# Collateral and bank screening as complements: A spillover effect

Sonny Biswas*

**Abstract**

I analyze a novel spillover effect from collateralized to uncollateralized loans. High-type borrowers have good projects while low-type borrowers do not know their project quality. High-type borrowers post collateral, and a monopolist bank screens only low-type borrowers' projects. Different from existing models, equilibrium collateral requirements are stricter than the minimum necessary to achieve separation, despite costly collateral. When high-type borrowers post more collateral, the bank charges a higher interest rate to low-type borrowers. This enhances the bank's incentives to screen the low-types' projects, thereby improving the average quality of uncollateralized loans issued.

JEL Classification: D82, G21

Keywords: Collateral, Bank screening, Adverse Selection, Countervailing incentives.

*University of Bristol Business School. 15-19 Tyndall's Park Road, Bristol, BS8 1PQ. s.biswas@bristol.ac.uk.
Declaration of Interests: None.

# 1 Introduction

Collateral in debt contracts serve a multitude of functions. One function of collateral is to protect lenders' interests in defaults (e.g., Tirole 2006). Modelling this particular function of collateral, Asriyan et al. (2022) show in a dynamic general equilibrium setting that credit booms associated with higher collateral values can crowd out bank screening leading to protracted economic crises (see also, Manove et al. 2001 and Gorton and Ordoñez 2014). Instead, I consider the screening function of collateral, which is that it solves an adverse selection problem. My analysis implies that, aside from being substitutes (as in existing models), collateral and direct screening can be complements since the use of collateral may enhance direct screening incentives through a spillover effect.

In the traditional adverse selection models of collateral (e.g., Bester 1985, Besanko and Thakor 1987a), collateral sorts borrowers into risk types, with no residual uncertainty regarding the borrowers' riskiness which makes direct screening by banks (e.g., through information acquisition) redundant. Posting collateral entails a deadweight loss and, hence, the minimum amount of collateral that achieves separation between types is used and no more. I present a setting in which the use of collateral eliminates information asymmetry but does not eliminate all uncertainties. Hence, direct bank screening plays a role despite the use of collateral. In equilibrium, collateral requirements can be higher than the minimum that is necessary to separate borrower-types. This result allows me to derive a new spillover effect which has not been considered previously.

I consider a model in which there are two types of borrowers who apply for credit from a monopolistic bank: a high-type borrower has a good project, while a low-type borrower may have a good or a bad project. Each borrower has personal assets which they may pledge as collateral to secure a loan; these assets are in addition to what is normally available to the lender in the case of default (as in Chan and Kanatas 1985). High-type borrowers post collateral which separates them from low-type borrowers, and hence, the bank does not screen the high-type's projects; this is the substitute relationship studied in existing models (e.g., Manove et al. 2001). The novelty in my model is as follows: conditional on separation between borrower-types, more collateral posted by the high-type allows the bank to charge a higher interest rate to the low-type, which provides improved incentives to the bank to screen the low-types' projects (i.e., screening and collateral are complements). Hence, the average quality of uncollateralized loans approved by the bank increases in the availability of collateral, i.e., there is a positive spillover effect from collateralized to uncollateralized loans. This effect arises when the level of collateral availability is intermediate and posting collateral is not too costly.

There are two distinguishing features of my model: First, even after resolution of information asymmetry between the monopolist lender and the borrowers, the lender cannot extract the full surplus from screening unless collateral availability is sufficiently high. What drives this result in my model is that the demand for the collateralized and uncollateralized debt is endogenous to contract terms, beyond separation. As a result of this, effort provision by the bank may be inefficiently low since it does not fully internalize the benefits of screening. Second, in contrast to existing screening models, equilibrium collateral requirements can be higher than the minimum necessary for separation. By increasing the collateral requirements for high-type borrowers, the bank makes it less attractive for low-type borrowers to mimic; this allows the bank to set a higher interest rate for the low-type. Since the bank retains more of the surplus from screening, the bank screens more diligently. The model delivers new empirical implications:

First, banks screen uncollateralized loans more diligently when economic conditions deteriorate; this prediction is consistent with Howes and Weitzner 2022 who find that in weaker economic conditions banks increase screening efforts, especially for loans with larger loss given defaults (which are the uncollateralized loans in my model). Second, an increase in collateral availability leads to a higher average quality of uncollateralized loans. This is the positive spillover effect of collateral, this prediction is yet to be tested.

The purpose of the model is to understand a credit market characterized by severe information asymmetry. I have in mind small private firms seeking to raise financing. These borrowers often have access to limited sources of funds, and hence, the banks providing financing to them have pricing power, reflecting my assumption that the bank is a monopolist. Unless banks can perform screening, the severity of information asymmetry may lead to an equilibrium in which no firms obtain financing. If the bank does not lend to them, borrowers may borrow from personal networks, which is an extremely common practice (see e.g., Lee and Persson (2016)). Since friends and family would arguably observe their type, high-type borrowers obtain more funds. I capture this feature in the model by assuming that high-type borrowers have higher outside options. Finally, small firms often borrow against personal assets (see e.g., Bahaj et al. 2020), which is how I model collateral here.

**Related literature.** In one set of theories collateral alleviates borrower moral hazard concerns and (observably) riskier borrowers pledge collateral (e.g., Boot et al. 1991 and Boot and Thakor 1994). In a signalling model, Ordonez et al. (2019) show that privately informed borrowers can use secured loans to signal their type when collateral values are uncertain. In dynamic models (e.g., Rajan and Winton 1995, Donaldson et al. 2020b and Donaldson et al. 2020a), collateral assigns priority to a loan over unsecured loans.

My model belongs to a complementary set of theories which show that collateral sorts borrowers into (unobservable) risk classes (e.g., Bester 1985, Chan and Kanatas 1985, Besanko and Thakor 1987a, Besanko and Thakor 1987b): these make up the screening theories of collateral. My model differs from the existing screening models on two grounds: first, equilibrium collateral requirements may be higher than necessary for separation and, second, the monopolist bank cannot extract the full surplus despite full resolution of asymmetric information unless collateral availability is sufficiently high.

In Manove et al. (2001), collateral lead to the sorting of borrowers into risk types and reduce costly screening by banks, even when it is socially optimal to screen (see also Hainz et al. 2013, Goel et al. 2014, Gorton and Ordoñez 2014, Gorton and Ordoñez 2020, Degryse et al. 2021, and Asriyan et al. 2022 who model collateral and bank screening as substitutes). Similar to these studies, in my model the bank does not screen the borrower who posts collateral (this is the classic substitution result). However, different to them, the greater use of collateral leads to more efficient screening of uncollateralized loans.

In my model, starting from a pooling equilibrium, the use of collateral alters the pool of projects being screened (the screening pool), which affects screening intensity. The importance of the quality of the screening pool in my model is reminiscent of several existing papers, e.g., Broecker (1990), Shaffer (1998), Marquez (2002), Vanasco (2017), and Hu (2022). Different from these papers, my main results are derived holding constant the screening pool: beyond achieving separation, a higher availability of collateral does not affect the screening pool, but affects screening incentives by diverting the surplus from low-type borrowers to the lender.

## 2 Model set-up

I consider a three-date economy, $t = 0, 1, 2$. There is a continuum of entrepreneurs (borrowers) with access to a project. The borrowers do not have corporate funds and seek financing for their projects from a monopolistic bank. All agents are risk-neutral and protected by limited liability. The risk-free rate is normalized to 0, so there is no discounting. The project is of fixed scale and requires an investment, normalized to 1, at $t = 1$. A borrower is either high-type, with probability $q_1$, or low-type, with probability $(1 - q_1)$. Borrower type is determined by nature, and it is the borrower's private information. If the borrower is low-type, the project is good with some probability, $q_2$, and bad with the complementary probability, $(1 - q_2)$. A low-type borrower does not know if her project is good or bad quality. If the borrower is high-type, the project is good with certainty. A good project either succeeds with probability, $p$, and produces

$X$, or fails with probability, $(1-p)$, and produces 0, at $t = 2$. A bad project produces 0 with certainty. Denote by $q_p \equiv q_1 + (1-q_1)q_2$ the unconditional probability of a good projects. The payoff structure for each borrower is illustrated in Figure 1. I make two parametric assumptions regarding the profitability of projects:

**A1:** $pX - 1 > 0$

**A2:** $q_p pX - 1 < 0$

Assumption *A1* indicates that the good projects are profitable. If types are separated, a high-type borrower's project is not screened as high-type borrowers have a good project with certainty. Therefore, the screening pool is made up of either only low-type borrowers' projects or both low- and high-type borrowers' projects. $q_2$ is the fraction of good project in a screening pool containing only low-type borrowers' projects, while $q_p$ is the fraction of good projects in a screening pool containing both high and low-type borrowers' projects. *A2* implies that it is not profitable to lend without screening unless it is known with certainty that the borrower is high-type. This assumption ties in well with the objective of the model to describe a credit market characterized by severe information asymmetry.

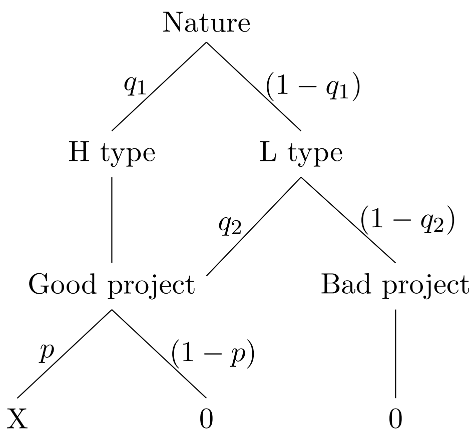


Figure 1: Payoff tree

The bank offers debt contracts at $t = 0$.[1] A borrower applies for a debt contract, which may

[1] The assumption that only debt contracts are offered is without loss of generality since, given the structure of returns in our model, all contracts (sharing rules) are equivalent.

or may not reveal her type. The borrower type does not directly reveal the project quality. The bank can screen the borrower's project to gauge the quality of the project. Screening produces a signal $s_g$ (project is good) or $s_b$ (project is bad), and the signal is informative but noisy:

$$Pr(s = s_g|\text{project} = good) = Pr(s = s_b|\text{project} = bad) = \beta \geq \frac{1}{2} \tag{1}$$

$\beta$ is the precision of the signal, which increases in screening intensity, $F$, as follows:

$$\beta = \frac{1}{2} + F \tag{2}$$

where $F \in (0, \frac{1}{2})$. If $F = 0$, the signal is pure noise and $\beta = \frac{1}{2}$. For $F = \frac{1}{2}$, $\beta = 1$ and screening perfectly reveals the project's quality. The cost of screening is given by $\frac{\tau}{2}F^2$, with $\tau > 0$ and sufficiently large, in a sense made precise later. The functional form implies that the screening cost is increasing and convex in screening intensity. Convexity reflects increasing difficulty for the bank to find out more and more about a project (see e.g., Song and Thakor 2010). Having incurred a non-zero screening cost such that $F > 0$, the bank must only agree to lend if the signal is good, $s = s_g$; otherwise, the bank might as well not have incurred the cost in the first place. Given an observed signal, the posterior probability of the project quality depends on the composition of the screening pool. The posterior probabilities are given as follows:

$$Pr(\text{project} = \text{good}|s_g, q_A) = \frac{\beta q_A}{[\beta q_A + (1-\beta)(1-q_A)]} \tag{3}$$

$$Pr(\text{project} = \text{good}|s_b, q_A) = \frac{(1-\beta) q_A}{[\beta(1-q_A) + (1-\beta)q_A]} \tag{4}$$

$q_A \in \{q_2, q_p\}$ is the unconditional probability of a good project in the screening pool. $q_A = q_2$ if the screening pool contains low-type borrowers only, while $q_A = q_p$ if the pool contains both types. The conditional probability of a bad project is $Pr(\text{project} = \text{bad}|s) = 1 - Pr(\text{project} = \text{good}|s)\ \forall\ s \in \{s_g, s_b\}$. For the pure noise signal, when $F = 0$ and $\beta = \frac{1}{2}$, the conditional probability that the project is good becomes equal to the prior, which is given by the fraction of the good projects in the screening pool, $q_A$.

If screening, the bank extends credit if the signal is good, $s = s_g$. Given the quality of the screening pool, $q_A$, the probability of the good signal is $Pr(s = s_g|q_A)$, which is given by the denominator in Equation (3). If the signal is positive the project quality is good with probability $Pr(\text{project} = \text{good}|s_g, q_A)$ (Equation (3)). If the project is good it succeeds with probability, $p$, and the bank receives a repayment of $R$. If the good project fails, or the project is bad, the repayment is 0. The bank's expected payoff from an uncollateralized loan (i.e., a contract featuring only interest rates) is:

$$
\begin{aligned}
&Pr(s = s_g|q_A)[Pr(\text{project} = \text{good}|s_g, q_A)pR - 1] - \frac{\tau}{2}F^2 \\
&= \beta q_A p R_l - \beta(2q_A - 1) - (1 - q_A) - \frac{\tau}{2}F^2
\end{aligned} \tag{5}
$$

**A3:** The cost of screening $\tau$ is sufficiently high:

$$
\frac{\tau}{2} > q_A(pX - 2) + 1 \tag{6}
$$

Assumption *A3* ensures that the solution of the bank's problem satisfies $F < \frac{1}{2}$.

Although the borrowers do not have any corporate funds to invest in the project, they have personal assets, $W$, which they can pledge as collateral. In collateralized loans, the bank can seize the collateral if the borrower fails to deliver the promised repayments. These assets are in addition to what is normally available to the lender in the case of a default, i.e., these could be personal assets or third-party guarantees (see Chan and Kanatas 1985). Posting collateral may be costly with the cost given by $k \geq 0$ (see Parlatore (2019) for a microfoundation for this cost), and the cost is proportional to the amount of collateral posted. Pledging collateral can be costly due to the disparity between the valuation of the borrower and the lender. The disparity can arise since the borrower may be the best user of the asset, while the lender lacks the expertise to use the asset. The disparity can also arise since the lender has to transport and store the asset on seizure (when more collateral is posted, the cost to store it is higher).

As I consider monopolistic banks, collateral would not play a role if high- and low-types have identical outside options (see page 677 in Besanko and Thakor 1987a, also Lengwiler and Rishabh 2017).[2] Therefore, I assume that a high-type borrower has a higher outside option than a low-type borrower (similar to Sengupta 2014).

**A4:** *High-type borrowers have outside option, $H$, with $H \in (0, pX - 1)$; while low-type borrowers' outside option is normalized to* $0$.

The higher outside option of high-type borrowers can be understood as follows: while borrower-type is hidden from the bank, borrowers' friends and family can observe their type. Then, they can fund the project, but at a smaller scale. In this case, the high-type will invest and produce $H$, while the low-type will not invest. These frictions are of particular relevance in the credit markets since new entrepreneurs often find it difficult to acquire bank loans but can raise seed funding from personal networks (see e.g., Lee and Persson 2016). That high-type borrowers have a higher outside option than low-type borrowers gives rise to countervailing incentives which implies that low-type borrowers potentially mimic high-type borrowers and makes collateral relevant in my model.[3]

**Timeline.** At $t = 0$, a bank offers a loan contract which consists of the interest rate and collateral requirement. Each borrower applies for a contract at $t = 1$. The bank decides whether or not to screen the borrower's project. If screening, the bank further decides the intensity of screening. Depending on the outcome of screening, the bank then approves or

[2] With symmetric outside options, the bank's objective is to deter the high-type from mimicking the low-type while extracting the full surplus. Since it is more attractive for high-type borrowers to post collateral than low-type borrowers, increasing collateral requirements is not effective in deterring the high-type.

[3] Laffont and Martimort (2002) (pages 101-115) discuss the impact of type-dependent reservation utilities on the principal-agent problem and give an overview of several applications that have appeared in the broader contract theory literature.

rejects the borrower's application. If the loan is approved, the bank funds the project. If the application is rejected, the borrower makes her type-specific outside option. The payoffs are realized at $t = 2$, when all agents consume the output. The timing is illustrated in Figure 2.

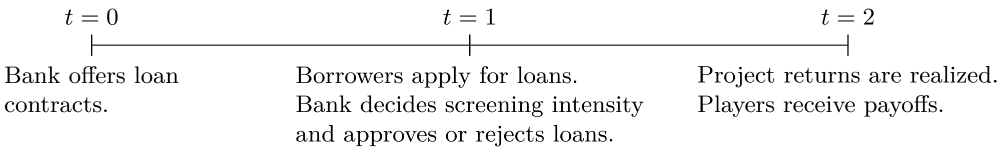


Figure 2: Timeline

**Equilibrium definition.** Since the uninformed player (the bank) moves first, this is a screening game. I look for the pure strategy subgame-perfect Nash equilibrium.[4] The bank maximizes its expected payoff subject to the individual rationality and incentive compatibility constraints of each borrower type. Additionally, the loan repayment rates must satisfy the borrowers' limited liability constraints and the non-negativity constraints (the feasibility constraints). We know from Rothschild and Stiglitz (1976) that a competitive equilibrium may not exist in a screening game due to banks undercutting one another. However, the non-existence problem does not arise in my setting since there is no threat of undercutting with a monopolistic bank.

# 3 Analysis

## 3.1 Symmetric information benchmark

In the symmetric information benchmark, a borrower's type is observable, but in the case of a low-type borrower, neither the bank nor the borrower knows the project quality. The bank offers type-contingent contracts, $(R_i, C_i),\ i \in \{l, h\}$. $R_i$ is the repayment rate and $C_i$ is the collateral requirement. Since it is assumed that collateral is costly, in the symmetric information benchmark, the bank sets $C_i = 0$ for each type to minimize deadweight loss.

[4]Despite the presence of private information, the appropriate solution concept is a non-Bayesian equilibrium concept since the uninformed player moves first by offering contracts. Therefore, there are no Bayesian inferences to be made when the contracts are offered.

If the borrower is high-type, the bank does not screen and always approves the loan application. If, on the other hand, the borrower is low-type, the bank incurs positive screening cost, say $F^{SI}$, and extends credit only if the signal is positive, i.e., $s = s_g$. The bank solves:

$$\begin{aligned}
&\underset{F}{\text{Max}} && \beta q_2 p R_l - \beta(2q_2 - 1) - (1 - q_2) - \frac{\tau}{2}F^2 \\
&\text{subject to} && \\
&\text{(IRH)} && p(X - R_h) \geq H \\
&\text{(IRL)} && \beta q_2 p(X - R_l) \geq 0 \\
&\text{(FCs)} && 0 \leq R_i \leq X \ \forall\ i \in \{l, h\}
\end{aligned} \tag{7}$$

The bank maximizes its expected payoff with respect to the choice of screening intensity, $F$. Since only low-type borrowers' projects are screened, the screening pool consists of $q_2$ (substitute $q_A = q_2$ in Equation (5) to derive the objective function). The bank sets the repayment rates such that the Individual Rationality (or IR) constraints of the borrowers are satisfied. These are IRH and IRL for high- and low-type borrowers, respectively. The left-hand side (LHS) of an IR constraint is the expected payoff of the corresponding borrower type and the right-hand side (RHS) is their outside option. The FCs are the feasibility constraints which are the non-negativity and limited liability constraints.

**Proposition 1 (Symmetric information)** *In the symmetric information benchmark, high-type borrowers always receive credit. The bank screens low-type borrowers' projects and grants credit only if the signal is positive. The equilibrium is characterized as follows:*

$$R_h = X - \frac{H}{p} \tag{8}$$

$$R_l = X \tag{9}$$

$$F^{SI} = \frac{1}{\tau}(q_2(pX - 2) + 1) \tag{10}$$

**Proof.** The proof is in the appendix. ■

The bank grants credit to all high-type borrowers. A low-type borrower with a good project receives credit with probability, $\beta^{SI} = \frac{1}{2} + F^{SI}$. Due to noisy screening, some bad projects are financed, and some good projects are denied credit. With probability, $(1 - \beta^{SI})$, the bank incorrectly rejects a low-type borrower with a good project and incorrectly grants credit to a low-type borrower with a bad project.

### 3.2 Equilibrium

In this section, I derive the equilibrium for the asymmetric information case. Before characterizing the equilibrium, I state the following Lemma:

**Lemma 1** *There does not exist a separating equilibrium in which only low-type borrowers obtain financing.*

**Proof.** The proof is in the appendix. ■

By the revelation principle of Myerson (1979), the bank can induce truth-telling from the borrowers by offering two incentive compatible contracts. Suppose that the bank offers type-contingent contracts, $(R_i, C_i),\ i \in \{l, h\}$. Since high-type borrowers are less likely to default, it is relatively less costly for the high-type to post collateral. Thus, the high-type posts more collateral than the low-type, i.e., $C_h > C_l$. Without loss of generality, the offered contracts are $(R_l, 0)$ and $(R_h, C_h)$. To start with, I assume that pledging collateral is costless, i.e., $k = 0$. In the separating equilibrium, the bank screens only low-type borrowers' projects and solves:

$$\begin{aligned}
&\underset{F}{\text{Max}} && \beta q_2 p R_l - \beta(2q_2 - 1) - (1 - q_2) - \frac{\tau}{2} F^2 \\
&\text{subject to} && \\
&\text{(IRH')} && p(X - R_h) - (1-p)C_h \geq H \\
&\text{(IRL)} && \beta q_2 p(X - R_l) \geq 0 \\
&\text{(ICH')} && p(X - R_h) - (1-p)C_h \geq \beta p(X - R_l) + (1-\beta)H \\
&\text{(ICL')} && \beta q_2 p(X - R_l) \geq q_2(p(X - R_h) - (1-p)C_h) - (1-q_2)C_h \\
&\text{(FCs)} && 0 \leq R_i \leq X \ \forall\ i \in \{l, h\}
\end{aligned} \tag{11}$$

The RHS of high-type borrowers' new IR constraint reflects that when the project fails, she loses the posted collateral. The LHS of the IC constraint for borrower type $i$ is her expected payoff when she tells the truth, while the RHS is her expected payoff from mimicking the other type. If mimicking low-type borrowers, a high-type borrower's project is screened. Due to noisy screening, the high-type borrower is denied credit with probability $(1 - \beta)$, in which case she makes her outside option, $H$. If mimicking high-type borrowers, a low-type borrower is always granted credit, and with probability $q_2$, the project is good.

First, consider the case that $W = 0$. In this case, borrowers cannot pledge collateral to secure loans and the equilibrium is pooling in which high- and low-type borrowers apply for the identical contract, i.e., the promised repayment is $R_h = R_l = \hat{R}$. In a pooling equilibrium, the low-type extract informational rents, which are increasing in the outside option of the high-type, $H$. Hence, bank profits in a pooling equilibrium are falling in $H$. There exists a threshold, $H_1$, such that for $H = H_1$, bank profits in a pooling equilibrium are 0. I state these results in the following proposition:

**Proposition 2 (No collateral)** *When neither borrower-type posts collateral, the equilibrium is pooling if $H \leq H_1$ or there is no financing if $H > H_1$. In a pooling equilibrium, the bank screens both high- and low-type borrowers' projects and grants credit if screening produces a good* *signal. The equilibrium is characterized as follows:*

$$\hat{R} = X - \frac{H}{p} \tag{12}$$

$$F^P = \frac{1}{\tau}(q_p(pX - H - 2) + 1) \tag{13}$$

**Proof.** The proof is in the appendix. ■

Next, consider the $W > 0$ case, implying that borrowers may post collateral. From the constraints in (11), we derive the feasible bounds on collateral requirements:

$$\underbrace{\frac{q_2 H(1-\beta^C)}{(1-q_2)}}_{\equiv \hat{C}} < C_h < \underbrace{\frac{q_2 H}{(1-q_2)}}_{\equiv \overline{C}} \tag{14}$$

The lower bound, $\hat{C}$, comes from violating the low-types' IC constraint and the upper bound, $\overline{C}$, comes from the feasibility constraint that the interest rate cannot be higher than $X$. Below the lower bound, there is no feasible interest rate for which the low-type does not mimic the high-type. At the upper bound, the surplus extracted by the low-type goes to 0, as in the symmetric information benchmark.

Suppose that $W = \gamma\overline{C} + (1-\gamma)\hat{C}$ where $\gamma$ is an exogenous parameter with $\gamma \in [0,1]$. $\gamma = 0$ if borrowers have just enough assets to achieve separation, but no more. A higher $\gamma$ allows a borrower to post more collateral, with $\gamma = 1$ implying that the borrower may post the maximum feasible level of collateral, $\overline{C}$. Therefore, an increase in $\gamma$ can be interpreted as an increase in the availability of collateral. Having introduced the parameter, $\gamma$, we are ready to fully characterize the equilibrium in Proposition 3.

**Proposition 3 (Costless collateral)** *High-type borrowers post collateral, $C_h = W$ and receive credit. The bank screens low-type borrowers' projects and grants credit only if the signal is* *positive. The equilibrium is characterized as follows:*

$$R_h = X - \frac{1}{p}(H + (1-p)W) \tag{15}$$

$$R_l = X - \frac{H}{p} + \frac{\gamma H}{p} \tag{16}$$

$$F^C = \frac{1}{\tau}(q_2(pX - H - 2) + 1 + \gamma q_2 H) \tag{17}$$

**Proof.** The proof is in the Appendix. ■

For $\gamma = 0$, the low-type extracts a fraction of the surplus since $R_l < X$. A higher $\gamma$ leads to an increase in $R_l$, which in turn improves the bank's incentive to screen the low-type's project, $F^C$. For this reason, if collateral is costless, the collateral requirement is as high as possible. Thus, different to existing screening models of collateral, the equilibrium level of collateral is uniquely pinned down in my model, even when collateral is costless. For $\gamma = 1$, the equilibrium becomes identical to the symmetric information benchmark in terms of the intensity with which the uncollateralized loans are screened. For $\gamma < 1$, the screening intensity, $F^C$, differs from the symmetric information benchmark, $F^{SI}$, since low-type borrowers extract a fraction of the surplus. $F^{SI} \geq F^C$ if $(1-\gamma) \geq 0$. This condition is always satisfied for any $\gamma$ (with equality for $\gamma = 1$). Since low-type borrowers extract a fraction of the surplus when the availability of collateral is limited, the screening incentives are lower.[5]

Next, I consider the case that pledging collateral is costly, i.e., $k > 0$. The lender faces the following trade-off. On the one hand, $R_l(C_h)$ is increasing in $C_h$, which implies that as the high-type posts a higher level of collateral, the bank extracts more of the surplus from lending to the low-type. On the other hand, collateral entails a deadweight loss (and the cost is borne the monopolist lender). Due to this trade-off, there exists a threshold, $\bar{k}$, such that for $k = \bar{k}$, bank profit in the separating equilibrium is the same whether $C_h = \hat{C}$ or $C_h = \overline{C}$. I state the

[5]Note that given the constraints of the economy, a regulator who maximizes net social surplus cannot improve upon this outcome. The regulator faces the same set of constraints as the bank and since the constraints bind to determine the contract terms, the regulator's solution coincides with that of the bank.

result in the following lemma:

**Lemma 2** *Suppose $k > 0$ and $\gamma = 1$. In a separating equilibrium, collateral requirements are high, $C_h = \overline{C}$, if $k \leq \bar{k}$, and low, $C_h = \hat{C}$, otherwise.*

**Proof.** The proof is in the appendix. ■

In the existing screening models of collateral, the equilibrium features the minimum level of collateral required to separate, whenever collateral entails a non-zero cost. My setting yields a different result for the following reason: as high-type borrowers post a higher level of collateral, the bank extracts more of the surplus from lending to low-type borrowers. This, in turn, improves the bank's incentives to screen the low-type and leads to higher bank profitability. That is, there is a positive spillover effect from collateralized to uncollateralized loans.

For given collateral requirements, bank profits are falling in the separating equilibrium when cost of posting collateral is higher. There exists a threshold $k^0$, such that for $k = k^0$, bank profits in a separating equilibrium with $C = \hat{C}$ is 0. There exist thresholds, $k^p_\gamma$, such that for $k = k^p_\gamma$, bank profits in a separating equilibrium with $\gamma \in \{0, 1\}$ equal bank profits in the pooling equilibrium. I characterize the equilibrium in the proposition below:

**Proposition 4 (Costly collateral)** *Suppose $\gamma = 1$. The equilibrium is separating with $C_h = \overline{C}$ for low cost of collateral, $k \leq \bar{k}$, separating with $C_h = \hat{C}$ for intermediate costs of collateral, $\bar{k} < k \leq \min(k^0, k^P_0)$, while for high cost of collateral, $k > \max(k^P_0, k^P_1)$, the equilibrium is pooling if $H \leq H_1$ or there is no financing if $H > H_1$ and $k > k^0$.*

**Proof.** The proof is in the appendix. ■

In Figure 3, I illustrate the different equilibria that arise in different parameter regions. I plot the bounds on $C_h$ from Equation (14) and the upper bound on $H$ from Assumption *A4* in the $(C_h, H)$ space. Both the bounds on $C_h$ start at the origin and are upward sloping and linear in this space, $\overline{C}$ being steeper than $\hat{C}$. The upper bound on $H$ is represented by the vertical line.

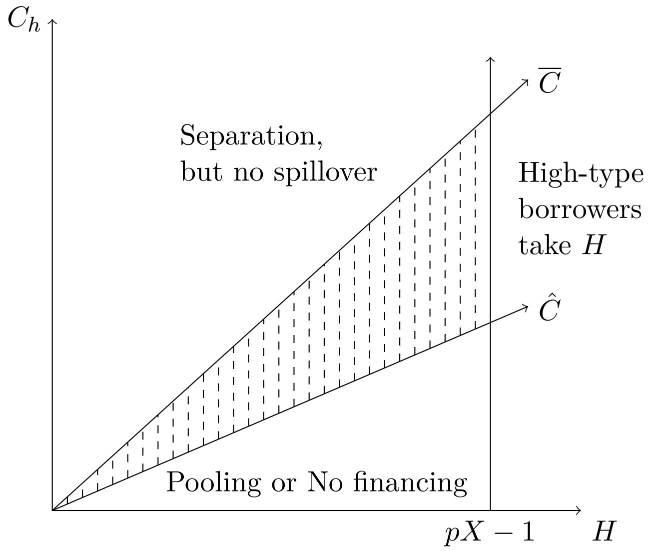


Figure 3: Spillover regions (shaded area)

To the right of the upper bound, the high-type takes their outside option and bank screens the low-types' projects and provides funds at $R_l = X$ if screening produces a positive signal.

The parameters of interest lie to the left of the upper bound on $H$. Below the shaded region, collateral requirements are insufficient to foster separation since ICH is violated: there is either no financing or a pooling equilibrium, depending on parameters. Consider the case that $k$ is small, $k < \bar{k}$. The spillover effect arises in the shaded region. Along the lower contour of the shaded region, collateral requirements are just enough to separate borrower-types as long as $C_h > 0$, but the corresponding $R_l$ from ICL is $R_l = \hat{R} < X$. For any $H$, an increase in $C_h$ allows the bank to charge a higher interest rate to the low-type (this rate is derived from the ICL constraint). At the upper contour, the corresponding $R_l$ from ICL equals $X$, i.e., the bank extracts all the surplus from screening. Above the shaded region, higher collateral requirements does not affect the equilibrium, since $R_l$ cannot be increased beyond $X$.

I leverage both an ex-ante screening device (i.e., collateral) and an ex-post screening device (i.e., direct screening by the bank) to deliver the spillover effect result. The two screening devices play distinct roles in my model since the ex-ante device resolves information asymmetry and the ex-post device solves (noisily) residual uncertainty. To illustrate the role of each, it is useful to consider benchmarks by shutting down the other screening device:

1. Assume that $W = 0$ or $k = \infty$. In this case, posting collateral is infeasible. This implies that there is either pooling of borrower-types or no bank financing in equilibrium. Either way, the spillover effect does not arise.

2. Assume that $q_2 = 0$. In this case, there is no scope for direct screening by the bank since conditional on separation, there is no residual uncertainty. High-type borrowers post collateral and obtain financing. Low-type borrowers do not obtain financing. Similarly, with $\tau = \infty$, $F \to 0$, and given Assumption *A2*, low-type borrowers do no obtain financing. Since collateral is costly and there are no benefits of collateral beyond separating borrower-types, the minimum amount of collateral necessary for separation is used, and no more (identical to standard screening models like Bester 1985).

Therefore, the spillover effect does not arise if either of the screening devices is shut down.

### 3.3 Extension: More than two types

In the baseline, I consider two types of borrowers. In Section 6.2, I consider an extension by introducing intermediate-type borrowers. Intermediate borrowers have a good project with probability $q_m \in (q_2, 1)$; they are more likely to have good projects than low-type borrowers, but may possess bad projects with some positive probability. Intermediate borrowers' outside options is normalized to 0. I show that there may be full or partial separation, depending on the wealth constraints. Suppose that collateral availability $W = \hat{C}$; then high-type and intermediate borrowers post collateral to separate from low-type borrowers. High-type and intermediate borrowers obtain financing at the same terms. With that as the starting point, an increase in collateral availability does not affect the screening pool but allows the bank to extract more of the surplus from low-type borrowers, which, in turn, positively affects the bank's screening incentives. Thus, as in the baseline model, in this extension there also arises a positive spillover effect from collateralized to uncollateralized loans.

# 4 Empirical relevance and implications

## 4.1 Empirical relevance

In this section, I discuss how realistic two of the key assumptions of my model are:

**Bank market power:** I have considered a monopolist bank. The results go through qualitatively if banks retain positive market power in the uncollateralized loan market. As long as banks have positive market power, they increase their surplus by increasing collateral requirements for high-type borrowers since it allows them to charge higher interest rates to low-type borrowers. Thus, with positive bank market power, I can perform the comparative static analyses with respect to the availability of collateral, which deliver the key results of my model. Conceptually, one could assume direct screening by banks is a scarce skill, which would allow banks to extract surplus in the uncollateralized loan market. Empirically, a large literature establishes that competition in the banking sector is not perfect. In a sample of 48 countries between 1995 and 2007, Forssbaeck and Shehzad (2015) estimate that the country-level loan market Lerner index has a mean close to 50% (see also Delis et al. 2016 and Beck et al. 2013). The estimates in these studies indicate that banks enjoy substantial market power.

**Role of collateral.** In the model, a high-type borrower uses assets unconnected to the firm (e.g., personal assets or third-party guarantees) as collateral to separate themselves from bad borrowers. The empirical relevance of my model hinges on whether personal assets or third-party guarantees are indeed used as corporate collateral and, if so, how widespread the phenomenon is. Bahaj et al. (2020) present compelling evidence that the use of personal assets as corporate collateral can mitigate financing frictions and that an increase in the value of directors' personal assets leads to higher firm-level investment (see also Anderson et al. 2022 and Beyhaghi 2022).[6]

[6]Most of the evidence on directors pledging personal assets to secure corporate loans relates to privately held

## 4.2 Empirical implications

My model shares with other screening models of collateral (e.g., Bester 1985) the prediction that safer borrowers post collateral to separate from riskier borrowers; this prediction has considerable empirical support. Berger et al. (2011) provide evidence that unobserved risk is negatively correlated with pledging collateral; see also, Ioannidou et al. (2022) and Godlewski and Weill (2011). I present below new predictions which other related models cannot deliver. In deriving these predictions, I consider the case that the cost of pledging collateral is small (e.g., $k = 0$) and the borrowers have intermediate levels of personal wealth, $\hat{C} \leq W < \overline{C}$.

**Prediction 1.** *An increase in high-type borrowers' outside option leads to a higher intensity with which banks screen uncollateralized loans.*

From Equation 17, equilibrium screening intensity, $F^C$, is falling in high-type borrowers' outside option, $H$.[7] The intuition is as follows: as high-type borrowers' outside options increase, it becomes more attractive for low-type borrowers to mimic the high-type, which lowers the interest rate that the low-type can be charged. In turn, this reduces the bank's incentive to screen low-type borrowers diligently since they retain less of the surplus from screening.

It seems reasonable to interpret high-type borrowers' outside option as reflecting the strength of local economic conditions: in weaker economic conditions, neither borrower type obtains financing from personal networks since friends and family are themselves financially constrained, while when economic conditions improve and financial constraints of friends and family ease, high-type borrowers will have better outside options since they can potentially obtain alternate financing more easily. With this interpretation in mind, the prediction is consistent with Howes and Weitzner (2022)[8], who find that the quality of information produced by banks at loan

companies, but there is anecdotal evidence that this practice is also prevalent in public companies. E.g., in 2020, Richard Branson pledged as collateral his private Caribbean island (Necker Island) to raise funds from the UK government to prevent the collapse of his Virgin Group empire.

[7] $\frac{\partial F^C}{\partial H} = -\frac{1}{\tau} q_2(1-\gamma) < 0$

[8] The sample in Howes and Weitzner (2022) contain many small and non-public firms which is the market segment that I model in this paper; in the same database, Beyhaghi (2022) finds that around 46% of loans carry third-party guarantees.

origination improves as local economic conditions deteriorate (see also, Lisowsky et al. 2017). Further, also consistent with my model, Howes and Weitzner (2022) find that the cyclical sensitivity of information quality is driven by loans which have larger potential losses (in my model these are the loans which are not backed by collateral). The findings in Howes and Weitzner (2022) can also be explained by Dang et al. (2012). My model offers an alternative explanation to Dang et al. (2012); in their model, information production is triggered by a fall in the value of collateral, while in my model, information production is triggered by a fall in borrowers' outside options.

As a direct implication of higher screening intensity in economic downturns, the model predicts that lending standards are countercyclical. Asea and Blomberg (1998), Dell'Ariccia et al. (2012), and Rodano et al. (2018) present empirical evidence in support of this prediction.

**Prediction 2.** *An increase in collateral availability leads to an improved average quality of uncollateralized loans issued.*

This prediction embodies the spillover effect from collateralized to uncollateralized loans. As high-type borrowers post more collateral, low-type borrowers are charged higher interest rates and, hence, their projects are screened more diligently. More diligent screening by the bank reduces errors, i.e., fewer good projects are rejected and more bad projects are rejected, leading to a higher average quality of the uncollateralized loans. To the best of my knowledge, this prediction has not been tested.[9]

# 5 Concluding remarks

I present a model of a credit market characterized by severe information asymmetry. As in existing screening models, high-type borrowers post collateral to separate from low-type borrowers; since the high-type borrower posts collateral, the bank does not screen their project

[9]Ioannidou et al. (2022) consider the spillover effect from secured to unsecured loans for a given borrower, which is not the case in my model.

(this is the well-known substitute relationship as in Manove et al. 2001). My model has two distinguishing features: first, a monopolist lender cannot extract the full surplus from screening even after resolution of information asymmetry unless collateral availability is sufficiently high, and second, the lender sets higher collateral requirements in equilibrium than the minimum necessary for separation (even if collateral is costly). When the cost of posting collateral is small and the level of collateral availability is intermediate, increasing collateral requirements for collateralized loans allows banks to charge a higher interest rate on uncollateralized loans, which, in turn, improves banks' incentives to screen uncollateralized loans. Thus, the model uncovers a new mechanism of how an increase in collateral availability may affect information production.

# 6 Appendix

## 6.1 Omitted proofs

**Proof of Lemma 1.** Suppose that the bank commits to serve low-type borrowers only. At $t = 0$ the bank offers an $R = X$ contract. This contract violates the high-types' participation constraint, so they stay out. Only low-type borrowers apply for this contract at $t = 1$. The bank screens the projects of the low-type and extends credit whenever the signal is positive. At the same time, at $t = 1$, the bank knows that the borrowers who are not served are the high-type (because they turned down the $R = X$ contract at $t = 0$). Then, the bank offers a contract to the high-type with interest rate, $R = X - \frac{H}{p}$. Moreover, anticipating this outcome the low-type will not apply for the $R = X$ contract at $t = 0$. Therefore, since the bank cannot credibly commit to not serve high-type borrowers at $t = 1$ (even if parameters are such that it would like to be able to commit), the conjectured equilibrium unravels.

**Proof of Proposition 1.** I solve for the equilibrium by backward induction and begin with the bank's screening decision at $t = 2$. An interior solution to the bank's problem is given by the first order condition:

$$q_2(pR_l - 2) + 1 - \tau F = 0 \tag{18}$$

At $t = 0$, the bank sets the repayment rates such that the IR constraints of the borrowers are satisfied. The IR constraints bind for both types. To see why this is the case, consider a candidate equilibrium in which the IR constraint for type $i$ is not binding. Then, the bank can increase its profits by increasing $R_i$ a little, without violating the other constraint. Hence the candidate equilibrium with non-binding IR constraint for either type is not stable. From the relevant IR constraints, the repayment rates, $R_h$ and $R_l$, are derived (Equations (8) and (9)). The non-negativity constraints are satisfied if $pX > H$, which holds due to Assumption *A4*, and the limited liability constraints are always satisfied, binding for low-type borrowers and slack for high-type borrowers. Substitute $R_l$ in the first order condition to derive the screening intensity, $F^{SI}$ (Equation (10)). Finally, I check that the solution is interior, i.e., $0 < F^{SI} < \frac{1}{2}$. Take the extreme value, $pX \to 1$ which does not violate any of the assumptions: $F^{SI}$ becomes $\frac{1}{\tau}(1 - q_2)$, which is strictly positive since $q_2 < 1$. Increasing $pX$ leads to a *higher* $F$, therefore, $F^{SI} > 0$ holds for all parameters. Assumption *A3* ensures $F^{SI} < \frac{1}{2}$.

**Proof of Proposition 2.** First, I show that when $C_h = 0$, a separating equilibrium cannot exist. Suppose that $C_h = 0$ and there is a separating equilibrium with $R_h \neq R_l$. The IC constraints are jointly satisfied only if $(1 - \beta) \leq 0$. However, this condition is never satisfied for any $\beta < 1$. For $\beta = 1$, it is satisfied for $R_h = R_l$, which violates the starting assumption of $R_h \neq R_l$. Thus, separation does not arise and both borrowers-types apply for the identical contract, i.e., the promised repayment is $R_h = R_l = \hat{R}$.

In a pooling equilibrium, the bank solves the following problem:

$$\begin{aligned}
&\underset{F}{\text{Max}} && \beta q_p p\hat{R} - \beta(2q_p - 1) - (1 - q_p) - \frac{\tau}{2}F^2 \\
&\text{subject to} && \\
&\text{(IRH)} && p(X - \hat{R}) \geq H \\
&\text{(IRL)} && \beta q_2 p(X - \hat{R}) \geq 0 \\
&\text{(FCs)} && 0 \leq \hat{R} \leq X
\end{aligned} \tag{19}$$

I solve for the equilibrium by backward induction and begin with the bank's screening decision at $t = 2$. An interior solution to the bank's problem is given by the first order condition:

$$q_p(p\hat{R} - 2) + 1 - \tau F = 0 \tag{20}$$

At $t = 0$, the bank sets the repayment rates such that the IR constraints of the borrowers are satisfied. In order to make sure that the high-type participates, her individual rationality constraint must be satisfied. This automatically satisfies low-type borrowers' individual rationality constraint. From the IRH constraint, derive the interest rate in the pooling equilibrium, $\hat{R}$ (Equation (12)). The condition is satisfied with equality. If it is slack, the bank can increase its profits by increasing $\hat{R}$ a little, without violating any other relevant constraints. Substitute $\hat{R}$ in the first order condition to derive the screening intensity in the pooling equilibrium, $F^P$ (Equation (13)). Finally, I check that the solution is interior, i.e., $0 < F^P < \frac{1}{2}$. Take the extreme values, $pX \to 1$ and $H \to pX - 1$: $F^P$ becomes $\frac{1}{\tau}(1 - q_p pX)$, which is strictly positive given Assumption *A2*. Increasing $pX$ and/or reducing $H$ leads to a *higher* $F$, therefore $F^P > 0$ holds for all parameters. Assumption *A3* ensures $F^P < \frac{1}{2}$.

Bank profits in a pooling equilibrium is given by:

$$\Pi_P = \beta^P(q_p(pX - H - 2) + 1) - (1 - q_p) - \frac{\tau}{2}F^{P2} \tag{21}$$

$\Pi_P$ is falling in $H$. There exists a threshold, $H_1$ such that for $H = H_1$, $\Pi_P = 0$. $H_1$ is given by:

$$H_1 = \frac{\tau - 4q_2 - 4q_1 - \tau((\tau - 8q_2 - 8q_1 + 8q_1q_2 + 8)/\tau)^{0.5} + 4q_1q_2 + 2pXq_1 + 2pXq_2 - 2pXq_1q_2 + 2}{2(q_1 + q_2 - q_1q_2)} \tag{22}$$

**Proof of Proposition 3.** To solve the problem I initially assume that the ICH' constraint is satisfied. After solving the modified problem, I verify that a solution exists which does not violate the starting assumption. In the relaxed problem, the IRH' constraint must bind; if not binding, the bank can increase $R_h$ a little to increase its profits without violating the other constraints. Next, note that either the IRL or ICL' constraint must bind. If neither constraint is binding the bank can increase $R_l$ a little to increase its profits without violating the other constraints. It is the ICL' constraint which binds, and not the IRL constraint, if the RHS of the ICL' constraint is greater than 0. Using the IRH' constraint, the ICL' constraint binds if:

$$C_h \leq \frac{q_2 H}{(1 - q_2)} \equiv \overline{C} \tag{23}$$

Equation (23) represents an upper bound on the amount of collateral used. If this condition is violated, then the IRL constraint binds which implies $R_l = X$.

From the IRH' constraint, high-type borrowers' repayment rate when the loan is collateralized becomes:

$$R_h(C_h) = X - \frac{1}{p}(H + (1 - p)C_h) \tag{24}$$

The interest rate is falling in the level of collateral, to allow the high-type to achieve her outside option, in expectation. The limited liability constraint ($R_h < X$) is always satisfied. The non-negativity constraint for a high-type borrower's repayment rate is satisfied, $R_h \geq 0$, if:

$$H \leq pX - (1-p)C_h \tag{25}$$

Notice that the RHS is falling in $C_h$, implying that a higher $C_h$ makes the condition more binding. This implies that if the condition is satisfied for the upper bound of $C_h$, it will be satisfied for smaller values of $C_h$. Substituting $C_h = \overline{C}$ and simplifying:

$$H \leq \frac{pX(1-q_2)}{1-pq_2} \tag{26}$$

Equation (25) is satisfied if the LHS above is greater than the upper bound on $H$, $pX-1$, from Assumption *A4*:

$$\frac{pX(1-q_2)}{1-pq_2} \geq pX - 1 \tag{27}$$

$$\implies pq_2(pX-1) \geq q_2 pX - 1 \tag{28}$$

The above condition is always satisfied since the LHS is positive (given Assumption *A1*) while the RHS is negative (given Assumption *A2* and $q_2 < q_p$). Thus, Equation (25) is always satisfied.

Supposing that Equation (23) is satisfied, i.e., the ICL' constraint binds and using $R_h(C_h)$, the repayment rate charged to low-type borrowers, $R_l(C_h)$, becomes:

$$R_l(C_h) = X - \frac{q_2 H - (1-q_2)C_h}{\beta(C_h)q_2 p} \tag{29}$$

The upper bound, $C_h \leq \overline{C}$, ensures that the limited liability constraint is satisfied, i.e., $R_l \leq X$.

The non-negativity constraint is satisfied if $R_l \geq 0$, which gives a lower bound as follows:

$$C_h \geq \frac{q_2(H - \beta(C_h)pX)}{(1 - q_2)} \equiv \underline{C} \tag{30}$$

Finally, I verify that bounds on $C_h$ do not violate the starting assumption that the ICH' constraint is satisfied. Substituting $R_h(C_h)$ and $R_l(C_h)$ in the ICH' constraint:

$$C_h \geq \frac{q_2 H(1 - \beta^C(C_h))}{(1 - q_2)} \equiv \hat{C}(\beta^C) \tag{31}$$

The ICH' constraint is satisfied only if collateral is sufficiently large, $C_h \geq \hat{C}$. Combining with the feasibility constraints of low-type borrowers' repayment rate, the collateral that high-type borrowers need to post, $C_h$, in order to achieve separation lies in the range, max $(\underline{C},\ \hat{C}) < C_h < \overline{C}$. It is easily verified that $\hat{C} < \overline{C}$ is always satisfied for any $\beta > 0$, and $\hat{C} > \underline{C}$ is satisfied as long as $pX > H$, which holds due to Assumption, *A4*. Therefore, the feasible range of collateral requirements for which separation is achieved is given by $C_h \in [\hat{C}, \overline{C}]$.

Consider the case that for a given $\gamma$, $C_h = \lambda\overline{C} + (1-\lambda)\hat{C}$ with $\lambda \in [0, \gamma]$; $\lambda$ is to be determined in equilibrium. Substituting $R_h$, $R_l$, and $C_h$ in the bank's objective function and taking the first order condition gives the equilibrium screening intensity:

$$F^C = \frac{1}{\tau}(q_2(pX - H - 2) + 1 + \lambda q_2 H) \tag{32}$$

Substituting $F^C$ into $C_h$, the equilibrium level of collateral is:

$$C_h = \frac{q_2 H}{1 - q_2}\left[1 - (1 - \lambda)\left[\frac{1}{2} + \frac{1}{\tau}(q_2(pX - H - 2) + 1 + \lambda q_2 H)\right]\right] \tag{33}$$

To fully characterize the equilibrium, substitute $F^C$ and $C_h$ into $R_h$ and $R_l$. High-type borrowers' repayment rate, $R_h$ is falling in the equilibrium amount of collateral (such that the high-type is indifferent with regards to the level of collateral requirement), while low-type borrowers' repayment is increasing in the amount of collateral, as follows:

$$R_l = X - \frac{H}{p} + \frac{\lambda H}{p} \tag{34}$$

For $\lambda = 0$, which corresponds to the minimum collateral requirement which achieves separation, $R_l = \hat{R}$. For $\lambda = 1$, a low-type borrower's repayment becomes $R_l = X$. Therefore, by setting $\lambda = \gamma$, the bank extracts the maximum surplus from lending to the low-type borrower (if $\gamma = 1$, the bank extracts the full surplus).

**Proof of Lemma 2.** Consider the case that for a given $\gamma$, $C_h = \lambda\overline{C} + (1-\lambda)\hat{C}$ with $\lambda \in [0, \gamma]$; $\lambda$ is to be determined in equilibrium. The lemma states that $\lambda$ is always given by a corner solution, i.e., either $\lambda = 0$ if posting collateral is sufficiently costly or $\lambda = \gamma$ if posting collateral is cheap. The bank's objective function in the separating equilibrium, including the cost of collateral, is:

$$\begin{aligned}\Pi_S =& q_1(pR_h + (1-p)C_h - 1 - kC_h) \\ &+ (1-q_1)\left(\beta^C(q_2(pR_l - 2) + 1) - (1-q_2) - \frac{\tau}{2}{F^C}^2\right)\end{aligned} \tag{35}$$

With probability $q_1$ the bank makes collateralized loans to high-type borrowers (the top line). The collateral cost is incurred whether or not the project fails (e.g., to transfer and store the collateral when the loan is approved); it could be adapted to the case that the cost of collateral is only incurred on the failure of the project without qualitatively affecting the results. With probability $(1 - q_1)$ the bank makes uncollateralized loans to the low-type (the second line).

Derivating $\Pi_S$ with respect to $\lambda$ and $k$,

$$\frac{\partial \Pi_S^2}{\partial \lambda \partial k} = -\frac{\partial C_h}{\partial \lambda} \tag{36}$$

$$= -\frac{q_2 H}{1-q_2}\left[\frac{1}{2} + \frac{1}{\tau}(q_2(pX - H - 2) + 1 + \lambda q_2 H - (1-\lambda) q_2 H)\right] \tag{37}$$

The derivative is negative if:

$$\underbrace{\frac{1}{2} + \frac{1}{\tau}(q_2(pX - H - 2) + 1 + \lambda q_2 H)}_{\beta^C} > \frac{1}{\tau}(1-\lambda) q_2 H \tag{38}$$

Taking the extreme values, $\beta = \frac{1}{2}$, $\lambda = 0$, and $H = pX - 1$, Equation (38) becomes:

$$\frac{\tau}{2} > q_2(pX - 1) = q_2(pX - 2) + q_2 \tag{39}$$

Equation (39) is satisfied due to Assumption *A3* since $q_2 \leq 1$. Given that Equation (39) is satisfied for the extreme values considered above, Equation (38) will be satisfied for any feasible values of $\beta$, $\lambda$, and $H$. Thus, the derivative, $\frac{\partial \Pi_S^2}{\partial \lambda \partial k}$, is negative.

Suppose that collateral is costless, i.e., $k = 0$. In this case, $\Pi_S$ is increasing in $\lambda$, since conditional on separation, the higher use of collateral allows the bank to extract the full surplus from lending to the low-type, which leads to higher profits. Since $\frac{\partial \Pi_S^2}{\partial \lambda \partial k} < 0$, as $k$ increases, $\frac{\partial \Pi_S}{\partial \lambda}$ is falling and it becomes negative for some $k$ which is sufficiently high. When the derivative is (weakly) positive, i.e., bank profit is (weakly) increasing in $\lambda$, we set $\lambda$ as high as possible (i.e., $\lambda = \gamma$); otherwise, we set $\lambda = 0$. Suppose $\gamma = 1$. There exists a threshold, $\bar{k}$, such that for $k = \bar{k}$, profits in the separating equilibrium with $\lambda = 1$ equals the profits in the separating equilibrium with $\lambda = 0$. The expression for $\bar{k}$ is:

$$\bar{k} = \frac{(q_1 - 1)(q_2 - 1)(\tau - 4q_2 - q_2 H + 2pq_2 X + 2)}{q_1(\tau - 4q_2 - 2q_2 H + 2pq_2 X + 2)} \tag{40}$$

The bank sets $\lambda = 1$ for $k \leq \bar{k}$ and sets $\lambda = 0$ for $k > \bar{k}$.

**Proof of Proposition 4.** There are four possibilities: separating with $\lambda = \gamma$ ($C_h = \overline{C}$ if $\gamma = 1$), separating with $\lambda = 0$ ($C_h = \hat{C}$), pooling in which no collateral is used, and no financing. Which emerges in equilibrium depend on exogenous parameters.

The exists a threshold $k^0$ such for $\lambda = 0$ and $k = k^0$, $\Pi_S = 0$. $k^0$ is given by:

$$k^0 = \frac{\begin{array}{c} -(2\tau((q_1-1)((2q_2+q_2H-q_2pX-1)^2/(2\tau)-q_2+ \\ ((2q_2+q_2H-q_2pX-1)(\tau-4q_2-2q_2H+2q_2pX+2))/(2\tau)+1)-q_1(H-pX+1))(q_2-1)) \end{array}}{q_1q_2H(4q_2+\tau+2q_2H-2q_2pX-2)} \tag{41}$$

While bank profits in a separating equilibrium are falling in the cost of collateral, $k$, bank profits in a pooling equilibrium, $\Pi_P$ are invariant in $k$. The exists thresholds $k^P_\gamma$ such that for $\gamma \in \{0, 1\}$ and $k = k^p_\gamma$, $\Pi_S = \Pi_P$. $k^p_\gamma$ are given by:

$$k^P_0 = \frac{\begin{array}{c} -(2\tau(q_2-1)((q_1-1)((2q_2+q_2H-pq_2X-1)^2/(2\tau)-q_2 \\ +((2q_2+q_2H-pq_2X-1)(\tau-4q_2-2q_2H+2pq_2X+2))/(2\tau)+1) \\ -((q_1+q_2-q_1q_2)(H-pX+2)-1)(((q_1+q_2-q_1q_2)(H-pX+2)-1)/\tau-1/2)-q_1 \\ -q_1(H-pX+1)+q_2(q_1-1)+((q_1+q_2-q_1q_2)(H-pX+2)-1)^2/(2\tau)+1)) \end{array}}{q_1q_2H(4q_2+\tau+2q_2H-2pq_2X-2)} \tag{42}$$

$$k^P_1 = \frac{\begin{array}{c} (q_2-1)(q_1+((q_1+q_2-q_1q_2)(H-pX+2)-1)(((q_1+q_2-q_1q_2)(H-pX+2)-1)/\tau-1/2) \\ +q_1(H-pX+1)-q_2(q_1-1)-((q_1+q_2-q_1q_2)(H-pX+2)-1)^2/(2\tau) \\ +((q_1-1)(4q_2^2-\tau-4q_2+2pXq_2+X^2p^2{q_2}^2-4pX{q_2}^2+pXq_2\tau+1))/(2\tau)-1) \end{array}}{Hq_1q_2} \tag{43}$$

Collateral requirements are high, $\lambda = \gamma$ (i.e., if $\gamma = 1$ then $C_h = \overline{C}$), for $k \leq \min(\bar{k}, k^P_1)$, and low, $C_h = \hat{C}$, for $\bar{k} < k \leq \min(k^0, k^P_0)$. For $k > k^P_\gamma$ (with $\gamma \in \{0, 1\}$), there is a pooling equilibrium if $H \leq H_1$. Finally, there is no financing in equilibrium if $k > k^0$ and $H > H_1$.

## 6.2 More than two types of borrowers

In this extension, I consider the case that there are three types of borrowers: high-type, intermediate, and low-type. An intermediate borrower has a good project with probability, $q_m \in (q_2, 1)$, i.e., intermediate borrowers may have bad projects, but they have good projects with a higher probability than low-type borrowers. To simplify exposition, I assume that $q_m = 1-\epsilon$ where $\epsilon$ is positive but arbitrarily small; this assumption implies that the bank would lend to intermediate borrowers without screening their project.[10] Intermediate borrowers have a lower outside option than high-type borrowers, $H_m < H$, which is normalized to 0. I consider the costless collateral case, $k = 0$. Define $C'$ as follows:

$$C' := \frac{q_m H}{(1-q_m)} \tag{44}$$

I characterize the equilibrium in the following proposition.

**Proposition 5 (Three-type case)** *There is partial pooling equilibrium for intermediate levels of collateral availability, $\hat{C} \leq W < C'$; in this case, the higher types post collateral to separate from the low-type. Full separation takes place when collateral is very high, $W > C'$.*

**Proof.** Suppose that the bank offers a contract intended for the intermediate borrowers, $(R_m, C_m)$, with $0 < C_m < C_h$. An intermediate borrower's IR constraint is as follows:

$$q_m(p(X - R_m) - (1-p)C_m) - (1-q_m)C_m \geq 0 \tag{45}$$

This IR constraint of intermediate borrowers must bind if they separate from high-type borrowers, but in a partial pooling equilibrium in which the two higher types pool, this constraint must be slack. In this case, the high-types' IR constraint will be the binding one. Intermediate borrowers mimic the high-type if two conditions are satisfied which are, (i) they are better off mimicking the high-type than truthfully revealing their type ($\text{ICM}_1$) and, (ii) they are better

[10] The cost of screening exceeds the gains from screening if: $\epsilon < \frac{pX(1-\beta)+\beta+\frac{\tau}{2}F^2}{pX(1-\beta)+2\beta-1}$. Since the RHS is positive, it is always possible to find an $\epsilon$ sufficiently small such that this condition holds.

off mimicking the high-type than mimicking the low-type ($\text{ICM}_2$):

$$(\text{ICM}_1) \qquad q_m(p(X-R_m)-(1-p)C_m) < q_m(p(X-R_h)-(1-p)C_h) \tag{46}$$

$$(\text{ICM}_2) \qquad q_m(p(X-R_h)-(1-p)C_h)-(1-q_m)C_h > \beta q_m p(X-R_l) \tag{47}$$

Using $q_m = 1-\epsilon$ and $R_l$ and $R_h$ from Equations 34 and 24, $\text{ICM}_2$ is satisfied if $\epsilon < (1-q_2)$ which holds since $\epsilon$ is arbitrarily close to 0. With the IR constraint of intermediate borrowers binding, $\text{ICM}_1$ is satisfied if $C_h \geq C'$. Comparing Equations (23) and (44), $\overline{C} < C'$ if:

$$\frac{(q_m-q_2)H}{1-q_2} > 0 \tag{48}$$

Since $q_m > q_2$, the above condition is always satisfied. Finally, for the partial pooling equilibrium to exist, we need to check that the high-type does not accept the $(R_m, C_m)$ offer. Given that IRH binds, high-type borrowers do not mimic intermediate borrowers if:

$$H \geq (p(X-R_m)-(1-p)C_m) \implies C_m \leq C' \tag{49}$$

Comparing $\text{ICM}_1$ and Equation 49, the lower bound on $C_h$ above which intermediate borrowers do not mimic the high-type is the same as the upper bound on $C_m$ below which high-type borrowers do not mimic intermediate borrowers. Thus, there exist $W \in (C_m, C_h)$ such that all constraints are satisfied for the partial pooling equilibrium. Full separation takes place for $W > C'$ since for any $C_h > C'$ intermediate borrowers are separated from the high-type. ■

Consider the parameters, $\hat{C} \leq W < C'$. For these parameters, low-type borrowers do not mimic high-type borrowers since their IC constraint is violated, but the intermediate borrowers do. Given that it is the high-type borrowers' IR constraint which must bind in this partial pooling equilibrium, the contract terms, $(R_h, C_h)$, is identical to the baseline. For $W = \hat{C}$, the higher types separate from the low-type, but the low-type extract a fraction of the surplus since

$R_l < X$. As the availability of collateral increases, the bank increases collateral requirements for the higher types, which allows the bank to charge a higher interest rate to the low-type and positively affects the bank's incentive to screen the low-types' projects. Thus, as in the baseline model, the positive spillover effect from collateralized to uncollateralized loans also arises in the three-type case. The analysis generalizes to any number of borrower types, but the analysis becomes more complicated without additional qualitative insights.

Table 1: Notations

| Notations | Definitions | Parametric restrictions |
|---|---|---|
| Exogenous parameters | | |
| $p$ | Probability that a good project succeeds, $p \in (0,1)$ | |
| $X$ | Output if the project succeeds | *A1:* $pX - 1 > 0$ |
| $q_1$ | Fraction of high-type borrowers, $q_1 \in (0,1)$ | |
| $q_2$ | Fraction of good projects among low-type borrowers, $q_2 \in (0,1)$ | |
| $q_p$ | $q_p = q_1 + (1-q_1)q_2$ | *A2:* $q_p pX - 1 < 0$ |
| $q_A$ | $q_A \in \{q_2, q_p\}$ | |
| $\tau$ | Cost of screening | *A3:* $\frac{\tau}{2} > q_A(pX-2)+1$ |
| $s_g,\ s_b$ | Screening produces a good or bad signal, resp. | |
| $k$ | Cost of posting collateral | |
| $H$ | High-type borrowers' outside option | *A4:* $H \in (0, pX-1)$ |
| $W$ | Borrower's personal assets | |
| Endogenous parameters | | |
| $R_i$ | Promised repayment for type $i$ borrower, $0 \leq R_i \leq X$ | |
| $\hat{R}$ | Promised repayment in pooling, $0 \leq \hat{R} \leq X$ | |
| $C_i$ | Collateral requirements for type $i$ borrower, $C_l = 0$ | |
| $\beta$ | Screening precision $\beta = \frac{1}{2} + F$ where $F \in (0, 0.5)$ | |
| $F^{SI},\ F^C,\ F^P$ | Screening intensity under symmetric info, separating, pooling | |
| Equilibrium entities | | |
| $\hat{C}$, $\overline{C}$ | Bounds on collateral requirements, $C_h \in [\hat{C}, \overline{C}]$ | |
| $\gamma$ (exogenous) | Collateral availability, $W = \gamma\overline{C} + (1-\gamma)\hat{C}$ where $\gamma \in [0,1]$ | |
| $\lambda$ | Collateral requirements, $\lambda \in [0,\gamma]$ | |
| $\Pi_S$, $\Pi_P$ | Profits in separating, pooling | |
| $H_1$ | $\Pi_P = 0$ when $H = H_1$ | |
| $\bar{k}$ | $\Pi_S(\hat{C}) = \Pi_S(\overline{C})$ when $k = \bar{k}$ | |
| $k^0$ | $\Pi_S(\hat{C}) = 0$ when $k = k^0$ | |
| $k_0^P$ | $\Pi_S(\hat{C}) = \Pi_P$ when $k = k_0^P$ | |
| $k_1^P$ | $\Pi_S(\overline{C}) = \Pi_P$ when $k = k_1^P$ | |

## References


Anderson, G., Bahaj, S., Chavaz, M., Foulis, A., and Pinter, G. (2022). Lending relationships and the collateral channel. *Review of Finance*.

Asea, P. and Blomberg, B. (1998). Lending cycles. *Journal of Econometrics*, 83(1).

Asriyan, V., Laeven, L., and Martin, A. (2022). Collateral booms and information depletion. *Review of Economic Studies*, 89(2):517–555.

Bahaj, S., Foulis, A., and Pinter, G. (2020). Home values and firm behaviour. *American Economic Review*, 110(7):2225–2270.

Beck, T., De Jonghe, O., and Schepens, G. (2013). Bank competition and stability: cross-country heterogeneity. *Journal of Financial Intermediation*, 22(2):218–244.

Berger, A., Frame, W. S., and Ioannidou, V. (2011). Tests of ex ante versus ex post theories of collateral using private and public information. *Journal of Financial Economics*, 100(1):85 – 97.

Besanko, D. and Thakor, A. (1987a). Collateral and rationing: Sorting equilibria in monopolistic and competitive credit markets. *International Economic Review*, 28(3):671–89.

Besanko, D. and Thakor, A. V. (1987b). Competitive equilibrium in the credit market under asymmetric information. *Journal of Economic Theory*, 42(1):167 – 182.

Bester, H. (1985). Screening vs. rationing in credit markets with imperfect information. *American Economic Review*, 75(4):850–55.

Beyhaghi, M. (2022). Third-party credit guarantees and the cost of debt: Evidence from corporate loans. *Review of Finance*, 26(2):87–317.

Boot, A., Thakor, A., and Udell, G. (1991). Secured lending and default risk: Equilibrium analysis, policy implications and empirical results. *Economic Journal*, 101(406):458–72.

Boot, A. W. A. and Thakor, A. V. (1994). Moral hazard and secured lending in an infinitely repeated credit market game. *International Economic Review*, 35(4):899–920.

Broecker, T. (1990). Credit-worthiness tests and interbank competition. *Econometrica*, 58(2):429–452.

Chan, Y.-S. and Kanatas, G. (1985). Asymmetric valuations and the role of collateral in loan agreements. *Journal of Money, Credit and Banking*, 17(1):84–95.

Dang, T. V., Gorton, G., and Holmstrom, B. (2012). Ignorance, debt and financial crises. *Working paper*.

Degryse, H., Karapetyan, A., and Karmakar, S. (2021). To ask or not to ask? collateral and screening in lending relationships. *Journal of Financial Economics*, 142(1):239–260.

Delis, M. D., Kokas, S., and Ongena, S. (2016). Foreign ownership and market power in banking: Evidence from a world sample. *Journal of Money, Credit and Banking*, 48(2-3):449–483.

Dell'Ariccia, G., Igan, D., and Laeven, L. (2012). Credit booms and lending standards: Evidence from the subprime mortgage market. *Journal of Money, Credit and Banking*, 44(367-384):367?384.

Donaldson, J., Gromb, D., and Piacentino, G. (2020a). Conflicting priorities: A theory of covenants and collateral. *Working paper*.

Donaldson, J., Gromb, D., and Piacentino, G. (2020b). The paradox of pledgeability. *Journal of Financial Economics*, 137(3):591–605.

Forssbaeck, J. and Shehzad, C. T. (2015). The conditional effects of market power on bank risk - cross country evidence. *Review of Finance*, 19:1997–2038.

Godlewski, C. J. and Weill, L. (2011). Does collateral help mitigate adverse selection? a cross-country analysis. *Journal of Financial Services Research*, 40:49–78.

Goel, A. M., Song, F., and Thakor, A. V. (2014). Correlated leverage and its ramifications. *Journal of Financial Intermediation*, 23(4):471 – 503.

Gorton, G. and Ordoñez, G. (2014). Collateral crises. *American Economic Review*, 104(2):343–78.

Gorton, G. and Ordoñez, G. (2020). Good booms, bad booms. *Journal of the European Economic Association*, 18(2):618–665.

Hainz, C., Weill, L., and Godlewski, C. J. (2013). Bank competition and collateral: Theory and evidence. *Journal of Financial Services Research*, 44:131–148.

Howes, C. and Weitzner, G. (2022). Bank information production over the business cycle. *Working paper.*

Hu, Y. (2022). A dynamic theory of bank lending, firm entry, and investment fluctuations. *Journal of Economic Theory*, 204:105515.

Ioannidou, V., Pavanini, N., and Peng, Y. (2022). Collateral requirements and adverse selection in lending markets. *Journal of Financial Economics*, 144(1):93–121.

Laffont, J.-J. and Martimort, D. (2002). *The theory of incentives.* Princeton University Press, Princeton, NJ.

Lee, S. and Persson, P. (2016). Financing from family and friends. *Review of Financial Studies*, 29(9):2341?2386.

Lengwiler, Y. and Rishabh, K. (2017). Credit from the monopoly bank. *Working paper.*

Lisowsky, P., Minnis, M., and Sutherland, A. (2017). Economic growth and financial statement verification. *Journal of Accounting Research*, 55:745–794.

Manove, M., Padilla, A. J., and Pagano, M. (2001). Collateral versus project screening: A model of lazy banks. *RAND Journal of Economics*, 32(4):726–744.

Marquez, R. (2002). Competition, adverse selection, and information in the banking industry. *Review of Financial Studies*, 15(3):901–926.

Myerson, R. B. (1979). Incentive compatibility and the bargaining problem. *Econometrica*, 47(1):61–73.

Ordonez, G., Perez-Reyna, D., and Yogo, M. (2019). Leverage dynamics and credit quality. *Journal of Economic Theory*, 183:183–212.

Parlatore, C. (2019). Collateralizing liquidity. *Journal of Financial Economics*, 131(2):299–322.

Rajan, R. and Winton, A. (1995). Covenants and collateral as incentives to monitor. *Journal of Finance*, 50(4):1113–1146.

Rodano, G., Serrano-Velarde, N., and Tarantino, E. (2018). Lending standards over the credit cycle. *Review of Financial Studies*, 31:2943?2982.

Rothschild, M. and Stiglitz, J. (1976). Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. *Quarterly Journal of Economics*, 90(4):629–649.

Sengupta, R. (2014). Lending to uncreditworthy borrowers. *Journal of Financial Intermediation*, 23(1):101–128.

Shaffer, S. (1998). The winner's curse in banking. *Journal of Financial Intermediation*, 7(4):359 – 392.

Song, F. and Thakor, A. (2010). Financial system architecture and the co-evolution of banks and capital markets. *Economic Journal*, 120(547):1021–1055.

Tirole, J. (2006). *The theory of corporate finance*. Princeton University Press, Princeton, NJ.

Vanasco, V. (2017). The downside of asset screening for market liquidity. *Journal of Finance*, 72(5):1937–1982.