CANDIDATE – PLEASE NOTE!

PRINT your name on the line below and return this booklet with the answer sheet. Failure to do so may result in disqualification.

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FORM TP 2012233

TEST CODE **02234010**

MAY/JUNE 2012

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

ANALYSIS, MATRICES AND COMPLEX NUMBERS

Unit 2 – Paper 01

90 minutes

01 JUNE 2012 (a.m)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

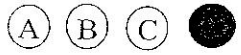
1. This test consists of 45 items. You will have 90 minutes to answer them.
2. In addition to this test booklet, you should have an answer sheet.
3. Do not be concerned that the answer sheet provides spaces for more answers than there are items in this test.
4. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.
5. On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

   Sample Item

   The expression $(1 + \sqrt{3})^2$ is equivalent to

   (A) 4
   (B) 10
   (C) $1 + 3\sqrt{3}$
   (D) $4 + 2\sqrt{3}$

   Sample Answer

   The best answer to this item is "$4 + 2\sqrt{3}$", so answer space (D) has been shaded.
6. If you want to change your answer, be sure to erase it completely before you fill in your new choice.
7. When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, omit it and go on to the next one. You can return later to the item omitted. Your score will be the total number of correct answers.
8. You may do any rough work in this booklet.
9. The use of silent, non-programmable scientific calculators is allowed.

   Examination Materials:

   A list of mathematical formulae and tables. **(Revised 2012)**

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

1. $\int \frac{2x}{(x-1)(x+3)}\,dx =$

(A) $\int \left(\frac{2}{x-1} + \frac{2}{3(x+3)} \right) dx$

(B) $\int \left(\frac{2}{3(x-1)} + \frac{2}{(x+3)} \right) dx$

(C) $\int \left(\frac{3}{2(x-1)} + \frac{1}{2(x+3)} \right) dx$

(D) $\int \left(\frac{1}{2(x-1)} + \frac{3}{2(x+3)} \right) dx$

2. $\frac{x+5}{x(x^2+6x+17)}$ may be expressed as

(A) $\frac{P}{x} + \frac{Q}{(x-1)} + \frac{R}{x+17}$

(B) $\frac{P}{x} + \frac{Qx+R}{(x-1)(x+17)}$

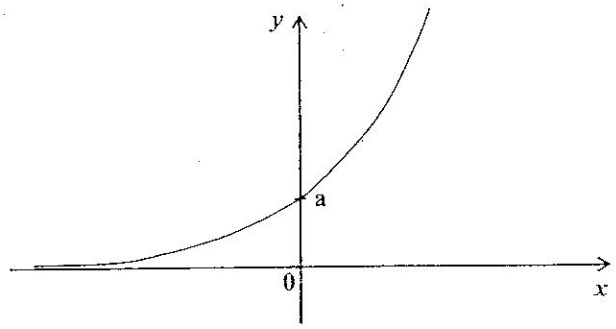
(C) $\frac{P}{x} + \frac{Qx+R}{x^2+6x+17}$

(D) $\frac{P}{x} + \frac{Qx}{x^2+6x+17}$

3. The general solution for the second-order differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ is

(A) $y = Ae^{2x} + Be^{x}$
(B) $y = Ae^{2x} + x\,Be^{x}$
(C) $y = e^{x}(A + Bx)$
(D) $y = e^{2x}(A + Bx)$

Item 4 refers to the following graph.



4. The graph represents

(A) $y = log_x a$
(B) $y = log_a x$
(C) $y = e^x + a$
(D) $y = ae^x$

5. $\int (\cos 5x \cos 3x)dx =$

(A) $\int (\cos 8x + \cos 2x)dx$

(B) $\int (\cos 8x - \cos 2x)dx$

(C) $\frac{1}{2}\int (\cos 8x - \cos 2x)dx$

(D) $\frac{1}{2}\int (\cos 8x + \cos 2x)dx$

6. If $x = \log_a m$, then $\log_m \frac{a^5}{m} =$

(A) $\frac{1}{x^5} - 1$

(B) $\frac{5}{x} - 1$

(C) $5x - 1$

(D) $x^5 - 1$

7. Given $e^{x+y} - x = 0$ then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{e^x + y}$

(B) $\frac{1}{e^{x+y}}$

(C) $\frac{1-e^{x+y}}{e^{x+y}}$

(D) $\frac{1-e^{x+y}}{e^x}$

8. $\frac{d}{dx}\sin^{-1}(2x)$ is equal to

(A) $\sqrt{1-4x^2}$

(B) $2\sqrt{1-4x^2}$

(C) $\frac{1}{\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$

9. $\int \frac{1}{x^2+4} dx =$

(A) $\tan^{-1}(\frac{x}{2}) + c$

(B) $\frac{1}{2}\tan^{-1}(\frac{x}{2}) + c$

(C) $\tan^{-1}(2x) + c$

(D) $2\tan^{-1}(2x) + c$

10. Given that $f(x) = \ln 3x^2$, then $f''(-2)$ equals

(A) -1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

11. If $\frac{dy}{dx} = 2xy$, then the value of $\frac{d^2y}{dx^2}$ at the point (1, 2) is

(A) 6
(B) 8
(C) 12
(D) 16

12. Given that $\ln 2 = a$, $\ln 3 = b$, $\ln 5 = c$, then $\ln 750$ is

(A) $a + b + c$
(B) $a + b + 3c$
(C) $a + 3b + c$
(D) $2a + 2b + 2c$

13. The gradient of the tangent to the curve $x^3 - 2xy + y^2 = 5$ at (2,1) is

(A) 2

(B) $\frac{13}{4}$

(C) 5
(D) 6

14. The expression $e^{3\ln x}$ may be written as

(A) $3x$
(B) x^3
(C) $\ln x^3$
(D) $\ln 3x$

15. The number of bacteria present in a culture is modelled by $y = y_0 e^{kt}$, where $k > 0$, y is the population after t hours, and y_0 is the initial population. The rate of growth, c, when $t = 5$ is given by

(A) $c = e^{5k}$
(B) $c = ke^5$
(C) $c = ky_0e^{5k}$
(D) $c = 5e^{5k}$

16. Which of the following sequences is the first four terms of an arithmetic progression?

(A) n, n – 2, n – 4, n – 6
(B) n, –(n + 1), (n + 2), –(n + 3)
(C) n, 2n + 1, 2n + 3, 2n + 8
(D) $n, \frac{n}{10}, \frac{n}{100}, \frac{n}{1000}$

17. For what values of x is the series $\sum_{r=0}^{\infty} x^r$ convergent?

(A) $x > 1$
(B) $x \le 1$
(C) $-1 < x < 1$
(D) $-1 \le x \le 1$

18. The coefficient of a^2b^5 in the expansion of $(a + b)^7$ is

(A) ${}^3C_2$
(B) ${}^5C_3$
(C) ${}^7C_3$
(D) ${}^7C_5$

19. The binomial coefficient $\binom{n}{2}$ is equivalent to

(A) $\binom{n}{1} + \binom{n}{1}$
(B) $\binom{n}{n-2}$
(C) $\binom{n-2}{n}$
(D) $\binom{n}{n+2}$

20. The sum to infinity of the geometric series $\frac{1}{2} - \frac{1}{3} + \frac{2}{9} - \frac{4}{27} + \cdots$ is

(A) $\frac{3}{10}$
(B) $\frac{3}{5}$
(C) $\frac{3}{2}$
(D) 3

21. A sequence is defined as $u_{n+1} = 1 - \dfrac{1}{1+u_n}$ where $u_1 = 1$ and $n \in \mathrm{N}$. The 20th term of the sequence is

(A) $\dfrac{1}{20}$

(B) $\dfrac{1}{21}$

(C) $\dfrac{19}{20}$

(D) $\dfrac{20}{21}$

22. $\displaystyle\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right) =$

(A) $\dfrac{n+2}{n+1}$

(B) $-\dfrac{n}{n+1}$

(C) $-\dfrac{1}{n+1}$

(D) $\dfrac{n}{n+1}$

23. The values of x for which the expansion of $\dfrac{1}{\sqrt{(100-50x)}}$ is valid are

(A) $-1 < x < 1$

(B) $-2 < x < 2$

(C) $-\dfrac{1}{2} < x < \dfrac{1}{2}$

(D) $x < -2$ and $x > 2$

24. ${}^8C_3$ equals

(A) ${}^8C_2 + {}^8C_1$

(B) ${}^8C_5 + {}^5C_3$

(C) $\dfrac{8}{3} \times {}^7C_3$

(D) $\dfrac{8}{3} \times {}^7C_2$

25. A sequence is defined as $u_1 = 2$, $u_2 = 4$, $u_n = u_{n-1} + u_{n-2}$, for $n \geq 3$. This sequence is

(A) convergent
(B) oscillating
(C) divergent
(D) periodic

26. The value of the term independent of x in the binomial expansion of $\left(x^2 + \dfrac{1}{x}\right)^{12}$ is

(A) $\binom{12}{4}$

(B) $\binom{12}{3}$

(C) $\binom{12}{1}$

(D) $\binom{12}{0}$

27. If the true length of a piece of pipe is 50 cm and its measured length is 51 cm, then the relative error in length is

(A) 1%
(B) 2%
(C) 3%
(D) 4%

28. A table of height 92.5 cm will have error bounds of

(A) ± 0.05
(B) ± 0.10
(C) ± 0.15
(D) ± 5

29. A music artist sells 48 000 CDs in the first year of release. Sales are halved each subsequent year. If each CD is sold for \$25, then the value of sales for a particular year, n, can be expressed as

(A) $P = 25(48000)\left(\frac{1}{2}\right)^n$

(B) $P = 25(48000)\left(\frac{1}{2}\right)^{n-1}$

(C) $P = \frac{25}{2}(48000)^n$

(D) $P = 25(24000)^{n-1}$

30. The function $f(x) = 3 + 4x - x^4$ has a root in the interval

(A) $0 < x < 1$
(B) $1 < x < 2$
(C) $2 < x < 3$
(D) $3 < x < 4$

31. In how many ways can the letters A, B, C, D and E be arranged so that the pair A and B, AND the pair C and D are always together?

(A) 3!

(B) 5!

(C) $5! \times 2$

(D) $3! \times 2 \times 2$

32. If $P = \begin{pmatrix} 1 & -2 & 0 \\ 3 & 1 & 5 \\ -1 & 2 & 3 \end{pmatrix}$ and

$Q = \begin{pmatrix} -7 & 6 & -10 \\ -14 & 3 & -5 \\ 7 & 0 & 7 \end{pmatrix}$ then

$PQ = \begin{pmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{pmatrix}$.

The matrix P^{-1} equals

(A) $(PQ)^{-1}$
(B) $(QP)^{-1}$
(C) $\frac{1}{21}P$
(D) $\frac{1}{21}Q$

33. The number of distinct permutations of the letters of the word POSSIBILITY is

(A) ${}^{11}P_{11}$

(B) $({}^{11}P_3)\,({}^{11}P_2)$

(C) $\frac{11!}{3!2!}$

(D) ${}^{11}P_8$

34. $\frac{2-i}{3+2i} =$

(A) $\frac{4-7i}{13}$

(B) $\frac{7-4i}{13}$

(C) $\frac{8-i}{13}$

(D) $\frac{8+i}{13}$

35. The roots of the equation $x^2 + 1 = 0$ are

(A) $x = -1, 1$
(B) $x = -1, i$
(C) $x = 1, -i$
(D) $x = -i, i$

36. The complex number $z = \sqrt{3} + i$ can be expressed as

(A) $\sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(B) $\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(C) $2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(D) $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

37. X and Y are mutually exclusive events. If $P(X) = \frac{1}{4}$ and $P(Y) = \frac{1}{5}$ then $P(X \cup Y) =$

(A) $\frac{1}{9}$

(B) $\frac{2}{5}$

(C) $\frac{9}{20}$

(D) $\frac{1}{20}$

38. The roots of a quadratic equation $ax^2 + bx + c = 0$, are the complex numbers $1 + 2i$ and $2 - i$. The equation is

(A) $x^2 - (3 + i)x + 4 + 3i = 0$
(B) $x^2 - (3 + i)x - 4 - 3i = 0$
(C) $x^2 - (3 - i)x + 4 + 3i = 0$
(D) $x^2 + (3 + i)x + 4 + 3i = 0$

39. The value of $(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})^2$ is

(A) -2
(B) -1
(C) 1
(D) 2

40. The expresssion $i[(1 + i)^2 - (1 - i)^2]$ is equal to

(A) -4
(B) -2
(C) 2
(D) 4

41. The roots of the equation $2x^2 + 6x + 17 = 0$ are

(A) $\frac{-3 \pm 5i}{2}$

(B) $\frac{3 \pm 5i}{2}$

(C) $\frac{-3}{2} \pm 5i$

(D) $3 \pm \frac{5i}{2}$

42. A is a 3 x 3 matrix with determinant 14. If the matrix of cofactors A is

$$\begin{pmatrix} 4 & -14 & -2 \\ 3 & -7 & -5 \\ 1 & 7 & 3 \end{pmatrix}$$

then A^{-1} =

(A) $\frac{1}{14}\begin{pmatrix} 4 & -14 & -2 \\ 3 & -7 & -5 \\ 1 & 7 & 3 \end{pmatrix}$

(B) $\frac{1}{14}\begin{pmatrix} 4 & 3 & 1 \\ -14 & -7 & 7 \\ -2 & -5 & 3 \end{pmatrix}$

(C) $\frac{1}{14}\begin{pmatrix} 4 & 14 & -2 \\ 1 & 7 & -5 \\ 1 & -7 & 3 \end{pmatrix}$

(D) $\frac{1}{14}\begin{pmatrix} 4 & -3 & 1 \\ -14 & 7 & 7 \\ -2 & -5 & 3 \end{pmatrix}$

43. The letters of the word I R R E G U L A R are to be arranged in a line. The number of possible arrangements in which the 3 Rs are NOT together is

(A) $7!$
(B) $9! - 7!$
(C) $\frac{9!}{3!} - 7!$
(D) $9! - 7! \times 3!$

Item 44 refers to the table below which shows the number of males and females and their preferences for Drink A and Drink B.

	Male	Female	Total
Drink A	12	18	30
Drink B	20	10	30
Total	32	28	60

44. One person is randomly selected. What is the probability that this person is female and prefers Drink B?

(A) $\frac{1}{6}$
(B) $\frac{1}{3}$
(C) $\frac{5}{14}$
(D) $\frac{1}{2}$

45. The locus of the points described by a complex number z is given by $|z-1-2i|=3$. The locus describes a circle with

(A) centre $(-1, -2)$ and radius 3 units
(B) centre $(-1, -2)$ and radius 9 units
(C) centre $(1, 2)$ and radius 3 units
(D) centre $(1, 2)$ and radius 9 units

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.