Can Information Imprecision Be Valuable? The Case Of Credit Ratings*

Sonny Biswas† Kostas Koufopoulos‡ Anjan Thakor§

September 17, 2023

Abstract

We develop a model that explains two stylized facts – the coarseness of credit ratings relative to the underlying default probabilities, and the countercyclical nature of ratings precision. Ratings coarseness arises from the revenue-maximizing behavior of rating agencies, but it may also maximize net social surplus. There is scope for regulation since the private outcome may differ from the socially desirable outcome – the planner puts a ceiling (floor) on the fee if the desired outcome is coarseness (precision). The analysis generates results consistent with existing empirical evidence as well as new predictions.

Keywords: Credit ratings, Coarseness, Information precision, Adverse selection, Effort moral hazard

Jel codes: D82, D83, G24, G28

*We thank S. Nageeb Ali, Vladimir Asriyan, Matthieu Bouvard, Michael Fishman, John Kuong, Alessio Piccolo, Joel Shapiro, Enrique Schroth, Giancarlo Spagnolo, Pavel Zryumov, conference participants at SFS Cavalcade, FINEST Spring Workshop, BSE Summer Forum, CEPR Endless Summer, and seminar participants at Essex, Bristol, Bangor, Cardiff, Groningen, Reading, Durham, LUISS, Luxembourg, CAFRAL (RBI) for their helpful comments on earlier drafts of the paper. We alone are responsible for any remaining errors.

†University of Bristol Business School. s.biswas@bristol.ac.uk.

‡Department of Economics and Related Studies, University of York. kostas.koufopoulos@york.ac.uk.

§ECGI, FTG Fellow, MIT LFE Associate, and Olin Business School, Washington University in St. Louis. thakor@wustl.edu.

"The traditional philosophy of science approach to prediction leaves little room for appreciating the value and potential of imprecise information. At least, they are considered a stepping stone to more precise predictions." – Elliott-Graves (2020)

1 Introduction

More precise information is typically regarded as being superior to less precise information. Perhaps the most obvious argument in favor of this proposition is a "free disposal of information" reasoning – if information of lower precision was more valuable, the receiver of the information could always use a coarser partition of the transmitted information. Yet, we see numerous examples of imprecise information being produced and communicated even when more precise information is technically feasible. A striking example of this is the assignment of credit ratings of debt instruments. There is a continuum of default probabilities, but only about two dozen or so credit ratings. This means that credit ratings are imprecise indicators of default probabilities.

Ratings are produced by credit rating agencies (CRAs) which are information intermediaries that acquire and process information about firms, thereby reducing firms' financing frictions. This suggests that CRAs should produce and communicate as precise information as possible. The fact that they do not do this suggests that the imprecision associated with ratings coarseness may have value. What is this value? Moreover, the precision of ratings seems to be dependent on the business cycle, with ratings displaying greater precision during downturns (e.g., Griffin and Tang 2012). Why? We address these questions theoretically in this paper and develop a model consistent with these stylized facts – ratings coarseness and the greater informativeness of ratings during downturns. We also examine how ratings coarseness interacts with market structure in the CRA industry. Consistent with the mixed nature of the empirical findings, we show that competition may lead to more or less precise ratings depending on model parameters.

The model we develop is quite simple. There are three types of observationally identical firms that can be either good, intermediate or bad in credit quality.[1] Each firm is privately informed about its type, whereas all other agents have common-knowledge priors captured

[1]Credit quality in the model corresponds to the firm's probability of default on its debt.

by a probability distribution over types. Good firms have positive-NPV projects, bad firms have negative-NPV projects, and intermediate firms have projects that are positive-NPV if the firm exerts (unobservable and costly) effort and negative otherwise.[2] Effort exertion incentives get weaker as the firm's debt repayment increases. Thus, there are only two frictions in the model – asymmetric information about firm types and effort-aversion moral hazard.

In the absence of certification by a CRA, securities issued by all firms are priced as a pool. If the fraction of bad borrowers is sufficiently high, the expected NPV becomes negative and financing is denied to all as the market breaks down. CRAs can learn firm-type perfectly and assign a rating at a flat fee, which is determined endogenously. If CRAs provide precise ratings, each firm-type is identified accurately by the market and firms issue debt that is accurately priced for each type. The interest rate comes from the participation constraints of investors which are just binding in equilibrium (zero expected profits). But, at this equilibrium interest rate, intermediate firms do not exert effort since the cost of this effort is too high. Thus, only good firms obtain financing, while intermediate and bad types are excluded.

If, on the other hand, CRAs provide coarse ratings, they pool good and intermediate firms into a single rating category. Given this, there is a partial pooling equilibrium in which both good and intermediate firms issue debt, and the debt is priced according to the average quality of the pool. As a result, debt issued by good firms is under-priced and the debt issued by intermediate firms is over-priced, i.e., good firms subsidize intermediate firms. When the fee charged by the CRA is sufficiently low, there is a large enough subsidy to make it incentive compatible for intermediate firms to exert effort. This effort exertion transforms projects of intermediate firms into positive-NPV. Both good and intermediate firms obtain financing and net social surplus is higher than attainable in the precise-ratings equilibrium. Coarseness delivers the socially desirable outcome that does not arise in the absence of coarseness. If the fee is higher, coarse ratings pool good and intermediate borrowers without eliciting effort exertion by intermediate firms, which lowers net social surplus.

The preceding analysis does not consider what the CRA would wish to do, so we turn

[2]The idea is that some firms may have the ability to undertake costly risk management activities to lower their default probabilities.

next to the incentives of the CRA. We assume that issuing precise ratings entails a cost, while the cost of issuing coarse ratings is normalized to 0. The ratings agency chooses a ratings policy (degree of coarseness) and the fee with the objective to maximize its fee revenue. A CRA offers coarse ratings if the cost of precision is sufficiently high, and precise ratings otherwise.

We begin by considering the case in which there is a single CRA. The ratings policy the monopolist CRA adopts is determined by the following trade-off – the CRA's ability to capture the full surplus under precise ratings versus its ability to capture a fraction of a potentially bigger surplus under coarse ratings. The surplus in the case of coarse ratings may be larger due to a number of reasons: (i) the CRA does not incur the cost of precision, (ii) coarse ratings induce effort exertion by intermediate borrowers through cross-subsidization, and (iii) both good and intermediate borrowers purchase ratings (as opposed to only good borrowers as is in the case of precise ratings).

Next, we consider multiple competing CRAs. In this case, CRAs charge a fee which equals the marginal cost of issuing a rating. The degree of coarseness is determined by the profit-maximization objective of good borrowers. If the subsidy to intermediate borrowers in a pooling equilibrium is smaller than the cost of precision, then coarseness arises in equilibrium. Depending on the parameters, competition may lead to more or less precision than in the case of a monopolist CRA. This is consistent with conflicting empirical evidence – while Doherty et al. (2012) and Kisgen and Strahan (2010) find that competition leads to greater precision, Becker and Milbourn (2011) find the opposite. Our analysis suggests that cross-sectional differences in borrower profitability – not appropriately controlled for in the empirical analysis – may generate these differences in empirical findings.

If the cost of effort is sufficiently small, the efficient outcome is obtained with coarse ratings and low fees charged by CRAs. Low fees leave sufficient subsidy to intermediate borrowers to induce effort exertion. When the cost of effort is higher, there are no positive fees that can induce effort exertion. In this case, the efficient outcome arises under precise ratings in which only good borrowers obtain financing. Depending on parameters, both types of inefficiencies may arise – ratings are coarse when the socially desirable outcome is precision or precise when the socially desirable outcome is coarseness. This gives rise to a possible role for regulation. The regulator puts a ceiling (floor) on the fee if the desired outcome is coarseness (precision). Strikingly, for any market structure in the CRA industry, when the cost of effort provision by intermediate borrowers is sufficiently small, if the cost of greater precision is very small (or it is costless) the planner *always* prefers coarse ratings, while the unregulated equilibrium features precise ratings.

In our theory, in addition to playing the usual role of reducing information asymmetry, credit ratings also have welfare-enhancing real effects because they can induce effort provision that would not be forthcoming in the absence of ratings. Further, our model generates the novel insight that it is *not necessarily welfare-enhancing for CRAs to fully eliminate all information asymmetry.* For some deep parameters, withholding some information allows CRAs to deliver higher net social surplus compared to the case in which investors have full information.

We also relate ratings coarseness to the business cycle. Assume that the ratio of good firms to intermediate firms is higher in an economic boom compared to an economic downturn.[3] In a downturn, the relative paucity of good borrowers implies that the subsidy from good to intermediate firms in the pooling equilibrium may not be sufficient to incentivize intermediate firms to exert effort. Then, with coarse ratings, the unique outcome will be a pooling one and no one in the pool will obtain financing. So, no firm obtains a rating. Clearly, this cannot be an equilibrium. To attract the good firms, the CRA will offer to precisely identify good firms and eschew pooling. This means that in a downturn, ratings coarseness disappears. This implication of the model that the precision of ratings may be higher in downturns than in booms is consistent with the empirical evidence in Ashcraft et al. (2010) and Griffin and Tang (2012). Bar-Isaac and Shapiro (2013) (see also, Bar-Isaac and Shapiro 2011) have a similar result regarding the countercyclicality of ratings precision, but it is driven by different forces in a dynamic model with reputational concerns – the source of imprecision in their model is costs associated with higher precision, while in our model imprecision can be valuable. As a result, our policy implications are different.

In addition to generating results that are consistent with existing stylized facts, our model also produces a new prediction – when the cost of information production for

[3] An alternate way to capture the business cycle would be to assume that the cost of effort for intermediate firms is higher in economic downturns. This interpretation would yield similar predictions.

CRAs declines, real investment by (rated) firms goes down. This is counter to the usual intuition that if information production by CRAs facilitates investments, then a lower information production cost should elevate investment.

This paper is related to the literature on the value of imprecise information. In oligopolistic models of incomplete information, there are conflicting results on the benefits or costs of observing more precise information. In Rotemberg and Saloner (1986) and Vives (1984), observing more precise information has value, whereas the opposite is true in Gal-or (1987). In these models, the firm cannot affect the precision of the information and the quantity of output produced has no effect on the precision. In contrast, Gal-or (1988) develops a model in which experience in production allows firms to internally generate private signals at no cost. When the firm is endowed with less precise information about cost, it has a greater incentive to produce. Information imprecision thus has value because it encourages production. In contrast to this literature, our model focuses on the external provision of information by either monopolistic or competitive CRAs, and shows that information imprecision in communication can mitigate moral hazard and elevate net social welfare even when the entity communicating the information has more precise information in its possession.

Our paper is also related to the credit ratings literature. Building on the foundations provided by the financial intermediation literature that rating agencies are an example of diversified information-production intermediaries (e.g., Allen 1990, Millon and Thakor 1985, and Ramakrishnan and Thakor 1984), a strand of the literature showed that credit ratings can resolve coordination problems in financial markets (e.g., Boot et al. 2006, Manso 2013, Goldstein and Huang 2020, and Terovitis 2022).[4] In this set of papers, new information arises following the production of ratings, which is not the case in our model. Parlour and Rajan (2020) show that ratings can be valuable in the presence of contract incompleteness. In our setting, welfare is non-monotonic in the precision of information communicated by CRAs.

Other papers have focused on failures in the credit rating process, including incentives for rating agencies to manipulate ratings (e.g., Bolton et al. 2012, Sangiorgi et al. 2009,

[4] Thakor and Merton (2023) view credit ratings in the presence of asymmetric information and product complexity as a third-party verification mechanism. In their model, such verification interacts with voluntary information disclosure by firms and influences the complexity of products that firms design.

Opp et al. 2013, Frenkel 2015, and Sangiorgi and Spatt 2017). While inflated ratings refer to incorrect ratings, which is only feasible in settings with naive investors, we consider coarse ratings, i.e., ratings are vague but, on average, correct. That is, ratings inflation and ratings coarseness are different phenomena. Ratings inflation refers to imprecise ratings that are on average too high relative to the underlying default probabilities, whereas coarse ratings are correct on average but not as finely partitioned as the underlying default probabilities.

Our contribution relative to this literature is that we develop a theory in which ratings coarseness arises endogenously as an equilibrium phenomenon to elevate net social surplus. That is, not only are credit ratings coarse, but this coarseness improves ex ante economic efficiency relative to a setting with precise ratings. This connects us to papers in which ratings are coarse, like Lizzeri (1999), Doherty et al. (2012), and Ali et al. (2022). In these papers, information communication is endogenously coarse, but coarseness does not impact allocative efficiency. In Kartasheva and Yilmaz (2020), coarseness also arises endogenously due to the monopolist CRA's revenue maximizing behavior but coarseness destroys welfare. In contrast to these models, coarseness *improves* welfare in ours (for some parameters).

Our result that coarseness can be welfare-improving is reminiscent of Spence (1973). In contrast to Spence (1973), the pooling equilibrium in our model may be inefficient from a social perspective for some parameters, but may still arise, while in Spence (1973), the pooling is always efficient from a social perspective but it does not survive standard equilibrium refinements. Several articles highlight that opacity can be valuable in different contexts (see, for example, Hirshleifer 1971, Bouvard et al. 2015, and Dang et al. 2017). In contrast to these models, the result in our model arises from an interaction between adverse selection and moral hazard; specifically, the presence of the adverse selection friction relaxes the moral hazard constraint.

The paper closest to ours is Goel and Thakor (2015), which also provides an endogenous theory of coarse ratings. Using a cheap-talk model, the paper shows that ratings coarseness can arise as a second-best equilibrium phenomenon even when higher precision can improve investment efficiency. The reason is that coarseness is the only incentive compatible mechanism for truthful communication by the CRA. In contrast, coarseness helps to achieve the first-best investment in our model when precision fails to do so. That is, even when the CRA can credibly communicate very precise information, welfare is higher when it chooses imprecision over precision. As a result of these differences, the predictions of the two models are also very different; see our discussion of the effect of competition in Section 4.3.

Our paper also relates to the literature on bank stress testing (see e.g., Goldstein and Leitner 2018 and Orlov et al. 2022). Using tools from the Bayesian persuasion literature, the optimal stress tests generally take a simple pass or fail form, rather than identifying each bank's asset quality precisely. This test design maximizes welfare by pooling stronger banks with vulnerable banks, only allowing the weakest banks to fail. Our theory differs in two respects: First, there is no relaxation of informational frictions in these models – this is what delivers the increase in welfare in our model. Second, unlike the bank stress test literature in which coarseness is the solution of the planner's problem, the coarseness in our model arises due to market forces.

Finally, our paper relates to the literature on competition among CRAs. Some theories explore the interaction between competing CRAs when borrowers may solicit multiple ratings (see e.g., Bar-Isaac and Shapiro 2011, Bouvard and Levy 2018, Farkas 2021, and Piccolo 2021). In contrast to these models, the borrower in our setting solicits ratings from a single CRA as there is no value-added of purchasing multiple ratings. Other theories study the ratings shopping phenomenon which feature naive investors. We derive conditions under which inefficiencies may arise under monopoly or competition and offer solutions to restore efficiency.

The rest of the paper is organized as follows. Section 2 develops the model. Section 3 presents an analysis of the base model and compares the outcomes across three choices by the CRA: (i) no ratings, (ii) precise ratings, and (iii) coarse ratings. In Section 4 we analyze the endogenous choice of ratings precision by the CRA. Empirical predictions and policy implications are examined in Section 5. Extensions of the base model are analyzed in Section 6. Section 7 concludes. All proofs are in the Appendix.

2 Model

2.1 Set-up

We consider an economy in which all agents are risk-neutral and the discount rate (risk-free rate) is zero. There are three types of agents: firms, CRAs, and investors. A firm has access to a project that needs investment and the scale of the investment is normalized to 1 unit. Each firm has zero initial endowment, so it seeks to raise funds from outside investors to invest in its project. Specifically, each firm raises $(1 + f)$ from the market; they invest 1 unit in the project, and either consume f right away or use f to pay a fee to a CRA (more details below). Investment occurs at $t = 0$ and returns are realized at $t = 1$, at which point all agents consume.

There are three types of firms and each firm privately knows its type. The common prior belief is that a fraction α of the firms have good projects, g, a fraction β have intermediate quality projects, m, and a fraction $1 - \alpha - \beta$ have bad projects, b. A firm succeeds with probability, $p_i \in \{p_g, p_m, p_b\}$, and fails with the complementary probability. If a project succeeds, it generates a cash flow, $X > 0$, and if it fails, it generates 0; firm types differ only on the probability of success. An intermediate firm can exert hidden effort at cost, c, to increase its success probability by δ.

A CRA can perfectly identify a bad firm at zero cost and incurs a cost $k \geq 0$ to distinguish the good from the intermediate firms, i.e., the marginal cost of producing coarse ratings is 0, while the marginal cost of producing precise ratings is k.[5] A firm pays an endogenously determined fixed fee, f, to the rating agency to certify its type. The fee is paid by firms from the money raised in the market. The modelling of the fee is consistent with the issuer-pays model and reflects the observation that issuers choose to pay for a rating only if the rating they obtain allows them to borrow in the market.

We make the following assumptions relating to the deep parameters:

A1: $p_g X - k > 1 > p_m X > p_b X$

Assumption *A1* states that good firms have positive-NPV projects after taking into account the marginal cost of producing precise information, while intermediate and bad

[5] Introducing noise in the CRA's default probability discovery does not qualitatively change the results.

firms have negative-NPV projects. An intermediate firm can exert effort at cost, c, to increase the success probability by δ, such that its project becomes positive-NPV:

A2: $p_g X - 1 > (p_m + \delta)X - 1 > c > \frac{\delta}{p_m+\delta}((p_m + \delta)X - 1) \equiv c_s$

Combined with *A1*, the second inequality of *A2* implies that exerting effort by intermediate firms is efficient, net of the cost of effort, i.e., $\delta X - c > 0$. The value generated from exerting effort makes intermediate firms' projects positive-NPV. The set-up is meant to reflect the possibility of risk management activities that could help intermediate firms to reduce their default probability. Nonetheless, good firms are still more likely to succeed than intermediate firms (the first inequality in *A2*). The final inequality of *A2*, $c > c_s$, implies that the cost of exerting effort is sufficiently large, such that intermediate firms do not exert effort given actuarially fair interest rates. This assumption simplifies the analysis by reducing the number of cases that we need to consider.

Bad firms cannot increase the probability of their success by exerting effort. Thus, while it is efficient to finance intermediate firms only if they exert effort, while it is always inefficient to finance bad firms. We could also allow good and bad firms to have such a hidden effort choice, but if we assume that a good firm is creditworthy regardless of its effort choice and a bad firm is never creditworthy regardless of its effort choice[6], our main results are sustained. So, in the interest of simplicity, we do not give good and bad firms this effort choice.

A3: $(\alpha p_g + \beta(p_m + \delta) + (1 - \alpha - \beta)p_b)X < 1 + \beta c$

A4: $(\alpha p_g + (1 - \alpha - \beta)p_b)X < 1 - \beta$

Assumption *A3* implies that the expected NPV across all three firm-types is negative, even if intermediate firms exert effort. *A4* implies that the expected NPV across good and bad firms is negative. *A3* and *A4* impose that the fraction of bad firms is so high that they must be excluded from the market.

2.2 The game

The stages of the game are as follows:

[6]This assumption would be necessary to meaningfully distinguish the intermediate firm from the good and bad firms.

Stage 1: CRAs announce which policy they will adopt and the fee that they will charge the issuers who purchase a rating. A policy is either no ratings, or precise ratings, or coarse ratings.[7] Without loss of generality, we assume that CRAs charge a flat fee across types. Under precise ratings, the ability to condition the fee on type does not affect the results, while under coarse ratings, it would be necessary to charge a flat fee to sustain the pooling equilibrium.

Stage 2: Given what has been announced by CRAs, firms decide whether to get rated. If they choose to get rated, they pay the fee to the CRA.

Stage 3: Regardless of whether the firm chose to obtain a rating or not, the firm can propose a debt contract with promised repayment, R, to competitive investors.

Stage 4: Investors form a belief about the firm type given the rating given to the firm (if any) and the offered interest rates. Given these beliefs, investors decide whether to accept or reject the proposed contract. Investment occurs only if the proposed contract is accepted, at which point intermediate firms decide whether to exert unobservable effort.

We look for the pure strategy Perfect Bayesian Nash equilibria of this game that satisfy the Intuitive criterion of Cho and Kreps (1987). We solve for the equilibrium backwards. First, we analyze the financing game, conditional on the ratings precision set by CRAs. Then, we derive the CRA's choice of ratings precision by fully anticipating the outcome of the financing game.

2.3 Benchmark 1: Observable types and effort

We now consider the first-best allocation which is the allocation that obtains in the absence of both informational frictions, i.e., firm-types are observable and intermediate firms' effort levels are observable and contractible. Due to competition among investors and full information, the interest rates are such that investors make zero expected profits.

Proposition 1 (Benchmark 1) *In the first-best equilibrium, good and intermediate firms obtain financing and intermediate firms exert effort. Bad firms do not obtain*

[7] We allow for all possible combinations of coarseness (all types together and three different pairs of two types). In Lemma 1, we show that the only coarse ratings on the equilibrium path is pooling good and intermediate firms.

financing. The interest rates are as follows:

$$R_g = \frac{1+f}{p_g} \tag{1}$$

$$R_m = \frac{1+f}{p_m+\delta} \tag{2}$$

Proof. The proof follows from the discussion in the text. ■

The marginal benefit of an increase in effort in terms of an increase in the expected return exceeds the marginal cost of that effort. Therefore, in the first-best, intermediate firms choose a contract which implements the efficient effort level. The reason why interest rates are as in Equations (1) and (2) is competition among investors which leads to zero expected returns for them.

2.4 Benchmark 2: Observable types and unobservable effort

In this section, we consider the case in which firm-types are observable but effort is not observable, and hence, not contractible. Given observable types, good firms receive financing at interest rate, R_g, and invest. Bad firms do not obtain financing since they have negative-NPV projects. Consider the effort incentive constraint of an intermediate firm:

$$(p_m+\delta)(X-R) - c \geq p_m(X-R) \tag{3}$$

The left-hand side (LHS) represents intermediate firms' expected profits when exerting effort, while the right-hand side (RHS) represents the expected profits when not exerting effort. From Equation (3), an intermediate firm exerts effort only if the interest rate is sufficiently small:

$$R \leq X - \frac{c}{\delta} \equiv \underline{R} \tag{4}$$

The difference between $\frac{c}{\delta}$ and c is the rent that intermediate firms should receive to exert effort, which drives the inefficiency. If the equilibrium interest rate is higher than $\underline{R}$, intermediate firms' incentive compatibility (IC) constraint for effort provision is violated.

Suppose that the investors believe that intermediate firms will exert effort. Intermediate firms will raise funds at interest rate, R_m, which sets the competitive investors' expected profits to zero. However, $\underline{R} < R_m$ holds since $c > c_s$ (Assumption *A2*). Thus, with observable types, investors' belief that intermediate firms exert effort is not fulfilled. This implies that the investment of intermediate firms will be negative-NPV and so for any $R \leq X$, investors make losses. As a result, intermediate firms do not receive financing.

Proposition 2 (Benchmark 2) *Suppose that firm-types are observable but effort is not. Good firms obtain credit at interest rate, R_g, while intermediate and bad firms do not obtain financing.*

Proof. The proof is in the Appendix. ■

3 Outcome under each CRA policy separately

In this section, we examine the case that both informational frictions are present. To ease exposition, prior to endogenizing CRAs' choice of ratings precision, we analyze the outcome for each of the following three cases: CRAs provide (1) no ratings or (2) precise ratings or (3) coarse ratings. In Section 4, CRAs optimally choose whether to provide precise or coarse ratings and the fee that they charge in order to maximize their profits.

3.1 Outcome under no ratings

First we consider the case in which CRAs do not provide ratings. If lenders offer an interest rate which is meant for good firms under full information, then intermediate and bad firms will mimic, and the contract will be loss-making for investors. This is because, by *A3*, the average NPV across all three types is negative even if intermediate borrowers exert effort. Thus, there cannot exist a pooling equilibrium in which all firms obtain financing at the actuarially-fair pooling interest rate. This suggests that, in the absence of ratings, the market breaks down and no firm obtains financing.

Proposition 3 (No ratings) *In the absence of credit ratings the unique equilibrium consistent with zero expected profits for investors is the one in which no firm obtains financing (market breakdown).*

Proof. The proof is in the Appendix. ■

3.2 Outcome under precise ratings

In this section, we consider the case in which CRAs assign precise ratings. Since we consider perfect learning by the CRAs at a cost k, the analysis in this case is similar to the case in which firm types are observable but effort is not observable. Because ratings are precise, investors know the firm type and they do not need to make any inferences about firm types from the offered contracts, i.e., investors' beliefs do not play a role in this case. The equilibrium is identical to the one derived in Proposition 2 in terms of allocation. The minimum fee that CRAs may charge is the marginal cost of precision, $f = k$, and the maximum fee is denoted f_p and it equals the NPV of a good borrower's project. A higher fee would violate the participation constraints of investors. The case of precise ratings improves upon the situation with no ratings since it eliminates the market breakdown equilibrium. The precise-ratings equilibrium is inefficient compared to the first best because, unlike in the first best, intermediate firms do not obtain financing.

Proposition 4 (Precise ratings) *Suppose that ratings are precise. Only good firms obtain credit at interest rate, R_g, and they pay a fee, $f \in [k, f_p]$, with, $f_p = p_g X - 1$.*

Proof. The proof is in the Appendix. ■

3.3 Outcome under coarse ratings

We now consider the case of the CRA assigning coarse ratings.

Lemma 1 *The only coarse categorization which is not equivalent to the no-ratings or precise-ratings allocation is the one which pools good and intermediate firms.*

Proof. The proof is in the Appendix. ■

Given Lemma 1, for the rest of the analysis, whenever we refer to coarse ratings we consider the case in which good and intermediate firms are pooled together in a single ratings category. Under coarse ratings, the debt issued by good and intermediate firms is priced according to the average quality of the pool. If intermediate firms exert effort, then the average NPV of the coarse-ratings pool is positive. If intermediate firms do not exert effort, the average NPV of the pool may be positive or negative; it is positive if the ratio of good firms to intermediate firms is sufficiently high:

$$\begin{aligned} &\left(\frac{\alpha}{\alpha+\beta}p_g + \frac{\beta}{\alpha+\beta}p_m\right) X - 1 \geq 0 \\ \implies &\frac{\alpha}{\beta} \geq \frac{1-p_m X}{p_g X - 1} \equiv \gamma \end{aligned} \tag{5}$$

The following lemmas derive the pooling interest rate and the parameters under which it is feasible to elicit effort provision by intermediate borrowers in a coarse-ratings equilibrium.

Lemma 2 *For a given fee, f, the interest rate in a coarse-ratings equilibrium is ${R^P}_e$ with $e = 1$ if intermediate borrowers exert effort and $e = 0$ if they do not.*

$${R^P}_e = \frac{(\alpha+\beta)(1+f)}{\alpha p_g + \beta(p_m + \delta(e))} \tag{6}$$

where $\delta(e = 1) = \delta$ and $\delta(e = 0) = 0$. ${R^P}_e < R_m$.

Proof. The proof is in the Appendix. ■

Under coarse ratings, by pooling with good borrowers, intermediate borrowers obtain a subsidy in terms of a lower interest rate compared to the precise-ratings case. This subsidy may induce intermediate borrowers to exert effort by relaxing their moral hazard constraint.

Lemma 3 *Suppose $f = 0$. Under coarse ratings, intermediate borrowers exert effort if $c \leq c_p$, where c_p is given by:*

$$c_p \equiv \delta X - \frac{\delta(\alpha+\beta)}{\alpha p_g + \beta(p_m + \delta)} \tag{7}$$

Moreover, $c_p > c_s$ always holds.

Proof. The proof is in the Appendix. ■

Under coarse ratings, subsidized debt may incentivize intermediate firms to exert effort. Thus, for some parameters, intermediate firms would exert effort when ratings are coarse, but not when the ratings are precise, implying that the coarse-ratings case involves a higher net social surplus. We refer to this as the "bright side" of coarseness.[8] Note that c_p is derived assuming $f = 0$. Hence, if $c > c_p$, there is no fee for which pooling will elicit effort exertion by intermediate borrowers.

We define an incentive compatible upper bound on the fee, $\bar{f}_c$, such that if $f \leq \bar{f}_c$, intermediate borrowers exert effort due to the subsidy that they obtain from good firms in a pool. If $f > \bar{f}_c$, intermediate firms do not exert effort since the subsidy is insufficient to elicit effort exertion, i.e., the pooling interest rate assuming that intermediate firms exert effort, ${R^P}_{e=1}$, is higher than the incentive compatible interest rate, $\underline{R}$. $\bar{f}_c$ is given by:

$$\bar{f}_c \equiv \frac{1}{\delta(\alpha+\beta)} \left[(\delta X - c)(\alpha p_g + \beta(p_m + \delta))\right] - 1 \tag{8}$$

Note that $\bar{f}_c$ is falling in c and $\bar{f}_c \geq 0$ holds if $c \leq c_p$. For $c > c_p$, $f = \bar{f}_c$ is not feasible since $\bar{f}_c$ becomes negative, and the CRA does not participate.

If intermediate borrowers do not exert effort, the maximum fee that CRAs may charge is denoted as $\hat{f}_c$, which reflects the average quality of the pool when intermediate borrowers do not exert effort:

$$\hat{f}_c \equiv \frac{1}{\alpha+\beta} \left[(\alpha p_g + \beta p_m) X\right] - 1 \tag{9}$$

Which equilibrium obtains depends on the deep parameters and the fee charged by the CRA as the following result shows:

Proposition 5 (Coarse ratings) *Suppose that ratings are coarse. If $c \leq c_p$ and the fee is sufficiently low, $f \in [0, \bar{f}_c]$, both good and intermediate firms obtain financing and intermediate firms exert effort. For higher fees, $f > \bar{f}_c$:*

1. *For $\frac{\alpha}{\beta} \geq \gamma$, both good and intermediate firms obtain financing but intermediate firms do not exert effort if $f \leq \hat{f}_c$, and the market breaks down if $f > \hat{f}_c$.*

2. *For $\frac{\alpha}{\beta} < \gamma$, the market breaks down.*

[8] The "dark side" of coarseness is that it may allow intermediate firms to invest even when they do not exert effort (which is value-destroying).

Proof. The proof is in the Appendix. ■

Under coarse ratings, intermediate firms exert effort when the fee charged by the CRA is sufficiently low, $f \leq \bar{f}_c$. However, if the fee charged under coarse ratings is higher, intermediate firms may obtain financing if the lenders can break even on average, which happens if the ratio of good to intermediate firms is sufficiently high (i.e., $\frac{\alpha}{\beta} \geq \gamma$), but they do not exert effort. If $c > c_p$, for any non-zero fee, the most efficient outcome is obtained with precise ratings, i.e., by completely eliminating the information asymmetry problem, since it is more efficient to not have intermediate firms invest than for them to invest but not exert effort.

4 Equilibrium

In this section, we allow the CRAs to optimally choose whether they offer precise or coarse ratings and set the fee, and we analyze the market equilibrium under different market structures in the CRA industry.

4.1 Equilibrium with a monopolist CRA

We begin by characterizing the equilibrium when there is a monopolist CRA. The CRA maximizes its revenue by jointly choosing the ratings policy and the fee. In the next two lemmas, we derive the equilibrium fee under different circumstances and order them, respectively.

Lemma 4 *The fee charged by the CRA depends on the deep parameters and the ratings policy chosen as follows:*

1. *If ratings are precise, then for all parameters the fee is* f_p.
2. *If ratings are coarse and* $\frac{\alpha}{\beta} \geq \gamma$, *the fee is* $\max(\bar{f}_c, \hat{f}_c)$.
3. *If ratings are coarse and* $\frac{\alpha}{\beta} < \gamma$, *the market breaks down.*

Proof. The proof is in the Appendix. ■

Lemma 5 *$f_p > \max(\bar{f}_c, \hat{f}_c)$ and $\bar{f}_c \geq \hat{f}_c$, if:*

$$c \leq \frac{\beta X \delta^2}{\alpha p_g + \beta(p_m + \delta)} \equiv \bar{c} \tag{10}$$

There exist feasible parameters for which $\bar{c} > c_s$.

Proof. The proof is in the Appendix. ■

The monopolist CRA charges a higher fee in the precise-ratings equilibrium. The reason is that good borrowers have a NPV higher than the average borrower in the coarse pool, whether or not intermediate borrowers exert effort. The incentive compatible fee, $\bar{f}_c$, may be higher than the incentive incompatible fee, $\hat{f}_c$, if the surplus created from effort exertion is sufficiently large which is the case if the cost of effort is sufficiently small, $c \leq \bar{c}$.

In maximizing its revenue, the constraint that the CRA faces is that investors, who are the consumers of the ratings, have rational expectations. Hence, if ratings are uninformative, investors will not use the ratings, and firms will consequently not purchase ratings. The CRA faces the following trade-off in choosing between precise and coarse ratings: On the one hand, the fee is higher with precise ratings, i.e., $f_p > \max(\bar{f}_c, \hat{f}_c)$. On the other hand, with precise ratings, only good borrowers obtain ratings, while both good and intermediate borrowers get rated when the CRA issues coarse ratings. Moreover, in choosing its coarse ratings policy, the CRA also needs decide whether to set the incentive compatible fee or the incentive incompatible fee. Below, we characterize the equilibrium of the game:

Proposition 6 (Monopolist CRA) *Coarseness arises if either one of the following conditions is met:*

$$k \geq \frac{1}{\alpha\delta}\left[(\alpha p_g + \beta(p_m + \delta))c - \beta\delta((p_m + \delta)X - 1)\right] \equiv k_1 \tag{11}$$

$$k \geq \frac{\beta(1 - p_m X)}{\alpha} \equiv k_2 \tag{12}$$

There exist feasible parameters for which $k > k_1$ and $k > k_2$. If $k < \min(k_1, k_2)$, the equilibrium features precise ratings and the fee is f_p. In a coarse equilibrium, the fee is $f = \bar{f}_c$ and intermediate borrowers exert effort if $c \leq \min(c_p, \bar{c})$, and the fee is $f = \hat{f}_c$

and intermediate borrowers do not exert effort, otherwise.

Proof. The proof is in the Appendix. ■

Proposition 6 implies that a CRA, motivated by the objective of maximizing its fee revenue, may issue imprecise ratings so that good and intermediate firms are pooled into a single ratings category. This pooling allows cross-subsidization from good to intermediate firms and provides the incentives to the latter to exert effort, even when these firms would not exert effort if their securities were accurately priced. Thus, pooling improves welfare. Note that the CRA's motivation to engage in this ratings pooling comes not from any social welfare considerations, but rather because doing so maximizes its profits. As a corollary to Proposition 6, we state the following:

Corollary 1 $\min(k_1, k_2) > 0$. *For k arbitrarily close to 0, the equilibrium with a monopolist CRA feature precise ratings.*

Proof. The proof is in the Appendix. ■

4.2 Perfect competition

In this section, we model competition by assuming that there is free entry of CRAs. Given the degree of ratings precision, if a CRA charges a fee above the marginal cost to produce it, a competing CRA will always offer an ϵ less to attract the issuing firms. Following the classic Bertrand argument, this iterates until the fee equals the marginal cost.

Lemma 6 *With free entry of CRAs, the fee always equals the marginal cost of producing a rating; i.e., $f = k$ for precise ratings and $f = 0$ for coarse ratings.*

Proof. The proof follows from the discussion in the text. ■

Given Lemmas 3 and 6, intermediate borrowers exert effort under coarse ratings if the cost of exerting effort is sufficiently small, $c \leq c_p$, and not otherwise. Suppose that intermediate firms exert effort with coarse ratings. Then the magnitude of the subsidy that a good firm provides to intermediate firms in the pool is:

$$p_g({R^P}_{e=1} - R_g) = \frac{\beta(p_g - (p_m + \delta))}{\alpha p_g + \beta(p_m + \delta)} \equiv \sigma_{e=1} \tag{13}$$

where ${R^P}_e$ is the pooling repayment rate for effort, $e \in \{0,1\}$ (see Equations (30) and (28)). For $\frac{\alpha}{\beta} \geq \gamma$, the coarse-ratings equilibrium arises also for $c > c_p$, but the subsidy is not sufficient to induce intermediate firms to exert effort. The magnitude of the subsidy in this case, $\sigma_{e=0}$, is derived by setting $\delta = 0$ in Equation (13). That is, we denote the subsidy as σ_e, where $e = 1$ if $c \leq c_p$ and $e = 0$ if $c > c_p$ and $\frac{\alpha}{\beta} \geq \gamma$; σ_e is always positive. We characterize the equilibrium below:

Proposition 7 (Competition) *With free entry of CRAs, the equilibrium is coarse if $k \geq \sigma_e$, and precise if $k < \sigma_e$. $e = 1$ if $c \leq c_p$ and $e = 0$ if $c > c_p$.*

Proof. The proof is in the Appendix. ■

This proposition says that whether the equilibrium features coarse or precise ratings depends on which categorization is preferred by good firms. If the cost to produce precise ratings is small, i.e., $k < \sigma_e$, the coarse-ratings equilibrium does not survive because a new entrant CRA can offer precise ratings to skim the cream and attract only good firms.[9] Since σ_e is positive, for k sufficiently small, the equilibrium features precise ratings. If $k \geq \sigma_e$, the precise-ratings equilibrium does not survive since the entrant CRA can attract good firms by offering coarse ratings. This deviation is costly for good firms since it entails subsidizing intermediate firms. Nonetheless, good firms benefit from the lower fee, $f = 0$, and this benefit exceeds the cost.

As a corollary of Proposition 7, we state the following counterintuitive result relating the cost of producing information by the CRA to net social welfare.

Corollary 2 *For $c \leq c_p$, net social surplus is (weakly) increasing in the cost of information production, k.*

For $c \leq c_p$, net social surplus is maximized under coarse ratings, which arises for $k \geq \sigma_e$. A lower cost, $k < \sigma_e$, leads to precise ratings and a lower net social surplus.

[9] Of course, competition ensures that the CRA just recovers its marginal cost of producing precise ratings, i.e., $f = k$.

4.3 Monopoly vs. competition

In this section, we show that competition among CRAs may lead to more or less coarseness. In the case of a monopolist CRA, the degree of coarseness is determined by the CRA's profit-maximizing objective. The monopolist CRA offers coarse ratings if k_1 and k_2 are small (i.e., $k > \min(k_1, k_2)$). By contrast, competitive CRAs behave in a way that maximizes the expected return of good borrowers which determines whether coarseness or precision arises. If the subsidy to intermediate borrowers, σ_e, is smaller than the cost of precision, then competition among CRAs results in coarse ratings. Whether competition leads to more or less coarseness depends on the deep parameters of the model.

Proposition 8 (Monopoly vs. Competition) *Suppose that $c \leq c_p$. Then, there exist values of k for which monopoly features coarse ratings and competition features precise ratings if $X > \min\{B_1, B_2\}$, where B_1 and B_2 are defined in the Appendix (see Equation* (39)*). If* (39) *is violated, then there exist values of k for which monopoly features precise ratings and competition features coarse ratings.*

The corresponding condition for the case of $c > c_p$ is $X > B_3$, where B_3 is defined in the Appendix (see Equation (40)*).*

Proof. The proof is in the Appendix. ■

The subsidy from good borrowers to bad borrowers in a coarse pool, σ_e, is unaffected by X, so X does not affect the condition under which coarseness arises when CRAs compete (i.e., $k > \sigma_e$). However, k_1 and k_2 are decreasing in X (since $\bar{f}_c$ and $\hat{f}_c$ are increasing in X), implying that the condition for coarseness under monopoly is easier to satisfy for higher values of X. Thus, for X sufficiently small, $\sigma_e < \min(k_1, k_2)$, and there are values of k for which there is precision under monopoly and coarseness under perfect competition. Similarly, for X sufficiently large, $\min(k_1, k_2) < \sigma_e$, and there are values of k for which there is precision under competition and coarseness under monopoly. The result that competition among CRAs may lead to more or less precision (depending on parameters) differs from existing papers – in Lizzeri (1999), competition leads to information revelation, while in Goel and Thakor (2015), competition leads to more coarseness.

5 Policy and empirical implications

In this section, we discuss the new policy implications and empirical predictions yielded by our baseline model.

5.1 Policy implications

Consider a planner whose objective is to maximize net social surplus. Can the planner intervene to improve upon the unregulated outcome? If so, show should the planner intervene? We show that the planner can obtain the efficient outcome by using two tools – influencing the degree of competition among CRAs and regulating the fee charged. In the unregulated equilibrium, two types of inefficiencies may arise:

First, only good borrowers obtain financing when ratings are precise, even though coarse ratings and low fees would elicit effort provision by intermediate borrowers when the cost of exerting effort is sufficiently small. Under monopoly, this inefficient outcome obtains if $k < k_1$ and $c < \bar{c}$. Under competition, this inefficient outcome obtains if $k < \sigma_{e=1}$ and $c \leq c_p$. In monopoly the inefficiency arises due to the CRA's revenue maximization motive, whereas, in competition the inefficiency arises due to the threat of undercutting. The planner obtains the efficient outcome by setting a ceiling on the fee, where the ceiling depends on the market structure in the CRA industry.

Second, suppose that the ratio of good to intermediate borrowers is high, i.e., $\frac{\alpha}{\beta} > \gamma$. If ratings are coarse, then both good and intermediate borrowers obtain financing, but intermediate borrowers may not exert effort which destroys value. Under monopoly this inefficient outcome obtains if $k > k_2$ and $c > \bar{c}$; note that in this case, the monopolist CRA charges a fee, $\hat{f}_c$ and obtains the full surplus generated. Under competition this inefficient outcome obtains if $k > \sigma_{e=0}$ and $c > c_p$. The efficient outcome may thus be obtained by putting a floor on the fee. The planner implements the floor if the value destruction due to the participation of intermediate firms is greater than the total cost of precision:

$$\beta(1 - p_m X) > \alpha k \tag{14}$$

The above condition becomes $k < k_2$. Noting that coarseness (without effort provision by intermediate borrowers) may arise under monopoly only if $k > k_2$, the planner cannot solve this inefficiency by setting a floor on the fee in the monopolist CRA case. The intuition is that since the monopolist CRA keeps the full surplus (when $f = \hat{f}_c$), it acts as the planner would, which rules out any scope for intervention by the planner. When CRAs compete, the floor obtains the efficient outcome when $k \in (\sigma_{e=0}, k_2)$.

We characterize the optimal intervention by the planner in different circumstances in the following proposition.

Proposition 9 (Optimal intervention)

1. *Suppose that $c \leq c_p$. The second-best outcome features coarse ratings and low fees. If ratings are precise, the planner puts a ceiling on the fee which depends on the structure of the CRA industry. The ceiling is $f \leq \bar{f}_c$ under a monopolist CRA, and $f < k$ when CRAs compete.*

2. *Suppose that $c > c_p$ and $k < k_2$. The second-best outcome features precise ratings. Ratings are precise under a monopolist CRA but may be coarse when CRAs compete. The planner puts a floor on the fee, $f \geq k$, if $k \in (\sigma_{e=0}, k_2)$.*

Proof. The proof is in the Appendix. ■

For $c \leq c_p$, the coarse ratings equilibrium is identical to the first-best, in terms of welfare. However, high fees charged by the monopolistic CRA can destroy this equilibrium. Interestingly, although competition among CRAs reduce fees, it does not necessarily lead to welfare-improving coarseness.

For $c > c_p$ and $\frac{\alpha}{\beta} \geq \gamma$, low fees can allow the existence of the coarse-ratings equilibrium. However, for these parameters, intermediate firms obtain financing without exerting effort. To eliminate this undesirable equilibrium, the planner can impose a floor on the fee. For a sufficiently high fee, the coarse-ratings equilibrium collapses, and ratings become precise.

As a corollary of Proposition 9, we present the following result:

Corollary 3 *For $k = 0$ and $c \leq c_p$, regardless of the market structure, the planner prefers coarse ratings, while the equilibrium features precise ratings.*

Strikingly, when the cost of effort provision by intermediate borrowers is sufficiently small, if the cost of information acquisition is very small, the planner *always* prefers coarse ratings, and optimal intervention (in the form of a ceiling on the fee) is necessary to obtain the efficient outcome.

5.2 Empirical implications

In this subsection, we discuss the empirical implication of the analysis.

1. Ratings precision is countercyclical.

   Suppose that there is one CRA. Both good and intermediate firms obtain credit. Suppose that the cost of exerting effort is high, $c > c_p$, such that intermediate firms do not exert effort. Then, as long as the ratio of good to intermediate firms is high, $\frac{\alpha}{\beta} \geq \gamma$, the coarse-ratings equilibrium obtains. However, as the ratio falls below γ, the coarse-ratings equilibrium is no longer viable since it will be characterized by no financing for any firms. In this case, the CRA will offer precise ratings to separate good firms from intermediate firms. Therefore, ratings become more precise as $\frac{\alpha}{\beta}$ falls. Assuming the ratio of good firms to intermediate firms is high in an economic boom and low in an economic downturn, it follows that the precision of ratings will be higher in downturns than in booms, as discussed earlier. The prediction is consistent with the findings in Ashcraft et al. (2010) and Griffin and Tang (2012).

   Griffin and Tang (2012) show that, during the boom period leading up to the global financial crisis of 2007-2009, a top CRA frequently made subjective (not model-based) adjustments to firms' ratings. This increased the number of securities in the highest ratings category, diluting the value of obtaining the highest rating because the highest credit-quality tranches subsidize the lower credit-quality tranches within the same rating category. Further, they find that firms whose ratings were most positively adjusted suffered the biggest downgrades in a future downturn. Viewed through the lens of our model, this observation is consistent with intermediate firms being pooled into the highest category with good firms in booms, but subsequently downgraded in downturns.[10]

[10] Of course, in the real world, we do not observe CRAs altering the number of ratings categories over the business cycle. The following illustrative example clarifies how our model would be applied in practical scenarios. Consider a situation where there are two ratings categories, namely A and B. Category A consists of firm-types p_1 and p_2, whereas category B comprises firm-types p_3 and p_4 (with

2. Higher competition among CRAs leads to more precise ratings when projects are very profitable, and it leads to coarseness, otherwise.

   From Proposition 8, if X is sufficiently large (see conditions (39) and (40)), then there are values of k such that monopoly features coarse ratings and competition features precise ratings, $\sigma_e > k > \min(k_1, k_2)$. For smaller X, there are values of k such that competition features coarse ratings and monopoly features precise ratings, $\min(k_1, k_2) > k > \sigma_e$. Intuitively, the differences between the monopoly and competition cases arise since σ_e (the subsidy from good to intermediate borrowers under perfect competition) is unaffected by X, while the fee charged in the monopoly case is increasing in X.

   The prediction that higher competition among CRAs may lead to more or less coarseness is consistent with conflicting empirical findings. On the one hand, Doherty et al. (2012) and Kisgen and Strahan (2010) find that higher competition among CRAs leads to more informative ratings. On the other hand, Becker and Milbourn (2011) show that increased competition leads to more issuer-friendly and less informative ratings. Our analysis implies that empirical tests of the effect of competition on the degree of ratings coarseness should control for borrower profitability.

3. A lower cost of information production by CRAs leads to lower investment.

   For a lower cost of information production ($k < \sigma_e$), ratings are precise, which drives out intermediate firms and leads to lower aggregate investment. An ideal test of this prediction would involve a shock which lowers the information production costs of CRAs, but not the market. This prediction is yet to be empirically tested.

6 Extensions

6.1 Randomization

In the baseline model, we only examine equilibria in pure strategies – the ratings policy states that all firms of a given type will obtain the same rating. In this extension, we allow for CRAs to randomize. Specifically, the policy states that all good borrowers

$p_1 > p_2 > p_3 > p_4$). During a recession, firm-type p_3 is elevated to category A, while in an economic upturn, it is downgraded.

and a fraction $q \in [0,1]$ of intermediate borrowers will be included in the pool, while the remaining intermediate borrowers will be identified precisely – $q = 0$ is the precise-ratings case, while $q = 1$ is the coarse-ratings case without randomization. With $q < 1$, if an intermediate borrower applies for a rating and is not included in the coarse pool, she will not raise financing, and hence, not pay the fee to the CRA.

Consider the case of the monopolist CRA. The first observation to make is that if $q \in (0,1)$, the CRA must incur the cost of precision, k. By incurring the cost, the CRA would be able to identify an issuer to be of intermediate quality, which is a necessary step before it can randomize.

Does the monopolist CRA randomize when intermediate borrowers do not exert effort? When $q = 0$, i.e., ratings are precise, the full surplus is extracted by the monopolist CRA. By increasing q beyond 0, if the fee is such that intermediate borrowers do not exert effort, the net surplus shrinks for two reasons – lack of effort exertion by the intermediate borrowers destroy value, and the overall increase in the cost of precision from αk (only good borrowers apply for a rating) to $(\alpha+\beta)k$ (both good and intermediate borrowers apply for a rating). Thus, randomization (i.e., $0 < q < 1$) will not be chosen if intermediate borrowers do not exert effort.

Next we show that the monopolist CRA does not randomize when intermediate borrowers exert effort. Assuming that intermediate borrowers exert effort, for $0 < q < 1$ and a given fee, f, the pooling interest rate becomes:

$$R^P{}_{e=1}(q) = \frac{(\alpha + q\beta)(1+f)}{\alpha p_g + q\beta(p_m + \delta)} \tag{15}$$

$R^P{}_{e=1}(q)$ is increasing in q since the inclusion of more intermediate borrowers in the coarse pool worsens the average quality of the pool. Thus, a higher q reduces the subsidy that each intermediate borrower obtains, which tightens their effort moral hazard constraint. We derive the maximum fee below which intermediate borrowers exert effort by solving $R^P{}_{e=1}(q) = \underline{R}$:

$$\bar{f}_c(q) = \frac{1}{\delta(\alpha + q\beta)}\left[(\delta X - c)(\alpha p_g + q\beta(p_m + \delta))\right] - 1 \tag{16}$$

The net fee revenue of the CRA is $\Pi(q) = (\alpha + q\beta)\bar{f}_c(q)$. CRAs choose q to maximize its revenue. Differentiating $\Pi(q)$ with respect to q:

$$\frac{\partial\Pi(q)}{\partial q} = \frac{1}{\delta}[\beta(p_m + \delta)(\delta X - c) - \delta\beta] \tag{17}$$

The derivative is negative for $c > c_s$ (which is assumed in *A2*). That the derivative is negative implies that a CRA's fee revenue is falling in q and the CRA sets q as small as possible, $q = 0$, i.e., there is no randomization.

Moving to the case in which CRAs compete, allowing for randomization does not affect the equilibrium in Proposition 7. Under $q \in (0, 1)$, the fee is $f = k$ which is the marginal cost of producing the rating. From the perspective of good borrowers, this outcome is strictly dominated by the precise-ratings case since the fee would be the same in both cases, but with precise ratings the quality of the rated pool is higher and investors charge a lower interest rate. A CRA can always deviate from a $q \in (0, 1)$ equilibrium by offering precise ratings and attracting all good borrowers. Thus, competition among CRAs would prevent randomization by CRAs.

The above discussion shows that randomization does not arise in either the monopoly or the competition cases. However, from the planner's perspective, it would always be feasible to improve upon a precise-ratings equilibrium by enforcing a $q > 0$ policy − if q is sufficiently small, the subsidy that the included intermediate borrowers obtain in the pool is large which elicits effort provision. The planner would implement this policy when $c > c_p$, since for $c \leq c_p$ coarse ratings and low fees achieve the first best outcome.

Proposition 10 (Optimal randomization by planner) *If $c > c_p$ and k is small, the optimal outcome sets $f = k$ and q such that the ${R^P}_{e=1}(q) = \underline{R}$:*

$$q^P = \frac{\alpha p_g(\delta X - c) - \delta\alpha(1 + k)}{\beta(\delta(1 + k) - (p_m + \delta)(\delta X - c))} \tag{18}$$

The necessary and sufficient condition for randomization to be optimal is the total benefit of randomization to the economy, $q^P\beta((p_m + \delta)X - 1)$, exceeds the total cost, $(\alpha + \beta)k$.

Proposition 10 implies that some degree of coarseness is always optimal if k is not too high.

6.2 A more general model

In the baseline model, firms in only one ratings category obtain financing. Below we present an alternative model in which firms in multiple ratings categories obtain financing and welfare-improving coarseness arises across the ratings spectrum.

There are $2N$ types of firms and each firm privately knows its type. A firms of type j has success probability p_j, with $j \in \{1, 2, ..2N\}$, with $p_1 < p_2 < .. < p_{2N}$, with $p_1 X - 1 > 0$. The fraction of each firm-type in the economy is $\frac{1}{2N}$. Additionally, there are infinitely many bad firms with success probability, $p_b = \epsilon$ where ϵ is arbitrarily close to 0. Any pool of borrowers containing these bad firms will have an expected NPV that is negative, implying that firms of type p_j must be rated to obtain financing in the market. Each firm of type j may improve its success probability by δ by exerting unobservable effort at a cost, $c(\delta) = \frac{\tau}{2}\delta^2$. The cost function is convex in δ, and τ is a strictly positive constant, reflecting the marginal value of effort.

CRAs are competitive and observe that any adjacent pair of firms, $2j-1$ and $2j$, will have a lower success probability than the next pair, $2j+1$ and $2j+2$, at zero cost. CRAs incur a cost of precision, $k > 0$, to distinguish the adjacent types, $2j-1$ and $2j$. That is, for example, a CRA observes at zero cost that firms of type $j = 1$ and $j = 2$ have a lower success probability than firms of type $j = 3$ and $j = 4$, but it must incur the cost of precision to distinguish firms of type $j = 1$ from firms of type $j = 2$ (and firms of type $j = 3$ from type $j = 4$). CRAs generate a rating that is either precise or coarse and CRAs' fee, f, equals the marginal cost of producing the rating, i.e., $f = k$ for precise ratings and $f = 0$ for coarse ratings.

If a firm of type j is precisely identified, it promises a repayment, R_j, and chooses δ subject to investors' participation constraint (which binds):

$$\delta_j{}^* = \arg\max(p_j + \delta_j)(X - R_j) - \frac{\tau}{2}\delta_j{}^2 \tag{19}$$

$$\text{subject to } (p_j + \delta_j{}^*)R_j - 1 = 0 \tag{20}$$

The first-best effort level is derived by ignoring moral hazard, and is given as follows:

$$\delta^{fb} = \frac{X}{\tau} \tag{21}$$

The solution of the problem taking into account the moral hazard friction is as follows[11]:

$$\delta_j{}^* = \frac{X - p_j\tau + \sqrt{(X - p_j\tau)^2 + 4\tau(p_jX - 1)}}{2\tau} \tag{22}$$

Lemma 7 *$\delta_j{}^*$ is increasing in p_j.*

Proof. The proof is in the Appendix. ■

Higher types have a higher non-effort component of success probability (i.e., p_j is increasing in j). This implies that the debt issued by higher types has a smaller face value. This leads to higher effort provision by higher types since they retain more of the surplus generated from exerting effort (i.e., $R_j < R_{j+1}$ leads to $\delta_j{}^* > \delta_{j+1}{}^*$). However, the equilibrium effort level is below the first-best level since the moral hazard constraint binds.

Consider any adjacent pair, $2j-1$ and $2j$. Under coarse ratings, both firms promise a repayment, $\hat{R}$, and choose δ subject to the participation constraint of investors (which binds):

$$\delta^P = \arg\max (p_j + \delta)(X - \hat{R}) - \frac{\tau}{2}\delta^2 \tag{23}$$

$$\text{subject to } (p_j + \delta^P)\hat{R} - 1 = 0 \tag{24}$$

The solution of this problem is as follows:

$$\delta^P = \frac{X - \frac{1}{2}(p_{2j-1} + p_{2j})\tau + \sqrt{(X - \frac{1}{2}(p_{2j-1} + p_{2j})\tau)^2 + 4\tau(\frac{1}{2}(p_{2j-1} + p_{2j})X - 1)}}{2\tau} \tag{25}$$

Given the same repayment, $\hat{R}$, both firms choose the same level of effort, improving their success probability by δ^P (independent of type). When there is pooling of two types, the lower type exerts more effort and the higher type exerts less effort than when types are precisely identified, i.e., $\delta_{2j}{}^* > \delta^P > \delta_{2j-1}{}^*$. The intuition is that in a coarse pool, resources are diverted from the higher type to the lower type, which negatively affects the higher type's incentives and positively affects the lower type's incentives. The subsidy

[11] Note that for $p_jX - 1 > 0$, as is assumed above, the smaller root is negative. Since δ must be positive, we disregard the negative root and consider only the positive root.

that firms of type $2j$ provide in the pool with firms of type $2j-1$ is:

$$p_{2j}(\hat{R}_{2j,2j-1} - R_{2j}) \equiv \sigma_{2j} \tag{26}$$

In the following proposition, we characterize the equilibrium.

Proposition 11 *Consider any adjacent pair, $2j-1$ and $2j$. The net social surplus generated is higher under coarse ratings in which the adjacent types are pooled than under precise ratings. Ratings are coarse if $\sigma_{2j} > k$.*

Proof. The proof is in the Appendix. ■

7 Conclusion

We have presented a model of credit ratings in which coarse ratings may arise as an equilibrium outcome and ratings precision is countercyclical. Compared to the precise-ratings case, coarse ratings introduce asymmetric information by forcing a pooling equilibrium. In existing models, the pooling generated by ratings coarseness is undesirable since it leads to inefficient investment (e.g., Goel and Thakor 2015). However, given the informational frictions we consider, the pooling equilibrium can enhance net social surplus. The pooling equilibrium leads to cross-subsidization across firm types, with intermediate firms benefiting from more favorable terms due to pooling with good firms. The cross-subsidization induces intermediate firms to exert effort, when they would not have done so if their securities were accurately priced.

Thus, our model delivers "a bright side of coarseness" – coarseness can increase net social surplus even if CRAs have very precise information. The critics of the issuer-pays model suggest that it may lead to coarser information communication by CRAs. Our model highlights that such coarseness need not be a negative outcome from a social welfare standpoint. We show that the efficient (second-best) outcome can be achieved by regulating the fee charged by CRAs. Depending on whether coarse or precise ratings are more socially desirable, the regulator would put a ceiling or a floor on the fee.

Appendix: Proofs

Proof of Proposition 2.

Proof. Given that firm-types are observable, each type's debt is priced such that investors' participation constraint facing that type is satisfied. In order to satisfy investors' participation constraint, the interest rate offered by good firms must be $R \geq R_g$. Suppose that good firms obtain financing in equilibrium at interest rate, $R = R' > R_g$. A good firm will deviate by offering an infinitesimally lower interest rate, $R' - \epsilon$, and this will be accepted by investors since they are strictly better off compared to their outside option for any $R > R_g$. By this argument, no interest rate other than $R = R_g$ survives in equilibrium. Given Assumption *A2* that $c > c_s$, the actuarially fair interest rate when intermediate borrowers exert effort, R_m, is bigger than the interest rate below which intermediate firms exert effort, $\underline{R} < R_m$. This implies that the investment of intermediate firms will be negative-NPV and so for any $R \leq X$, investors make losses. Hence, intermediate firms do not receive financing. Similarly, bad firms do not receive financing since their investment is negative-NPV and for any $R \leq X$, investors make losses. ■

Proof of Proposition 3.

Proof. We first show that an equilibrium with financing cannot exist. For any interest rate, $R < X$, all three types of firms are strictly better off if they obtain financing, compared to the case in which they do not obtain financing. Hence, all firm-types seek financing. By Assumption *A3*, the expected NPV across all three firm types is negative. Thus, the contract will be loss-making for the investors, and hence, investors will not offer financing.

Consider now the case that the market breaks down in equilibrium, i.e., no firm obtains financing. A firm may deviate by offering a contract with a promised repayment, $R < X$. Regardless of the firm-type, this deviation, if the offer is accepted, makes the deviant firm strictly better off compared to the no-financing equilibrium. Exactly because this deviation makes all types of firms strictly better off, the Intuitive criterion does not have a bite (step 1 does not rule out any type as a potential defector), so there exists a strictly positive set of beliefs that the offer comes from a bad firm. If the deviating firm is bad, the expected payoff for the investor is negative, so such an offer will be rejected. Thus, the market breakdown equilibrium is the unique equilibrium which survives the Intuitive criterion and is consistent with zero expected profits for investors. ■

Proof of Proposition 4.

Proof. Suppose that the fee is given by $f_p = p_g X - 1 + \epsilon$, where ϵ may be positive or negative. Substituting in R_g (Equation 1), investors' participation constraint is satisfied for any $\epsilon \leq 0$ and violated for $\epsilon > 0$. Additionally, the fee cannot be negative to ensure the participation of CRAs. ■

Proof of Lemma 1.

Proof. It is never an equilibrium for all firm types to be given the same rating since this does not produce any information and is equivalent to the no-ratings case. Thus, from Proposition 3, if all firm types are given the same rating the market breakdown equilibrium is the unique equilibrium. Given that there are three firm-types, coarse ratings in our model always entail two categories and can take the following forms:

1. Good and intermediate firms are pooled together in a single category, and bad firms are in a separate category.
2. Intermediate and bad firms are pooled together in a single category, and good firms are in a separate category.
3. Good and bad firms are pooled together in a single category, and intermediate firms are in a separate category.

Option 2 is equivalent to the precise-ratings case since good firms obtain financing and neither intermediate nor bad firms obtain financing. Option 3 is equivalent to the no-ratings case. Given Assumption *A4*, the average NPV of the pool consisting of good and bad firms is negative, so neither good nor bad firms obtain financing under option 3. Also, intermediate firms do not obtain financing, given Assumption *A2*. In option 1, both good and intermediate firms may obtain financing for some parameters (e.g., if $\beta \to 0$, then the pool of good and intermediate borrowers is comparable to the pool of only good borrowers, and will obtain financing), which implies that this option is not equivalent to the no-ratings or precise-ratings cases. ■

Proof of Lemma 2.

Proof. With coarse ratings, the debt is priced as being issued by a pool consisting of intermediate and good firms. Suppose that intermediate firm exerts effort, i.e., $e = 1$. Given a pooling interest rate, ${R^P}_{e=1}$, the zero profit condition of the investors is:

$$\frac{\alpha}{\alpha+\beta} p_g {R^P}_{e=1} + \frac{\beta}{\alpha+\beta}(p_m+\delta){R^P}_{e=1} - 1 - f = 0 \tag{27}$$

Solving, we derive the interest rate if the ratings are coarse and intermediate firms exert effort:

$${R^P}_{e=1} = \frac{(\alpha+\beta)(1+f)}{\alpha p_g + \beta(p_m+\delta)} \tag{28}$$

If intermediate firms do not exert effort, i.e., $e = 0$, then given a pooling interest rate, ${R^P}_{e=0}$, the zero profit condition of the investors is:

$$\frac{\alpha}{\alpha+\beta} p_g {R^P}_{e=0} + \frac{\beta}{\alpha+\beta} p_m {R^P}_{e=0} - 1 - f = 0 \tag{29}$$

Solving, we derive the interest rate if the ratings are coarse and intermediate firms do not exert effort:

$${R^P}_{e=0} = \frac{(\alpha+\beta)(1+f)}{\alpha p_g + \beta p_m} \tag{30}$$

■

Proof of Lemma 3.

Proof. The pooling interest rate, ${R^P}_{e=1}$, is consistent with the beliefs of investors that intermediate firm exerts effort if it is smaller than the incentive compatible interest rate, i.e., ${R^P}_{e=1} \leq \underline{R}$. Assuming $f = 0$, ${R^P}_{e=1} \leq \underline{R}$ if:

$$c \leq \delta X - \frac{\delta(\alpha+\beta)}{\alpha p_g + \beta(p_m+\delta)} \equiv c_p \tag{31}$$

If the cost of effort is sufficiently small, i.e., $c \leq c_p$, intermediate firm exerts effort if $f = 0$ and the equilibrium with $R = {R^P}_{e=1}$ exists. This implies that for $c > c_p$, the investors' beliefs that intermediate firms exert effort are not fulfilled which implies that the equilibrium with $R = {R^P}_{e=1}$ cannot exist for any non-zero f.

Using c_s from Assumption *A2* and (7):

$$
\begin{aligned}
& c_p > c_s \\
\implies & \delta X - \frac{\delta(\alpha+\beta)}{\alpha p_g + \beta(p_m+\delta)} > \delta X - \frac{\delta}{p_m+\delta} \\
\implies & \frac{1}{p_m+\delta} > \frac{\alpha+\beta}{\alpha p_g + \beta(p_m+\delta)} \\
\implies & p_g > p_m + \delta
\end{aligned} \tag{32}
$$

The above condition is always satisfied due to Assumption *A2*. ■

Proof of Proposition 5.

Proof. Suppose that the fee is given by $f = \bar{f}_c + \epsilon$, where ϵ may be positive or negative. Substituting in the effort moral hazard constraint, ${R^P}_{e=1}$ (Equation (28)), intermediate borrowers' incentive compatibility constraint is satisfied (i.e., ${R^P}_{e=1} \leq \underline{R}$) only if $\epsilon \leq 0$ and violated for $\epsilon > 0$. A fee lower than $\bar{f}_c$ makes the constraint less binding. However, the fee cannot be negative to ensure the participation of CRAs. Thus, a coarse-ratings equilibrium with effort provision by intermediate borrowers may arise if $f \in [0, \bar{f}_c]$. For $\frac{\alpha}{\beta} < \gamma$, the average NPV of the pool is negative unless intermediate borrowers exert effort, which implies that for $f > \bar{f}_c$ the market breaks down.

For $\frac{\alpha}{\beta} \geq \gamma$, it is possible that there is financing even if intermediate borrowers do not exert effort, i.e., the fee is high, $f > \bar{f}_c$. If investors hold the belief that intermediate firms do not exert effort, i.e., $e = 0$, then the pooling interest rate is ${R^P}_{e=0}$. Suppose that the fee is given by $f = \hat{f}_c + \epsilon$, where ϵ may be positive or negative. Substituting in ${R^P}_{e=0}$, investors' participation constraint is satisfied for any $\epsilon \leq 0$ and violated for $\epsilon > 0$. Thus, a coarse-ratings equilibrium without effort provision by intermediate borrowers may arise if $\bar{f}_c < f \leq \hat{f}_c$. ■

Proof of Lemma 4.

Proof. The monopolist CRA charges the maximum possible fee given the rating policy and subject to satisfying investors' participation constraints. Thus, if ratings are precise, $f = f_p$, where f_p is the maximum fee under precise ratings which satisfies investors' participation constraint (see Proposition 4). Under coarse ratings and for $\frac{\alpha}{\beta} < \gamma$, the fee must be such that intermediate borrowers exert effort, $f = \bar{f}_c$, where $\bar{f}_c$ is the maximum fee under coarse ratings consistent with effort exertion by intermediate borrowers (see Proposition 5). For $\frac{\alpha}{\beta} \geq \gamma$, the fee is $f = \bar{f}_c$ if $\bar{f}_c \geq \hat{f}_c$, and $f = \hat{f}_c$, otherwise; intermediate borrowers exert effort in the former case, while not in the latter. ■

Proof of Lemma 5.

Proof. $f_p > \max(\bar{f}_c, \hat{f}_c)$ follows directly from Assumptions *A1* and *A2*. $\bar{f}_c > \hat{f}_c$ if $c \leq \bar{c}$ (see Equation (10)). $\bar{c} > c_s$ if:

$$\beta\delta X(p_m + \delta) > (\alpha p_g + \beta(p_m + \delta))((p_m + \delta)X - 1) \tag{33}$$

$$\implies \alpha < \frac{1}{p_g}\frac{\beta\delta X(p_m + \delta) - \beta(p_m + \delta)((p_m + \delta)X - 1)}{(p_m + \delta)X - 1} \tag{34}$$

$$\implies \alpha < \frac{1}{p_g}\frac{\beta(p_m + \delta)\overbrace{(1 - p_m X)}^{>0 \text{ from } A1}}{\underbrace{(p_m + \delta)X - 1}_{>0 \text{ from } A2}} \tag{35}$$

α lies between 0 and 1, while the RHS is positive. The above condition is satisfied for α sufficiently small. Assumptions *A1* and *A2* do not feature α, while *A3* and *A4* put upper bounds on it. Thus, it is always feasible to set α to be sufficiently small such that the above condition is satisfied without violating any of the parametric restrictions. ■

Proof of Proposition 6.

Proof. The monopolist CRA chooses ratings policy and fee to maximize its net profits. Under precise ratings, the CRA incurs a cost, k, and only good borrowers purchase ratings. The total profit is $\alpha(f_p - k)$. Under coarse ratings, the CRA does not incur a cost, and both good and intermediate borrowers purchase ratings. The total profit is $(\alpha + \beta)\max(\bar{f}_c, \hat{f}_c)$. $(\alpha + \beta)\bar{f}_c > \alpha(f_p - k)$ if $k \geq k_1$ (see Equation (11)) and $(\alpha + \beta)\hat{f}_c > \alpha(f_p - k)$ if $k \geq k_2$ (see Equation (12)). In a coarse-ratings equilibrium, the CRA chooses the incentive compatible fee if $\bar{f}_c > \hat{f}_c$ which holds if the cost of effort provision is sufficiently small, $c \leq \bar{c}$ (see Equation (10)).

For $\frac{\alpha}{\beta} \geq \gamma$, the average NPV of good and intermediate firms is positive, whether or not intermediate firms exert effort. If $k > \min(k_1, k_2)$, the CRA maximizes its net profits by offering coarse ratings and charging a fee which depends on the cost of exerting effort: the fee is $f = \bar{f}_c$ if $c \leq \bar{c}$ and $f = \hat{f}_c$ if $c > \bar{c}$. Rational investors hold the correct beliefs in equilibrium and set the interest rate such that they break even, on average.

Intermediate firms certainly prefer coarse ratings because they are subsidized by good firms. Regarding good firms, they are worse off with coarse ratings compared to precise ratings, but given that only coarse ratings are offered, they prefer coarse ratings to no ratings. The reason is that if a good firm chooses no rating, it will get no financing. Therefore, good firms will also purchase coarse ratings. If the cost of producing precise ratings is small, $k < \min(k_1, k_2)$, the CRA maximizes its net profits by offering precise ratings and charging high fees, $f = f_p$.

If, on the other hand, $\frac{\alpha}{\beta} < \gamma$, coarse ratings and high fees, $f > \bar{f}_c$ imply that there is market breakdown and no firm obtains credit. This leads to a total fee revenue of 0, which is less than the total fee with precise ratings or coarse ratings and low fees, $f = \bar{f}_c$. Hence, for $\frac{\alpha}{\beta} < \gamma$, the equilibrium with one CRA features precise ratings and $f = f_p$ if $k < k_1$ and coarse ratings with $f = \bar{f}_c$ if $k > k_1$.

Finally, we need to show that there exist feasible parameters for which coarseness, with or without effort provision by intermediate borrowers, may arise. To show that coarseness with (resp. without) effort provision is feasible we need to check that $k > k_1$ (resp. $k > k_2$) are feasible without violating the upper bound on k in Assumption *A1*, $k < p_g X - 1$.

To see that parameters exist such that coarseness with effort provision arises, substitute $c = c_s$ (this is the lower bound of c from Assumption *A2*) in k_1. k_1 becomes:

$$k_1 = p_g X - \frac{p_g}{p_m + \delta} < p_g X - 1 \tag{36}$$

Thus, for $c = c_s$, k_1 is smaller than the upper bound on k since $p_g > p_m + \delta$ (which is true by Assumption *A2*). Hence, there may exist $k > k_1$ which does not violate Assumption *A1*. For these k, the equilibrium features coarse ratings and intermediate borrowers exert effort if $c < \bar{c}$.

To see that parameters exist such that coarseness without effort provision arises, consider the case $\frac{\alpha}{\beta} \geq \gamma$ and $c > \bar{c}$. Assuming $1 - p_m X = \epsilon$ (where ϵ is positive but arbitrarily close to 0) does not violate any assumptions. Given this assumption, $k_2 \to 0$ as $\epsilon \to 0$. Since the upper bound of k from Assumption *A1*, $p_g X - 1$, is strictly positive, there may exist $k < p_g X - 1$ such that $k > k_2$. For these k, the equilibrium features coarse ratings and intermediate borrowers do not exert effort. ■

Proof of Corollary 1.

Proof. To prove the corollary, we need to show that $\min(k_1, k_2) > 0$. k_2 is always positive since $1 - p_m X > 0$ (by Assumption *A1*). To see that k_1 is always positive, first note that k_1 is increasing in c:

$$\frac{\partial k_1}{\partial c} = \frac{1}{\alpha\delta}(\alpha p_g + \beta(p_m + \delta)) > 0 \tag{37}$$

Next, we substitute in k_1 the lower bound of c from Assumption *A2*, $c = c_s$. k_1 becomes:

$$k_1 = \frac{p_g c_s}{\delta} > 0 \tag{38}$$

k_1 is positive when $c = c_s$. Therefore, since k_1 is increasing in c, it must be that case that k_1 is positive for any $c > c_s$. ■

Proof of Proposition 7.

Proof. Suppose that $k > \sigma_e$, where $e = 1$ if $c \leq c_p$, and $e = 0$ if $c > c_p$ and $\frac{\alpha}{\beta} \geq \gamma$. First, we show that precise ratings cannot be an equilibrium. Under precise ratings, firms pay a fee $f = k$. Suppose that a CRA deviates by offering coarse ratings and a lower fee, ϵ, such that $k - \epsilon > \sigma_e$. Then, good firms find it profitable to deviate since the fall in the fee is higher than the subsidy it would provide by deviating to the coarse-ratings equilibrium, thereby eliminating the precise-ratings equilibrium. Now we show that coarse-ratings is an equilibrium. Under coarse ratings, firms pay a fee $f = 0$. Suppose that a CRA deviates by offering precise ratings. The lowest feasible fee that the CRA charges is $f = k$. Even for this fee, a good firm is worse off with precise ratings since the increase in the fee would be higher than the subsidy it provides, i.e., $k > \sigma_e$. Thus, the deviating CRA cannot attract good firms and the equilibrium sustains.

Suppose that $k < \sigma_e$. First, we show that coarse ratings cannot be an equilibrium. Under coarse ratings, firms pay a fee $f = 0$. Suppose that a CRA deviates by offering precise ratings and a fee, $f = k + \epsilon$, such that $k + \epsilon < \sigma_e$. Then, good firms find it profitable to deviate since the increase in the fee is lower than the subsidy it provides, thereby eliminating the coarse-ratings equilibrium. Next, we show that precise ratings is an equilibrium. Under precise ratings, firms pay a fee $f = k$. Suppose that a CRA deviates by offering coarse ratings and the lowest feasible fee, $f = 0$. Even for this fee, a good firm is worse off with coarse ratings since the fall in the fee would be lower than the subsidy it would provide, i.e. $k < \sigma_e$. Thus, the deviating CRA cannot attract good firms and the equilibrium sustains. ■

Proof of Proposition 8.

Proof. If $c \leq c_p$, under perfect competition in the CRA industry, coarse ratings arise if $k > \sigma_{e=1}$ and are characterized by effort provision by intermediate firms. If $\min(k_1, k_2) < \sigma_{e=1}$, then for $k \in (\min(k_1, k_2), \sigma_{e=1})$, monopoly features coarse ratings and competition features precise ratings. $\min(k_1, k_2) < \sigma_{e=1}$ exists if the following condition is satisfied:

$$\begin{aligned} X > \min\Bigg[& -\frac{\alpha((\alpha - (c(\alpha p_g + \beta(p_m + \delta)) + \delta(\alpha + \beta))/\delta)/\alpha + (\beta(p_g - (p_m + \delta)))/(\alpha p_g + \beta(p_m + \delta)))}{\beta(p_m + \delta)} \equiv B_1, \\ & \frac{(\alpha + \beta)(p_m + \delta)}{p_m(\alpha p_g + \beta(p_m + \delta))} \equiv B_2 \Bigg] \end{aligned} \tag{39}$$

If, on the other hand, condition (39) is violated, there exists $\sigma_{e=1} < \min(k_1, k_2)$, implying that for $k \in (\sigma_{e=1}, \min(k_1, k_2))$, monopoly features precise ratings and competition features coarse ratings.

If $c > c_p$, under perfect competition in the CRA industry, coarse ratings arise if $k > \sigma_{e=0}$ and are characterized by no effort provision by intermediate firms. Given that $c > c_p$, $f = \bar{f}_c$ is not feasible, i.e., we consider the case, $k_1 > k_2$. If $k_2 < \sigma_{e=0}$, then for $k \in (k_2, \sigma_{e=0})$, monopoly features coarse ratings and competition features precise ratings. $k_2 < \sigma_{e=0}$ exists if the following condition is satisfied:

$$X > \frac{\alpha + \beta}{\alpha p_g + \beta p_m} \equiv B_3 \tag{40}$$

If, on the other hand, condition (40) is violated, there exists $\sigma_{e=0} < k_2$, implying that for $k \in (\sigma_{e=0}, k_2)$, monopoly features precise ratings and competition features coarse ratings.

Next, we show through different examples that there exist feasible parameters for which each of the conditions in Equations (39) and (40) is satisfied.

Example 1:

Suppose that $c < \bar{c}$, implying that $k_1 < k_2$. Assume $c = c_s$ and $(p_m + \delta)X - 1 = \epsilon$, where ϵ is positive but arbitrarily small. k_1 and $\sigma_{e=1}$ become:

$$k_1 = \frac{p_g \epsilon}{p_m + \delta} \tag{41}$$

$$\sigma_{e=1} = \frac{\beta(p_g X - 1 - \epsilon)}{\alpha p_g X + \beta(1+\epsilon)} \tag{42}$$

As $\epsilon \to 0$, $k_1 \to 0 < \sigma_{e=1} \to \frac{\beta(p_g X - 1)}{\alpha p_g X + \beta} > 0$. This implies that for $k \in (k_1, \sigma_{e=1})$, there is coarseness with effort provision by intermediate borrowers in the monopoly case and precision in the competition case.

Example 2:

Assume that $k_1 = k_2$ and $p_g = p_m + \delta + \epsilon$, where ϵ is positive but arbitrarily small. $\sigma_{e=1}$ becomes:

$$\sigma_{e=1} = \frac{\beta \epsilon}{\alpha p_g + \beta(p_g - \epsilon)} \tag{43}$$

As $\epsilon \to 0$, $\sigma_{e=1} \to 0 < k_1 = k_2 = \frac{\beta(1 - p_m X)}{\alpha} > 0$. This implies that for $k \in (\sigma_{e=1}, k_1)$, there is coarseness with effort provision by intermediate borrowers in the competition case and precision in the monopoly case.

Example 3:

Suppose that $c > \bar{c}$, implying that $k_2 < k_1$. $k_2 < \sigma_{e=1}$ if $X > B_2$ (see Equation (39)). The upper bound on X comes from Assumption *A1*, $p_m X - 1 < 0$ (assuming that α and β are small enough that Assumptions *A3* and *A4* are satisfied). Suppose that $p_m X - 1 = -\epsilon, where \epsilon$ is positive but arbitrarily small. $X > B_2$ becomes:

$$X = \frac{1-\epsilon}{p_m} > \frac{(\alpha+\beta)(p_m+\delta)}{p_m(\alpha p_g + \beta(p_m+\delta))} \tag{44}$$

$$\implies \alpha p_g - \epsilon \underbrace{(\alpha p_g + \beta(p_m+\delta))}_{\to 0 \text{ when } \epsilon \to 0} > \alpha(p_m+\delta) \tag{45}$$

As $\epsilon \to 0$, the above condition is satisfied, i.e., $k_2 < \sigma_{e=1}$ exists. This implies that for $k \in (k_2, \sigma_{e=1})$, there is coarseness without effort provision by intermediate borrowers in the monopoly case and precision in the competition case.

Using the same steps as above when $c > c_p$, it can be shown that $k_2 < \sigma_{e=0}$ exists. This implies that for $k \in (k_2, \sigma_{e=0})$, there is coarseness without effort provision by intermediate borrowers in the monopoly case and precision in the competition case.

Example 4:

Suppose that $c > \bar{c}$, implying that $k_2 < k_1$. $k_2 > \sigma_{e=1}$ if $X < B_2$ (see Equation (39)). The lower bound on X comes from Assumption *A1*, $p_g X - 1 > 0$. Suppose that $p_g X - 1 = \epsilon$, where ϵ is positive but arbitrarily small. $X < B_2$ becomes:

$$X = \frac{1+\epsilon}{p_g} < \frac{(\alpha+\beta)(p_m+\delta)}{p_m(\alpha p_g + \beta(p_m+\delta))} \tag{46}$$

$$\implies \beta \underbrace{(p_m - p_g)}_{-ve}(p_m+\delta) + \epsilon \underbrace{(p_m(\alpha p_g + \beta(p_m+\delta)))}_{\to 0 \text{ when } \epsilon \to 0} < \alpha p_g \delta \tag{47}$$

As $\epsilon \to 0$, the above condition is satisfied since the LHS is negative while the RHS is positive, i.e., $k_2 > \sigma_{e=1}$ exists. This implies that for $k \in (\sigma_{e=1}, k_2)$, there is coarseness with effort provision by intermediate borrowers in the competition case and precision in the monopoly case.

Using the same steps as above when $c > c_p$, it can be shown that $k_2 > \sigma_{e=0}$ exists. This implies that for $k \in (k_2, \sigma_{e=0})$, there is coarseness without effort provision by intermediate borrowers in the competition case and precision in the monopoly case. ■

Proof of Proposition 9.

Proof. The proof mostly follows from the discussion in the text. It remains to be shown that $\sigma_{e=0} < k_2$ exists. Using Equations (13) (setting $\delta = 0$) and (12), $\sigma_{e=0} < k_2$ simplifies as follows:

$$\alpha < \frac{(\alpha p_g + \beta p_m)\overbrace{(1 - p_m X)}^{>0 \text{ from A1}}}{p_g - p_m} \tag{48}$$

α lies between 0 and 1, while the RHS is positive. The above condition is satisfied for α sufficiently small. Assumptions *A1* and *A2* do not feature α, while *A3* and *A4* put upper bounds on it. Thus, it is always feasible to set α to be sufficiently small such that the above condition is satisfied without violating any of the parametric restrictions. ■

Proof of Lemma 7.

Proof. Taking the derivative of ${\delta_j}^*$ with respect to p_j, we obtain:

$$\frac{\partial {\delta_j}^*}{\partial p_j} = -\frac{1}{2} + \frac{1}{2}\frac{X + p_j\tau}{((X + p_j\tau)^2 - 4\tau)^{0.5}} \tag{49}$$

The derivative is positive if:

$$X + p_j\tau > ((X + p_j\tau)^2 - 4\tau)^{0.5} \tag{50}$$

$$\implies (X + p_j\tau)^2 > (X + p_j\tau)^2 - 4\tau \tag{51}$$

$$\implies \tau > 0 \tag{52}$$

This condition is always satisfied. Hence, ${\delta_j}^*$ is increasing in p_j. ■

Proof of Proposition 11.

Proof. The surplus generated by a firm of type j under precise ratings is ${U^*}_j$.

$${U^*}_j = (p_j + {\delta_j}^*)X - 1 - \frac{\tau}{2}{\delta_j}^2 \tag{53}$$

Moving from precise to coarse ratings, a transfer from the higher to the lower type implies that firms of type $2j-1$ (the lower type in the pooling) exert a higher effort, $\delta^P > {\delta_{2j-1}}^*$, while firms of type $2j$ exert a lower effort, $\delta^P < {\delta_{2j}}^*$. However, given the convexity of the cost function, for any given transfer, the increase in δ for the lower type would exceed in absolute terms the fall in δ of the higher type. Thus, the average δ increases, while remaining lower than the first-best level (since the moral hazard constraint binds). This implies that coarseness leads to a higher net social surplus. If $\sigma_{2j} > k$, then in the unique equilibrium, firms of type $2j-1$ and $2j$ are pooled together through coarse ratings. A further coarsening of ratings is not feasible due to competition among CRAs. ■

References

Ali, S. N., Haghpanah, N., Lin, X., and Siegel, R. (2022). How to sell hard information. *Quarterly Journal of Economics*, 137(1):619–678.

Allen, F. (1990). The market for information and the origin of financial intermediation. *Journal of Financial Intermediation*, 1(1):3–30.

Ashcraft, A., Goldsmith-Pinkham, P., and Vickery, J. (2010). Mbs ratings and the mortgage credit boom. *Working paper*.

Bar-Isaac, H. and Shapiro, J. (2011). Credit ratings accuracy and analyst incentives. *American Economic Review, Papers and Proceedings*, 101(3):120–124.

Bar-Isaac, H. and Shapiro, J. (2013). Ratings quality over the business cycle. *Journal of Financial Economics*, 108(1):62–78.

Becker, B. and Milbourn, T. (2011). How did increased competition affect credit ratings? *Journal of Financial Economics*, 101(3):493–514.

Bolton, P., Freixas, X., and Shapiro, J. (2012). The credit ratings game. *Journal of Finance*, 67(1):85–111.

Boot, A., Milbourn, T., and Schmeits, A. (2006). Credit ratings as coordination mechanisms. *Review of Financial Studies*, 19(1):81–118.

Bouvard, M., Chaigneau, P., and de Motta, A. (2015). Transparency in the financial system: Rollover risk and crises. *Journal of Finance*, 70(4):1805–1837.

Bouvard, M. and Levy, R. (2018). Two-sided reputation in certification markets. *Management Science*, 64(10):4755–4774.

Cho, I.-K. and Kreps, D. M. (1987). Signaling games and stable equilibria. *Quarterly Journal of Economics*, 102(2):179–221.

Dang, T. V., Gorton, G., Holmstrom, B., and Ordonez, G. (2017). Banks as secret keepers. *American Economic Review*, 107(4):1005–1029.

Doherty, N., Kartasheva, A., and Phillips, R. (2012). Information effect of entry into credit ratings market: The case of insurers' ratings. *Journal of Financial Economics*, 106(2):308–330.

Elliott-Graves, A. (2020). The value of imprecise prediction. *Philosophy Theory and Practice in Biology*, 12(4).

Farkas, M. (2021). Competition, communication and rating bias. *Journal of Economic Behavior & Organization*, 189:637–656.

Frenkel, S. (2015). Repeated interaction and rating inflation: A model of double reputation. *American Economic Journal: Microeconomics*, 7(1):250–80.

Gal-or, E. (1987). First-mover disadvantages with private information. *Review of Economic Studies*, 54(2):279–292.

Gal-or, E. (1988). The advantages of imprecise information. *Rand Journal of Economics*, 19(2):266–275.

Goel, A. and Thakor, A. (2015). Information reliability and welfare: A theory of coarse credit ratings. *Journal of Financial Economics*, 115(3):541–557.

Goldstein, I. and Huang, C. (2020). Credit rating inflation and firms' investments. *Journal of Finance*, 75(6):2929–2972.

Goldstein, I. and Leitner, Y. (2018). Stress tests and information disclosure. *Journal of Economic Theory*, 177:34–69.

Griffin, J. and Tang, D. Y. (2012). Did subjectivity play a role in cdo credit ratings? *Journal of Finance*, 67:1540–6261.

Hirshleifer, J. (1971). The private and social value of information and the reward to inventive activity. *American Economic Review*, 61(4):561–574.

Kartasheva, A. and Yilmaz, B. (2020). Precision of ratings. *Working paper.*

Kisgen, D. and Strahan, P. (2010). Do regulations based on credit ratings affect a firm's cost of capital? *Review of Financial Studies*, 23(12):4324–4347.

Lizzeri, A. (1999). Information revelation and certification intermediaries. *RAND Journal of Economics*, 30(2):214–231.

Manso, G. (2013). Feedback effects of credit ratings. *Journal of Financial Economics*, 109(2):535–548.

Millon, M. and Thakor, A. (1985). Moral hazard and information sharing: A model of financial information gathering agencies. *Journal of Finance*, 40(5):1403–1422.

Opp, C., Opp, M., and Harris, M. (2013). Rating agencies in the face of regulation. *Journal of Financial Economics*, 108(1):46–61.

Orlov, D., Zryumov, P., and Skrzypacz, A. (2022). The design of macroprudential stress tests. *Review of Financial Studies*.

Parlour, C. and Rajan, U. (2020). Contracting on credit ratings: Adding value to public information. *Review of Financial Studies*, 33(4):1412–1444.

Piccolo, A. (2021). Credit ratings and competition. *Working paper*.

Ramakrishnan, R. and Thakor, A. (1984). Information reliability and a theory of financial intermediation. *Review of Economic Studies*, 51(3):415–432.

Rotemberg, J. and Saloner, G. (1986). Price leadership. *MIT Working paper*.

Sangiorgi, F., Sokobin, J., and Spatt, C. (2009). Credit-rating shopping selection and the equilibrium structure of ratings. *Working paper*.

Sangiorgi, F. and Spatt, C. (2017). Opacity, credit rating shopping, and bias. *Management Science*, 63(12):3999–4446.

Spence, M. (1973). Job market signaling. *Quarterly Journal of Economics*, 87(3):355–374.

Terovitis, S. (2022). Information disclosure and the feedback effect in capital markets. *Journal of Financial Intermediation*, 49:100897.

Thakor, R. T. and Merton, R. C. (2023). Trust, transparency, and complexity. *Review of Financial Studies*, page to appear.

Vives, X. (1984). Duopoly information equilibrium: Cournot and bertrand. *Journal of Economic Theory*, 34(1):71–94.