1. The conjugate of the complex number $7+\frac{1}{2}i$ is

(A) $-7-\frac{1}{2}i$

(B) $-7+\frac{1}{2}i$

(C) $7-\frac{1}{2}i$

(D) $\frac{1}{2}+7i$

2. The complex number $z=\sqrt{3}+i$ can be expressed as

(A) $\sqrt{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

(B) $\sqrt{2}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

(C) $2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

(D) $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

3. Given that $\cos 2x = 1 - 2\sin^2 x$,
then $\int_0^{\pi} \sin^2\left(\frac{x}{4}\right)dx$ is

(A) $2-\pi$
(B) $\pi-2$
(C) $1-\frac{\pi}{2}$
(D) $\frac{\pi}{2}-1$

4. One square root of $3-4i$ is

(A) $\sqrt{3}-2i$

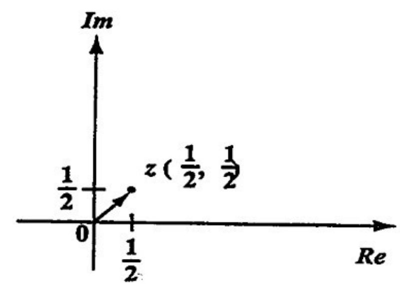
(B) $\sqrt{3}+2i$

(C) $2-i$

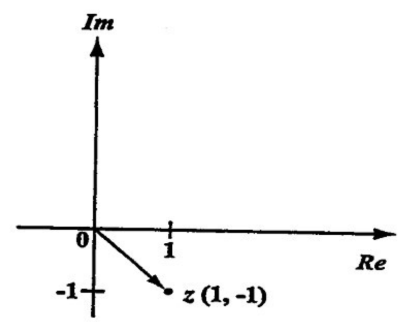
(D) $2+i$

5. The complex number $z=\frac{1}{1-i}$ can be represented on an Argand diagram as

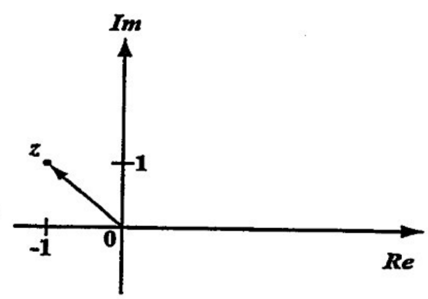
(A)



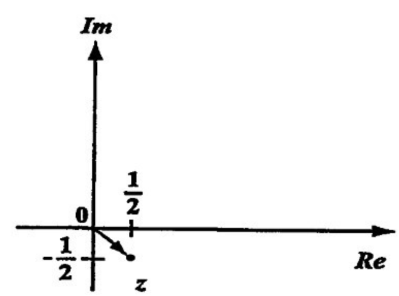
(B)



(C)



(D)



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6. If $f(x) = \ln 2x$, then $f'(x) =$

(A) $\frac{2}{x}$

(B) $\frac{1}{x^2}$

(C) $\frac{-2}{x}$

(D) $\frac{1}{x}$

7. The equation $e^x - x^4 = 0$ has a root between

(A) 0 and 1
(B) 1 and 2
(C) 2 and 3
(D) 3 and 4

8. The number of bacteria present in a culture is modelled by $y = y_0 e^{kt}$, where $k > 0$, y is the population after t hours, and y_0 is the initial population. The rate of growth, c, when $t = 5$ is given by

(A) $c = e^{5k}$

(B) $c = ke^5$

(C) $c = 5e^{5k}$

(D) $c = ky_0e^{5k}$

9. $\frac{d}{dx}(\ln x)^3 =$

(A) $\frac{3}{x}$

(B) $3x$

(C) $\frac{3}{x}(\ln x)^2$

(D) $3(\ln x)^2$

10. Given that a, b, c and k are constants, then $\int \frac{3}{x^2(x-1)}\,dx$ can be expressed as

(A) $\frac{a}{x} + b\ln|x-1| + k$

(B) $a\ln|x| + b\ln|x-1| + k$

(C) $a\ln|x| + \frac{b}{x} + \frac{c}{x-1} + k$

(D) $a\ln|x| - \frac{b}{x} + c\ln|x-1| + k$

11. The integral of $\frac{1}{1-\sin^2 x}$ with respect to x is

(A) $\tan x + c$
(B) $\tan^2 x + c$
(C) $\sec^2 x + c$
(D) $\sec x \tan x + c$

12. The partial fractions of $\frac{x+3}{(2x+5)(x-1)^2}$ may be expressed in the form

(A) $\frac{P}{2x+5} + \frac{Q}{(x-1)^2}$

(B) $\frac{Px}{(2x+5)} + \frac{Q+R}{(x-1)^2}$

(C) $\frac{Px}{(2x+5)} + \frac{Q}{(x-1)} + \frac{Rx}{(x-1)^2}$

(D) $\frac{P}{(2x+5)} + \frac{Q}{(x-1)} + \frac{R}{(x-1)^2}$

13. A curve is given parametrically by the equations $x = t^2 - 2t$, $y = t^2 + 2t$. The **simplest** expression for the gradient of the tangent in terms of t is

(A) $\frac{t-1}{t+1}$

(B) $\frac{2t-2}{2t+2}$

(C) $\frac{t+1}{t-1}$

(D) $\frac{t+1}{2t-2}$

14. $\int \frac{dx}{\sqrt{1-9x^2}} =$

(A) $\frac{1}{9}\sin^{-1} x + c$

(B) $\frac{1}{3}\sin^{-1} 3x + c$

(C) $\frac{1}{9}\sin^{-1} \frac{x}{3} + c$

(D) $\frac{1}{9}\sin^{-1} 3x + c$

15. The argument of the complex number $z = \frac{1}{2} + i\frac{\sqrt{3}}{2}$ is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

16. For $-1 < 2n < 1$, $\sum_{r=0}^{\infty}(2n)^r =$

(A) $\frac{2}{1-n}$

(B) $\frac{1}{1-2n}$

(C) $\frac{2n}{1+2n}$

(D) $\frac{2n}{1-2n}$

17. If the terms of the sequence $u_1, u_2, u_3 \ldots, u_n \ldots$ satisfy the recurrence relation $u_{n+1} = u_n + 3$, $n \geq 1$, then the n^{th} term may be expressed as

(A) $u_1 + 3(n-1)$
(B) $u_1 + 6(n-1)$
(C) $u_1 + 3n$
(D) $u_1 + 6n$

18. The 5th term in the sequence that is defined by the relation $u_n = (-1)^{n+1}\frac{n}{3n-1}$, $n \geq 1$, is

(A) $\frac{-10}{14}$

(B) $\frac{-5}{14}$

(C) $\frac{5}{14}$

(D) $\frac{10}{14}$

19. Which of the following sequences, $\{u_n\}$, converges?

(A) $\left\{\left(\frac{1}{2}\right)^n\right\}$

(B) $\left\{\left(\frac{1}{n}\right)^{-n}\right\}$

(C) $\left\{(-1)^n\right\}$

(D) $\left\{2^n\right\}$

20. Which of the following series are arithmetic series?

I. $\sum_{r=1}^{n}(7+3r)$

II. $\sum_{r=1}^{n}2(3^r)$

III. $\sum_{r=1}^{n}\log_{10}(r+1)$

IV. $\sum_{r=1}^{n}\log_{10}3^{(r+1)}$

(A) I and III only
(B) I and IV only
(C) III and IV only
(D) I, II and IV only

21. The sum to infinity of a geometric series is $\frac{1}{1-2x}$. The range of x is

(A) $-1 < x < 1$

(B) $-2 < x < 2$

(C) $-\frac{1}{2} < x < \frac{1}{2}$

(D) $x > \frac{1}{2}$

22. Let a_n and S_n denote respectively, the value of the n^{th} term and the n^{th} partial sum of a series. The value of $S_{n+2} - S_n$ when calculated on the series is

(A) a_{n+1}
(B) a_{n+2}
(C) $a_{n+1} + a_{n+2}$
(D) $a_{n+1} - a_{n+2}$

23. The binomial coefficient $\binom{n}{2}$ is equivalent to

(A) $\binom{n}{1}+\binom{n}{1}$

(B) $\binom{n}{n-2}$

(C) $\binom{n-2}{n}$

(D) $\binom{n}{n+2}$

24. Given that $S_n = \sum_{i=1}^{n}\left(\frac{1}{i}-\frac{1}{i+1}\right)$, $\lim_{n\to\infty} S_n$ is

(A) 0

(B) $\frac{2}{3}$

(C) 1

(D) ∞

25. The value of the term that is independent of x in the binomial expansion of $\left(x^2+\frac{1}{x}\right)^{12}$ is

(A) $\binom{12}{4}$

(B) $\binom{12}{3}$

(C) $\binom{12}{1}$

(D) $\binom{12}{0}$

26. Given that the coefficient of the term in b^3 in the binomial expansion of $(a+b)^5$ is 40, then a =

(A) 2
(B) 4
(C) 10
(D) 20

27. If $\sum_{n=2}^{\infty} 2^{-n} = a$, then a is

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 2

(D) 4

28. The Maclaurin series for sin x, up to the term in x^3, is

(A) $x-\frac{x^3}{6}$

(B) $x+\frac{x^3}{6}$

(C) $1+x+\frac{x^2}{2}-\frac{x^3}{6}$

(D) $1+x-\frac{x^2}{2}-\frac{x^3}{6}$

29. Let f be a continuous function with $f(0) = 1$ and $f(0.8) = -0.76$.

The first approximation to the root in $[0, 0.8]$, to three decimal places, using linear interpolation is

(A) 0.000
(B) 0.400
(C) 0.444
(D) 0.455

30. Given that the nth approximation of the root of the equation $x^5 = x^3 + 25$ based on the Newton–Raphson method is x_n, then x_{n+1} may be expressed as

(A) $\frac{x_n - x_n^5 - x_n^3 + 25}{5x_n^4 - 3x_n^2}$

(B) $\frac{x_n - x_n^5 - s_n^3 - 25}{5x_n^4 - 3x_n^2}$

(C) $\frac{4x_n^5 - 4x_n^3 - 25}{5}$

(D) $\frac{4x_n^5 - 2x_n^3 + 25}{5x_n^4 - 3x_n^2}$

31. In how many ways can the letters ABCDE be arranged so that the A and B are always together?

(A) $5!$

(B) $4! \times 2$

(C) $4!$

(D) $\frac{5!}{2}$

32. The number of distinct permutations of the letters of the word P O S S I B I L I T Y is

(A) $\frac{11!}{3!2!}$

(B) ${}^{11}P_{11}$

(C) ${}^{11}P_{8}$

(D) $({}^{11}P_{3})\ ({}^{11}P_{2})$

33. A relay team of five teachers is to be chosen from a group of 15 teachers.

In how many ways could this relay team be chosen?

(A) $\frac{15!}{10!5!}$

(B) $\frac{15!}{10!}$

(C) $\frac{15!}{5!}$

(D) $15!$

34. X and Y are mutually exclusive events. If $P(X) = \frac{1}{4}$ and $P(Y) = \frac{1}{5}$ then $P(X \cup Y) =$

(A) $\frac{1}{9}$

(B) $\frac{2}{5}$

(C) $\frac{9}{20}$

(D) $\frac{1}{20}$

35. The matrix A is a 3×3 matrix with determinant 14. If the matrix of cofactors of A is $\begin{pmatrix} 4 & -14 & -2 \\ 3 & -7 & -5 \\ 1 & 7 & 3 \end{pmatrix}$ then $A^{-1} =$

(A) $\frac{1}{14}\begin{pmatrix} 4 & -14 & -2 \\ 3 & -7 & -5 \\ 1 & 7 & 3 \end{pmatrix}$

(B) $\frac{1}{14}\begin{pmatrix} 4 & 3 & 1 \\ -14 & -7 & 7 \\ -2 & -5 & 3 \end{pmatrix}$

(C) $\frac{1}{14}\begin{pmatrix} 4 & 14 & -2 \\ 3 & 7 & -5 \\ 1 & -7 & 3 \end{pmatrix}$

(D) $\frac{1}{14}\begin{pmatrix} 4 & -3 & 1 \\ -14 & 7 & 7 \\ -2 & -5 & 3 \end{pmatrix}$

36. The number of possible values of x which satisfy the system of simultaneous equations,

$$2x + 3y + 2z = -5$$
$$4x + 6y + 4z = -10$$
$$6x + 9y + 6z = -16$$

is

(A) 0

(B) 1

(C) 2

(D) 3

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37. If $M = \begin{pmatrix} 1 & 1 & 4 \\ 3 & 2 & -1 \\ 6 & 0 & 5 \end{pmatrix}$, then the cofactor of the element 3 in M above may be written as

(A) $\begin{vmatrix} 1 & 4 \\ 0 & 5 \end{vmatrix}$

(B) $-\begin{vmatrix} 1 & 4 \\ 0 & 5 \end{vmatrix}$

(C) $\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix}$

(D) $-\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix}$

38. If $P = \begin{pmatrix} 1 & -2 & 0 \\ 3 & 1 & 5 \\ -1 & 2 & 3 \end{pmatrix}$ and

$Q = \begin{pmatrix} -7 & 6 & -10 \\ -14 & 3 & -5 \\ 7 & 0 & 7 \end{pmatrix}$ then

$PQ = \begin{pmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{pmatrix}$.

The matrix P^{-1} equals

(A) $(PQ)^{-1}$

(B) $(QP)^{-1}$

(C) $\frac{1}{21}P$

(D) $\frac{1}{21}Q$

39. The letters of the word I R R E G U L A R are to be arranged in a line. The number of possible arrangements in which the 3 Rs are NOT together is

(A) $7!$

(B) $\frac{9!}{3!} - 7!$

(C) $9! - 7!$

(D) $9! - 7! \times 3!$

40. Two coins and a die with faces numbered 1 to 6 are thrown together once. Assuming that the die and coins are fair, the probability of obtaining 2 heads and a number less than 4 is

(A) $\frac{2}{3}$

(B) $\frac{1}{24}$

(C) $\frac{1}{8}$

(D) $\frac{3}{4}$

41. The determinant of the matrix

$M = \begin{pmatrix} 3 & -1 & 5 \\ 2 & 3 & -2 \\ 0 & 5 & 4 \end{pmatrix}$ is

(A) 20
(B) 64
(C) 104
(D) 124

42. If the auxiliary equation for a homogeneous second order differential equation with real, constant coefficients is given by $\lambda^2 + 6\lambda + 50 = 0$, then the general solution of the differential equation may be given by

(A) $y = e^{\lambda x}(A + Bx)$

(B) $ye^{\int \lambda(x)dx} = \int Q(x)e^{\int \lambda(x)dx}\,dx + C$

(C) $y = Ae^{\lambda_1 x} + Be^{\lambda_2 x}$

(D) $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

43. The general solution of the differential equation $(x-2)\frac{dy}{dx} = y$ is

(A) $y = e^{ln(x-2)} + c$
(B) $y = e^{ln(x-2)} \cdot e^c$
(C) $y = 2e^{ln(x-2)} + c$
(D) $y = -2e^{ln(x-2)} + ln\,c$

44. Given that $y = \frac{\pi}{4}$ and $x = \frac{1}{2}$, then the particular solution of $\frac{dy}{dx} = 2x\cos^2 y$ is

(A) $\tan y = x^2 + \frac{3}{4}$

(B) $\tan y = 2x + \frac{1}{4}$

(C) $\sin y = 2x + 1$

(D) $2\cos y = x^2 + c$

45. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

(A) $y = e^x$
(B) $y = kx$
(C) $y = x + k$
(D) $y = ln\,x + k$

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.