For $W < \overline{C}$, the collateral equilibrium bears some similarities with both the full information benchmark and the pooling equilibrium. The bank screens only low-type borrowers' projects, as in the full information benchmark. The screening intensity, F^S, differs from the full information benchmark, F^{fi}, since low-type borrowers extract a fraction of the surplus, as in the pooling equilibrium. $F^{fi} \geq F^S$ if $(1-\gamma) \geq 0$. This condition is always satisfied for any γ (with equality for $\gamma = 1$). Since low-type borrowers extract a fraction of the surplus when the availability of collateral is limited, the screening incentives are negatively distorted.[7]

So far, I have assumed that collateral is costless to pledge. Suppose that there is a cost of posting collateral, k (see Parlatore (2019) for a microfoundation for this cost), and the cost is proportional to the amount of collateral posted. Pledging collateral can be costly due to the disparity between the valuation of the borrower and the lender. The disparity can arise since the borrower may be the best user of the asset, while the lender lacks the expertise to use the asset. The disparity can also arise since the lender has to transport and store the asset on seizure and there may be additional legal costs (when more collateral is posted, the cost to store it is higher; thus the cost is increasing in the amount of collateral posted).

The lender faces the following trade-off. On the one hand, $R_l(C_h)$ is increasing in C_h, which implies that as the high-type posts a higher level of collateral, the bank extracts more of the surplus from lending to the low-type. On the other hand, collateral entails a deadweight loss (and the cost is borne the monopolist lender). From this trade-off, I present:

Proposition 4 ***(Costly collateral)*** *Suppose that $W \geq \overline{C}$ and $\Pi_S|_{\gamma=0,k=\bar{k}} > \Pi_P > 0$. Collateral requirements are high, $C_h = \overline{C}$, if the cost of collateral is small, $k < \bar{k}$, and low, $C_h = \hat{C}$, if collateral is moderately costly, $\bar{k} < k < k^P$. If collateral is very costly, $k > k^P$, collateral is not used and the equilibrium is pooling.*

[7]Note that given the constraints of the economy, a regulator who maximizes net social surplus cannot improve upon this outcome. The regulator would like to set $R_l = X$ to induce efficient effort provision. However, the regulator faces the same set of constraints as the bank and since the constraints bind to determine the contract terms, the regulator's solution coincides with that of the bank.

Proof. The proof is in the appendix, where I also define and provide expressions for Π_S, Π_P, $\bar{k}$ and k^P. ■

Additionally, from the proof of Lemma 1, a separating equilibrium in which the bank only serves the low-type is time-inconsistent. Thus, the only equilibrium possible is the one described in Proposition 4. In the existing screening models of collateral, the equilibrium features the minimum level of collateral required to separate, whenever collateral entails a non-zero cost. My setting yields a different result for the following reason: as high-type borrowers post a higher level of collateral, the bank extracts more of the surplus from lending to low-type borrowers. This, in turn, improves the bank's screening incentives, and pushes the equilibrium towards the full information benchmark.

2.5 Graphical illustration

In this section, I illustrate the main results with a numerical example. The benchmark parameter values are: $q_1 = 0.5$, $q_2 = 0.55$, $p = 0.32$, $X = 4$, $H = 0.15$, $\tau = 1.5$. For these parameters, all the assumptions on the exogenous parameters, *A1-A4*, are satisfied, and Equation (26) holds.

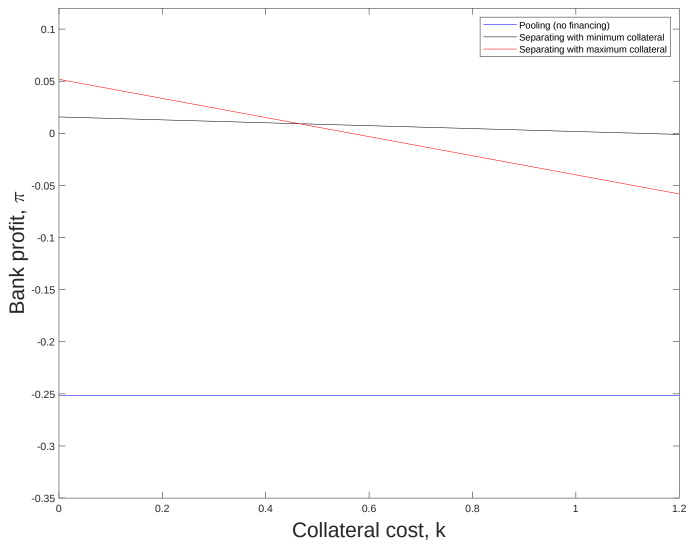

Figure 3: Bank Profit

In figure 3, I plot bank profits under pooling (blue line), separation with the minimum amount of collateral (black line), and separation with the maximum amount of collateral (red line), against the cost of collateral, k. For the benchmark parameters, $\bar{k} = 0.465$. For $k < 0.465$, bank profit is maximized for the separating with the maximum amount of collateral and the spillover effect arises. If collateral is moderately costly, $0.465 < k < 1.124$, bank profit is higher in the separating with the minimum amount of collateral and profit is positive. For very costly collateral, $k > 1.124$, there is no financing.

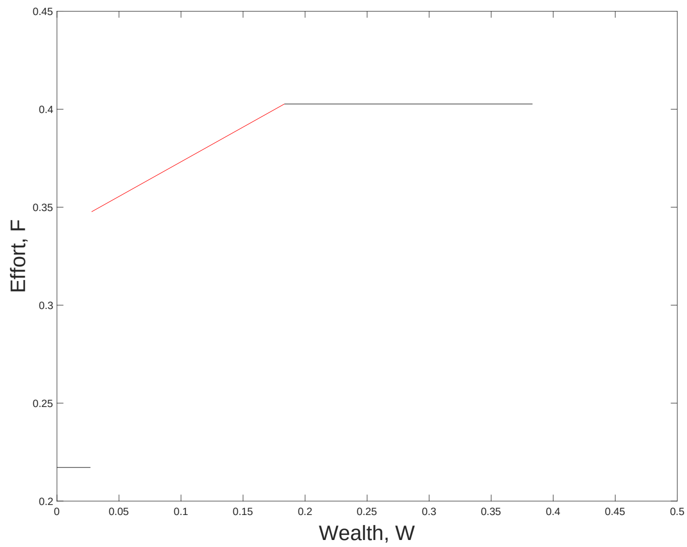

Figure 4: Effort

Figure 4 plots the level of bank effort against borrower wealth, W (or the availability of collateral). The minimum amount of collateral necessary for separation is $\hat{C} = 0.028$. For $W < 0.028$, the equilibrium is pooling and there is no financing. At $W = 0.028$, there is separation, which leads to financing, but the outcome is still inefficient relative to the full information benchmark. The amount of collateral which achieves the efficient outcome is $\overline{C} = 0.183$.

The interesting set of parameters is $0.028 \leq W < 0.183$ (red line). For these parameters, an increase in collateral availability leads to higher collateral requirements for high-type borrowers, with a concurrent fall in their interest rate. In turn, the low-type pay a higher interest rate and the bank exerts more effort to evaluate low-type borrowers' projects. Therefore, these parameters represent the spillover effect.

3 Empirical relevance and implications

In this section, I provide some indirect/partial empirical evidence in support of the mechanism of the model and outline the new testable implications.

In the model, a high-type borrower uses assets unconnected to the firm (e.g., personal assets or third-party guarantees) as collateral to separate themselves from bad borrowers. The first step to assess the empirical content of the model is to check whether personal assets or third-party guarantees are indeed used as corporate collateral and, if so, how widespread the phenomenon is. Bahaj et al. (2020) present compelling evidence that the use of personal assets as corporate collateral can mitigate financing frictions and that an increase in the value of directors' personal assets leads to higher firm-level investment (see also Anderson et al. 2022). Further, they provide survey evidence that, for larger firms, directors often pledge personal assets. Beyhaghi (2022) finds that over 46% of corporate loans are fully or partially guaranteed by a legal entity separate from the borrowing firm.[8]

The next step is to assess to what extent collateralized and uncollateralized debt influence the pricing of each other, i.e., is there a spillover effect? In their structural model, Ioannidou et al. (2022) find that a shock to collateral values leads banks to adjust the pricing of not only collateralized loans, but also uncollateralized loans.[9] Further, as the terms of collateralized loans become more stringent, so do the terms of uncollateralized loans. Their results are consistent with the interpretation that adjusting the pricing of collateralized loans affects the marginal borrower who chooses an uncollateralized loan, and therefore affects the quality of each pool (which is a key differentiating feature in my model). For a more direct test of my model, it

[8]Most of the evidence on directors putting up collateral for firms relates to privately held companies, but there is anecdotal evidence that this practice is prevalent in public companies too. E.g., in 2020, Richard Branson pledged as collateral his private Caribbean island (Necker Island) to raise funds from the UK government to prevent the collapse of his Virgin Group empire.

[9]Note that the sample in Ioannidou et al. (2022) contains both personal and corporate assets as collateral.

would be necessary to hold constant the average qualities of the pools of the collateralized and uncollateralized loan applications, which is not the case in Ioannidou et al. (2022).

I present below two new predictions which are potentially testable. In deriving the predictions, I consider the case that the cost of collateral is small, $k < \bar{k}$. Borrower wealth is $\hat{C} \leq W < \overline{C}$ which implies that the availability of collateral is sufficient to induce separation between borrower-types, but it is not unlimited.

Prediction 1. *Higher collateral availability leads to an improved average quality of uncollateralized loans issued.*

This prediction embodies the spillover effect from collateralized to uncollateralized loans. As high-type borrowers post more collateral, low-type borrowers are charged higher interest rates and, hence, their projects are screened more diligently. More diligent screening by the bank reduces errors, i.e., fewer good projects are rejected and more bad projects are rejected, leading to a higher average quality of the uncollateralized loans.

Prediction 2. *Uncollateralized lending increases in collateral availability if low-type borrowers have many good projects (e.g., economic booms), and falls in collateral availability, otherwise.*

The total amount of uncollateralized lending is L^S:

$$L^S = (1 - q_1)(q_2\beta^S + (1 - q_2)(1 - \beta^S)) \tag{20}$$

The availability of collateral impacts uncollateralized lending through its effect on the screening precision. Differentiating L^S with respect to γ:

$$\frac{\partial L^S}{\partial \gamma} = \frac{q_2 H}{\tau}(1 - q_1)(2q_2 - 1) \lesseqgtr 0 \tag{21}$$

Since the sign of the derivative is not clear, the probability of uncollateralized lending may increase or decrease in the availability of collateral. Specifically, uncollateralized lending increases in the availability of collateral if low-type borrowers have a sufficiently high fraction of good projects, $q_2 > 0.5$. The intuition is as follows. As the availability of collateral increases, the bank exerts a higher screening effort and identifies project quality more accurately. Therefore, if there are many (resp. few) good projects in the pool of uncollateralized loan applications, the bank increases (resp. reduces) its supply of uncollateralized credit.

In Figure 5, I plot uncollateralized lending against collateral availability, W. The benchmark parameter values are the same as in Section 2.5, except $k = 0$ and q_2 takes different values. For $q_2 = 0.55$ (blue line), lending increases in collateral availability, while it falls in collateral availability for $q_2 = 0.45$ (yellow line). For $q_2 = 0.5$ (red line), lending is unaffected.

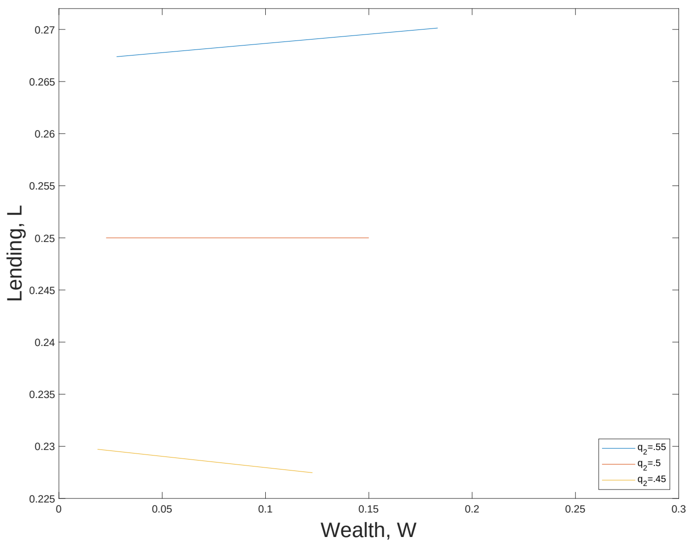

Figure 5: Uncollateralized lending

4 Conclusion

I present a model in which the use of collateral complements bank screening via a spillover effect from collateralized to uncollateralized loans. I establish the relevance of my model using existing empirical findings and offer new predictions. I conclude by discussing two key assumptions.

4.1 More than two types of borrowers

In the baseline, I consider two types of borrowers. In Section 5.2, I consider an extension by introducing intermediate-type borrowers. Intermediate borrowers are more likely to have good projects than low-type borrowers, but may possess bad projects with some positive probability. Intermediate borrowers have intermediate outside options. I show that there may be full or partial separation, depending on the wealth constraints. Importantly, as in the baseline model, in this extension there also arises a positive spillover effect from collateralized to uncollateralized loans. The analysis generalizes to any number of borrower-types, but it becomes complicated since there are more cases to consider.

4.2 Bank market power

I have considered a monopolist bank. The results go through qualitatively if banks retain positive market power in the uncollateralized loan market. The reason is that, as long as banks have positive market power, they increase their surplus by increasing collateral requirements for high-type borrowers since it allows them to charge higher interest rates to low-type borrowers. Thus, with positive bank market power, I can perform the comparative static analyses with respect to the availability of collateral, which deliver the key results of my model.

Conceptually, one could assume direct screening by banks is a scarce skill, which would allow banks to extract surplus in the uncollateralized loan market. Empirically, a large literature establishes that competition in the banking sector is not perfect. In a sample of 48 countries between 1995 and 2007, Forssbaeck and Shehzad (2015) estimate that the country-level loan market Lerner index, a measure of market power, has a mean close to 50% (see also Delis et al. 2016 and Beck et al. 2013).[10] The estimates in these studies indicate that banks enjoy substantial market power, which gives empirical relevance to my model.

[10] The Lerner index is the percentage mark-up charged over the marginal cost. It takes a value of 0 in the case of perfect competition and a value of 1 under monopoly; intermediate values reflect positive market power.

5 Appendix

5.1 Omitted proofs

Proof of Lemma 1. The analysis in the text shows that interest rates alone cannot separate borrower-types if both high- and low-type borrowers obtain financing since the low-type mimics the high-type. It remains to be shown that the bank cannot commit to serve the low-type only. Suppose that at $t = 0$, the bank offers an $R = X$ contract. This contract violates the high-types' participation constraint, so they stay out. Only low-type borrowers apply for this contract at $t = 1$. The bank screens the projects of the low-type and extends credit whenever the signal is positive. At the same time, at $t = 1$, the bank knows that the borrowers who are not served are the high-type (because they turned down the $R = X$ contract at $t = 0$). Then, the bank offers a contract to the high-type with interest rate, $R = X - \frac{H}{p}$. Moreover, anticipating this outcome the low-type will not apply for the $R = X$ contract at $t = 0$. Therefore, since the bank cannot credibly commit to not serve high-type borrowers at $t = 1$ (even if parameters are such that it would like to be able to commit), the conjectured equilibrium unravels.

Proof of Proposition 1. I solve for the equilibrium by backward induction and begin with the bank's screening decision at $t = 2$. An interior solution to the bank's problem is given by the first order condition:

$$q_2(pR_l - 2) + 1 - \tau F = 0 \tag{22}$$

At $t = 0$, the bank sets the repayment rates such that the IR constraints of the borrowers are satisfied. The IR constraints bind for both types. To see why this is the case, consider a candidate equilibrium in which the IR constraint for type i is not binding. Then, the bank can increase its profits by increasing R_i a little, without violating the other constraint. Hence the candidate equilibrium with non-binding IR constraint for either type is not stable. From the relevant IR constraints, the repayment rates, R_h and R_l, are derived (Equations (8) and (9)). The non-negativity constraints are satisfied if $pX > H$, which holds due to Assumption *A4*, and the limited liability constraints are always satisfied, binding for low-type borrowers and slack for high-type borrowers. Substitute R_l in the first order condition to derive the screening intensity, F^{fi} (Equation (10)). Finally, I check that the solution is interior, i.e., $0 < F^{fi} < \frac{1}{2}$. Take the extreme value, $pX \to 1$ which does not violate any of the assumptions: F^{fi} becomes $\frac{1}{\tau}(1 - q_2)$, which is strictly positive since $q_2 < 1$. Increasing pX leads to a *higher* F, therefore, $F^{fi} > 0$ holds for all parameters. Assumption *A3* ensures $F^{fi} < \frac{1}{2}$.

Proof of Proposition 2. I solve for the equilibrium by backward induction and begin with the bank's screening decision at $t = 2$. An interior solution to the bank's problem is given by the first order condition:

$$q_p(p\hat{R} - 2) + 1 - \tau F = 0 \tag{23}$$

At $t = 0$, the bank sets the repayment rates such that the IR constraints of the borrowers are satisfied. In order to make sure that the high-type participates, her individual rationality constraint must be satisfied. This automatically satisfies low-type borrowers' individual rationality constraint. From the IRH constraint, derive the interest rate in the pooling equilibrium, $\hat{R}$ (Equation (14)). The condition is satisfied with equality. If it is slack, the bank can increase its profits by increasing $\hat{R}$ a little, without violating any other relevant constraints. Substitute $\hat{R}$ in the first order condition to derive the screening intensity in the pooling equilibrium, F^P (Equation (15)). Finally, I check that the solution is interior, i.e., $0 < F^P < \frac{1}{2}$. Take the extreme values, $pX \to 1$ and $H \to pX - 1$, which does not violate any of the assumptions: F^P becomes $\frac{1}{\tau}(1 - q_p pX)$, which is strictly positive given Assumption *A2*. Increasing pX and/or reducing H leads to a *higher* F, therefore $F^P > 0$ holds for all parameters. Assumption *A3* ensures $F^P < \frac{1}{2}$.

Proof of Proposition 3. To solve the problem I initially assume that the ICH' constraint is satisfied. After solving the modified problem, I verify that a solution exists which does not violate the starting assumption. In the relaxed problem, the IRH' constraint must bind; if not binding, the bank can increase R_h a little to increase its profits without violating the other constraints. Next, note that either the IRL or ICL' constraint must bind. If neither constraint is binding the bank can increase R_l a little to increase its profits without violating the other constraints. It is the ICL' constraint which binds, and not the IRL constraint, if the RHS of the ICL' constraint is greater than 0. Using the IRH' constraint, the ICL' constraint binds if:

$$C_h \leq \frac{q_2 H}{(1-q_2)} \equiv \overline{C} \tag{24}$$

Equation (24) represents an upper bound on the amount of collateral used. If this condition is violated, then the IRL constraint binds which implies $R_l = X$.

From the IRH' constraint, high-type borrowers' repayment rate when the loan is collateralized becomes:

$$R_h(C_h) = X - \frac{1}{p}(H + (1-p)C_h) \tag{25}$$

The interest rate is falling in the level of collateral, to allow the high-type to achieve her outside option, in expectation. The limited liability constraint ($R_h < X$) is always satisfied. The non-negativity constraint for a high-type borrower's repayment rate is satisfied, $R_h \geq 0$, if:

$$H \leq pX - (1-p)C_h \tag{26}$$

Assume that (26) is not violated. Supposing that Equation (24) is satisfied, i.e., the ICL' constraint binds and using $R_h(C_h)$, the repayment rate charged to low-type borrowers, $R_l(C_h)$, becomes:

$$R_l(C_h) = X - \frac{q_2 H - (1-q_2)C_h}{\beta(C_h) q_2 p} \tag{27}$$

The upper bound, $C_h \leq \overline{C}$, ensures that the limited liability constraint is satisfied, i.e., $R_l \leq X$. The non-negativity constraint is satisfied if $R_l \geq 0$, which gives a lower bound as follows:

$$C_h \geq \frac{q_2(H - \beta(C_h)pX)}{(1-q_2)} \equiv \underline{C} \tag{28}$$

Finally, I verify that bounds on C_h do not violate the starting assumption that the ICH' constraint is satisfied. Substituting $R_h(C_h)$ and $R_l(C_h)$ in the ICH' constraint:

$$C_h \geq \frac{q_2 H(1 - \beta^S(C_h))}{(1-q_2)} \equiv \hat{C}(\beta^S) \tag{29}$$

The ICH' constraint is satisfied only if collateral is sufficiently large, $C_h \geq \hat{C}$. Combining with the feasibility constraints of low-type borrowers' repayment rate, the collateral that high-type borrowers need to post, C_h, in order to achieve separation lies in the range, max $(\underline{C},\ \hat{C}) < C_h < \overline{C}$. It is easily verified that $\hat{C} < \overline{C}$ is always satisfied for any $\beta > 0$, and $\hat{C} > \underline{C}$ is satisfied as long as $pX > H$, which holds due to Assumption, *A4*. Therefore, the feasible range of collateral requirements for which separation is achieved is given by $C_h \in [\hat{C}, \overline{C}]$.

Consider the case that $C_h = \gamma\overline{C} + (1-\gamma)\hat{C} \equiv C^\gamma$ with $\gamma \in [0,1]$. $\gamma = 0$ if borrowers have just enough assets to achieve separation, but no more. γ is increasing in the amount of collateral that the borrowers can post, and $\gamma = 1$ if the borrower is unconstrained, i.e., $W \geq \overline{C}$. Therefore, an increase in γ can be interpreted as an increase in the availability of collateral.

Substituting R_h, R_l, and C^γ in the bank's objective function and taking the first order condition gives the equilibrium screening intensity:

$$F^S = \frac{1}{\tau}(q_2(pX - H - 2) + 1 + \gamma q_2 H) \tag{30}$$

Substituting F^S into C^γ, the equilibrium level of collateral is:

$$C^\gamma = \frac{q_2 H}{1 - q_2}(1 - (1-\gamma)\beta^S) \tag{31}$$

To fully characterize the equilibrium, substitute F^S and C^γ into R_h and R_l. High-type borrowers' repayment rate, $R_h(C^\gamma)$ is falling in the equilibrium amount of collateral (such that the high-type is indifferent with regards to the level of collateral requirement), while low-type borrowers' repayment is increasing in the amount of collateral, as follows:

$$R_l = \underbrace{X - \frac{H}{p}}_{\hat{R}} + \frac{\gamma H}{p} \tag{32}$$

For $\gamma = 0$, which corresponds to the minimum collateral requirement which achieves separation, $R_l = \hat{R}$. For $\gamma = 1$, a low-type borrower's repayment becomes $R_l = X$. Therefore, by setting $\gamma = 1$, the bank extracts the full surplus from lending to the low-type borrower. As long as collateral is costless, the bank increases the requirement to the full extent, $C_h = \overline{C}$, whenever borrowers have sufficient pledgeable wealth, $W \geq \overline{C}$. Thus, γ takes a value less than 1 only if the borrower is wealth-constrained.

Proof of Proposition 4. Following the discussion in the text, in a separating equilibrium the collateral requirements lie in the range $C_h \in (\hat{C},\ \overline{C})$. Re-write C_h as a convex combination of the two corner values, $C_h = (1-\gamma)\hat{C} + \gamma\overline{C}$, with $\gamma \in [0,1]$. The first part of the proposition states that γ is always given by a corner solution, i.e., either $\gamma = 0$ or $\gamma = 1$. The bank's objective function in the separating equilibrium, including the cost of collateral, is:

$$
\begin{aligned}
\Pi_S = & q_1(pR_h + (1-p)C_h - 1 - kC_h) \\
& + (1-q_1)\left(\beta^S(q_2(pR_l - 2) + 1) - (1-q_2) - \frac{\tau}{2}{F^S}^2\right)
\end{aligned} \tag{33}
$$

With probability q_1 the bank makes collateralized loans to high-type borrowers (the top line). The collateral cost is incurred whether or not the project fails (e.g., to transfer and store the collateral when the loan is approved); it could be adapted to the case that the cost of collateral is only incurred on the failure of the project without qualitatively affecting the results. With probability $(1-q_1)$ the bank makes uncollateralized loans to the low-type (the second line). Derivating with respect to γ and k,

$$
\frac{\partial \Pi_S^2}{\partial \gamma \partial k} = -\frac{\partial C_h}{\partial \gamma} = \hat{C} - \overline{C} < 0 \tag{34}
$$

Suppose that collateral is costless, i.e., $k = 0$. In this case, Π_S is increasing in γ, since conditional on separation, the higher use of collateral allows the bank to extract the full surplus from lending to the low-type, which leads to higher profits. From Equation (34), as k increases, $\frac{\partial \Pi_S}{\partial \gamma}$ is falling. There must exist $k = \bar{k}$ such that:

$$
\left.\frac{\partial \Pi_S}{\partial \gamma}\right|_{k=\bar{k}} = 0 \tag{35}
$$

When $k = \bar{k}$, the bank is indifferent in the choice of γ. For $k < \bar{k}$, the derivative is positive, i.e., bank profit is increasing in γ, while for $k > \bar{k}$, bank profit is falling in γ. Hence, we set γ as high as possible, i.e., $\gamma = 1$ whenever $k < \bar{k}$ and set $\gamma = 0$ for $k > \bar{k}$. The expression for $\bar{k}$ is:

$$
\bar{k} = \frac{(q_1 - 1)(q_2 - 1)(\tau - 4q_2 - q_2 H + 2pq_2 X + 2)}{q_1(\tau - 4q_2 - 2q_2 H + 2pq_2 X + 2)} \tag{36}
$$

The profit in the separating equilibrium is falling in k, while the profit in the pooling equilibrium, Π_P, is unaffected in k:

$$\Pi_P = \beta^P(q_p(pX - H - 2) + 1) - (1 - q_2) - \frac{\tau}{2}F^{P2} \tag{37}$$

Suppose that k_γ^P is the value of k which makes the bank indifferent between the pooling equilibrium and the separating equilibrium, given γ:

$$\Pi_P = \Pi_S|_{\gamma\in\{0,1\},k=k_\gamma^P} \tag{38}$$

If the cost of direct screening, τ, is very large, there is no financing for any parameters. Suppose that τ is small such that $\Pi_S|_{\gamma=1,k=0} > 0$. Given the assumptions made, $\Pi_S|_{\gamma=1,k=0} > \max\left(\Pi_S|_{\gamma=0,k=0},\ \Pi_P\right)$. If $\Pi_S|_{\gamma=0,k=0} > \Pi_P$, then $k^P = k_0^P$. If $\Pi_S|_{\gamma=0,k=0} < \Pi_P$, then $k^P = k_1^P$. For $k > k^P$ it is profitable for the bank to forego the separating in favour of the pooling (as long as $\Pi_P > 0$). The expressions for k_γ^P are as follows:

$$k_0^P = \frac{\begin{array}{c}2\tau(q_2-1)(q_2+((q_1+q_2-q_1q_2)(H-pX+2)-1)(((q_1+q_2-q_1q_2)(H-pX+2)-1)/\tau-1/2)-(q_1-1)\\ ((2q_2+Hq_2-pXq_2-1)^2/(2\tau)-q_2+((2q_2+Hq_2-pXq_2-1)(\tau-4q_2-2Hq_2+2pXq_2+2))/(2\tau)+1)\\ +q_1(H-pX+1)-((q_1+q_2-q_1q_2)(H-pX+2)-1)^2/(2\tau)-1)\end{array}}{Hq_1q_2(4q_2+\tau+2Hq_2-2pXq_2-2)} \tag{39}$$

$$k_1^P = \frac{\begin{array}{c}(q_2-1)(q_2+((q_1+q_2-q_1q_2)(H-pX+2)-1)(((q_1+q_2-q_1q_2)(H-pX+2)-1)/\tau-1/2)\\ +q_1(H-pX+1)-((q_1+q_2-q_1q_2)(H-pX+2)-1)^2/(2\tau)\\ +((q_1-1)(4q_2^2-\tau-4q_2+2pXq_2+X^2p^2q_2^2-4pXq_2^2+pXq_2\tau+1))/(2\tau)-1)\end{array}}{Hq_1q_2} \tag{40}$$

Note that the statement of the proposition considers the case that $\Pi_S|_{\gamma=0,k=\bar{k}} > \Pi_P > 0$. There could be other cases. E.g., if $\max\left(\Pi_S|_{\gamma=0,k=\bar{k}},\ \Pi_P\right) < 0$, then financing occurs only with $\gamma = 1$ and up to the point that k is sufficiently low such that $\Pi_S|_{\gamma=1} \geq 0$.

5.2 More than two types of borrowers

In this extension, I consider the case that there are three types of borrowers: high-type, intermediate, and low-type. The purpose of this extension is to show that there exist parameters under which the spillover effect result in the baseline model arises even if we consider that there are more than two types of borrowers. Interestingly, in this extension, the use of collateral need not lead to full separation among borrower-types.

An intermediate borrower has a good project with probability, $q_m \in (q_2, 1)$, i.e., intermediate borrowers may have bad projects, but they have good projects with a higher probability than low-type borrowers. To simplify exposition, I assume that q_m is sufficiently close to 1 such that the bank would lend to intermediate borrowers without screening. Intermediate borrowers have intermediate outside options, $H_m \in (0, H)$.

The bank offers a contract intended for the intermediate borrowers, (R_m, C_m). An intermediate borrower's IR constraint is as follows:

$$q_m(p(X - R_m) - (1-p)C_m) - (1-q_m)C_m \geq H_m \tag{41}$$

Since intermediate borrowers have a lower outside option than high-type borrowers, intermediate borrowers mimic the high-type. An intermediate borrower's IC constraint is as follows:

$$q_m(p(X - R_m) - (1-p)C_m) - (1-q_m)C_m \geq q_m(p(X - R_h) - (1-p)C_h) - (1-q_m)C_h \tag{42}$$

With the IR constraint of intermediate borrowers binding, their IC constraint simplifies to:

$$C_h \geq \frac{q_m H - H_m}{(1-q_m)} \equiv C' \tag{43}$$

The high-type separate from intermediate borrowers only if they post $C_h \geq C'$. Comparing

Equations (24) and (43), $\overline{C} < C'$ if:

$$H_m < \frac{q_m H(1-q_2) - q_2 H(1-q_m)}{1-q_2} \tag{44}$$

For q_m sufficiently close to 1 (as assumed above), the above condition will be satisfied. Consider the parameters, $\hat{C} \leq W < \overline{C} < C'$. For these parameters, low-type borrowers do not mimic high-type borrowers since their IC constraint is violated, but the intermediate borrowers do. For $W = \hat{C}$, the higher types separate from the low-type, but the low-type extract a fraction of the surplus since $R_l < X$. As the availability of collateral increases, the bank increases collateral requirements for the higher types, which allows the bank to charge a higher interest rate to the low-type and positively affects the bank's incentive to screen the low-types' projects. I summarize this result in the following proposition:

Proposition 5 *Suppose that $\hat{C} \leq W < \overline{C} < C'$. There is a partial pooling in which high-type and intermediate borrowers pool and only the low-types' projects are screened. An increase in the availability of collateral allows the bank to extract a higher fraction of the surplus from the low-type, which positively affects its incentives to screen the low-types' projects.*

Analogous to the baseline model, in this extension, the bank's effort to screen the low-types' projects increases in the availability of collateral beyond $W = \hat{C}$, even though $\hat{C}$ already separates the high-type and intermediate borrowers from the low-type (since the higher types post collateral, but not the low-type). Thus, extending to the three-type case does not affect the main results qualitatively since the positive spillover effect from collateralized to uncollaterlized loans still arises.

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