Can Information Imprecision Be Valuable? The Case Of Credit Ratings

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Abstract

We develop a model that explains two stylized facts – the coarseness of credit ratings relative to the underlying default probabilities, and the countercyclical nature of ratings precision. The imprecise nature of coarse ratings arises from the revenue-maximizing behavior of rating agencies, but it may maximize net social surplus. When there is competition among rating agencies, for some parameter values welfare is increased by restricting competition, and for others it is increased by encouraging competition. In addition to the novel result that coarseness improves welfare relative to greater precision, our model also generates new testable predictions.

Keywords: Credit ratings, Coarseness, Information precision, Adverse selection, Effort moral hazard

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"The traditional philosophy of science approach to prediction leaves little room for appreciating the value and potential of imprecise information. At least, they are considered a stepping stone to more precise predictions." – Elliott-Graves (2020)

1 Introduction

More precise information is typically regarded as being superior to less precise information. Perhaps the most obvious argument in favor of this proposition is a "free disposal of information" reasoning – if information of lower precision was more valuable, the receiver of the information could always use a coarser partition of the transmitted information. Yet, we see numerous examples of imprecise information being produced and communicated even when more precise information is technically feasible. A striking example of this is credit ratings of debt instruments. There is a continuum of default probabilities, but only about two dozen or so credit ratings. This means that credit ratings are imprecise indicators of default probabilities.

Ratings are produced by credit rating agencies (CRAs) which are information intermediaries that acquire and process information about firms, thereby reducing firms' financing frictions. This suggests that CRAs should produce and communicate as precise information as possible. The fact that they do not do this suggests that the imprecision associated with ratings coarseness may have value. What is this value? Moreover, the precision of ratings seems to be dependent on the business cycle, with ratings displaying greater precision during downturns (e.g., Griffin and Tang 2012). Why? We address these questions theoretically in this paper and develop a model consistent with these stylized facts – ratings coarseness and the greater informativeness of ratings during downturns. We also examine how ratings coarseness interacts with market structure in the CRA industry. In our model, the regulator can achieve the efficient equilibrium by influencing competition among CRAs.

The model we develop is quite simple. There are three types of observationally identical firms that can be either good, intermediate or bad in credit quality.[1] Each firm is privately informed about its type, whereas all other agents have common-knowledge priors captured by a probability distribution over types. Good firms have positive-NPV projects, bad firms have negative-NPV projects, and intermediate firms have projects that are positive-NPV if the firm exerts (privately costly) effort and negative otherwise.[2] Effort exertion incentives get weaker as the firm's debt repayment increases. Thus, there are only two frictions in the model – asymmetric information about firm types and effort-aversion moral hazard.

In the absence of certification by a CRA, securities issued by all firms are priced as a pool. If the fraction of bad borrowers is sufficiently high, the expected NPV becomes negative and there is no financing at all (i.e., the market breaks down). CRAs can costlessly learn firm type perfectly and assign a rating at a flat fee.

If CRAs provide precise ratings, each firm type is identified accurately by the market and firms issue debt that is accurately priced for each type. The interest rate comes from the (binding in equilibrium) participation constraints of investors whose zero-expected-return participation constraints must be satisfied. But, at this equilibrium interest rate, the intermediate firms do not exert effort when the cost of this effort is sufficiently high. Thus, for a sufficiently high cost of exerting effort, only the good firms obtain finance, while the intermediate and bad types are excluded.

If, on the other hand, the CRAs provide coarse ratings, they pool the good and intermediate firms into a single rating category. Given this, there is a partial pooling equilibrium in which both the good and intermediate firms issue debt, and the debt is priced according to the average quality of the pool. As a result, debt issued by the good firms is under-priced and the debt issued by the intermediate firms is over-priced, i.e., the good firms subsidize the intermediate firms. A large enough subsidy makes it incentive compatible for the intermediate firms to exert

[1] Credit quality in the model corresponds to the firm's probability of default on its debt.

[2] The idea is that some firms may have the ability to undertake costly risk management activities to lower their default probabilities.

effort. This effort exertion transforms projects of the intermediate firms into positive NPV. Both good and intermediate firms obtain financing and net social surplus is higher than attainable in the precise-ratings equilibrium. Coarseness delivers the socially desirable outcome which does not arise in the absence of coarseness.

The preceding analysis does not consider what the CRA would wish to do, so we turn next to the incentives of the CRA. We begin by considering the case in which there is a single CRA. The rating agency's objective is to maximize its fee revenue. For this reason, as long as the average NPV across the good and intermediate firms is positive, so financing can be raised by the pool (whether or not the intermediate firms exert effort), the CRA provides coarse ratings because this maximizes fee revenue. If the cost of effort is sufficiently low, the coarse ratings induce the intermediate firms to exert effort due to the cross-subsidization from the good firms. However, for some parameters, the intermediate firms may obtain finance, but do not exert effort. In sum, coarse ratings arise as an equilibrium outcome due to the fee-maximizing behavior of the CRA. In those cases in which the coarse ratings elicit effort provision by the intermediate firms, net social surplus is also maximized and the first-best is achieved.

Next, consider the case of multiple competing CRAs. Suppose there is a coarse-ratings equilibrium. If the proportion of intermediate firms in the pool is small, a competing CRA can maximize its fee revenue by unilaterally deviating from the coarse-ratings equilibrium and poaching the good firms by offering precise ratings. Thus, competition can cause the coarse-ratings equilibrium to break down, leading to more precise ratings. We show that whether net social surplus is maximized by coarse or precise ratings depends on the deep parameters of the model. The regulator can achieve the desired equilibrium by restricting or encouraging competition among CRAs.

In our theory, in addition to playing the usual role of reducing information asymmetry, credit ratings also have welfare-enhancing real effects because they can induce effort provision that would not be forthcoming in the absence of ratings. Further, our model generates the novel insight that it is not necessarily welfare-enhancing for CRAs to fully eliminate all information asymmetry. For some deep parameters, withholding some information allows CRAs to deliver higher net social surplus compared to the case in which investors have full information.

We also relate ratings coarseness to the business cycle. Assume that the ratio of good firms to intermediate firms is high in an economic boom and low in an economic downturn.[3] In a downturn, the relative paucity of good borrowers implies that the subsidy from the good to the intermediate firms in the pooling equilibrium may not be sufficient to incentivize the intermediate firms to exert effort. Then, with coarse ratings, the unique outcome will be a pooling one and no one in the pool will obtain financing. So, no firm obtains a rating. Clearly, this cannot be an equilibrium. To attract the good firms, the CRA will offer to precisely identify the good firms and eschew pooling. This means that in a downturn, ratings coarseness disappears. This implication of the model that the precision of ratings may be higher in downturns than in booms is consistent with the empirical evidence in Griffin and Tang (2012).[4]

In addition to generating results that are consistent with existing stylized facts, our model also produces two new predictions. One prediction is that greater competition among CRAs reduces investments by (rated) firms. The other prediction is that, somewhat surprisingly, when the cost of information production for CRAs declines, real investment by (rated) firms go down. This is counter to the usual intuition that if information production by CRAs facilitates investments, then a lower production cost should elevate investment.

This paper is related to the literature on the value of imprecise information. In oligopolistic models of incomplete information, there are conflicting results on the benefits or costs of observing more precise information. In Rotemberg and Saloner (1986) and Vives (1984), ob-

[3]An alternate way to capture the business cycle would be to assume that the cost of effort for the intermediate firms is higher in economic downturns. This interpretation would yield similar predictions.

[4]Bar-Isaac and Shapiro (2013) have a similar result but it is driven by different forces in a dynamic model with reputational concerns. Additionally, Bar-Isaac and Shapiro (2011) suggest that in economic booms CRAs have the smallest incentives to be accurate.

serving more precise information has value, whereas the opposite is true in Gal-or (1987). In these models, the firm cannot affect the precision of the information and the quantity of output produced has no effect on the precision. In contrast, Gal-or (1988) develops a model in which experience in production allows firms to internally generate private signals at no cost. When the firm is endowed with less precise information about cost, it has a greater incentive to produce. Information imprecision thus has value because it encourages production. In contrast to this literature, our model focuses on the external provision of information in either monopolistic or competitive markets, and shows that information imprecision in communication can resolve moral hazard and elevate net social welfare even when the entity communicating the information has more precise information in its possession.

Our paper is also related to the credit ratings literature. Building on the foundations provided by the financial intermediation literature that rating agencies are an example of diversified information-production intermediaries (e.g., Allen 1990, Millon and Thakor 1985, and Ramakrishnan and Thakor 1984), a strand of the literature showed that credit ratings can resolve coordination problems in financial markets (e.g., Boot et al. 2006, and Manso 2013).[5] Parlour and Rajan (2020) show that ratings can be valuable in the presence of contract incompleteness. In our setting, welfare is non-monotonic in the precision of information communicated by the CRA.

Other papers have focused on failures in the credit rating process, including incentives for rating agencies to manipulate ratings (e.g., Bolton et al. 2012, Sangiorgi et al. 2009, Opp et al. 2013, Frenkel 2015, Sangiorgi and Spatt 2017, and Farkas 2021). Ratings inflation is also studied by Goldstein and Huang (2020), where the focus is on the feedback effect of ratings – creditors respond to CRA's inflated but informative rating and this affects the firm's investment decision and credit quality. While inflated ratings refer to incorrect ratings, which is only feasible in

[5]Thakor and Merton (2023) view credit ratings in the presence of asymmetric information and product complexity as a third-party verification mechanism.

settings with naive investors, we consider coarse ratings, i.e., ratings are vague but, on average, correct. That is, ratings inflation and ratings coarseness are different phenomena. Ratings inflation refers to imprecise ratings that are on average too high relative to the underlying default probabilities, whereas coarse ratings are correct on average but not as finely partitioned as the underlying default probabilities.

Our contribution relative to this literature is that we develop a theory in which ratings coarseness arises endogenously as an equilibrium phenomenon to elevate net social surplus. That is, not only are credit ratings coarse, but this coarseness improves ex ante economic efficiency relative to a setting with precise ratings. This connects us to earlier papers in which ratings are coarse, like Lizzeri (1999) and Doherty et al. (2012). In these papers, information communication is endogenously coarse, but coarseness does not impact allocative efficiency. In Kartasheva and Yilmaz (2020), coarseness also arises endogenously due to the monopolist CRA's revenue maximizing behavior but coarseness destroys welfare. In contrast to these models, coarseness improves welfare in ours (for some parameters).

The paper closest to ours is Goel and Thakor (2015), which also provides an endogenous theory of coarse ratings. Using a cheap-talk model, the paper shows that ratings coarseness can arise as an equilibrium phenomenon even when higher precision can improve investment efficiency because that is the only incentive compatible mechanism for truthful communication by the CRA. In contrast, coarseness helps to achieve the first-best investment in our model when precision fails to do so. That is, even when the CRA can credibly communicate very precise information, welfare is higher when it chooses imprecision over precision. In this sense, ours is the first paper to show that ratings coarseness increases welfare.

In our model, the CRA's ratings assignment seeks to maximize the CRA's (fee) revenue. Ali et al. (2022) develop a model in which the CRA also issues ratings to maximize its fee revenue. They show that the revenue-maximizing scheme involves the CRA producing noisy information about firms, but, unlike our model, ratings may lie in a continuum, so the paper does not deal with ratings coarseness. Also, in contrast to our model, the information produced by the CRA is neither socially valuable nor ex ante valuable to the firm being rated. Our result that coarseness can be welfare-improving is reminiscent of Spence (1973). In contrast to Spence (1973), the pooling equilibrium in our model may be inefficient from a social perspective for some parameters, but may still arise, while in Spence (1973), the pooling is always efficient from a social perspective but it does not survive standard equilibrium refinements.

Finally, our paper also relates to the literature on bank stress testing (see e.g., Goldstein and Leitner 2018 and Orlov et al. 2022). Using tools from the Bayesian persuasion literature, the optimal stress tests generally take a simple pass or fail form, rather than identifying each bank's asset quality precisely. This test design maximizes welfare by pooling stronger banks with vulnerable banks, only allowing the weakest banks to fail. Our theory differs in two respects: First, there is no relaxation of informational frictions in these models – this is what delivers the increase in welfare in our model. Second, unlike the bank stress test literature, the coarseness in our model arises due to market forces.

2 Model

2.1 Set-up

We consider an economy in which all agents are risk-neutral and the discount rate (risk-free rate) is zero. There are three types of agents: firms, CRAs, and investors. A firm has access to a project that needs investment and the scale of the investment is normalized to 1 unit. Each firm has zero initial endowment, so it seeks to raise funds from outside investors to invest in its project. Specifically, each firm raises $(1 + f)$ from the market; they invest 1 unit in the project, and either consume f right away or use f to pay a fee to a CRA (more details below). Investment occurs at $t = 0$ and returns are realized at $t = 1$, at which point all agents consume.

There are three types of firms and each firm privately knows its type. The common prior belief is that a fraction α of the firms have good projects, g, a fraction β have intermediate quality projects, m, and a fraction $1-\alpha-\beta$ have bad projects, b. A firm succeeds with probability, $p_i \in \{p_g, p_m, p_b\}$, and fails with the complementary probability. If a project succeeds, it generates a cash flow, $X > 0$, and if it fails, it generates 0; firm types differ only on the probability of success. We make the following assumptions relating to the deep parameters:

A1: $p_g X - f > 1 > p_m X > p_b X$

Assumption *A1* states that the good firms have positive-NPV projects after taking into account a fee to the CRA (if they have to pay it), while the intermediate and bad firms have negative-NPV projects. An intermediate firm can exert hidden effort at cost, c, to increase its success probability by δ, such that its project becomes positive-NPV:

A2: $p_g X > (p_m + \delta)X > 1 + c + f$

Combined with Assumption *A1*, the second inequality of Assumption *A2* implies that exerting effort by intermediate firms is efficient, net of the cost of effort, i.e., $\delta X - c > 0$. The value generated from exerting effort makes intermediate firms' projects positive-NPV. Nonetheless, Assumption *A2* also states that good firms are still more likely to succeed than intermediate firms. The bad firms cannot increase the probability of their success by exerting effort. Thus, it is efficient to finance intermediate firms only if they exert effort, while it is always inefficient to finance bad firms.

The set-up is meant to reflect the possibility of risk management activities that could help the intermediate firms to reduce its default probability. We could also allow the good and bad firms to have such a hidden effort choice, but if we assume that a good firm is creditworthy regardless of its effort choice and a bad firm is never creditworthy regardless of its effort choice[6], our main results are sustained. So, in the interest of simplicity, we do not give the good and

[6]This assumption would be necessary to meaningfully distinguish the intermediate firm from the good and bad firms.

bad firms this effort choice.

A3: $(\alpha p_g + \beta(p_m + \delta) + (1 - \alpha - \beta)p_b)X < 1 + \beta c$

A4: $(\alpha p_g + (1 - \alpha - \beta)p_b)X < 1 - \beta$

Assumption *A3* implies that the expected NPV across all three firm types is negative, even if the intermediate firms exert effort. Assumption *A4* implies that the expected NPV across the good and bad firms is negative, i.e., the fraction of bad firms is sufficiently high.

A CRA can perfectly identify the firm type at zero cost and a firm pays an exogenously given fixed fee, f, to the rating agency to certify its type.[7] The fee is paid by the firms from the money raised in the market. The modelling of the fee is consistent with the issuer-pays model and reflects the observation that issuers choose to pay for a rating only if the rating they obtain allows them to borrow in the market.

2.2 The game

The stages of the game are as follows:

Stage 1: CRAs announce policies and a policy is either no ratings, precise ratings, or coarse ratings.[8]

Stage 2: Given what has been announced by the CRAs, the firms decide whether to get rated. If they choose to get rated, they pay a fee, f, to the CRA.

Stage 3: Regardless of whether the firm has chosen to be rated or not, the firm can propose a debt contract with gross interest rate (promised repayment), R, to competitive investors.

Stage 4: Investors form a belief about the firm type given the rating given to the firm (if any) and the offered interest rates. Given these beliefs, investors decide whether to accept or reject

[7]Introducing noise in the CRA's default probability discovery does not qualitatively change the results. We recognize that the exogenous fee assumption is strong, so we consider an extension in Section 5.1, in which observing firm types is costly for the CRAs and the fee is determined endogenously.

[8]We allow for all possible combinations of coarseness (all types together and three different pairs of two types). In Lemma 1, we show that the only coarse ratings on the equilibrium path is pooling good and intermediate firms.

the proposed contract. Investment occurs only if the proposed contract is accepted, at which point intermediate-type firms decide whether to exert unobservable effort.

We look for the pure strategy Perfect Bayesian Nash equilibria of this game that satisfy the Intuitive criterion of Cho and Kreps (1987). We solve for the equilibrium backwards. First, we analyze the financing game, conditional on the ratings precision set by CRAs. Then, we derive the CRA's choice of ratings precision by fully anticipating the outcome of the financing game.

2.3 Benchmark 1: Observable types and effort

We now consider the first-best allocation. The first-best is the allocation that obtains in the absence of both informational frictions, i.e., firm types are observable and the intermediate firms' effort levels are observable and contractible. Due to competition among investors and full information, the interest rates are such that the investors make zero expected profits.

Proposition 1 (Benchmark 1) *In the first-best equilibrium, the good and intermediate firms obtain finance and the intermediate firms exert effort. The bad firms do not obtain finance. The interest rates are as follows:*

$$R_g = \frac{1+f}{p_g} \tag{1}$$

$$R_m = \frac{1+f}{p_m+\delta} \tag{2}$$

The marginal return on an increase in effort in terms of an increase in the expected return exceeds the marginal cost of that effort. Therefore, in the first-best, the intermediate firms choose a contract which implements the efficient high effort level. The reason why interest rates are as in Equations (1) and (2) is competition among investors which leads to zero expected returns for them.

2.4 Benchmark 2: Observable types and unobservable effort

In this section, we consider the case in which firm types are observable but effort is not observable, and hence, not contractible. Given observable types, good firms receive financing at interest rate, R_g, and invest. The bad firms do not obtain financing since they have negative-NPV projects. Whether intermediate firms obtain financing depends on the deep parameters. Consider the effort incentive constraint of an intermediate firm:

$$(p_m + \delta)(X - R) - c \geq p_m(X - R) \tag{3}$$

The left-hand side (LHS) represents the intermediate firms' expected profits when exerting effort, while the right-hand side (RHS) represents the expected profits when not exerting effort. From Equation (3), an intermediate firm exerts effort only if the interest rate is sufficiently small:

$$R \leq X - \frac{c}{\delta} \equiv \underline{R} \tag{4}$$

The difference between $\frac{c}{\delta}$ and c is the rent that the intermediate firms should receive to exert effort, which drives the inefficiency. If the equilibrium interest rate is higher than $\underline{R}$, the intermediate firms' incentive compatibility (IC) constraint for effort provision is violated. Suppose that the investors believe that intermediate firms will exert effort. The intermediate firms will raise funds at interest rate, R_m, which sets the competitive investors' expected profits to zero. $R_m \leq \underline{R}$ if:

$$c \leq \delta X - \frac{\delta(1 + f)}{p_m + \delta} \equiv c_s \tag{5}$$

If the cost of exerting effort is sufficiently small, i.e., $c \leq c_s$, then the intermediate firms exert effort, consistent with the beliefs of investors. However, for $c > c_s$, investors' belief that the intermediate firms exert effort is not fulfilled. This implies that the investment of the intermediate firms will be negative-NPV and so for any $R \leq X$, the investors make losses. As a result, when $c > c_s$, the intermediate firms do not receive financing.

Proposition 2 (Benchmark 2) *Suppose that firm types are observable but effort is not.*

1. *For $c \leq c_s$, both good and intermediate firms obtain financing at the promised repayment, R_g and R_m, respectively, and the intermediate firms exert effort.*
2. *For $c > c_s$, the good firms obtain credit at interest rate, R_g, while the intermediate firms do not obtain financing.*

Proof. The proof is in the Appendix. ■

3 Outcome under each CRA policy separately

In this section, we examine the case that both informational frictions are present. To ease exposition, prior to endogenizing CRAs' choice of ratings precision, we analyze the outcome for each of the following three cases: CRAs provide (1) no ratings or (2) precise ratings or (3) coarse ratings. In Section 4, CRAs optimally choose whether to provide precise or coarse ratings in order to maximize their profits.

3.1 Outcome under no ratings

First we consider the case that CRAs do not provide ratings. Under Assumption *A2*, $p_m + \delta < p_g$, if lenders offer an interest rate which is meant for the good firms under full information, then the intermediate and bad firms will mimic and the contract will be loss-making for the investors. This is because the average NPV across all three types is negative (Assumption *A3*). Thus, there cannot exist a pooling equilibrium in which all firms obtain financing at the actuariallyfair pooling interest rate. This suggests that, in the absence of ratings, the market breaks down and no firm obtains financing.

Proposition 3 (No ratings) *In the absence of credit ratings the unique equilibrium consistent with zero expected profits for investors is the one in which no firm obtains financing (market breakdown).*

Proof. The proof is in the Appendix. ■

3.2 Outcome under precise ratings

In this section, we consider the case that CRAs assign precise ratings. Since we consider perfect learning by the CRAs, the analysis in this case is similar to case in which firm types are observable but effort is not observable. Because ratings are precise, investors know the firm type and they do not need to make any inferences about firm types from the offered contracts, i.e., investors' beliefs do not play a role in this case. The equilibrium is identical to the one derived in Proposition 2.

The case of precise ratings improves upon the situation with no ratings since it eliminates the market breakdown equilibrium. For $c \leq c_s$, the precise ratings equilibrium coincides with the outcome in the first-best. However, for $c > c_s$, the precise-ratings equilibrium is inefficient compared to the first-best because, unlike in the first-best, the intermediate firms do not obtain financing in the precise-ratings equilibrium.

3.3 Outcome under coarse ratings

We now consider the case of the CRA assigning coarse ratings.

Lemma 1 *The only coarse categorization which is not Pareto-dominated by either the precise-ratings allocation or another coarse-ratings allocation is the one which pools together the good* *and the intermediate firms.*

Proof. The proof is in the Appendix. ■

Given Lemma 1, for the rest of the analysis, whenever we refer to coarse ratings we consider the case in which the good and the intermediate firms are pooled together in a single ratings category.

Under coarse ratings, the debt issued by the good and intermediate firms is priced according to the average quality of the pool. If the intermediate firms exert effort, then the average NPV of the pool is positive. If the intermediate firms do not exert effort, the average NPV of the pool may be positive or negative; it is positive if:

$$\left(\frac{\alpha}{\alpha+\beta}p_g + \frac{\beta}{\alpha+\beta}p_m\right)X - 1 \geq 0$$
$$\implies \frac{\alpha}{\beta} \geq \frac{1-p_mX}{p_gX-1} \equiv \gamma \tag{6}$$

$\frac{\alpha}{\beta}$ represents the ratio of good firms to intermediate firms. If this ratio is sufficiently high, i.e., $\frac{\alpha}{\beta} \geq \gamma$, the average NPV across the good and intermediate firms is positive; otherwise, it is negative.

To characterize the coarse-ratings equilibrium, we define c_p, such that if $c > c_p$, the intermediate firms do not exert effort even if they pool with the good firms, i.e., the pooling repayment rate assuming that the intermediate firms exert effort is higher than the incentive compatible repayment rate, $\underline{R}$ (the derivation is in the proof Proposition 4 in the Appendix). c_p is given by:

$$c_p \equiv \delta X - \frac{\delta(\alpha+\beta)(1+f)}{\alpha p_g + \beta(p_m+\delta)} \tag{7}$$

Which equilibrium obtains depends on the deep parameters as the following result shows:

Proposition 4 (Coarse ratings) *Suppose that ratings are coarse.*

1. *For $c \leq c_p$, both the good and intermediate firms obtain financing, and the intermediate firms exert effort.*

2. *For $c > c_p$ and $\frac{\alpha}{\beta} \geq \gamma$, both the good and intermediate firms obtain financing, but the intermediate firms do not exert effort.*

3. *For $c > c_p$ and $\frac{\alpha}{\beta} < \gamma$, coarse ratings lead to market breakdown in which none of the firms obtain financing.*

Proof. The proof is in the Appendix. ■

Under coarse ratings, there is pooling across the good and intermediate firms, and as a result, the debt issued by the good firm is under-priced and the debt issued by the intermediate firm is over-priced. The consequent mispricing subsidy incentivizes intermediate firms to exert effort for $c \leq c_p$. In terms of net social surplus, this case is identical to the first-best, except that there is no cross-subsidization in the first-best and good and intermediate firms have different financing costs. For $c > c_p$, the subsidy is insufficient to induce the intermediate firms to exert effort. In this case, the pooling equilibrium may still be feasible if the lenders can break even on average, which happens if the ratio of good to intermediate firms is sufficiently high, i.e., $\frac{\alpha}{\beta} \geq \gamma$.

3.4 Welfare comparison

Observe that precise ratings eliminate the no-financing equilibrium (see Proposition 3) for all parameters, and hence, the precise-ratings case strictly dominates the no-ratings case in terms of welfare, implying that having no ratings is never socially desirable. Whether precise ratings generate higher net social surplus than coarse ratings depends on the deep parameters. In order to compare the precise and coarse ratings cases, we present the following lemma:

Lemma 2 *The cost above which the intermediate firms do not exert effort is higher in the coarse-ratings case than in the precise-ratings case, i.e.,* $c_p > c_s$*.*

Proof. The proof is in the Appendix. ∎

Lemma 2 implies that the intermediate-type firms exert effort for a larger set of parameters when ratings are coarse compared to when the ratings are precise. With precise ratings, the intermediate firms' debt is priced accurately, $R = R_m$. For this price of debt, it is not incentive compatible for the intermediate firms to exert costly effort, unless the cost of effort is small, $c \leq c_s$. With coarse ratings, financing subsidy due to pooling with good firms elicits effort for a larger set of effort costs, namely for all $c \leq c_p$, where $c_p > c_s$.

We compare the welfare properties of the precise and coarse ratings cases in the proposition below.

Proposition 5 (Coarseness and Welfare) *The welfare comparison between the precise and coarse ratings cases depends on the deep parameters as follows:*

1. *For* $c \leq c_s$*, social welfare in the precise and coarse ratings cases are identical to the first-best.*

2. *For* $c_p \geq c > c_s$*, social welfare is higher under coarse ratings, in which case it is identical to the first-best.*

3. *For* $c > c_p$*, social welfare is higher under precise ratings, but not identical to the first-best.*

Proof. The proof is in the Appendix. ∎

We obtain our most interesting results for the parameters, $c_p \geq c > c_s$. For these parameters, the cost of exerting effort is such that the intermediate firms do not exert effort and thus fail to obtain financing if the ratings are precise, but they obtain financing and exert effort if ratings are coarse. Thus, for these parameters, the coarse-ratings case involves a higher net social surplus than the precise-ratings case. We refer to this as the "bright side" of coarseness. For $c > c_p$, coarse ratings cannot induce the intermediate-type firms to exert effort. In this case, the most efficient outcome is obtained with precise ratings, i.e., by completely eliminating the information asymmetry problem, since it is more efficient to not have the intermediate firms invest than for them to invest but not exert effort.

4 Equilibrium

In this section, we allow the CRAs to optimally choose whether they offer precise or coarse ratings, and we analyze the market equilibrium under different market structures. Specifically, we examine how competition affects whether the coarse-ratings or the precise-ratings equilibrium arises.

4.1 Equilibrium with one CRA

In this section, we characterize the equilibrium when there is a single CRA. The CRA charges a fixed fee, f, for providing a rating to a firm. The objective of the CRA is to maximize its total fee revenue, taking the fee per firm, f, as a given.[9] Thus, the CRA would like to rate firms in a way that induces as many firms as possible to obtain ratings and raise financing. The constraint is that the investors, who are the consumers of the ratings, have rational expectations. Hence, if ratings are uninformative, investors will not use the ratings, and firms will consequently not purchase the ratings. The choice facing the CRA is to either issue precise ratings or coarse ratings. Below, we characterize the equilibria of the game:

Proposition 6 (One CRA) For $c \leq c_s$, precise and coarse ratings generate the same fee for the ratings agency, $(\alpha + \beta)f$. Suppose that $c > c_s$:

[9]While we treat f as exogenous, it should be noted that the results hold for any $f > 0$ that satify participation constraints. In section 5.1, we address the endogenization of f.

1. For $\frac{\alpha}{\beta} \geq \gamma$, the equilibrium with one CRA features coarse ratings and the total fee revenue is $(\alpha + \beta)f$.

2. For $\frac{\alpha}{\beta} < \gamma$, the equilibrium with one CRAs features precise ratings and the total fee revenue is αf.

Proof. The proof is in the Appendix. ■

For a sufficiently small cost of effort, $c \leq c_s$, both intermediate and good firms obtain ratings to separate themselves from the bad firms. Whether ratings are precise or coarse has distributional consequences only. When the cost of effort is sufficiently high, $c > c_s$, the pooling equilibrium sustains if the ratio of good to intermediate firms is sufficiently high, i.e., $\frac{\alpha}{\beta} \geq \gamma$. Thus, for these parameters, the CRA offers coarse ratings. When the ratio of good to intermediate firms is sufficiently low, i.e., $\frac{\alpha}{\beta} < \gamma$, the CRA offers precise ratings to attract only the good firms because coarse ratings would lead to a market breakdown in which case it would generate zero fee revenue.

Note that the equilibrium outcome may or may not be the efficient one. Consider first the case of $\frac{\alpha}{\beta} \geq \gamma$. If $c \leq c_p$, the equilibrium is identical to the first-best. If $c > c_p$, the intermediate firms do not exert effort, which reduces net social surplus. In this case, precise ratings would improve upon the coarse-ratings equilibrium outcome by removing the intermediate firms from the pool of rated firms, but this is still not the first-best. For $\frac{\alpha}{\beta} < \gamma$, the equilibrium is inefficient since the intermediate firms do not obtain credit, but this equilibrium cannot be improved upon.

The novel contribution of our model is captured in part 1 of Proposition 6. A CRA, motivated by the objective of maximizing its fee revenue, issues imprecise ratings so that good and intermediate firms are pooled into a single ratings category. This pooling allows cross-subsidization from good to intermediate firms and provides the incentives to the latter to exert effort, even when these firms would not exert effort if their securities were accurately priced. Thus, pooling improves welfare. But the CRA's motivation to engage in this ratings pooling comes not from any social welfare considerations, but rather because doing so is profit maximizing for the CRA.

4.2 Competition among CRAs

In this section, we model competition by assuming that there are $N \geq 2$ CRAs. We solve for the symmetric equilibrium, which is characterized in the result below:

Proposition 7 (Competition) *With N CRAs, the symmetric equilibrium features coarse ratings if* $\frac{\alpha}{\beta} < \frac{1}{N-1}$*, and precise ratings, otherwise.*

Proof. The proof is in the Appendix. ■

This proposition says that competition among CRAs may lead to precise ratings. To see why this is the case, notice that starting from a symmetric equilibrium with coarse ratings, a competing CRA may unilaterally deviate to increase its market share by offering precise ratings and poaching the good firms' business. So competition works to unravel pooling.[10] This proposition also says that higher competition makes precise ratings more likely (i.e., precision will obtain with more values of firm pool quality, $\frac{\alpha}{\beta}$). As competition increases, each CRA's market share in the symmetric equilibrium is smaller, so the incentive to deviate and capture all of the good firms is stronger. This result contrasts with the earlier literature in which competition may increase ratings coarseness (e.g., Goel and Thakor 2015).

Next, we discuss the welfare consequences of competition among CRAs. The coarse-ratings equilibrium is more efficient than the precise-ratings equilibrium, only if coarseness induces the intermediate firms to exert effort. This is the case for $c \leq c_p$. When $\frac{\alpha}{\beta} \geq \gamma$ and $c > c_p$, the coarse-ratings equilibrium can arise for sufficiently small N, but it is less efficient than the precise-ratings equilibrium since the intermediate firms do not exert effort. We discuss the various cases in Proposition 8.

[10] As, for example, in Rothschild and Stiglitz (1976).

Proposition 8 (Competition and welfare) *The relationship between competition among CRAs and net social surplus depends on parameters as follows:*

1. *For $c \leq c_s$, the degree of competition in the CRA industry does not affect the net social surplus, so any equilibrium is identical to the first-best in terms of the net social surplus.*

2. *For $c_s < c \leq c_p$, the net social surplus is maximized for $\frac{\alpha}{\beta} < \frac{1}{N-1}$.*

3. *Suppose that $\frac{\alpha}{\beta} \geq \gamma$. For $c \leq c_p$, the net social surplus is maximized for $\frac{\alpha}{\beta} < \frac{1}{N-1}$ which leads to coarse ratings. For $c > c_p$, the net social surplus is maximized for $\frac{\alpha}{\beta} \geq \frac{1}{N-1}$ which leads to precise ratings.*

4. *Suppose that $\frac{\alpha}{\beta} < \gamma$. Then the unique equilibrium is characterized by precise ratings.*

5 Extensions

In this section, we present two extensions to the baseline model which show that our central idea holds true in more general settings.

5.1 Endogenous fee

So far, we have assumed that CRAs charge an exogenously given fee, f. In the competition case, CRAs compete on ratings precision. In this section, we endogenize the fee charged by the CRA and model competition by assuming that there is free entry of CRAs. Unlike the previous section, CRAs compete on two dimensions – ratings precision and fee. For this analysis, we extend the baseline model by assuming that a CRA identifies a bad firm at no cost but incurs a cost, $k \geq 0$, to distinguish the good from the intermediate types (by setting $k = 0$, we return to the baseline model), i.e., the marginal cost of producing coarse ratings is 0, while the marginal cost of producing precise ratings is k.

Lemma 3 *With free entry of CRAs, the fee always equals the marginal cost of producing a rating; i.e., $f = k$ for precise ratings and $f = 0$ for coarse ratings.*

The intuition for the above result is the classic Bertrand argument. Given the degree of ratings precision, if a CRA charges a fee above the marginal cost to produce it, a competing CRA will always offer an ϵ less to attract the issuing firms. This iterates until the fee equals the marginal cost.

Suppose that $c \leq c_p$, i.e., intermediate firms exert effort with coarse ratings. Then the magnitude of the subsidy that a good firm provides to intermediate firms in the pool is:

$$p_g({R^P}_{e=1} - R_g) = \frac{\beta(p_g - (p_m + \delta))(1 + f)}{\alpha p_g + \beta(p_m + \delta)} \equiv \sigma_{e=1} \tag{8}$$

where ${R^P}_e$ is the pooling repayment rate for effort, $e \in \{0, 1\}$ (see Equations (16) and (13)). For $\frac{\alpha}{\beta} \geq \gamma$, the coarse-ratings equilibrium arises also for $c > c_p$, but the subsidy is not sufficient to induce the intermediate firms to exert effort. The magnitude of the subsidy in this case, $\sigma_{e=0}$, is derived by setting $\delta = 0$ in Equation (8). That is, we denote the subsidy as σ_e, where $e = 1$ if $c \leq c_p$ and $e = 0$ if $c > c_p$ and $\frac{\alpha}{\beta} \geq \gamma$. We characterize the endogenous fee equilibrium below:

Proposition 9 (Endogenous fee) *With free entry of CRAs and an endogenous fee, the equilibrium is coarse if $k \geq \sigma_e$, and precise if $k < \sigma_e$.*

Proof. The proof is in the Appendix. ■

Whether the equilibrium features coarse or precise ratings depends on which categorization is preferred by the good firms. If the cost to produce precise ratings is small, i.e., $k < \sigma_e$, the coarse-ratings equilibrium does not survive since a new entrant CRA can offer precise ratings to skim the cream and attract only the good firms.[11] Similarly, if $k \geq \sigma_e$, the precise-ratings

[11] Of course, competition ensures that the CRA just recovers its marginal cost of producing precise ratings, i.e., $f = k$.

equilibrium does not survive since the entrant CRA can attract the good firms by offering coarse ratings. Although this deviation is costly for the good firms since it entails subsidizing the intermediate firms, the good firms benefit from the lower fee, $f = 0$, and this benefit exceeds the cost.

As a corollary of Proposition 9, we state the following counter-intuitive result relating the cost of producing information by the CRA to net social welfare.

Corollary 1 *For $c_s < c \leq c_p$, net social surplus is (weakly) increasing in the cost of information production, k.*

For $c_s < c \leq c_p$, net social surplus is maximized under coarse ratings, which arises for $k \geq \sigma_e$. A lower cost, $k < \sigma_e$, leads to precise ratings and a lower net social surplus.

Overall, with free entry and an endogenously determined fee, we have qualitatively similar results to those in the case of competition with exogenously given fee (section 4.2) − coarse ratings may arise with the free entry of CRAs, and for some parameters, competition can lead to precise ratings. Different from the case of competition with an exogenously given fee, we cannot perform a comparative static analysis with respect to the degree of competition. For k sufficiently large, ratings are coarse in equilibrium, and precise otherwise, for any $N \geq 2$. The reason for this result is the classic Bertrand insight that the competition effects are fully revealed with two competitors.

5.2 A more general model

In the baseline model, only firms in one ratings category obtain financing. In this section, we present an illustrative extension in which firms in multiple ratings categories obtain financing. We show that, even in this case, welfare-improving coarseness may arise along the spectrum of ratings.

There are five types of firms and each firm privately knows its type. A fraction α_j of the firms have a positive-NPV project each, with success probability p_j, with $j \in \{1, 2, 3\}$. To keep it comparable with the baseline model, we assume that $p_1 > p_2 > p_3$, $\alpha_j = \frac{1}{3}\alpha \; \forall j$, and $\frac{1}{3}(p_1 + p_2 + p_3) = p_g$. A fraction β of the firms have an intermediate project each; the m projects are positive-NPV only if monitored. A fraction $1 - \alpha - \beta$ of the firms have a bad project each and the bad project has negative NPV. Like m firms, p_2 and p_3 firms can exert hidden effort at cost, c, to increase the success probability by δ, with $p_j + \delta < p_{j-1}$. c is such that p_2 firms only exert effort if sufficiently subsidized (e.g., through pooling with the adjacent higher type)[12]:

$$\delta X - \frac{\delta(1+f)}{p_2 + \delta} < c \leq \delta X - \frac{2\delta(1+f)}{p_1 + p_2 + \delta} \tag{9}$$

We make an additional assumption in this version of the model which is that p_1 and p_2 firms have outside options, ω_j, with $\omega_1 > \omega_2$. We impose the following parametric restrictions to reduce the number of cases to be considered:

$$\frac{1}{2}(p_1 + p_2)X - 1 - f > \omega_1 \geq \frac{1}{3}(p_1 + p_2 + p_3 + 2\delta)X - 1 - f \tag{10}$$

$$\omega_2 \geq \frac{1}{2}(p_2 + p_3 + \delta)X - 1 - f \tag{11}$$

To keep things as simple as possible, the outside option of p_3 firms and lower firm-types are normalized to 0. Finally, we consider the case that $\frac{\alpha_3}{\beta} \geq \frac{1 - p_m X}{p_3 X - 1}$, implying that the average NPV across the p_3 and m firms is positive.

Due to the outside options of p_1 and p_2 firms, it is not feasible to pool either/both of these firm-types with p_3 firms and/or lower types. The CRA is indifferent between the following policies: p_1 and p_2 firms are identified precisely and there is a single pool, $\{p_3, m\}$ or there are two pools, made up of adjacent types, $\{p_1, p_2\}$ and $\{p_3, m\}$. In both cases, bad forms do not

[12] The lower bound on c is derived by replacing p_m with p_2 in c_s (Equation (5)) and the upper bound on c is derived by replacing α with α_1, β with α_2, p_g with p_1, and p_m with p_2 in c_p (Equation (7)).

get rated and the CRA generates a revenue of $(\alpha + \beta)f$. However, the latter is more efficient from a social perspective since p_2 firms exert effort in the latter case and not in the former. We characterize the equilibrium below:

Proposition 10 (Five-type case) *There exists an equilibrium in which there are two coarse ratings categories where rated firms get financing:* $\{p_1, p_2\}$ *and* $\{p_3, m\}$*. The bad firms do not get rated and do not obtain financing. Moreover, this equilibrium is unique if we assume that the CRA chooses the socially efficient outcome in case of indifference.*

Proof. The proof is in the Appendix. ■

The equilibrium described in Proposition 10 is an example which shows that the basic intuition that we developed in the baseline model – ratings coarseness can be welfare improving – extends to more general settings (e.g., more types). This equilibrium arises under certain assumptions regarding firm types and their outside options. These assumptions reduce the number of cases which need to be considered in the five-type model, while still allowing us to present the equilibrium of interest. By relaxing these assumptions, we could obtain equilibria with bigger/same/smaller number of categories and with different composition of types within each category; in many of these equilibria, firms in multiple categories obtain financing and coarseness arises along the spectrum of ratings.

6 Empirical and policy implications

In this section, we discuss the new empirical predictions and policy implications yielded by our model. In stating the implications of our model, we note that precise ratings in our context corresponds to perfect revelation of firm types, which is counterfactual to what is observed in reality. However, this extreme result is an artifact of our assumption that CRAs observe firm types at zero cost.[13] Introducing a convex cost will ensure (for some parameters) that some

[13]This is also unrealistic, but makes our case for coarseness stronger.

coarseness will be retained even under high degrees of competition.

6.1 Empirical implications

1. Ratings precision is countercyclical.

   Suppose that there is one CRA. Both the good and the intermediate firms obtain credit. Suppose that the cost of exerting effort is high, $c > c_p$, such that the intermediate firms do not exert effort. Then, as long as the ratio of good to intermediate firms is high, $\frac{\alpha}{\beta} \geq \gamma$, the coarse-ratings equilibrium obtains. However, as the ratio falls below γ, the coarse-ratings equilibrium is no longer viable since it will be characterized by no financing for any firms. In this case, the CRA will offer precise ratings to separate the good firms from the intermediate firms. Therefore, ratings become more precise as $\frac{\alpha}{\beta}$ falls. Assuming the ratio of good firms to intermediate firms is high in an economic boom and low in an economic downturn, it follows that the precision of ratings will be higher in downturns than in booms, as discussed earlier. The prediction is consistent with the findings of Griffin and Tang (2012). Bar-Isaac and Shapiro (2013) have a similar result but it is driven by different forces – the source of imprecision in their model is costs associated with higher precision, while in our model imprecision may arise even if the cost of higher precision is 0. As a result, our policy implications are different.

2. Higher competition among CRAs leads to more precise ratings.

   The prediction comes directly from Proposition 7. The prediction is consistent with empirical findings in Doherty et al. (2012) and Kisgen and Strahan (2010). Doherty et al. (2012) find that the entry of S&P into the market for insurance ratings previously covered by a monopolist, A.M. Best, led to more precise information since S&P offered ratings which attracted the higher quality issuers within each of the categories of A.M. Best. In Kisgen and Strahan (2010), the entry of a fourth agency, Dominion Rating Agency Service, leads to significant changes in the pricing of bonds, suggesting that the rating by the new entrant carries information over and above that provided by the incumbents, i.e., information precision is higher.

3. Higher competition among CRAs leads to lower investment.

   Building on the above prediction, since higher competition leads to precise ratings, the intermediate firms do not obtain financing in bond markets. Thus, increased competition can lower the level of aggregate investment. To the best of our knowledge, this prediction is yet to be empirically tested.

4. A lower cost of information production by CRAs leads to lower investment.

   This prediction comes from Proposition 9. For a higher cost of information production ($k \geq \sigma_e$), ratings are coarse, which drives out intermediate firms and leads to lower aggregate investment. An ideal test of this prediction would involve a shock which lowers the information production costs of CRAs, but not the market. This prediction is yet to be empirically tested.

6.2 Policy implications

What should a social planner do with respect to competition among CRAs?

1. For $c_p \geq c > c_s$, competition should be restricted to ensure that the coarse-ratings equilibrium arises.

2. For $c > c_p$ and $\frac{\alpha}{\beta} \geq \gamma$, competition should be encouraged to ensure that the precise-ratings equilibrium arises.

For $c_p \geq c > c_s$, the coarse ratings equilibrium is identical to the first-best, in terms of welfare. Excessive competition can destroy this equilibrium, which is why, for these parameters, competition should be restricted. Several papers make the case against inter-CRA competition due to the fact that it encourages ratings shopping. However, as noted by Spatt (2009), ratings shopping is a concern only if issuers can cherry-pick which ratings to disclose to the market. In the US, cherry-picking ratings is not always possible since Standard and Poor's and Moody's rate all taxable corporate bonds, regardless of whether the issuer pays the fee. In our model, we derive a new dark side of competition in the CRA industry. Competition can be harmful as it eliminates the socially desirable coarse ratings equilibrium and reduces net social surplus.

For $c > c_p$ and $\frac{\alpha}{\beta} \geq \gamma$, low levels of competition can allow the existence of the coarse-ratings equilibrium. However, for these parameters, the intermediate firms obtain finance, but do not exert effort. To eliminate this undesirable equilibrium, the regulators can encourage more competition. For N sufficiently large, the coarse-ratings equilibrium collapses, and ratings become precise.

7 Conclusion

We present a model of credit ratings in which coarse ratings may arise as an equilibrium outcome and ratings precision is countercyclical. Compared to the precise-ratings case, coarse ratings introduce asymmetric information by forcing a pooling equilibrium. In existing models, the pooling generated by ratings coarseness is undesirable since it leads to inefficient investment (e.g., Goel and Thakor 2015). However, given the informational frictions we consider, the pooling equilibrium can enhance net social surplus. The pooling equilibrium leads to cross-subsidization across firm types, with the intermediate firms benefiting from more favorable terms due to pooling with the good firms. The cross-subsidization induces the intermediate firms to exert effort, when they would not have done so if their securities were accurately priced. Thus, our model delivers "a bright side of coarseness" – it can increase net social surplus. We believe ours is the first paper to show that ratings coarseness can be better for welfare than precision even when the constraints of the environment permit greater precision.

We derive the welfare properties of the coarse-ratings equilibrium vis-á-vis the precise-ratings equilibrium. We show that net social surplus is not monotonically increasing in information precision. Which equilibrium produces higher net social surplus depends on the deep parameters of the model. Since CRAs offer coarse ratings in a low-competition environment and precise ratings in high-competition environment, the regulator can achieve the efficient outcome by influencing competition among CRAs. Finally, we obtain a counter-intuitive result that a lower cost of information production by CRAs can lead to lower investment and lower welfare.

Appendix: Omitted proofs

Proof of Proposition 2.

Proof. In order to satisfy the investors' participation constraint, the interest rate offered by good firms must be $R \geq R_g$. Suppose that the good firms obtain financing in equilibrium at interest rate, $R = R' > R_g$. A good firm will deviate by offering an infinitesimally lower interest rate, $R' - \epsilon$, and this will be accepted by the investors since they are strictly better off compared to their outside option for any $R > R_g$. By this argument, no interest rate other than $R = R_g$ survives in equilibrium.

For $c \leq c_s$, the intermediate-type firms exert effort, given interest rate, $R = R_m$. Following the same arguments as above, there are no deviations from the $R = R_m$ contract. For $c > c_s$, the intermediate firms will exert effort only if $R < R_m$. However, this implies that investors' participation constraints are violated. Also, when the intermediate firms do not exert effort, their project is negative-NPV (Assumption *A1*). Hence, for these parameter values, the intermediate firms do not receive financing. ■

Proof of Proposition 3.

Proof. We first show that an equilibrium with financing cannot exist. For any interest rate, $R < X$, all three types of firms are strictly better off if they obtain financing, compared to the case in which they do not obtain financing. Hence, all firm types seek financing. By Assumption *A3*, the expected NPV across all three firm types is negative. Thus, the contract will be loss-making for the investors, and hence, investors will not offer financing.

Consider now the case that that the market breaks down in equilibrium, i.e., no firm obtains financing. A firm may deviate by offering a contract with a promised repayment, $R < X$. Regardless of the firm type, this deviation, if the offer is accepted, makes the deviant firm strictly better off compared to the no-financing equilibrium. Exactly because this deviation makes all types of firms strictly better off, the Intuitive criterion does not have a bite (step 1 does not rule out any type as a potential defector), so there exists a strictly positive set of beliefs that the offer comes from a bad firm. If the deviating firm is bad, the expected payoff for the investor is negative, so such an offer will be rejected. Thus, the market breakdown equilibrium is the unique equilibrium which survives the Intuitive criterion and is consistent with zero expected profits for investors. ■

Proof of Proposition 4.

Proof. With coarse ratings, the debt is priced as being issued by a pool consisting of intermediate and good firms. Suppose that the intermediate firm exerts effort, i.e., $e = 1$. Given a pooling interest rate, ${R^P}_{e=1}$, the zero profit condition of the investors is:

$$\frac{\alpha}{\alpha+\beta} p_g {R^P}_{e=1} + \frac{\beta}{\alpha+\beta}(p_m+\delta){R^P}_{e=1} - 1 - f = 0 \tag{12}$$

Solving, we derive the interest rate if the ratings are coarse and the intermediate firms exert effort:

$${R^P}_{e=1} = \frac{(\alpha+\beta)(1+f)}{\alpha p_g + \beta(p_m+\delta)} \tag{13}$$

The pooling interest rate, ${R^P}_{e=1}$, is consistent with the beliefs of investors that the intermediate firm exerts effort if the ${R^P}_{e=1}$ is smaller than $\underline{R}$. ${R^P}_{e=1} \leq \underline{R}$ if:

$$c \leq \delta X - \frac{\delta(\alpha+\beta)(1+f)}{\alpha p_g + \beta(p_m+\delta)} \equiv c_p \tag{14}$$

If the cost of effort is sufficiently small, i.e., $c \leq c_p$, the intermediate firm exerts effort and the equilibrium with $R = {R^P}_{e=1}$ exists. However, for $c > c_p$, the investors' beliefs that the intermediate firms exert effort are not fulfilled which implies that the equilibrium with $R = {R^P}_{e=1}$ cannot exist.

For $c > c_p$, the investors hold the correct belief that the intermediate firms do not exert effort, i.e., $e = 0$. Given a pooling interest rate, ${R^P}_{e=0}$, the zero profit condition of the investors is:

$$\frac{\alpha}{\alpha+\beta} p_g {R^P}_{e=0} + \frac{\beta}{\alpha+\beta} p_m {R^P}_{e=0} - 1 - f = 0 \tag{15}$$

Solving, we derive the interest rate if the ratings are coarse and the intermediate firms do not exert effort:

$${R^P}_{e=0} = \frac{(\alpha+\beta)(1+f)}{\alpha p_g + \beta p_m} \tag{16}$$

Whether the coarse ratings equilibrium with ${R^P}_{e=0}$ exists or not depends on the distribution of firms, as we show below.

If the ratio of good to intermediate firms is sufficiently large, i.e., $\frac{\alpha}{\beta} \geq \gamma$, then the average NPV of the pool consisting of good and intermediate firms is positive, and investors can recover their funds, in expectation (see Equation (6)). As a result, the equilibrium in which both good and intermediate firms obtain credit and the intermediate firms do not exert effort can exist. Consider the case that ratings are coarse and the repayment rate is $R = {R^P}_e$. Suppose that a firm deviates and offers a lower interest rate, $R < {R^P}_e$. Such a deviation, if the offer is accepted, makes both the good and the intermediate firms strictly better off. This implies that the Intuitive criterion does not have a bite and there exists a strictly positive set of beliefs that the offer comes from an intermediate firm. If the deviating firm is intermediate, the expected payoff for the investor is negative, so such an offer will be rejected. Thus, the conjectured equilibrium is indeed an equilibrium.

If $\frac{\alpha}{\beta} < \gamma$, then the average NPV of the pool consisting of good and intermediate firms becomes negative which implies that investors cannot break even at any interest rate $R \leq X$. Using the identical arguments as in the proof of Proposition 3, the market breakdown equilibrium is the unique equilibrium which survives the Intuitive criterion and is consistent with zero expected profits for investors. ■

Proof of Lemma 1.

Proof. It is never an equilibrium for all firm types to be given the same rating since this does not produce any information and is equivalent to the no-ratings case. Thus, from Proposition 3, if all firm types are given the same rating the market breakdown equilibrium is the unique equilibrium.

Given that there are three firm types, coarse ratings in our model always entail two categories and can take the following forms:

1. The good and the intermediate firms are pooled together in a single category, and the bad firms are in a separate category.

2. The intermediate and bad firms are pooled together in a single category, and the good firms are in a separate category.

3. The good and bad firms are pooled together in a single category, and the intermediate firms are in a separate category.

Note that option 2 above is Pareto-dominated by the precise-ratings equilibrium. For $c \leq c_s$, only the good firms obtain financing under option 2. This is a worse outcome than the precise-ratings equilibrium since, for these parameters, the intermediate firms obtain financing and exert effort under precise ratings. For $c > c_s$, option 2 becomes identical to the precise-ratings equilibrium.

Similarly, option 3 above is Pareto-dominated by option 1. Given Assumption *A4*, the average NPV of the pool consisting of good and bad firms is negative, so neither good or bad firms obtain financing under option 3. Also, intermediate firms obtain financing only for $c \leq c_s$. Whereas in option 1, both the good and intermediate firms obtain financing for a larger set of parameters, i.e., $c \leq c_p$ where $c_p > c_s$ (see Lemma 2). ■

Proof of Lemma 2.

Proof. Using Equations (5) and (7):

$$\begin{aligned}
& c_p > c_s \\
\implies & \delta X - \frac{\delta(\alpha+\beta)(1+f)}{\alpha p_g + \beta(p_m+\delta)} > \delta X - \frac{\delta(1+f)}{p_m+\delta} \\
\implies & \frac{1}{p_m+\delta} > \frac{\alpha+\beta}{\alpha p_g + \beta(p_m+\delta)} \\
\implies & p_g > p_m + \delta \qquad (17)
\end{aligned}$$

The above condition is always satisfied due to Assumption *A2*. ■

Proof of Proposition 5.

Proof. For $c \leq c_s$, both intermediate and good firms obtain financing and the intermediate firms exert effort, regardless of the whether ratings are precise or coarse (see Propositions 2 and 4). For these parameters, while there are distributional consequences, the net social surplus in the economy is unaffected by whether ratings are precise or coarse.

For $c_p \geq c > c_s$, the intermediate firms do not exert effort if the ratings are precise (and do not obtain financing), while they obtain finance and exert effort with coarse ratings (see Propositions 2 and 4). For these parameters, coarse ratings generate a higher net social surplus than if ratings are precise.

Next, we consider the case of $c > c_p$. With precise ratings, good firms obtain financing and invest, while intermediate firms do not obtain financing (see Proposition 2). The equilibrium with coarse ratings depends on parameters (see Proposition 4). If the ratio of good to intermediate firms is sufficiently high, i.e., $\frac{\alpha}{\beta} \geq \gamma$, the average NPV of the pool of good and intermediate firms is positive, although the intermediate firms do not exert effort. For these parameters, with coarse ratings both good and intermediate firms obtain credit, but investment by the intermediate firms is welfare-reducing (because the intermediate firms' NPV is negative when they do not exert effort). Therefore, precise ratings generate a higher net social surplus than if the ratings are coarse. If $\frac{\alpha}{\beta} < \gamma$, then the average NPV of the pool of good and intermediate firms is negative. Thus, with coarse ratings, none of the firms obtain credit. Putting these together, for $c > c_p$, the precise ratings equilibrium welfare-dominates the coarse ratings equilibrium, for any distribution of firm types. ■

Proof of Proposition 6.

Proof. Suppose that $c > c_s$. With precise ratings, only the good firms obtain financing and pay for ratings, generating a total fee of αf for the ratings agency. Under coarse ratings, the fee is $(\alpha + \beta)f$.

For $\frac{\alpha}{\beta} \geq \gamma$, the average NPV of the good and intermediate firms is positive, whether or not the intermediate firms exert effort. The CRA maximizes its fee revenue by offering coarse ratings. Rational investors hold the correct beliefs in equilibrium and set the interest rate such that they break even, on average. The intermediate firms certainly prefer coarse ratings because they are subsidized by good firms. Regarding good firms, they are worse off with coarse ratings compared to precise ratings, but given that only coarse ratings are offered, they prefer coarse ratings to no ratings. The reason is that if a good firm chooses no rating, it will get no financing (the argument is similar to). Therefore, the good firms will also purchase coarse ratings. The CRA's total fee under coarse ratings, $(\alpha+\beta)f$, exceeds the fee under precise ratings, αf. Hence, for $\frac{\alpha}{\beta} \geq \gamma$, the equilibrium with one CRA features coarse ratings.

If, on the other hand, $\frac{\alpha}{\beta} < \gamma$, coarse ratings imply that there is market breakdown and no firm obtains credit. This leads to a total fee revenue of 0, which is less than the total fee with precise ratings. Hence, for $\frac{\alpha}{\beta} < \gamma$, the equilibrium with one CRA features precise ratings. ■

Proof of Proposition 7.

Proof. Suppose that the ratings agencies issue coarse ratings. The total fee revenue for the industry is $(\alpha + \beta)f$, and in a symmetric equilibrium, each agency gets $\frac{1}{N}(\alpha + \beta)f$. Is there a profitable deviation from this equilibrium? A ratings agency could unilaterally deviate from the conjectured equilibrium by offering precise ratings which will attract all good firms since the good firms will benefit from being separated from the intermediate firms. The deviating agency will raise total fee revenue of αf. Thus, deviation is profitable only if:

$$\begin{aligned} \alpha f &> \frac{1}{N}(\alpha + \beta)f \\ \Longrightarrow \frac{\alpha}{\beta} &> \frac{1}{N-1} \end{aligned} \tag{18}$$

If the above condition does not hold, then the symmetric equilibrium with coarse ratings will sustain. ■

Proof of Proposition 9.

Proof. Suppose that $k > \sigma_e$, where $e = 1$ if $c \leq c_p$, and $e = 0$ if $c > c_p$ and $\frac{\alpha}{\beta} \geq \gamma$. First, we show that precise ratings cannot be an equilibrium. Under precise ratings, firms pay a fee $f = k$. Suppose that a CRA deviates by offering coarse ratings and a lower fee, ϵ, such that $k - \epsilon > \sigma_e$. Then, the good firms find it profitable to deviate since the fall in the fee is higher than the subsidy it would provide by deviating to the coarse-ratings equilibrium, thereby eliminating the precise-ratings equilibrium. Now we show that coarse-ratings is an equilibrium. Under coarse ratings, firms pay a fee $f = 0$. Suppose that a CRA deviates by offering precise ratings. The lowest feasible fee that the CRA charges is $f = k$. Even for this fee, a good firm is worse off with precise ratings since the increase in the fee would be higher than the subsidy it provides, i.e., $k > \sigma_e$. Thus, the deviating CRA cannot attract good firms and the equilibrium sustains.

Suppose that $k < \sigma_e$. First, we show that coarse ratings cannot be an equilibrium. Under coarse ratings, firms pay a fee $f = 0$. Suppose that a CRA deviates by offering precise ratings and a fee, $f = k + \epsilon$, such that $k + \epsilon < \sigma_e$. Then, the good firms find it profitable to deviate since the increase in the fee is lower than the subsidy it provides, thereby eliminating the coarse-ratings equilibrium. Next, we show that precise ratings is an equilibrium. Under precise ratings, firms pay a fee $f = k$. Suppose that a CRA deviates by offering coarse ratings and the lowest feasible fee, $f = 0$. Even for this fee, a good firm is worse off with coarse ratings since the fall in the fee would be lower than the subsidy it would provide, i.e. $k < \sigma_e$. Thus, the deviating CRA cannot attract good firms and the equilibrium sustains. ■

Proof of Proposition 10.

Proof. Given the assumptions, bad firms are never pooled with higher types since the fraction of bad firms is such that including them in a pool leads to no financing for any firm in that pool (same as in the baseline). We consider the feasibility of CRA policies involving the p_1, p_2, p_3, and m (intermediate) firms:

1. Given the outside option of the p_1 firms, the following pools are infeasible: $\{p_1, p_2, p_3, m\}$, $\{p_1, p_2, m\}$, $\{p_1, p_3, m\}$, and $\{p_1, m\}$. If any of these policies are announced by the CRA, the expected profits of p_1 firms fall below their outside option, which implies that they will choose to stay out and not get rated.

2. Given the outside option of the p_2 firms, the following pools are infeasible: $\{p_2, p_3, m\}$, and $\{p_2, m\}$. If either of these policies is announced by the CRA, the expected profits of p_2 firms fall below their outside option, which implies that they will choose to stay out and not get rated.

3. There is a single pool, $\{p_1, p_2, p_3\}$, or each firm-type is identified precisely: under these

policies, p_1, p_2, and p_3 firms choose to get rated. In the pool, p_2 and/or p_3 firms may or may not exert effort depending on parameters. In the precise-ratings case, neither p_2 nor p_3 firms exert effort. Intermediate and bad firms do not get rated and do not obtain financing. CRA revenue is αf.

4. There are two pools made up of adjacent types, $\{p_1, p_2\}$ and $\{p_3, m\}$: under this policy, p_1 and p_2 firms choose to get rated and obtain financing. Also, p_3 and m firms choose to get rated since we consider the case that the pool of p_3 and m firms has a positive NPV. p_2 firms exert effort and m firms may or may not exert effort depending on parameters, although p_3 firms do not. Bad firms do not get rated and do not obtain financing. CRA revenue is $(\alpha + \beta)f$.

5. p_1 and p_2 firms are identified precisely and there is a pool, $\{p_3, m\}$: under this policy, all four firm-types, p_1, p_2, p_3, and m, choose to get rated. However, different from the case above, p_2 firms do not exert effort. Bad firms do not get rated and do not obtain financing. CRA revenue is $(\alpha + \beta)f$.

Of the five alternatives, the revenue-maximizing ones are 4 and 5. The CRA is indifferent between these policies since they both generate a revenue of $(\alpha + \beta)f$, but 4 is the efficient alternative since p_2 firms exert effort in this case. Given the CRA's indifference, either of these policies may be chosen by the CRA. ■

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