Cross-subsidization across projects and investment efficiency: A new theory of financial intermediation

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Abstract

We consider a setting in which a bank seeks financing for two unrelated projects; the first suffers from effort moral hazard, while the second suffers from adverse selection. On-balance sheet financing with cross-subsidization across projects arises as the unique equilibrium and the outcome is efficient (second-best). Although banks do not possess superior skills, bank financing strictly dominates market financing and off-balance sheet financing for all parameter values. Different from the well-known cross-pledging effect, the cross-subsidization benefits in our model are independent of the correlation structure across project returns, which generates new empirical implications.

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1 Introduction

In most existing theories of financial intermediation banks possess special skills in acquiring and processing information and exist to resolve informational frictions which impede financing (e.g., Diamond 1984 Ramakrishnan and Thakor 1984, Allen 1990). The explosion in the development of information technology in recent decades has meant that the comparative advantage of banks in acquiring and processing information compared to direct market financing has shrunk considerably. Does bank financing still retain any advantages over direct market financing? We present a model in which bank financing dominates market financing even though banks do not possess superior skills. The key observation behind our result is the following: In market financing, financiers must make non-negative profits on a project-by-project basis. In contrast, when multiple projects are undertaken under a single umbrella (such as in the case of a bank), this constraint is relaxed, and investors must make non-negative profits across all projects, i.e., there can be cross-subsidization across projects. This cross-subsidization can be value-enhancing since it facilitates more efficient mobilization of limited resources.

We present a model in which there are two investment opportunities for a bank; the first suffers from an effort moral hazard problem, while the second suffers from an adverse selection problem. Regarding the project associated with adverse selection, we show that a fully separating equilibrium always exists in which only good banks invest. Separation arises through the violation of the bad bank's participation constraint rather than through the standard incentive compatibility constraint. In this equilibrium, investors make strictly positive expected profits even though the bankers make a take-it-or-leave-it offer to investors (see e.g., Biswas and Koufopoulos 2022, Bernhardt et al. 2022b, Bernhardt et al. 2022a). Rationally anticipating the positive expected profits from the second project, investors are willing to accept a lower promised repayment for the debt issued to finance the first project (the effort moral hazard project). A lower promised repayment relaxes the bank's effort moral hazard constraint and boosts effort provision in the first project. Our theory is consistent with evidence on the existence of cross-subsidization across unrelated activities in financial institutions (see e.g., Griffin et al. 2007, Santikian 2014, and Jenkinson et al. 2018). Finally, we show that our equilibrium allocation coincides with the planner's solution (i.e., the equilibrium allocation is second-best).

Our model generates a number of interesting testable empirical predictions: First, the spread on debt is lower and bank monitoring intensity is higher for banks with better growth prospects. Second, the spread on debt falls and bank monitoring intensity increases if the macroeconomic outlook improves. Third, the spread on debt also falls in the expected return on the bank's core activities; importantly, after controlling for the mean, the variance of returns on the core activities has no effect on the spread. Finally, the spread on debt falls in the bank's growth opportunities if these are positively correlated with its core activities, while the direction of the effect is not clear if there is a negative correlation.

We contrast our solution against two benchmarks:

The first benchmark is direct financing. Both bank and direct financing deliver full separation in the case of the second project (efficiency), while bank financing delivers a strictly more efficient outcome in the case of the first project. Effort provision in the first project is higher under bank financing since resources from the second project are used to reduce the face value of debt issued for this project, which eases the moral hazard constraint. Under market financing, the profits made by investors on the second project are just a re-distribution of wealth, and they are not used productively. Thus, under direct financing cross-subsidization across projects does not occur, and hence, the solution is inefficient.

The second benchmark comes from Segura and Zeng (2020) who also consider a model very similar to ours. In their paper, they use this setting to provide a rationale for off-balance sheet financing by banks. They consider two modes of financing the first project: on-balance sheet and off-balance sheet with voluntary support. With on-balance sheet financing, investors have unlimited recourse to the bank's existing assets, while with off-balance sheet financing, the bank has the option, but not the obligation, to provide support in case of the first project's failure. Compared to on-balance sheet financing, off-balance sheet financing reduces effort provision in the first project, but allows a good bank to signal the quality of the second project to a greater extent. The very arrangement that relaxes the adverse selection friction in the second project makes the moral hazard friction in the first project more binding since it drives up the repayment in the first project which reduces the bank's incentive to exert effort.

The off-balance sheet solution in Segura and Zeng (2020) is inefficient for three reasons: First, they exogenously assume that investors make zero expected profits in the project associated with adverse selection[1] which leads to either a partial pooling equilibrium in which some bad banks participate which destroys value or a market breakdown equilibrium with no financing at all. Second, in this equilibrium the bank uses all cash flows generated by assets-in-place for signaling purposes (money-burning) which makes the effort moral hazard problem more binding and leads to lower effort provision. In contrast, we do not impose this assumption and we show that there exists a unique equilibrium which is always separating where only good banks invest (efficient outcome). In this equilibrium, outside investors make strictly positive profits (in fact, they extract the full NPV of the second project). Third, our solution increases effort provision even further because competition among investors implies that the profits they expect to make in the second project are used to reduce the face value of debt issued for the project associated with moral hazard which relaxes the moral hazard constraint. Hence, given the informational frictions in this model, the off-balance sheet financing mode is inefficient and does not maximize bank profit. Therefore, a rational profit-maximizing bank will never choose it.

The solution of Segura and Zeng (2020) can be interpreted as a form of disintermediation since the first project is moved off-balance sheet. In sharp contrast, our solution suggests that their setting is unsuitable to provide an explanation for the use of off-balance sheet financing. Our analysis implies that which combination of frictions rationalizes off-balance sheet financing with voluntary support remains an open question.

In most existing theories, financial intermediaries have special skills in acquiring or processing information which facilitates financing.[2] For example, Diamond (1984), Von Thadden (1995), and Holmstrom and Tirole (1997) emphasize the role of banks as monitors who positively affect the success probability, while Ramakrishnan and Thakor (1984) and Allen (1990) focus on the screening role of banks in which banks can distinguish between good and bad borrowers. In these models, banks have a monitoring or screening cost advantage over non-intermediated outcomes (depending on the model, this advantage is either assumed, or it arises as an equilibrium outcome). In contrast, banks in our model

[1]This is an unjustifiable assumption since, in a game, one cannot assume that competitive investors make zero expected profits in equilibrium. It must be proved.

[2]There is a separate strand of the literature which focuses on the liquidity creation/provision role of banks (see e.g., Diamond and Dybvig 1983, Gorton and Penacchi 1990, Diamond and Rajan 2001, and Donaldson et al. 2018).

do not have superior skills in resolving any informational frictions, yet they generate value through two novel channels:

First, in our model an efficient separating equilibrium arises through socially costless signaling where as in existing banking theories separation is achieved through costly information production. Second, in our model there is cross-subsidization across projects which is absent in existing models: the solution of the adverse selection problem requires that investors make strictly positive expected profits which are transferred to relax the moral hazard constraint. Coval and Thakor (2005) also do not assume that banks have special information processing skills; rational banks form a beliefs-bridge between optimistic entrepreneurs and pessimistic investors. In our theory, all agents are rational, so we explore a different role of banks than the one considered by Coval and Thakor (2005).

Improved effort incentives due to cross-subsidization across projects is not a new idea. In Tirole (2006), cross-pledging across imperfectly correlated projects subject to effort moral hazard leads to greater effort provision by the manager (see also Diamond 1984, Cerasi and Daltung 2000, Laux 2001, Axelson et al. 2009, and Maurin et al. 2023) – the optimal contract entails that the bank is only compensated when all projects succeed; this relaxes the limited liability constraint of the agent and induces the efficient level of effort provision. The cross-pledging mechanism relies on exploiting the diversification effect resulting from imperfectly correlated projects and it is ineffective if projects are perfectly correlated. Our mechanism works differently – the bank effectively pledges the expected cash flows from the second project to the first, which reduces the cost of external financing, thereby boosting effort provision. Thus, in contrast to existing models, the correlation structure across projects does not play a role which implies that the cross-subsidization channel in our model is distinct from the well-known diversification effect.

2 Model

2.1 Set-up

There are four dates $t \in \{0, 1, 2, 3\}$. There is a bank which has no funds of its own at $t = 0$ and obtains a pay-off from assets-in-place at $t = 1$. The bank has two investment opportunities, the first at $t = 0$ and the second at $t = 2$; each project generates a payoff in the period following the investment. The funds are raised from competitive external investors. All agents are risk-neutral and the discount rate is set to 0.

The first project is always good (g), while the second project may be either good or bad (b). In either case, a project produces R per unit of investment if it succeeds and zero if it fails. A project succeeds with probability p_i. We assume that only good projects are profitable:

A1: $p_g R > 1 > p_b R$

The first project to be undertaken at $t = 0$ is of scale 1 unit and is subject to effort moral hazard. Its failure may be due to systematic reasons, with probability $q < 1 - p_g$, or due to idiosyncratic reasons, with probability $1 - p_g - q$. While the probability of systematic failure is beyond the bank's control, the bank can exert unobservable effort $e \in [0, 1]$ at a cost $c(e)$ to reduce the probability of idiosyncratic failure. Taking into account the bank's effort provision, the first project's success probability is $p_g + me$ and idiosyncratic failure occurs with probability $1 - p_g - q - me$, with $m \leq 1 - p_g - q$ to ensure non-negative probability of idiosyncratic failure. m is a constant which is interpreted as the marginal value of effort. The cost of effort $c(e)$ satisfies:

A2: i. $c(0) = 0$, ii. $c'(0) = 0$ and $c'(1) > mR$ and iii. $c''(e) > \left(\frac{m}{p_g}\right)^2$

It is deemed the systematic state if the first project fails due to systematic reasons (state $\sigma = S$) and non-systematic state if either the first project succeeds or it fails due to idiosyncratic reasons (state $\sigma = \bar{S}$). At $t = 1$ the state is publicly observed. The bank obtains pay-off Y_σ at $t = 1$ from the assets-in-place; we assume $Y_S = Y_{\bar{S}} = Y$.[3] We assume that Y is not so large such that the $t = 0$ debt becomes riskless, as otherwise the first-best effort level could be implementable:

A3: $Y < 1 - (1 - q + \alpha q)(p_g R - 1)$

At $t = 1$, the bank privately learns whether the second project is good or bad. The second project to be undertaken at $t = 2$ is of scale I units, which we normalize to 1 unit. It is good with certainty in the non-systematic state. In the systematic state, it is of type g with probability $\alpha \in (0, 1)$. In this case, the bank privately observes the project's type. We assume that α is small such that the average NPV of the pool is negative:

[3] Our results go through qualitatively if we assume that $Y = 0$, but we keep it in the model to facilitate comparisons of our solution with related benchmarks.

A4: $\alpha < \frac{1-p_b}{p_g R - p_b R}$

The timeline is illustrated in Figure 1.

$t=0$	$t=1$	$t=2$	$t=3$
Bank raises financing for the 1st project. After receiving financing, the bank chooses effort level.	Assets-in-place generate cash flows, Y. State σ is publicly observed. 1st project returns realize. Bank privately learns second project's type.	Bank seeks financing for the 2nd project. Investors provide funds if they believe project is good.	2nd project returns realize.

Figure 1: Timeline

2.2 The Game

We consider a two-stage sequential game, the second stage of which itself is a standard two-stage signaling game.

- Stage 1 (which occurs at $t = 0$): the bank (ignorant of its future type) seeks to raise 1 unit of funds immediately to invest in the first project, and potentially, additional funds at $t = 2$ to invest in the second project. Competitive investors propose the following contract to the bank: $(D_1, K, j, l, D_{3,\bar{S}}(j), D_{3,S}(l))$ where $D_1 \leq R$, $K \geq 0$, $j \in \{0, 1\}$, $l \in \{0, 1\}$, and $D_{3,\sigma} \leq R$. $j = 1$ (resp. $l = 1$) indicates that the contract includes a provision for financing the second project in the idiosyncratic state (resp. systematic state) and if investors offer funds at $t = 2$ then the bank must accept it. $j = 0$ (resp. $l = 0$) indicates that there is no provision. The contract states that investors get D_1 if the first project succeeds. If the first project fails, investors get $Y - K$ in the systematic state and Y in the idiosyncratic state. In relation to the second project, suppose that $l = j = 1$, i.e., investor may offer financing for the second project. In the idiosyncratic case, if investors provide 1 unit to the banks, there is investment in the second project and the promised repayment is $D_{3,\bar{S}}(j = 1)$. In the systematic state, the bank either invests K directly in the second project or consumes it. If the bank invests K in the second project, then investors need to provide an additional $1-K$ to undertake the project. Conditional on investors providing funds to undertake the second project, they receive a promised repayment, $D_{3,S}(l=1)$. If $j=0$ (resp. $l=0$), then $D_{3,\bar{S}}=0$ (resp. $D_{3,S}=0$). In this case, the bank will try to raise financing for the second project using a different contract playing a new game as if the funding of the two projects is unconnected. The game will be identical to the two-stage signaling game which follows.

- Stage 2a (which occurs at $t=2$): the bank (now aware of its type) seeks to raise funds to invest in the second project. It can use K either to invest directly in the second project or consume it, which sends a signal about its type.

- Stage 2b: given the realization of the state, the terms of the contract signed in the first stage of the game, and the use of K by the bank if it is the systematic state, uninformed investors form beliefs about the bank's type and decide whether they will provide funds or not.

We look for the pure strategy Perfect Bayesian equilibria of this game that satisfy the Intuitive Criterion of Cho and Kreps (1987).[4] Note that we do not impose exogenously whether investors offer funds for both projects at $t=0$ or only the first project at $t=0$. The latter is the special case in which $K=0$, $l=j=0$, and $D_{3,\sigma}=0$.

2.3 First-best

In the first-best, the effort level exerted by the bank is observable and verifiable, and the type of the second project is publicly known. In this case, the bank always undertakes the first project and, given Assumption 2, the level of effort is:

$$c'(e^{FB}) = mR \tag{1}$$

The bank invests in the second project only if it is good.

[4]The appropriate solution concept is Perfect Bayesian since in the second stage of the game banks may signal about their type through the use of K, which implies that inferences are made.

2.4 Equilibrium

The two key dates of strategic interaction between the bank and investors are $t = 0$ and $t = 2$ when the bank raises funds for the projects. We solve for the equilibrium using backward induction.

2.4.1 Financing at $t = 2$

In the non-systematic state, it is common knowledge that the project is good. If the initial contract has provision for financing the second project in the non-systematic state, i.e. $j = 1$, then investors provide funds and the face value of debt $D_{3,\bar{S}}$ is determined in stage 1 of the game. If the initial contract does not include provision for financing the second project in the non-systematic state, i.e. $j = 0$, then the bank can obtain financing for this project using a different contract written at $t = 2$. In this case, the new contract will price the debt competitively such that the bank extracts the full surplus from this project.

The more interesting case is the systematic state in which there is an asymmetric information problem. First, consider the case that the initial contract has provision for financing the second project in the systematic state, i.e. $l = 1$. Below, we list all the candidate equilibria of the subgame which starts at $t = 2$ (Stage 2a), given the terms of the contract signed in Stage 1 of the game:

1. A candidate separating equilibrium in which only bad banks obtain financing.

2. A candidate pooling equilibrium in which both firm types obtain financing.

3. A candidate pooling equilibrium in which neither firm type obtains financing (market breakdown).

4. A candidate separating equilibrium in which both good and bad banks obtain financing, but at different terms.

5. A candidate separating equilibrium in which only good banks obtain financing.

Below we consider each candidate equilibrium.

Lemma 1 *There cannot exist a separating equilibrium in which only bad banks obtain financing or both good and bad banks obtain financing but at different terms.*

Proof. The proof is in the Appendix. ■

Lemma 2 *There cannot exist a pooling equilibrium in which both good and bad banks obtain financing.*

Proof. The proof is in the Appendix. ■

Suppose that the contract in the first stage specifies a pair, $(K, D_{3,S})$. Bank i's profit is denoted Π_i and its participation constraint is given by:

$$\Pi_i = p_i(R - D_{3,S}) - K \geq 0 \tag{2}$$

We derive the slope of $\Pi_i = 0$ in the $(K, D_{3,S})$ space by totally differentiating Equation (2) with respect to $D_{3,S}$ and K:

$$\left.\frac{dD_{3,S}}{dK}\right|_{\Pi_i=0} = -\frac{1}{p_i} < 0 \tag{3}$$

Investors facing a type i bank make an expected profit which is denoted γ_i, and they provide the necessary funds for investment if $\gamma_i \geq 0$:

$$\gamma_i = p_i D_{3,S} - (1 - K) \geq 0 \tag{4}$$

We derive the slope of $\gamma_i = 0$ in the $(K, D_{3,S})$ space by totally differentiating Equation (4) with respect to $D_{3,S}$ and K:

$$\left.\frac{dD_{3,S}}{dK}\right|_{\gamma_i=0} = -\frac{1}{p_i} < 0 \tag{5}$$

In Figure 2, we plot the participation constraints of banks (red lines) and investors (blue lines) in the $(K, D_{3,S})$ space. Both $\Pi_i = 0$ and $\gamma_i = 0$ have a negative slope and are linear in the $(K, D_{3,S})$ space. Additionally, since the slopes are the same, $\Pi_i = 0$ and $\gamma_i = 0$ do not cross for any i. $\Pi_i = 0$ implies that $D_{3,S} = R$ for $K = 0$ and $D_{3,S} = R - \frac{1}{p_i}$ for $K = 1$. $\gamma_i = 0$ implies that $D_{3,S} = \frac{1}{p_i}$ for $K = 0$ and $D_{3,S} = 0$ for $K = 1$. A bank of type i is willing to undertake the project in the region below its participation constraint, while an investor facing a bank of type i is willing to provide funds in the region above the corresponding participation constraint.

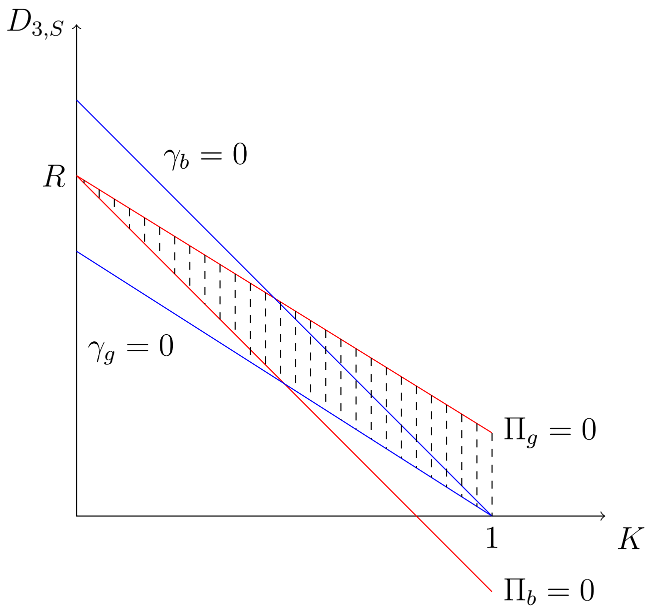

Figure 2: Feasible equilibria

Lemma 3 *Depending on the terms of the contract set in Stage 1 of the game, there either exists a pooling equilibrium in which neither bank obtains financing (market breakdown) or a financing equilibrium in which only good banks invest.*

Proof. We present a graphical proof and refer to Figure 2.

There exists a financing equilibrium in the shaded region: here, investors' participation constraints are slack and they make positive profits when they face good banks (i.e., $\gamma_g > 0$), a good bank's participation constraint is satisfied (i.e., $\Pi_g \geq 0$), while a bad bank's participation constraint is (weakly) violated (i.e., $\Pi_b \leq 0$). Financing is infeasible above the shaded region since the good bank's participation constraint is violated. Below the shaded region, the market breaks down since either the bad bank's participation constraint is satisfied and/or investors' participation constraint is violated.

Both the breakdown equilibrium and the separating equilibrium with only good banks participating are feasible along the bad bank's participation constraint bordering the shaded region up to $(K = 0, D_{3,S} = R)$. For these parameters, a bad bank is indifferent between participating or not which implies that the breakdown equilibrium cannot be ruled out for these parameters in this subgame. ■

Therefore, the subgame at $t = 2$ potentially has two equilibria: one is the separating equilibrium in which only good banks invest and another in which no banks invest (market breakdown). Below, we show that in the whole game the only equilibrium that survives is the separating equilibrium in which only good banks invest.

In Lemmas 1, 2, and 3, we have considered the case that the initial contract has provision for financing the second project in the systematic state, i.e. $l = 1$. Suppose, instead, that $l = 0$. The equilibrium set is exactly the same as the $l = 1$ case. The results in Lemmas 1, 2, and 3 still hold. The arguments need modifying because in this case $D_{3,S}$ may be chosen in stage 2A of the game. If $K \in (0, 1]$, because $D_{3,S}$ can be chosen, there exists a unique separating equilibrium in which only good banks invest. If $K = 0$, there may also exist the market breakdown equilibrium.

2.4.2 Financing at $t = 0$

At $t = 0$, investors offer a contract to the bank which specifies the terms and conditions under which they will provide the bank with the funds required for investment in the first, and potentially, the second project.

If the first project succeeds, investors obtain the promised repayment, D_1. In the case of idiosyncratic failure, they obtain the cash flow from the assets-in-place, Y, while in the case of systematic failure, investors obtain $Y - K$ and the bank retains K. In the idiosyncratic state (whether there is success or failure), it is common knowledge that the second project is good. The contract offered at $t = 0$ may provide all the funds necessary to undertake the second project in return for the promised repayment, $D_{3,\bar{S}}$. In the systematic state, the bank privately knows the second project's type to be good or bad. If the pair, $(K, D_{3,S})$, violates the bad bank's participation constraint while satisfying the good bank's one (see Figure 2), investors will be convinced that the bank investing K (instead of consuming it) has a good project, and will provide this bank with the remaining $(1 - K)$ units to undertake the second project in return for the promised repayment, $D_{3,S}$, as long as they make weakly positive profits (i.e., $\gamma_g \geq 0$). Given the contract offered, investors' zero profit condition at $t = 0$ is as follows:

$$
\begin{aligned}
&(p_g + me)D_1 + (1 - q - p_g - me)Y + q(Y - K) \\
&+ (1 - q)(p_g D_{3,\bar{S}} - 1) + \alpha q(p_g D_{3,S} - (1 - K)) = 1
\end{aligned} \tag{6}
$$

From the zero profit condition, we derive the repayment, ${D_1}^*$. The expected return of investors consist of two parts. The first part (top line) is the expected repayment from the first project. The second part (second line) is investors' net expected profits from the second project. This expected profit lowers the face value of debt, D_1.

The bank's expected profits is:

$$
\begin{aligned}
\Pi_1 =& (p_g + me)(R - {D_1}^* + Y) - c(e) + \\
& (1 - q)p_g(R - D_{3,\bar{S}}) + \alpha q p_g(R - D_{3,S}) + (1 - \alpha)qK
\end{aligned} \tag{7}
$$

The top line is the expected profit from the first project, net of the cost of effort. The second line is the profit from the second project depending on whether the state is idiosyncratic or systematic and the bank's use of K in the systematic state. The bank maximizes its expected profit from the first project by choosing the level of effort. Taking the first order condition with respect to the effort level, solving the investor's zero profit condition (Equation (6)) for D_1, and substituting it into the first order condition, we obtain the equilibrium effort level exerted by the bank:

$$
\begin{aligned}
c'(e^*) &= m(R - {D_1}^* + Y) \\
&= m\left(R - \frac{1 - Y}{p_g + me} - \frac{(1 - \alpha)qK - \alpha q(p_g D_{3,S} - 1) - (1 - q)(p_g D_{3,\bar{S}} - 1)}{p_g + me}\right)
\end{aligned} \tag{8}
$$

Because $c(e)$ is strictly convex in e, $c'(e)$ is strictly increasing in e. Since the right hand side of Equation (8) is falling in K and increasing in $D_{3,\sigma}$, it implies that the level of effort is maximized by setting K as small as possible and $D_{3,\sigma}$ as large as possible. The intuition is as follows: The bank exerts a higher level of effort in the first project if it retains more of the surplus generated due to effort which relaxes the effort moral hazard constraint. A lower direct bank investment in the second project and a high promised repayment for the debt issued at $t = 2$ boosts investors' expected profits in that project. Anticipating the expected profits, investors accept a lower promised repayment in the first project. By maximizing investors' expected profits in the second project, the promised repayment on the first project is reduced to the maximum extent possible.

Below, we present a series of lemmas which allows us to characterize the equilibrium in Proposition 1.

Lemma 4 *There cannot exist an equilibrium in which the equilibrium contract offers financing only for the first project, i.e., $j = 0$ and/or $l = 0$ cannot be an equilibrium. In any equilibrium, the contract must specify $j = l = 1$.*

Proof. The proof is in the Appendix. ■

$j = 1$ (resp. $l = 1$) implies that the $t = 0$ contract includes provision for financing the second project if the idiosyncratic (resp. systematic) state arises at $t = 2$. The intuition behind Lemma 4 is that if the initial contract includes provision for financing the second project, transfer of profits from the second project to the first relaxes the effort moral hazard in the first project. Given competition among investors, the bank captures all of the surplus created from the additional effort, which implies that the choice of j and l in equilibrium must maximize the surplus, i.e., $j = l = 1$.

Lemma 5 *From the $t = 0$ perspective, there cannot exist an equilibrium in which a good project at $t = 2$ does not obtain financing.*

Proof. The proof is in the Appendix. ■

Starting from a non-financing equilibrium, an investor can profitably deviate by offering a new pair, $(K, D_{3,S})$, which satisfies the good bank's and investors' participation constraints and violates the bad bank's participation constraint at $t = 2$.

Lemma 6 *There cannot exist an equilibrium in which the contract specifies $K > 0$. In any equilibrium, the contract must specify $K = 0$ and $D_{3,S} = R$.*

Proof. The proof is in the Appendix. ■

Bank profit is maximized for the smallest possible K (from Equation (8)). Competition among investors drives K to 0 and $D_{3,S}$ to R. For any $K > 0$ (and corresponding $D_{3,S}$ on the good bank's participation constraint), there exists a deviation in which investors offer a smaller K to move up along the good bank's participation constraint (see Figure 2), increase their expected profits in the second project, and use it to lower D_1 which, in turn, relaxes the bank's effort moral hazard constraint and increases bank value. Note from Lemma 3, $(K = 0, D_{3,S} = R)$ can potentially lead to two equilibria in the second stage subgame: one is the separating one considered above and the other is the market breakdown equilibrium. But from the perspective of $t = 0$, the market breakdown equilibrium cannot exist due to the possibility of profitable deviations. Thus, the separating equilibrium with $(K = 0, D_{3,S} = R)$ is the unique equilibrium of the full game. While K equals 0 on the equilibrium path, it is the off-path threat of positive K that sustains the equilibrium.

Lemma 7 *There cannot exist an equilibrium in which the contract specifies $D_{3,\bar{S}} < R$. In any equilibrium, the contract must specify $D_{3,\bar{S}} = R$.*

Proof. The proof is in the Appendix. ■

Competition among investors sets $D_{3,\bar{S}} = R$. For any $D_{3,\bar{S}} < R$, an investor can deviate by increasing $D_{3,\bar{S}}$ and using it to lower D_1 which, in turn, relaxes the bank's effort moral hazard constraint and increases bank value.

Substituting the equilibrium values, $(K = 0,\ D^*_{3,\sigma} = R)$ in Equation (6) we derive the promised repayment in the first project, D^*_1:

$$D_1{}^* = \frac{1 - (1 - p_g - me)Y}{p_g + me} - \frac{(1 - q + \alpha q)(p_g R - 1)}{p_g + me} \tag{9}$$

Lemmas 4–7 rule out all candidate equilibria, but one: $(D_1 = D_1{}^*,\ K = 0,\ j = l = 1,\ D^*_{3,\sigma} = R)$. Now we need to show that there is no profitable deviation from this equilibrium. To see why this is the case, note that any deviation would have to increase bank profits by relaxing the moral hazard constraint further. To do so, D_1 must be lower than $D_1{}^*$. However, since the full surplus from the second project is already used to minimize D_1 and investors are on their participation constraints, $D_1 < D_1{}^*$ is not feasible., Thus, this candidate equilibrium exists. Using $D_1 = D_1{}^*$, $K = 0$, and $D^*_{3,\sigma} = R$ in Equation (8), the equilibrium effort level is given by:

$$c'(e^*) = m\left(R - \frac{1 - Y}{p_g + me} + \frac{(1 - q + \alpha q)(p_g R - 1)}{p_g + me}\right) \tag{10}$$

Given Assumption *A3*, $D^*_1 > Y$, implying that the debt is risky. In turn, this prevents equilibrium level of effort from reaching the first-best level, i.e., $e^* < e^{FB}$. We characterize the equilibrium in Proposition 1.

Proposition 1 *There exists a unique equilibrium in which the contract specifies* $(D_1 = D_1{}^*,\ K = 0,\ j = l = 1,\ D^*_{3,\sigma} = R)$ *at* $t = 0$. *There is full separation in the second project and only good banks participate. Effort provision in the first project is given by* $c'(e^*)$.

The $t = 0$ contract in Segura and Zeng (2020) does not include provision for financing the second project. Their contract corresponds to the $j = l = 0$ case, which we allow for in our analysis, and show that it is off the equilibrium path. That is, a rational profit-maximizing bank will not choose $j = 0$ and/or $l = 0$. Also note that the contract in Proposition 1 is renegotiation-proof. For a contract to be renegotiated, it is necessary that both parties are willing to renegotiate it at an interim date ($t = 2$, in this model). At $t = 2$, good banks would like to renegotiate and capture the surplus from the second project. However, investors are not willing to renegotiate the original contract as they would be giving up the surplus without any benefits in return.

2.5 The planner's solution

The objective of the planner is to maximize the net social surplus (the bank value). The net social surplus consists of two parts: the value created by the project associated with adverse selection and the value created by the project associated with effort moral hazard. With regards to the project which is subject to adverse selection, maximization of the net social surplus requires that only good projects are undertaken. The planner can achieve this separation by giving K to the bank and allow banks to play a two-stage signaling game similar to the one we consider above (stages 2a and 2b) where the role of the investors is played by the planner. Furthermore, in order to maximize the aggregate net social surplus, the planner will extract the full NPV of the second project and transfer it to the first project to relax the effort moral hazard constraint as much as possible. Hence, at the optimum, he will set $K = 0$. The planner does the same also in the idiosyncratic state. Formally, the planner's problem reduces to:

$$
\begin{aligned}
&\underset{\tau_1,\tau_{3,\sigma},K}{\text{Max}} && (p_g + me)(R - \tau_1 + Y) - c(e) \\
&\text{subject to} && \\
&\text{(IC)} && c'(e^*) = m(R - \tau_1 - Y) \\
&\text{(PC)} && p_g(R - \tau_{3,S}) - K \geq 0 \geq p_b(R - \tau_{3,S}) - K \\
&\text{(FC)} && (p_g + me)\tau_1 + (1 - q - p_g - me)Y + q(Y - K) \\
& && + (1-q)(p_g \tau_{3,\bar{S}} - 1) + \alpha q(p_g \tau_{3,S} - (1 - K)) \geq 1 \\
&\text{(LL)} && \tau_{3,\sigma} \leq R
\end{aligned}
\tag{11}
$$

The planner provides the required funds and sets transfers from the bank to himself as τ_1 which is the repayment at $t = 1$ and $\tau_{3,\sigma}$ which is the repayment at $t = 3$ in state σ, where $\sigma \in \{S, \bar{S}\}$. The planner maximizes the objective function with respect to the effort exerted by the bank subject to three constraints. The first constraint is the effort moral hazard constraint (IC). The second constraint is related to truth-telling about the second project. The third constraint is the planner's feasibility constraint (FC) (analogous to Equation (6) in the equilibrium analysis), where the second line of the FC is the transfer of profits from the second to the first project. The final constraint is the limited liability constraint.

The planner's objective is to maximize the net social surplus. Given that full separation can be achieved on the second project and the planner can extract the full surplus (NPV) on this project, the planner's problem reduces to maximizing the bank's effort in the first project. From the IC, this can be done by reducing τ_1 to the maximum extent possible (given Assumption *A3*, the first-best cannot be reached). To minimize τ_1, the FC must bind. The choice variables are $\tau_{3,\sigma}$ and K. From the FC the aggregate effect of K is $-(1 - \alpha)K$, implying that a strictly positive K diverts resources which could be used to reduce τ_1 and increase effort. Thus, at the optimum, the planner will set $K = 0$. Also, from the second line of the FC, higher the $\tau_{3,\sigma}$, lower is the τ_1 consistent with the planner's FC being satisfied. Thus, the planner will set the transfer in both states to the maximum possible consistent with the limited liability constraint, i.e., $\tau_{3,S} = \tau_{3,\bar{S}} = R$. $\tau_{3,S} = R$ and $K = 0$ satisfies the truth-telling constraint, and hence, is consistent with a separating equilibrium which allows the planner to extract the full surplus in the systematic state.[5] The planner's allocation becomes ($\tau_1 = D_1{}^*,\ K = 0,\ j = l = 1,\ \tau^*_{3,\sigma} = R$), which coincides with the competitive equilibrium allocation, and hence, the equilibrium allocation is optimal (second-best).

Proposition 2 *The equilibrium allocation of our game coincides with the planner's solution, and hence, it is efficient (second-best).*

3 Benchmarks

In this section, we contrast our solution with two benchmarks (market financing and dis-intermediation).

3.1 Market versus intermediated financing

Instead of a bank seeking joint financing for the two projects, suppose that there are two separate firms, each managing one of the projects, which obtain financing directly from the market. We assume that the firm managing the first (resp. second) project has assets-in-place which produce $Y_1 > 0$ (resp. $Y_2 > 0$), with $Y_1 + Y_2 = Y$.

With regards to the second project, good firms achieve separation by investing an amount, K, in the project. The good firm offers a pair (K, D_3) along the lower contour of the shaded region in Figure 2. The offer is on the bad bank's participation constraint for $K \in [0, \bar{K}]$ and on investors' participation constraint (when facing good borrowers) for $K \in (\bar{K}, 1]$, where $\bar{K}$ is the intersection point of Π_b and γ_g. It is assumed that bad banks stay out in case of indifference between investing or not. Despite perfect competition, market investors obtain positive profits in expectation and the expected profits are smaller as direct investment by the firm increases (for $K > \bar{K}$, investors' expected profits become 0). In this case, the firm invests all available cash flows, $K = Y_2$, into the project to minimize sharing profits with investors. Regardless of the split of the surplus, the outcome is efficient since the equilibrium is separating and only good projects obtain financing for any $Y_2 > 0$.

[5] By an argument similar to the one developed in Lemma 5 we can show that this is indeed the unique equilibrium in the game played by the planner and the investors.

With regards to the first project, the zero profit condition of investors is given as follows:

$$(p_g + me)D_1 + (1 - p_g - me)Y_1 - 1 = 0 \tag{12}$$

From the zero profit condition, we derive the repayment, ${D_1}^D$. The firm maximizes its expected profit from the first project by choosing the level of effort:

$${\Pi_1}^D = (p_g + me)(R - {D_1}^D + Y_1) - c(e) \tag{13}$$

Taking the first order condition, and substituting the investor's zero profit condition (Equation (12)), we obtain the equilibrium effort exerted:

$${D_1}^D = \frac{1 - (1 - p_g - me)Y_1}{p_g + me} \tag{14}$$

$$c'(e^D) = m(R - D_1^D + Y_1) \tag{15}$$

Proposition 3 *Intermediated financing strictly dominates market financing for all parameter values.*

Proof. The proof is in the Appendix. ■

$e^D < e^*$ for all parameter values. The reason is that with direct financing, the two projects are separately financed and the borrower cannot bring forward the expected profits to investors in the second project to reduce the repayment for the first project, i.e., $D_1^D < D_1^*$ for any $Y_1 \leq Y$. Since the provider of effort retains more of the surplus from effort provision in the case of intermediated financing, effort is higher in this case compared to the market financing case.

Intermediated and direct financing deliver equivalent outcomes (in terms of efficiency) in the case of the second project, while intermediation delivers a strictly more efficient outcome in the case of the first project. The result is driven by cross-subsidization across projects in the intermediation case.

3.2 Intermediation vs. disintermediation

Segura and Zeng (2020) consider a very similar setting and consider the case that the first project may be funded on- or off-balance sheet (disintermediation). First, we recap their analysis in brief, and then we provide a comparison.

Under the on-balance sheet financing mode, investors in the first project have unlimited recourse to cash flows from the assets-in-place. In the absence of cross-subsidization across projects, the promised repayment to investors in the first project, D_1^{on}, and effort provision, e^{on}, are given by:

$$D_1{}^{on} = \frac{1-(1-p_g-me)Y}{p_g+me} \tag{16}$$

$$c'(e^{on}) = m(R - D_1^{on} + Y) \tag{17}$$

Different from us, Segura and Zeng (2020) assume that investors make zero profits on each project. For this reason, there is a partial pooling equilibrium in which all good banks and a fraction of bad banks invest in the second project.

Under the off-balance sheet financing mode, in the event of failure of the first project, there is no obligation for the bank to make repayments using cash flows arising from the bank's assets-in-place. However, if it fails for systematic reasons, the good bank can signal its second project's type by voluntarily using cash flows from the assets-in-place to repay the investors in the first project. The promised repayment to the investors in the first project, D_1^{off}, and effort provision, e^{off}, are given by:

$$D_1{}^{off} = \frac{1-\alpha qY}{p_g+me} \tag{18}$$

$$c'(e^{off}) = m(R - D_1^{off}) \tag{19}$$

The ratio of marginal cost of providing voluntary support to the marginal benefit is smaller for good banks compared to bad banks, implying that voluntary support can be an effective signalling device. There is a partial pooling equilibrium in which all good banks and a fraction of bad banks invest; due to the voluntary support, the fraction of bad banks investing in the off-balance sheet case is always smaller than in the on-balance sheet case.[6]

The trade-off between on-balance sheet financing (without cross-subsidization across projects) and off-balance sheet financing is the following: under on-balance sheet financing, the effort provision in the first project is higher, while under off-balance sheet financing signalling in the second project is stronger. The optimal financing mode is determined from this trade-off. Our solution differs since we allow (but do not impose) for the two projects to be jointly financed. We do not assume that investors make zero profits on a project-by-project basis; instead, investors must make weakly positive profits on aggregate. To facilitate comparisons, we present the following Lemma:

Lemma 8 *The face value of debt and the corresponding effort levels under different financing modes are as follows:* $D_1^* < D_1^{on} < D_1^{off}$ *and* $e^* > e^{on} > e^{off}$*.*

Proof. The proof is in the Appendix. ■

Our solution, (D_1^*, e^*), strictly dominates both the on-balance sheet solution without cross-subsidization, (D_1^{on}, e^{on}), and the off-balance sheet solution, (D_1^{off}, e^{off}).

1. $e^* > e^{on} > e^{off}$ (Lemma 8). Effort provision in our solution dominates the effort provision in either case above for all parameter values since $D_1^* < D_1^{on} < D_1^{off}$. Intuitively, the first-period debt is priced by rationally anticipating that investors will make positive expected profits in the second project; this reduces the promised repayment D_1 and, since the bank retains more of the surplus from exerting effort, effort provision is higher.

2. In relation to the second project, Segura and Zeng (2020) obtain a partial pooling equilibrium in which all good banks and a fraction of bad banks invest if cash flows from the assets-in-place are sufficiently high (inefficiency); otherwise, there is a market breakdown equilibrium with no financing at all (inefficiency). In contrast, our equilibrium is always separating in which only good banks invest (efficiency) even if the cash flows generated by the assets-in-place is 0.

Proposition 4 *For all parameter values, on-balance sheet financing with cross-subsidization*

[6] If cash flows from assets-in-place are large enough to make the second project riskless, there is full separation.

across projects strictly dominates on-balance sheet financing without cross-subsidization and off-balance sheet financing.

In the choice of the financing mode, Segura and Zeng (2020) trade-off the efficiency gains from mitigating the adverse selection friction with the efficiency gains from relaxing the moral hazard friction; mitigating one makes the other friction more binding. In our case, the solution of the adverse selection friction relaxes the moral hazard friction; indeed, as we show in Section 2.5, our solution implements the second-best outcome.

In summary, the off-balance sheet solution of Segura and Zeng (2020) is inefficient for three reasons: First, they impose the unjustifiable assumption that investors make zero expected profits in the second project which leads to either an inefficient partial pooling equilibrium or a complete market breakdown equilibrium. Second, in this equilibrium the bank uses all cash flows generated by assets-in-place for signaling purposes (money-burning) which makes the effort moral hazard problem more binding and leads to lower effort provision. In contrast, we do not impose this assumption and we show that there exists a unique equilibrium which is always separating where only good banks invest. Third, our solution increases effort provision even further by transferring profits made from the second project to reduce the promised repayment for the first project. Hence, given the informational frictions in this model, the off-balance sheet financing mode is inefficient and does not maximize bank profit. Therefore, a rational profit-maximizing bank will never choose it.

4 Empirical implications

In this section, we present the key implications arising out of our analysis. First, we present a lemma which forms the basis the empirical predictions below:

Lemma 9 *The increase in bank profits is higher than the direct increase in profits due to an increase in its expected growth opportunities, α, or an increase in the probability of the idiosyncratic state, $(1-q)$.*

The intuition is as follows: At $t = 0$, a higher α and/or a lower q implies higher expected returns on the second project which are fully captured by investors. This has two effects:

a direct one and an indirect one. The increase in the expected profits made by investors directly reduces the face value of debt, D_1. This fall in D_1 relaxes the effort moral hazard constraint in the first project and leads to higher effort provision which, in turn, reduces the bank's default probability, and hence, the spread on debt falls further.

We state the testable empirical predictions below:

1. *The spread on debt is lower and bank monitoring intensity is higher for banks with higher expected growth opportunities.*

   Banks which have stronger growth prospects, i.e., a higher α, can use the future profits to reduce the spread on debt due to the two effects discussed above. A key feature of this prediction is that the lower spread is, in part, driven by higher effort provision by the bank. This is potentially testable. One can proxy bank monitoring with borrower site visits, hiring third party appraisers, or the frequency with which banks demand loan-specific information (see Gustafson et al. 2021).

2. *The spread on debt falls and monitoring intensity increases if the macroeconomic prospects improve.*

   The systematic state in our model corresponds to the crisis state. If the probability of the systematic state is lower, i.e., q falls, the spread on debt falls and effort provision is higher. The intuition is similar to the one described above.

3. *The spread on debt falls and monitoring intensity increases as the expected return from the bank's core activities (assets-in-place) increases. However, holding constant the expected return on the assets-in-place, the variance of the return on assets-in-place has no effect on the spread.*

   First, it is clear to see that an increase in the expected return on the assets-in-place, Y, reduces the face value of debt, D_1 (see Equation (9)), since more can be pledged to relax the effort moral hazard constraint. Suppose that $Y_S \neq Y_{\bar{S}}$ and Y is the weighted average across the states, $Y \equiv qY_S + (1-q)Y_{\bar{S}}$. If $Y_{\bar{S}}$ increases and Y_S falls, such that Y is held constant, D_1 is unaffected. The variance of Y across states does not play a role in our model because both Y_S and $Y_{\bar{S}}$ are fully captured by the investors. This result is different from the prediction in Segura and Zeng (2020) that the variance, holding constant the mean, is relevant for the outcome since the assets-in-place affects the spread only in the systematic state in the case of off-balance sheet financing.

4. *The spread on debt falls in the bank's growth opportunities if these are positively correlated with its core activities. If they are negatively correlated, the effect may reverse.*

   If there is a positive correlation between the bank's growth opportunities, α, and the return on its assets-in-place, Y, then an increase in one is accompanied by an increase in the other. The effect of both changes are in the same direction, which is that the spread on debt falls. If on the other hand, Y and α are negatively correlated, then an increase in one is accompanied by a fall in the other. As a result, these two changes have opposite effects on the spread and which effect dominates depends on the parameters.

5 Conclusion

We present a model in which there are two investment opportunities for a bank; the first suffers from an effort moral hazard problem, while the second suffers from an adverse selection problem. We show that on-balance sheet financing with cross-subsidization across projects strictly dominates direct financing, on-balance sheet financing without cross-subsidization and off-balance sheet financing for all parameter values, and delivers the second-best outcome. In our solution, expected profits from projects which suffer from adverse selection are used to induce higher effort provision in projects subject to effort moral hazard. Griffin et al. 2007, Santikian 2014, and Jenkinson et al. 2018 provide empirical evidence consistent with cross-subsidization across unrelated activities in financial institution. Our analysis implies that, given the informational frictions in the present setting, the optimal solution requires intermediation rather than off-balance sheet financing. Hence, in order to provide a rationale for disintermediation (off-balance sheet financing), we need to identify a different combination of frictions.

Appendix: Omitted proofs

Proof of Lemma 1:

Proof. Due to limited liability, $D_{3,S} \leq R$. In a separating equilibrium in which bad banks obtain financing without pooling with good banks, investors' participation constraint is violated since for any $D_{3,S}$ satisfying the limited liability constraint, $p_b D_{3,S} < 0$ (Assumption *A1*). Therefore, there cannot exist a separating equilibrium in which bad banks obtain financing. ■

Proof of Lemma 2:

Proof. Due to limited liability, $D_{3,S} \leq R$. In a pooling equilibrium in which both good and bad banks obtain financing, investors' participation constraint is violated since for any $D_{3,S}$ satisfying the limited liability constraint investors make strictly negative profits ($\alpha p_g D_{3,S} + (1-\alpha) p_b D_{3,S} < 0$, by assumptions *A1* and *A4*). Therefore, there cannot exist an equilibrium in which both good and bad banks obtain financing. ■

Proof of Lemma 4:

Proof. Suppose that the contract at $t = 0$ specifies $j = l = 0$. The investor's zero profit condition is:

$$(p_g + me)D_1 + (1 - q - p_g - me)Y + q(Y - K) = 1 \tag{20}$$

Competition among investors sets $K = 0$ which minimizes D_1 and maximizes the effort level and the value of the first project. Let's denote the solution $D_1(j, l = 0)$.

With regards to the second project, the best case scenario for the good bank is that it separately finds financing for the second project and extracts all the surplus generated from it (this will be the case in the idiosyncratic case, but not in the systematic case). Starting from this equilibrium, an investor can profitably deviate by offering a contract with $l = j = 1$ and $K = \epsilon$ on the good bank's participation constraint, where ϵ is positive but arbitrarily close to 0. If, upon observing the use of K, funding is provided for the second project, the face value of debt in the systematic state, $D_{3,S}$, becomes arbitrarily close to R. That is, the deviating contract will allow the investor to achieve separation and capture (almost) all of the surplus from the second project.

The expected profits that investors make in the second project can then be used to reduce the promised repayment for the first project, $D_1 < D_1(j, l = 0)$. In itself, the transfer between projects is value-irrelevant for the bank. However, the transfer relaxes the bank's effort moral hazard constraint in the first project, implying that the bank exerts a higher effort compared to the case in which the expected profits from the second project is not transferred to the first; higher effort, in turn, increases bank value. The surplus created from the additional effort can be split such that both the bank and the investor is strictly better off compared to the conjectured equilibrium, and hence, the $j = l = 0$ contract cannot be an equilibrium. Following the same logic as above that transferring resources from the the second to the first project increases bank effort and value, $(j = 0,\ l = 1)$ and $(j = 1,\ l = 0)$ contracts can also be ruled out. ■

Proof of Lemma 5:

Proof. In the idiosyncratic state, there is no uncertainty about project type, so the bank always obtains financing. Next, we show that good projects always obtain financing also in the systematic state. Suppose that there exists a market breakdown equilibrium in the systematic state at $t = 2$. The investor's zero profit condition is:

$$(p_g + me)D_1 + (1 - q - p_g - me)Y + (1 - q)(p_g D_{3,\bar{S}} - 1) = 1 \tag{21}$$

Let's denote the solution D_1^b. Given the beliefs associated with the breakdown equilibrium, the expected profits made by investors in the second project is 0 when $\sigma = S$. Under competition any profits made by investors on the second project will be transferred to reduce the face value of debt on the first project. Thus, the face value of debt in the first project in this case is higher than in the case when there is financing of the good second project and investors make a strictly positive profit on it. From Lemma 3, there exist pairs, $(K, D_{3,S})$, in the shaded region in Figure 2, such that investors can offer financing in the second project and make strictly positive profits. Given this offer, good banks at $t = 2$ will invest since their participation constraint is satisfied, while bad banks will not invest since their participation constraint is violated in this region. By the Intuitive Criterion, investors assign a probability 1 to the event that only good banks will invest. Also, given that this offer can lie above investors' zero profit line corresponding to good banks, this deviation implies strictly positive profits for them. As a result, there is scope that an investor can offer such a pair and a $D_1 < D_1^b$. This lower D_1 will induce higher effort provision and lead to higher profits which the bank will fully capture. Therefore, this deviation is strictly profitable both for the investor and the bank, and hence, the market breakdown equilibrium cannot exist. ■

Proof of Lemma 6:

Proof. In the shaded region of Figure 2, consider an equilibrium contract below the good bank's participation constraint. Noting from Equation (8) that bank profits (over the two projects) is maximized for the smallest possible K and the largest possible $D_{3,S}$, this equilibrium cannot survive competition among investors. Competition among investors pushes the equilibrium vertically up all the way till the good bank's participation constraint binds; this maximizes $D_{3,S}$ for any given $K \geq 0$. Next, suppose that the equilibrium contract is along the good bank's participation constraint and specifies a strictly positive investment, $K > 0$. There is a deviation from this conjectured equilibrium since an investor can offer a contract with a lower K and higher $D_{3,S}$ (going up along the good bank's participation constraint); such an contract will drive down D_1, which will lead to higher effort in the first project and increase bank value. Thus, competition among investors sets $K = 0$, which implies $D_{3,S}^* = R$. ■

Proof of Lemma 7:

Proof. Suppose that $D_{3,\bar{S}} < R$. An investor will deviate by offering a higher $D_{3,\bar{S}}$ and use the expected profits in the second project to offer a lower D_1. The transfer will relax the bank's effort moral hazard constraint and increase bank value. As long as the new D_1 splits the surplus created from the additional effort, the new contract will make both the investor and the bank strictly better off compared to the conjectured equilibrium, which therefore cannot be sustained. ■

Proof of Proposition 3:

Proof. With regards to the adverse selection project, there is full separation between good and bad banks in both intermediated and market financing cases. Thus, to show that intermediated financing dominates market financing in terms of efficiency, we need to show that $e^* > e^D$ for all parameter values. First, we show that $D_1^* > D_1^D$. Using

Equations 9 and 14:

$$D_1^* = \frac{1-(1-p_g-me)Y}{p_g+me} - \underbrace{\frac{(1-q+\alpha q)(p_gR-1)}{p_g+me}}_{>0} < \frac{1-(1-p_g-me)Y_1}{p_g+me} = D_1^D \tag{22}$$

Using $D_1^* < D_1^D$ and Equations 10 and 15:

$$c'(e^*) = m(R - D_1^* + Y) > m(R - D_1^D + Y) = c'(e^D) \tag{23}$$

Since $c'(e)$ is increasing in e, it holds that $e^* > e^D$. ■

Proof of Lemma 8:

Proof. First we show that $D_1^* < D_1^{on} < D_1^{off}$. Noting that $1-p_g-\alpha q > me$ since it is assumed that $1-p_g-q \geq me$ and using Equations 9, 16, and 18:

$$\begin{aligned} D_1^* &= \frac{1-(1-p_g-me)Y}{p_g+me} - \underbrace{\frac{(1-q+\alpha q)(p_gR-1)}{p_g+me}}_{>0} < \underbrace{\frac{1-(1-p_g-me)Y}{p_g+me}}_{D_1^{on}} \\ &= \underbrace{\frac{1-(1-p_g-\alpha q-me)Y-\alpha qY}{p_g+me}}_{\text{add and subtract } \alpha qY} < \frac{1-\alpha qY}{p_g+me} = D_1^{off} \end{aligned} \tag{24}$$

Using $D_1^* < D_1^{on} < D_1^{off}$ and Equations 10, 17, and 19:

$$c'(e^*) = m(R - D_1^* + Y) > \underbrace{m(R - D_1^{on} + Y)}_{c'(e^{on})} > m(R - D_1^{off}) = c'(e^{off}) \tag{25}$$

Since $c'(e)$ is increasing in e, it holds that $e^* > e^{on} > e^{off}$. ■

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