



CARIBBEAN EXAMINATIONS COUNCIL

CAPE® Pure Mathematics

**SYLLABUS
SPECIMEN PAPER
MARK SCHEME
SUBJECT REPORTS**

Macmillan Education
4 Crinan Street, London, N1 9XW
A division of Macmillan Publishers Limited
Companies and representatives throughout the world

www.macmillan-caribbean.com

ISBN 978-0-230-48247-0

© Caribbean Examinations Council (CXC®) 2015
www.cxc.org
www.cxc-store.com

The author has asserted their right to be identified as the author of this work in accordance with the Copyright, Design and Patents Act 1988.

First published 2014
This revised version published 2015

Permission to copy
The material in this book is copyright. However, the publisher grants permission for copies to be made without fee. Individuals may make copies for their own use or for use by classes of which they are in charge; institutions may make copies for use within and by the staff and students of that institution. For copying in any other circumstances, prior permission in writing must be obtained from Macmillan Publishers Limited. Under no circumstances may the material in this book be used, in part or in its entirety, for commercial gain. It must not be sold in any format.

Designed by Macmillan Publishers Limited
Cover design by Macmillan Publishers Limited and Red Giraffe

CAPE® Pure Mathematics Free Resources

LIST OF CONTENTS

CAPE® Pure Mathematics Syllabus Extract	3
--	----------

CAPE® Pure Mathematics Syllabus	4
--	----------

CAPE® Pure Mathematics Specimen Papers:

Unit 1 Paper 01	68
Unit 1 Paper 02	76
Unit 1 Paper 032	82
Unit 2 Paper 01	86
Unit 2 Paper 02	94
Unit 2 Paper 032	99

CAPE® Pure Mathematics Mark Schemes:

Unit 1 Paper 01	103
Unit 1 Paper 02	105
Unit 1 Paper 032	118
Unit 2 Paper 01	123
Unit 2 Paper 02	125
Unit 2 Paper 032	138

CAPE® Pure Mathematics Subject Reports:

2004 Subject Report	143
2005 Subject Report	181
2006 Subject Report	213
2007 Subject Report	243
2008 Subject Report Trinidad and Tobago	275
2008 Subject Report Rest of Caribbean	294
2009 Subject Report	313
2010 Subject Report	332
2011 Subject Report	355
2012 Subject Report	379
2013 Subject Report	402
2014 Subject Report	422

Pure Mathematics

Mathematics is one of the oldest and most universal means of creating, communicating, connecting and applying structural and quantitative ideas. Students doing this syllabus will have already been exposed to Mathematics in some form mainly through courses that emphasise skills in using mathematics as a tool, rather than giving insight into the underlying concepts.

This syllabus will not only provide students with more advanced mathematical ideas, skills and techniques, but encourage them to understand the concepts involved, why and how they "work" and how they are interconnected. It is also to be hoped that, in this way, students will lose the fear associated with having to learn a multiplicity of seemingly unconnected facts, procedures and formulae. In addition, the course should show them that mathematical concepts lend themselves to generalisations, and that there is enormous scope for applications to solving real problems. The course is therefore intended to provide quality in selected areas rather than in a large number of topics.

The syllabus is arranged into two (2) Units, each Unit consists of three Modules.

Unit 1: Algebra, Geometry and Calculus

- Module 1 – Basic Algebra and Functions
- Module 2 – Trigonometry, Geometry and Vectors
- Module 3 – Calculus I

Unit 2: Complex Numbers, Analysis and Matrices

- Module 1 – Complex Numbers and Calculus II
- Module 2 – Sequences, Series and Approximations
- Module 3 – Counting, Matrices and Differential Equations



CARIBBEAN EXAMINATIONS COUNCIL

Caribbean Advanced Proficiency Examination®

CAPE®

PURE MATHEMATICS

Effective for examinations from May–June 2013

Published by the Caribbean Examinations Council

© 2012, Caribbean Examinations Council

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form, or by any means electronic, photocopying, recording or otherwise without prior permission of the author or publisher.

Correspondence related to the syllabus should be addressed to:

The Pro-Registrar
Caribbean Examinations Council
Caenwood Centre
37 Arnold Road, Kingston 5, Jamaica

Telephone Number:+1(876) 630-5200
Facsimile Number: +1(876) 967-4972
E-mail Address: cxcwzo@cxc.org
Website: www.cxc.org

Copyright © 2012 by Caribbean Examinations Council
The Garrison, St Michael BB14038, Barbados



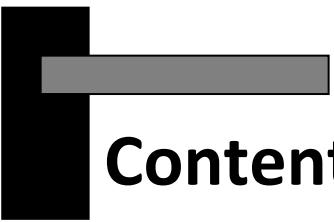
This document CXC A6/U2/12 replaces CXC A6/U2/07 issued in 2007.

Please note that the syllabus has been amended and amendments are indicated by italics.

First issued 1999
Revised 2004
Revised 2007
Amended 2012

Please check the website www.cxc.org for updates on CXC's syllabuses.





Contents

INTRODUCTION	i
RATIONALE	1
AIMS	2
SKILLS AND ABILITIES TO BE ASSESSED	3
PRE-REQUISITES OF THE SYLLABUS.....	3
STRUCTURE OF THE SYLLABUS.....	3
RECOMMENDED 2-UNIT OPTIONS.....	4
<i>MATHEMATICAL MODELLING</i>	4
UNIT 1: ALGEBRA, GEOMETRY AND CALCULUS	
MODULE 1 : BASIC ALGEBRA AND FUNCTIONS	7
MODULE 2 : <i>TRIGONOMETRY, GEOMETRY AND VECTORS</i>	16
MODULE 3 : CALCULUS I	21
UNIT 2: <i>COMPLEX NUMBERS, ANALYSIS AND MATRICES</i>	
MODULE 1 : <i>COMPLEX NUMBERS AND CALCULUS II</i>	27
MODULE 2 : SEQUENCES, SERIES AND APPROXIMATIONS.....	33
MODULE 3 : COUNTING, MATRICES AND DIFFERENTIAL EQUATIONS	40
OUTLINE OF ASSESSMENT	46
REGULATIONS FOR PRIVATE CANDIDATES	54
REGULATIONS FOR RE-SIT CANDIDATES	54
ASSESSMENT GRID.....	55
MATHEMATICAL NOTATION	56



Introduction

The Caribbean Advanced Proficiency Examination (CAPE) is designed to provide certification of the academic, vocational and technical achievement of students in the Caribbean who, having completed a minimum of five years of secondary education, wish to further their studies. The examinations address the skills and knowledge acquired by students under a flexible and articulated system where subjects are organised in 1-Unit or 2-Unit courses with each Unit containing three Modules. Subjects examined under CAPE may be studied concurrently or singly.

The Caribbean Examinations Council offers three types of certification. The first is the award of a certificate showing each CAPE Unit completed. The second is the CAPE Diploma, awarded to candidates who have satisfactorily completed at least six Units including Caribbean Studies. The third is the CXC Associate Degree, awarded for the satisfactory completion of a prescribed cluster of seven CAPE Units including Caribbean Studies and Communication Studies. For the CAPE Diploma and the CXC Associate Degree, candidates must complete the cluster of required Units within a maximum period of five years.

Recognised educational institutions presenting candidates for CXC Associate Degree in one of the nine categories must, on registering these candidates at the start of the qualifying year, have them confirm in the required form, the Associate Degree they wish to be awarded. Candidates will not be awarded any possible alternatives for which they did not apply.

Pure Mathematics Syllabus

◆ RATIONALE

Mathematics is one of the oldest and most universal means of creating, communicating, connecting and applying structural and quantitative ideas. The discipline of Mathematics allows the formulation and solution of real-world problems as well as the creation of new mathematical ideas, both as an intellectual end in itself, as well as a means to increase the success and generality of mathematical applications. This success can be measured by the quantum leap that occurs in the progress made in other traditional disciplines once mathematics is introduced to describe and analyse the problems studied. It is therefore essential that as many persons as possible be taught not only to be able to use mathematics, but also to understand it.

Students doing this syllabus will have already been exposed to Mathematics in some form mainly through courses that emphasise skills in using mathematics as a tool, rather than giving insight into the underlying concepts. To enable students to gain access to mathematics training at the tertiary level, to equip them with the ability to expand their mathematical knowledge and to make proper use of it, it is necessary that a mathematics course at this level should not only provide them with more advanced mathematical ideas, skills and techniques, but encourage them to understand the concepts involved, why and how they "work" and how they are interconnected. It is also to be hoped that, in this way, students will lose the fear associated with having to learn a multiplicity of seemingly unconnected facts, procedures and formulae. In addition, the course should show them that mathematical concepts lend themselves to generalisations, and that there is enormous scope for applications to solving real problems.

Mathematics covers extremely wide areas. However, students can gain more from a study of carefully selected, representative areas of Mathematics, for a "mathematical" understanding of these areas, rather than a superficial overview of a much wider field. While proper exposure to a mathematical topic does not immediately make students into experts in it, proper exposure will certainly give the students the kind of attitude which will allow them to become experts in other mathematical areas to which they have not been previously exposed. The course is therefore intended to provide quality in selected areas rather than in a large number of topics.

This syllabus will contribute to the development of the Ideal Caribbean Person as articulated by the CARICOM Heads of Government in the following areas: "demonstrate multiple literacies, independent and critical thinking and innovative application of science and technology to problem solving. Such a person should also demonstrate a positive work attitude and value and display creative imagination and entrepreneurship".

◆ AIMS

The syllabus aims to:

1. provide understanding of mathematical concepts and structures, their development and the relationships between them;
2. enable the development of skills in the use of mathematical and *information, communication and technology (ICT)* tools;
3. develop an appreciation of the idea of mathematical proof, the internal logical coherence of Mathematics, and its consequent universal applicability;
4. develop the ability to make connections between distinct concepts in Mathematics, and between mathematical ideas and those pertaining to other disciplines;
5. develop a spirit of mathematical curiosity and creativity, as well as a sense of enjoyment;
6. enable the analysis, abstraction and generalisation of mathematical ideas;
7. develop in students the skills of recognising essential aspects of concrete, real-world problems, formulating these problems into relevant and solvable mathematical problems and mathematical modelling;
8. develop the ability of students to carry out independent or group work on tasks involving mathematical modelling;
9. *integrate ICT tools and skills;*
10. provide students with access to more advanced courses in Mathematics and its applications at tertiary institutions.

◆ SKILLS AND ABILITIES TO BE ASSESSED

The assessment will test candidates' skills and abilities in relation to three cognitive levels.

1. **Conceptual knowledge** is the ability to **recall, select** and **use** appropriate facts, concepts and principles in a variety of contexts.
2. **Algorithmic knowledge** is the ability to **manipulate** mathematical expressions and procedures using appropriate symbols and language, logical deduction and inferences.
3. **Reasoning** is the ability to **select** appropriate strategy or select, **use** and **evaluate** mathematical models and **interpret** the results of a mathematical solution in terms of a given real-world problem and engage in problem-solving.

◆ PRE-REQUISITES OF THE SYLLABUS

Any person with a **good** grasp of the contents of the syllabus of the Caribbean Secondary Education Certificate (CSEC) General Proficiency course in Mathematics, *and/or the Caribbean Secondary Education Certificate (CSEC) General Proficiency course in Additional Mathematics*, or equivalent, should be able to undertake the course. However, successful participation in the course will also depend on the possession of good verbal and written communication skills.

◆ STRUCTURE OF THE SYLLABUS

The syllabus is arranged into two (2) Units, Unit 1 which will lay the foundation, and Unit 2 which expands on, and applies, the concepts formulated in Unit 1.

It is therefore recommended that Unit 2 be taken after satisfactory completion of Unit 1 or a similar course. Completion of each Unit will be separately certified.

Each Unit consists of three Modules.

Unit 1: Algebra, Geometry and Calculus, contains three Modules each requiring approximately 50 hours. The total teaching time is therefore approximately 150 hours.

- | | | |
|----------|---|------------------------------------|
| Module 1 | - | Basic Algebra and Functions |
| Module 2 | - | Trigonometry, Geometry and Vectors |
| Module 3 | - | Calculus I |

Unit 2: Complex Numbers, Analysis and Matrices, contains three Modules, each requiring approximately 50 hours. The total teaching time is therefore approximately 150 hours.

- | | | |
|----------|---|---|
| Module 1 | - | Complex Numbers and Calculus II |
| Module 2 | - | Sequences, Series and Approximations |
| Module 3 | - | Counting, Matrices and Differential Equations |

◆ RECOMMENDED 2-UNIT OPTIONS

1. Pure Mathematics Unit 1 **AND** Pure Mathematics Unit 2.
2. Applied Mathematics Unit 1 **AND** Applied Mathematics Unit 2.
3. Pure Mathematics Unit 1 **AND** Applied Mathematics Unit 2.

◆ MATHEMATICAL MODELLING

Mathematical Modelling should be used in both Units 1 and 2 to solve real-world problems.

- A. The topic Mathematical Modelling involves the following steps:
 1. identification of a real-world situation to which modelling is applicable;
 2. carry out the modelling process for a chosen situation to which modelling is applicable;
 3. discuss and evaluate the findings of a mathematical model in a written report.
- B. The Modelling process requires:
 1. a clear statement posed in a real-world situation, and identification of its essential features;
 2. translation or abstraction of the problem, giving a representation of the essential features of the real-world;
 3. solution of the mathematical problem (analytic, numerical, approximate);
 4. testing the appropriateness and the accuracy of the solution against behaviour in the real-world;
 5. refinement of the model as necessary.
- C. Consider the two situations given below.
 1. A weather forecaster needs to be able to calculate the possible effects of atmospheric pressure changes on temperature.
 2. An economic adviser to the Central Bank Governor needs to be able to calculate the likely effect on the employment rate of altering the Central Bank's interest rate.

In each case, people are expected to predict something that is likely to happen in the future. Furthermore, in each instance, these persons may save lives, time, and money or change their actions in some way as a result of their predictions.

One method of predicting is to set up a mathematical model of the situation. A mathematical model is not usually a model in the sense of a scale model motor car. A mathematical model is a way of describing an underlying situation mathematically, perhaps with equations, with rules or with diagrams.

D. Some examples of mathematical models are:

1. Equations

(a) Business

A recording studio invests \$25 000 to produce a master CD of a singing group. It costs \$50.00 to make each copy from the master and cover the operating expenses. We can model this situation by the equation or mathematical model,

$$C = 50.00x + 25\,000$$

where C is the cost of producing x CDs. With this model, one can predict the cost of producing 60 CDs or 6 000 CDs.

Is the equation $x + 2 = 5$ a mathematical model? Justify your answer.

(b) Banking

Suppose you invest \$100.00 with a commercial bank which pays interest at 12% per annum. You may leave the interest in the account to accumulate. The equation $A = 100(1.12)^n$ can be used to model the amount of money in your account after n years.

2. Table of Values

Traffic Management

The table below shows the safe stopping distances for cars recommended by the Highway Code.

Speed m/h	Thinking Distance m	Braking Distance m	Overall Stopping Distance m
20	6	6	12
30	9	14	23
40	12	24	36
50	15	38	53
60	18	55	73
70	21	75	96

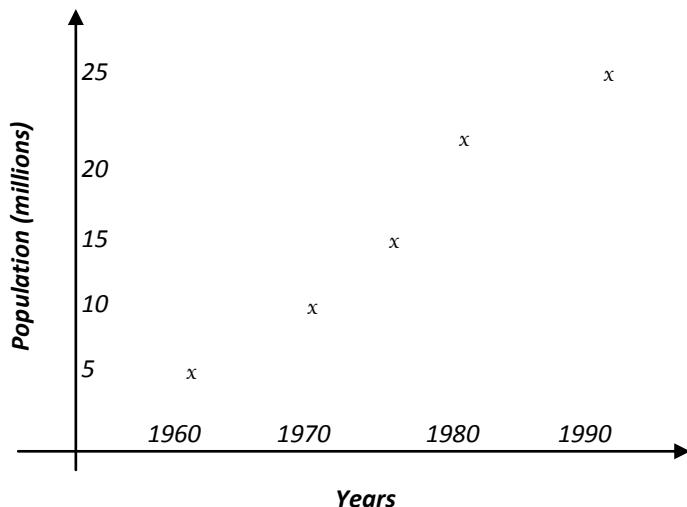
We can predict our stopping distance when travelling at 50 m/h from this model.

3. Rules of ThUMB

You might have used some mathematical models of your own without realising it; perhaps you think of them as “rules of thumb”. For example, in the baking of hams, most cooks use the rule of thumb that “bake ham fat side up in roasting pan in a moderate oven (160°C) ensuring 25 to 40 minutes per $\frac{1}{2}$ kg”. The cook is able to predict how long it takes to bake his ham without burning it.

4. Graphs

Not all models are symbolic in nature; they may be graphical. For example, the graph below shows the population at different years for a certain country.



RESOURCE

Hartzler, J. S. and Swetz, F.

Mathematical Modelling in the Secondary School Curriculum, A Resource Guide of Classroom Exercises, Vancouver, United States of America: National Council of Teachers of Mathematics, Incorporated, Reston, 1991.

◆ UNIT 1: ALGEBRA, GEOMETRY AND CALCULUS

MODULE 1: BASIC ALGEBRA AND FUNCTIONS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *develop the ability to construct simple proofs of mathematical assertions;*
2. *understand the concept of a function;*
3. *be confident in the manipulation of algebraic expressions and the solutions of equations and inequalities;*
4. *understand the properties and significance of the exponential and logarithm functions;*
5. *develop the ability to use concepts to model and solve real-world problems.*

SPECIFIC OBJECTIVES

(A) ***Reasoning and Logic***

Students should be able to:

1. *identify simple and compound propositions;*
2. *establish the truth value of compound statements using truth tables;*
3. *state the converse, contrapositive and inverse of a conditional (implication) statement;*
4. *determine whether two statements are logically equivalent.*

CONTENT

(A) ***Reasoning and Logic***

1. *Simple statement (proposition), connectives (conjunction, disjunction, negation, conditional, bi-conditional), compound statements.*
2. *Truth tables.*
3. *Converse and contrapositive of statements.*
4. *Logical equivalence.*
5. *Identities involving propositions.*

UNIT 1

MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

SPECIFIC OBJECTIVES

(B) The Real Number System – \mathbb{R}

Students should be able to:

1. *perform binary operations;*
2. *use the concepts of identity, closure, inverse, commutativity, associativity, distributivity addition, multiplication and other simple binary operations;*
3. *perform operations involving surds;*
4. construct simple proofs, specifically direct proofs, or proof by the use of counter examples;
5. *use the summation notation (Σ);*
6. establish simple proofs by using the principle of mathematical induction.

CONTENT

(B) The Real Number System – \mathbb{R}

1. *Definition of binary operation.*
2. *Applications of the concepts of commutativity, associativity, distributivity, identity, inverse and closure.*
3. Axioms of the system - including commutative, associative and distributive laws; non-existence of the multiplicative inverse of zero.
4. Methods of proof - direct, counter-examples.
5. Simple applications of mathematical induction.

SPECIFIC OBJECTIVES

(C) Algebraic Operations

Students should be able to:

1. *apply the Remainder Theorem;*
2. use the Factor Theorem to find factors and to evaluate unknown coefficients;
3. extract all factors of $a^n - b^n$ for positive integers $n \leq 6$;
4. use the concept of identity of polynomial expressions.

UNIT 1

MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

CONTENT

(C) Algebraic Operations

1. Factor Theorem.
2. Remainder Theorem.

SPECIFIC OBJECTIVES

(D) Exponential and Logarithmic Functions

Students should be able to:

1. define an exponential function $y = a^x$ for $a \in \mathbb{R}$;
2. sketch the graph of $y = a^x$;
3. define a logarithmic function as the inverse of an exponential function;
4. define the exponential functions $y = e^x$ and its inverse $y = \ln x$, where $\ln x \equiv \log_e x$;
5. use the fact that $y = \ln x \Leftrightarrow x = e^y$;
6. simplify expressions by using laws of logarithms;
7. use logarithms to solve equations of the form $a^x = b$;
8. solve problems involving changing of the base of a logarithm.

CONTENT

(D) Exponential and Logarithmic Functions

1. Graphs of the functions a^x and $\log_a x$.
2. Properties of the exponential and logarithmic functions.
3. Exponential and natural logarithmic functions and their graphs.

UNIT 1

MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

4. Laws of logarithms applied to problems:

(a) $\ln (PQ) = \ln P + \ln Q;$

(b) $\ln (P/Q) = \ln P - \ln Q;$

(c) $\ln P^a = a \ln P.$

SPECIFIC OBJECTIVES

(E) Functions

Students should be able to:

1. define mathematically the terms: function, domain, range, one-to-one function (injective function), onto function (surjective function), many-to-one, one-to-one and onto function (bijective function), composition and inverse of functions;
2. prove whether or not a given simple function is one-to-one or onto and if its inverse exists;
3. use the fact that a function may be defined as a set of ordered pairs;
4. use the fact that if g is the inverse function of f , then $f[g(x)] = x$, for all x , in the domain of g ;
5. illustrate by means of graphs, the relationship between the function $y = f(x)$ given in graphical form and $y = |f(x)|$ and the inverse of $f(x)$, that is, $y = f^{-1}(x)$.

CONTENT

(E) Functions

1. Domain, range, many-to-one, composition.
2. Injective, surjective, bijective functions, inverse function.
3. Transformation of the graph $y = f(x)$ to $y = |f(x)|$ and, if appropriate, to $y = f^{-1}(x)$.

UNIT 1

MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

SPECIFIC OBJECTIVES

(F) *The Modulus Function*

Students should be able to:

1. define the modulus function;

2. use the properties:

(a) $|x|$ is the positive square root of x^2 ,

(b) $|x| < |y|$ if, and only if, $x^2 < y^2$,

(c) $|x| < |y| \Leftrightarrow -y < x < y$,

(d) $|x + y| \leq |x| + |y|$;

3. solve equations and inequalities involving the modulus function, using algebraic or graphical methods.

CONTENT

(F) *The Modulus Function*

1. Definition of the modulus function.

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases};$$

2. Equations and inequalities involving simple rational and modulus functions.

SPECIFIC OBJECTIVES

(G) *Cubic Functions and Equations*

Students should be able to use the relationship between the sum of the roots, the product of the roots, the sum of the product of the roots pair-wise and the coefficients of $ax^3 + bx^2 + cx + d = 0$.

UNIT 1

MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

CONTENT

(G) Cubic Functions and Equations

Sums and products, with applications, of the roots of cubic equations.

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. The Real Number System

- (a) *The teacher should do a brief review of the number system before starting this section.*
- (b) *The teacher should encourage students to practice different methods of proof, for example, to prove that the product of two consecutive integers is an even integer.*

2. Proof by Mathematical Induction (MI)

Typical Question

Prove that some formula or statement P is true for all positive integers $n \geq k$, where k is some positive integer; usually $k = 1$.

Procedure

Step 1: Verify that when $k = 1$: P is true for $n = k = 1$. This establishes that P is true for $n = 1$.

Step 2: Assume P is true for $n = k$, where k is a positive integer > 1 . At this point, the statement k replaces n in the statement P and is taken as true.

Step 3: Show that P is true for $n = k + 1$ using the true statement in step 2 with n replaced by k .

Step 4: At the end of step 3, it is stated that statement P is true for all positive integers $n \geq k$.

Summary

Proof by MI: For $k > 1$, verify Step 1 for k and proceed through to Step 4.

UNIT 1

MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

Observation

Most users of MI do not see how this proves that P is true. The reason for this is that there is a massive gap between Steps 3 and 4 which can only be filled by becoming aware that Step 4 only follows because Steps 1 to 3 are repeated an infinity of times to generate the set of all positive integers. The focal point is the few words “for all positive integers $n \geq k$ ” which points to the determination of the set S of all positive integers for which P is true.

Step 1 says that $1 \in S$ for $k = 1$.

Step 3 says that $k + 1 \in S$ whenever $k \in S$, so immediately $2 \in S$ since $1 \in S$.

Iterating on Step 3 says that $3 \in S$ since $2 \in S$ and so on, so that $S = \{1, 2, 3 \dots\}$, that is, S is the set of all positive integers when $k = 1$ which brings us to Step 4.

When $k > 1$, the procedure starts at a different positive integer, but the execution of steps is the same. Thus, it is necessary to explain what happens between Steps 3 and 4 to obtain a full appreciation of the method.

Example 1: Use Mathematical Induction to prove that $n^3 - n$ is divisible by 3, whenever n is a positive integer.

Solution: Let P (n) be the proposition that “ $n^3 - n$ is divisible by 3”.

Basic Step: P (1) is true, since $1^3 - 1 = 0$ which is divisible by 3.

Inductive Step: Assume P (n) is true: that is, $n^3 - n$ is divisible by 3.

We must show that P ($n + 1$) is true, if P(n) is true. That is, is, $(n + 1)^3 - (n + 1)$ is divisible by 3.

$$\begin{aligned} \text{Now, } (n + 1)^3 - (n + 1) &= (n^3 + 3n^2 + 3n + 1) - (n + 1) \\ &= (n^3 - n) + 3(n^2 + n) \end{aligned}$$

Both terms are divisible by 3 since $(n^3 - n)$ is divisible by 3 by the assumption and $3(n^2 + n)$ is a multiple of 3. Hence, P ($n + 1$) is true whenever P (n) is true.

Thus, $n^3 - n$ is divisible by 3 whenever n is a positive integer.

UNIT 1

MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

Example 2: Prove by Mathematical Induction that the sum S_n of the first n odd positive integers is n^2 .

Solution: Let $P(n)$ be the proposition that the sum S_n of the first n odd positive integers is n^2 .

Basic Step: For $n = 1$ the first odd positive integer is 1, so $S_1 = 1$, that is: $S_1 = 1 = 1^2$, hence $P(1)$ is true.

Inductive Step: Assume $P(n)$ is true. That is, $S_n = 1 + 3 + 5 + \dots + (2n - 1) = n^2$.

$$\begin{aligned} \text{Now, } S_{n+1} &= 1 + 3 + 5 + \dots + (2n - 1) + (2n + 1) \\ &= [1 + 3 + 5 + \dots + (2n - 1)] + (2n + 1) \\ &= n^2 + (2n + 1), \text{ by the assumption,} \\ &= (n + 1)^2 \end{aligned}$$

Thus, $P(n+1)$ is true whenever $P(n)$ is true.

Since $P(1)$ is true and $P(n) \rightarrow P(n+1)$, the proposition $P(n)$ is true for all positive integers n .

3. Functions (Injective, surjective, bijective) – Inverse Function

Mathematical proof that a function is one-to-one (injective), onto (surjective) or (one-to-one and onto function) bijective should be introduced at this stage.

Teacher and students should explore the mapping properties of quadratic functions which:

- will, or will not, be injective, depending on which subset of the real line is chosen as the domain;
- will be surjective if its range is taken as the co-domain (completion of the square is useful here);
- if both injective and surjective, will have an inverse function which can be constructed by solving a quadratic equation.

Example: Use the function $f : A \rightarrow B$ given by $f(x) = 3x^2 + 6x + 5$, where the domain A is alternatively the whole of the real line, or the set $\{x \in \mathbb{R} \mid x \geq -1\}$, and the co-domain B is \mathbb{R} or the set $\{y \in \mathbb{R} \mid y \geq 2\}$.

UNIT 1

MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

4. Cubic Equations

Teachers should first review the theory of the quadratic equation and the nature of its roots.

RESOURCES

Bostock, L. and Chandler, S.	<i>Core Mathematics for A-Levels</i> , United Kingdom: Stanley Thornes Publishing Limited, 1997.
Campbell, E.	<i>Pure Mathematics for CAPE, Vol. 1</i> , Jamaica: LMH Publishing Limited, 2007.
Caribbean Examinations Council	<i>Injective and Surjective Functions</i> , Barbados: 1998.
<i>Caribbean Examinations Council</i>	<i>Proof by Mathematical Induction (MI)</i> , Barbados: 2004.
Caribbean Examinations Council	<i>The Real Number System</i> , Barbados: 1997.
Greaves, Y.	<i>Solution of Simultaneous Linear Equations by Row Reduction</i> , Barbados: Caribbean Examinations Council, 1998.
Hartzler, J. and Swetz, F.	<i>Mathematical Modelling in the Secondary School Curriculum, A Resource Guide of Classroom Exercises</i> , Vancouver, United States of America: National Council of Teachers of Mathematics, Incorporated Reston, 1991.
Martin, A., Brown, K., Rigby, P. and Ridley, S.	<i>Advanced Level Mathematics Tutorials Pure Mathematics CD-ROM sample (Trade Edition)</i> , Cheltenham, United Kingdom: Stanley Thornes (Publishers) Limited, Multi-user version and Single-user version, 2000.

UNIT 1

MODULE 2: TRIGONOMETRY, GEOMETRY AND VECTORS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *develop the ability to represent and deal with objects in two and three dimensions through the use of coordinate geometry and vectors;*
2. *develop the ability to manipulate and describe the behaviour of trigonometric functions;*
3. *develop the ability to establish trigonometric identities;*
4. *acquire the skills to solve trigonometric equations;*
5. *acquire the skills to conceptualise and to manipulate objects in two and three dimensions;*
6. *develop the ability to use concepts to model and solve real-world problems.*

SPECIFIC OBJECTIVES

(A) Trigonometric Functions, Identities and Equations (all angles will be assumed to be in radians unless otherwise stated)

Students should be able to:

1. *use compound-angle formulae;*
2. *use the reciprocal functions of $\sec x$, $\cosec x$ and $\cot x$;*
3. *derive identities for the following:
 - (a) $\sin kA$, $\cos kA$, $\tan kA$, for $k \in \mathbb{Q}$,
 - (b) $\tan^2 x$, $\cot^2 x$, $\sec^2 x$ and $\cosec^2 x$,
 - (c) $\sin A \pm \sin B$, $\cos A \pm \cos B$;*
4. *prove further identities using Specific Objective 3;*
5. *express $a \cos \theta + b \sin \theta$ in the form $r \cos(\theta \pm \alpha)$ and $r \sin(\theta \pm \alpha)$, where r is positive, $0 < \alpha < \frac{\pi}{2}$;*

UNIT 1**MODULE 2: TRIGONOMETRY, GEOMETRY AND VECTORS (cont'd)**

6. find the general solution of equations of the form:
 - (a) $\sin k\theta = s,$
 - (b) $\cos k\theta = c,$
 - (c) $\tan k\theta = t,$
 - (d) $a \cos \theta + b \sin \theta = c,$
for $a, b, c, k, s, t, \in \mathbb{R};$
7. find the solutions of the equations in Specific Objectives 6 above for a given range;
8. obtain maximum or minimum values of $f(a \cos \theta + b \sin \theta)$ for $0 \leq \theta \leq 2\pi.$

CONTENT

- (A) **Trigonometric Functions, Identities and Equations (all angles will be assumed to be radians)**
1. The functions $\cot x, \sec x, \operatorname{cosec} x.$
 2. Compound-angle formulae for $\sin(A \pm B), \cos(A \pm B), \tan(A \pm B).$
 3. *Multiple-angle formulae.*
 4. Formulae for $\sin A \pm \sin B, \cos A \pm \cos B.$
 5. *Expression of $a \cos \theta + b \sin \theta$ in the forms $r \sin(\theta \pm \alpha)$ and $r \cos(\theta \pm \alpha)$, where r is positive, $0 < \alpha < \frac{\pi}{2}.$*
 6. General solutions of simple trigonometric equations.
 7. Trigonometric identities $\cos^2 \theta + \sin^2 \theta \equiv 1, 1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta, 1 + \tan^2 \theta \equiv \sec^2 \theta.$
 8. Maximum and minimum values of functions of $\sin \theta$ and $\cos \theta.$

UNIT 1

MODULE 2: TRIGONOMETRY, GEOMETRY AND VECTORS (cont'd)

SPECIFIC OBJECTIVES

(B) Co-ordinate Geometry

Students should be able to:

1. find equations of tangents and normals to circles;
2. find the points of intersection of a curve with a straight line;
3. find the points of intersection of two curves;
4. obtain the Cartesian equation of a curve given its parametric representation;
5. obtain the parametric representation of a curve given its Cartesian equation;
6. *determine the loci of points satisfying given properties.*

CONTENT

(B) Co-ordinate Geometry

1. Properties of the circle.
2. Tangents and normals.
3. Intersections between lines and curves.
4. Cartesian equations and parametric representations of curves including the parabola and ellipse.
5. *Loci.*

SPECIFIC OBJECTIVES

(C) Vectors

Students should be able to:

1. *express a vector in the form* $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ *or* $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$
where \mathbf{i} , \mathbf{j} and \mathbf{k} are unit vectors in directions of x -, y - and z -axis respectively;
2. define equality of two vectors;
3. add and subtract vectors;
4. multiply a vector by a scalar quantity;

UNIT 1

MODULE 2: TRIGONOMETRY, GEOMETRY AND VECTORS (cont'd)

5. derive and use unit vectors, position vectors and displacement vectors;
6. find the magnitude and direction of a vector;
7. find the angle between two given vectors using scalar product;
8. find the equation of a line in vector form, parametric form, Cartesian form, given a point on the line and a vector parallel to the line;
9. determine whether two lines are parallel, intersecting, or skewed;
10. find the equation of the plane, in the form $x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = d$, $\mathbf{r} \cdot \mathbf{n} = d$, given a point in the plane and the normal to the plane.

CONTENT

(C) Vectors

1. Expression of a given vector in the form $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.
2. Equality, addition and subtraction of vectors; multiplication by a scalar.
3. Position vectors, unit vectors, displacement vectors.
4. Length (magnitude/modulus) and direction of a vector.
5. Scalar (Dot) Product.
6. Vector equation of a line.
7. Equation of a plane.

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. Trigonometric Identities

Much practice is required to master proofs of Trigonometric Identities using identities such as the formulae for:

$$\sin(A \pm B), \cos(A \pm B), \tan(A \pm B), \sin 2A, \cos 2A, \tan 2A$$

UNIT 1

MODULE 2: TRIGONOMETRY, GEOMETRY AND VECTORS (cont'd)

Example: The identity $\frac{1 - \cos 4\theta}{\sin 4\theta} \equiv \tan 2\theta$ can be established by realising that $\cos 4\theta \equiv 1 - 2 \sin^2 2\theta$ and $\sin 4\theta \equiv 2 \sin 2\theta \cos 2\theta$.

Derive the trigonometric functions $\sin x$ and $\cos x$ for angles x of any value (including negative values), using the coordinates of points on the unit circle.

2. Vectors

Teachers should introduce students to the three dimensional axis and understand how to plot vectors in three dimensions.

RESOURCE

Bostock, L. and Chandler, S.	<i>Mathematics - The Core Course for A-Level</i> , United Kingdom: Stanley Thornes (Publishers) Limited, 1997.
Campbell, E.	<i>Pure Mathematics for CAPE, Vol. 1</i> , Jamaica: LMH Publishing Limited, 2007.

UNIT 1

MODULE 3: CALCULUS I

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *understand the concept of continuity of a function from its graph;*
2. *develop the ability to find the limits (when they exist) of functions in simple cases;*
3. *know the relationships between the derivative of a function at a point and the behaviour of the function and its tangent at that point;*
4. *know the relationship between integration and differentiation;*
5. *know the relationship between integration and the area under the graph of the function;*
6. *develop the ability to use concepts to model and solve real-world problems.*

SPECIFIC OBJECTIVES

(A) Limits

Students should be able to:

1. *use graphs to determine the continuity and discontinuity of functions;*
2. *describe the behaviour of a function $f(x)$ as x gets arbitrarily close to some given fixed number, using a descriptive approach;*
3. *use the limit notation $\lim_{x \rightarrow a} f(x) = L$, $f(x) \rightarrow L$ as $x \rightarrow a$;*
4. *use the simple limit theorems:*
If $\lim_{x \rightarrow a} f(x) = F$, $\lim_{x \rightarrow a} g(x) = G$ and k is a constant,
then $\lim_{x \rightarrow a} kf(x) = kF$, $\lim_{x \rightarrow a} f(x)g(x) = FG$, $\lim_{x \rightarrow a} \{f(x) + g(x)\} = F + G$,
and, provided $G \neq 0$, $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{F}{G}$;
5. *use limit theorems in simple problems;*
6. *use the fact that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, demonstrated by a geometric approach;*
7. *identify the point(s) for which a function is (un)defined;*
8. *identify the points for which a function is continuous;*

UNIT 1

MODULE 3: CALCULUS I (cont'd)

9. identify the point(s) where a function is discontinuous;
10. use the concept of left-handed or right-handed limit, and continuity.

CONTENT

(A) Limits

1. Concept of limit of a function.
2. Limit Theorems.
3. Continuity and Discontinuity.

SPECIFIC OBJECTIVES

(B) Differentiation I

Students should be able to:

1. define the derivative of a function at a point as a limit;
2. differentiate, from first principles, functions such as:
 - (a) $f(x) = k$ where $k \in \mathbb{R}$,
 - (b) $f(x) = x^n$, where $n \in \{-3, -2, -1, -\frac{1}{2}, \frac{1}{2}, 1, 2, 3\}$,
 - (c) $f(x) = \sin x$,
 - (d) $f(x) = \cos x$;
3. use the sum, product and quotient rules for differentiation;
4. differentiate sums, products and quotients of:
 - (a) polynomials,
 - (b) trigonometric functions;
5. apply the chain rule in the differentiation of
 - (a) composite functions (substitution),
 - (b) functions given by parametric equations;

UNIT 1

MODULE 3: CALCULUS I (cont'd)

6. *solve problems involving rates of change;*
7. *use the sign of the derivative to investigate where a function is increasing or decreasing;*
8. *apply the concept of stationary (critical) points;*
9. *calculate second derivatives;*
10. *interpret the significance of the sign of the second derivative;*
11. *use the sign of the second derivative to determine the nature of stationary points;*
12. *sketch graphs of polynomials, rational functions and trigonometric functions using the features of the function and its first and second derivatives (including horizontal and vertical asymptotes);*
13. *describe the behaviour of such graphs for large values of the independent variable;*
14. *obtain equations of tangents and normals to curves.*

CONTENT

(B) Differentiation I

1. The Gradient.
2. The Derivative as a limit.
3. Rates of change.
4. Differentiation from first principles.
5. Differentiation of simple functions, product, quotients.
6. *Stationary points, the chain rule and parametric equations.*
7. Second derivatives of functions.
8. Curve sketching.
9. Tangents and Normals to curves.

UNIT 1**MODULE 3: CALCULUS I (cont'd)****SPECIFIC OBJECTIVES****(C) Integration I**

Students should be able to:

1. recognise integration as the reverse process of differentiation;
2. demonstrate an understanding of the indefinite integral and the use of the integration notation $\int f(x) dx$;
3. show that the indefinite integral represents a family of functions which differ by constants;
4. demonstrate use of the following integration theorems:

(a) $\int cf(x)dx = c \int f(x)dx$, where c is a constant,

(b) $\int \{f(x) \pm g(x)\} dx = \int f(x) dx \pm \int g(x) dx$;

5. find:

(a) indefinite integrals using integration theorems,

(b) integrals of polynomial functions,

(c) integrals of simple trigonometric functions;

6. integrate using substitution;

7. use the results:

(a) $\int_a^b f(x)dx = \int_a^b f(t)dt$,

(b) $\int_0^a f(x)dx = \int_0^a f(x-a)dx$ for $a > 0$,

(c) $\int_a^b f(x) dx = F(b) - F(a)$, where $F'(x) = f(x)$;

UNIT 1

MODULE 3: CALCULUS I (cont'd)

8. apply *integration* to:
 - (a) finding areas under the curve,
 - (b) finding areas between two curves,
 - (c) *finding volumes of revolution by rotating regions about both the x- and y-axes;*
9. given a rate of change with or without initial boundary conditions:
 - (a) formulate a differential equation of the form $y' = f(x)$ or $y'' = f(x)$ where f is a polynomial or a trigonometric function,
 - (b) solve the resulting differential equation in (a) above and interpret the solution where applicable.

CONTENT

(c) Integration I

1. Integration as the reverse of differentiation.
2. Linearity of integration.
3. Indefinite integrals (concept and use).
4. Definite integrals.
5. Applications of integration – areas, volumes and solutions to elementary differential equations.
6. Integration of polynomials.
7. Integration of simple trigonometric functions.
8. Use of $\int_a^b f(x) dx = F(b) - F(a)$, where $F'(x) = f(x)$.
9. Simple first or second order differential equations of the type $y' = f(x)$ or $y'' = f(x)$.

UNIT 1

MODULE 3: CALCULUS I (cont'd)

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

The Area under the Graph of a Continuous Function

Class discussion should play a major role in dealing with this topic. Activities such as that which follows may be performed to motivate the discussion.

Example of classroom activity:

Consider a triangle of area equal to $\frac{1}{2}$ units, bounded by the graphs of $y = x$, $y = 0$ and $x = 1$.

- (a) Sketch the graphs and identify the triangular region enclosed.
- (b) Subdivide the interval $[0, 1]$ into n equal subintervals.
- (c) Evaluate the sum, $s(n)$, of the areas of the inscribed rectangles and $S(n)$, of the circumscribed rectangles, erected on each subinterval.
- (d) By using different values of n , for example, for $n = 5, 10, 25, 50, 100$, show that both $s(n)$ and $S(n)$ get closer to the required area of the given region.

RESOURCES

Bostock, L., and Chandler, S.

Mathematics - The Core Course for A-Level, United Kingdom:
Stanley Thornes Publishing Limited, (Chapters 5, 8 and 9), 1991.

Campbell, E.

Pure Mathematics for CAPE, Vol. 1, Jamaica: LMH Publishing Limited, 2007.

Caribbean Examinations Council

Area under the Graph of a Continuous Function, Barbados: 1998.

Caribbean Examinations Council

Differentiation from First Principles: The Power Function, Barbados: 1998.

◆ UNIT 2: COMPLEX NUMBERS, ANALYSIS AND MATRICES

MODULE 1: COMPLEX NUMBERS AND CALCULUS II

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *develop the ability to represent and deal with objects in the plane through the use of complex numbers;*
2. *be confident in using the techniques of differentiation and integration;*
3. *develop the ability to use concepts to model and solve real-world problems.*

SPECIFIC OBJECTIVES

(A) Complex Numbers

Students should be able to:

1. recognise the need to use complex numbers to find the roots of the general quadratic equation $ax^2 + bx + c = 0$, when $b^2 - 4ac < 0$;
2. use the concept that complex roots of equations with constant coefficients occur in conjugate pairs;
3. write the roots of the equation in that case and relate the sums and products to a , b and c ;
4. calculate the square root of a complex number;
5. express complex numbers in the form $a + bi$ where a , b are real numbers, and identify the real and imaginary parts;
6. add, subtract, multiply and divide complex numbers in the form $a + bi$, where a and b are real numbers;
7. find the principal value of the argument θ of a non-zero complex number, where $-\pi < \theta \leq \pi$;
8. find the modulus and conjugate of a given complex number;
9. interpret modulus and argument of complex numbers on the Argand diagram;
10. represent complex numbers, their sums, differences and products on an Argand diagram;
11. find the set of all points z (locus of z) on the Argand Diagram such that z satisfies given properties;

MODULE 1: COMPLEX NUMBERS AND CALCULUS II (cont'd)

12. apply De Moivre's theorem for integral values of n ;
13. use $e^{ix} = \cos x + i \sin x$, for real x .

CONTENT

(A) Complex Numbers

1. Nature of roots of a quadratic equation, sums and products of roots.
2. *Conjugate pairs of complex roots.*
3. Addition, subtraction, multiplication and division of complex numbers in the form $a + bi$ where a, b are the real and imaginary parts, respectively, of the complex number.
4. The modulus, argument and conjugate of a complex number.
5. Representation of complex numbers on an Argand diagram.
6. Locus of a point.
7. De Moivre's theorem for integral n .
8. *Polar-argument and exponential forms of complex numbers.*

SPECIFIC OBJECTIVES

(B) Differentiation II

Students should be able to:

1. find the derivative of $e^{f(x)}$, where $f(x)$ is a differentiable function of x ;
2. find the derivative of $\ln f(x)$ (to include functions of x – polynomials or trigonometric);
3. apply the chain rule to obtain gradients and equations of tangents and normals to curves given by their parametric equations;
4. use the concept of implicit differentiation, with the assumption that one of the variables is a function of the other;
5. differentiate any combinations of polynomials, trigonometric, exponential and logarithmic functions;

UNIT 2

MODULE 1: COMPLEX NUMBERS AND CALCULUS II (cont'd)

6. differentiate inverse trigonometric functions;
7. obtain second derivatives, $f''(x)$, of the functions in 3, 4, 5 above;
8. *find the first and second partial derivatives of $u = f(x, y)$.*

CONTENT

(B) Differentiation II

1. Application of the chain rule to differentiation.
2. Chain rule and differentiation of composite functions.
3. Gradients of tangents and normals.
4. Implicit differentiation.
5. First derivative of a function which is defined parametrically.
6. Differentiation of inverse trigonometric functions.
7. Differentiation of combinations of functions.
8. Second derivative, that is, $f''(x)$.
9. *First partial derivative.*
10. *Second partial derivative.*

SPECIFIC OBJECTIVES

(C) Integration II

Students should be able to:

1. express a rational function (**proper and improper**) in partial fractions in the cases where the denominators are:
 - (a) distinct linear factors,
 - (b) repeated linear factors,

UNIT 2

MODULE 1: COMPLEX NUMBERS AND CALCULUS II (cont'd)

- (c) quadratic factors,
- (d) repeated quadratic factors,
- (e) combinations of (a) to (d) above (repeated factors will not exceed power 2);
- 2. express an improper rational function as a sum of a polynomial and partial fractions;
- 3. integrate rational functions in Specific Objectives 1 and 2 above;
- 4. integrate trigonometric functions using appropriate trigonometric identities;
- 5. integrate exponential functions and logarithmic functions;
- 6. find integrals of the form $\int \frac{f'(x)}{f(x)} dx$;
- 7. use substitutions to integrate functions (the substitution will be given in all but the most simple cases);
- 8. use integration by parts for combinations of functions;
- 9. integrate inverse trigonometric functions;
- 10. derive and use reduction formulae to obtain integrals;
- 11. use the trapezium rule as an approximation method for evaluating the area under the graph of the function.

CONTENT

(C) Integration II

- 1. Partial fractions.
- 2. Integration of rational functions, using partial fractions.
- 3. Integration by substitution.
- 4. Integration by parts.
- 5. Integration of inverse trigonometric functions.
- 6. Integration by reduction formula.
- 7. Area under the graph of a continuous function (Trapezium Rule).

UNIT 2

MODULE 1: COMPLEX NUMBERS AND CALCULUS II (cont'd)

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

Principal Argument of a Complex Number

The representation of the complex number $z = 1 + i$ on the Argand diagram may be used to introduce this topic. Encourage students to indicate and evaluate the argument of z . The students' answers should be displayed on the chalkboard.

Indicate that the location of z on the Argand diagram is unique, and therefore only one value of the argument is needed to position z . That argument is called the principal argument, $\arg z$, where:

$$-\pi < \text{principal argument} \leq \pi.$$

Students should be encouraged to calculate the principal argument by either solving:

- (a) *the simultaneous equations*

$$\cos \theta = \frac{\operatorname{Re}(z)}{|z|} \text{ and } \sin \theta = \frac{\operatorname{Im}(z)}{|z|}, \text{ with } -\pi < \theta \leq \pi;$$

or,

- (b) *the equation*

$$\tan \theta = \frac{\operatorname{Im}(z)}{\operatorname{Re}(z)} \text{ for } \operatorname{Re}(z) \neq 0 \text{ and } -\pi < \theta \leq \pi,$$

together with the representation of z on the Argand diagram.

- (c) *Students should be encouraged to find the loci of z -satisfying equations such as:*

- (i) $|z - a| = k;$
- (ii) $|z - c| = |z - b|;$
- (iii) $\arg(z - a) = \alpha.$

RESOURCES

Bostock, L. and Chandler, S.

Core Mathematics for A-Levels, United Kingdom:
Stanley Thornes Publishing Limited, 1997.

Bradie, B.

Rate of Change of Exponential Functions: A Precalculus Perspective, Mathematics Teacher Vol. 91(3), p. 224 – 237.

UNIT 2

MODULE 1: COMPLEX NUMBERS AND CALCULUS II (cont'd)

Campbell, E.

Pure Mathematics for CAPE, Vol. 2, Jamaica: LMH Publishing Limited, 2007.

Caribbean Examinations Council

The Exponential and Logarithmic Functions – An Investigation, Barbados: 1998.

Martin, A., Brown, K., Rigby, P. and Ridley, S.

Pure Mathematics, Cheltenham, United Kingdom: Stanley Thornes (Publishers) Limited, 2000.

UNIT 2

MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *understand* the concept of a sequence as a function from the natural numbers to the real numbers;
2. *understand* the difference between sequences and series;
3. distinguish between convergence and/or divergence of some standard series or sequences;
4. apply successive approximations to roots of equations;
5. *develop* the ability to use concept to model and solve real-world problems.

SPECIFIC OBJECTIVES

(A) Sequences

Students should be able to:

1. define the concept of a sequence $\{a_n\}$ of terms a_n as a function from the positive integers to the real numbers;
2. write a specific term from the formula for the n^{th} term, or from a recurrence relation;
3. describe the behaviour of convergent and divergent sequences, through simple examples;
4. apply mathematical induction to establish properties of sequences.

CONTENT

(A) Sequences

1. Definition, convergence, divergence *and* limit of a sequence.
2. Sequences defined by recurrence relations.
3. Application of mathematical induction to sequences.

UNIT 2

MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS (cont'd)

SPECIFIC OBJECTIVES

(B) Series

Students should be able to:

1. use the summation (Σ) notation;
2. define a series, as the sum of the terms of a sequence;
3. identify the n^{th} term of a series, in the summation notation;
4. define the m^{th} partial sum S_m as the sum of the first m terms of the sequence, that is,

$$S_m = \sum_{r=1}^m a_r;$$

5. apply mathematical induction to establish properties of series;
6. find the sum to infinity of a convergent series;
7. apply the method of differences to appropriate series, and find their sums;
8. use the Maclaurin theorem for the expansion of series;
9. *use the Taylor theorem for the expansion of series.*

CONTENT

(B) Series

1. Summation notation (Σ).
2. *Series as the sum of terms of a sequence.*
3. *Convergence and/or divergence of series to which the method of differences can be applied.*
4. *The Maclaurin series.*
5. *The Taylor series.*
6. Applications of mathematical induction to series.

UNIT 2

MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS (cont'd)

SPECIFIC OBJECTIVES

(C) The Binomial Theorem

Students should be able to:

1. explain the meaning and use simple properties of $n!$ and $\binom{n}{r}$, that is, ${}^n C_r$, where $n, r \in \mathbb{Z};$
2. recognise that ${}^n C_r$, that is, $\binom{n}{r}$, is the number of ways in which r objects may be chosen from n distinct objects;
3. expand $(a + b)^n$ for $n \in \mathbb{Q};$
4. apply the Binomial Theorem to real-world problems, for example, in mathematics of finance, science.

CONTENT

(C) The Binomial Theorem

1. Factorials and Binomial coefficients; their interpretation and properties.
2. The Binomial Theorem.
3. Applications of the Binomial Theorem.

SPECIFIC OBJECTIVES

(D) Roots of Equations

Students should be able to:

1. test for the existence of a root of $f(x) = 0$ where f is continuous using the Intermediate Value Theorem;
2. use interval bisection to find an approximation for a root in a given interval;
3. use linear interpolation to find an approximation for a root in a given interval;
4. explain, in geometrical terms, the working of the Newton-Raphson method;

UNIT 2

MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS (cont'd)

5. use the Newton-Raphson method to find successive approximations to the roots of $f(x) = 0$, where f is differentiable;
6. use a given iteration to determine a root of an equation to a specified degree of accuracy.

CONTENT

(D) Roots of Equations

Finding successive approximations to roots of equations using:

1. *Intermediate Value Theorem;*
2. *Interval Bisection;*
3. *Linear Interpolation;*
4. *Newton - Raphson Method;*
5. *Iteration.*

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the learning activities listed below.

1. The Binomial Theorem

Students may be motivated to do this topic by having successive expansions of $(a + x)^n$ and then investigating the coefficients obtained when expansions are carried out.

$$\begin{aligned}(a + b)^1 &= a + b \\(a + b)^2 &= a^2 + 2ab + b^2 \\(a + b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\(a + b)^4 &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + a^4\end{aligned}$$

and so on.

UNIT 2

MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS (cont'd)

By extracting the coefficients of each term made up of powers of a , x or a and x .

$$\begin{array}{ccccc} & & 1 & & \\ & 1 & & 1 & \\ & 1 & 2 & & 1 \\ & 1 & 3 & 3 & 1 \\ 1 & 4 & 6 & 4 & 1 \end{array}$$

Students should be encouraged to use the emerging pattern to generate further expansions of $(a + x)^n$. This can be done by generating the coefficients from Pascal's Triangle and then investigating other patterns. For example, by looking at the powers of a and x (powers of x increase from 0 to n , while powers of a decrease from n to 0; powers of a and x add up to n).

In discussing the need to find a more efficient method of doing the expansions, the Binomial Theorem may be introduced. However, this can only be done after the students are exposed to principles of counting, with particular reference to the process of selecting. In so doing, teachers will need to guide students through appropriate examples involving the selection of r objects, say, from a group of n unlike objects. This activity can lead to defining ${}^n C_r$ as the number of ways of selecting r objects from a group of n unlike objects.

In teaching this principle, enough examples should be presented before
$${}^n C_r = \frac{n!}{(n - r)! r!}$$
 formula is developed.

The binomial theorem may then be established by using the expansion of $(1 + x)^n$ as a starting point. A suggested approach is given below:

Consider $(1 + x)^n$.

To expand, the student is expected to multiply $(1 + x)$ by itself n times, that is,
$$(1 + x)^n = (1 + x)(1 + x)(1 + x) \dots (1 + x)$$
.

The result of the expansion is found as given below:

The constant term is obtained by multiplying all the 1's. The result is therefore 1.

The term in x is obtained by multiplying $(n - 1)$ 1's and one x . This x , however, may be chosen from any of the n brackets. That is, we need to choose one x out of n different brackets. This can be done in ${}^n C_1$ ways. Hence, the coefficient of x is ${}^n C_1$.

UNIT 2

MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS (cont'd)

Similarly, the term in x^2 may be obtained by choosing two x 's and $(n - 2)$ 1's. The x 's may be chosen from any two of the n brackets. This can be done in ${}^n C_2$ ways. The coefficient of x^2 is therefore ${}^n C_2$.

This process continues and the expansion is obtained:

$$(1 + x)^n = 1 + {}^n C_1 x + {}^n C_2 x^2 + {}^n C_3 x^3 + \dots + x^n.$$

This is known as the binomial theorem. The theorem may be written as

$$(1 + x)^n = \sum_{r=0}^n {}^n C_r x^r.$$

The generalisation of this could be done as a class activity where students are asked to show that:

$$(a + b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + {}^n C_3 a^{n-3} b^3 + \dots + b^n.$$

This is the binomial expansion of $(a + b)^n$ for positive integral values of n . The expansion terminates after $(n + 1)$ terms.

2. The Intermediate Value Theorem

- (a) Motivate with an example.

Example: A taxi is travelling at 5 km/h at 8:00 a.m. Fifteen minutes later the speed is 100 km/h. Since the speed varies continuously, clearly at some time between 8:00 a.m. and 8:15 a.m. the taxi was travelling at 75 km/h.

Note that the taxi could have traveled at 75 km/h at more than one time between 8:00 a.m. and 8:15 a.m.

- (b) Use examples of continuous functions to illustrate the Intermediate Value Theorem.

Example: $f(x) = x^2 - x - 6$ examined on the intervals $(3.5, 5)$ and $(0, 4)$.

3. Existence of Roots

Introduce the existence of the root of a continuous function $f(x)$ between given values a and b as an application of the Intermediate Value Theorem.

UNIT 2

MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS (cont'd)

Emphasis should be placed on the fact that:

- (a) f must be continuous between a and b ;
- (b) The product of $f(a)$ and $f(b)$ is less than zero, that is, $f(a)$ and $f(b)$ must have opposite signs.

4. *Interval Bisection*

Initially students should be able to determine an interval in which a real root lies. If $f(a)$ and $f(b)$ are of opposite signs, and f is continuous, then $a < x < b$, for the equation $f(x) = 0$.

Students may be asked to investigate $x = \frac{\alpha + \beta}{2}$ and note the resulting sign to determine which side of $\frac{\alpha + \beta}{2}$ the root lies. This method can be repeated until same answer to the desired degree of accuracy is obtained.

5. *Linear Interpolation*

*Given the points (x_0, y_0) and (x_1, y_1) on a continuous curve $y = f(x)$, students can establish that for $f(x_0)$ and $f(x_1)$ with opposite signs and that f is continuous, then $x_0 < x < x_1$, for the equation $f(x) = 0$. If $|f(x_0)| < |f(x_1)|$ say, students can be introduced to the concept of similar triangles to find successive approximations, holding $f(x_1)$ constant. This intuitive approach is formalised in **linear interpolation**, where the two points (x_0, y_0) and (x_1, y_1) can be joined by a straight line and the x -value of the point on this line is calculated. A first approximation for x can be found using*

$$\frac{x}{f(x_0)} = \frac{x_1}{f(x_1)}.$$

Successive approximations can be found with this approach until the same answer to the desired degree of accuracy is obtained.

RESOURCE

Bostock, L. and Chandler, S.

Core Mathematics for A-Levels, United Kingdom:
Stanley Thornes Publishing Limited, 1997.

Campbell, E.

Pure Mathematics for CAPE, Vol. 2, Jamaica: LMH
Publishing Limited, 2007.

UNIT 2

MODULE 3: COUNTING, MATRICES AND DIFFERENTIAL EQUATIONS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *develop* the ability to analyse and solve simple problems dealing with choices and arrangements;
2. *develop an understanding of* the algebra of matrices;
3. *develop* the ability to analyse and solve systems of linear equations;
4. *develop skills to model some real-world phenomena by means of differential equations, and solve these;*
5. *develop* the ability to use concepts to model and solve real-world problems.

SPECIFIC OBJECTIVES

(A) Counting

Students should be able to:

1. state the principles of counting;
2. find the number of ways of arranging n distinct objects;
3. find the number of ways of arranging n objects some of which are identical;
4. find the number of ways of choosing r distinct objects from a set of n distinct objects;
5. identify a sample space;
6. identify the numbers of possible outcomes in a given sample space;
7. use Venn diagrams to illustrate the principles of counting;
8. use possibility space diagram to identify a sample space;
9. define and calculate $P(A)$, the probability of an event A occurring as the number of possible ways in which A can occur divided by the total number of possible ways in which all equally likely outcomes, including A , occur;
10. use the fact that $0 \leq P(A) \leq 1$;

UNIT 2

MODULE 3: COUNTING, MATRICES AND *DIFFERENTIAL EQUATIONS* (cont'd)

11. demonstrate and use the property that the total probability for all possible outcomes in the sample space is 1;
12. use the property that $P(A') = 1 - P(A)$ is the probability that event A does not occur;
13. use the property $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for event A and B ;
14. use the property $P(A \cap B) = 0$ or $P(A \cup B) = P(A) + P(B)$, where A and B are mutually exclusive events;
15. use the property $P(A \cap B) = P(A) \times P(B)$, where A and B are independent events;
16. use the property $P(A|B) = \frac{P(A \cap B)}{P(B)}$ where $P(B) \neq 0$.
17. use a tree diagram to list all possible outcomes for conditional probability.

CONTENT

(A) Counting

1. Principles of counting.
2. Arrangements with and without repetitions.
3. Selections.
4. Venn diagram.
5. Possibility space diagram.
6. Concept of probability and elementary applications.
7. Tree diagram.

SPECIFIC OBJECTIVES

(B) Matrices and Systems of Linear Equations

Students should be able to:

1. operate with conformable matrices, carry out simple operations and manipulate matrices using their properties;
2. evaluate the determinants of $n \times n$ matrices, $1 \leq n \leq 3$;

UNIT 2

MODULE 3: COUNTING, MATRICES AND *DIFFERENTIAL EQUATIONS* (cont'd)

3. reduce a system of linear equations to echelon form;
4. row-reduce the augmented matrix of an $n \times n$ system of linear equations, $n = 2, 3$;
5. determine whether the system is consistent, and if so, how many solutions it has;
6. find all solutions of a consistent system;
7. invert a non-singular 3×3 matrix;
8. solve a 3×3 system of linear equations, having a non-singular coefficient matrix, by using its inverse.

CONTENT

(B) Matrices and Systems of Linear Equations

1. $m \times n$ matrices, for $1 \leq m \leq 3$, and $1 \leq n \leq 3$, and equality of matrices.
2. Addition of conformable matrices, zero matrix and additive inverse, associativity, commutativity, distributivity, transposes.
3. Multiplication of a matrix by a scalar.
4. Multiplication of conformable matrices.
5. Square matrices, singular and non-singular matrices, unit matrix and multiplicative inverse.
6. $n \times n$ determinants, $1 \leq n \leq 3$.
7. $n \times n$ systems of linear equations, consistency of the systems, equivalence of the systems, solution by reduction to row echelon form, $n = 2, 3$.
8. $n \times n$ systems of linear equations by row reduction of an augmented matrix, $n = 2, 3$.

SPECIFIC OBJECTIVES

(C) *Differential Equations and Modeling*

Students should be able to:

1. solve first order linear differential equations $y' - ky = f(x)$ using an integrating factor, given that k is a real constant or a function of x , and f is a function;

UNIT 2

MODULE 3: COUNTING, MATRICES AND DIFFERENTIAL EQUATIONS (*cont'd*)

2. solve first order linear differential equations given boundary conditions;
3. solve second order ordinary differential equations with constant coefficients of the form

$ay'' + by' + cy = 0 = f(x)$, where $a, b, c \in \mathbb{R}$ and $f(x)$ is:

- (a) a polynomial,
- (b) an exponential function,
- (c) a trigonometric function;

and the complementary function may consist of

- (a) 2 real and distinct root,
- (b) 2 equal roots,
- (c) 2 complex roots;

4. solve second order ordinary differential equation given boundary conditions;
5. use substitution to reduce a second order ordinary differential equation to a suitable form.

CONTENT

(C) Differential Equations and Modeling

1. Formulation and solution of differential equations of the form $y' - ky = f(x)$, where k is a real constant or a function of x , and f is a function.
2. Second order ordinary differential equations.

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. Counting

Consider the three scenarios given below.

- (a) Throw two dice. Find the probability that the sum of the dots on the uppermost faces of the dice is 6.
- (b) An insurance salesman visits a household. What is the probability that he will be successful in selling a policy?

UNIT 2

MODULE 3: COUNTING, MATRICES AND DIFFERENTIAL EQUATIONS (cont'd)

- (c) A hurricane is situated 500km east of Barbados. What is the probability that it will hit the island?

These three scenarios are very different for the calculation of probability. In 'a', the probability is calculated as the number of successful outcomes divided by the total possible number of outcomes. In this classical approach, the probability assignments are based on equally likely outcomes and the entire sample space is known from the start.

The situation in 'b' is no longer as well determined as in 'a'. It is necessary to obtain historical data for the salesman in question and estimate the required probability by dividing the number of successful sales by the total number of households visited. This frequency approach still relies on the existence of data and its applications are more realistic than those of the classical methodology.

For 'c' it is very unclear that a probability can be assigned. Historical data is most likely unavailable or insufficient for the frequency approach. The statistician might have to revert to informed educated guesses. This is quite permissible and reflects the analyst's prior opinion. This approach lends itself to a Bayesian methodology.

One should note that the rules and results of probability theory remain exactly the same regardless of the method used to estimate the probability of events.

2. Systems of Linear Equations in Two Unknowns

- (a) In order to give a geometric interpretation, students should be asked to plot on graph paper the pair of straight lines represented by a given pair of linear equations in two unknowns, and to examine the relationship between the pair of straight lines in the cases where the system of equations has been shown to have:
- (i) one solution;
 - (ii) many solutions;
 - (iii) no solutions.
- (b) Given a system of equations with a unique solution, there exist equivalent systems, obtained by row-reduction, having the same solution. To demonstrate this, students should be asked to plot on the same piece of graph paper all the straight lines represented by the successive pairs of linear equations which result from each of the row operations used to obtain the solution.

UNIT 2

MODULE 3: COUNTING, MATRICES AND DIFFERENTIAL EQUATIONS (cont'd) RESOURCES

Bolt, B. and Hobbs, D. *101 Mathematical Projects: A Resource Book*, United Kingdom: Cambridge University Press, 1994.

Bostock, L. and Chandler, S. *Core Mathematics for A-Levels*, United Kingdom: Stanley Thornes Publishing Limited, 1997.

Campbell, E. *Pure Mathematics for CAPE, Vol. 2, Jamaica*: LMH Publishing Limited, 2007.

Crawshaw, J. and Chambers, J. *A Concise Course in A-Level Statistics*, Cheltenham, United Kingdom: Stanley Thornes (Publishers) Limited, 1999.

◆ OUTLINE OF ASSESSMENT

Each Unit of the syllabus is assessed separately. The scheme of assessment for each Unit is the same. A candidate's performance on each Unit is reported as an overall grade and a grade on each Module of the Unit. The assessment comprises two components, one external and one internal.

EXTERNAL ASSESSMENT (80 per cent)

The candidate is required to sit two written papers for a total of 4 hrs.

Paper 01 This paper comprises forty-five, compulsory multiple-choice items. **30 per cent**
(1 hour 30 minutes)

Paper 02 This paper comprises six, compulsory extended-response questions. **50 per cent**
(2 hours 30 minutes)

SCHOOL-BASED ASSESSMENT (20 per cent)

School-Based Assessment in respect of each Unit will contribute 20 per cent to the total assessment of a candidate's performance on that Unit.

Paper 03/1

This paper is intended for candidates registered through a school or other approved educational institution.

The School-Based Assessment comprises three class tests designed and assessed internally by the teacher and externally by CXC. The duration of each test is 1 to 1½ hours. The tests must span, individually or collectively, the three Modules, and must include mathematical modelling.

Paper 03/2 (Alternative to Paper 03/1)

This paper is an alternative to Paper 03/1 and is intended for private candidates.

The paper comprises three questions. The duration of the paper is 1½ hours.

MODERATION OF SCHOOL-BASED ASSESSMENT (PAPER 03/1)

School-Based Assessment Record Sheets are available online via the CXC's website www.cxc.org.

All School-Based Assessment Record of marks must be submitted online using the SBA data capture module of the Online Registration System (ORS). A sample of assignments will be requested by CXC for moderation purposes. These assignments will be re-assessed by CXC Examiners who moderate the School-Based Assessment. Teachers' marks may be adjusted as a result of moderation. The Examiners' comments will be sent to schools. All samples must be delivered to the specified marking venues by the stipulated deadlines.



Copies of the students' assignment that are not submitted must be retained by the school until three months after publication by CXC of the examination results.

ASSESSMENT DETAILS FOR EACH UNIT

External Assessment by Written Papers (80 per cent of Total Assessment)

Paper 01 (1 hour 30 minutes – 30 per cent of Total Assessment)

1. Composition of the Paper

- (a) This paper consists of forty-five multiple-choice items, with fifteen items based on each Module.
- (b) All items are compulsory.

2. Syllabus Coverage

- (a) Knowledge of the entire syllabus is required.
- (b) The paper is designed to test a candidate's knowledge across the breadth of the syllabus.

3. Question Type

Questions may be presented using words, symbols, tables, diagrams or a combination of these.

4. Mark Allocation

- (a) Each item is allocated 1 mark.
- (b) Each Module is allocated 15 marks.
- (c) The total marks available for this paper is 45.
- (d) This paper contributes 30 per cent towards the final assessment.

5. Award of Marks

Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

Reasoning: Selection of appropriate strategy, evidence of clear thinking, explanation and/or logical argument.

Algorithmic Knowledge: Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner.

Conceptual Knowledge: Recall or selection of facts or principles; computational skill, numerical accuracy, and acceptable tolerance limits in drawing diagrams.

6. Use of Calculators

- (a) Each candidate is required to have a silent, non-programmable calculator for the duration of the examination, and is entirely responsible for its functioning.
- (b) The use of calculators with graphical displays will not be permitted.
- (c) Answers found by using a calculator, without relevant working shown, may not be awarded full marks.
- (d) Calculators must not be shared during the examination.

7. Use of Mathematical Tables

A booklet of mathematical formulae will be provided.

Paper 02 (2 hours 30 minutes – 50 per cent of Total Assessment)

1. Composition of Paper

- (a) The paper consists of six questions. Two questions are based on each Module (Module 1, Module 2 and Module 3).
- (b) All questions are compulsory.

2. Syllabus Coverage

- (a) Each question may be based on one or more than one topic in the Module from which the question is taken.
- (b) Each question may develop a single theme or unconnected themes.

3. Question Type

- (a) Questions may require an extended response.
- (b) Questions may be presented using words, symbols, tables, diagrams or a combination of these.

4. Mark Allocation

- (a) Each question is worth 25 marks.
- (b) The number of marks allocated to each sub-question will appear in brackets on the examination paper.
- (c) Each Module is allocated 50 marks.
- (d) The total marks available for this paper is 150.
- (e) This paper contributes 50 per cent towards the final assessment.

5. Award of Marks

- (a) Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

Reasoning: Selection of appropriate strategy, evidence of clear thinking, explanation and/or logical argument.

Algorithmic Knowledge: Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner.

Conceptual Knowledge: Recall or selection of facts or principles; computational skill, numerical accuracy, and acceptable tolerance limits in drawing diagrams.

- (b) Full marks will be awarded for correct answers and presence of appropriate working.
- (c) Where an incorrect answer is given, credit may be awarded for correct method provided that the working is shown.
- (d) If an incorrect answer in a previous question or part-question is used later in a section or a question, then marks may be awarded in the latter part even though the original answer is incorrect. In this way, a candidate is not penalised twice for the same mistake.
- (e) A correct answer given with no indication of the method used (in the form of written working) will receive no marks. Candidates are, therefore, advised to show all relevant working.

6. Use of Calculators

- (a) Each candidate is required to have a silent, non-programmable calculator for the duration of the examination, and is responsible for its functioning.
- (b) The use of calculators with graphical displays will not be permitted.
- (c) Answers found by using a calculator, without relevant working shown, may not be awarded full marks.
- (d) Calculators must not be shared during the examination.

7. Use of Mathematical Tables

A booklet of mathematical formulae will be provided.

SCHOOL-BASED ASSESSMENT

School-Based Assessment is an integral part of student assessment in the course covered by this syllabus. It is intended to assist students in acquiring certain knowledge, skills, and attitudes that are associated with the subject. The activities for the School-Based Assessment are linked to the syllabus and should form part of the learning activities to enable the student to achieve the objectives of the syllabus.

During the course of study for the subject, students obtain marks for the competence they develop and demonstrate in undertaking their School-Based Assessment assignments. These marks contribute to the final marks and grades that are awarded to students for their performance in the examination.

The guidelines provided in this syllabus for selecting appropriate tasks are intended to assist teachers and students in selecting assignments that are valid for the purpose of School-Based Assessment. In order to ensure that the scores awarded by teachers are in line with the CXC standards, the Council undertakes the moderation of a sample of the School-Based Assessment assignments marked by each teacher.

School-Based Assessment provides an opportunity to individualise a part of the curriculum to meet the needs of students. It facilitates feedback to the students at various stages of their experience. This helps to build the self-confidence of students as they proceed with their studies. School-Based Assessment also facilitates the development of the critical skills and abilities emphasised by this CAPE subject and enhance the validity of the examination on which candidate performance is reported. School-Based assessment, therefore, makes a significant and unique contribution to both the development of relevant skills and the testing and rewarding of students for the development of those skills.

The Caribbean Examinations Council seeks to ensure that the School-Based Assessment scores are valid and reliable estimates of accomplishment. The guidelines provided in this syllabus are intended to assist in doing so.

Paper 03/1 (20 per cent of Total Assessment)

This paper comprises three tests. The tests, designed and assessed by the teacher, are externally moderated by CXC. The duration of each test is 1 to 1½ hours.

1. Composition of the Tests

The three tests of which the School-Based Assessment is comprised must span, individually or collectively, the three Modules and include mathematical modelling. At least 30 per cent of the marks must be allocated to mathematical modelling.

2. Question Type

Paper 03/2 may be used as a prototype but teachers are encouraged to be creative and original.

3. Mark Allocation

- (a) There is a maximum of 20 marks for each test.
- (b) There is a maximum of 60 marks for the School-Based Assessment.

- (c) The candidate's mark is the total mark for the three tests. One-third of the total marks for the three tests is allocated to each of the three Modules. (See 'General Guidelines for Teachers' below.)
- (d) For each test, marks should be allocated for the skills outlined on page 3 of this Syllabus.

4. Award of Marks

- (a) Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

For each test, the 20 marks should be awarded as follows:

Reasoning:	Selection of appropriate strategy, evidence of clear thinking, explanation and/or logical argument.
	(3 – 5 marks)

Algorithmic Knowledge:	Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner.
	(10 – 14 marks)

Conceptual Knowledge:	Recall or selection of facts or principles; computational skill, numerical accuracy, and acceptable tolerance limits in drawing diagrams.
	(3 – 5 marks)

- (b) If an incorrect answer in an earlier question or part-question is used later in a section or a question, then marks may be awarded in the later part even though the original answer is incorrect. In this way, a candidate is not penalised twice for the same mistake.
- (c) A correct answer given with no indication of the method used (in the form of written working) will receive no marks. Candidates should be advised to show all relevant working.

Paper 03/2 (20 per cent of Total Assessment)

1. Composition of Paper

- (a) This paper consists of three questions, each based on one of the three Modules.
- (b) All questions are compulsory.

2. Question Type

- (a) Each question may require an extended response.
- (b) A part of or an entire question may focus on mathematical modeling.
- (c) A question may be presented using words, symbols, tables, diagrams or a combination of these.

3. Mark Allocation

- (a) Each question carries a maximum of 20 marks.
- (b) The Paper carries a maximum of 60 marks.
- (c) For each question, marks should be allocated for the skills outlined on page 3 of this Syllabus.

4. Award of Marks

- (a) Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

For each test, the 20 marks should be awarded as follows:

Reasoning:	Selection of appropriate strategy, evidence of clear reasoning, explanation and/or logical argument. (3 – 5 marks)
------------	---

Algorithmic Knowledge:	Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner. (10 – 14 marks)
------------------------	---

Conceptual Knowledge:	Recall or selection of facts or principles; computational skill, numerical accuracy, and acceptable tolerance limits in drawing diagrams. (3 – 5 marks)
-----------------------	--

- (b) If an incorrect answer in a previous question or part-question is used later in a section or a question, then marks may be awarded in the later part even though the original answer is incorrect. In this way, a candidate is not penalised twice for the same mistake.
- (c) A correct answer given with no indication of the method used (in the form of written working) will receive no marks. Candidates should be advised to show all relevant working.

GENERAL GUIDELINES FOR TEACHERS

1. Teachers should note that the reliability of marks awarded is a significant factor in the *School-Based Assessment*, and has far-reaching implications for the candidate's final grade.
2. Candidates who do not fulfill the requirements of the *School-Based Assessment* will be considered absent from the whole examination.
3. Teachers are asked to note the following:
 - (a) the relationship between the marks for the assignment and those submitted to CXC on the *school-based assessment* form should be clearly shown;

- (b) the teacher is required to allocate one-third of the total score for the *School-Based Assessment* to each Module. Fractional marks should not be awarded. In cases where the mark is not divisible by three, then:
- (i) when the remainder is 1 mark, the mark should be allocated to Module 3;
 - (ii) when the remainder is 2, then a mark should be allocated to Module 3 and the other mark to Module 2;
- for example, 35 marks would be allocated as follows:
- $35/3 = 11$ remainder 2 so 11 marks to Module 1 and 12 marks to each of Modules 2 and 3.
- (c) the standard of marking should be consistent.
4. Teachers are required to submit a copy of EACH test, the solutions and the mark schemes with the sample.

◆ REGULATIONS FOR PRIVATE CANDIDATES

Candidates who are registered privately will be required to sit Paper 01, Paper 02 and Paper 03/2. Paper 03/2 will be 1½ hours' duration and will consist of three questions, each worth 20 marks. Each question will be based on the objectives and content of one of the three Modules of the Unit. Paper 03/2 will contribute 20 per cent of the total assessment of a candidate's performance on that Unit.

Paper 03/2 (1 hour 30 minutes)

The paper consists of three questions. Each question is based on the topics contained in one Module and tests candidates' skills and abilities to:

1. recall, select and use appropriate facts, concepts and principles in a variety of contexts;
2. manipulate mathematical expressions and procedures using appropriate symbols and language, logical deduction and inferences;
3. select and use a simple mathematical model to describe a real-world situation;
4. simplify and solve mathematical models;
5. interpret mathematical results and their application in a real-world problem.

◆ REGULATIONS FOR RE-SIT CANDIDATES

Candidates who have earned a moderated score of at least 50 per cent of the total marks for the Internal Assessment component, may elect not to repeat this component, provided they re-write the examination no later than TWO years following their first attempt. These resit candidates must complete Papers 01 and 02 of the examination for the year in which they register.

Resit candidates must be entered through a school or other approved educational institution.

Candidates who have obtained less than 50 per cent of the marks for the School-Based Assessment component must repeat the component at any subsequent sitting or write Paper 03/2.

◆ ASSESSMENT GRID

The Assessment Grid for each Unit contains marks assigned to papers and to Modules and percentage contributions of each paper to total scores.

Units 1 and 2

Papers	Module 1	Module 2	Module 3	Total	(%)
External Assessment					
Paper 01 (1 hour 30 minutes)	15 (30 weighted)	15 (30 weighted)	15 (30 weighted)	45 (90 weighted)	(30)
Paper 02 (2 hours 30 minutes)	50	50	50	150	(50)
School-Based Assessment					
Paper 03/1 or Paper 03/2 (1 hour 30 minutes)	20	20	20	60	(20)
Total	100	100	100	300	(100)

◆ MATHEMATICAL NOTATION

The following list summarises the notation used in the Mathematics papers of the Caribbean Advanced Proficiency Examination.

Set Notation

\in	is an element of
\notin	is not an element of
$\{x: \dots\}$	the set of all x such that ...
$n(A)$	the number of elements in set A
\emptyset	the empty set
U	the universal set
A'	the complement of the set A
\mathbb{W}	the set of whole numbers $\{0, 1, 2, 3, \dots\}$
\mathbb{N}	the set of natural numbers $\{1, 2, 3, \dots\}$
\mathbb{Z}	the set of integers
\mathbb{Q}	the set of rational numbers
$\overline{\mathbb{Q}}$	<i>the set of irrational numbers</i>
\mathbb{R}	the set of real numbers
\mathbb{C}	the set of complex numbers
\subset	is a <i>proper</i> subset of
$\not\subset$	is not a proper subset of
\subseteq	<i>is a subset of</i>
$\not\subseteq$	is not a subset of
\cup	union
\cap	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leq b\}$

Logic

\wedge	conjunction
\vee	(inclusive) injunction
$\vee\!\vee$	exclusive disjunction
\sim	negation
\rightarrow	conditionality
\leftrightarrow	bi-conditionality
\Rightarrow	implication
\Leftrightarrow	equivalence

Miscellaneous Symbols

\equiv	is identical to
\approx	is approximately equal to
\propto	is proportional to
∞	infinity

Operations

$\sum_{i=1}^n x_i$	$x_1 + x_2 + \dots + x_n$
\sqrt{x}	the positive square root of the real number x
$ x $	the modulus of the real number x
$n!$	n factorial, $1 \times 2 \times \dots \times n$ for $n \in \mathbb{N}$ ($0! = 1$)
${}^n C_r \binom{n}{r}$	the binomial coefficient, $\frac{n!}{(n-r)! r!}$, for $n, r \in \mathbb{N}$, $0 \leq r \leq n$
${}^n P_r$	$\frac{n!}{(n-r)!}$

Functions

f	the function f
$f(x)$	the value of the function f at x
$f: A \rightarrow B$	the function f under which each element of the set A has an image in the set B
$f: x \rightarrow y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
fg	the composite function $f(g(x))$
$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as x tends to a
$\Delta x, \delta x$	an increment of x
$\frac{dy}{dx}, y'$	the first derivative of y with respect to x
$\frac{d^n y}{dx^n}, y^{(n)}$	the n th derivative of y with respect to x
$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, ..., n th derivatives of $f(x)$ with respect to x
\dot{x}, \ddot{x}	the first and second derivatives of x with respect to time t
e	the exponential constant
$\ln x$	the natural logarithm of x (to base e)
$\lg x$	the logarithm of x to base 10

Complex Numbers

i	$\sqrt{-1}$
z	a complex number, $z = x + yi$ where $x, y \in \mathbb{P}$
$\operatorname{Re} z$	the real part of z
$\operatorname{Im} z$	the imaginary part of z
$ z $	the modulus of z
$\arg z$	the argument of z , where $-\pi < \arg z \leq \pi$
\bar{z}, z^*	the complex conjugate of z

Vectors

\underline{a} , \mathbf{a} , \vec{AB}	vectors
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector \mathbf{a}
$ \mathbf{a} $	the magnitude of the vector \mathbf{a}
$\mathbf{a} \cdot \mathbf{b}$	the scalar product of the vectors \mathbf{a} and \mathbf{b}
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the positive Cartesian coordinate axes
$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	$x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

Probability

S	the sample space
A, B, \dots	the events A, B, \dots
$P(A')$	the probability that the event A <i>does not occur</i>

Matrices

M	a matrix M
(M^{-1})	inverse of the non-singular square matrix M
M^T, M_T	transpose of the matrix M
$\det M, M $	determinant of the square matrix M

Western Zone Office

CARIBBEAN EXAMINATIONS COUNCIL

Caribbean Advanced Proficiency Examination®



PURE MATHEMATICS

Specimen Papers and Mark Schemes/Keys

Specimen Papers:

- Unit 1, Paper 01
- Unit 1, Paper 02
- Unit 1, Paper 032
- Unit 2, Paper 01
- Unit 2, Paper 02
- Unit 2, Paper 032

Mark Schemes and Keys:

- Unit 1, Paper 01
- Unit 1, Paper 02
- Unit 1, Paper 032
- Unit 2, Paper 01
- Unit 2, Paper 02
- Unit 2, Paper 032

FORM TP 02134010/SPEC

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

ADVANCED PROFICIENCY EXAMINATION

SPECIMEN PAPER
MULTIPLE CHOICE QUESTIONS
FOR

PURE MATHEMATICS

UNIT 1 – Paper 01

ALGEBRA, GEOMETRY AND CALCULUS

90 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This test consists of 45 items. You will have 90 minutes to answer them.
2. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.
3. Look at the sample item below.

Sample Item

The lines $2y - 3x - 13 = 0$ and $y + x + 1 = 0$
intersect at the point

Sample Answer

- (A) (-3, -2)
(B) (-3, 2)
(C) (3, -2)
(D) (3, 2)



The best answer to this item is “(-3, 2)”, so answer space (B) has been shaded.

4. You may do any rough work in this booklet.
5. The use of silent, non-programmable scientific calculators is allowed.

Examination Materials Permitted

A list of mathematical formulae and tables (provided) – **Revised 2010**

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

Copyright © 2011 Caribbean Examinations Council
All rights reserved

1. Let p , q and r be the propositions
 p : Students have a driving licence,
 q : Students have a passport,
 r : Students have an identification card.

The compound proposition, Students have a driving licence or identification card (but not both) together with a passport is expressed as

- (A) $((p \wedge r) \vee \sim(p \wedge r)) \wedge q$
(B) $((p \vee r) \vee \sim(p \wedge r)) \vee q$
(C) $((p \vee r) \vee \sim(p \vee r)) \wedge q$
(D) $((p \vee r) \wedge \sim(p \wedge r)) \wedge q$

2. The compound proposition $p \wedge q$ is true can be illustrated by the truth table

(A)

<u>p</u>	<u>q</u>	<u>$p \wedge q$</u>
0	0	0
0	1	0
1	0	0
1	1	1

0	0	0
0	1	0
1	0	0
1	1	1

(B)

<u>p</u>	<u>q</u>	<u>$p \wedge q$</u>
0	0	1
0	1	0
1	0	0
1	1	1

0	0	1
0	1	0
1	0	0
1	1	1

(C)

<u>p</u>	<u>q</u>	<u>$p \wedge q$</u>
0	0	0
0	1	0
1	0	1
1	1	1

0	0	0
0	1	0
1	0	1
1	1	1

(D)

<u>p</u>	<u>q</u>	<u>$p \wedge q$</u>
0	0	1
0	1	0
1	0	1
1	1	0

0	0	1
0	1	0
1	0	1
1	1	0

3. The contrapositive for the conditional proposition $p \rightarrow q$ is

- (A) $q \rightarrow p$
(B) $\sim p \rightarrow q$
(C) $\sim q \rightarrow \sim p$
(D) $p \rightarrow \sim q$

4. The proposition $q \rightarrow p$ is logically equivalent to

- (A) $\sim p \wedge \sim q$
(B) $p \vee \sim q$
(C) $\sim q \wedge p$
(D) $q \wedge \sim p$

5. The expression $\frac{5\sqrt{45} - \sqrt{80}}{\sqrt{5} - \sqrt{125}}$ is equal to

(A) $\frac{5\sqrt{35}}{\sqrt{120}}$

(B) $\frac{11\sqrt{5}}{5}$

(C) $\frac{11}{4\sqrt{5}}$

(D) $-\frac{11}{4}$

6. If $2x^3 + ax^2 - 5x - 1$ leaves a remainder of 3 when divided by $(2x + 1)$, then a is

(A) -7

(B) 7

(C) $-\frac{1}{2}$

(D) -5

(E)

7. Given that $x = 3^y$, $y > 0$ then $\log_x 3$ is equal to
- (A) y
 (B) $3y$
 (C) $\frac{1}{y}$
 (D) $\frac{3}{y}$
8. Given that $f(x) = 2 - e^{2x}$, the inverse function, $f^{-1}(x)$, for $x < 2$ is
- (A) $\ln(2-x)$
 (B) $\ln(2-2x)$
 (C) $2 \ln(2-x)$
 (D) $\frac{1}{2} \ln(2-x)$
9. The function $f(x) = 2x^2 - 4x + 5$, for $x \in \mathbb{R}$, is one-to-one for $x > k$, where $k \in \mathbb{R}$. The value of k is
- (A) -2
 (B) -1
 (C) 1
 (D) 3
10. Given that $fg(x) = x$, where $g(x) = \frac{2x+1}{3}$, $f(x) =$
- (A) $\frac{3x-1}{2}$
 (B) $\frac{3}{2x+1}$
 (C) $\frac{2}{3x+1}$
 (D) $\frac{3x}{2x+1}$
11. The values of x for which $|2x - 3| = x + 1$ are
- (A) $x = -\frac{2}{3}$, $x = -4$
 (B) $x = -\frac{2}{3}$, $x = 4$
 (C) $x = \frac{2}{3}$, $x = 4$
 (D) $x = \frac{2}{3}$, $x = -4$
12. Given that $x^2 + 4x + 3$ is a factor of $f(x) = 2x^3 + 7x^2 + 2x - 3$, the values of x for which $f(x) = 0$ are
- (A) $-3, 1, -\frac{1}{2}$
 (B) $-3, -1, -\frac{1}{2}$
 (C) $3, 1, \frac{1}{2}$
 (D) $-3, -1, \frac{1}{2}$
13. The cubic equation $2x^3 + x^2 - 22x + 24 = 0$ has roots α , β and γ . The value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ is
- (A) $-\frac{1}{12}$
 (B) $-\frac{1}{11}$
 (C) $-\frac{1}{2}$
 (D) $\frac{11}{12}$
14. The range(s) of values of x for which $\frac{3x+2}{x-1} > 0$ are
- (A) $x > -\frac{2}{3}$, $x > 1$
 (B) $-\frac{2}{3} < x < 1$
 (C) $x < \frac{2}{3}$, $x > 1$
 (D) $x < -\frac{2}{3}$, $x > 1$

- 15.** The values of x for which $|x + 5| > 3$ are
- (A) $x < -8, x < -2$
 (B) $x > 0, x < 1$
 (C) $x > -2, x < -8$
 (D) $x > -2, x > -8$
- 16.** $\frac{1 + \cot^2 \theta}{\sec \theta \cosec \theta} \equiv$
- (A) $\tan \theta$
 (B) $\cos \theta$
 (C) $\cot \theta$
 (D) $\cosec \theta$
- 17.** The general solution of the equation $\cos 2\theta = 1$ is
- (A) $n\pi + \frac{\pi}{4}$
 (B) $n\pi$
 (C) $n\pi + \frac{\pi}{2}$
 (D) $\frac{(2n+1)\pi}{4}$
- 18.** $\cos \theta + 3 \sin \theta = 2$ can be expressed as
- (A) $4 \cos \left(\theta - \tan^{-1} \left(\frac{1}{3} \right) \right) = 2$
 (B) $2 \cos \left(\theta + \tan^{-1} (3) \right) = 2$
 (C) $\sqrt{10} \cos \left(\theta - \tan^{-1} (3) \right) = 2$
 (D) $\sqrt{10} \cos \left(\theta + \tan^{-1} \left(\frac{1}{3} \right) \right) = 2$
- 19.** If $\cos A = \frac{3}{5}$ and A is acute, then $\sin 2A$ is equal to
- (A) $\frac{6}{25}$
 (B) $\frac{8}{25}$
 (C) $\frac{12}{25}$
 (D) $\frac{24}{25}$
- 20.** The minimum value of $\frac{1}{2 \cos(\theta + \frac{\pi}{4})}$ is
- (A) -1
 (B) 0
 (C) $\frac{1}{2}$
 (D) 2
- 21.** A curve C_1 is given by the equation $y = x^2 + 1$, and a curve C_2 is given by the equation $\frac{16}{x^2} + 1$, $x \in \mathbb{R}, x > 0$. The value of x for which $C_1 = C_2$ is
- (A) -4
 (B) 2
 (C) -2
 (D) 4
- 22.** The tangent to the circle, C , with equation $x^2 + y^2 + 4x - 10y - 5 = 0$ at the point $P(3, 2)$ has equation
- (A) $3x + 5y - 19 = 0$
 (B) $5x + 3y + 19 = 0$
 (C) $3x - 5y + 19 = 0$
 (D) $5x - 3y - 9 = 0$

- 23.** The x -coordinates of the points where the line, l , with equation $y = 2x + 1$ cuts the curve C with equation $\frac{6}{x}$ are
- (A) $\frac{3}{2}, -2$
 (B) $-\frac{3}{2}, -2$
 (C) $\frac{3}{2}, 2$
 (D) $-\frac{3}{2}, 2$
- 24.** The Cartesian equation of the curve C given by the parametric equations $x = 3 \sin \theta - 2$, $y = 4 \cos \theta + 3$ is
- (A) $x^2 + y^2 = 30$
 (B) $9x^2 + 16y^2 = 13$
 (C) $(x - 3)^2 + 4(y - 4)^2 = 36$
 (D) $16(x + 2)^2 + 9(y - 3)^2 = 144$
- 25.** Relative to a fixed origin, O , the position vector of A is $\overrightarrow{OA} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and the position vector of B is $\overrightarrow{OB} = 9\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$. $|\overrightarrow{AB}|$ is
- (A) 1 unit
 (B) 7 units
 (C) $3\sqrt{21}$ units
 (D) 49 units
- 26.** Relative to a fixed origin, O , the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$, and point B has position vector $(-5\mathbf{i} + 9\mathbf{j} - 5\mathbf{k})$. The line, l , passes through the points A and B . A vector equation for the line l is given by
- (A) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \lambda(-7\mathbf{i} + 6\mathbf{j} - \mathbf{k})$
 (B) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \lambda(-5\mathbf{i} + 9\mathbf{j} - 5\mathbf{k})$
 (C) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \lambda(-3\mathbf{i} + 12\mathbf{j} - 9\mathbf{k})$
 (D) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \lambda(-10\mathbf{i} + 27\mathbf{j} + 20\mathbf{k})$
- 27.** Relative to a fixed origin, O , the line, l_1 , has position vector $\begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, and the line, l_2 , has position vector $\begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix}$, where λ and μ are scalars. The cosine of the acute angle between l_1 and l_2 is given by
- (A) $\cos \theta = -\frac{2}{3}$
 (B) $\cos \theta = \frac{2}{3}$
 (C) $\cos \theta = \frac{8}{27}$
 (D) $\cos \theta = \frac{2}{27}$
- 28.** Relative to a fixed origin, O , the point A has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point B has position vector $(5\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$. Given that a vector \mathbf{v} is of magnitude $3\sqrt{6}$ units in the direction of $|\overrightarrow{AB}|$, then $\mathbf{v} =$
- (A) $3\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$
 (B) $-3\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$
 (C) $-3\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$
 (D) $3\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$
- 29.** The point $P(3, 0, 1)$ lies in the plane π with equation $\mathbf{r} \cdot (\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}) = d$. The constant d is
- (A) $\sqrt{21}$
 (B) 5
 (C) $\sqrt{41}$
 (D) 10

- 30.** The line, l_1 , has equation $\mathbf{r} = 6\mathbf{i} - 3\mathbf{j} - 2\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})$ and the line, l_2 , has equation $-5\mathbf{i} + 15\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + a\mathbf{k})$. Given that l_1 is perpendicular to l_2 the value of a is
- (A) 2
(B) -2
(C) -4
(D) 6
- 31.** $\lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x^2 - 2x - 3}$ is
- (A) 0
(B) 1
(C) $\frac{7}{4}$
(D) ∞
- 32.** $\lim_{\theta \rightarrow 0} \frac{\sin x}{\frac{x}{2}}$ is
- (A) 0
(B) $\frac{1}{2}$
(C) 1
(D) 2
- 33.** Given that $\lim_{x \rightarrow -1} \{3f(x) + 2\} = 11$, where $f(x)$ is real and continuous, the $\lim_{x \rightarrow -1} \{2f(x) + 5x\}$ is
- (A) -11
(B) 1
(C) 4
(D) 13
- 34.** Given that $f(x) = (6x + 4) \sin x$, then $f'(x)$ is
- (A) $6 \cos x$
(B) $2(3x + 2) \cos x + 6 \sin x$
(C) $6x \cos x + 6 \sin x$
(D) $3 + 2 \sin x + (3x + 2) \cos x$
- 35.** The derivative by first principles of the function $f(x) = \frac{1}{x^2}$ is given by
- (A) $\lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$
(B) $\lim_{h \rightarrow 0} \frac{\frac{1}{(x^2+h)} - \frac{1}{x^2}}{h}$
(C) $\lim_{h \rightarrow 0} \frac{\frac{1}{x^2} - \frac{1}{(x^2+h)}}{h}$
(D) $\lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$
- 36.** Given $f(x) = 3 \cos 2x$, then $f'(x) =$
- (A) $6 \sin 2x$
(B) $-3 \sin 2x$
(C) $-6 \sin 2x$
(D) $-\frac{3}{2} \sin 2x$
- 37.** The curve, C , with equation $y = x^3 - 6x^2 + 9x$ has stationary points at $P(3, 0)$ and $Q(1, 4)$. The nature of these stationary points is
- (A) $(3, 0)_{\max}$ $(1, 4)_{\min}$
(B) $(3, 0)_{\min}$ $(1, 4)_{\max}$
(C) $(3, 0)_{\text{infl}}$ $(1, 4)_{\max}$
(D) $(3, 0)_{\text{infl}}$ $(1, 4)_{\min}$
- 38.** Given that the gradient function to a curve, C , at the point $P(2, 3)$ is $6x^2 - 14x$, the equation of the normal to C at P is given by the equation
- (A) $y - 3 = -2(x - 2)$
(B) $y - 3 = -4(x - 2)$
(C) $y - 3 = 4(x - 2)$
(D) $y - 3 = \frac{1}{4}(x - 2)$

39. $\int \frac{(2x+1)^2}{\sqrt{x}} dx =$

- (A) $\int \left(4x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + x^{-\frac{1}{2}} \right) dx$
 (B) $\int \left(4x^{\frac{3}{2}} + x^{-\frac{1}{2}} \right) dx$
 (C) $\int (4x + 4 + x^{-1}) dx$
 (D) $\int (4x^{\frac{5}{2}} + 4x^{\frac{3}{2}} + x^{\frac{1}{2}}) dx$

40. Given that $\int_1^3 f(x) dx = 8$, then

$$\int_1^3 [2f(x) - 5] dx =$$

- (A) 6
 (B) 11
 (C) 13
 (D) 21

41. The area of the finite region, R , enclosed by the curve $y = x - \frac{1}{\sqrt{x}}$,

the lines $x = 1$ and $x = 4$ is

- (A) $\frac{5}{2}$
 (B) $\frac{11}{2}$
 (C) $\frac{27}{4}$
 (D) $\frac{19}{2}$

42. The region, R , enclosed by the curve with equation $y = 4 - x^2$ in the first quadrant is rotated completely about the y -axis. The volume of the solid generated is given by

- (A) $\pi \int_0^4 (4-y) dy$
 (B) $\pi \int_0^4 (4-y)^2 dy$
 (C) $\pi \int_0^2 (4-y) dy$
 (D) $\pi \int_0^2 (4-y)^2 dy$

43. Given that $\frac{d}{dx} \frac{2x-1}{3x+2} = \frac{7}{(3x+2)^2}$ then

$$\int_1^2 \frac{21}{(3x+2)^2} dx =$$

- (A) $\left. \frac{3(7)}{(3x+2)^2} \right|_1^2$
 (B) $\left. \frac{2x-1}{(3x+2)} \right|_1^2$

- (C) $\left. \frac{3(2x-1)}{(3x+2)} \right|_1^2$

- (D) $\left. \frac{-21}{(3x+2)} \right|_1^2$

44. Given that $\int_{-2}^0 f(x) dx = \frac{16}{3}$ and water is pumped into a large tank at a rate that is proportional to its volume, V ,
 $\int_{-2}^2 f(x) dx = \frac{32}{3}$ where $f(x)$ is a real continuous function in the closed interval $[-2, 2]$, then $\int_0^2 f(x) dx =$

- (A) $\frac{16}{3}$
 (B) 16
 (C) $\frac{64}{3}$
 (D) 32

45. Water is pumped into a large tank at a rate that is proportional to its volume, V , at time, t , seconds. There is a small hole at the bottom of the tank and water leaks out at a constant rate of $5 \text{ m}^3/\text{s}$. Given that k is a positive constant, a differential equation that satisfies this situation is

- (A) $\frac{dV}{dt} = kV - 5$
 (B) $\frac{dV}{dt} = -kV - 5$
 (C) $\frac{dV}{dt} = kV + 5$
 (D) $\frac{dV}{dt} = kV$

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

TEST CODE **02134020/SPEC**

FORM TP 02134020/SPEC

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – Paper 02

ALGEBRA, GEOMETRY AND CALCULUS

SPECIMEN PAPER

2 hours 30 minutes

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.

The maximum mark for each Module is 50.

The maximum mark for this examination is 150.

This examination consists of 8 printed pages.

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **THREE** significant figures.

Examination Materials

Graph paper (provided)

Mathematical formulae and tables (provided) – **Revised 2010**

Mathematical instruments

Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

Copyright © 2011 Caribbean Examinations Council

All rights reserved

SECTION A (MODULE 1)

Answer BOTH questions.

1. (a) Let **p** and **q** be given propositions.

- (i) Copy and complete the table below to show the **truth tables** of $p \rightarrow q$ and $\sim p \vee q$. [3 marks]

p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$

- (ii) Hence, state whether the compound propositions $p \rightarrow q$ and $\sim p \vee q$ are logically equivalent, giving reasons for your answer. [2 marks]

- (iii) Use the **algebra of propositions** to show that $p \wedge (p \rightarrow q) = p \wedge q$. [3 marks]

- (b) The binary operation * is defined on the set of real numbers, \mathbb{R} , as follows:

$$x * y = x + y - 1 \text{ for all } x, y \text{ in } \mathbb{R}.$$

Prove that *

- (i) is closed in \mathbb{R} , [3 marks]

- (ii) is commutative in \mathbb{R} , [2 marks]

- (iii) is associative in \mathbb{R} . [4 marks]

- (c) Let $y = \frac{2x}{x^2 + 4}$.

- (i) Show that for all real values of x , $-\frac{1}{2} \leq y \leq \frac{1}{2}$ [5 marks]

- (ii) Hence, sketch the graph of y for all x such that $-2 \leq x \leq 2$. [3 marks]

Total 25 marks

2. (a) Two of the roots of the cubic equation $2x^3 + px^2 + qx + 2 = 0$ are -1 and $\frac{1}{2}$.
Find

(i) the values of the constants p and q [4 marks]

(ii) the third root of the equation. [3 marks]

(b) Prove by Mathematical Induction that $\sum_{r=1}^n (6r + 5) = n(3n + 8)$. [10 marks]

(c) Solve for x the following equation $e^{2x} + 2e^{-2x} = 3$. [8 marks]

Total 25 marks

SECTION B (MODULE 2)

Answer BOTH questions.

3. (a) (i) Prove the identity

$$\frac{\sin 3\theta + \sin \theta}{\cos 3\theta + \cos \theta} \equiv \tan 2\theta .$$

[4 marks]

- (ii) Solve the equation

$$\sin 3\theta + \sin \theta + \sin 2\theta = 0, 0 \leq \theta \leq 2\pi.$$

[7 marks]

- (b) (i) Express $f(\theta) = 8\cos \theta + 6\sin \theta$ in the form $r \cos(\theta - \alpha)$

$$\text{where } r > 0, 0^\circ < \alpha < 90^\circ.$$

[3 marks]

- (ii) Determine the minimum value of

$$g(\theta) = \frac{10}{10 + 8\cos \theta + 6\sin \theta}$$

stating the value of θ for which $g(\theta)$ is a minimum.

[4 marks]

- (c) Let $A = (2, 0, 0)$, $B = (0, 0, 2)$ and $C = (0, 2, 0)$.

- (i) Express the vectors \overrightarrow{BC} and \overrightarrow{BA} in the form $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

[2 marks]

- (ii) Show that the vector $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ is perpendicular to the plane through A, B, and C.
[2 marks]

- (iii) Hence, find the Cartesian equation of the plane through A, B and C. [3 marks]

Total 25 marks

4. The equation of the line L is $x + 2y = 7$ and the equation of the circle C is $x^2 + y^2 - 4x - 1 = 0$.
- (a) Show that the line, L , is a tangent to the circle C . [8 marks]
- (b) Find,
- (i) the equation of the tangent, M , diametrically opposite to the tangent, L , of circle C .
- (ii) the equation of the diameter of C which is parallel to L [4 marks]
- (iii) the coordinates of its points of intersection with C . [2 marks]
- (c) The parametric equations of a curve, C , are given by
$$x = \frac{t}{1+t} \text{ and } y = \frac{t^2}{1+t}.$$
- Determine the Cartesian equation of C in the form $y = f(x)$. [6 marks]

Total 25 marks

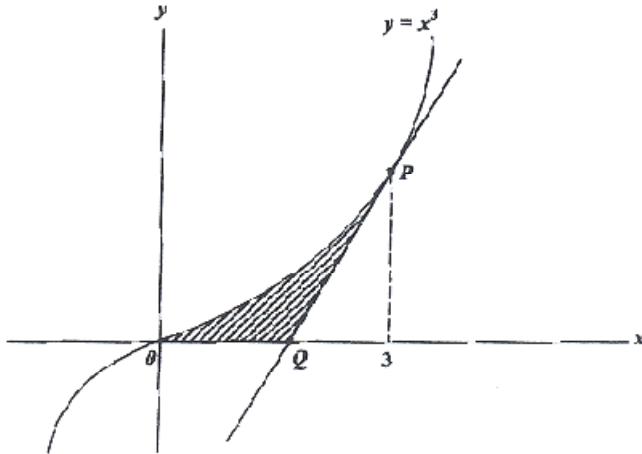
SECTION C (MODULE 3)

Answer BOTH questions.

5. (a) Show that $\lim_{h \rightarrow 0} \frac{h}{\sqrt{x+h} - \sqrt{x}} = 2\sqrt{x}$. [6 marks]
- (b) The function $f(x)$ is such that $f(x) = 18x + 4$. Given that $f(2) = 14$ and $f(3) = 74$, find the value of $f(4)$. [8 marks]
- (c) If $y = \frac{x}{1+x^2}$, show that
- (i) $\frac{dy}{dx} = \frac{1}{(1+x^2)^2} - y^2$ [5 marks]
- (ii) $\frac{d^2y}{dx^2} = \frac{2y(x^2 - 3)}{(1+x^2)^3}$ [6 marks]

Total 25 marks

6. (a) The diagram below, **not drawn to scale** is a sketch of the curve $y = x^3$. The tangent, PQ, meets the curve at $P(3, 27)$.



- (i) Find
- the equation of the tangent PQ [4 marks]
 - the coordinates of Q . [1 mark]
- (ii) Calculate
- the area of the shaded region, [5 marks]
 - the volume of the solid generated when the shaded region is rotated completely about the x -axis, giving your answer in terms of π . [5 marks]

[The volume, V , of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.]

- (b) The gradient of a curve which passes through the point $(0, 3)$ is given by

$$\frac{dy}{dx} = 3x^2 - 8x + 5.$$

- (i) Determine the equation of the curve. [3 marks]
- (ii) Find the coordinates of the TWO stationary points of the curve in (b) (i) above and distinguish the nature of EACH point. [7 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

TEST CODE **02134032/SPEC**

FORM TP 02134032/SPEC

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – Paper 032

ALGEBRA, GEOMETRY AND CALCULUS

SPECIMEN PAPER

1 hour 30 minutes

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.

The maximum mark for each Module is 20.

The maximum mark for this examination is 60.

This examination consists of 4 printed pages.

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **THREE** significant figures.

Examination Materials Permitted

Graph paper (provided)

Mathematical formulae and tables (provided) – **Revised 2010**

Mathematical instruments

Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

Copyright © 2011 Caribbean Examinations Council
All rights reserved

SECTION A (MODULE 1)

Answer this question.

1. (a) **p** and **q** are two given propositions.
- (i) State the converse of $p \rightarrow q$. **[1 mark]**
- (ii) Show that the contrapositive of the inverse of $p \rightarrow q$ is the converse of $p \rightarrow q$. **[2 marks]**
- (b) $f(n) = 2^n + 6^n$
- (i) Show that $f(k+1) = 6f(k) - 4(2^k)$ **[3 marks]**
- (ii) Hence, or otherwise, prove by mathematical induction that, for $n \in \mathbb{N}$, $f(n)$ is divisible by 8. **[4 marks]**
- (c) (i) On the same diagram, sketch the graphs of $y = x + 2$ and $y = \left| \frac{1}{x-2} \right|$, showing clearly on your sketch the coordinates of any points at which the graphs cross the axes. **[4 marks]**
- (ii) Find the range of values of x for which $x + 2 < \left| \frac{1}{x-2} \right|$. **[6 marks]**

Total 20 marks

SECTION B (MODULE 2)

Answer this question.

2. (a) (i) Using $\sin^2 \theta + \cos^2 \theta \equiv 1$ show that $\operatorname{cosec}^2 \theta - \cot^2 \theta \equiv 1$. [2 marks]
- (ii) Hence, or otherwise, prove that $\operatorname{cosec}^4 \theta - \cot^4 \theta \equiv \operatorname{cosec}^2 \theta + \cot^2 \theta$. [2 marks]

- (b) A curve, C , has parametric equations

$$x = \sin^2 \theta, y = 2 \tan \theta, 0 \leq \theta < 90^\circ.$$

Find the Cartesian equation of C . [4 marks]

- (c) The line, l_1 , has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line, l_2 , has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C , find

- (i) the coordinates of C [3 marks]
- (ii) the angle between l_1 and l_2 , correct to 2 decimal places. [4 marks]
- (iii) Show that the vector $\mathbf{n} = 4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}$ is perpendicular to l_1 and l_2 . [2 marks]
- (iv) Hence find the vector equation of the plane, $\mathbf{r} \cdot \mathbf{n} = d$, through the point $\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$. [3 marks]

Total 20 marks

SECTION C (MODULE 3)

3. (a) $S_n = 1 \times 1! + 2 \times 2! + 3 \times 3! + \dots + n \times n! = (n+1)! - 1.$

Show that $\lim_{n \rightarrow \infty} \frac{S_n}{S_{n+1}} = 0.$ [4 marks]

- (b) Using $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1,$ differentiate from first principles $f(x) = \cos x.$ [5 marks]

- (c) A circular patch of oil has radius, $r,$ metres at time, $t,$ hours after it was spilled. At time 2:00 p. m., one hour after the spillage, the radius of the patch of oil is 5 metres. In a model, the rate of increase of r is taken to be proportional to $\frac{1}{r}.$

- (i) Form a differential equation for r in terms of $t,$ involving a constant of proportionality, $k.$ [1 mark]

- (ii) Solve the differential equation in (c) (i) above and hence show that the radius of the patch of oil is proportional to the square root of the time elapsed since the spillage. [7 marks]

- (iii) Determine the time, to the nearest minute, at which the model predicts that the radius of the patch of oil will be 12 metres. [3 marks]

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

SPECIMEN PAPER
MULTIPLE CHOICE QUESTIONS
FOR

PURE MATHEMATICS

UNIT 2 – Paper 01

COMPLEX NUMBERS, ANALYSIS AND MATRICES

90 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This test consists of 45 items. You will have 90 minutes to answer them.
2. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.
3. Look at the sample item below.

Sample ItemIf the function $f(x)$ is defined by $f(x) \cos x$ then $f'(x)$ is

(A) $-\frac{1}{2\sqrt{x}} \sin \sqrt{x}$

Sample Answer

(B) $-\frac{1}{2} \sin \sqrt{x}$

 A B C

(C) $\frac{1}{\sqrt{x}} \sin \sqrt{x}$

(D) $\frac{1}{2\sqrt{x}} \sin \sqrt{x}$

The best answer to this item is " $-\frac{1}{2\sqrt{x}} \sin \sqrt{x}$ ", so answer space (D) has been shaded.

4. You may do any rough work in this booklet.
5. The use of silent, non-programmable scientific calculators is allowed.

Examination Materials Permitted

A list of mathematical formulae and tables (provided) – Revised 2010

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

 Copyright © 2011 Caribbean Examinations Council
 All rights reserved

1. If z and z^* are two conjugate complex numbers, where $z = x + iy$, $x, y \in \mathbb{R}$, then $zz^* =$
- (A) $x^2 + y^2$
 (B) $x^2 - y^2 - 2xyi$
 (C) $x^2 - y^2$
 (D) $x^2 + y^2 - 2xyi$
2. If $|z + i| = |z + 1|$, where z is a complex number, then the locus of z is
- (A) $y = 0$
 (B) $y = 1$
 (C) $y = x$
 (D) $y = \frac{x}{2}$
3. Given that $z + 3z^* = 12 + 8i$, then $z =$
- (A) $-3 - 4i$
 (B) $3 - 4i$
 (C) $3 + 4i$
 (D) $-3 + 4i$
4. One root of a quadratic equation with real coefficients is $2 - 3i$. The quadratic equation is
- (A) $x^2 + 4x + 13 = 0$
 (B) $x^2 - 4x - 13 = 0$
 (C) $x^2 + 4x - 13 = 0$
 (D) $x^2 - 4x + 13 = 0$
5. If $z = \cos \theta + i \sin \theta$, then $z^4 + \frac{1}{z^4} =$
- (A) $2 \cos 4\theta$
 (B) $2i \sin 4\theta$
 (C) $\cos 4\theta + i \sin 4\theta$
 (D) $4 \cos \theta - i(4 \sin \theta)$
6. The gradient of the normal to the curve with the equation $xy^3 + y^2 + 1 = 0$ at the point $(2, -1)$ is
- (A) -4
 (B) $-\frac{1}{4}$
 (C) $\frac{1}{4}$
 (D) 4
7. If $\frac{dy}{dx} = \frac{5x}{y}$, then $\frac{d^2y}{dx^2} =$
- (A) $\frac{5}{y^3} - \frac{25x^2}{y^2}$
 (B) $\frac{5}{y} - \frac{25x^2}{y^3}$
 (C) $\frac{1}{y} - \frac{25}{xy}$
 (D) $\frac{5}{xy} - \frac{25}{x^3 y^3}$
8. The curve C is given by the parametric equations $x = t + e^{-t}$, $y = 1 - e^{-t}$. The gradient function for C at the point (x, y) is given as
- (A) $\frac{1}{1 - e^t}$
 (B) $\frac{1}{e^t - 1}$
 (C) $\frac{-1}{1 + e^t}$
 (D) $\frac{1}{e^t + 1}$

9. Given $y = a \arccos(ax)$, where a is a constant, $\frac{dy}{dx} =$
- (A) $\frac{a^2}{\sqrt{(1 - a^2x^2)}}$
 (B) $-\frac{1}{\sqrt{(1 - a^2x^2)}}$
 (C) $-\frac{a^2}{\sqrt{(1 - a^2x^2)}}$
 (D) $\frac{1}{\sqrt{(1 - a^2x^2)}}$
10. Given that $f(x, y, z) = x^2y + y^2z - z^2x$ then $\frac{\partial f}{\partial y} =$
- (A) $x^2y + 2yz$
 (B) $x^2 + 2yz$
 (C) $x^2 + y^2$
 (D) $x^2 + y^2 + z^2$
11. $\int \frac{x^3}{(x^2 - 3x + 2)} dx$ may be expressed as
- (A) $\int \left(\frac{Px + Q}{x^2 - 3x + 2} \right) dx$
 (B) $\int \left(\frac{P}{x-1} + \frac{Q}{x-2} \right) dx$
 (C) $\int \left(x + 3 + \frac{Px}{x^2 - 3x + 2} \right) dx$
 (D) $\int \left(x + 3 + \frac{P}{x-1} + \frac{Q}{x-2} \right) dx$
12. $\int \frac{1}{\sqrt{(a^2 - x^2)}} dx =$
- (A) $a \sqrt{(a^2 - x^2)} + C$
 (B) $\arcsin(x^2) + C$
 (C) $\arcsin\left(\frac{x}{a}\right) + C$
 (D) $a \arcsin(ax) + C$
13. Given that $y = \frac{\pi}{2} - x$, then $\int_0^{\frac{\pi}{2}} \sin^2 x dx =$
- (A) $\int_0^{\frac{\pi}{2}} \cos^2 y dy$
 (B) $\int_0^{\frac{\pi}{2}} \sin^2 y dy$
 (C) $\int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx$
 (D) $\int_0^{\frac{\pi}{2}} (\sin y + \cos y) dy$

15. The value of $\int_0^{\pi/2} x \cos dx$
is

(A) 1
(B) $\frac{\pi}{2}$
(C) $\frac{\pi}{2} - 1$
(D) $\frac{\pi}{2} + 1$

16. Given that a sequence of positive integers $\{U_n\}$ is defined by $U_1 = 2$ and $U_{n+1} = 3U_n + 2$, then $U_n =$

(A) $3n - 1$
(B) $3^n + 1$
(C) $3^n - 1$
(D) $3n + 2$

17. The sequence $a_n = \frac{3n^2 - n + 4}{2n^2 + 1}$

(A) converges
(B) diverges
(C) is periodic
(D) is alternating

18. The n th term of a sequence is given by

$u_n = 9 - 4\left(\frac{1}{2}\right)^{n-1}$. The 5th term of the sequence is

(A) $\frac{9}{4}$
(B) $\frac{35}{4}$
(C) $\frac{37}{4}$
(D) $\frac{71}{8}$

19. $\sum_{r=1}^{m-1} 3\left(\frac{1}{2}\right)^r =$

(A) $3 - 3 \times 2^{-m}$
(B) $3 - 3 \times 2^{(1-m)}$
(C) $6 - 3 \times 2^{(m-1)}$
(D) $6 - 3 \times 2^{(1-m)}$

20. Given that $\sum_{r=1}^n u_n = 5n + 2n^2$, then $u_n =$

(A) $4n + 3$
(B) $5n + 2$
(C) $2n^2 + n - 3$
(D) $4n^2 + 4n + 7$

21. The sum to infinity, $S(x)$, of the series $1 + \left(\frac{2}{1+x}\right) + \left(\frac{2}{1+x}\right)^2 + \left(\frac{2}{1+x}\right)^3 +$ is

(A) 1
(B) $\frac{x+2}{x+1}$
(C) $\frac{x+1}{x-1}$
(D) $1 + \left(\frac{2}{x+1}\right)^n$

22. The Maclaurin's series expansion for $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots$ has a general term BEST defined as

(A) $(-1)^n \frac{x^{n+1}}{(n+1)!}$
(B) $(-1)^{n+1} \frac{x^{n+1}}{(n+1)!}$
(C) $(-1)^n \frac{x^{2n+1}}{(n+1)!}$
(D) $(-1)^{n+1} \frac{x^{2n-1}}{(2n-1)!}$

23. The first 2 non-zero terms of the expansion of $\sin(x + \pi/6)$ are

(A) $\frac{1}{2} + \frac{\sqrt{3}}{x}x$

(B) $\frac{1}{2} - \frac{\sqrt{3}}{2}x$

(C) $\frac{1}{2} + \frac{1}{2}x$

(D) $\frac{\sqrt{3}}{2} + \frac{1}{2}x$

24. If ${}^n C_r = {}^{n-1} C_{r-1}$, then

(A) $n = r$

(B) $n - r = 1$

(C) $r - n = 1$

(D) $n - 1 = r + 1$

25. If $\left(2x^2 - \frac{2}{x}\right)^6 = \dots + k + \dots$, where k is independent of x , then $k =$

(A) -960

(B) -480

(C) 480

(D) 960

26. An investment prospectus offers that for an initial deposit of \$5 000 at January 1st an interest rate of 3% will be applied at December 31st on the opening balance for the year. Assuming that no withdrawals are made for any year, the value of the investment n years after the initial deposit is given by

(A) $(5\ 000)(1.03)^n$

(B) $(5\ 000)(0.03)^n$

(C) $(5\ 000)(1.03)^{n-1}$

(D) $(5\ 000)(0.03)^{n-1}$

27. The coefficient of x^3 in the expansion of $(1 + x + x^2)^5$ is

(A) 5

(B) 20

(C) 30

(D) 40

28. The equation $\sin x^2 + 0.5x - 1 = 0$ has a real root in the interval

(A) (0.8, 0.9)

(B) (0.7, 0.8)

(C) (0.85, 0.9)

(D) (0.9, 0.10)

29. $f(x) = x^3 - \frac{7}{x} + 2$, $x > 0$. Given that $f(x)$ has a real root α in the interval (1.4, 1.5), using the interval bisection once α lies in the interval

(A) (1.45, 1.5)

(B) (1.4, 1.45)

(C) (1.425, 1.45)

(D) (1.4, 1.425)

30. $f(x) = x^3 - x^2 - 6$. Given that $f(x) = 0$ has a real root α in the interval [2.2, 2.3], applying linear interpolation once on this interval an approximation to α , correct to 3 decimal places, is

(A) 2.216

(B) 2.217

(C) 2.218

(D) 2.219

31. Taking 1.6 as a first approximation to α , where the equation $4 \cos x + e^{-x} = 0$ has a real root α in the interval (1.6, 1.7), using the Newton-Raphson method a second approximation to α (correct to 3 decimal places) \approx

(A) 1.602

(B) 1.620

(C) 1.622

(D) 1.635

32. $f(x) = 3x^3 - 2x - 6$. Given that $f(x) = 0$ has a real root, α , between $x = 1.4$ and $x = 1.45$, starting with $x_0 = 1.43$ and using the iteration $x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$, the value of x_1 correct to 4 decimal places is

- (A) 1.4369
 (B) 1.4370
 (C) 1.4371
 (D) 1.4372

33. Ten cards, each of a different colour, and consisting of a red card and a blue card, are to be arranged in a line. The number of different arrangements in which the red card is not next to the blue card is

- (A) $9! - 2 \times 2!$
 (B) $10! - 9! \times 2!$
 (C) $10! - 2! \times 2!$
 (D) $8! - 2! \times 2!$

34. The number of ways in which all 10 letters of the word STANISLAUS can be arranged if the Ss must all be together is

- (A) $\frac{8! \times 3!}{2!}$
 (B) $8! \times 3!$
 (C) $\frac{8!}{3!}$
 (D) $\frac{8!}{2!}$

35. A committee of 4 is to be chosen from 4 teachers and 4 students. The number of different committees that can be chosen if there must be at least 2 teachers is

- (A) 36
 (B) 45
 (C) 53
 (D) 192

36. A and B are two events such that $P(A) = p$ and $P(B) = \frac{1}{3}$. The probability that neither occurs is $\frac{1}{2}$. If A and B are mutually exclusive events then $p =$

- (A) $\frac{5}{6}$
 (B) $\frac{2}{3}$
 (C) $\frac{1}{5}$
 (D) $\frac{1}{6}$

37. On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$ respectively. The probability that on a randomly chosen day Bill travels by foot and is late is

- (A) $\frac{1}{30}$
 (B) $\frac{1}{10}$
 (C) $\frac{3}{10}$
 (D) $\frac{13}{30}$

38. Given $\begin{vmatrix} 6 & 0 & 1 \\ 7 & 7 & 0 \\ 0 & -12 & x \end{vmatrix} = 0$, the value of x is

- (A) -2
(B) 2
(C) 7
(D) 12

Items 39 – 40 refer to the matrix below.

$$A = \begin{pmatrix} 2 & -7 & 8 \\ 3 & -6 & -5 \\ 4 & 0 & -1 \end{pmatrix}$$

39. The transpose of matrix, A , results in $|A|$ being

- (A) 0
(B) squared
(C) negative
(D) unchanged

40. The matrix resulting from adding Row 1 to Row 2 is

$$(A) \begin{pmatrix} -1 & -1 & 13 \\ 3 & -6 & -5 \\ 4 & 0 & -1 \end{pmatrix}$$

$$(B) \begin{pmatrix} 2 & -7 & 8 \\ -1 & -1 & -13 \\ 4 & 0 & -1 \end{pmatrix}$$

$$(C) \begin{pmatrix} -5 & 7 & 8 \\ -3 & -6 & -5 \\ 4 & 0 & -1 \end{pmatrix}$$

$$(D) \begin{pmatrix} -5 & 5 & -8 \\ 3 & -3 & 5 \\ 4 & -0 & 1 \end{pmatrix}$$

41. Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & 0 & 2 \\ 0 & 6 & -4 \\ 2 & -4 & 2 \end{pmatrix}$, by considering AB , then $A^{-1} =$

- (A) $2B$
(B) B
(C) $\frac{1}{2}B$
(D) $\frac{1}{2}AB$

42. The general solution of the differential equation $\sin x \frac{dy}{dx} - y \cos x = \sin 2x \sin x$ is found by evaluating

$$(A) \int \frac{d}{dx} y \sin x \, dx = \int 2 \cos x \, dx$$

$$(B) \int \frac{d}{dx} \frac{y}{\sin x} \, dx = \int 2 \cos x \, dx$$

$$(C) \int \frac{d}{dx} \frac{y}{\sin x} \, dx = \int \sin 2x \, dx$$

$$(D) \int \frac{d}{dx} \frac{y}{\sin x} \, dx = \int \cos x \, dx$$

43. The general solution of the differential equation $\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = 3e^x$ is of the form

- (A) $y = Ae^x + Be^{2x} + ke^x$
(B) $y = Ae^x + Be^{2x} - 3e^x$
(C) $y = Ae^{-x} + Be^{-2x} + kxe^x$
(D) $y = Ae^x + Be^{2x} + kxe^x$

- 44.** A particular integral of the differential equation $\frac{d^2y}{dx^2} + 25y = 3 \cos 5x$ is of the form $y = \lambda x \sin 5x$. The general solution of the differential equation is
- (A) $y = A \cos 5x - B \sin 5x - \lambda x \sin 5x$
 (B) $y = A \cos 5x + B \sin 5x + \lambda x \sin 5x$
 (C) $y = A \cos 5x + B \sin 5x - \lambda x \sin 5x$
 (D) $y = A \cos 5x - B \sin 5x + \lambda x \sin 5x$
- 45.** The general solution of the differential equation $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 4y = 2x^2 + x - 1$ is
- (A) $y = e^{2x} (A + Bx) + ax^2 + bx + c$
 (B) $y = e^{-2x} (A + Bx) + ax^2 + bx + c$
 (C) $y = e^{2x} (A + Bx) + 2x^2 + x - 1$
 (D) $y = e^{2x} (A - Bx) + ax^2 + bx + c$

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST

TEST CODE **02234020/SPEC**
FORM TP 02234020/SPEC

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – Paper 02

COMPLEX NUMBERS, ANALYSIS AND MATRICES

SPECIMEN PAPER

2 hours 30 minutes

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.

The maximum mark for each Module is 50.

The maximum mark for this examination is 150.

This examination consists of 5 printed pages.

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **THREE** significant figures.

Examination Materials

Graph paper (provided)

Mathematical formulae and tables (provided) – **Revised 2010**

Mathematical instruments

Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

Copyright © 2011 Caribbean Examinations Council
All rights reserved

SECTION A (MODULE 1)

Answer BOTH questions.

1. (a) (i) Express the complex number $\frac{4-2i}{1-3i}$ in the form of $a + ib$ where a and b are real numbers. [4 marks]

- (ii) Show that the argument of the complex number in (a) (i) above is $\frac{\pi}{4}$. [1 mark]

- (b) (i) Find the complex number $u = x + iy$, $x, y \in \mathbb{I}$, such that $u^2 = -5 + 12i$. [8 marks]

- (ii) Hence, solve the equation that $z^2 + iz + (1 - 3i) = 0$. [6 marks]

- (c) Find the complex number that $z = a + ib$ such that

$$(1 + 3i)z + (4 - 2i)z^* = 10 + 4i \quad [6 \text{ marks}]$$

Total 25 marks

2. (a) Find $\int e^{3x} \sin 2x \, dx$. [7 marks]

- (b) (i) a) Find $\frac{dy}{dx}$ when $y = \tan^{-1}(3x)$. [4 marks]

- b) Hence, find $\int \frac{(x+2)}{1+9x^2} \, dx$. [4 marks]

- (ii) Show that if $y = \frac{\ln(5x)}{x^2}$ then $\frac{dy}{dx} = \frac{1 - \ln(25x^2)}{x^3}$ [5 marks]

- (c) Let $f(x, y) = x^2 + y^2 - 2xy$.

- (i) Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ [2 marks]

- (ii) Show that

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 2f(x, y) \quad [3 \text{ marks}]$$

Total 25 marks

SECTION B (MODULE 2)

Answer BOTH questions.

3. (a) (i) Find constants A and B such that

$$\frac{1}{(2r-1)(2r+1)} \equiv \frac{A}{2r-1} + \frac{B}{2r+1}. \quad [5 \text{ marks}]$$

- (ii) Hence, find the value of S where

$$S = \sum_{r=1}^n \frac{1}{(2r-1)(2r+1)}. \quad [5 \text{ marks}]$$

- (iii) Deduce the sum to infinity of S . [3 marks]

- (b) (i) Find the r^{th} term of the series

$$1(2) + 2(5) + 3(8) + \dots \quad [2 \text{ marks}]$$

- (ii) Prove, by Mathematical Induction, that the sum to n terms of the series in (b) (i) above is $n^2(n+1)$. [10 marks]

Total 25 marks

4. (a) Given the series

$$\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + \dots$$

- (i) show that the series is geometric [3 marks]
- (ii) find the sum of the series to n terms. [4 marks]

- (b) Use Maclaurin's Theorem to find the FIRST three non-zero terms in the power series expansion of $\cos 2x$. [7 marks]

- (c) (i) Expand $\sqrt{\left(\frac{1+x}{1-x}\right)}$ up to and including the term in x^3 stating the values of x for which the expansion is valid. [7 marks]

- (ii) By taking $x = 0.02$ find an approximation for $\sqrt{51}$, correct to 5 decimal places. [4 marks]

Total 25 marks

SECTION C (MODULE 3)

Answer BOTH questions.

5. (a) Two cards are drawn without replacement from ten cards which are numbered 1 to 10. Find the probability that

(i) the numbers on BOTH cards are even [4 marks]

(ii) the number on one card is odd and the number on the other card is even. [4marks]

- (b) A journalist reporting on criminal cases classified 150 criminal cases by the age (in years) of the criminal and by the type of crime committed, violent or non-violent. The information is presented in the table below.

Type of Crime	Age (in years)		
	Less than 20	20 to 39	40 or older
Violent	27	41	14
Non-violent	12	34	22

What is the probability that a case randomly selected by the journalist

(i) is a violent crime? [2 marks]

(ii) was committed by someone LESS than 40 years old? [4 marks]

(iii) is a violent crime OR was committed by a person LESS than 20 years old? [3 marks]

- (c) On a particular weekend, 100 customers made purchases at Green Thumb Garden supply store. Of these 100 customers;

30 purchased tools

45 purchased fertilizer

50 purchased seeds

15 purchased seeds and fertilizer

20 purchased seeds and tools

15 purchased tools and fertilizer

10 purchased tools, seeds and fertilizer.

(i) Represent the above information on a Venn diagram. [4 marks]

(ii) Determine how many customers purchased:

- a) only tools
b) seeds and tools but not fertilizer,
c) tools and fertilizer but not seeds,
d) neither seeds, tools, nor fertilizer. [4 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

6. (a) Solve for x the following equation

$$\begin{vmatrix} 5 & x & 3 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0$$

[8 marks]

- (b) Solve the first order differential equation

$$y \tan x \frac{dy}{dx} = (4 + y^2) \sec^2 x$$

[4 marks]

- (c) Given that $y = u \cos 3x + v \sin 3x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 3y = 30 \sin 3x,$$

find

- (i) the values of the constants u and v ,

[8 marks]

- (ii) the general solution of the differential equation.

[5 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

TEST CODE **02234032/SPEC**

FORM TP 02234032/SPEC

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – Paper 032

COMPLEX NUMBERS, ANALYSIS AND MATRICES

SPECIMEN PAPER

1 hour 30 minutes

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.

The maximum mark for each Module is 20.

The maximum mark for this examination is 60.

This examination consists of 4 printed pages.

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **THREE** significant figures.

Examination Materials Permitted

Graph paper (provided)

Mathematical formulae and tables (provided) – **Revised 2010**

Mathematical instruments

Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

Copyright © 2011 Caribbean Examinations Council
All rights reserved

SECTION A (MODULE 1)

Answer this question.

1. (a) (i) Given that $z = 8 + (8\sqrt{3})i$, find the modulus and argument of z . [3 marks]
(ii) Using de Moivre's theorem, show that z^3 is real, stating the value of z^3 . [2 marks]
- (b) A complex number is represented by the point, P , in the Argand diagram.
(i) Given that $|z - 6| = |z|$ show that the locus of P is $x = 3$. [2 marks]
(ii) Determine the TWO complex numbers which satisfy both
 $|z - 6| = |z|$ and $|z - 3 - 4i| = 5$. [5 marks]
- (c) Given $I_n = \int_0^8 x^n (8-x)^{\frac{1}{x}} dx$, $n \geq 0$, show that
 $I_n = \frac{24n}{3n+4} I_{n-1}$, $n \geq 1$. [8 marks]

Total 20 marks

SECTION B (MODULE 2)

Answer this question.

2. (a) (i) Show that $(r+1)^3 - (r-1)^3 = 6r^2 + 2$. **[2 marks]**

- (ii) Hence, show that $\sum_{r=1}^n r^2 = \frac{n}{6}(n+1)(2n+1)$. **[5 marks]**

- (iii) Show that $\sum_{r=n}^{2n} r^2 = \frac{n}{6}(n+1)(an+b)$, where a and b are unknown constants. **[4 marks]**

- (b) The displacement, x metres, of a particle at time, t , seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + x + \cos x = 0.$$

When $t = 0$, $x = 0$ and $\frac{dx}{dt} = 0.5$.

Find a Taylor series solution for x in ascending powers of t , up to and including the term in t^3 . **[5 marks]**

- (c) Given that α is the only real root of the equation

$$x^3 - x^2 - 6 = 0,$$

- (i) Show that $2.2 < \alpha < 2.3$. **[2 marks]**

- (ii) Use linear interpolation once on the interval $[2.2, 2.3]$ to find another approximation to α , giving your answer to 3 decimal places. **[2 marks]**

Total 20 marks

SECTION C (MODULE 3)

Answer this question.

3. (a) Three identical cans of cola, two identical cans of tea and two identical cans of orange juice are arranged in a row.

Calculate the number of arrangements if the first and last cans in the row are of the same type of drink. **[3 marks]**

- (b) Kris takes her dog for a walk every day. The probability that they go to the park on any day is 0.6. If they go to the park, there is a probability of 0.35 that the dog will bark. If they do not go to the park, there is probability of 0.75 that the dog will bark.

Find the probability that the dog barks on any particular day. **[2 marks]**

- (c) A committee of six people, which must consist of at least 4 men and at least one woman, is to be chosen from 10 men and 9 women, including Albert and Tracey.

Find the number of possible committees that include either Albert or Tracey but not both. **[3 marks]**

- (d) A and B are two matrices such that

$$A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ 0 & 1 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} -3 & 4 & -7 \\ -2 & 1 & 2 \\ 2 & -1 & 3 \end{pmatrix}$$

(i) Find AB . **[2 marks]**

(ii) Deduce A^{-1} . **[2 marks]**

(iii) Given that $B^{-1} = \frac{1}{5} \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ 0 & 1 & 1 \end{pmatrix}$, prove that $(AB)^{-1} = B^{-1}A^{-1}$. **[2 marks]**

- (e) Find the general solution of the differential equation

$$\frac{dy}{dx} + y \cot x = \sin x. \quad \text{[6 marks]}$$

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1, PAPER 01

MARK SCHEME

Key

Unit 1 Paper 01

Module	Item	Key	Module	Item	Key
1	1	D	3	31	C
	2	A		32	D
	3	C		33	B
	4	B		34	B
	5	D		35	A
	6	D		36	C
	7	C		37	B
	8	D		38	D
	9	C		39	A
	10	A		40	A
	11	A		41	B
	12	B		42	A
	13	D		43	C
	14	D		44	A
	15	C		45	A
2	16	C			
	17	B			
	18	D			
	19	C			
	20	C			
	21	C			
	22	D			
	23	A			
	24	D			
	25	B			
	26	A			
	27	B			
	28	B			
	29	B			
	30	C			

02134020/CAPE/MS/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1

ALGEBRA, GEOMETRY AND CALCULUS

SPECIMEN PAPER

PAPER 02

SOLUTIONS AND MARK SCHEMES

SECTION A

(MODULE 1)

Question 1

(a)

		1 mark	1 mark	1 mark
p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

T = true F = false

[1 may be used for T and 0 for F]

[3 marks]

(ii) $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent since columns 4 and 5 are identical.

[2 marks]

$$(iii) \quad p \wedge (p \rightarrow q) \equiv p \wedge (\sim p \vee q) \quad (1 \text{ mark})$$

$= (\mathbf{p} \wedge \neg\mathbf{p}) \vee (\mathbf{p} \wedge \mathbf{p}) \dots$ distribute \wedge over \vee

$$= f \vee (p \wedge \neg q) \quad (1 \text{ mark})$$

$$= (\mathbf{p} \wedge \mathbf{q}) \quad (1 \text{ mark})$$

[3 marks]

$$(b) \quad (i) \quad x * y = x + y - 1, \forall x, y \text{ in } \mathbb{R}$$

$$x, y \in \mathbb{R} \Rightarrow x + y \in \mathbb{R} \text{ (sum of 2 real numbers)} \quad (1 \text{ mark})$$

$\Rightarrow x + y - 1 \in \mathbb{R}$ (difference of 2 real numbers)

$\Rightarrow x * y \in \mathbb{R}$ (1 mark)

$\Rightarrow *$ is closed in real numbers

(1 mark)

[3 marks]

$$(ii) \quad x * y = x + y - 1 = y + x - 1 \text{ (addition is commutative)}$$

(1 mark)

$\Rightarrow y * x$ (1 mark)

$\Rightarrow *$ is commutative in \mathbb{R}

[2 marks]

(b) (iii) $(x * y) * z = (x + y - 1) * z$ for $x, y, z \in \mathbb{R}$

$$\begin{aligned} &= (x + y - 1) + z - 1 \\ &= x + y + z - 1 - 1 \\ &= x + y + z - 2 \end{aligned}$$

[2 marks]

$$\begin{aligned} x * (y * z) &= x * (y + z - 1) \\ &= x + (y + z - 1) - 1 \\ &= x + y + z - 2 \end{aligned}$$

[1 mark]

$$\begin{aligned} \Rightarrow (x * y) * z &= x * (y * z) \text{ for all } x, y, z \in \mathbb{R} \\ \Rightarrow * \text{ is associative in } \mathbb{R} \end{aligned}$$

[1 mark]

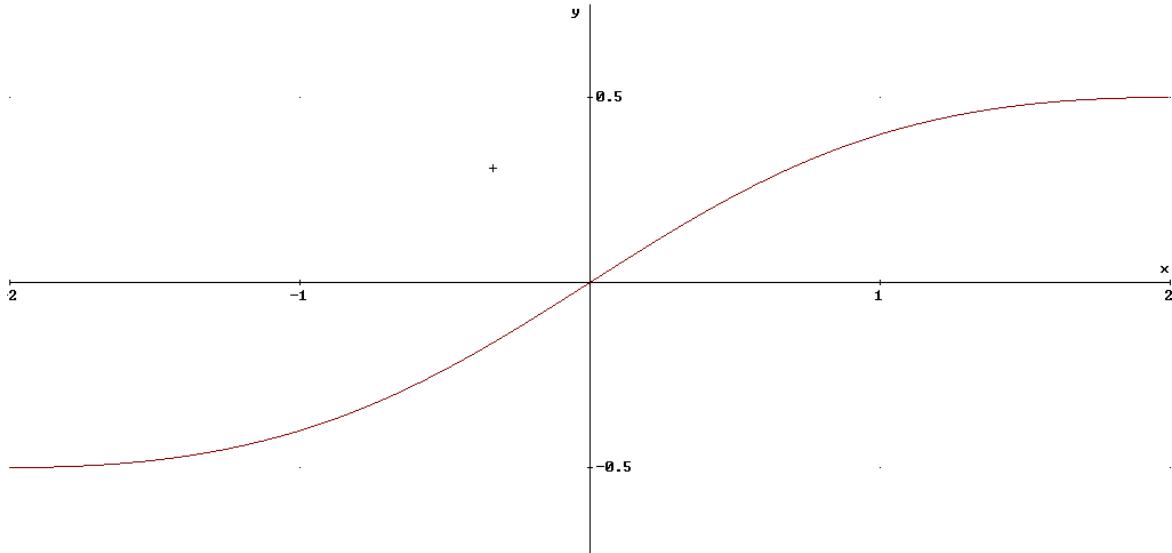
(c) (i) $y = \frac{2x}{x^2+4} \Rightarrow y(x^2 + 4) = 2x$ (1 mark)

$$\Rightarrow yx^2 - 2x + 4y = 0$$

$$\begin{aligned} \text{For } x \text{ real, } (-2)^2 - 4y(4y) &\geq 0 & (1 \text{ mark}) \\ \Rightarrow 4y^2 - 1 &\leq 0 & (1 \text{ mark}) \\ \Rightarrow (2y - 1)(2y + 1) &\leq 0 & (1 \text{ mark}) \\ \Rightarrow -\frac{1}{2} &\leq y \leq \frac{1}{2} & (1 \text{ mark}) \end{aligned}$$

[5 marks]

(ii) $|x| < 2 \Rightarrow -2 \leq x \leq 2$



[3 marks]

Total 25 marks

Question 2

- (a) Let $f(x) = 2x^3 + px^2 + qx + 2$
- (i) $f(-1) = 0 \Rightarrow -2 + p - q + 2 = 0 \Rightarrow p = q$ (1 mark)
- $$f\left(\frac{1}{2}\right) = 0 \Rightarrow \frac{1}{4} + \frac{p}{4} + \frac{q}{2} + 2 = 0 \Rightarrow p + 2q = -9 \quad (2 \text{ mark})$$
- $$\Rightarrow p = q = -3 \quad (1 \text{ mark})$$
- (ii) $f(x) = (2x-1)(x+1)(x-k)$ (1 mark)
- $$\equiv 2x^3 + px^2 + qx + 2 \quad (1 \text{ mark})$$
- $$\Rightarrow k = 2 \quad (1 \text{ mark})$$
- $$\Rightarrow \text{the remaining root is } 2 \quad (1 \text{ mark})$$

Alternatively

- (a) Let d be the third root of $f(x) = 0$
- (i) Then $-1 \times \frac{1}{2} \times d = -\frac{2}{2} \Rightarrow \frac{d}{2} = 1 \Rightarrow d = 2$ [3 marks]
- $$d = 2 \Rightarrow -1 + \frac{1}{2} + 2 = -\frac{p}{2} \Rightarrow p = -3$$
- $$\text{And } -\frac{1}{2} - 2 + 1 = \frac{q}{2} \Rightarrow q = -3 \quad [4 \text{ marks}]$$
- (b) Let P_n be the statement $\sum_{r=1}^n (6r + 5) = n(3n + 8)$
- For $n = 1$, L.H.S. of P_1 , is $6 + 5 = 11$ and R.H.S. of P_1
 $= 1(3 + 8)$
 $= 11 \quad (1 \text{ mark})$
- So P_n is true for $n = 1 \quad (1 \text{ mark})$

Assume that P_n is true for $n = k$, i.e.

$$11 + 17 + \dots + (6k + 5) = k(3k + 8) \quad (1 \text{ mark})$$

Then, we need to prove P_n is true for $n = k + 1$

$$\begin{aligned} \text{Now } \sum_{r=1}^{k+1} (6r+5) &= 11 + 17 + \dots + (6k + 5) + [6(k + 1) + 5] && (1 \text{ mark}) \\ &= k(3k + 8) + [6(k + 1) + 5] && (1 \text{ mark}) \\ &= 3k^2 + 8k + 6k + 6 + 5 \\ &= 3k^2 + 14k + 11 && (1 \text{ mark}) \\ &= (3k + 11)(k + 1) && (1 \text{ mark}) \\ &= (k + 1)[3(k + 1) + 8] && (1 \text{ mark}) \end{aligned}$$

Thus, if P_n is true when $n = k$, it is also true with $n = (k + 1)$; (1 mark)

$$\text{i.e. } \sum_{r=1}^n (6r + 5) = n(3n + 8) \quad \forall n \in \mathbb{N}. \quad (1 \text{ mark})$$

[10 marks]

$$\begin{aligned} (c) \quad e^{2x} + 2e^{-2x} = 3 &\Rightarrow e^{2x} + \frac{2}{e^{2x}} = 3 && (1 \text{ mark}) \\ &\Rightarrow (e^{2x})^2 - 3e^{2x} + 2 = 0 && (1 \text{ mark}) \\ &\Rightarrow (e^{2x} - 2)(e^{2x} - 1) = 0 && (2 \text{ mark}) \\ &\Rightarrow e^{2x} = 2 \text{ or } e^{2x} = 1 && (2 \text{ mark}) \\ &\Rightarrow 2x = \ln 2 \text{ or } 2x = 0 && \\ &\Rightarrow x = \frac{1}{2} \ln 2 \text{ or } x = 0 && (2 \text{ mark}) \end{aligned}$$

[8 marks]

Alternatively Let $y = e^{2x}$, giving $y^2 - 3y + 2 = 0$ etc.

Total 25 marks

SECTION B
(MODULE 2)

Question 3

$$\begin{aligned}
 \text{(a) (i)} \quad \text{LHS} &\equiv \frac{\sin 3\theta + \sin \theta}{\cos 3\theta + \cos \theta} && (\text{1 mark}) \\
 &\equiv \frac{2 \sin\left(\frac{3\theta + \theta}{2}\right) \cos\left(\frac{(3\theta - \theta)}{2}\right)}{2 \cos\left(\frac{3\theta + \theta}{2}\right) \cos\left(\frac{3\theta - \theta}{2}\right)} && (\text{1 mark}) \\
 &\equiv \frac{\sin 2\theta}{\cos 2\theta} && (\text{1 mark}) \\
 &\equiv \tan 2\theta && (\text{1 mark}) \\
 &\equiv \text{RHS} && \\
 &&& [\text{4 marks}]
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad 2 \sin\left(\frac{3\theta + \theta}{2}\right) \cos\left(\frac{3\theta - \theta}{2}\right) + \sin 2\theta &= 0 && (\text{1 mark}) \\
 \Rightarrow 2 \sin 2\theta \cos \theta + \sin 2\theta &= 0 && (\text{1 mark}) \\
 \Rightarrow \sin 2\theta(2 \cos \theta + 1) &= 0 && (\text{1 mark}) \\
 \Rightarrow \sin 2\theta = 0, \text{ that is } \theta &= 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \text{ or} && (\text{3 marks}) \\
 \cos \theta = \frac{-1}{2}, \text{ that is } \theta &= \frac{2\pi}{3}, \frac{4\pi}{3} && (\text{1 mark})
 \end{aligned}$$

[7 marks]

$$\begin{aligned}
 \text{(b) (i)} \quad r &= \sqrt{(6^2 + 8^2)} = 10, \quad x = \tan^{-1}\left(\frac{3}{4}\right) = 36.9^\circ && (\text{2 marks}) \\
 \therefore f(\theta) &= 10 \cos(\theta - 36.9^\circ) && (\text{1 mark})
 \end{aligned}$$

[3 marks]

$$\text{(ii)} \quad g(\theta) = \frac{10}{10 + 10 \cos(\theta - 36.9^\circ)} && (\text{1 mark})$$

Minimum value of $g(\theta)$ occurs when denominator has maximum value i.e.

$$\cos(\theta - 36.9) = 1. && (\text{1 mark})$$

$$\text{Min } g(\theta) = \frac{10}{10 + 10} = \frac{1}{2} \text{ occurs}$$

(when $\theta - 36.9 = 0$), when $\theta = 36.9$. && (2 marks)

[4 marks]

- (c) (i) $\overrightarrow{BC} = 2\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{BA} = 2\mathbf{i} - 2\mathbf{k}$ [2 marks]
(ii) $\mathbf{n} \cdot \overrightarrow{BC} = (\mathbf{i} + \mathbf{j} + \mathbf{k}) \cdot (2\mathbf{j} - 2\mathbf{k}) = 0 + 2 - 2 = 0$ (1 mark)
 $\mathbf{n} \cdot \overrightarrow{BA} = (\mathbf{i} + \mathbf{j} + \mathbf{k}) \cdot (2\mathbf{i} - 2\mathbf{k}) = 2 + 0 - 2 = 0$ (1 mark)

So \mathbf{n} is perpendicular to the plane through A, B and C.

[2 marks]

- (iii) Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ with $\mathbf{r} \cdot \mathbf{n} = d$ (1 mark)

represent the plane through A, B and C.

At the point A, $\mathbf{r} = 2\mathbf{i}$ so $\mathbf{r} \cdot \mathbf{n} = d$ (1 mark)

$$\Rightarrow (2\mathbf{i}) \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k}) = d \Rightarrow d = 2$$

Hence the Cartesian equation of the plane is $(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k}) = 2$ (1 mark)

$$\Rightarrow x + y + z = 2$$

[3 marks]

[Total 25 marks]

Question 4

- (a) $L: x + 2y = 7$ is a tangent to $x^2 + y^2 - 4x = 0$ if $x + 2y = 7$ if L touches the circle
 at 2 coincident points (1 mark)
 Now, $x = 7 - 2y$ (1 mark)
- $$\Rightarrow (7 - 2y)^2 - 4(7 - 2y) + y^2 - 1 = 0 \quad (1 \text{ mark})$$
- $$\Rightarrow y^2 - 4y + 4 = 0 \quad (1 \text{ mark})$$
- $$\Rightarrow (y - 2)^2 = 0 \quad (1 \text{ mark})$$
- $$\Rightarrow y = 2(\text{twice}) \quad (1 \text{ mark})$$
- $$\Rightarrow \text{when } y = 2, x = 3 \quad (1 \text{ mark})$$
- So L touches the circle at (3,2) (1 mark)
- [8 marks]

- (b) (i) Let $Q \equiv$ point diametrically opposite to (3, 2).
 The centre of C is (2,0)
 so $\frac{3+x}{2} = 2, x = 1$ (1 mark)
 and, $\frac{2+y}{2} = 0, y = -2$ (1 mark)
 $\therefore Q = (1, -2)$
 Tangent M at $Q: y + 2 = \frac{-1}{2}(x - 1)$ (2 marks)
 $2y + x + 3 = 0$ (1 mark)
- [5 marks]

- (ii) The equation of the diameter is $x + 2y = 2 + 0 = 2$ (1 mark)
 This meets C where
 $(2 - 2y)^2 + y^2 - 4(2 - 2y) - 1 = 0$ (1 mark)
 $\Rightarrow 4 - y + 4y^2 + y^2 - 8 + y - 1 = 0$ (1 mark)
 $\Rightarrow 5y^2 - 5 = 0$ (1 mark)
 $\Rightarrow y = \pm 1$ [4 marks]
- (iii) Coordinates of points of intersection are (1, 0), (-1, 4) [2 marks]

$$\begin{aligned}
 (c) \quad & x(1+t) = t & y(1+t) = t^2 \\
 \Rightarrow & \frac{y(1+t)}{x(1+t)} = \frac{t^2}{t} & (1 \text{ mark}) \\
 \Rightarrow & \frac{y}{x} = t & (1 \text{ mark}) \\
 \therefore & x = \frac{y}{1 + \frac{y}{x}} & (1 \text{ mark}) \\
 \Rightarrow & x = \frac{y}{x+y} & (1 \text{ mark}) \\
 \Rightarrow & y = \frac{x^2}{1-x} & (2 \text{ marks})
 \end{aligned}$$

[6 marks]

Total 25 marks

SECTION C
(MODULE 3)

Question 5

$$\begin{aligned}
 \text{(a)} \quad \lim_{h \rightarrow 0} \frac{h}{\sqrt{x+h} - \sqrt{x}} &= \lim_{h \rightarrow 0} \frac{h(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})} && (1 \text{ mark}) \\
 &= \lim_{h \rightarrow 0} \frac{h(\sqrt{x+h} + \sqrt{x})}{(x+h) - x} && (1 \text{ mark}) \\
 &= \lim_{h \rightarrow 0} \frac{h(\sqrt{x+h} + \sqrt{x})}{h} && (1 \text{ mark}) \\
 &= \lim_{h \rightarrow 0} (\sqrt{x+h} + \sqrt{x}) && (1 \text{ mark}) \\
 &= \sqrt{x} + \sqrt{x} && (1 \text{ mark}) \\
 &= 2\sqrt{x} && (1 \text{ mark})
 \end{aligned}$$

[6 marks]

$$\begin{aligned}
 \text{(b)} \quad f''(x) &= 18x + 4 \\
 \Rightarrow f'(x) &= 9x^2 + 4x + c && (1 \text{ mark}) \\
 \Rightarrow f(x) &= 3x^3 + 2x^2 + cx + d && (2 \text{ marks}) \\
 \text{Now } f(2) = 14 \Rightarrow 32 + 2c + d &= 14 \\
 \Rightarrow 2c + d &= -18 \dots \text{(i)} && (1 \text{ mark}) \\
 f(3) = 74 \Rightarrow 99 + 3c + d &= 74 \\
 \Rightarrow 3c + d &= -25 \dots \text{(ii)} && (1 \text{ mark})
 \end{aligned}$$

$$\begin{aligned}
 \text{From (i) + (ii),} \quad c &= -7 && (1 \text{ mark}) \\
 d &= -4 && (1 \text{ mark}) \\
 \text{So } f(4) &= 3(4^3) + 2(4^2) - 7(4) - 4 \\
 &= 192 + 32 - 32 && (1 \text{ mark}) \\
 &= 192
 \end{aligned}$$

[8 marks]

$$\begin{aligned}
 \text{(c) (i)} \quad y &= \frac{x}{1+x^2} \\
 \frac{dy}{dx} &= \frac{(1+x^2)1-x(2x)}{(1+x^2)^2} && (2 \text{ marks}) \\
 &= \frac{1-x^2}{(1+x^2)^2} && (1 \text{ mark}) \\
 &= \frac{1}{(1+x^2)^2} - \frac{x^2}{(1+x^2)^2} \\
 &= \frac{1}{(1+x^2)^2} - \left(\frac{x}{1+x^2}\right)^2 && (1 \text{ mark}) \\
 &= \frac{1}{(1+x^2)^2} - y^2 && (1 \text{ mark})
 \end{aligned}$$

[5 marks]

$$\begin{aligned}
 \text{(ii)} \quad \frac{d^2y}{dx^2} &= \frac{(1+x^2)^2(-2x)-(1-x^2)2(1+x^2)(2x)}{(1+x^2)^4} && (3 \text{ marks}) \\
 &= \frac{(1+x^2)(2x)[-(1+x^2)-2+2x^2]}{(1+x^2)^4} && (2 \text{ marks}) \\
 &= \frac{2y(x^2-3)}{(1+x^2)^2} && (1 \text{ mark})
 \end{aligned}$$

[6 marks]

Total 25 marks

Question 6

(a) (i) Finding the equation of the tangent PQ.

$$\text{a) } \frac{dy}{dx} = 3x^2 \quad (1 \text{ mark})$$

$$\left(\frac{dy}{dx} \right)_{x=3} = 3(3)^2 = 27 \quad (1 \text{ mark})$$

Equation of tangent:

$$y - 27 = 27(x - 3) \quad (1 \text{ mark})$$

$$y = 27x - 54 \quad (1 \text{ mark})$$

b) Q has coordinates (2, 0) (1 mark)

[5 marks]

$$\text{(ii) a) Area} = \int_0^3 y \, dx - \frac{1}{2}(3-2)(27) \quad (2 \text{ marks})$$

$$= \frac{1}{4} x^4 \Big|_0^3 - \frac{27}{2} \quad (2 \text{ marks})$$

$$= \frac{81}{4} - \frac{27}{2} \quad (1 \text{ mark})$$

$$= \frac{27}{4} \text{ units}^2 \quad (1 \text{ mark})$$

[5 marks]

(b) Required Volume

$$= \int_0^3 \pi y^2 \, dx - \text{Volume of the cone with radius 27 units and height 1 unit.} \quad (1 \text{ mark})$$

$$= \pi \int_0^3 x^6 \, dx - \frac{1}{3} \pi (27)^2 \quad (1 \text{ mark})$$

$$= \pi \frac{x^7}{7} \Big|_0^3 - \frac{1}{3} \pi (3^6) \quad (1 \text{ mark})$$

$$= \pi \left(\frac{3^7}{7} \right) - \frac{1}{3} \pi (3^6) \quad (1 \text{ mark})$$

$$= \frac{\pi}{7} (3^7 - 7(3^5)) \quad (1 \text{ mark})$$

$$= \frac{\pi}{7} (2 \times 3^5) \text{ units}^3 \quad (1 \text{ mark})$$

[5 marks]

(b) (i) $\frac{dy}{dx} = 3x^2 - 8x + 5$

 $y = x^3 - 4x^2 + 5x + C$ (1 mark)

substituting; $y = 3$ at $x = 0$

 $C = 3$ (1 mark)
 $y = x^3 - 4x^2 + 5x + 3$ (1 mark)

[3 marks]

(ii) $\frac{dy}{dx} = 0 \Rightarrow 3x^3 - 8x + 5 = 0$ (1 mark)

 $(3x - 5)(x - 1) = 0$ (1 mark)
 $x = \frac{5}{3}, 1$ (1 mark)
 $y = \frac{131}{27}, 5$ (1 mark)

co-ordinates are $\left(\frac{5}{3}, \frac{131}{27}\right), (1, 5)$

 $\frac{d^2y}{dx^2} = 6x - 8$ (1 mark)
 $\left(\frac{d^2y}{dx^2}\right)_{x=\frac{5}{3}} > 0 \Rightarrow \left(\frac{5}{3}, \frac{131}{27}\right)_{\text{max}}$ (1 mark)
 $\left(\frac{d^2y}{dx^2}\right)_{x=1} < 0 \Rightarrow (1, 5)_{\text{min}}$ (1 mark)

[7 marks]

Total 25 marks

02134032/CAPE/MS/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1

ALGEBRA, GEOMETRY AND CALCULUS

SPECIMEN PAPER

PAPER 032

SOLUTIONS AND MARK SCHEMES

Question	Details	Marks
1 (a) (i)	The converse of $p \rightarrow q$ is $q \rightarrow p$	1
(ii)	The inverse of $p \rightarrow q$ is $\sim p \rightarrow \sim q$	1
	The contrapositive of $\sim p \rightarrow \sim q$ is $\sim \sim q \rightarrow \sim \sim p = q \rightarrow p$	1
(b) (i)	$\begin{aligned} f(k+1) &= 2^{k+1} + 6^{k+1} \\ &= 2(2^k) + 6(6^k) \\ &= 6(2^k + 6^k) - 4(2^k) \\ &= 6f(k) - 4(2^k) \end{aligned}$	1 1 1
(ii)	<p>Assume $f(k)$ is divisible by 8 $f(1) = 2 + 6 = 8$ (true)</p> $\begin{aligned} f(k+1) &= 6f(k) - 2 \times 2(2^k) = 6f(k) - 2(2^{k+1}) \\ &= 6f(k) - 8\left(\frac{1}{4}2^{k+1}\right) \text{ which is divisible by 8} \end{aligned}$ <p>Since $f(k)$ is true for $k = 1$ and true for $f(k+1)$ then true for all $n \in \mathbb{N}$</p>	1 1 1 1
(c) (i)	$\frac{x+2}{ x-2 }$ <p>line , positive gradient, intercepts $(-2, 0), (0, 2)$ curve.....branch > 2 curve.....branch < 2 intercept $(0, \frac{1}{2})$</p>	1 1 1 1
(ii)	$x+2 = \frac{1}{x-2} \quad x^2 = 5 \quad x = \sqrt{5} \quad (-\sqrt{5} \text{ not applicable})$ $x+2 = \frac{1}{2-x} \quad x^2 = 3 \quad x = \sqrt{3} \quad (-\sqrt{3} \text{ not applicable})$ $\sqrt{3} < x < \sqrt{5}$ <p>(1) (1)</p>	2 2 1 1

[20]

S. O. (A) 3, (B) 5, (C) 5, (F) 3

Question	Details	Marks
2 (a) (i)	Dividing throughout by $\sin^2 \theta$ gives $\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$ $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \quad \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$	1
(ii)	$\operatorname{cosec}^4 \theta - \cot^4 \theta = (\operatorname{cosec}^2 \theta - \cot^2 \theta)(\operatorname{cosec}^2 \theta + \cot^2 \theta)$ $= \operatorname{cosec}^2 \theta + \cot^2 \theta$	1 1
(b)	$\frac{1}{x} = \operatorname{cosec}^2 \theta \quad \frac{2}{y} = \cot \theta$ (1) (1)	1 1
	From (a) (i) $\frac{1}{x} - \frac{4}{y^2} = 1$ $y^2 - 4x = xy^2$	1 1
(c) (i)	Comparing components of \mathbf{j} : $3 + 2\lambda = 9 \quad \lambda = 3$ Comparing components of \mathbf{i} : $2 + 3 = 5\mu \quad \mu = 1$ $C(5, 9, -1)$	1 1 1
(ii)	$\cos \theta = \frac{(\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \cdot (5\mathbf{i} + 2\mathbf{k})}{\sqrt{6} \times \sqrt{29}}$ (1 mark each for numerator and denominator) $\theta = \cos^{-1} \left(\frac{7}{\sqrt{174}} \right)$ $= 57.95^\circ$	1 1 1 1
(iii)	$(4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}) \cdot (5\mathbf{i} + 2\mathbf{k}) = 0$ $(4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}) \cdot (\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = 0$	1 1
(iv)	$\mathbf{r} \cdot (4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}) = (2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) \cdot (4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k})$ (1) (1) $\mathbf{r} \cdot (4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}) = 57$	1 1 1

Question	Details	Marks
S. O. (A) 5 (ii), 6 (B) 5, (C) 7, 9, 10		
3 (a)	$\lim_{n \rightarrow \infty} \frac{s_n}{s_{n+1}} = \lim_{n \rightarrow \infty} \frac{(n+1)!-1}{(n+2)!-1}$ $= \frac{\lim_{n \rightarrow \infty} (n+1)!-1}{\lim_{n \rightarrow \infty} (n+2)!-1} = \frac{\lim_{n \rightarrow \infty} (n+1)!-1}{\lim_{n \rightarrow \infty} (n+2)(n+1)!-1} \quad (1 \text{ mark each for limit})$ $= \lim_{n \rightarrow \infty} \frac{1}{n+2}$ $= 0$	1 1 1
(b)	$\frac{d}{dx} \cos x = \lim_{\delta x \rightarrow 0} \frac{\cos(x + \delta x) - \cos x}{\delta x}$ $= \lim_{\delta x \rightarrow 0} \frac{-2 \sin\left(\frac{x + \delta x + x}{2}\right) \sin\left(\frac{x + \delta x - x}{2}\right)}{\delta x}$ $= \lim_{\delta x \rightarrow 0} \frac{-2 \sin\left(x + \frac{\delta x}{2}\right) \sin\left(\frac{\delta x}{2}\right)}{\delta x}$ <p>Given $\lim_{\delta x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \frac{1}{2} \lim_{\delta x \rightarrow 0} \frac{\sin \frac{\delta x}{2}}{\left(\frac{\delta x}{2}\right)} = \frac{1}{2}$</p> $\lim_{\delta x \rightarrow 0} \frac{-2 \sin\left(x + \frac{\delta x}{2}\right)}{2} = -\sin x$	1 1 1 1 1
(c) (i)	$\frac{dr}{dt} = \frac{k}{r}$	1
(ii)	$\int r \, dr = \int k \, dt$ $\frac{1}{2} r^2 = kt + A$ $(1) \quad (1)$ $r = 0 \quad t = 0 \text{ gives } A = 0$ $r = 5 \quad t = 1 \text{ gives } k = \frac{25}{2}$ $r^2 = 25t \quad r = 5\sqrt{t}$	1 1 1 1 1

Question	Details	Marks
3 (c) (iii)	$12 = 5\sqrt{t} \quad t = \left(\frac{12}{5}\right)^2 = 5.76 \text{h} = 5 \text{ hrs } 46 \text{ mins}$ (1) (1) Time when radius is 12 metres is 6:46 p. m	1 1 1 20
	S. O. (A) 3, 4, 5, (B) 2 (iv), (C) 9 (i) (ii)	

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2, PAPER 01

MARK SCHEME

Key

Unit 2 Paper 01

Module	Item	Key	S.O.	Module	Item	Key	S.O.
1	1	A	A2	3	31	B	D5
	2	C	A11		32	C	D6
	3	B	A5		33	D	A2
	4	D	A3		34	D	A3
	5	A	A12		35	C	A4
	6	A	B4		36	D	A14
	7	B	B9		37	A	A16
	8	B	B3		38	B	B2
	9	C	B8		39	D	B1
	10	B	B6		40	A	B3
	11	D	C3		41	C	B7
	12	C	C9		42	B	C1
	13	A	C7		43	D	C3(ii)(i)
	14	D	C10		44	B	C3(iii)(iii)
	15	C	C8		45	A	C3(i)(ii)
2	16	C	A1				
	17	A	A3				
	18	B	A2				
	19	D	B4				
	20	A	B3				
	21	C	B6				
	22	D	B8				
	23	A	B9				
	24	A	C1				
	25	D	C3				
	26	C	C4				
	27	C	C3				
	28	A	D1				
	29	B	D2				
	30	C	D3				

02234020/CAPE/MS/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

SPECIMEN PAPER

UNIT 2

COMPLEX NUMBERS, ANALYSIS AND MATRICES

PAPER 02

SOLUTIONS AND MARK SCHEMES

SECTION A**(MODULE 1)****Question 1**

$$\begin{aligned}
 \text{(a) (i)} \quad & \frac{4-2i}{1-3i} = \frac{(4-2i)(1+3i)}{(1-3i)(1+3i)} \\
 &= \frac{4+12i-2i-6i^2}{1+9} \quad (1 \text{ mark}) \\
 &= \frac{4+10i+6}{10} \quad (1 \text{ mark}) \\
 &= \frac{10+10i}{10} \quad (1 \text{ mark}) \\
 &= 1+i \quad (1 \text{ mark}) \\
 &\qquad\qquad\qquad [4 \text{ marks}]
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \arg \text{ is } \tan^{-1}(1) = \frac{\pi}{4} \quad (1 \text{ mark}) \\
 &\qquad\qquad\qquad [1 \text{ mark}]
 \end{aligned}$$

(b) (i) Let $u = x + iy$, where x, y are real nos.

$$\begin{aligned}
 u^2 = -5 + 12i \Rightarrow (x+iy)^2 = -5 + 12i \\
 \Rightarrow x^2 - y^2 + 2ixy = -5 + 12i \quad (1 \text{ mark}) \\
 \Rightarrow x^2 - y^2 = -5, \quad 2xy = 12 \quad (1 \text{ mark}) \\
 \Rightarrow x^2 - \left(\frac{6}{x}\right)^2 = -5, \quad y = \frac{6}{x} \\
 \Rightarrow x^2 - \frac{36}{x^2} = -5 \quad (1 \text{ mark}) \\
 \Rightarrow (x^2)^2 + 5x^2 - 36 = 0 \quad (1 \text{ mark}) \\
 \Rightarrow (x^2 + 9)(x^2 - 4) = 0 \quad (1 \text{ mark}) \\
 \Rightarrow x^2 = -9 (\text{inadmissible}), x^2 = 4 \quad (1 \text{ mark}) \\
 \Rightarrow x = \pm 2, y = \mp 3 \quad (1 \text{ mark}) \\
 \Rightarrow = 2 - 3i \text{ or } -2 + 3i \quad (1 \text{ mark}) \\
 &\qquad\qquad\qquad [8 \text{ marks}]
 \end{aligned}$$

$$(b) \quad (ii) \quad z^2 + iz + (1 - 3i) = 0 \Rightarrow z = \frac{-i \pm \sqrt{i^2 - 4(1-3i)}}{2} \quad (1 \text{ mark})$$

$$\Rightarrow z \frac{-i \pm \sqrt{-1-4+12i}}{2} \quad (1 \text{ mark})$$

$$\Rightarrow z \frac{-i \pm \sqrt{-5+12i}}{2} \quad (1 \text{ mark})$$

$$\Rightarrow z \frac{-i \pm 2-3i}{2} \quad (1 \text{ mark})$$

$$\Rightarrow z \frac{2-4i}{2} \text{ or } \frac{2+2i}{2} \quad (1 \text{ mark})$$

$$\Rightarrow z = 1 - 2i \text{ or } -1 + i \quad (1 \text{ mark})$$

[6 marks]

$$(c) \quad (1 + 3i)z + (4 - 2i)z = 10 + 4i, \text{ and } z = a + bi$$

$$\Rightarrow (1 + 3i)(a + bi) + (4 - 2i)(a - bi) = 10 + 4i \quad (1 \text{ mark})$$

$$\Rightarrow (a - 3b) + i(3a + b) + (4a - 2b) + i(-4b - 2a) = 10 + 4i \quad (1 \text{ mark})$$

$$\Rightarrow a - 3b + 4a - 2b = 10 \text{ and } 3a + b - 4b - 2a = 4 \quad (1 \text{ mark})$$

$$\Rightarrow 5a - 5b = 10 \text{ and } a - 3b = 4 \quad (1 \text{ mark})$$

$$\Rightarrow a = 1, b = -1 \quad (1 \text{ mark})$$

$$\Rightarrow z = 1 - i \quad (1 \text{ mark})$$

[6 marks]

Total 25 marks

Question 2

(a) Let $I = \int e^{3x} \sin 2x \, dx$

$$= \frac{1}{3} e^{3x} \sin 2x - \int \frac{e^{3x}}{3} (2\cos 2x) \, dx \quad (2 \text{ marks})$$

$$= \frac{1}{3} e^{3x} \sin 2x - \frac{2}{3} \int e^{3x} (2\cos 2x) \, dx$$

$$= \frac{1}{3} e^{3x} \sin 2x - \frac{2}{3} \left[\frac{1}{3} e^{3x} \cos 2x + \int \frac{e^{3x}}{3} (2\sin 2x) \, dx \right] \quad (2 \text{ marks})$$

$$= \frac{1}{3} e^{3x} \sin 2x - \frac{2}{9} e^{3x} \cos 2x - \frac{4}{9} \int e^{3x} \sin 2x \, dx$$

$$= \frac{1}{3} e^{3x} \sin 2x - \frac{2}{9} e^{3x} \cos 2x - \frac{4}{9} I \quad (1 \text{ mark})$$

$$\Rightarrow I + \frac{4}{9} I = \frac{1}{9} (3 \sin 2x - 2\cos 2x) \quad (1 \text{ mark})$$

$$\Rightarrow I = \frac{1}{13} (3 \sin 2x - 2\cos 2x) + \text{constant} \quad (1 \text{ mark})$$

[7 marks]

Alternatively

$$\int e^{3x} e^{2ix} \, dx = \int e^{(3+2i)x} \, dx \quad (2 \text{ marks})$$

$$\Rightarrow \text{Im} \left[\frac{e^{(3+2i)x}}{3+2i} \right] + \text{constant} \quad (2 \text{ marks})$$

$$\Rightarrow \int e^{3x} \sin 2x \, dx = \text{Im} \left[\frac{(3-2i)}{13} e^{3x} (\cos 2x + i\sin 2x) \right] \quad (2 \text{ marks})$$

$$\frac{e^{3x}}{13} (3 \sin 2x - 2\cos 2x) + \text{const.} \quad (1 \text{ mark})$$

[7 marks]

(b) (i) a) $y = \tan^{-1}(3x) \Rightarrow \tan y = 3x \quad (1 \text{ mark})$

$$\Rightarrow \sec^2 y \frac{dy}{dx} = 3$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{\sec^2 y} \quad (1 \text{ mark})$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{1+\tan^2 y} \quad (1 \text{ mark})$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{1+9x^2} \quad (1 \text{ mark})$$

[4 marks]

b) $\int \frac{x+2}{1+9x^2} \, dx = \int \frac{x}{1+9x^2} \, dx + 2 \int \frac{1}{1+9x^2} \, dx \quad (1 \text{ mark})$

$$= \frac{1}{18} \ln(1+9x^2) + \frac{2}{3} \tan^{-1}(3x) + \text{constant} \quad (3 \text{ marks})$$

[4 marks]

(b) (ii) $y = \frac{\ln(5x)}{x^2}$, Using the product rule: $y = \frac{1}{x^2} \ln(5x)$ (1 mark)

$$\Rightarrow \frac{dy}{dx} = -\frac{2}{x^3} \ln(5x) + \frac{1}{x^2} \times \frac{1}{x} \quad (2 \text{ marks})$$

$$= \frac{1-2 \ln(5x)}{x^3} \quad (1 \text{ mark})$$

$$= \frac{1-\ln(25x^2)}{x^3} \quad (1 \text{ mark})$$

[5 marks]

c) $f(x, y) = x^2 + y^2 - 2xy$

(i) $\frac{\partial f}{\partial x} = 2x - 2y \quad \frac{\partial f}{\partial y} = 2y - 2x$ [2 marks]

(ii) $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = x(2x - 2y) + y(2y - 2x)$
 $= 2x^2 + 2y^2 - 4xy$ (1 mark)
 $= 2(x^2 + y^2 - 2xy)$ (1 mark)
 $= 2f(x, y)$ (1 mark)

[3 marks]

Total 25 marks

SECTION B

(MODULE 2)

Question 3

(a) (i) $\frac{1}{(2r-1)(2r+1)} \equiv \frac{A}{2r-1} + \frac{B}{2r+1}$

$$\Rightarrow 1 = A(2r+1) + B(2r-1) \quad (1 \text{ mark})$$

$$\Rightarrow 0 = 2A + 2B \text{ and } A - B = 1 \quad (2 \text{ marks})$$

$$\Rightarrow A = \frac{1}{2} \text{ and } B = -\frac{1}{2} \quad (2 \text{ marks})$$

[5 marks]

(ii) $S = \sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} = \sum_{r=1}^n \frac{1}{2} \left(\frac{1}{2r-1} - \frac{1}{2r+1} \right) \quad (1 \text{ mark})$

$$= \frac{1}{2} \left(\frac{1}{1} - \frac{1}{3} \right) + \frac{1}{2} \left(\frac{1}{3} - \frac{1}{5} \right) + \frac{1}{2} \left(\frac{1}{5} - \frac{1}{7} \right) + \dots + \frac{1}{2} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right) \quad (3 \text{ marks})$$

$$= \frac{1}{2} \left(1 - \frac{1}{2n+1} \right) \quad (1 \text{ mark})$$

[5 marks]

(iii) As $n \rightarrow \infty$, $\frac{1}{2n+1} \rightarrow 0 \quad (2 \text{ marks})$

Hence $S_\infty = \frac{1}{2} \quad (1 \text{ mark})$

[3 marks]

(b) (i) $S = 1(2) + 2(5) + 3(8) + \dots$

In each term, 1st factor is in the natural sequence and the second factor differs by 3 (1 mark)

\Rightarrow the r^{th} term is $r(3r-1)$ (1 mark)

[2 marks]

(ii) $S_n = \sum_{r=1}^n r(3r-1)$

for $n = 1 \quad S_1 = \sum_{r=1}^1 r(3r-1) = 1 \times 2 = 2$

and $1^2 (1+1) = 1 \times 2 = 2 \quad (1 \text{ mark})$

hence, $S_n = n^2 (n+1)$ is true for $n = 1 \quad (1 \text{ mark})$

Assume $S_n = n^2 (n+1)$ for $n = k \in \mathbb{N} \quad (1 \text{ mark})$

that is, $S_k = k^2 (k+1) \quad (1 \text{ mark})$

$$\begin{aligned} \text{Then, } S_{k+1} &= \sum_{r=1}^{k+1} r(3r-1) = S_k + (k+1)(3k+2) && (1 \text{ mark}) \\ &= k^2(k+1) + (k+1)(3k+2) && (1 \text{ mark}) \\ &= (k+1)[k^2 + 3k + 2] && (1 \text{ mark}) \end{aligned}$$

$$\begin{aligned} \Rightarrow S_{k+1} &= (k+1)[(k+1)(k+2)] \\ &= (k+1)^2 [(k+1)+1] && (1 \text{ mark}) \end{aligned}$$

\Rightarrow true for $n = k + 1$ whenever it is assumed true for $n = k$,
 \Rightarrow true for all $n \in \mathbb{N}$

$$\Rightarrow S_n = n^2(n+1) \quad n \in \mathbb{N}. \quad (1 \text{ mark})$$

[10 marks]

Total 25 marks

Question 4

(a) (i) Let $S \equiv \frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + \dots$

$$\frac{\frac{1}{2^4}}{\frac{1}{2}} = \frac{\frac{1}{2^7}}{\frac{1}{2^4}}$$

$$= \frac{1}{2^3}$$

(1 mark)

$\therefore S$ is geometric with common ratio $r = \frac{1}{2^3}$

(1 mark)

[3 marks]

(ii) $S_n = \frac{\frac{1}{2} \left[1 - \left(\frac{1}{2} \right)^{3n} \right]}{1 - \left(\frac{1}{2} \right)^3}$

$$= \frac{\frac{1}{2} \left[1 - \frac{1}{2^{3n}} \right]}{1 - \frac{1}{8}}$$

$$= \frac{1}{2} \times \frac{8}{7} \left[1 - \frac{1}{2^{3n}} \right]$$

$$= \frac{4}{7} \left[1 - \frac{1}{2^{3n}} \right]$$

(1 mark)

(1 mark)

[4 marks]

$$\begin{aligned}
 (b) \quad (i) \quad f(x) = \cos 2x &\Rightarrow f^1(x) = -2 \sin 2x & (1 \text{ mark}) \\
 &\Rightarrow f^{11}(x) = -4 \cos 2x & (1 \text{ mark}) \\
 &\Rightarrow f^{111}(x) = 8 \sin 2x & (1 \text{ mark}) \\
 &\Rightarrow f^{iv}(x) = 16 \cos 2x & (1 \text{ mark})
 \end{aligned}$$

$$\text{so, } f(0) = 1, \quad f^1(0) = 0, \quad f^{11}(x) = -4, \quad f^{111}(0) = 0, \quad f^{iv}(0) = 16 \quad (1 \text{ mark})$$

Hence, by Maclaurin's Theorem,

$$\cos 2x = 1 - \frac{4x^2}{2!} + \frac{16x^4}{4!} - \dots \quad (1 \text{ mark})$$

$$= 1 - 2x^2 + \frac{2}{3}x^4 \dots \quad (1 \text{ mark})$$

[7 marks]

$$\begin{aligned}
 (c) \quad (i) \quad \sqrt{\left(\frac{1+x}{1-x}\right)} &= (1+x)^{\frac{1}{2}} (1-x)^{-\frac{1}{2}} & (1 \text{ mark}) \\
 &= \left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 \dots\right) \left(1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 \dots\right) & (3 \text{ marks})
 \end{aligned}$$

$$= 1 + x + \frac{1}{2}x^2 + \frac{1}{2}x^3 \quad (2 \text{ marks})$$

$$\text{for } -1 < x < 1 \quad (1 \text{ mark})$$

[7 marks]

$$(ii) \quad \sqrt{\frac{1.02}{0.98}} = \sqrt{\frac{102}{98}} = \frac{1}{7}\sqrt{51} \quad (1 \text{ mark})$$

$$\sqrt{51} = 7\sqrt{\frac{1+x}{1-x}} \text{ where } x = 0.02 \quad (1 \text{ mark})$$

$$\Rightarrow \sqrt{51} = 7 \left\{ 1 + 0.02 + \frac{1}{2}(0.02)^2 + \frac{1}{2}(0.02)^3 \right\} \quad (1 \text{ mark})$$

$$= 7.14141 \text{ (5 d.p.)} \quad (1 \text{ mark})$$

[4 marks]

Total 25 marks

SECTION C
(MODULE 3)

Question 5

$$(a) (i) P(\text{First card drawn has even number}) = \frac{5}{10} = \frac{1}{2} \quad (1 \text{ mark})$$

$$P(\text{Second card drawn has even number}) = \frac{4}{9} \quad (2 \text{ marks})$$

$$\therefore P(\text{Both cards have even numbers}) = \left(\frac{1}{2}\right)\left(\frac{4}{9}\right)$$

$$= \frac{2}{9} \quad (1 \text{ mark})$$

[4 marks]

$$(ii) P(\text{Both cards have odd numbers}) = \frac{2}{9} \quad (1 \text{ mark})$$

$$P\left[\begin{array}{l} \text{One card has odd and the other has even} \\ \text{i.e. both cards do not have odd} \end{array}\right] = 1 - 2\left(\frac{2}{9}\right) \quad (2 \text{ marks})$$

$$\left.\begin{array}{l} \text{or do not have even numbers} \end{array}\right] = \frac{5}{9} \quad (1 \text{ mark})$$

[4 marks]

$$(b) (i) \frac{82}{150} = 0.547 \quad [2 \text{ marks}]$$

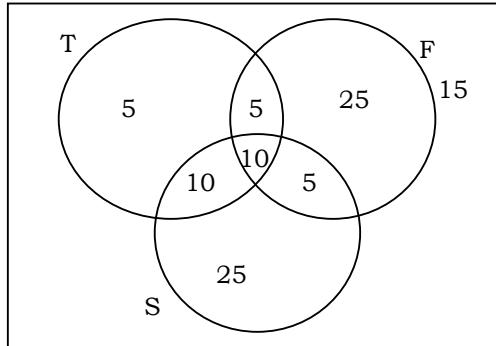
$$(ii) \frac{39}{150} + \frac{75}{150} = 0.76 \quad [4 \text{ marks}]$$

$$(iii) \frac{82}{150} + \frac{39}{150} - \frac{27}{150} = 0.267 \quad [3 \text{ marks}]$$

- (c) Let T, S and F represent respectively the customers purchasing tools, seeds and fertilizer.

(i) One mark for any two correct numbers

[4 marks]



Venn diagram

- (ii) a) 5 (1 mark)
b) 10 (1 mark)
c) 5 (1 mark)
d) 15 (1 mark)

[4 marks]

Total 25 marks

Question 6

$$(a) \begin{vmatrix} 5 & x & 3 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0$$

$$5(2x-2) - x(x^2+2x+3) + 3(2x+4+6) = 0 \quad (3 \text{ marks})$$

$$x^3 + 2x^2 - 13x - 20 = 0 \quad (1 \text{ mark})$$

$$\text{Subs } x = -4, \quad (-4)^3 + 2(-4)^2 - 13(-4) - 20 = 0$$

$$(x+4)(x^2 - 2x - 5) = 0 \quad (2 \text{ marks})$$

$$x = -4$$

$$x = \frac{2 \pm \sqrt{24}}{2}$$

$$x = 1 \pm \sqrt{6} \quad (2 \text{ marks})$$

[8 marks]

Alternatively

$$\begin{vmatrix} 5 & x & 3 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 4+x & x+4 & x+4 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0 \quad (\text{Add rows 2 and 3 to row 1})$$

$$\Rightarrow (x+4) \begin{vmatrix} 1 & 1 & 1 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0 \Rightarrow (x+4) \begin{vmatrix} 0 & 1 & 0 \\ x & 2 & -1 \\ -5 & 2 & x-2 \end{vmatrix} = 0$$

(subtract columns 2 from Columns 1 and 3).

$$\Rightarrow (x+4)x - (x^2 - 2x - 5) = 0 \Rightarrow x = -4 \text{ or } 1 \pm \sqrt{6} \quad [8 \text{ marks}]$$

$$(b) \quad y \tan x \frac{dy}{dx} = (4+y^2)\sec^2 x \quad (1 \text{ mark})$$

$$\frac{y}{4+y^2} \frac{dy}{dx} = \frac{\sec^2 x}{\tan x}$$

$$\frac{y dy}{4+y^2} = \frac{\sec^2 x dx}{\tan x}$$

$$\int \frac{y dy}{4+y^2} = \int \frac{\sec^2 x dx}{\tan x} \quad (1 \text{ mark})$$

$$\frac{1}{2} \ln(4+y^2) = \ln|\tan x| + c \quad (2 \text{ marks})$$

[4 marks]

(c) (i) $y = u \cos 3x + v \sin 3x$

$$\Rightarrow \frac{dy}{dx} = -3u \sin 3x + 3v \cos 3x \quad (2 \text{ marks})$$

$$\Rightarrow \frac{d^2y}{dx^2} = -9u \cos 3x - 9v \sin 3x \quad (2 \text{ marks})$$

$$\text{so, } \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 3y = -30 \sin 3x$$

$$\Rightarrow -(6v - 12u)\sin 3x + (-6u + 12v)\cos 3x = -30\sin 3x \quad (1 \text{ mark})$$

$$\Rightarrow 2u + v = 5 \text{ and } u = 2v \quad (1 \text{ mark})$$

$$\Rightarrow u = 2 \text{ and } v = 1 \quad (2 \text{ marks})$$

[8 marks]

(ii) the auxiliary equation of the different equation is

$$k^2 + 4k + 3 = 0 \quad (1 \text{ mark})$$

$$\Rightarrow (k + 3)(k + 1) = 0$$

$$\Rightarrow k = -3 \text{ or } -1 \quad (2 \text{ marks})$$

\Rightarrow the complementary function is

$$y = Ae^{-x} + Be^{-3x}; \text{ where } A, B \text{ are constants} \quad (1 \text{ mark})$$

$$\text{General solution is } y = Ae^{-x} + Be^{-3x} + \sin 3x + 2 \cos 3x \quad (1 \text{ mark})$$

[5 marks]

Total 25 marks

02234032/CAPE/MS/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2

COMPLEX NUMBERS, ANALYSIS AND MATRICES

SPECIMEN PAPER

PAPER 032

SOLUTIONS AND MARK SCHEMES

Question	Details	Marks
1 (a) (i)	$ z = \sqrt{8^2 + (8\sqrt{3})^2} = 16$	1
	$\arg(z) = \pi - \tan^{-1}(\sqrt{3}) = \frac{2\pi}{3}$ (1) (1)	1 1
(ii)	$z^3 = 16^3 (\cos 2\pi/3 + i \sin 2\pi/3)^3$ $= 16^3 (\cos 2\pi + i \sin 2\pi) = 4096$	1 1
(b) (i)	$(x-6)^2 + y^2 = x^2 + y^2$	1
	$x = 3$	1
(ii)	$ z - 3 - 4i = 5$ is a circle with centre $(3, 4)$ and radius 5	1
	Subs $x = 3$ gives $(3-3)^2 + (y-4)^2 = 5^2$	1
	$y^2 - 8y - 9 = 0$ $(y-9)(y+1) = 0$ $y = 9, -1$	1
	$z = 3 + 9i, 3 - i$ (1) (1)	1 1
(c)	$I_n = \frac{3}{4} [x^n (8-x)^{4/3}] + \frac{3}{4} \int nx^{n-1} (8-x)^{4/3} dx$ (1) (1)	1 1
	$= 0 + \frac{3}{4} \int nx^{n-1} (8-x)(8-x)^{1/3} dx$ (1) (1)	1 1
	$= \frac{3}{4} \int nx^{n-1} 8(8-x)^{1/3} dx - \frac{3}{4} \int nx^{n-1} x(8-x)^{1/3} dx$ (1) (1)	1 1
	$I_n = 6n I_{n-1} - \frac{3n}{4} I_n$	1
	$I_n = \frac{24n}{3n+4} I_{n-1}$	1

Question	Details	Marks
2 (a)(i)	$r^3 + 3r^2 + 3r + 1 - (r^3 - 3r^2 + 3r - 1) \\ = 6r^2 + 2$	1 1
(ii)	$r = 1: 2^3 - 0^3 = 6(1)^2 + 2 \\ r = 2: 3^3 - 1^3 = 6(2)^2 + 2 \\ \dots \\ r = n: (n+1)^3 - (n-1)^3 = 6n^2 + 2$	1 1 1
	summing gives $(n+1)^3 + n^3 - 1 = 6 \sum_1^n r^2 + 2n$	1 1
	$\sum_{r=1}^n r^2 = \frac{n}{6} (n+1)(2n+1)$	1
(iii)	$\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n-1} r^2 \\ = \frac{2n}{6} (2n+1)(4n+1) - \frac{n-1}{6} (n)(2n-1)$	1 1 1
	$= \frac{n}{6} (n+1)(14n+1)$	1
(b)	$f'(t) = -x - \cos x \quad f'(0) = -1$	1 1
	$f''(t) = (-1 + \sin x) \frac{dx}{dt} \quad f''(0) = -0.5$	1 1
	$f(t) = \frac{1}{2}t - \frac{1}{2}t^2 - \frac{1}{12}t^3 + \dots$	1
(c) (i)	$f(2.2) = -0.192 \quad f(2.3) = 0.877 \text{ By IVT and continuity root exists}$	1 1
(ii)	$\frac{\alpha - 2.2}{0.192} = \frac{2.3 - \alpha}{0.877}$	1
	$\alpha = 2.218$	1

S. O. (B) 1, 2, 4, 9, (D) 1, 3

Question	Details	Marks
3 (a)	Ends cola: $\frac{5!}{2!2!} = 30$ ways Ends green tea: $\frac{5!}{3!2!} = 10$ ways Ends orange juice: $\frac{5!}{3!2!} = 10$ ways Total = $30 + 10 + 10 = 50$ ways	1 1 1
(b)	$P(\text{bark}) = P(\text{park \& bark}) + P(\text{no park \& bark})$ $= (0.6)(0.35) + (0.4)(0.75) = 0.51$	1 1
(c)	Albert not Tracey = $(9 \text{ C } 3) \times (8 \text{ C } 2) + (9 \text{ C } 4) \times (8 \text{ C } 1) = 3360$ Tracey not Albert = $(9 \text{ C } 4) \times (8 \text{ C } 1) + (9 \text{ C } 5) = 1134$ # of selections = $3360 + 1134 = 4494$	1 1 1
(d) (i)	$AB = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ 0 & 1 & 1 \end{pmatrix} \times \begin{pmatrix} -3 & 4 & -7 \\ -2 & 1 & 2 \\ 2 & -1 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} = 5I$	1 1
(ii)	$AA^{-1}B = 5A^{-1}$ $A^{-1} = \frac{1}{5} \begin{pmatrix} -3 & 4 & -7 \\ -2 & 1 & 2 \\ 2 & -1 & 3 \end{pmatrix}$	1 1
(iii)	$(AB) = 5I \quad (AB)(AB)^{-1} = 5(AB)^{-1}$ $(AB)^{-1} = \frac{1}{5} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $B^{-1} A^{-1} = \frac{1}{5} \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ 0 & 1 & 1 \end{pmatrix} \times \frac{1}{5} \begin{pmatrix} -3 & 4 & -7 \\ -2 & 1 & 2 \\ 2 & -1 & 3 \end{pmatrix}$ $= \frac{1}{5} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	1 1 1

Question	Details	Marks
3 (e)	$I = \int \cot x \, dx$	1
	$= \sin x$	1
	$\sin x \frac{dy}{dx} + y \cos x = \sin^2 x$	1
	$\int \frac{d}{dx} (y \sin x) \, dx = \frac{1}{2} \int (1 - \cos 2x) \, dx$ (1) (1)	1 1
	$y \sin x = \frac{1}{2} \left(x - \frac{1}{2} \sin 2x \right) + C$	1
		[20]

CARIBBEAN EXAMINATIONS COUNCIL

**REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION**

MAY/JUNE 2004

MATHEMATICS

MATHEMATICS

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2004

INTRODUCTION

This is the sixth year that Mathematics Unit 1 was examined on open syllabus and the fifth year for Unit 2. Fifteen hundred and eighty-two candidates registered for Unit 1 examinations, and six hundred and eighty registered for Unit 2.

Each Unit comprised three papers, Paper 01, Paper 02 and Paper 03. Papers 01 and 02 were assessed externally and Paper 03 was assessed internally by the teachers and moderated by CXC.

Paper 01 in each Unit consisted of 15 compulsory short-answer questions. There were five questions in each of three sections, Sections 1, 2 and 3 corresponding to Modules 1, 2 and 3 respectively. Each question carried marks in the range of 4 – 8. Candidates could earn a maximum of 90 marks for this paper in each Unit, representing 30 per cent of the assessment for the respective Unit.

Paper 02 in each Unit consisted of six compulsory extended response questions. There were two questions in each Section/Module. Each question was worth 25 marks. Candidates could obtain a maximum of 150 marks on Paper 02, representing 50 per cent of the assessment for the Unit. Marks were awarded for Reasoning, Method and Accuracy.

Paper 03 was compulsory. It was assessed internally by the teacher and moderated by CXC. For Unit 1, candidates wrote three tests, one for each Section/Module. For Unit 2, candidates wrote one test for each of Modules 1 and 2, and were required to submit a project based on any aspect of the syllabus. This paper represented 20 per cent of the assessment for the Unit. This is the second year that Paper 03/2 was written by private candidates for Unit I and the first year for Unit 2.

GENERAL COMMENTS

UNIT 1

The performance of candidates showed improvement over previous years and in particular, there were several very good scores in Paper 02. Questions on such topics as Curve-sketching, Remainder Theorem, Sectors and Basic Calculus were

very well done. However, there is still room for improvement in dealing with indices, limits, aspects of Coordinate Geometry and Trigonometry, particularly in obtaining solutions to equations involving the basic trigonometric ratios. General algebraic manipulation also requires close attention and improvement in order to complement the progress gained by the candidates in assimilating the formal content of any topic. By and large, the candidates seemed well prepared this year for the experience.

DETAILED COMMENTS

UNIT I

PAPER 01 SECTION A (Module 1.1: Basic Algebra and Functions)

Question 1

Specific Objective (s): (c) 4, 5; (f) 2 (i)

This question tested the use of the Remainder Theorem in finding the possible values of the constant p.

Most candidates handled the question well but some had difficulties solving the equation $p^2 + p - 2 = 0$ for p. A common error was $(-1)^3 = 1$.

Answers: p = -2 or 1.

Question 2

(a) Specific Objective (s): (a) 3, 4, 5, 7

This part of the question tested basic knowledge of real numbers and the ability of candidates to use simple properties of inequalities as they relate to real numbers. There were very few good solutions to this question. Most candidates substituted specific values of x, y and k to obtain the required answer.

(b) Specific Objective (s): (b) 3; (f) 2 (i)

The question involved the knowledge of the modulus of real numbers and of the quadratic equations.

Most candidates were familiar with both topics, but some answers showed weaknesses in factorization of quadratic equations.

Answers: $x = -4, -2, 2, 4$.

Question 3

(a) Specific Objective (s): (e)

This part of the question tested knowledge of indices and was very well done.

(b) Specific Objective (s): (e); (f) 2(i); (c) 1, 2

This part of the question required substituting the result of Part (a) above in the given equation and then solving by means of basic knowledge of quadratic equations. Many candidates did not see the relevance of Part (a) to finding a solution and some who did, experienced difficulties in solving the resulting equation. In some instances, candidates showed a lack of understanding of the laws of indices.

Answers: $x = 2, x = 0$

Question 4

Specific Objective (s): (d) 1, 3; (f) 1

This question tested the ability of candidates to form the composite of functions and to solve the resulting equations. Several good solutions were submitted, but too many simple algebraic errors were noted, all of which spoiled the general performance on the question.

Answer: $x = -8$

Question 5

Specific Objective (s): (g) (i) 1, 2, 3, 4, 5

There were several attempts on Part (a) of this question. Many candidates obtained the correct answer; quite a few expressed their result in degrees and not radians.

Answer: 1.86 rad

Part (b) was well done by many candidates.

Answer: Approx. 22.1 m

SECTION B (Module 1.2: Geometry and Trigonometry)

Question 6

Specific Objective (s): (a) 9

This question involved converting the parametric representation of a curve to a Cartesian equation of the curve. There were several attempts at this question with many candidates obtaining about 50 per cent of the marks. The method of substitution presented inordinate difficulties due to weaknesses in the algebraic manipulation of the resulting expressions.

A few candidates spoilt otherwise good responses by simplifying

$$\frac{3x^2 - 18x + 27}{4} + 2 \quad \text{as} \quad \frac{3x^2}{4} - \frac{9x}{2} + \frac{29}{4}$$

Some others used the identity $3t^4 + 2 = A(2t^2 + 3)^2 + B(2t^2 + 3) + C$ but were unable to arrive at the correct values of A, B and C thereafter.

Answers: $A = \frac{3}{4}$, $B = \frac{-9}{2}$, $C = \frac{35}{4}$

Question 7

Specific Objective (s): (b)

This question examined linear, fractional and rational inequalities. Most candidates were able to determine critical values associated with the given inequality by employing a variety of methods appropriate for the purpose (for example, graphical, tabular). Some candidates were unable to find the solution after converting the original problem to a more manageable form.

Answers: $x > 2$ or $x < -3$

Question 8

(a) Specific Objective (s): (d) 2, 3

This question involved the use of simple trigonometric identities for $\cos 2A$ and $\sin 2A$. Many candidates used inappropriate forms for $\cos 2A$ and $\sin 2A$ to establish the required identity. This approach made the manipulation of the given expression(s) cumbersome, frequently leading to unnecessary complications. In general, the question was not well done.

(b) Specific Objective (s): (d) 3, 4, 7

The question required the use of the identity $\cos 2\theta = 2 \cos^2 \theta - 1$ to convert the given equation to a quadratic equation in $\cos \theta$. The solution of quadratic equations is also involved in obtaining the values for θ . Several candidates attempted this question but many displayed weaknesses in obtaining the correct quadratic equation in $\cos \theta$, in solving that equation, and in finding the correct values of θ within the specific range.

Answer(s): $\theta = 0$ or $\frac{\pi}{3}$

Question 9

Specific Objective(s): (e) 1

The theory of the roots of quadratic equations was examined in this question. In attempting this question, candidates made several simple errors in algebraic manipulation. There were too many mistakes made with signs, simplification of terms, and confusion between a quadratic expression and a quadratic equation. Many marks were lost due to carelessness.

Answer: $x^2 - 5x + 3 = 0$ is required equation

Question 10

(a) Specific Objective(s): (f) 5, 7

This part of the question tested the concept of unit vector. Many candidates were successful in obtaining the correct solution, but too often simple errors were made in calculating the modulus of the position vector of the point P.

Answer: $\frac{1}{\sqrt{10}} (\underline{i} + 3\underline{j})$

(b) Specific Objective(s): (f) 4, 8

The position vector of the point Q on \overrightarrow{OP} produced is required here. Many candidates did not know how to use the information $|\overrightarrow{OQ}| = 5$ and this proved to be a hindrance in obtaining the correct answer.

Answer: $\frac{5}{\sqrt{10}} (\underline{i} + 3\underline{j})$

(c) Specific Objective(s): (f) 10

This question tested knowledge of the condition for two vectors to be perpendicular. Several candidates stated the required condition but some failed to obtain the correct value of t because of faulty manipulation.

Answer(s): $t = -4$

SECTION C
(Module 1.3: Calculus 1)

Question 11

Specific Objective(s): 1 – 5

This question examined some of the basic concepts of limits. Several candidates attempted the question and approximately one-third of them obtained full marks. Some candidates lost credit by omitting reference to limits or by errors in expressing

$\frac{5}{4} - 4$ as a fraction.

Answer: $-\frac{11}{4}$ (Intermediate result: $\lim_{x \rightarrow -2} f(x) = \frac{5}{4}$)

Question 12

Specific Objective(s): (b) 6

This was a standard question on differentiation from first principles. There were several attempts at this question with a high degree of success. Many efforts suffered from errors in simplification.

Answer: $3x^2$

Question 13

Specific Objective(s): (b) 5, 7, 9, 13, 14, 17, 18, 19.

This question tested the concept of derivative of a function and conditions for maxima and minima.

Part (a) was quite well done. Both derivatives were found quite successfully by approximately 95 per cent of the candidates.

Several methods including the completion of the square, were used to solve Parts (b) and (c). Some candidates had difficulty completing the square.

Answer(s): (a) (i) $2rx + s$, (ii) $2r$

$$(b) \quad x = -\frac{s}{2r}, \quad r < 0$$

$$(c) \quad t = \frac{s^2}{4r} \text{ or } \frac{4rt - s^2}{4r}$$

Question 14

Specific Objective(s): (b) 2

This question examined the candidates' ability to use calculus methods to obtain the equation of a curve from given conditions. Approximately 60 per cent of the candidates responded favorably to this question. From the attempts, the following observations emerged:

- (i) Many candidates substituted $x = 1$ into the equation of the curve but did not substitute $y = 2$.
- (ii) The gradient of the curve was not correctly calculated.
- (iii) Some candidates did not equate the value of the gradient (7) when $x = 1$ was substituted.
- (iv) Usually, those candidates who formed two equations from the data were able to gain full marks.

Answer(s): $p = 10$, $q = -13$

Question 15

Specific Objective(s): (c) 4, 5, 6, 9

The question combined the modulus function with the concept of volume. Very few candidates responded to this question. Some candidates seemed unfamiliar with the modulus function and with the formula $\int y^2 dx$ for volume.

- (a) This part was done fairly well by those who attempted the question although answers were frequently not expressed in coordinate form.
- (b) Many candidates lost marks because they used the wrong upper limit for the volume. They used 4 instead of 2. About 5 per cent of the candidates noted that the volume generated was a right circular cone with base radius 2 and height 2 units.

Answer(s): (a) A (0, 2), B (2, 0)

(b) $\frac{8\pi}{3}$ units³

**PAPER 02
SECTION A
(Module 1.1: Basic Algebra and Functions)**

Question 1

(a) Specific Objective(s): (c) 1, 3 - 6

This question sought to test the candidates' knowledge of the Factor/Remainder theorem as it applies to polynomials. This question was generally well done. Most candidates obtained full marks.

Some candidates did not specify $f(1) = 0$ and $f(2) = 0$ thus improperly setting out the relevant equations to solve m and n . It may be useful to emphasize the importance of proper mathematical statements. One unique method of solving for m and n was demonstrated by a candidate who used the fact that since $(x - 1)$ and $(x - 2)$ are factors of $f(x)$ then the remainder for $f(x)$ when divided by $(x^2 - 3x + 2)$ in terms of m and n must be equal to zero. By deduction, the values of m and n were determined. The understanding of the Factor/Remainder Theorem was clearly established.

Answer(s): $m = -7$, $n = 6$; third factor is $x + 3$.

(b) Specific Objective(s): (c) 1, 6

This question tested knowledge of polynomial identities and the methods involved in determining unknown coefficients. Generally, the performance on this question was unsatisfactory. Many candidates did not get past the expansion of the expression on the right-hand side. The lack of algebraic skills was clearly evident. Grouping of terms and comparing coefficients with the left-hand side were beyond the abilities of a significant number of candidates.

Answers: $p = 2$, $q = -3$, $r = 7$

(c) Specific Objective(s): (a) 4, 5; (b) 1, 2, 3; (c) 1.

This part of the question examined the modulus function and inequalities.

In Part (i), a number of candidates showed that $-5 \leq (2x - 3) \leq 5$. However, as commonly seen in the classroom, solution sets included $x \leq 4$ and $x \leq -1$. Some candidates used the approach of squaring both sides. Obtaining $(x + 1)(x - 4) \leq 0$, candidates solved $x \leq 4$ and $x \leq -1$. Some candidates showed the region for the range of values of x on a quadratic graph and thus were able to state the correct solution set.

Parts (ii) and (iii) were correctly given by those candidates who obtained the correct expressions for x .

Answers: (i) $-1 \leq x \leq 4$, (ii) least value = 0, (iii) greatest value = 5

Question 2

(a) Specific Objective(s): (f) 2 (ii), (iii); (a) 6

This part of the question involved completion of the square for a quadratic function and the identification of the relevant coefficients. The maximum value of the function was also determined.

In Part (i), completion of the square continues to be problematic to many candidates. Particularly noticeable was the difficulty experienced when the quadratic expression involved the coefficient of x^2 being less than 0. Some candidates expanded the right-hand side and compared coefficients to determine the values of the constants A, B and C. It was not uncommon to see candidates giving the value of $x = -3$ as the maximum value of the function.

Part (ii) was generally well done. However, it was noted that some candidates plotted a graph from a table of values of x as would occur at the CSEC examinations. Many candidates lost marks for not “showing clearly its main features”.

In Part (iii) a significant number of candidates wrote in essay form the reasons for a function not being one-to-one. Some candidates showed $f(0) = 0$ and $f(6) = 0$. Very few of them showed that $f(x)$ was not one-to-one by using other values of x . A considerable number of them used the horizontal line test to show that a common point on the graph corresponded to distinct values of x .

Answers: (i) $A = 18$, $B = -2$, $p = -3$; max. pt $f(x) = 18$

(b) Specific Objective(s): (g) (i) 1, 2, (ii) 1, 2, 3; (a) 6, (d) 5.

This part of the question tested some features of the graphs of the functions $\sin x$ and $|\sin x|$ by means of sketches.

Parts (i), (a) and (b) were generally well done.

In Part (ii), very few candidates experienced any difficulty.

In Part (iii), candidates gave solution sets as $x = 0$, $x = \frac{\pi}{2}$, $x = 2\delta$. Some candidates gave their answers as $0 \leq x \leq 2\delta$. Candidates rarely stated the solution set correctly.

Answers: (iii) Solution set is $\{0 \leq x \leq \delta\} \cup \{2\delta\}$

SECTION B (Module 1.2: Geometry and Trigonometry)

Question 3

(a) Specific Objective(s): (a) 2; 5 (i), (ii)

This part of the question covered the topic of co-ordinate geometry as it applies to coordinates of points, perpendicularity of lines, gradients and points of intersection of lines. This part of the question was very popular with the candidates and there were many good answers. However, there were a few wrong answers due to faulty manipulation of correct equations.

Answer(s): (i) $2y = 3x - 4$, (ii) $3y + 2x = 7$, (iii) Q (2, 1)

(b) Specific Objective(s): (d) 2, 7

This question tested the ability of the candidates to solve a quadratic equation involving trigonometric functions and to find solutions within a given range. There were many excellent responses to this question. Most candidates were able to do this question completely. Faulty factorization of the quadratic equation was the main cause of errors in the solution.

Answer(s): $\theta = 41.8^\circ, 138.2^\circ$

(c) Specific Objective(s): (d) 5, 7

This question focused on the solution of trigonometric functions and equations based on formula for $\sin A + \sin B$. In general, candidates seemed unaware of the formula for $\sin A + \sin B$ in terms of half-angles, $\frac{A+B}{2}$ and $\frac{A-B}{2}$, so that the question was poorly done. Some candidates expanded $\sin 3x$ in terms of $\sin x$ as a means of obtaining solutions.

Answer(s): $x = 0, \pi/2, \pi$

Question 4

(a) Specific Objective(s): (a) 7, (e) 4, 5

This question focused on complex numbers.

In Part (a), the responses were very poor indeed. Insurmountable difficulties arose in separating the real and imaginary parts of the complex number w in terms of x and y .

$$\text{Answer(s): } w = \frac{(x-1)(x+2)+y^2}{(x+2)^2+y^2} + \frac{3y}{(x+2)^2+y^2} i$$

The difficulties encountered in Part (a) adversely affected the progress made in doing Part (b). In some instances, $\arg w = \frac{\pi}{4}$ was not properly interpreted.

Answer: $x^2 + y^2 + x - 3y - 2 = 0$

Not many candidates reached Parts (b) (ii) and (iii) due to problems arising in the earlier parts.

Answer(s): (ii) Equation of C is $(x + \frac{1}{2})^2 + (y - \frac{3}{2})^2 = \frac{9}{2}$

(iii) Centre of C = $(-\frac{1}{2}, \frac{3}{2})$, radius of C = $\frac{3}{\sqrt{2}}$

(c) Specific Objective(s): (f) 2, 4, 6, 7, 8

This question concentrated on vectors in the context of a parallelogram. The responses were satisfactory but few in number. Some candidates observed that P is the mid-point of MO or \vec{LN} and were able to use that insight to calculate \vec{MO} or \vec{LN} on the way to obtaining \vec{OP} .

Answer: $\vec{OP} = -\frac{1}{2}\underline{i} + \frac{9}{2}\underline{j}$

SECTION C (Module 1.3: Calculus 1)

Question 5

(a) Specific Objective(s): (a) 2, 4, 5, 6

This question dealt with limits of a rational function in which both numerator and denominator were quadratic functions of x with a common linear factor. This question was attempted by several candidates in a variety of ways. Many tried the straightforward method of factorizing numerator and denominator but often spoiled the analysis with faulty factorization. Use of L'Hôpital's (or L'Hospital's) rule was also evident. Those candidates who substituted $x = 3$ in the original form obtained $\frac{0}{0}$ and then were unable to continue. A few obtained full marks.

Answer: 2

(b) Specific Objective(s): (a) 7, 8

This question was a simple application on continuity. This part was very well done, however, a few of the weaker candidates equated both numerator and denominator to zero, an action that created an unwanted result.

Answer(s): Not continuous at $x = 0, x = -1$

(c) Specific Objective(s): (b) 7, 8, 10

This question dealt with the differential calculus, in particular, the formation of a differential equation from a given function.

In Part (i), there were good responses. Basic algebraic errors occurred in the simplifications of expressions involving removal of brackets and collection of similar terms.

A few candidates quoted the quotient rule for $\frac{u}{v}$ incorrectly.

Answer: $\frac{dy}{dx} = \frac{4x}{(x^2 + 1)^2}$

In Part (ii), many candidates did not demonstrate the correct approach to obtaining the result required. They often started with the equation on the question paper rather than by using the result of Part (c)(i) above, forming the expression on the left hand side and proceeding to simplify to the term on the right hand. Some treated it as an equation to be solved for y although y was given at the start, while others expanded $x(x^2 + 1)$ and so lost sight of common terms $x^2 + 1$ in $x(x^2 + 1)$ and $(x^2 + 1)^2$ in

$$\frac{dy}{dx}.$$

(d) Specific Objective(s): (b) 7, 8, 9, 12

This part dealt with increasing functions using the differential calculus. Generally, candidates were able to find $f'(x)$ and solve for x the equation $f'(x) = 0$.

Many stopped at that point and diverted to sketching the curve by investigating the factors of $f'(x)$. In some cases, the correct range emerged with careful reasoning. The factorization of $5x^4 - 5$ presented difficulties for many candidates.

Answer: $-1 < x < 1$

Question 6

Specific Objective(s): 13 – 19, 21

This question dealt with curve-sketching using the methods of calculus. Candidates performed capably on Part (a) of this question.

Answer: Stationary pts. are (1, 0) and (-1, 4)

In Part (b), the majority of candidates used the second derivative effectively, however, some continue to find the conditions for maxima and minima confusing. The change of sign was also used but candidates did not always make the correct inference from the change in sign.

Answer: (1, 0) is a minimum pt; (-1, 4) is a maximum pt.

In Part (c), many candidates did not recognize the emphasis placed on the condition that the curve touched the x -axis at $x = 1$. Most were able to show that $x - 1$ was a factor of $f(x)$ but the repeated nature of the factor $x - 1$ and the fact that (1, 0) was a minimum was not recognised, so that the shape of the curve at $x = 1$ was not always correct.

Part (d) of the question evoked many good responses but was spoiled by some of the considerations identified in Part (c).

Although the majority of candidates were able to identify that integration was required in Part (e), few candidates were able to arrive at the correct value. Many errors occurred at the point of substitution while some had difficulty with the limits of integration. Some candidates attempted to use the trapezium rule but were unable to make the correct conclusion.

Answer: $6 \frac{3}{4}$ sq. units

PAPER 03 INTERNAL ASSESSMENT

In general, the internally assessed tests were relevant and appropriate to the objectives stated in both Unit 1 and Unit 2 of the CAPE Mathematics Syllabus. In most cases, questions from past CAPE Mathematics examination papers were used for these examinations. There were instances where one examination paper was set to test all three modules and in most of these cases, the scores of candidates were lower than for those candidates tested on individual modules. The range of difficulty in the tests varied significantly. In some cases, the tests were too detailed in content to be conducted over a 1 to 1½ hour duration. In other instances, the tests did not reflect a sufficiently adequate coverage of the syllabus.

Most of the sample tests were submitted with question papers, solutions and detailed mark scheme indicating clearly the distribution of the marks; nevertheless, there were still too many samples submitted which did not contain all of these components. In one extreme case, the copy of the question paper was not submitted which slowed down the moderation process. Such situations must be avoided in the future.

Assessment was generally excellent with a high level of consistency but there were few occasions, where the allocation of marks was difficult to follow. In the future, fractional (or decimal) scores should not be submitted.

PAPER 03/2

SECTION A (Module 1.1: Basic Algebra and Functions)

Question 1

(a) Specific Objective(s): (a) 1, 2; (c) 1, 2, 4; (f) 2 (i)

The Remainder/Factor Theorem was tested in this part of the question. There were few candidates for this paper, nevertheless, there was evidence of familiarity with the topic.

Answers: $p = 6$ or $p = -3$

(b) Specific Objective(s): (c) 1; (g) 1, 2, 4

The sector and its properties were the focus of this part of the question. The candidates had difficulties in finding the perimeter of the sector and as a consequence could not obtain the expression for θ in Part (i). However assuming that θ in Part (i) was correct, some were able to find the expression for the area, A , in Part (ii).

Answer: Perimeter = $2r + r\theta$ cms

(c) Specific Objective(s): (f) 1, 3

Simultaneous equations, one linear and one quadratic, were examined in this part of the question. There were some good attempts but weaknesses in algebra resulted in incorrect solutions.

Answers: $x = 1, y = 2$; $x = -\frac{3}{4}, y = \frac{-13}{4}$

SECTION B **(Module 1.2: Geometry and Trigonometry)**

Question 2

(a) Specific Objective(s): (a) 1, 2, 5 (ii), 6

The coordinate geometry of a given line in the Cartesian plane was explored.

In Part (i), some difficulties were experienced in finding the coordinates of p.

With incorrect coordinates for p, the equation of the line in Part (ii) proved difficult to find.

Answer: (i) $p = (1, 1)$

(b) Specific Objective(s): (d) 3, 4, 7

The question involved finding solutions to equations for trigonometric functions of multiple angles.

Simple errors were made in Part (i). The connection $\sin 4\theta = \sin 2(2\theta)$ was not noticed by some candidates.

There were few correct solutions to Part (ii).

Answers: (i) $\sin 4\theta = 2 \sin 2\theta \cos 2\theta$

(ii) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$

(c) Specific Objective(s): (e) 1, 3

This part required knowledge of the product of the roots of a quadratic equation in terms of its coefficients. There were a few valiant attempts some of which were spoiled by interpreting i^2 as 1 instead of -1.

Answer: $c = 5$

(d) Specific Objective(s): (f) 1, 2, 4, 6, 7

This exercise involved vectors. Few candidates completed the major part of the solution.

Answer: Position vector of E is $\mathbf{i} - \mathbf{j}$.

SECTION C (Module 1.3: Calculus 1)

Question 3

(a) Specific Objective(s): (a) 2, 4, 6

The question examined the limit of a rational function in which the numerator and denominator had a common factor. There were good attempts at the solution.

Answer: limit = 8

(b) Specific Objective(s): (b) 7, 8, 9 ,10

Differential of a rational function was tested here. The candidates failed to simplify the integrand by dividing through by t^2 and then proceed by the quotient formula. There were few completed answers.

Answer:
$$\begin{aligned}\frac{dE}{dt} &= t - \frac{1}{t^3} \\ &= \frac{t^4 - 1}{t^3}\end{aligned}$$

(c) Specific Objective(s): (c) 3, 4, 5, 6

The evaluation of an integral by using a given result is examined in this part. This was done quite satisfactorily, however, some credit was lost for weakness in properly separating the integrand into two parts.

Answer: $\int_0^4 (4x - f(x)) dx = 26$

(d) Specific Objective(s): (b) 11; (c) 2, 3, 4, 6, 10

This question examined rate of change by means of the differential and integral calculus. There were a few attempts but none was completely successful due to faulty algebraic manipulations.

Answer(s) (i) $\frac{dp}{dt} = \frac{10}{t^2} + t$

(ii) $P = \frac{-10}{t} + \frac{t^2}{2} + k$, k is constant of integration

(iii) Change in P is $8\frac{1}{2}$

GENERAL COMMENTS UNIT 2

The overall performance of candidates in Unit 2 was very good. Some excellent performances were recorded in both papers, a clear indication that both teachers and the candidates are becoming comfortable with the requirements of the syllabus. However, there were candidates taking the examination who were not yet ready for such an in-depth examination process. There were also weaknesses in the ability of candidates to recall relevant information from Unit 1 or even CSEC and this has, on occasion, reduced their capability in completing standard processes for which the major tenets have been started. The analytical skills of some candidates need to be extended and improved considerably in order to maximize their potential at this level.

By and large, topics such as integration and differentiation in Calculus, partial fractions, Newton-Raphson estimation of roots of equations, and probability theory are familiar to most candidates but there is still uncertainty among many in dealing with mathematical induction, mathematical modeling questions leading to A.P and G.P, and general algebraic manipulation. Limits, indices and logarithms, and solutions to equations involving trigonometric functions still continue to present too many difficulties for candidates and require substantial consolidation.

DETAILED COMMENTS

UNIT 2 PAPER 01 SECTION A (Module 2.1: Calculus II)

Question 1

Specific Objective(s): (a) 3, 8, 9, 10

This question focused on exponential functions.

Most candidates responded favorably to this question and several good answers were submitted, nevertheless, the following weaknesses were noted:

The solution $x = \frac{1}{2} \ln 3$ was obtained but no explicit mention of this as the value of 'a' was made.

Candidates incurred unnecessary work by substituting $x = \frac{1}{2} \ln 3$ into $y = e^{2x}$ to obtain $y = 3$ or an approximate value (2.98 or 2.99). Some failed to see that $b = 3$ from the graph.

A few candidates used $\log_{10} 3$ in place of $\ln 3$.

Answer(s): $a = \frac{1}{2} \ln 3$ or 0.549, $b = 3$

Question 2

Specific Objective(s): (b) 4, 5, 7, 8

This question examined the product rule, implicit differentiation and the chain rule.

The question was done very well. Candidates should be encouraged to simplify their solutions, for example, the expression $\frac{6x}{3x^2}$ to obtain the simpler form $\frac{2}{x}$.

Some weaknesses were observed, for example,

- (i) incorrect use of the rules for logs – some candidates wrote $y = \ln(3x^2) \Rightarrow y = 2 \ln 3x$, others interpreted $\ln(3x^2)$ as the product $(\ln)(3x^2)$ and this led to a wrong result for $\frac{dy}{dx}$
- (ii) a common error in differentiation was $\frac{d}{dx}(\sin^2 x) = \cos^2 x$.

Answers: (a) $\frac{2}{x}$, (b) $2 \sin x \cos^2 x - \sin^3 x$

Question 3

Specific Objectives: (a) 2, 3, 4; (b) 4, 5, 6

This question dealt with exponential functions, quadratic equations, product rule and implicit differentiation.

The majority of the candidates obtained full marks for this question. Some excellent solutions were presented. Some of the weaknesses noted were as follows:

- (i) There was some confusion in finding $\frac{d}{dx}(2xy)$.
Typical examples are $\frac{d}{dx}(2xy) = 2 + 2y \frac{dy}{dx}$ or $2x + 2y$.
- (ii) There were cases of substituting (1, 1) before differentiation.

- (iii) Factorization of the quadratic equation $y^2 - 3y - 4 = 0$ presented difficulty to some candidates.
- (iv) Some candidates used $\log 4$ instead of $\ln 4$ while others seemed not to be aware that $\ln(-1)$ does not exist.

Answers: (a) $\frac{-1}{2}$, (b) $x = \ln 4$ or 1.39

Question 4

Specific Objective(s): (2, 3)

The topics tested were partial fractions and indefinite integrals.

This question was very well done. Several candidates obtained full marks showing excellent preparation. There were a few blemishes to solutions as follows:

- (i) Simplification of $\frac{A}{x} + \frac{B}{x+2}$ to give $\frac{Ax + B(x+2)}{x(x+2)}$
- (ii) The omission of dx after the integrand and the integral sign \int , and the omission of the constant of integration.

Answers: (a) $\frac{1}{2} \left[\frac{1}{x} + \frac{1}{x+2} \right]$, (b) $\frac{1}{2} \ln x(x+2) + k$ (constant).

Question 5

This question focused on integration by parts.

There were several attempts at this question. Overall, the question was well done, however, subtle weaknesses were evident in some of the solutions presented by the candidates.

In Part (i) a few candidates put $u = x^2$, $\frac{dv}{dx} = \ln x$ and this led to awkwardness in proceeding further; some used $u = x^2$, $v = \ln x$ to calculate $\frac{du}{dx} = 2x$, $\frac{dv}{dx} = \frac{1}{x}$ and then could proceed no further.

In Part (ii), the constant of integration was frequently omitted.

Answers: $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + k$ (constant)

SECTION B **(Module 2.2: Sequences, Series and Approximations)**

Question 6

Specific Objective(s): (C) 1, 2

This question dealt with the binomial expansiontheorem.

It was not well done despite the several attempts. Many candidates had serious difficulties in solving this problem correctly. It was evident from the responses that the preparation for this topic was limited. Candidates did not apply the binomial theorem to determine the term independent of x . A common error noted was that candidates

did not pay adequate attention to the term $\left(\frac{-1}{2x^2}\right)^3$ and in fact, got their answer as 7654.5 instead of -7654.5. Also noted was that some candidates expressed the term $\left(\frac{-1}{2x^2}\right)^3$ as $(2x^{-2})^3$. Of course, this resulted in the wrong coefficient, again exhibiting the weaknesses of candidates in working with indices.

Answer: $-\left(\frac{7}{2}\right)(3^7)$ or -7654.5

Question 7

Specific Objective(s): (b) 3

The topic examined in this question was arithmetic progressions.

Most candidates identified the common difference as 3. Algebra continues to pose difficulties to candidates. Consequently, many were not able to find the values for x and y correctly.

Answer: $x = \frac{7}{4}$, $y = \frac{-5}{2}$

Question 8

Specific Objective(s): (b) 4, 5, 6

Geometric series and sum to infinity were examined in this question.

This question was generally well done. However, the simplification of $\frac{4\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}}$ presented difficulties to candidates and in some cases was not pursued beyond this point. A significant number of candidates went on to answer Part (b) using the formula for the sum to infinity, $S_{\infty} = \frac{a}{1-r}$. Candidates did not demonstrate the use of the deduction $\lim_{n \rightarrow \infty} \left(\frac{1}{2}\right)^n \rightarrow 0$.

Answer: (a) $8(1 - (\frac{1}{2})^n)$, (b) 8

Question 9

Specific Objective(s): (c) 1, 2

The topics examined were binomial expansion and equating of coefficients.

This question was done poorly. The majority of candidates scored no more than three marks. Candidates stated the formula as given on the formula sheet but could not expand in terms of a and n. Consistent with the observations in Question 6, it seems that there was a problem in the preparation of this topic on the binomial theorem. Very few candidates were able to do this question properly.

Answers: $a = \frac{2}{3}$, $n = 9$

Question 10

Specific Objective(s): (d) 1

The topics tested were errors and approximations.

Generally this question was done satisfactorily. Some candidates experienced difficulty in determining the denominator to calculate the percentage error. Many of them used the estimated measurement instead of the true value.

Answer: 1.73%

SECTION C

(Module 2.3: Probability and Mathematical Modelling)

Question 11

Specific Objective(s): (a) 1, 2, 3

The topics examined were sample space and probability

There were several candidates who attempted this question and obtained some credit for what was presented. Nevertheless, there appear to be gaps in their understanding of some of the fundamental concepts associated with the topic(s). Some of these gaps resulted in the following weaknesses in the respective parts of the question:

- (a) Many candidates could not describe the sample space.
- (b) (i) Some candidates failed to recognize that the required probability that both balls are of the same color is based on the event set $\{R_1R_2, B_1B_2\}$.
- (ii) Many candidates experienced difficulty in setting out the probability that at least one ball is black. Some candidates attempted to use the complementary event giving probability $1 - P(\text{no black})$ without success while others did not recognize the listing in the set $\{R_1B_1, R_1B_2, R_2B_1, R_2B_2, GB_1, GB_2, B_1B_2\}$ as appropriate for this part of the question.

Answer(s): (a) $\{R_1R_2, R_1G, R_1B_1, R_1B_2, R_2G, R_2B_1, R_2B_2, GB_1, GB_2, B_1B_2\}$

(b) $\frac{2}{10}$ or $\frac{1}{5}$

(c) $\frac{7}{10}$

Question 12

Specific Objective(s): (a) 3, 4, 5

In this question, the topic examined was probability.

Most candidates attempted this question. There were few good responses. Some candidates were unable to formulate an algebraic expression linking the probabilities of A, B and C. Others demonstrated very little understanding of probability theory often producing solutions with probabilities greater than 1. Some candidates used a ratio method to determine the solution and those familiar with the law of total probability were able to solve the question successfully.

Answers: $P(A) = \frac{4}{7}$, $P(B) = \frac{2}{7}$, $P(C) = \frac{1}{7}$

Question 13

Specific Objective(s): (a) 5, 6, 7

The topics tested were probability and events.

This question was very well done with several candidates obtaining excellent scores. For the majority of candidates, Part (a) was straightforward. In Part (b), the basic rule of probability was known to a large number of candidates but algebraic errors occurred in many calculations giving rise to incorrect answers.

Part (c) was the most popular part of this question and a variety of methods was employed, including Venn diagrams, to produce correct results. However, a few candidates multiplied the probabilities $\frac{1}{3}$ and $\frac{1}{4}$ to obtain $\frac{1}{12}$ instead of $\frac{1}{3} - \frac{1}{4}$ which was the correct route.

Answers: (a) $P(A) = \frac{1}{3}$, (b) $P(B) = \frac{2}{3}$, (c) $P(A \cap B') = \frac{1}{12}$

Question 14

Specific Objective(s): (a) 3,4, 8, 10

The topics tested were probability – mutually exclusive and independent events. In Part (a), most candidates were able to identify accurately a mutually exclusive event. As a consequence, this part was very well done.

Several candidates attempted Part (b). Most of them were able to arrive at a correct solution. However, some candidates still had difficulty identifying that for independent events A and B, $P(A \cap B) = P(A) \times P(B)$. Generally, candidates were very familiar with the content of the question.

Answers: (a) (ii), (b) $P(A \cap B) = \frac{1}{10}$

Question 15

Specific Objective(s): (a) 3 – 6, 10, 12, 13; (b) 3

The topics tested were sample space and probability

Part (a) was poorly done. Most candidates did not understand the question and instead of writing down the sample space for the event as {HHM, HMH, MHH}, they tried to calculate the probability for the event occurring. Those candidates who attempted to write down the sample space were successful but a few wrote the outcomes descriptively rather than by symbols, H and M, a weakness which should be corrected for the future.

There were many successful attempts at Part (b). The majority of candidates correctly used the Binomial Distribution to calculate the probability of hitting the target at least once. Few candidates properly defined their random variable x as either a hit or a miss. A few candidates had problems in understanding that $P(x = 1)$ was equivalent to $1 - P(x = 0)$ with x defined as a hit. Some candidates used the tree diagram to calculate the probability although in the process some of the outcomes were omitted.

Answer(s): (a) {HHM, HMH, MHH}, (b) Prob. = $1 - (0.6)^3 = 0.784$

PAPER 02
SECTION A
(Module 2.1: Calculus II)

Question 1

Specific Objective(s): (a) 9, 10

The topics examined were logarithms, exponential function, equations and graphs. Parts (a) and (b) of this question were well done by most candidates, however, there were a few errors in algebraic manipulation which led to spurious results.

Some of these errors are as follows:

$$3 \log_a 3 = \log_a 9$$

$$\log_3(x+6) = \log_3 x + \log_3 6$$

$$-3 \log_a 3 - \log_a 5 = -\log_a \frac{27}{5}$$

Parts (c) and (d) were also well done. A few candidates did not use the given scales to draw the graphs but otherwise there were some very good responses.

Part (e) (i) was well done.

Many candidates tried to do Part (e) (ii) without reference to the graphs despite the instruction “use your graphs....”.

Answer(s): (a) $a = \frac{2}{3}$; (b) $x = 3$; (e) (i) $x = 0$, (iii) $x = 2$

Question 2

Specific Objective(s): (b) 3, 4, 5, 7, 8; (c) 4.

The topics tested were differentiation, integration by substitution, and parametric equations.

Part (a) (i) was very well done by many candidates.

In Part (ii), a number of candidates experienced difficulties with the form of the function $\tan^2(x^3)$ and did not seek to use a substitution to simplify this. A common response to this question was $6x^2 \sec^2(x^3)$. Another variant was $6x^2 \tan x \sec^2 x$.

Part (b) was not well done. Steps were often omitted, thus making the logic incomprehensible in many cases. There were also instances where there was not complete replacement of x by u in the integrand so that the simplicity of the integral in u was missed. The constant of integration was very frequently omitted.

The elementary calculus in Part (c) was very well done, however, some candidates seemed to have forgotten the connection between tangent and normal, and so could not obtain the equation of the normal.

Answer(s): (a) (i) $\frac{xe^x}{(1+x)^2}$ (ii) $6x^2 \tan(x^3) \sec^2(x^3)$

(b) $e^{\sin x} + k$ (constant)

(c) $\frac{dy}{dx} = \frac{1}{2}(2t-1)$, equation of normal: $4y + 4x + 3 = 0$

SECTION B (Module 2.2: Sequences Series and Approximations)

Question 3

Specific Objective(s): (a) 1, 2, 3, 5; (b) 3, 5

The topics tested were sequences, mathematical induction and A.P.

Responses in Part (a) of this question were generally poor and indicated extensive unfamiliarity with many of the basic concepts involved. Weaknesses which could be identified were as follows:

Unfamiliarity with the suffix notation u_{n+1} .

The concepts of convergent, divergent and periodic seemed not to be clearly understood by many candidates.

Many solutions were expressed in terms of u_n and not explicitly as required.

Answer(s): (ii) $\{u_n\} = \{1, 2, 1, 2, \dots\}$; $\{a_n\} = \{3, 3, 3, 3, \dots\}$;
 $\{b_n\} = \{1, -1, 1, -1, \dots\}$

(ii) $\{u_n\}$ is periodic; $\{a_n\}$ is convergent; $\{b_n\}$ is periodic

In Part (b), the induction step was not always set out clearly in the responses, nevertheless, there were some reasonable attempts.

In Part (c), the value of $d = -3$ (some, $d = 3$) was obtained by most candidates who attempted this question but then they experienced difficulty in finding the value of n . Whatever legitimate value was obtained for n was used to compute the sum of the AP.

Answers: $d = -3$, $n = 34$, sum of AP = 765

Question 4

Specific Objective(s): (e) 1, 2, 3

The topics examined were functions, approximation of roots and binomial expansion.

Generally, Part (a) was well done with candidates securing most of the marks allocated. Most candidates overlooked the concept of continuity in Parts (i), b) and c). A few candidates established Part (i) b) by considering the point of intersection of the two graphs $y = x^3$ and $y = 2 - 2x$.

Very few candidates explicitly mentioned the Intermediate Value Theorem but used the concept, nevertheless.

The Newton-Raphson formula for the second approximation in (ii) was sometimes written incorrectly as $a - \left[\frac{f'(x)}{f(x)} \right]$, $a \left[\frac{f'(x)}{f(x)} \right]$ or $a \left[\frac{f(x)}{f'(x)} \right]$. A few candidates proceeded beyond the 2nd approximation, quite unnecessarily.

Part (b) was generally well done with full marks being awarded in many cases. Weaknesses were observed in the following instances:

- (i) Incorrect expansion of the Binomial Theorem although this was given on the formula sheet
- (ii) Inability to collect the like terms together in order to obtain the coefficients of x^2 and x^3

In general, candidates seemed to have had practice in handling problems of this type.

Answers: (a) (ii) 0.818; (b) $a = 4$, coeff. of x^3 is - 80

SECTION C
(Module 2.3 Probability and Mathematical Modelling)

Question 5

Specific Objective(s): (a) 1 – 6, 8, 12; (b) 3

This question focused on probability.

This question was well done. Most candidates obtained full marks. Those who did not, lost marks for incorrect labeling of the branches of the tree diagram and for stating the relevant probabilities wrongly. Many candidates incorporated Parts (a) and (b) by completing a single tree diagram with the branches correctly labeled and the correct probabilities stated.

Answers to Part (c) (i) and (ii) were generally well done. Candidates used the tree diagram to deduce the correct probabilities rather than to re-calculate the probabilities at each stage.

Answers: (c) (i) $\frac{37}{240}$ (ii) $\frac{53}{144}$

Question 6

Specific Objective(s): (b) 3

This question focused on mathematical modelling

Overall, this question was poorly done.

In Part (a), a significant number of candidates could not obtain the expression for the area of the container despite drawing a diagram of the figure. Also evident was the fact that those candidates who did not obtain the expression for the area of the container did not go on to use the given expression to solve the problem.

A common error in the differentiation was

$$\begin{aligned}\frac{d}{dx} \left(\frac{216}{x} \right) &= \frac{d}{dx} (216x^{-1}) \\ &= 216 x^{-2} \\ &= \frac{216}{x^2}.\end{aligned}$$

This resulted in candidates getting a negative value for x . Realizing that this was not valid as a solution, they simply changed the value found to a positive value.

Candidates were completely lost with Part (b) of this question. In fact, most of them were awarded one mark for stating that profit is selling price less cost price. Those candidates who proceeded beyond that point incorrectly subtracted

$$\$(\frac{1}{2}x^2 + 50x + 50) \text{ from } \$(80 - \frac{1}{4}x).$$

No attention was paid to the cost of making x articles per day and the relationship of the selling price of each article. Some students attempted to solve the equation

$$\$(\frac{1}{2}x^2 + 50 + 50) = \$(80 - \frac{1}{4}x). \text{ Much work is still needed in the area of modelling.}$$

Most candidates gained full marks on Part (c) of this question. However, as commonly seen in the classroom, expressing answers in specific terms, for example, significant figures, decimal places or exactly, is either ignored by candidates or perhaps not clearly understood. Despite the instruction, ‘to the nearest dollar’, in bold print, candidates gave their answers to one and two decimal places.

Few candidates recognized the geometric progression and used this method to calculate the total amount of rent paid.

Answers: (a) (ii) $x = 3$

(b) (i) $\$(30x - \frac{3x^2 - 50}{4})$

(ii) $x = 20$

(iii) $\$250$

(c) $\$443$

PROJECTS

In general, the standard of the project reports was very high and the topics selected were both interesting and relevant to the respective syllabus content.

Candidates exercised good initiative and judgement in reporting their findings. The variety of tasks executed was very encouraging, although, as has been the case in the past, there was a definite bias towards statistical analysis. A few projects also included applications of differential equations.

The assessment criteria appeared to be well understood by the teachers and candidates, and achievements levels were rewarded in a consistent manner, except for a few cases where there was evidence of overgenerous compensation. In such instances, the mathematical model was developed but the data generated were insufficient to ensure the reliability of the model. In a few cases, the time constraint seemed to have militated against the execution of what would otherwise have been interesting conclusions to well-designed project proposals.

PAPER 03/2 SECTION A (Module 2.1: Calculus II)

Question 1

Specific Objective(s): (a) 2, 5; (b) 3, 4, 8; (c) 6

The topics tested in this question were exponential function, differentiation and integration.

Part (a) of the question was well done. The few candidates who took the examination performed well here.

Good responses were received for Part (b).

Part (c) was not well done. In too many instances, the question was treated as an equation to be solved. Candidates experienced tremendous difficulty in using

$\frac{dy}{dx}$ from Part (a) to obtain the exact result to this question.

The success rate was very low on Part (d). Very often $\sqrt{1 + \left(\frac{dy^2}{dx}\right)}$ was equated incorrectly to $1 + \frac{dy}{dx}$.

Answers: (b) $\frac{dy}{dx} = \frac{1}{2} (e^x - e^{-x})$

SECTION B (Module 2.2: Sequences, Series and Approximations)

Question 2

Specific Objective(s): (a) 1, 2

In this question, the topics tested were sequences, series and mathematical modelling.

In Part (a), almost all the candidates were able to find p_1 and p_2 .

In Part (b), few of a very small number of candidates were able to express p_{n+1} in terms of p_n .

Generally, Part (c) was not well done but there was one good response to this question.

Part (d) was done satisfactorily.

Answers: (a) $p_1 = 1100$, $p_2 = 1220$

(b) $p_{n+1} = (1.2) p_n - 100$

(d) $n = 17$

Question 3

Specific Objective(s): (a) 1, 2, 3, 4, 7, 10, 12

This question tested probability.

There were some good responses for Parts (a) and (b).

In Part (c), candidates experienced great difficulty in determining the number of favorable cases in this problem. It seemed not to be known that for any number formed from the digits 1, 2, 3, 4, 5 to be divisible by 5 then its last digit must be 5.

Answers: (a) (i) 0.35 (ii) 0.43 (iii) 0.2

$$(b) (i) P(B) = \frac{3}{4}, P(A \cup B) = \frac{5}{6}$$

$$(c) \text{Prob.} = \frac{1}{5}$$

**MODEL ANSWERS
UNIT I, PAPER 01**

Question 2

(a) Given that $x > y$ and $k < 0$ for the real numbers x , y and k , show that $kx < ky$.

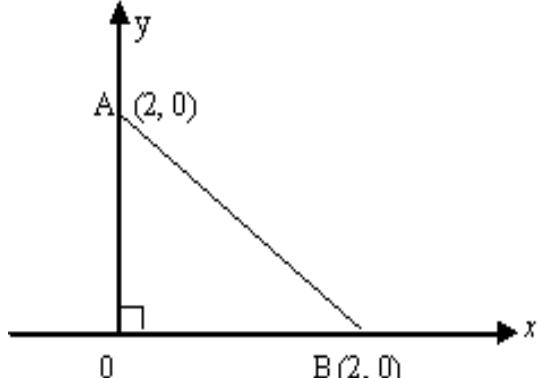
$$\begin{aligned}\text{Solution: } x, y \in \mathbb{R} \text{ and } x > y \Rightarrow x - y > 0 \\ \Rightarrow k(x - y) < 0 \quad \text{for } k < 0 \\ \Rightarrow kx - ky < 0 \\ \Rightarrow kx < ky\end{aligned}$$

$$\begin{aligned}\text{OR} \quad x, y \in \mathbb{R} \text{ and } x > y \Rightarrow y - x < 0 \\ \Rightarrow k(y - x) > 0 \quad \text{for } k < 0 \\ \Rightarrow ky - kx > 0 \\ \Rightarrow ky > kx \\ \Rightarrow kx < ky\end{aligned}$$

Question 15 (b)

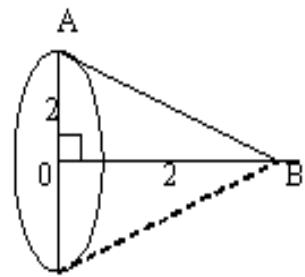
Solution: Reqd. volume = $\int_0^2 \pi y^2 dx$

$$\begin{aligned}
 &= \int_0^2 \pi |x-2|^2 dx \\
 &= \int_0^2 \pi (x-2)^2 dx \\
 &= \pi \frac{(x-2)^2}{3} \Big|_0^2 \\
 &= \frac{8\pi}{3} \text{ units}^3
 \end{aligned}$$



OR Base radius OA = 2 units
 Height = 2 units
 Reqd volume = volume of cone

$$\begin{aligned}
 &= \frac{1}{3} \pi (2^2) (2) \\
 &= \frac{8\pi}{3} \text{ units}^3
 \end{aligned}$$



UNIT II, PAPER 01Question 9

$$\text{Solution } (1 + ax)^n = 1 + n a x + \frac{n(n-1)}{2} a^2 x^2 + \dots$$

$$\Rightarrow na = 6, \frac{n(n-1)}{2} a^2 = 16$$

$$\text{Now, } \frac{n(n-1)}{2} a^2 = 16 \quad \Rightarrow na \left[\left(\frac{n-1}{2} \right) a \right] = 16$$

$$\Rightarrow 6 \left(\frac{n-1}{2} \right) a = 16$$

$$\Rightarrow 3(n-1)a = 16$$

$$\Rightarrow 3na - 3a = 16$$

$$\Rightarrow 18 - 3a = 16$$

$$\Rightarrow a = \frac{2}{3}$$

$$\Rightarrow n = 6 \times \frac{3}{2} = 9$$

Question 6 (c)

Solution

The rent paid after 15 years is

$$\$ \left[64 + 64 \times \left(\frac{7}{8} \right) + 64 \times \left(\frac{7}{8} \right)^2 + \dots + 64 \times \left(\frac{7}{8} \right)^{14} \right]$$

$$= \$ 64 \left(1 + \frac{7}{8} + \left(\frac{7}{8} \right)^2 + \dots + \left(\frac{7}{8} \right)^{14} \right)$$

$$= \$ 64 \left(\frac{1 - \frac{7^{15}}{8}}{1 - \frac{7}{8}} \right)$$

$$= \$64 \times 8 \times 0.865063$$

$$= \$443 \text{ to nearest \$}.$$

CARIBBEAN EXAMINATIONS COUNCIL

**REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION**

MAY/JUNE 2005

PURE MATHEMATICS

**Copyright © 2005 Caribbean Examinations Council
St Michael Barbados
All rights reserved**

PURE MATHEMATICS

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2005

INTRODUCTION

This is the seventh year that Mathematics Unit 1 was examined on open syllabus and the sixth year for Unit 2. The revised syllabus for each Unit was examined for the first time. Just over two thousand, five hundred candidates registered for Unit 1 and nine hundred registered for Unit 2.

Each Unit comprised three papers, Paper 01, Paper 02, and Paper 03. Papers 01 and Paper 02 were assessed externally and Paper 03 was assessed internally by the teachers and moderated by CXC.

Paper 01 in each Unit consisted of 15 compulsory, short-response questions. There were five questions in each of three sections, Section A, B and C corresponding to Modules 1, 2 and 3 respectively. The maximum number of marks for each question ranged from six to ten. In each Unit, candidates could earn a maximum of 120 marks for this paper representing 40 per cent of the assessment for the respective Unit.

Paper 02 in each Unit consisted of six compulsory extended response questions. There were two questions in each Section/Module. Each question was worth 20 marks. Candidates could obtain a maximum of 120 marks on Paper 02, representing 40 per cent of the assessment for the Unit. For each paper, marks were awarded for Reasoning, Method and Accuracy.

Paper 03 was compulsory. It was assessed internally by the teacher and moderated by CXC. For each Unit, candidates wrote three tests which assessed individually or collectively the three Sections/Modules. At least one test must exclusively assess mathematical modeling. This paper represented 20 per cent of the assessment for the Unit. This is the third year that Paper 03/2 was written by private candidates for Unit 1 and the second year for Unit 2.

GENERAL COMMENTS

UNIT 1

The performance of candidates continues to show improvement over previous years. This is particularly encouraging this year since a revised syllabus was being examined for the first time. Questions on topics involving Curve-Sketching, Coordinate Geometry of the straight line and circle, the Factor Theorem, Mensuration of Conic Sections, and Basic Differential and Integral Calculus were well done. Occasionally, amidst this broad coverage, there were pockets of weakness within these topics, this suggested that more practice is needed, at the level of preparation of candidates for the examination(s), to strengthen performance. As observed last year, general algebraic manipulation also requires further attention in order to eliminate faulty follow-through in problem solving.

A few topics continue to present challenges to candidates. These are Inequalities, Indices, Limits and Continuity/Discontinuity, and aspects of the Integral Calculus. Such topics should be targeted at the school level for special treatment. It was also observed that substitution in all forms was not treated very well.

The results showed some excellent performances in both papers and were generally very encouraging.

DETAILED COMMENTS

**UNIT 1
PAPER 01
SECTION A
(Module 1: Basic Algebra and Functions)**

Question 1

Specific Objective(s): (d) 4, 6; (c) 1, 3, 4.

The question sought to examine the candidates' ability to relate to its equation certain characteristics of the given graph of $f(x)$ such as its intercepts with the axes. The Remainder/Factor theorem was also relevant to the complete solution.

- (a) Many candidates substituted $x = 0$ and $x = 2$, obtaining expressions in terms of k and h . These candidates did not use the graph as required and failed to deduce the value of k as the y -intercept.

The mathematical use of words including STATE should be made clear to candidates in their tutorials. This would serve to make candidates aware of the requirements that such specific words may infer.

- (b) Some candidates followed through with the correct value of k from Part (a) and easily found the value of h .
- (c) Having obtained the correct value for k , and the correct value for h , candidates proceeded to use the remainder / factor theorem to factorize the expression completely. No instance was observed where candidates deduced that $f(2)$ resulted in coincident roots, thus allowing for determining the third factor by simple inspection.

- Answer(s): (a) $f(0) = 12$, $f(2) = 0$
(b) $h = -1$, $k = 12$
(c) $f(x) = (x-2)^{[OBJ]} (x+3)$

Question 2

Specific Objective(s): (a) 1, 3, 4, 5, 6, 8; (b) 1, 2

This question tested basic properties of two real numbers, both of which are negative, or one negative and one positive, and of the modulus function. Inequalities with quadratic expressions were also involved.

- (a) A significant number of candidates failed to apply correctly the concept of a modulus function. Some of them separated the inequality to read

$$x^{\text{OBJ}} - 3 < 2^{\text{OBJ}}$$

and proceeded to square both sides. Having obtained a quartic equation, they could not make a suitable substitution for $x^{\hat{d}}$ and hence find the correct solution set. Some candidates used the positive value for x only and gave the incorrect range of values of x . Other candidates, who used both positive and negative values for x , gave the solution set over both ranges of values of x , and did not use the fact that $x < 0$ was given condition.

- (b) Many candidates used suitable real numbers to represent k , x and y . Very few used a purely algebraic approach.

Answer(s): (a) $-3 < x < 0$

Question 3

Specific Objective(s): (c) 1; (e) 1, 2.

Surds and small powers of the variable x , its reciprocal OBJ and $(x + \text{OBJ})$ were the focus of this question.

- (a) It was most surprising to see a significant number of candidates who found it difficult to rationalize denominators in surd form. This question should not be beyond the mathematical skills of candidates at this level. Few obtained the maximum mark.
- (b) (i) The expansion of OBJ proved difficult for some candidates. Errors were made in obtaining the correct expansion $x^2 + 2 + \text{OBJ}$.
- (ii) Several attempts at simplification of OBJ some of which involved the term containing 3OBJ failed. Few candidates were able to obtain the maximum for this question.

Answer(s): (b) (ii) $x^3 + \text{OBJ} = -2$

Question 4

Specific Objectives: (f) 1, 7(i), 8, 10

The theme of this question was simultaneous equations in two unknowns, one equation linear and one quadratic.

Except for minor errors in simplification the overall performance was very good. It was not uncommon, however, to see candidates stating that $(2y - 3)^2 = 4y^2 \pm 9$.

Answer(s): $x = 1, y = 2$ and $x = -5/2, y = 1/4$

Question 5

Specific Objectives: (d) 1, 6; (f) 1

This question tested the basic properties of injectiveness, substitution and solution of equations in a single unknown.

- (a) Candidates showed a satisfactory understanding of the concept of a one-to-one function.
- (b) In spite of this topic being covered at CSEC level, many candidates found it difficult to obtain the correct expression for $f[f(x)]$.

Answer(s): (b) $x = 2$

SECTION B (Module 2: Plane Geometry)

Question 6

Specific Objective(s): (a) 1, 2, 3, 4, 7(ii), 8

This question tested some of the basic properties of perpendicular lines, as well as the coordinates of a point which divides a line segment in a given ratio.

- (a) This part was generally well done. Some candidates seemed unaware of the perpendicularity relationship between gradients. Some did not use the mid-point correctly.
- (b) This part was generally not well done. Some candidates used an incorrect formula ($p = \frac{a+b}{2}$ instead of $\frac{a+b}{2}$). Others used an alternative solution $p = b + \frac{1}{4} [obj]$ or $p = a + \frac{3}{4} [obj]$.

A few candidates observed that p was the mid-point of MP and used the formula $(p = \frac{1}{2} (m + b))$.

Common error: $\frac{a+b}{2}$

Answer(s): (a) (i) $M \equiv (1, -1)$ (ii) gradient = $-4/5$ (iii) equation:
 $4y = 5x - 9$
(b) Cord. of P: $(\underline{\underline{\underline{\underline{\underline{}}}}})$

Question 7

Specific Objective(s): (b) 18, 20, 21

This question examined the minimum value of a trigonometric function by converting it to the form $R \cos(\theta + \alpha)$. Most candidates attempted this question, but very few obtained the maximum marks.

A common source of error was the incorrect expansion of $R \cos(\theta + \infty)$, leading to $\infty = -35.3^\circ (0.615^\circ)$ instead of $\infty = 0.615^\circ$. A few candidates wrote down the minimum value of $f(\theta)$ as $\underline{\underline{\underline{\underline{\underline{}}}}}$, instead of $\underline{\underline{\underline{\underline{\underline{}}}}}$.

Another common error was solving $\underline{\underline{\underline{\underline{\underline{}}}}} \cos(\theta + 0.615) = -1$ instead of solving $\theta + 0.615 = \pi$. The use of degrees instead of radians was a common feature.

Answer(s): (a) $f(\theta) = \underline{\underline{\underline{\underline{\underline{}}}}}$ as $(\theta + \infty), \hat{\imath} = 0.615^\circ$
(b) minimum $f(\theta) = \underline{\underline{\underline{\underline{\underline{}}}}}$
(c) $\theta = 2.53^\circ$

Question 8

Specific Objective(s): (c) 1, 4, 5

This question dealt with the condition for the existence of complex roots of a quadratic equation as well as the expression of a complex number in the form $x + iy$, $i^2 = -1$.

Most candidates attempted this question.

(a) Most candidates were aware of the discriminant “ $b^2 - 4ac$ ” but some tried to solve $b^2 - 4ac > 0$. Some solved the inequality $k^2 < 9$ by writing down the solution $-3 < k < 3$, but failed to observe that $k^2 = \underline{\underline{\underline{\underline{\underline{}}}}}$, for all k , hence $\underline{\underline{\underline{\underline{\underline{}}}}} < 9$ implies $\underline{\underline{\underline{\underline{\underline{}}}}} < 3$.

A few candidates solved the inequality $k\tilde{o} < 9$ by using the graphical method. Some candidates wrote down the solution of the quadratic equation, using the quadratic formula, and then proceeded to solve the inequality. Again, a few candidates presumed that there were two equal complex roots and wrote down $(x+iy)\tilde{o} = x\underline{\underline{\underline{\underline{\underline{}}}}} 2kx + 9$.

Most candidates attempted this part and obtained full marks. Some used the alternative method $\underline{\underline{\underline{\underline{\underline{}}}}} = x+yi$ and cross multiplied to obtain real and imaginary

components. A few candidates did not separate the real imaginary components in their response.

Answer(s) : (a) $-3 < k < 3$

(b) $\tilde{\theta} + \text{[OBJ]}$

Question 9

Specific Objective(s) : (d) 1, 2, 3, 4, 5, 7, 8

This question dealt with the expression of a vector in the form $xi+yj$ and of a point as a position vector.

(a) Most candidates attempted this part and scored full marks. Some candidates used an incorrect i, j notation example $\text{[OBJ]} = \text{[OBJ]} = \text{[OBJ]}$

They very rarely used the notation $\text{[OBJ]}, \text{[OBJ]}, \text{[OBJ]}$ inserted of A,B,C.

(b) This part was generally poorly done.

Common Errors: $\text{[OBJ]} = A + B$
 $= Ax B$
 $\text{[OBJ]} = C + D = C \text{ [OBJ]} D$

The use of the correct vector notation was generally poor. Many careless mistakes such as $-\underline{i} - 2\underline{j} + 2\underline{i} + 5\underline{j} = -\underline{i}$ and $3\underline{j}$ and $-\underline{i} - 2\underline{j} + 5\underline{j} = \underline{i} - 3\underline{j}$ were made.

The following were made alternative method was also used.

$$\underline{d} = \underline{b} - \underline{a} + \underline{c}$$

$$\underline{d} = \underline{b} - \underline{a} + \underline{c} = (2,5) - (1,2) + (0, -4)$$

$$= (1, -1) = \underline{i} - \underline{j}$$

Answer(s) : (a) $\underline{i} - 2\underline{j}, 2\underline{i} + 5\underline{j}, -4\underline{j}$

(b) $\underline{i} - \underline{j}$

Question 10

Specific Objective(s): (d) 9, 10; (b) 5, 13, 19

This question dealt with parallel vectors that are expressed in terms of a position in i , j form.

The very large number of no responses and zeros, attested to the unfamiliarity of candidates with this form of question. Many candidates did not know how to respond. Several candidates formed the trigonometric equation correctly but thereafter were unable to solve it correctly.

Several candidates were familiar with the dot product.

A significant number of candidates claimed that the vectors were either equal or perpendicular, that is, $\underline{a} = \underline{b}$ or that $\underline{a} \cdot \underline{b} = 0$.

Answer(s) : $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$

SECTION C (Module 3: Calculus 1)

Question 11

Specific Objective(s): (a) 1, 2, 3, 4; (b) 5

This question provided a means of differentiating $y = \tilde{o}$, with respect to x , from principles, by expressing

$\underline{obj} - \underline{obj}$ as \underline{obj} from the given result

$$\underline{obj} + \underline{obj} \cdot \underline{obj} = h.$$

- (a) Many candidates did not rearrange the terms in the given result to express \underline{obj} as \underline{obj} and did not benefit from the lead provided. Those candidates who did, obtain the majority of the credit for this part of the question.
- (b) The majority of candidates did not see the connection of this part with part (a).

Question 12

Specific Objective(s): (a) 8, 9, 10.

This question tested discontinuity of a rational function over the real numbers and the location of roots in a closed interval. Many candidates showed familiarity with the topics.

- (a) Most candidates knew the condition for discontinuity of the function but a few did not factorize the quadratic expression $x^2 - 2x - 8$ correctly.
- (b) Very many candidates used the values $f(2)$ and $f(3)$ correctly to set up the change of sign of $f(x)$ in the interval but did not use the Intermediate Value Theorem and the continuity of $f(x)$ to the task.

Answer(s) (a) $x = 4$, $x = -2$

Question 13

Specific Objective(s): (b) 2, 7, 8, 18, 19, 25

The question tested the candidates' ability to find the first and second derivatives of a polynomial function and to relate these derivatives to using the gradient of the curve and to finding the equation of the normal at a point P on the curve.

- (a) Several candidates were successful in answering this part correctly but a few had difficulty in finding the value of the constant k .
- (b) Most candidates who attempted this question obtained the correct answer.
- (c) There were several good answers to this part; however, some candidates lost marks by using the wrong gradient, others, through faulty algebraic manipulation.

Answer(s)

- (a) $k = -8$
- (b) [OB] at $x = 1$
- (c) Equation : $2y = x - 23$

Question 14

Specific Objective(s): (b) 14, 15, 16, 17, 18, 19, 20, 21.

The question tested knowledge about stationary points and the nature of such points of a polynomial function f .

- (a) This part was very well done by many candidates. Some difficulties were

experienced by a few candidates due to faulty algebraic manipulation.

- (b) As for part (a) this part was well done but errors were made by a few candidates in obtaining the second derivative, and some confusion in distinguishing the maximum and minimum points was evident.

Answer(s)

- (a) Stationary points: $(3, -21)$, $(-1, 11)$
(b) Minimum at $x = 3$, maximum at $x = -1$.

Question 15

Specific Objective(s): (b) 23; (c) 3, 4, 7(i), 8, 10(i).

The question tested the candidates knowledge on areas between a curve and the x-axis for a specific range of values of x .

- (a) The majority of candidates were successful in obtaining the coordinates of the points P, Q and R.
(b) There were several very good attempts at this part of the question. The main stumbling block in obtaining correct answers resulted from candidates not taking the absolute value of the area O Q R.

Answer(s)

- (a) $P \boxed{\text{OBJ}} (-1, 3)$, $Q \boxed{\text{OBJ}} (1, -1)$, $R \boxed{\text{OBJ}} (2, 0)$
(b) Area = $\boxed{\text{OBJ}}$ square units.

**UNIT 1
PAPER 02
SECTION A
(Module 1: Basic Algebra and Functions)**

Question 1

Specific Objective(s): (c) 1; (d) 4, 6, 9; (e); (f) 6, 7.

The question tested the candidates' knowledge on graphs of the modulus function, indicies and surds.

- (a) There were many good solutions to this part of the question. Several candidates obtained full marks for this part.

- (b) There were some very good solutions but equally so many attempts failed because of faulty manipulation of indices.
- (c) This part was not well done, but a few candidates did present good solutions gaining full credit.

Answer(s)

- (a) Table Values (x , $f(x)$) : (-1, 3), (0, 0), (1, 1), (3, 3)
- (b) $k = 3$ or :
- (c) (i) $x = 2$ (repeated),
(ii) $\boxed{0}$

Question 2

Specific Objective(s): (a) 10; (f) 3; (g)

This question tested the principle of mathematical induction, solutions of simultaneous equations and solutions of inequalities involving rational functions.

- (a) This part of the question was not well done. The initial step for $n = 1$ was as far as many candidates were able to reach in setting out the induction process. More practice in proceeding from $n = k$ to $n = k+1$ appears to be needed.
- (b) There were a few very good efforts at this part but in general it was not well done.
- (c) This part of the question proved to be the most manageable for the candidates although errors in the algebra marred the attainment of correct answers in some cases.

Answer(s)

- (b) (i) $= p-2$
(ii) solution set = $\boxed{0}$
- (c) $2 < x < \boxed{0}$

SECTION B
(Module 2: Plane Geometry)

Question 3

Specific Objective(s) : (a) 1, 2, 3, 7, 11, 14, 8.

This question dealt with the geometric of the circle and tested knowledge of the centre, radius and tangent.

- (a) Very well done. The vast majority of the candidates found this part easy.
- (b) As for part (a), in view of the connection between the two parts.
- (c) Well done, but for a small number candidates who made a few arithmetic blunders.
- (d) Very many good answers were presented. That the tangent at A was perpendicular to OA was not appreciated by a few candidates.
- (e) This part was found to be easy when parts (a) to (d) above were well established.

Answer(s)

- (a) $a \approx 1, b = -1$ (b) (i) $O \approx (1, -1)$, (ii) radius = 5 units
- (d) equation: $4y + 3x = 24$
- (e) $B \approx (-2, -5)$.

Question 4

Specific Objective(s): (b) 1, 3, 12, 13, 14, 16, 17; (c) 4, 5, 6; (d) 4, 9, 10, 11 9(i).

The question tested elementary properties of the sector and cone, as well as trigonometric functions, vectors and basic manipulation of complex numbers.

- (a) Candidates found this part easy and obtained full marks for their efforts.
- (b) (i) There were many good derivations of this identity.
- (ii) The majority of candidates were not able to obtain the condition for the perpendicularity of \underline{a} and \underline{b} in terms of θ and so did not see the relevance of part (b) (i) above. At that point, their efforts as solution fell apart.
- (c) There were several very good solution but there were too many faulty algebra produced wrong solutions (e.g. $i\hat{\theta} = -1$).

Answer(s)

- (a) (i) arc ABC = [OBJ] cm
(b) [OBJ] = [OBJ]
(c) $z = 17 + 6i \Leftrightarrow 18.0$

SECTION C (Module 3: Calculus 1)

Question 5

Specific Objective(s): (a) 3, 4, 6 (ii); (b), 4, 7, 8, 9, 10, 18 19; (c) 4, 7, 10(ii)

This question tested the topics of limits involving the trigonometric function $\sin kx$, differential equations and volume generated by rotating the area between curves about the x -axis.

(a) (i) Many candidates showed a lack of knowledge of the concepts of limits. Instances of [OBJ] or [OBJ] were seen as responses.

- (ii) The substitution $x = [OBJ]$ was used to obtain [OBJ] but candidates were unable to obtain this limit, having failed at (i).
(iii) This part did not prove any easier than (i) and (ii). The concepts of products and quotients of limits escaped many candidates.

(b) This question was well done.

- (i) There was no difficulty of note in the responses to this question.
(ii) Interestingly many candidates quoted the formula for the volume of solid generated by rotating a function of x -axis as $2\pi [OBJ]$. Some candidates calculated the volume as $\pi [OBJ]$ and failed to subtract the volume of the cone which may have been either by simple mensuration or by integration.

Answer(s):

- (i) [OBJ]
(ii) [OBJ]
(c) (i) P[OBJ]
(ii) $V = [OBJ]$ cubic units.

Question 6

Specific Objective(s): (b) 8, 10, 12, 13, 14, 17; (c) 4, 8, 9

This question tested the chain rule for differentiating a function of a function, stationary points of functions and values for which functions are increasing/decreasing, and substitution as a means of relating one integral to another.

- (a) Some candidates experienced difficulties in parts of this question. Differentiation of a composite function of x and the multiple angle of trigonometric functions were instances of such difficulties. The simplification of the final answer was also badly done in some cases.
- (b) (i) This question was well done.
- (i) Many candidates stated $\frac{dy}{dx} > 0$ and gave the solution as $x > 5$ and $x > 1$ for y increasing, and $\frac{dy}{dx} < 0$ with $x < 5$ and $x < 1$ for y decreasing. A few instances of a graphical approach were observed.
- (c) (i) This question was poorly done. Not surprisingly the weakness in algebra were evident.
- (ii) Candidates found the deduction $\frac{dy}{dx}$ beyond their understanding. Performances were generally poor.

Answer(s)

- (a) $10x^{\frac{9}{2}}$
- (b) (i) $x = 1$, $x = 5$
- (i) $x > 5$ or $x < 1$
- (c) $\frac{dy}{dx}$

UNIT 1
PAPER 03/B
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

- (a) Specific Objective(s): (c) 1, 6; (f) 5

This part of the question covered completion of the square involving the determination of given constants.

A few candidates obtained full marks on this part. Others who attempted it lost marks through faulty algebra.

Answer(s)

- (a) $h = 2, k = 3$

- (b) Specific Objective(s): (a) 1, 3, 5, 6;

Aspects of inequalities were tested in this part which was well-done. In (ii) some candidates did not seem to know how to use the condition $p^{[OBJ]} + q^{[OBJ]} = 1$.

- (c) Specific Objective(s): (d) 3, 4, 6.

This part of the question used a quadratic functions to represent a mathematics model. Most of the small number of candidates sketched the graph of the function correctly but were unable to determine correct answers to part (ii).

Answer(s)

- (i) a 11

- (b) \$25

- (c) 5

SECTION B
(Module 2: Plane Geometry)

Question 1

- (a) Specific Objective(s): (a) 1, 2, 4, 5, 7 (ii), 9

This part of the question covered the coordinate geometry of the straight line, perpendicular lines and the intersection of straight lines. This part of the question was well done.

Answer(s)

- (a) (i) N [OBJ] (ii) PN = [OBJ] units

(b) Specific Objective(s): (b) 5, 13, 17, 19

A mathematical model was portrayed by a trigonometric function of the parameter t. Few candidates used the complete information in the table to obtain the four equations necessary for the answer and as a consequence there were few correct solutions.

Answer(s)

$$P = 5 \quad , \quad q = 30$$

SECTION C (Module 3: Calculus 1)

Question 3

(a) Specific Objective(s): (a) 3, 5, 7

Limits and continuity were examined in this part of the question.

Candidates showed familiarity with both concepts but poor factorization of the functions (numerator and/or denominator) led to incorrect solutions. There were a few good solutions, nevertheless.

Answer(s)

(i) [09]

(ii) $f(x)$ continuous : :

(b) Specific Objective(s): (b) 5(i)

Differentiating from first principles was examined in this part.

There were good attempts at solution of this part, but several candidates did not use the limit procedure correctly.

Answer(s)

$$2x + 2$$

(c) Specific Objective(s): (b) 9; (c) 5, 6 (ii), 8, 11

The topic of mathematical modelling was tested in this part by means of the differential and internal calculus.

Few candidates obtained the correct differential equation for the model, and of those, only a small percentage was able to solve the problem completely.

Answer(s)

(i) [OBJ]

(ii) 4 minutes

GENERAL COMMENTS UNIT 2

The general caliber of candidates in Unit 2 was of a very high standard with a small number of candidates recording outstanding performances in this first year of a revised syllabus. Indeed, there were very encouraging responses to the additional topics which were examined.

Notwithstanding the semblance of an improved performance, there were a few candidates who were inadequately prepared for the occasion. The lack of preparation showed up in the inability to carry out processes previously covered in Unit 1 particularly, but also in CSEC, and if executed successfully, would have led to greater accomplishment at this level.

Topics such as integration and differentiation in Calculus, simple probability and counting problems, and approximation to roots of equations, evoked favourable responses but general weakness continue to manifest themselves in areas such as mathematical modeling, indices and logarithms, and series. Indeed, substitution also reared its head as a new area of weakness. Most of these deficiencies can be rectified by extended practice on the respective themes. Teachers should also ensure that the preparation of candidates for the examinations is not excessively formula-driven or too heavily dependent on the use of calculators, but that basic principles and processes are emphasized.

On the whole the performance of the candidates was very satisfactory.

DETAILED COMMENTS

UNIT 2 PAPER 01 SECTION A (Module 1: Calculus II)

Question 1

(a) Specific Objective(s): (a) 1

The question tested the ability of candidates to differentiate a function $f\tilde{o}$ and its derivative $f'\tilde{o}$.

(i) Almost all candidates were able to obtain $f'\tilde{o}$ successfully.

- (ii) Most candidates were able to $\lambda f(x)$ and equate it to $f'(x)$. However many of them did not equate the coefficients of the constant term, x_1 , x_2 to obtain a_1 , a_2 and a_3 in terms of $\boxed{a_0}$ and a_0 . Pool substitution was the cause for incorrect results.

Answer(s)

$$a_1 = \boxed{a_0}, a_0, a_2 = \boxed{a_0}, a_3 = \boxed{a_0}$$

- (b) Specific Objective(s): (b) 1, 5

The question tested differentiation of functions of a function.

This question was done well by most candidates who attempted it.

In (i), many candidates obtained the correct solution and in (ii) most candidates used the product rule to obtain the correct result.

Answer(s)

(i) $\boxed{a_0}$

(ii) $\boxed{a_0}$

Question 2

- (a) Specific Objective(s): (b) 5; (c) 4, 6

This question, involved the use of logarithms in the topic of change of base.

- (i) This part was surprisingly very poorly done.
- (ii) Several candidates had difficulty converting from base e to base 10. Many others did not use the fact that $9 = 3^{\log_e 9}$ to obtain a connection between $\log 9$ and $\log 3$.

Answer(s)

(i) $\log_{10} \boxed{a_0}$

(ii) $\log_e 9 = \boxed{a_0}$

- (b) Specific Objective(s): (a) 10

This question examined the use of logarithms in solving equations.

Very many candidates performed well on this question. It was well done.

Answer(s)

$$X = \text{?}$$

Question 3

Specific Objective(s): (b) 5; (c) 4, 6

This question tested the use of substitution in differentiating and integrating trigonometric functions.

- (a) Some candidates did not use the identity $\tan^2\theta + 1 = \sec^2\theta$ and so failed to apply the quotient rule effectively in obtaining the desired result. In some cases, the substitution was not done properly.
- (b) Many candidates did not observe the connection between parts (a) and (b) and hence did not obtain the required result. A very few cleverly used integration by parts for this part of the question.

In many cases, the constant of integration was omitted.

Answer(s)

- (b) $\frac{1}{2}x^2 + C$ (constant of integration).

Question 4

Specific Objective(s): (b) 5, 6

The question covered the topic of the differentiation of products of trigonometric functions and the formation of differential equations involving such functions.

- (a) This part of the equation was very well done by many candidates although some did have difficulties applying the product rule. A few other had problems expressing $\sin 2x$ in the required form.
- (b) Many candidates used alternative forms of $\cos 2x$ to obtain $\sin 2x$. The question was well done.

Question 5

Specific Objective(s): (c) 5, 6, 7

The question tested aspects of the integral calculus including and involving the use of substitution.

- (a) There were many good solutions to this part of the question once the

substitution

$u = x^{\frac{1}{3}} + 1$ or some such was used. There were some problems with the constants in the solution. Some candidates observed that the question was of the form $\int u^2 du$ and dealt with it accordingly.

- (b) Too many candidates did not use the given substitution properly but attempted integration by parts instead. In many such cases the integrand in the variable x was not replaced completely by a new integrand involving u only. Otherwise, there were some good solutions to this part.

Answer(s)

- (a) $3 \ln|u| + C$ (Constant Of Integration)
(b) $-\frac{1}{u} + 9$ (c. o. i.).

SECTION B (Module 2: Sequences, Series and Approximations)

Question 6

Specific Objective(s): (a) 2

This question examined sequences.

Many candidates attempted this question with approximately half of them scoring full marks. Some candidates experienced great difficulty with substitution in the given recurrence relation and this reduced considerably their chances of success.

- (a) Candidates who did not observe the condition $n \geq 0$ had many problems obtaining definitive answers. Some others made mistakes in the algebraic manipulation of the equations involved.
- (b) Many candidates who got past part (a) were able to complete this part successfully.

Answer(s)

- (a) $u_1 = 4$ or $u_1 = -1$
(b) $u_3 = 1$ or $u_3 = -1$

Question 7

Specific Objective(s): (b) 4, 7

This question examined the topic on series with particular reference to the A.P.

Many candidates attempted this question but several of them did not know how to find the n^{th} term a_n . As a consequence there were not many complete solutions.

- (a) This part was not well done. Too many candidates found it difficult to obtain a_n .
- (b) For those who were able to do part (a), this part was well done.
- (c) This part was done very well by those candidates who successfully answered parts (a) and (b).

Answer(s)

- (a) $a_n = 4n - 6$
- (c) (i) -2
(ii) 4

Question 8

Specific Objective(s): (c) 1

This question tested simple properties of equality and sums of binomial coefficients nC_k , $k \geq 1$.

Not many candidates were able to express nC_k correctly in terms of factorials. This suggests that several were only exposed to nC_k in the context of a calculator key. As a consequence, this question its entirety was not done as well as was anticipated.

- (a) These basic building blocks for questions on the binomial coefficients seemed not to be familiar to many candidates.
- (b) Unfamiliarity with the coefficients in (a) expressed in general terms made it difficult for some candidates to do this part successfully.
- (c) There were not many good solutions to this part since it depended on the earlier parts (a) and (b).

Answer(s)

(a) ${}^nC_k = [OBJ]$ ${}^nC_{k-1} = [OBJ]$

Question 9

Specific Objective(s): (c) 1, 3

This question tested the binomial expansion of the expression $x^{\frac{1}{4}}$ with a view to finding the value of the constant a based on a given relationship between specific terms in the expansion.

There were not many good solutions to such a basic question on the binomial expansion of this sort. Several candidates did not write down the coefficients of $x^{\frac{1}{4}}$ and $x^{\frac{3}{4}}$ in the expansion correctly and this spoiled their chances of obtaining correct answers to the constant a . Some others ignored the value -2 for a as the fourth root of 16.

Answer(s)

$$A = \pm 2$$

Question 10

Specific Objective(s): (d) 1, 2

This question examined the topic of errors.

There were many good solutions to this question. On the whole it was quite well done.

- (a) Most candidates were able to find the upper and lower boundaries correctly. A few made minor errors in calculation but were able to follow through quite successfully.
- (b) (i) Several candidates were able to find the maximum and minimum areas correctly, and this led naturally to the correct minimum absolute error of 24.75.
 - (ii) In finding the maximum percentage error, many candidates got the area of 625 but had some difficulty in finding the error of 25.25.

Answer(s)

- (a) $24.5 \leq \text{length} < 25.5$
- (b) (i) Min. abs. error = 24.75
 - (ii) Max % error = $\frac{25.25 - 24.75}{24.75} \times 100\% = 4.04$

SECTION C (Module 3: Counting, Matrices and Modelling)

Question 11

Specific Objective(s): (a) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

This question explored basic counting principles and simple probability.

- (a) Not many candidates used basic counting principles to solve this problem. Many tried to used nC_r or nP_r to find a solution.
- (b) (i) There were several attempts at this part but few applied straightforward techniques to arrive at the correct solution.
 - (ii) This part depended on (a) above, so without the correct solution to (a) and possibly (b)(i), not many correct answers were achieved.

Answer(s)

- (a) 625
- (b) (i) 250
 - (ii) 0.6

Question 12

Specific Objective(s): (a) 4, 5, 6, 7, 10

This question examined the topics of sample spaces and probability.

This question was quite well done. Many candidates were able to gain close to full marks on this question.

- (a) Very well done by many candidates but a few did not write out the full space and lost some credit as a consequence.
- (b) (i) This part was very well done.
 - (ii) Many candidates obtained full marks for this part.

Answer(s)

- (a) [OBJ]
- (b) (i) [OBJ] (ii) [OBJ]

Question 13

Specific Objective(s): (b) 1

This question examined the basic properties of product and transpose of matrices.

Candidates found both parts of question easy and many scored full marks. The question was very well done.

Answer(s)

(a) (i) $AB = \boxed{\text{OBJ}}\boxed{\text{OBJ}}$

(ii) $B^T A^T = \boxed{\text{OBJ}}\boxed{\text{OBJ}}$

(b) $(AB) = \boxed{\text{OBJ}} = B^T A^T$

Question 14

Specific Objective(s): (b) 1, 3, 4, 5, 6

This question covered the topic of augmented matrix, and echelon form as they apply to the solution of systems of linear equations.

There were several attempts by candidates but many candidates failed to obtain complete solutions because of minor errors in arithmetic and algebra. Despite these blemishes, many good solutions were achieved.

Answer(s)

(a) $= \boxed{\text{OBJ}}$

(b) $\boxed{\text{OBJ}}$

(c) $x = 1, y = 0, z = 2.$

Question 15

Specific Objective(s): (c)

The topic examined in this question is mathematical modeling, posed here in the context of the rate of change of the volume V of a sphere of radius r .

The quality of responses to this question was very poor. Many candidates who attempted it found it difficult to obtain the differential equation in (a) without which a solution for r in (b) could not be reached.

Answer(s)

- (a) $\text{[OBJ]} \quad (k \text{ is constant of proportionality})$
- (b) (included on question paper)

UNIT 2
PAPER 02
SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (a) 2, 3, 4, 5, 6, 9, 10

This question tested the relationship from the logarithmic and exponential functions, including their graphical relationship.

- (a) This part was generally well done. Common Errors: reflection about the x or y -axis or the origin. Some who tried to reflect about the line $y = x$, failed to draw the asymptote to the y -axis. Some candidates did not understand reflection about $y = x$.

Most candidates calculated the value of v but not of p .

- (b) This part was very well done. A few candidates used the relationship $a^{\log_b a} = b$ to solve the problem.
- (c) This part was generally well done.
 - (i) Misinterpreting e^{-x} as $-e^x$
 - (ii) Expressing e^{2x} as e^{x^2}
 - (iii) Expressing $3e^{-x}$ as [OBJ]
 - (iv) Using the same letter x in the substitution of e^x i.e. $x = e^x$.
 - (v) Failure to state $\ln 1 = 0$

Many students obtained full marks.

Answer(s)

- (a) (i) Sketch graph
(ii) Reflection of each other about $y = x$, and inverse function
(iii) $v = 33.1 p = -1.00$
(b)
(c) $x = 0$ or $x = \ln 3$

Question 2

Specific Objective(s): (b) 3, 4; (c) 1, 3

- (a) This part tested the use of the chain rule to find the gradient of and normal to, a curve given by its parametric equations.

Most candidates attempted this question and were comfortable using the chain rule

[OBJ] = [OBJ]

Common Errors:

- (i) [OBJ]
(ii) Gradient of normal [OBJ] gradient of tangent = 1.
(b) This part tested the expression and integration of a proper rational function that was decomposed as partial functions.

This part was generally well done. Many candidates found it very difficult to calculate $\frac{dy}{dx}$. Some evaluated it variously as $\ln \tilde{o}$, [OBJ] $\ln x \tilde{o}$, $2x$ kn [OBJ]. Evaluation of a definite integral was understood by most candidates. Many mistakes with the algebra were seen.

Answer(s)

- (a) (i) [OBJ] (ii) [OBJ]
(b) (i) [OBJ]
(ii) [OBJ][OBJ]

SECTION B

(Module 2: Sequences, Series and Approximations)

Question 3

Specific Objective(s): (b) 1, 10; (c) 1, 2, 3

This question examined the summation of series by the method of differences, convergences of geometric series and properties of the binomial coefficients.

- (a) (i) This part of the question was not well done. Some candidates used Mathematical Induction to solve this problem. Many did not realize that by simply writing out the terms for each and summing the desired result would follow.
- (ii) Most candidates did not appreciate that as $n \rightarrow \infty$, $\frac{1}{n^2} \rightarrow 0$.
- (b) Only a few correct solutions to this part were seen. Again, this part was very poorly done. The main weakness was the inability to solve inequalities.
- (c) Most candidates made the two correct substitutions in this part to obtain the required results.

Answer(s):

- (b) $x > 4$ or $-1 < x < 1$ or $x < -4$
- (c) (i) Substitution is $x = 1$
- (ii) Substitution is $x = -1$

Question 4

Specific Objective(s): (e) 1, 3

This question examined the behaviour of a given polynomial function in a specific interval and the Newton–Raphson method for finding approximations of a root of the function in the interval.

Several candidates attempted this question with varying degrees of success. Many of them obtained at least half marks for their efforts.

- (a) (i) Most candidates were able to correctly differentiate the given function but did not recognize a range of values for x was required, in this case, $x \in (-\infty, 0)$. They also did not pay attention to the word ‘strictly’ when describing the behaviour of the function.
- (ii) This part was well done but the majority of the candidates lost credit for

neglecting to mention the I.V.T. and the continuous nature of the function over the interval.

- (iii) Again the continuous nature of the function was ignored.
- (b) Candidates were able to improve their scores with this Newton-Raphson question. The formula and manipulation of variables were well done.

SECTION C **(Module 3: Counting, Matrices and Modelling)**

Question 5

Specific Objective(s): (a) 1-7

The question tested some basic tenets of counting principles as well as fundamental concepts of probability.

The vast majority of candidates attempted this question and many obtained at least 40 per cent of the marks.

- (a) (i) This part was well done.
- (ii) This part was not so well done. Many candidates had difficulty in obtaining the correct denominator in calculating the probability in each case.
- (b) (i) This part was well done.
- (ii) This part presented problems to several candidates.
- (iii) Several partial answers were seen. Many candidates obtained correct answers to parts of this question.

Answer(s)

(a) (i) (ii) (a) [OBJ] (b) [OBJ] (c) [OBJ]

(b) (i) 2024 (ii) [OBJ] (iii) [OBJ]

Question 6

Specific Objective(s): (b) 2; (c)

This question covered determinants and mathematical modelling.

Several candidates attempted this question but there were few completely correct answers.

- (a) Many candidates had difficulty in expanding the determinant to obtain the cubic equation $x^3 - 7x - 6 = 0$. Some who reached that stage did not recognize that the Factor Theorem would be of assistance in solving for x . Not many candidates used any form of the Gauss-Jordan method of solution.
- (b) Not many candidates relished this modelling question. Several interpreted the information in a manner which led to an A.P and not a G.P. Among those candidates who recognized the G.P model some incorrectly used the common ratio r as 0.04 and not 0.96.

Answer(s)

- (a) $x = -3, 1 \text{ or } 2$
- (b) (i) 1200^[09]
- (ii) 30000^[09]
- (iii) 30000^[09]
- (iv) Maximum number = 30,000

**UNIT 2
PAPER 03
(Module 1: Calculus)**

Question 1

Specific Objective(s): (a) 9, 10

This question examined the use of logarithms to investigate a mathematical model in the form of a straight line of fit data derived from a biological experiment.

There were very few candidates registered for this paper. At least one candidate performed creditably.

Answer(s)

- (a) (ii) (a) $\boxed{\text{OBJ}}$ (b) $x = \log_{10} x$ (c) $d = \log_{10} b$
(b) Table 2: $\boxed{\text{OBJ}}$, $(1.48, 3.21)$, $(1.60, 3.40)$, $(1.70, 3.57)$
(c) (i) (a) gradient = $\boxed{\text{OBJ}}$
 (b) $b = 10$,
 (c) $n = 1.5$, $d = 1$
 (ii) $x = 31.6$ for $y = 1800$

**SECTION B
(Module 2: Sequences, Series and Approximations)**

Question 2

Specific Objective(s): (b) 4, 8

This question geometric series in the context of mathematical modelling.

The few candidates who registered for this paper all attempted this question with some success.

Answer(s)

- (a) (i) Beginning of year 4 – $A + AR + AR^{\boxed{\text{OBJ}}} + AR^3$
Beginning of year 5 – $A + AR + AR^2 + AR^3 + AR^4$
- (ii) End of year 4 – $AR + AR^2 + AR^3 + AR^4$

End of year 5 – $A + AR + AR^2 + AR^3 + AR^4 + AR^5$

(b) Beginning of nth year – $A + AR + AR^2 + \dots + AR^{n-1}$

(c) Payout at end of nth year

$$AR + AR^2 + AR^3 + \dots + AR^n$$

$$= AR(1 + R + R^2 + \dots + R^{n-1})$$

$$= \$[\text{OBJ}], R > 1$$

(d) $n = 20, r = 5$ $[\text{OBJ}]$ Amount of payout = \$8975

SECTION C (Module 3: Counting, Matrices and Modelling)

Question 3

Specific Objective(s): (b) 1, 2, 5, 6, 7, 8; (c)

This question covers the topic of matrices in the context of mathematical modelling of a testing process in a chemical plant.

Attempts at this question were partially successful.

Answer(s)

(a) $[\text{OBJ}]$, so A is non-singular

(c) $A^{-1} = [\text{OBJ}]$

(d) $X = A^{-1} Y = [\text{OBJ}]$ $[\text{OBJ}] = [\text{OBJ}]$

PAPER 03
INTERNAL ASSESSMENT
MODULE TESTS

In general the exercises used in the internal assessment tests were relevant to and appropriate for the objectives stated in both Units 1 and 2 of the CAPE Mathematical syllabus. Like in the previous years, most of the questions used in the tests were taken from past CAPE Mathematics examination papers. Unfortunately, there were instances where one examination was set to test all three modules. The range of difficulty also varied significantly. In a number of cases, the tests were very detailed in content and the time allocated for completion was either too long (some were in excess of two hours) or too short. There were a few cases where the tests did not reflect adequately a sufficient coverage of the syllabus and the topic of mathematical modelling was not formally included.

Most tests were submitted with question papers, solutions and detailed marking schemes indicating clearly the distribution of the marks; however, there were far too many samples submitted which did not contain all the components necessary for the conduct of a complete analysis. In some extreme cases, question papers were not submitted in accordance with the guidelines in the syllabus and this made the moderation process far too cumbersome. Strict adherence to the guidelines for module tests is encouraged in order to enhance the efficiency and accuracy of the process.

Assessment was generally of a good standard except in a few cases where the allocations of final marks was difficult to follow. There appeared to be no major problems with the new topics introduced into the syllabus and tested for the first time in 2005.

CARIBBEAN EXAMINATIONS COUNCIL

**REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION**

MAY/JUNE 2006

PURE MATHEMATICS

PURE MATHEMATICS

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION MAY/JUNE 2006

INTRODUCTION

This is the second year that the current syllabus has been examined. There has been a significant increase in the number of candidates writing the examinations, approximately 4430 for Unit 1 compared to 2405 in 2005 and 1500 compared to 885 for Unit 2. Performances varied across the spectrum of candidates with an encouraging number obtaining excellent grades, but there continues to be a large cadre of candidates who seem unprepared for the examination.

GENERAL COMMENTS

UNIT 1

Overall performance in this Unit was satisfactory with a number of candidates excelling in such topics as the Factor/Remainder Theorem, Coordinate Geometry as contained in the Unit, Basic Differential and Integral Calculus, and Curve-sketching. However, many candidates continue to find Indices, Limits, Continuity/Discontinuity and Inequalities challenging. These topics should be given special attention by teachers if improvements in performance are to be achieved. General algebraic manipulation of simple terms, expressions and equations also require attention.

DETAILED COMMENTS

**UNIT 1
PAPER 01
SECTION A
(Module 1: Basic Algebra and Functions)**

Question 1

This question sought to examine, in Part (a), knowledge relating to substitution and the factor/remainder theorems, and in Part (b), to the use of summation via the Σ notation.

- (a) (i), (ii) Candidates continue to demonstrate weaknesses in making substitutions for values in algebraic expressions. Substituting $x=1$ resulted in the expression $1^4 - (p+1)^2 + p$. However, beyond this result some candidates failed to conclude that _____ is a factor of $f(x) = x^4 - (p+1)x^2 + p$, for _____.

Many candidates attempted the division

$$x-1 \overline{)x^4 - (p+1)x^2 + p}$$

but could not proceed beyond the point of obtaining

$$\begin{array}{r} x^3 \\ x-1 \overline{)x^4 - (p+1)x^2 + p} \\ \underline{x^4 - x^3} \end{array}$$

A few candidates earned full marks in this question, demonstrating that they fully understood the concepts of substitution and the remainder/factor theorems.

- (b) The majority of candidates who could not show the required result failed to use the fact that

$$\sum_{r=1}^n 1 = n.$$

Substitution for $\sum_{r=1}^n 3r = 3 \frac{n}{2}(n+1)$ posed little difficulties.

Answer: (a) (ii) $p = 4$

Question 2

This question tested the modulus function, sets and simple identities on real numbers:

- (a) Generally, this question was poorly done. Many candidates solved the equation $|x-4|=h$, using the values of $x=2$ and $x=7$, to obtain values of $h=2$ and $h=3$. However, they could not proceed to the largest value of h .

Some candidates attempted to solve

$$(x-4)^2 = h^2$$

and had difficulties in answering the question. Very few candidates gained full marks on this question.

- (b) Expansion of algebraic expressions continues to be an area of weakness, particularly those involving mixed terms such as x, y and $\frac{1}{2}y$. Those candidates who expanded the

left-hand side correctly and simplified the result were able to find the correct value of k .

Answers: (a) Largest $h = 2$

(b) $k = \frac{3}{4}$

Question 3

The topics examined in this question were indices, surds and inequalities.

(a) (i), (ii) Rational inequalities that involve solving $\frac{f(x)}{g(x)} \leq (\geq)c$, generally result in

candidates solving $f(x) \leq (\geq)c[g(x)]$.

Partial solutions do not give the full range of values of x for which the inequality is true. A few candidates showed a fair understanding of the methods required for finding the correct solutions. Many candidates used the technique requiring a table with change of signs for $ax+b$ and $x+1$ to obtain the range of values of x for the solution. No candidate used the technique of the number line and the region of like signs for greater than or equal to zero (≥ 0) or the region of unlike signs for less than or equal to zero (≤ 0).

(b) It is surprising that many candidates are still finding it difficult to use simple manipulation of indices to evaluate results. Evidence of failing to express $\sqrt{2}$ as $2^{1/2}$ and to simplify $8^{-1/3}$ were observed. Many candidates failed to obtain full marks for showing the final expression $2^4(\sqrt{2})$.

Answer(s): (a) (i) $a = 1, b = -2$; (ii) $x > 2$ or $x < -1$.

Question 4

This question focused on functions and their properties.

(a) (i) Several candidates were able to get the correct values for p and q .

Many candidates were able to simply read the values from the graph by using the boundary values of the domain and the expression for $f(x)$.

(ii) The concept of range was not fully understood by many candidates.

(b) (i),(ii),(iii) Candidates continued to show poor understanding of surjective, injective and bijective functions. Some candidates used the horizontal line test for the injective function. Others used the vertical line test for the surjective function.

Many of the responses were in fact essays explaining when a function is surjective and/or injective. More mathematical examples should be practised by candidates to enhance understanding of these concepts as they relate to functions.

- Answer(s): (a) (i) $p = 2, q = 1$; (ii) $1 \leq f(x) \leq 2$
(b) (i) surjective since $1 \leq f(x) \leq 2$ for each $-1 \leq x \leq 1$
(ii) not injective since $f(-1) = f(1) = 2$
(iii) no inverse since f is not injective

Question 5

This question examined the solution of a system of two simultaneous linear equations with two unknowns.

- (a) Not many candidates used the method of the non-singular matrix to obtain the condition for a unique solution of a system of equations. Candidates used the idea of the coefficients of x and y not being equal in order to find the condition for a unique solution. However, candidates failed to reason that $n \in R$ hence lost marks for this question.
- (b), (c) It was not difficult for several candidates to deduce the values of m for inconsistent and infinitely many solutions, after finding the correct solution to Part (a). Some incorrect values were given for n .

Answer(s): (a) $m \neq 4$ for any $n \in R$

(b) $m = 4, n \neq 2$

(c) $n = 2, m = 4$

SECTION B **(Module 2: Plane Geometry)**

Question 6

This question tested some of the salient properties of the intersection of perpendicular lines and the perpendicular distance of a given point from a given line.

- (a) Most candidates recognized that the gradient of PQ was 2 and correctly found the equation of the line PQ. Few candidates found it difficult to determine the required gradient but using the gradient obtained, they were able to find an equation.

- (b) Several candidates obtained full marks for this part. Those who followed through from Part (a) also recognized that by solving the pair of equations simultaneously, they would find the coordinates of point Q.
- (c) Most candidates used the correct formula in finding the exact length of the line segment PQ. However, the majority, after having found $\sqrt{5}$ proceeded to approximate to 3 significant figures.

In general, the question was well answered.

Answer(s): (a) $y = 2x + 3$; (b) $Q = (1, 5)$; (c) $PQ = \sqrt{5}$ units

Question 7

The question tested knowledge of the cosine formula.

Many candidates did not know how to find $\cos \frac{2\pi}{3}$. Some seemed unfamiliar with the cosine rule while others did not know the meaning of ‘exact length’. For a topic which has been examined so often, too many candidates found the question difficult; however, despite the shortcomings of some candidates, there were a few excellent answers.

Answer(s): (a) $AC = 13$; (b) $AB = 13\sqrt{2}$

Question 8

The focus of this question was quadratic equations in trigonometric functions, and trigonometric identities.

In general, this question was reasonably well-done; however, poor algebraic manipulation hindered several candidates’ progress in both parts. Due care is required in writing signs when a substitution is made. Some candidates did not give the answers in Part (a) in radians while a few had difficulties in solving a quadratic equation involving a trigonometric function.

Answer(s): (a) $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Question 9

The question examined complex numbers and the roots of quadratic equations as they relate to the coefficients of the equations.

The overall performance on the question was poor. Again poor algebraic manipulation was evident and in some instances the roots of the equation were not multiplied to obtain a value for k . In Part (b), multiplying numerator and denominator of the left hand side by $3+4i$ did not occur to many candidates while to some, the rationalisation and equating of real and imaginary parts presented insurmountable challenges. A few candidates did well on this question.

Answer(s): (a) $k = 13$ (b) $u = 11$, $v = 2$

Question 10

This question tested properties of vectors including perpendicularity.

The response to this question was satisfactory, nevertheless, several candidates lost credit for faulty algebraic manipulation. While the basic concepts seem to be understood, errors due to carelessness spoiled the correctness of answers for many.

Answer(s): $x = -3$, $y = 1$

SECTION C

Question 11

The question tested basic knowledge of limits and continuity/discontinuity.

Factorization and substitution were the two main areas of weakness in the efforts of the candidates. Most candidates showed familiarity with the concepts but errors in factorization of either numerator or denominator were the main obstacles towards achieving complete and correct solutions.

Answer(s): (a) -3 (b) $x = \pm\sqrt{3}$

Question 12

The focus of this question was differentiation and the critical values of a function.

Most candidates attempted this question but not many obtained maximum marks. Several did not

seem to be familiar with the term ‘critical’ as it applies to curves. Many errors occurred due to faulty algebra and weakness in simplifications.

Answer(s): (a) critical value at $(4, \frac{1}{8})$, a minimum

(b) $f'(x) = 4x \sin x^2 \cos x^2$

Question 13

This question tested knowledge of stationary points of a cubic curve and of methods used to obtain the equation of a normal to the curve at a given point.

- (a) Many candidates obtained full marks on this part of the question but others had difficulties in separating out the values of x for the stationary points.
- (b) There were many good answers to this question but again faulty algebraic manipulation was problematic in a few cases.

Answer(s): (a) $A \equiv (-2, 16)$, $B \equiv (2, -16)$

(b) Equation of the normal is $y = \frac{1}{12}x$ or $12y = x$

Question 14

This question focused on area under the curve and the integration of simple functions to obtain such an area.

Several candidates obtained full marks for this question. A few had difficulty expressing the shaded region in Part (a) as a difference of two integrals while others ignored $\int_2^3 dx$ in the calculations in Part (c). A few candidates chose to find the area of the triangle as a means of calculating the required area in Part (c).

Answer(s): (a) $\int_2^3 \frac{16}{x^2} dx - \int_2^3 \left(\frac{1}{2}x - 1\right) dx$

(b) $A = 2.42 \text{ units}^2$

Question 15

This question focused on a fundamental principle of definite integrals and the use of substitution towards the evaluation of such integrals.

Not many candidates attempted this question. However, among those who attempted it, a few

earned full marks and most of the others at least 4 marks. The main difficulty seemed to be a misinterpretation of the question which required direct use of

- (i) the given result in Part (a) applied to $f(x) = x \sin x$
- (ii) the identity $\sin(\pi - x) = \sin x$

The only integration involved required candidates to find in Part (c).

**UNIT 1
PAPER 02
SECTION A
(Module 1: Basic Algebra and Functions)**

Question 1

This question tested the candidates' ability to solve two simultaneous equations in two unknowns, one being quadratic and one being linear, as well as to demonstrate the relationship between the sum and product of roots and coefficients of $ax^2 + bx + c = 0$.

The question was generally well answered with many candidates scoring the maximum 20 marks.

- (a) In cases where candidates attempted to make y the subject of either formula there were problems in expanding the brackets after the substitution was made.

For example, $x - 3\left(\frac{6-x^2}{x}\right) + 1$ was **incorrectly** expanded as

$$x - \frac{18 - 3x^2}{3x} + 1$$

- (b) Many candidates
- (i) equated $\alpha + \beta$ incorrectly
 - (ii) represented $\alpha^2 + \beta^2$ incorrectly
 - (iii) failed to put the expression $x^2 + 2x - 2$ equal to zero as required (that is, a quadratic equation).

Answer(s): (a) $x = -\frac{9}{4}, y = -\frac{5}{12}$ and $x = 2, y = 1$

(b) (i) $\alpha + \beta = -4, \alpha\beta = 1$

(ii) $\alpha^2 + \beta^2 = 14$

(iii) Equation:

$$x^2 + 2x - 2 = 0$$

Question 2

This question tested the principle of mathematical induction and the use of the sigma (Σ) notation.

Many candidates performed below average in this question, especially in (b) and (c). Only a very small percentage of candidates earned the maximum 20 marks.

- (a) From the responses it was evident that candidates showed some improvement over previous years. However, few candidates earned full marks.

Some weaknesses observed were:

- The use of the right hand (RHS) only (instead of both LHS and RHS) to establish that $P(1)$ is true.
- The inductive step was incorrectly obtained by some candidates who replaced k with $k + 1$, thus obtaining $P(k+1) = \frac{1}{2}(k+1)(k+2)$ instead of $P(k+1) = \frac{1}{2}k(k+1) + k + 1$.
- Incorrect conclusions involving $\forall n \in Z, \forall n \in R$ instead of $\forall n \in Z^+, \forall n \in N$ or equivalent.

- (b) (i) There were poor responses to this part. Candidates demonstrated a lack of understanding of the concept tested in this part of the question. Some candidates multiplied the expression by 2 thereby **incorrectly** obtaining

$$2\left(\frac{1}{2}n(n+1)\right) \text{ for } \sum_{r=1}^{2n} r .$$

- (ii) This part of the question was poorly done by the majority of candidates who failed to

recognize that $\sum_{r=n+1}^{2n} r = \sum_{r=1}^{2n} r - \sum_{r=1}^n r$.

- (c) Many candidates failed to see that this part was a continuation from (b) (i) and (ii), so that very few correct answers were obtained.

For example, $\sum_{r=n+1}^{2n} r = 100$ was incorrectly interpreted by the weaker candidates as

$$n+1=100, \text{ that is, } n=99.$$

Answer(s): (b) (i) $\sum_{r=1}^{2n} r = n(2n+1)$

(ii) $\sum_{r=n+1}^{2n} r = \frac{1}{2}n(3n+1)$

(c) $n=8$

SECTION B **(Module 2: Plane Geometry)**

Question 3

This question dealt with the geometry of the circle and tested the candidates' ability to find the centre and radius, given the equation of a circle in the Cartesian form, to obtain parametric equations from the Cartesian form and to find the points of intersection of a curve with a straight line.

- (i) This part was generally well done, however, some candidates encountered problems converting the given equation into the form $(x - a)^2 + (y - b)^2 = r^2$. Problems arose when candidates had to complete the square. Candidates who expanded to find 'f' and 'g' usually forgot to change signs for the coordinates of the centre. Others factorised **incorrectly** to find the centre usually by grouping like terms, for example, $x(x+2) + y(y-4) = 4$ to obtain **incorrectly** that radius = 4 and centre = (2,-4).
- (ii) This part was poorly done. Candidates generally did not show any understanding of the concepts involved. A preferred method was substitution of the parametric equations using $\sin^2 \theta + \cos^2 \theta = 1$ to obtain the original equation given in (i).
- (iii) This part of the question was generally well done; however, substitution of $y = 1 - x$ into $x^2 + 2x + y^2 - 4y = 4$ was the preferred method leading to the quadratic equation $2x^2 + 4x - 7 = 0$ from which was obtained the required solution.
- (b) This question was well done. Most candidates, however, did not exhibit a full understanding of 'General Solution' and stopped after finding the principal values of θ .

Answer(s): (a) (i) Centre (-1, 2); radius = 3 units

$$(b) \theta = 2n\pi \pm \frac{\pi}{3}, \quad 2n\pi \pm \pi$$

Question 4

This question covered topics related to trigonometric functions of the form $a \cos x + b \sin x$ and complex numbers.

Most candidates showed familiarity with the concepts involved in both parts of this question, however, in Part (a) the notion of stationary point confused some candidates.

In Part (b), many candidates did not see the connection between (iii), (i) a and b), and the roots of quadratic equations.

Some good answers were received:

Answer(s): (a) (i) $R=4.1, \alpha=14^\circ$;
 (ii) $x=104^\circ$

$$(b) (i) a) 5+i, b) 18-i, c) -\frac{6}{25} - \frac{17}{25}i$$

(ii) Equation: $z^2 - (5+i)z + (18-i) = 0$

SECTION C (Module 3: Calculus 1)

Question 5

This question covered the topics of differentiation from first principles of the function $y = \sin 2x$ and the application of differentiation in obtaining the gradient and equation of a tangent to a given curve.

This question was generally well done by several candidates. Part (a)(iii) proved to be challenging for some candidates, particularly those who experienced difficulties in obtaining A and B correctly in (a) (ii).

In Part (b), some candidates had minor difficulties in differentiating $y = hx^2 + \frac{k}{x}$ but apart from those, many candidates found this part easy.

Answer(s): (a) (i) $\lim_{\delta x \rightarrow 0} \frac{\sin \delta x}{\delta x} = 1$; (ii) $A = 2x + \delta x, B = \delta x$

(b) (i) $h = 2, k = -1$; (iii) Equation: $2y = 12x - 9$

Question 6

The topics tested in this question involved integration by means of the rectangular rule, and differentiation and integration of rational functions.

- (a) (i) Most candidates used the trapezium rule instead of the required rectangular rule.
- (ii) Many candidates were unable to show the required equation for the approximate area S using the given sum $\sum_{r=1}^{n-1} r = \frac{1}{2}n(n-1)$. The factorization of the expression for S in (a) (i) was clearly not recognized by the candidates.
- (b) (i) This part was well done. Students correctly identified that the quotient rule was needed. There were some instances where candidates rearranged the expression for $f(x)$ and used the product rule as an alternative.
- (ii) Candidates' performance on this part of the question was satisfactory. Most students took notice of the "Hence" part of the question and realized that the integral of the expression given involved using a scalar multiple of $f(x)$.
- (c) There were some good results for this question. Some candidates, however, did not notice that the integration process involved a negative index and proceeded to treat the index as a positive number. For most candidates, the 'solving process' was well done.

Answer(s): (b) (ii) $\frac{6}{5} = 1.2$; (c) $u = 2$.

UNIT 1
PAPER 03/B (ALTERNATE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

This question focused on the modulus function, indices, quadratic equations and properties of functions in general.

- (a) (i) There were a few good responses to this part among the small number of candidates, nevertheless, some candidates obtained only one value of x because one or other of the two possible equations $x+4 = 2x-1$, $x+4 = -(2x-1)$ was ignored.

- (ii) Indices continue to present difficulties to many candidates. Too often candidates **incorrectly** obtained $\frac{x^2}{4}$ from the expression $\frac{3^{x^2}}{81}$.

- (b) Candidates seemed not to understand the basic definition of a function and so had difficulty in doing Parts (i) and (ii), and in recognising a function as a set of ordered pairs.

Answer(s): (a) (i) $x = 5$ or $x = -1$ (ii) $x = 4$ or $x = -2$

(b) (i) v maps to both 1, 3 or w is not mapped to any $b \in B$.

(ii) Delete one of the arrows from v to 1 or 3 and map w to any $b \in B$.

(iii) $g = \{(u,1), (v,1), (x,2), (y,4), (w,b), \text{any } b \in B\}$ or
 $g = \{(u,1), (v,3), (x,2), (y,4), (w,b), \text{any } b \in B\}$

SECTION B (Module 2: Plane Geometry)

Question 2

This question tested a linear function model of an experiment, trigonometrical identities and some basic properties of complex numbers.

- (a) The initial value $d = 0$ corresponding to $w = 500$ in the table in (i) was routinely missed by the candidates and this adversely affected the outcomes to the entire Part (a). Few correct answers were received.
- (b) Both identities presented difficulties due mainly to faulty algebra, however, there were one or two correct derivations in both cases.
- (c) Candidates showed some familiarity with complex numbers but a few seemed not to know how to find $\arg(z)$ in Part (ii). None used the fact that $\bar{z} \cdot \bar{\bar{z}} = |\bar{z}|^2$ which connected Part (i) with Part (iii).

Answer(s): (a) (i)

d (day)	0	25
w (gm)	500	1500

(ii) (a) $w = f(d) = 40d + 500$

(b) $w = 900$

(iii) $d = 42$

(c) (i) $|\bar{z}| = 1$

(ii) $\arg(\bar{z}) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

(iii) $\bar{z}\bar{\bar{z}} = 1$

SECTION C (Module 3: Calculus 1)

Question 3

The topics covered in this question were limits, integration and volume of rotation.

- (a) There were some good answers to the limits posed in this part.
- (b) This part was not well done. The separation of $\int_2^3 [f(x) + 4] dx$ did not come readily to all but a very few of the small number of candidates.
- (c) That rotation was around the y-axis was ignored by almost all of the candidates.

Answer(s):

$$(a) (i) \lim_{x \rightarrow 4} \frac{\sqrt{x-2}}{x-4} = \lim_{x \rightarrow 4} \frac{1}{\sqrt{x+2}} = \frac{1}{4}$$

$$\begin{aligned} (ii) \lim_{x \rightarrow 4} \frac{\sqrt{x-2}}{x^2 - 5x + 4} &= \lim_{x \rightarrow 4} \frac{\sqrt{x-2}}{x-4} \lim_{x \rightarrow 4} \frac{1}{x-1} \\ &= \frac{1}{4} \times \frac{1}{3} \\ &= \frac{1}{12} \end{aligned}$$

(b) $\int_2^3 [f(x) + 4] dx + \int_3^5 f(x) dx = \int_2^5 f(x) dx + \int_2^3 4 dx$

(c) (i) $\frac{\pi}{2}$ (ii) π

GENERAL COMMENTS

UNIT 2

In general, the performance of candidates in Unit 2 was of a high standard with a small number of candidates reaching an outstanding level of proficiency. However, there were some candidates who were inadequately prepared for the examination.

Topics in Calculus, simple probability, approximation to roots of equations and series seemed well covered but weaknesses continue to manifest themselves at the level of algebraic manipulation, including substitution, which frustrate the processes required to complete the problem-solving exercises posed in the questions. As recommended in previous years, extended practice on respective themes needs to be undertaken in order to eradicate such deficiencies and raise the level of performance in the identified areas of weakness.

The results on the whole were very encouraging.

DETAILED COMMENTS

UNIT 2 PAPER 01 SECTION A (Module 1: Calculus II)

Question 1

This question examined the use of logarithms in solving equations. The majority of the candidates performed well on this question.

- (a) Candidates performed better in this part of the question than in Part (b). Despite this, some errors were evident. For instance, $(\log x)^2$ was incorrectly interpreted as $\log x^2$ or $2 \log x$.
- (b) Many candidates were aware of the principle/procedure involved, but premature rounding off of values affected the accuracy of the answer. In some cases, $\log 5 - \log 3$ was incorrectly represented as $\frac{\log 5}{\log 3}$ instead of $\log(5/3)$

Answer(s): (a) $x = 4$ or $x = 2$

(b) $x = 3.15$

Question 2

This question tested the candidates' ability to differentiate (using the chain rule or otherwise), combinations of trigonometric and logarithmic functions as well as to find the derivative of $e^{f(x)}$ and $\ln f(x)$, where $f(x)$ is a differentiable function of x .

The question was generally well done by the many candidates who attempted it, with approximately half of them scoring the maximum mark.

- (a) Many candidates omitted the brackets and wrote

$$e^{2x+\sin x} 2 + \cos x \text{ or } 2 + \cos x e^{2x+\sin x}$$

instead of $(2 + \cos x) e^{2x+\sin x}$

- (b) The most common errors were:

$$(i) \quad \frac{d}{dx}(\tan 3x) = \sec^2 x \text{ or } \sec^2 3x$$

$$(ii) \quad \frac{d}{dx}(\ln(x^2 + 4)) = \ln\left(\frac{2x}{x^2 + 4}\right) \text{ or } \frac{1}{x^2 + 4}$$

Answer(s): (a) $\frac{dy}{dx} = e^{2x+\sin x} (2 + \cos x)$

(b) $\frac{dy}{dx} = 3 \sec^2 3x + \frac{2x}{x^2 + 4}$

Question 3

This question tested the ability of candidates to use the concept of implicit differentiation to obtain the gradient of the curve at a point P and to use it to find the equation of the normal at a point P on the curve.

- (a) There were many good solutions to this part of the question. However, some candidates experienced problems with the implicit differentiation and the product rule. The transposing of terms in the equation posed a challenge in a few cases as well.
- (b) This part of the question was generally very well done. About 95% of the candidates were able to obtain the correct gradient of the normal from the gradient of the curve in Part (a), and the subsequent equation.

Answer(s): (a) $\frac{dy}{dx} = \frac{-1}{2}$

(b) Equation: $y = 2x + 5$ or $y - 2x = 5$

Question 4

This question examined the candidates' knowledge about applying the chain rule to find the first and second derivatives of trigonometric functions involving $\sin 2A$ and $\cos 2A$.

The response to this question was very good with many candidates earning the maximum mark.

- (a) Many candidates differentiated $\sin 2A + \cos 2A$ as composite functions. This was efficient and full marks were obtained. However, few candidates transformed $\sin 2A + \cos 2A$ using the trigonometric identities $\sin 2A = 2\sin A \cos A$, and $\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$. This was longwinded and often included errors where candidates were unable to complete the solution successfully. Some candidates used the chain rule effectively in this regard.
- (b) Candidates had a good level of success in obtaining $\frac{d^2y}{dx^2}$ using composite functions. Some candidates used other methods which were complex/complicated, without success. The proof of $\frac{d^2y}{dx^2} + 4y = 0$ was well done using the correct substitution for $\frac{d^2y}{dx^2}$ and y .

Answer(s): (a) $\frac{dy}{dx} = 2\cos 2x - 2\sin 2x$

(b) Since $\frac{d^2y}{dx^2} = -4\sin 2x - 4\cos 2x$

$$= -4y$$

$$\therefore \frac{d^2y}{dx^2} + 4y = 0$$

Question 5

Candidates were required to use given substitutions to integrate functions.

- (a) This part of the question was generally well answered with the majority of candidates earning the maximum of 4 marks for this part. There were some attempts at integration by parts although the question specifically stated "use the substitution given".
- (b) This part proved to be a bit more problematic, as it involved three substitutions. There were difficulties in obtaining $dx = u du$, with some candidates incorrectly obtaining

$$dx = \frac{du}{u} \text{ instead.}$$

The omission of the constant of integration was not frequently seen. The manipulation of indices, and transposition continue to be challenging to some candidates.

Answer(s): (a) $\frac{1}{9} \sin^9 x + k$ (constant of integration)

(b) $\frac{1}{10}(2x+1)^{\frac{5}{2}} - \frac{1}{6}(2x+1)^{\frac{3}{2}} + k$ (constant of integration).

or $\frac{1}{10}\sqrt{(2x+1)^5} - \frac{1}{6}\sqrt{(2x+1)^3} + k$ (constant of integration).

SECTION B

(Module 2: Sequences, Series and Approximations)

Question 6

- (a) This part of the question tested candidates' knowledge and ability to manipulate recurrence relations. Most candidates attempted this question, however, there was a high percentage of candidates who responded incorrectly. The main source of error encountered was that candidates substituted values for n and showed by example that $u_{n+2} = -u_n$ and $u_{n+4} = u_n$. Most candidates showed $u_{n+4} = u_n$ by first finding u_{n+3} and substituting (a long method). They generally did not note the relation

$$u_{n+4} = -u_{n+2} \rightarrow u_{n+4} = -(-u_n).$$

- (b) This part of the question required candidates to write specific terms from the recurrence relations. Most candidates were able to identify correctly the required terms. However, a common error encountered was the use of u_1 as -3 (disregarding the given data; $u_1 = 1$).

Answer(s): (b) $u_1 = 1$ (given), $u_2 = -3$, $u_3 = -1$, $u_4 = 3$

Question 7

This question tested knowledge about summing a geometric series to n terms as well as finding x and d given the sum and product of three consecutive terms, $x-d$, x and $x+d$ of an arithmetic series.

Most candidates attempted this question with about ninety percent gaining at least six marks.

- (a) Most candidates identified the common ratio and were able to use the sum formula for a GP.
- (b) Many candidates who attempted this part obtained the maximum marks. The majority summed the terms and found the value of x . They then substituted this value in the product equation and solved correctly for d . However, a few candidates ignored the fact that $d > 0$ and left their answer as $d = \pm 2$.

Answer(s): (b) (i) $x = 7$, $d = 2$

Question 8

The question tested the use of the binomial term ${}^n C_r$, quadratic equations and inequalities.

Approximately 80% of the candidates attempted this question in which 60% were able to earn full marks. However, there were some candidates who experienced difficulties in parts of this question, for example, with the binomial expansion of ${}^{x-2} C_2$.

Such candidates lacked the basic building block on binomial coefficients which seemed not to be known by some of the candidates. Some candidates also failed to answer the questions asked in Part (a) and went straight ahead to solve for x , for example, ${}^{x-2} C_2 = {}^5 C_2 \Rightarrow x-2=5 \Rightarrow x=7$.

Answer(s): (b) $x=7$

Question 9

This question focused on the expansion of the expression $(1+ux)(2-x)^3$ in powers of x up to the term in x^2 . Candidates were required to find the value of the constant 'u', based on a given condition of a specific coefficient in the expansion.

- (a) Many candidates were able to expand the expression properly but a significant number of them did not stop at x^2 ; instead they expanded the expression completely.
- (b) Several candidates had elementary problems with signs and as a result, they were unable to obtain the correct coefficient of x^2 and lost marks. Thus, although they identified the terms properly, they failed to give the correct answer for u .

Answer(s): (a) $8 + (8u - 12)x + (6 - 12u)x^2 + \dots$

$$(b) u = \frac{1}{2}$$

Question 10

The question tested the candidates' knowledge of intersecting graphs and the algebraic equation represented at the point of intersection. It also covered roots in a specific interval of the real number line.

Most candidates were able to write down the equation required. The majority of candidates were able to determine that the function (equation) had values with different signs at the end points of the interval.

However, a large number of candidates did not point out that the function should be continuous. Some were insightful enough to say that the function was both continuous and differentiable.

While candidates established the existence of the root within the interval through a difference in signs of the value of the functions, a large number of them did not use the words ‘intermediate value theorem’ seemingly unaware that this was the theorem in use.

Generally, this question was well done.

Answer(s): (a) $e^x = -x$ or $e^x + x = 0$

SECTION C **(Module 3: Counting, Matrices and Modelling)**

Question 11

This question tested the candidates’ skills in using the binomial term ${}^n C_r$ in counting problems.

This question was generally well done by the majority of candidates. A few candidates who were unable to answer the question, applied the concept of ‘permutation’ rather than ‘combination’ that was required.

Answer(s): (a) 70 (b) 224 (c) 425

Question 12

The question tested arrangements of objects.

- (a) This question was generally well answered. However, some candidates had difficulty in distinguishing between permutations and combinations.
- (b) This part proved to be more difficult than Part (a) as candidates could not determine the denominator as $(7!)$, in the calculation of the required probability.

Answer(s): (a) (i) 3600 (ii) 2400 (b) $\frac{3600}{7!} = \frac{5}{7} = 0.714$

Question 13

The topics covered related to determinants and methods of their evaluation.

This question was successfully answered by a large majority of the candidates. Two methods were used by the candidates to answer the question. Of these, the more popular method related to the use of ‘minors’.

Question 14

The topics covered in this question were systems of equations, cofactors of a matrix, transpose of a matrix, matrix multiplication and determinants.

- (a) Almost all candidates earned full marks on this part.
- (b) (i) Most candidates confused the matrix of cofactors of the matrix A with the determinant of A.
- (ii) Almost all candidates were able to write the transpose of B but too many were unable to form the matrix product $B^T A$.
- (iii) Few candidates, even among those who calculated $B^T A$ correctly, were able to deduce the $\det A$.

Answer(s): (a)
$$\begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \\ 3 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

(b) (i) $B = \begin{pmatrix} -2 & -1 & 3 \\ -2 & 5 & 3 \\ 0 & -3 & -3 \end{pmatrix}$ (ii) $B^T A = \begin{pmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{pmatrix}$

(iii) $|A| = -6$

Question 15

The question tested the candidates' knowledge of the differential calculus in determining rates of change. Few candidates attempted this question. The majority of those who attempted the question earned very low marks.

- (a) For Part (a) most candidates earned 1 mark.
- (b) For Part (b) most candidates failed to differentiate the function correctly. Most candidates substituted the values of h and r directly into $A = 2\pi r^2 + 2\pi rh$. Candidates attempted to use various methods to answer this question which had no relationship whatsoever to the question.

Answer(s): (a) $\frac{dr}{dt} = 1.5$

(b) $\frac{dA}{dt} = 54\pi \text{ cm}^2/\text{sec}$ when $r = 4, h = 10$

UNIT 2
PAPER 02
SECTION A
(Module 1: Calculus II)

Question 1

This question tested differentiation of a function of a function by means of the product rule as well as mathematical modelling related to an exponential function.

- (a) (i) Basically, most candidates were able to apply the Product Rule for differentiation. However, many cases were observed where the differential of

$\ln^2 x$ was **incorrectly** given as $\frac{2}{x}$ and $2 \ln x$.

In addition, many candidates lost marks for inability to simplify the differential to show the required result.

- (ii) Those candidates who attempted to apply the Product Rule for the result at Part (a) found it difficult to differentiate the expression $\ln x (3 \ln x + 2)$. Many candidates used the result at Part (a) as $3x^2 \ln^2 x + 2x^2 \ln x$ and proceeded to apply the Product Rule. Failure of these candidates to differentiate $\ln^2 x$ correctly resulted in their inability to show the required result. Many weaknesses in algebra were evident in the candidates' work.

- (b) (i) A number of candidates gave the correct answer to this part of the question. Many of them substituted the values of $N = 50$ when $t = 0$ but failed to determine $e^{-r(0)} = 1$. Instead they carried forward the expression $1 + ke$. This resulted in the value of k given in terms of e . Failure to give the limit of ke^{-rt} for large t complicated the answers.
- (ii) A common error seen in the responses to this question was NOT calculating the EXACT value of r . The majority of students got the equation $e^{-r} = 0.2$ but used the calculator to find r .

- (iii) Having obtained the values for k and r there were no difficulties getting the correct answer to this question.

A few candidates gained full marks for the entire question.

Answer(s): (b) (i) $N = 800$ (ii) $k = 15$, $r = \ln 5$ (iii) $N = 714$

Question 2

The topics tested in this question related to partial fractions, integration of rational functions and reduction formulae.

- (a) (i) A few candidates **erroneously** used the result

$$\frac{1+x}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x^2+1}$$

to find the partial fractions.

Other errors included wrong calculations for the values of x which were substituted.

- (ii) A few candidates incorrectly evaluated $\int \frac{-x}{x^2+1} dx$, particularly with respect to the minus sign. Many candidates failed to include the constant of integration.
- (b) (i) Some candidates found this part of the question difficult. It was clear that integration by parts was not fully understood by these candidates. Other candidates had difficulty evaluating.
- $$(xe^x - e^x)_0^1$$
- (ii) Generally most candidates demonstrated that they knew how to proceed with this question. However, writing the integral as

$$I_n = x^n e^x - \int_0^1 e^x nx^{n-1} dx$$

without including the limits of integration in the first integral, resulted in

$$I_n = x^n e^x - nI_{n-1}.$$

Candidates simply quoted the given result for I_n thus obtaining partial credit.

- (iii) Many candidates attempted to find $\int_0^1 x^3 e^x dx$. The attendant difficulties were expected. Some candidates found it very difficult to use the formula for I_n and the link to I_1 .

A few candidates gained full marks for the question.

Answer(s): (a) (i) $\frac{1+x}{(x-1)(x^2+1)} = \frac{1}{x-1} - \frac{x}{x^2+1}$

(ii) $\int \frac{1+x}{(x-1)(x^2+1)} dx = \ln|x-1| - \frac{1}{2}\ln(x^2+1) + k \text{ (const)}$

(b) (i) $I_1 = 1$ (iii) $I_3 = 6 - 2e$

SECTION B

(Module 2: Sequence, Series and Approximations)

Question 3

This question tested arithmetic progressions, mathematical induction and sequences.

- (a) (i) This part required that candidates show that $\sum_{r=1}^m \ln 3^r$ is an arithmetic progression.

Many candidates listed the terms in the progression with a comma between them, for example, $\ln 3, \ln 3^2, \ln 3^3$, and not $\ln 3 + \ln 3^2 + \ln 3^3 + \dots + \ln 3^m$ as was expected. Many answers did not indicate the m^{th} term of the progression. Approximately 80 per cent of the candidates showed working to indicate the first term ‘a’ and the common difference, ‘d’. Approximately 60 per cent of these candidates calculated the numerical value of $\ln 3$, rounding it off to the 3 s.f for the most part, but some also rounded off to 1 s.f.

It must be noted that it is **unnecessary** to use 10 significant figures in the value for $\ln 3$. Almost fifty per cent of the candidates made a final statement to indicate that they understood that the progression was arithmetic in nature.

- (ii) Few candidates added all 20 terms.

Approximately eighty per cent of candidates earned full marks for this question.

- (iii) Many candidates gained full marks for this part.

- (b) (i) Approximately five per cent of the candidates performed exceptionally well on this item.

The remaining ninety per cent had a vague idea as to the steps involved in proof by mathematical induction.

Steps such as:

Prove true for $n = 1$, assume true for $n = k$, were missing for the most part.

The final statement was also missing.

- (ii) Approximately ninety-five per cent of the candidates did not do this question correctly. Most of them did not understand that they should apply the technique of completing the square.

- Answer(s): (a) (i) A.P. with first term $\ln 3$ and common difference $\ln 3$.
 (ii) Sum to 20 terms is $210 \ln 3$.

$$(b) (ii) x_{n+1} - x_n = (x_n - \frac{1}{2})^2 > 0 \\ \Rightarrow x_n < x_{n+1}$$

Question 4

This question tested the ability of the candidate to sketch curves and use the Newton-Raphson method to find the non-zero root of $\sin x - x^2$.

Part (a) was poorly done as all but a few students had problems sketching $y = \sin x$ and $y = x^2$. Most drew $y = x^2$ as either the straight line $y = x$ or as a V-shaped curve. Candidates also had some problems sketching $y = \sin x$. Candidates need to be reminded of the need to use an appropriate scale on each axis as many did not show the point of $(\frac{\pi}{2}, 1)$ for $\sin x$ nor the $(1, 1)$ for x^2 . In too many cases, the points of intersection were way off. Candidates should also have stated the domain for the sketches.

Part (b) was attempted by several candidates although they just mentioned that the 2 curves ‘intersected at 2 points, hence there were 2 real roots’. Hardly anyone wrote that $x = 0$ and $x = \alpha$ (the non-zero root).

Part (c) was also attempted by many candidates. Most of them knew that they were supposed to use the Intermediate Value Theorem, but many used values of $x < 0$ and values other than $\frac{\pi}{4}, \frac{\pi}{2}$. Some used 0 and then said that 0 was positive. Many did not mention that the function was continuous.

Part (d) was attempted by almost all candidates. This was handled well by many, but the common mistake was the use of 0.7 as degrees instead of radians, hence, an incorrect answer was obtained. A few quoted the Newton-Raphson formula incorrectly. Many candidates did not notice that only one iteration was required and went on to find x_2, x_3, \dots and up to x_8 (in a few cases).

Answer(s): (c) Interval: $[0, \frac{\pi}{2}]$, (d) $x_2 = 0.943$

SECTION C

(Module 3: Counting, Matrices and Modelling)

Question 5

This question tested the candidates’ knowledge of simple permutations and probability in Part (a) while Part (b) examined their knowledge based on a probability model.

- (a) (i) The majority of candidates who attempted this question realised that they were dealing with a 4-digit number, although a few considered a 6-digit number. Many candidates realised that the number must start with the digits 3, 4 or 5, but a few included the digit 6.
- (ii) In general this question was poorly answered, however, responses of candidates indicated that they were familiar with the concept of probability.
- (b) (i) It was clear that many candidates were not prepared to apply an unfamiliar formula to calculate the required probability, for example, binomial model. Some candidates exhibited serious misconceptions of probability as values were given outside of the interval $[0, 1]$.

- (ii) Some candidates gave no consideration to possibilities that would have exhausted the sample space and chose to perform longer, rigorous calculations in order to arrive at their solution.

Answer(s): (a) (i) 180 (ii) Prob = $\frac{96}{180} = 0.533$
 (b) (i) 0.543 (ii) 0.457

Question 6

The question tested basic knowledge about the product of conformable matrices and of finding the inverses of invertible matrices. Modelling is also included.

Almost all candidates attempted this question with many good responses. Part (a) focused on standard routine matrix operations while Part (b) focused on mathematical modelling incorporating matrices.

In Part (a), candidates frequently made arithmetic errors in calculating AB and this made it difficult to deduce A^{-1} from (a)(i). Others resorted to alternative methods of finding A^{-1} .

In Part (b), the weaker candidates seemed to have been challenged by the wording of the problem. The majority of them interchanged c, b, z , with p, q, r , and generated meaningless equations.

Another common error made by candidates was attempting to find M^{-1} although it was given in Part (b)(v).

Answer(s): (a) (i) $AB = 4I$ (ii) $A^{-1} = \frac{1}{4} \begin{pmatrix} 2 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

(b) (i) $\cancel{z} - \text{grass} = 2p + 2q + 6r$

(ii) $2p + 4q = c$

$2p + 2q + 6r = z$

$6p + 4q + 4r = b$

(iii) $\begin{pmatrix} 2 & 4 & 0 \\ 2 & 2 & 6 \\ 6 & 4 & 4 \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} c \\ z \\ b \end{pmatrix}$

(iv) $x = M^{-1}D$

(v) $p = 3, q = 6, r = 2.$

UNIT 2
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)

SECTION A
(Module 1: Calculus II, Algebra and Fractions)

Question 1

The question tested the candidates' knowledge of functions, natural logarithms and differential equations based on a simple biomathematical model.

There were some good responses to this question from among the few candidates who wrote the examination. Some experienced difficulty in writing down the differential equation in Part (a), but for those candidates who got past Part (a) the results were encouraging. Part (b)(i) also posed some minor challenges.

Answer(s):

(a) $f'(t) = k f(t)$

(b)(i) $f(0) = 10^6 = 1\ 000\ 000 ; f(2) = 2 \times 10^6 = 2\ 000\ 000.$

(iii) $f(7) = 10^6(2^{\frac{7}{2}}) \approx 11\ 313\ 709$

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 2

This question examined sequences by means of mathematical modelling.

There were some excellent solutions to this problem from the small number of candidates. The only aspects of real difficulty for one or two candidates were Parts (a)(ii) and (iii).

Answer(s): (a)(i) row 1 - entries: Year 4 - $P(1 - \frac{1}{q})^3$; Year 5 - $P(1 - \frac{1}{q})^4$

row 2 - entries: Year 3 - $P(1 - \frac{1}{q})^3$; Year 4 - $P(1 - \frac{1}{q})^4$

Year 5 - $P(1 - \frac{1}{q})^5$

(ii) G.P. with common ratio $1 - \frac{1}{q}$

(iii) $P(1 - \frac{1}{q})^n$.

(b)(i) $q = 5$

(ii) \$6553.60

(iii) $n = 17$

SECTION C
(Module 3: Counting, Matrices and Modelling)

Question 3

The question covered the topic of random selections and row reduction of the augmented matrix of a given system of equations.

The candidates found this question manageable. Only one or two had difficulty concluding the inconsistency of the system in Part (b).

Answer(s): (a) (i) $\frac{56}{2002} = 0.028$ (ii) $\frac{560}{2002} = 0.280$ (iii) 0.972

PAPER 03

INTERNAL ASSESSMENT

Module Tests

Approximately 138 centres (161 Teachers) in Unit 1 and 90 centres (98 Teachers) in Unit 2 were moderated.

In general, there was a marked improvement in the quality, consistency of marks awarded and the presentation of the Internal Assessment (Module Tests) by the teachers throughout the participating territories. Unit 2 was of a very good standard.

Although most of the questions used were taken from past CAPE Pure Mathematics Examination Papers, it was evident that few teachers made a conscientious effort to be original and creative in the tests designed.

This year, the majority of the samples of tests were submitted with question papers, solutions and detailed marking schemes with the marks allocated to the cognitive levels as specified in the syllabus. There were eight of 161 teachers in Unit 1 and two out of 98 teachers of Unit 2 who submitted samples without the required documents, therefore, making the moderation process more difficult.

It should be noted that the majority of teachers satisfied the objectives outlined by CXC CAPE Pure Mathematics Syllabus, Unit 1 and Unit 2. However, some common mistakes observed throughout the moderation process included:

- (1) Test items examined were not consistent with the allotted time for the examination as some were either too long or too short. In one instance, there were 15 items with several parts to be completed by the candidates in 1 hour.

Teachers are reminded that the module test should be of 1 1/2 hours' duration.

- (2) A few teachers continued to award fractional marks.

- (3) On the question papers, teachers should indicate the time allotted, the total score for the examination, as well as instructions for the test.
- (4) In a few cases, teachers tested topics in Unit 1 that were not in the Unit 1 Syllabus, for example, Logarithms, Partial Fractions and Implicit Differentiation. It must be noted that ‘3-dimensional vectors’ is not on the CAPE syllabus.
- (5) A maximum of 5 samples is required for moderation. Please note that additional samples are not needed, unless there is a specific request from the Council. (Refer to FORM PMATH - 2)
- (6) A few teachers are using the incorrect PMATH form to record marks. The scores for the three Modules (1, 2 and 3) scores must be recorded on the same form. (Form PMATH 2-3 Unit 2).
- (7) Marks for the candidates should be clearly identified for each question at the side of the student’s solution; and the total at the top.
- (8) The maximum mark that is allocated to each question on the question paper should be reflected in the allocation of marks on the solution and the mark scheme.

Overall, for the efficiency of the moderation process, teachers should make every effort to adhere to the guidelines provided in the CAPE Pure Mathematics Syllabus.

CARIBBEAN EXAMINATIONS COUNCIL

**REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION**

MAY/JUNE 2007

PURE MATHEMATICS

**CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2007**

INTRODUCTION

This is the third year and the final time that the current syllabus has been examined. **A revised syllabus will be examined in 2008.** There has been a significant increase in the number of candidates writing the examinations with approximately 5021 writing Unit 1 compared to 4 430 in 2006 and 2 521 compared to 1 500 for Unit 2. Performances varied across the entire spectrum of candidates with an encouraging number obtaining excellent grades, but there continues to be a large number of candidates who seem unprepared to write the examinations particularly for Unit 1; a more effective screening process needs to be instituted to reduce the number of ill-prepared candidates.

GENERAL COMMENTS

UNIT 1

The overall performance in this Unit was satisfactory with a number of candidates excelling in such topics as the Factor/Remainder Theorem, Coordinate Geometry, Basic Differential and Integral Calculus and Surds. However, many candidates continue to find Indices, Limits, Continuity/Discontinuity and Inequalities challenging. These topics should be given special attention if improvements in performance are to be achieved. Other areas needing consolidation are general algebraic manipulation of simple terms; expressions and equations; substitution, either as a substantive topic in the syllabus or as a convenient tool for problem solving.

DETAILED COMMENTS

**UNIT 1
PAPER 01
SECTION A
(Module 1: Basic Algebra and Functions)**

Question 1

Specific Objective(s): (c) 1, 2, 3, 4, 6.

This question tested the use of the factor theorem to find factors and to evaluate an unknown coefficient in a polynomial given one of the factors. This question was attempted by almost all candidates. The majority of the candidates performed well on this question.

- (a) The correct answer was obtained by the majority of candidates, however, a few of the candidates substituted incorrectly using $x = -1$ instead of $x = 1$ for the factor $x - 1$.
- (b) This part of the question was generally well done by candidates, with various methods being used to determine the remaining factors such as long division, factor theorem, comparing coefficients and synthetic division. However, many candidates solved (for roots) after factorising and expressed the factors as $x = -1, -2$. Also, a small percentage of candidates expressed the remaining factors in quadratic form, failing to factorise completely.

Some candidates did not recognize that a non-zero remainder after division implied that no factors existed.

Answer(s):

- (a) $p = 2$
- (b) $(x + 1), (x + 2)$

Question 2

Specific Objective(s): (e) 1, 2; (f) 2.

This question examined the candidates' ability to solve an equation as well as to perform operations involving surds.

The question was generally well done by the many candidates (approximately 90 per cent) who attempted it, with approximately half of them scoring the maximum mark.

- (a) Most candidates were able to express 27 as 3^3 . However, approximately 25 per cent of the candidates were unable to express $(3^x)^2$ as 3^{2x} , expressing it instead as 3^{x+2} or, as 3^{x^2} , in a few cases. Despite the errors made with the indices, most candidates showed competence in their ability to equate the indices and to solve for x .
- (b) Most candidates (approximately 75%) recognized the need to rationalize the denominator of the fraction by using its conjugate. However, some careless errors were made in the expansion, grouping and simplification of the surds, hence preventing some candidates from obtaining full marks in this part of the question. There were errors with fundamental operations such as $\sqrt{3} \sqrt{3} = \sqrt{6}$ and $\sqrt{3} \sqrt{3} = 9$.

Answer(s):

- (a) $x = 6$
- (b) $x = 13, y = -7$

Question 3

Specific Objective(s): (d) 1, 3, 9.

This question tested the ability of candidates to obtain coordinates of a given point after translations of the graph $y = f(x)$ and applying the principles of one-to-one functions in solving an equation involving composite functions.

This question was popular among the majority of candidates.

- (a) Candidates were aware that a translation was involved. However, many failed to effect the correct translation; applying it to the wrong coordinate in some cases and to the wrong direction in other cases.

- (b) The proof that $f(x)$ is a one-to-one function was poorly done by the majority of candidates. Much practice is needed with these proofs from first principles, that is, the general proof (instead of the proof for specific/particular values of x which was popular among candidates). Some candidates used proof by counter example for NOT one-to-one.

In the second part, the more confident candidates opted for the “hence” approach, while the majority used the “otherwise” approach in solving for x . The composite functions posed a challenge for the weaker candidates, with many simplifying and substituting incorrectly, for example, replacing x with $(3x - 2)$ to work out the composite as in $f(x - 3) = 3x - 2 - 3$ instead of the **correct approach** $f(x - 3) = 3(x - 3) - 2$.

Answer(s):

- (a) (i) $A'(1, 1)$
(ii) $A''(-2, 3)$
(b) (ii) $x = -5$

Question 4

Specific Objective(s): (a) 6; (f) 5.

This question examined the candidates’ ability to obtain the solution sets of inequalities involving the modulus function and to express a quadratic function in the form of a completed square.

The response to this question was good with approximately 80 per cent of the candidates scoring at least 6 marks.

- (a) Some candidates encountered problems in performing the operations necessary for the removal of the modulus sign. A common error observed was $(x - 4)^2 - 6 > 0$ instead of the correct inequality $(x - 4)^2 > 6^2$ (or equivalent) which is obtained by squaring both sides.

Final answers were given as equations (roots) in some instances and not as a solution set in inequality form. A number of the candidates who used a graphical method to obtain their answer had difficulty expressing their final answer as an inequality.

A number of **errors** occurred with transpositions and sign changes, for example:-
 $(x - 4) - 6 > 0 \Rightarrow x - 4 > 6 \Rightarrow x > 2$ instead of $x > 10$.

- (b) The question on completing the square was better manipulated than Part (a), however, a few errors were made with simple fraction addition such as $w = 2 + \frac{1}{12} = \frac{1}{6}$. Some candidates equated the coefficients and **incorrectly** obtained $w = 2$.

Answer(s): (a) $x > 10$ or $x < -2$ (b) $u = -3, v = \frac{1}{6}, w = \frac{25}{12}$

Question 5

Specific Objective(s): (f) 1, 7(i), 8(i), 8(ii), 10.

This question tested the candidates' ability to solve simultaneous equations in two unknowns, one being quadratic and one being linear.

The majority of the candidates attempted this question and a large percentage of them earned at least 5 out of 7 marks. The candidates who used the substitution $y = 5 - 3x$ were more successful than the

ones who used $x = \frac{5-y}{3}$ which presented a challenge to weaker candidates in terms of simplifying to obtain the correct quadratic equation.

A few candidates tried the elimination method without much success and in most cases, could not obtain the correct quadratic equation. Some candidates who obtained the quadratic equation in the form $-2x^2 + 5x - 2 = 0$ had difficulty factorizing it. Some candidates incorrectly solved $-2x + 1 = 0$ as

$x = -\frac{1}{2}$. Also, a significant number of candidates solved for x and forgot to solve for y to complete the solutions.

Simple multiplication errors such as $3 \times \frac{1}{2} = 3\frac{1}{2}$ are not expected at the CAPE level.

Several errors in factorizing quadratic equations were noted as well, the most popular being:-

$$-2x^2 + 5x = 2 \Rightarrow x(-2x + 5) = 2 \Rightarrow x = 2 \text{ or } x = \frac{3}{2}$$

Answer(s): $x = \frac{1}{2}, y = 3\frac{1}{2}$ and $x = 2, y = -1$

SECTION B **(Module 2: Plane Geometry)**

Question 6

Specific Objective(s): (a) 1, 2, 7 (i), 8, 9

The question tested the candidates' ability to solve problems in coordinate geometry dealing with equations of lines, points of intersection and perpendicularity of lines.

Approximately 85 per cent of the candidates attempted the question with the majority earning 4 or 5 marks out of the maximum 9 marks. A common mistake made by candidates related to treating M as the mid-point of one or both of the lines AC or BD . A few candidates also used BD perpendicular to AC and obtained incorrect solutions.

Answer(s): (a) (i) Equation of AC is $3y = x + 2$
(ii) Equation of BD is $y = 2x - 6$
(b) $M \equiv (4, 2)$

Question 7

Specific Objective(s): (b) 1, 5, 18, 20

This question was a test of the trigonometric form $R \cos(\theta + \alpha)$.

Many candidates attempted the question but there was limited success in most cases.

- (a) Some candidates ignored the form $R \cos(\theta + \alpha)$ in attempting to manipulate $\cos \theta - \sin \theta$ within the parameters specified. A few performed well, nevertheless.
- (b) Candidates who were familiar with the form in Part (a) performed well in Part (b). However, some were unable to obtain the general solution which suggests that this form of the solution needs further practice.

Answer(s): (a) $\cos \theta - \sin \theta = \sqrt{2} \cos\left(\theta + \frac{\pi}{4}\right)$

(b) $\theta = 2n\pi, 2n\pi - \frac{\pi}{2}, n \in \mathbb{Z}$

Question 8

Specific Objective(s): (b) 3, 6

Knowledge of the area of a sector and of the area of a triangle was required to solve this problem.

In general, the question was popular with the candidates. However, many seemed unwilling to use radian measure, thus reducing the maximum mark they could achieve on the question. Several candidates experienced difficulties in using the formulae for the respective areas, in terms of π , of the sector and the triangle.

Answer(s): (a) $\pi \text{ cm}$ (b) $\left(3\pi - \frac{9}{4}\right) \text{cm}^2$.

Question 9

Specific Objective(s): (c) 4, 5, 7

The question tested the candidates' ability to obtain the conjugate of a complex number, to manipulate a complex number and its conjugate and to find the modulus of a complex number.

Approximately 90 per cent of the candidates responded to this question. Many of them were unable to earn the maximum seven marks, mainly because they seemed unfamiliar with the conjugate of a complex number. Several **incorrect** representations of the conjugate of $4 + 3i$ such as $-4 - 3i, 3 - 4i$;

$3i + 4, -4 + 3i$ and $\frac{1}{5}(4 + 3i)$ were seen.

A significant number of candidates did not express $\frac{\bar{z}}{z}$ in the form $a + bi$ as required; they left the answer as $\frac{7 - 24i}{25}$. Others who correctly obtained $\frac{7}{25} - \frac{24}{25}i$ disregarded the fractions for a and b , and found the modulus in Part (b) as $\sqrt{7^2 + 24^2}$.

Answer(s): (a) $\frac{\bar{z}}{z} = \frac{7}{25} - \frac{24}{25}i$ (b) $\left| \frac{\bar{z}}{z} \right| = 1$

Question 10

Specific Objective(s): (d) 1, 3, 5, 8, 9, 10

This question tested basic elements of vector algebra in terms of magnitude, position vectors and perpendicularity.

In manipulating the vectors in this question, candidates made several errors, many of which were numerical in nature. As a consequence, several candidates did not earn maximum marks for their efforts.

Answer(s): (a) (i) $\vec{AB} = -\mathbf{i} - 6\mathbf{j}$ (ii) $|\vec{AB}| = \sqrt{37}$ (iii) $\vec{OM} = \frac{8}{3}\mathbf{i}$

(b) \vec{OA} is not perpendicular to \vec{OB} since $\vec{OA} \cdot \vec{OB} \neq 0$.

SECTION C **(Module 3: Calculus I)**

Question 11

Specific Objective(s): (a) 3, 4, 5, 6, 7

This question tested knowledge of limits and continuity or discontinuity of functions.

- (a) The majority of candidates recognised the indeterminate form when $x = -2$ was substituted in the original expression. However, a large number of them experienced difficulty in factorizing the cubic expression in the numerator. Some candidates factorized and cancelled correctly but then encountered difficulties with substituting $x = -2$ in the rational function which remained after cancellation.

Some candidates used L'Hopital's rule with a few doing so satisfactorily.

- (b) Most candidates associated “continuity” with putting the denominator equal to zero but some of these had difficulty in drawing the correct conclusion when the results $x \neq 6$ and $x \neq -3$ were obtained.

Answer(s): (a) $\frac{3}{2}$ (b) Continuous for all $x \neq 6, x \neq -3$

Question 12

Specific Objective(s): 4, 7, 8, 9, 10, 11

This question examined differentiation of rational functions and integration of a trigonometric function.

- (a) About 95 per cent of the candidates attempted this part of the question and were able to apply the quotient rule for differentiation, however, many errors were made in simplifying the resulting algebraic expression to arrive at the correct answer.

Some candidates wrote $f(x)$ as $(x^2 - 4)(x^3 + 1)^{-1}$ and successfully applied the product rule to obtain the answer.

- (b) Only about 60 per cent of the candidates attempted this part of the question which is based on Specific Objectives (c) 7 and 8. Many candidates did not seem to know how to differentiate $u = \sin 2x$ to obtain $du = 2 \cos 2x dx$, while others among those who did obtain the expression for du , failed to substitute correctly to transform the original integral to $\int \frac{1}{2}u du$; the point seemed to have been missed that in substituting $u = \sin 2x$ all occurrences of x in the integrand $\sin 2x \cos 2x$ and dx should be replaced with some function of u so that a new integrand appears under the integral sign expressed entirely in u and du .

Not many candidates used the “or otherwise” approach, but a few observed that

$$\sin 2x \cos 2x = \frac{1}{2} \sin 4x \text{ and proceeded to obtain the correct solution of } \frac{1}{4}.$$

Answer(s): (a) $f'(x) = \frac{-x^4 + 12x^2 + 2x}{(x^3 + 1)^2}$

(b) $\frac{1}{4}$

Question 13

Specific Objective(s): (b) 2, 7, 8, 9, 25

This question required candidates to obtain constant coefficients in a cubic equation, given points on the curve and the gradient at one of the points.

- (a) This part of the question was attempted by almost all of the candidates, not all of whom earned maximum marks because of weaknesses in the algebraic manipulation. Several candidates did not show that $r = 0$ since the curve passed through the origin and so complicated the calculations which depended on this fact. Other candidates failed to differentiate and could not make use of the gradient at P being equal to 8 in order to obtain a second equation.
- (b) Many candidates did not use the fact that the gradient was given as 8 at P but derived it. Others substituted $x = 2$, $y = 1$ at $P(1, 2)$ instead of $x = 1$ and $y = 2$ to find the equation of the normal.

Answer(s): (a) $p = 3$, $q = -1$, $r = 0$ (b) $8y + x = 17$

Question 14

Specific Objective(s): (b) 7, 8, 9, 16, 17, 19, 20, 21

The question tested candidates' ability to find coordinates of stationary points of a curve and to determine the nature of the stationary points.

- (a) The majority of candidates who attempted this question knew that the function had to be differentiated, however, after finding the first derivative and equating the result to zero only 60 per cent of them were able to arrive at the correct solution to the quadratic equation obtained. Of those who got the x -values correct, a considerable number could not correctly find the y -values.
- (b) Most candidates knew that the nature of the stationary points revolved around "something" being "positive" or "negative", but some were uncertain what that "something" was.

Approximately 20 per cent of the candidates gained full marks on this question.

Answer(s): (a) $(2, 16)$ and $(-2, -16)$
(b) $(2, 16)$, a maximum; $(-2, -16)$, a minimum

Question 15

Specific Objective(s): (c) 3, 6 (ii), 7(ii), 10 (i)

The question tested the candidates' ability to use the integral calculus to find the area between two curves.

- (a) Most candidates knew that the points of intersection of the straight line and the parabola had to be found but a few made mistakes in factorisation. Several candidates earned the maximum marks.
- (b) There were many good answers to this part, however, a few candidates made mistakes in using the incorrect form of the integrand representing the difference between the equations of the straight line and the parabola. Some others used wrong limits and obtained incorrect results.

Answer(s): $P \equiv (-1, 1)$, $Q \equiv (3, 9)$

(b) $\frac{32}{3}$ units²

**UNIT 1
PAPER 02
SECTION A
(Module 1: Basic Algebra and Functions)**

Question 1

Specific Objective(s): (a) 1, (c) 1, 5, (e) 2, (f) 8

The question tested knowledge of real roots and real factors. Overall, the question was attempted by more than 90 per cent of the candidates. However, only a few managed to earn maximum marks.

- (a) (i) The majority of the candidates recognised that the expression $x^4 - 9$ is the difference of two squares and hence were successful in factorising to give $(x^2 - 3)(x^2 + 3)$. Many candidates failed to state the factors as $(x - \sqrt{3})(x + \sqrt{3})$ and $(x^2 + 3)$.
- (ii) It was evident that some candidates had difficulty with the terms 'real factors' and 'real roots'. Most of the candidates gave $\sqrt{3}$ as the only root.
- (b) (i) Although all of the candidates attempted this question, some of them experienced difficulty squaring an equation involving a fraction. Instead of getting,

$$u^2 = x^2 + 8 + \frac{16}{x^2}, \text{ most candidates wrote } u^2 = x^2 + \frac{16}{x^2} \text{ or } u^2 = x^4 + 8x^2 + 16.$$

A few candidates failed to simplify u^2 , rather leaving it as $x^2 + \frac{4x}{x} + \frac{4x}{x} + \frac{16}{x^2}$.

- (ii) Many candidates misinterpreted this question. They used the method of substitution to obtain $u^2 - 9u + 20 = 0$. In other words they used an ‘or otherwise’ approach rather than a deductive approach as suggested. Most of the candidates failed to recognise that since $x \neq 0$, then $x^2 \neq 0$, but rather ignored the presence of x^2 in $f(x)$.
- (iii) A poor interpretation of Part (ii) led to much difficulty in solving Part (iii). Nevertheless, most candidates managed to utilize the factor theorem in order to solve for x when $f(x) = 0$. A number of candidates gave 0 as one of the possible values.

Answer(s): (a) (i) $(x + \sqrt{3})$, $(x - \sqrt{3})$ and $(x^2 + 3)$ (ii) $x = \sqrt{3}$, $x = -\sqrt{3}$

$$(b) (i) u^2 = x^2 + 8 + \frac{16}{x^2} \quad (iii) x = 1, 2, 4$$

Question 2

Specific Objective(s): (a) 1, 4, 10, (c) 2

This question tested the use of the summation notation, sums and products of quadratic equations and the principle of Mathematical Induction.

- (a) Many candidates attempted to solve this question using the principle of mathematical induction. This misinterpretation led to incorrect solutions. Also, most candidates failed to equate correctly $3 S_{2n}$ and $11 S_n$ as a result of some careless mistakes such as:
 - (i) Incorrectly substituting n instead of $2n$ in finding S_{2n}
 - (ii) Inaccurately expanding brackets for instance, $3(2n^2 + n) = 6n^2 + 6n$ and $2\left(\frac{1}{2}(n^2 + n)\right) = 11n^2 + n$.

Several candidates had problems solving the quadratic equation $n^2 - 5n = 0$.

Many of the candidates who solved the quadratic equation $n^2 - 5n = 0$ correctly to obtain $n = 0$ and $n = 5$, failed to state that only $n = 5$ satisfies $n \in N$.

- (b) Most of the candidates attempted this question and were able to earn maximum marks. However, a few failed to identify correctly the sums and roots of both equations. For example, instead of $(2\alpha + \beta) + (2\alpha - \beta) = 8$, they wrote $\alpha + \beta = 8$.
- (c) Most of the candidates attempted this question. However, the majority performed extremely poorly. A number of candidates could not indicate all the steps clearly. The ability to use critical thinking and make logical deduction continues to be a major weakness.

Candidates preparing to write the examinations should find the “Pure Mathematics Resource Material” document recently published by CXC helpful in studying Mathematical Induction, and other topics as well.

Answer(s): (a) $n = 5$

$$(b) (i) p = \alpha + \beta, q = 4\alpha^2 - \beta^2 \quad (ii) \alpha = 2, \beta = 12 \\ (iii) p = 14, q = -128$$

SECTION B
(Module 2: Plane Geometry)

Question 3

Specific Objective(s): 1, 2, 5, 7 (ii), 8, 9, 10, 13, 14

This question dealt with the geometry of the circle and tested the candidates' ability to find the lengths of the radius and of the tangents, the equation of a circle and points of intersection of the circle with the x -axis from given data about the circle.

Approximately 80 per cent of the candidates attempted this question. There were several good solutions. Varying methods were used to obtain results, showing that candidates had a good grasp of the relevant material.

Answer(s): (a) (i) radius = 5 units (ii) $(x - 5)^2 + (y + 4)^2 = 5^2$
(iii) $A \equiv (2, 0)$, $B \equiv (8, 0)$ (iv) $3x + 4y = 24$ (v) $P \equiv (0, 6)$

Question 4

Specific Objective(s): (b) 5, 7, 8, 12, 13, 14, 15, 16

This question challenged the candidates' ability to apply basic trigonometric ratios in establishing identities, and in evaluating angles in the context of a triangle.

Overall performance was below expectation. The main difficulty seems to stem from candidates' inability to recall information from CSEC level on basic trigonometric ratios. While many candidates readily used the identity $\cos 2\theta = \cos^2\theta - \sin^2\theta$ in Part (a) (i), many did not make the connection with Part (a) (ii). In Part (b), the use of the ratios for the triples (3, 4, 5) and (5, 12, 13) in right-angled triangles, the fact that the exterior angle r is equal to the sum of the interior opposite angles p and q , and the sum of the angles in the large triangle being 180 giving $p + t = 120 - q$, all proved difficult for the majority of candidates.

Answer(s): (b) (i) $\frac{4}{5}$ (ii) $\frac{12}{13}$ (iii) $\frac{63}{65}$ (iv) $\frac{3\sqrt{3} - 4}{10}$

SECTION C

(Module 3: Calculus 1)

Question 5

Specific Objective(s): (b) 8, 9, 10, 11, 12, 13

This question examined differentiation and relationships between first and second derivatives of a given function with a hint of mathematical modelling.

- (a) (i) This question was generally well done by most candidates, however a few candidates demonstrated weaknesses in differentiation, especially in the concept of the chain rule.

(ii) Most candidates who got the right answer for Part (i) were able to follow through to show this part.

(iii) The overall performance on this part was poor. Candidates did not recognize **the use of the product rule**. Most candidates attempted to differentiate the expression for dy/dx to obtain d^2y/dx^2 , but failed to obtain the correct expression as a result of the algebraic manipulation involved.

(b) Generally, this question was poorly done. A few candidates were able to attain full marks. Most candidates did not seem to understand the maximum and minimum concept of cosine.

Many weaknesses were seen in the differentiation of the expression $h = 2(1 + \cos(\pi t/450))$. A common error was $dh/dt = -2 \sin(\pi t/450)$.

Answer(s): (a) (i) $\frac{dy}{dx} = \frac{5x}{\sqrt{5x^2 + 3}}$
 (b) (i) 4 metres (ii) $t = 450$ min. (iii) $\frac{\pi}{450}$ m/min

Question 6

Specific Objective(s): (b) 23, (c) 7 (ii), (iii), 8, 9, 10 (ii)

This question focused on a fundamental principle of definite integrals and the use of substitution towards the evaluation of such integrals. Curve sketching and the calculation of volume is also included.

- (a) As in the previous year, the result $\int_0^a f(x)dx = \int_0^a f(a-x) dx$ was not fully understood by the candidates.

(i) This question was poorly done by most candidates. Very few candidates recognized the relationship between the constants a and $\pi/2$.

- (ii) A few candidates obtained the correct answer, but most candidates had difficulty obtaining the answer $\pi/4$. Not many candidates used the formula for $\cos 2x$ in terms of $\sin^2 x$ or $\cos^2 x$ as a means of calculating I.
- (b) (i) This question was well done by most candidates, however, a few did not recognize that the graph was a parabola which concaved upwards.
- (ii) This question was generally well done by the majority of candidates. Some candidates used $V = \pi \int y^2 dx$ instead of $\pi \int x^2 dy$. A few candidates calculated incorrect values for the limits.

Answer(s): (b) (ii) $\frac{\pi}{2}$ units³

UNIT 1
PAPER 03/B (ALTERNATE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (a) 6, (c) 1, (d) 1, 3, 5, (g)

This question focused on properties of functions and the solution of inequalities.

- (a) Most candidates were unable to substitute competently into the given function.
- (i) Most candidates made simple errors such as: $h\left(\frac{1}{t}\right) = \frac{1}{\frac{1}{t} + t}$ and $h(t^2) = (t + \frac{1}{t})^2$
- (ii) A number of candidates failed to simplify $h(t)h\left(\frac{1}{t}\right) - h(t^2)$
- (b) The response to this question showed that most candidates had difficulty in solving inequalities. A number of candidates multiplied both sides of the inequality by $(x + 2)$ instead of the more direct approach of multiplying both sides by $(x + 2)^2$. A few candidates accurately ensured that zero was on one side of the equality. However, a few demonstrated poor algebraic manipulations in attempting to simplify the inequality.
- (c) Most of the candidates demonstrated a poor understanding of key terms used in functions.

Answer(s): (a) (i) $\left(t + \frac{1}{t}\right)^2$ (ii) 2
(b) $-3 < x < -2$
(c) (i) range of $h = \{x \in \mathbb{R}: x \geq 2\}$ (iii) domain of $h^{-1} = \{x \in \mathbb{R}: x \geq 2\}$

SECTION B
(Module 2: Plane Geometry)

Question 2

Specific Objective(s): (a) 5, 6, 8, 9, 14, 16

The question focused on the ellipse represented in terms of parametric equations and by means of its Cartesian equation. The equation of a tangent to the ellipse was also examined.

- (a) Several candidates attempted this question, many of whom recognised that the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ was the means to obtaining the desired equation. Some candidates who were not informed of the correction ($y = 2 \sin\theta$) obtained instead the form $\frac{x^2}{9} + \frac{y^2}{16} = 1$; they were awarded full credit.
- (b) Only a few candidates were able to solve correctly this equation. The majority of the candidates were unable to derive the gradient of the tangent which they could have obtained by using the chain rule $\left(\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} \right)$ or with knowledge of implicit functions. Candidates' inability to derive the equation of the tangent led to some problems in Part (c).
- (c) (i) A number of candidates assumed a certain equation for the tangent. Consequently, a few of them earned a reasonable number of marks due to their knowledge of some basic geometric concepts (x and y intercepts, the area of a triangle, the length of a given line and perpendicular lines).
- (ii) Those candidates who obtained the coordinates of Q and R were able to find the area of ΔQOR quite easily.
- (iii) Candidates were able to calculate the length of QR by means of the formula for the distance between two points with known coordinates.
- (iv) Candidates successfully utilized the formula for the length of perpendicular from O to QR as
$$\sqrt{\frac{|ax + by + c|}{a^2 + b^2}}.$$

No candidate utilized the result of (c) (ii) (that is, the area of triangle $A = \frac{1}{2}bh$) to find the length of the perpendicular.

Answer(s): (b) $\frac{\sqrt{3}}{6}x + \frac{y}{4} = 1$

- (c) (i) $Q \equiv (2\sqrt{3}, 0)$, $R \equiv (0, 2)$ (ii) Area of $\Delta QOR = 2\sqrt{3}$ units²
(iii) $QR = 4$ units (iv) Perp. = $\sqrt{3}$ units in length

SECTION C
(Module 3: Calculus 1)

Question 3

Specific Objective(s): (a) 3, 4, 5, 6 (b) 9, 11, 15 - 17, 19 - 21

This question deals with limits, critical values of a given cubic function and differentiation leading to a maxima and minima.

- (a) (i) The majority of the candidates who attempted this question were unable to factorise the expression $(x - 1)$ in terms of $x^{1/3}$.
(ii) A number of candidates attempted to solve this question using the “hence” approach, although their results to Part (i) were incorrect. As a result, only a few candidates were able to obtain maximum marks.
- (b) (i) Most candidates attempted this question and were successful in obtaining full marks. Many candidates were confused with critical values of y and critical point; in most cases candidates gave the latter, that is, $\left(\frac{-1}{3}, \frac{32}{27}\right), (1, 0)$.
- (c) (i) Most of the candidates were successful in showing that $S = 2\pi r^2 + \frac{20}{r}$. However, a few candidates were unable to show that S has a minimum value when $r^3 = \frac{5}{\pi}$.

At least 90 per cent of the candidates attempted this question. With the exception of Part (a), the question appeared manageable to several candidates.

Answer(s): (a) (ii) $\frac{1}{3}$

(b) (i) $\left(-\frac{1}{3}, \frac{32}{27}\right)$ and $(1, 0)$ (ii) $\left(-\frac{1}{3}, \frac{32}{27}\right)$ – a max.; $(1, 0)$ – a min.

GENERAL COMMENTS

UNIT 2

In general, the performance of candidates in Unit 2 was of a high standard with a small number of candidates reaching an outstanding level of proficiency. However, there were some candidates who were inadequately prepared for the examination.

Topics in Calculus, Simple Probability, Approximation to Roots of Equations, and Series seemed well covered. Weaknesses continue to manifest themselves at the level of algebraic manipulation, including substitution, which frustrate the processes required to complete the problem-solving exercises posed in the questions. As indicated in previous years, extended practice on respective themes needs to be undertaken in order to eradicate such deficiencies and raise the level of performance in the areas of weakness identified.

However, the results on the whole were very encouraging.

DETAILED COMMENTS

UNIT 2 PAPER 01 SECTION A (Module 1: Calculus II)

Question 1

Specific Objective(s): (a) 3, 10

This question examined the exponential function and the use of logarithms in solving equations.

- (a) Almost all candidates attempted this question with approximately 30 per cent obtaining full marks, however, some candidates experienced difficulties in factorising the expressions $e^{2p} - 2 e^p$ and $e^{-p} - 2 e^{-2p}$.
- (b) Candidates were asked to solve an equation of the form $a^x = b$.

Most candidates attempted this part of the question with a satisfactory degree of success.

Overall, about 35 per cent of the candidates scored between 6 and 8 marks on this question.

Answer(s): (a) $p = \ln 2$, $q = \frac{1}{2}$ (b) $x = \frac{\log 3}{2 \log 2} = 0.792$

Question 2

Specific Objective(s): (b) 2, 3, 5

This question focused on parametric equations of a curve, gradients and differentiation of functions.

- (a) There were several good responses from the candidates on this part of the question, nevertheless, a few candidates had difficulties differentiating $\frac{4}{t}$ while others seemed unaware of the chain rule and were unable to finish the question.
- (b) The quality of responses to this question was mixed. Some candidates had difficulty differentiating one or both of $\tan^2(3x)$ and $\ln(x^3)$. A few candidates obtained full marks for Part (b).

Answer(s): (a) gradient = -1 (b) $\frac{dy}{dx} = 6 \tan 3x \sec^2 3x + \frac{3}{x}$

Question 3

Specific Objective(s): (c) 1, 3

This question involved the use of partial fractions in finding integrals.

- (a) This question was well done with a success rate of approximately 96 per cent.
- (b) Most candidates were aware that $\int \frac{1}{b+x} dx = \ln |b+x|$ but too many seemed unaware that $\int \frac{1}{b-x} dx = -\ln |b-x|$.

Some candidates did not include a ‘constant of integration’ in each case.

Answer(s): (a) $P = 1, Q = 1$ (b) $\ln |3+x| - \ln |2-x| + \text{constant}$

Question 4

Specific Objective(s): (c) 4, 5, 6, 7

This calculus question covered the topics of integration by substitution and integration by parts.

- (a) This part of the question was generally well done, however, many candidates who did not use the substitution completely were confronted with the task of manipulating an integrand which was a function of both u and x .

- (b) Several candidates attempted this part but 80 per cent of them could only reach as far as $x \tan x - \int \tan x \, dx$, not being able to evaluate $\int \tan x \, dx$.

In both parts, the constant of integration was omitted.

Answer(s): (a) $\left(\frac{(2x - 5)}{24} \right)^6 + \frac{(2x - 5)^5}{4} + \text{const.}$ (b) $x \tan x + \ln(\cos x) + \text{const.}$

Question 5

Specific Objective(s): (c) 7, 9

This question models a manufacturing process by means of a first order differential equation. The use of integrating factors was tested.

Approximately 85 per cent of the candidates attempted this question with about 9 per cent obtaining full marks.

The major difficulty encountered by candidates was the failure in obtaining the correct integrating factor. Some candidates used 'c' as the constant of integration which confused their otherwise correct solution.

Answer: $c = 5x + \frac{205}{2} e^{-2x} - \frac{5}{2}$

SECTION B (Module 2: Sequences, Series and Approximations)

Question 6

Specific Objective(s): (a) 1, 2

This question tested the candidates' knowledge of, and ability to manipulate, recurrence relations as they apply to sequences.

About 98 per cent of candidates, attempted this question and performed extremely well. Most scored full marks which indicated that they were well prepared in this area.

Candidates provided a variety of approaches to obtain the solution.

Answer(s): (b) $u_3 = 2^2, u_5 = 2^3$

Question 7

Specific Objective(s): (b) 4, 8

Geometric progressions were covered in this question.

Approximately 95 per cent of the candidates attempted this question. Many were able to generate the equations $a + ar^2 = 50$, $ar + ar^3 = 150$ connecting the first term a and common ratio r , but some encountered difficulties in solving these two equations to find a and r . Those candidates who obtained the correct values of a and r were able to complete the question and earn maximum marks.

Answer(s): (a) $r = 3$ (b) $a = 5$ (c) sum = 605

Question 8

Specific Objective(s): (c) 2, 3

This question examined the candidates' ability to extract the independent term in a binomial expansion.

Candidates found this question easy with about 95 per cent submitting good answers and approximately 90 per cent of those obtaining maximum marks.

Answer(s): $10C_6 2^6(-5)^4$

Question 9

Specific Objective(s): (c) 1, 2, 3

This question on the binomial theorem tested the candidates' knowledge about binomial coefficients in terms of factorials.

Most candidates who attempted the question were able to do both parts of (a) but had considerable difficulty in obtaining the result in (c) which suggests that more practice is needed in the area of the manipulation of expressions involving factorials.

Answer(s): (a) (i) $\frac{(2n)!}{n! n!}$ (ii) $\frac{(2n - 1)!}{n! (n - 1)!}$

Question 10

Specific Objective(s): (b) 10

The topic covered the method of differences in the summation of series.

Approximately 80 per cent of the candidates attempted this question and of these 85 per cent obtained less than 5 marks while 15 per cent obtained between 5 and 8 marks.

Candidates were comfortable with the method of finding the solution to Part (a) but had some difficulties with substitution. A few candidates were unable to cope with the summation notation Σ .

Answer(s): (b)
$$\sum_{r=3}^n \frac{r+3}{r(r-1)(r-2)} = \frac{1}{2} \left[\frac{7}{2} - \frac{2n+3}{n(n-1)} \right]$$

SECTION C
(Module 3: Counting, Matrices and Modelling)

Question 11

Specific Objective(s): (a) 2, 3, 4

This question tested arrangements of objects.

- (a) Many candidates failed to deduce the number of combinations of the vowels. Some candidates confused combination with permutation.
- (b) The question was generally well done by most candidates. Some candidates confused addition with multiplication. Candidates were unable to differentiate between AND and OR.

Answer(s): (a) $\frac{8!}{3! 2!}$ (b) 186

Question 12

Specific Objective(s): (a) 5, 6, 7, 8, 9

This question tested some basic properties of counting and probability.

- (a) Many candidates wrote the number 36 instead of representing the answer as ordered pairs.
- (b) (i) Many candidates did not subtract 1/36 from 12/36.
- (ii) This part was well done by many candidates.
- (iii) Many candidates did not subtract 1/36 from $(11/36) + (6/36)$ and obtained the correct answer.

Answer(s): (a) $\{(a, b) : a, b \in \mathbb{N}, 1 \leq a, b \leq 6\}$

$$(b) \quad (i) \quad \frac{11}{36} \quad (ii) \quad \frac{6}{36} \quad (iii) \quad \frac{4}{9}$$

Question 13

Specific Objective(s): (b) 1, 2

This question focused on the products, transposes and determinants of square matrices.

This question was reasonably well done by most candidates. A few candidates showed weaknesses in calculating the determinant.

Answer(s): (a) $|X| = -365$ (b) $Y^T X = \begin{pmatrix} 7 & 68 & 29 \\ 56 & 78 & 4 \end{pmatrix}$

Question 14

Specific Objective(s): (b) 7, 8

This question covers the calculation of the inverse of non-singular matrices and the solution of matrix equations.

- (a) Most candidates who used row reduction were unable to attain the final answer for A^{-1} .
- (b) Many candidates wrote $X = YA^{-1}$ instead of $X = A^{-1}Y$. This was a common error.

A few candidates attempted to derive the answer using simultaneous equations but experienced difficulties.

Answer(s): (a) $A^{-1} = \frac{1}{2} \begin{pmatrix} 5 & 6 & -15 \\ -7 & -8 & 21 \\ -1 & -2 & 5 \end{pmatrix}$ (b) $X = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

Question 15

Specific Objective(s): (c) 1, 2

This question examined rate of increase in the context of mathematical modelling of properties of a sphere.

This question was generally well done by most candidates. A few students showed weaknesses in algebraic operations.

Incorrect differentiation was seen by a few candidates.

Answer(s): (a) $\frac{dr}{dt} = \frac{11}{16\pi} \text{ cm s}^{-4}$ (b) $\frac{ds}{dt} = 55 \text{ cm}^2 \text{ s}^{-4}$

**UNIT 2
PAPER 02
SECTION A
(Module 1: Calculus II)**

Question 1

Specific Objective(s): (a) 6, 8, 10, 11

This question related to logarithmic and exponential functions, and their graphs.

All candidates attempted this question with varying degrees of success.

- (a) About 90 per cent of the candidates attempted this part. Change of base and the value of x satisfying the equation $\log_8 x = -2$ presented some candidates with major challenges.
- (b) The table was generally well done in (i). In (ii), most candidates changed the scale to accommodate the range of values for the graph while others did not plot the point at $x = 3$; in both instances, candidates were credited with maximum marks.

About 96 per cent of the candidates who attempted this part of the question obtained at least 4 of the 5 marks that were allocated.

In Part (iii) b), about 50 per cent of the candidates obtained the end points for the range and far less got the inequality completely correct.

Answer(s): (a) $x = 2, \frac{1}{64}$ (b) (iii) a) $x = 0$, b) $-1 \leq x < 0$

Question 2

Specific Objective(s): (c) 7, 8

This question tested the candidates' ability to use reduction formulae.

- (a) About 50 per cent of the candidates who attempted this question earned maximum marks. Many were familiar with the identity $\sec^2 x = 1 + \tan^2 x$ and applied it correctly.
- (b) There were several good responses to this part, with many candidates obtaining full marks.
- (c) (i) About 25 per cent of the many candidates who attempted this part of the question obtained full marks. The majority of candidates earned at least 4 of the 7 marks.
- (ii) About 15 per cent of the candidates who attempted this part obtained full marks. Some candidates tried to use I_n for $n = 0$ even though it was stated earlier that $n \geq 2$.

Answer(s): (b) $n \tan^{n-1} x \sec^2 x$ (c) (ii) $I_4 = \frac{\pi}{4} - \frac{2}{3}$

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 3

Specific Objective(s): (a) 2, 5; (b) 4, 8, 9; (c) 1

This question tested the candidates' ability to apply the principle of mathematical induction to factorials; identifying and obtaining the general term of a geometric series as well as its sum to infinity.

The majority of the candidates attempted this question. However, a minority was able to gain near maximum marks.

- (a) The Mathematical Induction was very poorly done. It was very clear that most students lacked the understanding of the **process** of induction; not even being able to recognize what was to be proved.

The basic step of proving that $n! = 1! = u_1 = 1$ (given) was hardly seen. Most candidates wrote a very clear and concise memorized conclusion even though a proper inductive step was absent. Many candidates added the $(k+1)^{th}$ term to the k^{th} term although the question dealt with $n!$ – a clear lack of understanding.

Some other weaknesses observed were:

- The inductive step was attempted by some students who replaced k with $k+1$ thus obtaining $u_{k+1} = (k+1)!$ instead of $u_{k+1} = (k+1)u_k = (k+1)k! = (k+1)!$
- Incorrect conclusions involving for all $n \in Z$, for all $n \in R$, were frequently seen, instead of for all $n \in Z^+$, for all $n \in N$ or equivalent.

- (b) In this part, many candidates failed to recognize that the question asked for the n^{th} term which should be simplified into a single fraction in this case. This prevented some candidates from obtaining full marks. Careless simplification errors were made, especially with the indices because of the lack of brackets, for example, $2 - (n - 1)$ was incorrectly written as $2 - n - 1 = 1 - n$.

In Part (ii), many candidates used a particular solution approach, when a general solution approach was necessary to show that the series S is a geometric progression.

Answer(s): (b) (i) $a_n = \frac{18}{3^n}$ OR $6\left(\frac{1}{3}\right)^{n-1}$ OR $2 \times 3^{2-n}$

(ii) $\frac{a_n}{a_{n-1}} = \frac{1}{3}$, a constant

(iii) $a = 6, r = \frac{1}{3}$

(iv) $S_\infty = 9$

Question 4

Specific Objective(s): (b) 4, 7; (c) 1; (e) 1, 2, 4

This question examined the use of the Intermediate Value Theorem in testing for the existence of a root in an equation; the Newton-Raphson Method in finding successive approximations to a root in an equation and mathematical modelling involving an arithmetic series.

For this question, Part (b) proved more challenging to the candidates than Part (a).

- (a) (i) The main difficulties candidates encountered with this part are as follows:-

- Not stating that the function is “continuous”
- Attempting to use the given formula to get the root
- Using the derivatives $f'(0)$ and $f'(1)$ in an attempt to show that $f(x)$ has a root in the interval $(0,1)$.

However, many candidates correctly used $f(0) \cdot f(1) < 0$ or the sign change criterion between $f(0)$ and $f(1)$, as well as mentioning the Intermediate Value Theorem, in this part of the question.

- (ii) The main areas of concern regarding the candidates’ approach were as follows:

- Writing the Newton-Raphson Formula incorrectly as

$$x_2 = \frac{f(x_1)}{f'(x_1)} \text{ or } x_2 = \frac{f(x_1)}{f'(x_1)} \text{ instead of the correct form } x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}.$$

- Having difficulty in simplifying the expression $x_n \frac{f(x_n)}{f'(x_n)}$ due mainly to the omission of the necessary brackets, that is, $\frac{x_1(4x_1^3 - 4) - x_1^4 - 4x + 1}{4x_1^3 - 4}$ instead of
$$\frac{x_1(4x_1^3 - 4) - (x_1^4 - 4x + 1)}{4x_1^3 - 4}$$
 - Substitution of particular values in the Newton-Raphson Formula in order to prove the given expression.
- (b) (i) • Not recognising the loan repayment as an A.P., hence writing **all 12** instalments, then summing.
 • Using $P = \$570$ to work out the 12 payments then summing to prove that $A = \$10\ 800$ rather than using $A = \$10\ 800$ to prove that $P = \$570$.
- (ii) It was noted that the students had more difficulty with (b) (ii) than with (b) (i).
- Writing the loan balance as $(10\ 800 - \text{nth instalment})$ instead of $(10\ 800 - S_n)$ where S_n is the sum of the first n instalments.
 - Using the sum of a G.P. instead of the sum of an A.P.
 - Incorrect expansion and simplification of the brackets when forming an expression for the sum of the instalments.
 - Using $10\ 800 = S_n$ instead of $(10\ 800 - S_n)$ as an attempt to find an expression for the remaining debt.

Answer(s): (b) (ii) $D = 10\ 800 - 540n - 30n^2$

SECTION C (Module 3: Counting, Matrices and Modelling)

Question 5

Specific Objective(s): (a) 4, 7, 9, 10

This question tested the candidates' knowledge of simple counting principles and probability.

There were several good responses to this question with many candidates scoring between 12 and 20 marks. A significant number earned maximum marks.

- (a) Some candidates experienced confusion between permutation and combination. More practice is recommended to clarify the distinction.
- (b) A few candidates included the column totals in their calculation of the number of males and the number of females.

In Part (ii), some candidates did not recognise the problem as inclusive and so many found $P(\text{males}) + P(\text{news})$ rather than $P(\text{males}) + P(\text{news}) - P(\text{males and news})$.

Generally, attention should also be paid to computational correctness in problems of this kind.

Answer(s): (a) (i) 210 (ii) $\frac{55}{210} = 0.262$

(b) (i) $\frac{48}{100} = 0.48$ (ii) $\frac{70}{100} = 0.70$ (iii) $\frac{20}{100} = 0.20$

(iv) $1 - \frac{30}{100} = \frac{70}{100} = 0.70$

(c) (i) $p = 0.20$ (ii) 0.45

Question 6

Specific Objective(s): (b) 1, 2, 3, 4, 5, 6

The question examined solutions to systems of linear equations by matrix methods, as well as properties of matrices and determinants.

Although almost all the candidates attempted the question, generally, it was not well done. Very few candidates were able to obtain full marks in this question.

- (a) (i) In a few cases, candidates were not able to express the system of equations in complete matrix form, that is, $AX = Y$.
- (ii) Although candidates were able to identify the augmented matrix, in many instances this was not written using valid notation. Some candidates confused the augmented matrix with the adjugate matrix.
- (iii) Most candidates were able to perform elementary row operations. There were, however, many arithmetic errors. There also seemed to be a problem in identifying when the matrix was in echelon form with candidates getting a simpler matrix to work with but not a matrix in the desired echelon form.
- (iv) Most candidates were able to find the value of α from their reduced matrix.
- (v) The majority of candidates were able to solve the system of equations up to a point. Many got as far as stating $y = -1$, $x + z = 11$. Only a few candidates were able to go further to choose x or z arbitrarily and state the final required answer.

- (b) (i) This was generally well done, however, many candidates seemed not to be able to identify the 3×3 identity matrix, I . Some students misinterpreted $KI - A$ as $K(I - A)$.
- (ii) Most candidates who correctly found $KI - A$ were able to at least make a valid attempt at finding the determinant. In many instances, there were errors in the simplification of the determinant and this led to the wrong cubic equation. Most candidates were able to use the factor theorem correctly to factorise the cubic. There were cases of candidates correctly factorising the valid cubic equations but losing marks by making errors such as $k^2 - 3 = 0 \Rightarrow k = \pm 3$ or by simply stating that $k^2 - 3 = 0 \Rightarrow k^2 = \sqrt{3}$.

Answer(s): (a) (i)
$$\begin{pmatrix} 1 & 1 & 1 \\ 3 & -2 & 3 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 35 \\ \alpha \end{pmatrix}$$
 (ii)
$$\begin{pmatrix} 1 & 1 & 1 & | & 10 \\ 3 & -2 & 3 & | & 35 \\ 2 & 1 & 2 & | & \alpha \end{pmatrix}$$

(iii)
$$\begin{pmatrix} 1 & 1 & 1 & | & 10 \\ 0 & -1 & 0 & | & 1 \\ 0 & 0 & 0 & | & \alpha - 21 \end{pmatrix}$$
 (iv) $\alpha = 21$

(v) $x = 11 - z, y = -1, z$ arbitrary

(b) (i) $kI - A = \begin{pmatrix} k & 1 & -1 \\ 1 & k & -1 \\ -1 & -1 & k-1 \end{pmatrix}$ (ii) $|kI - A| = 0 \Rightarrow k = 1, k = \pm \sqrt{3}$

UNIT 2
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)

SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (c) 1, 2, 3

The question posed a mathematical modelling problem based on proportionality, exponentials and the solution of a differential equation.

Most of the candidates who took the paper were unable to form the differential equation required in Part (a).

While 10% of the candidates gained maximum marks, the majority earned less than 10% of the 20 marks.

Several candidates were unable to manipulate the exponential and logarithmic functions.

Answer(s): (a) $\frac{dA}{dt} = kA, \quad k < 0 \quad \text{or} \quad \frac{dA}{dt} = -kA, \quad k > 0$

SECTION B
(Module 2: Sequences, Series and Approximation)

Question 2

Specific Objective(s): (b) 4, 6, 8, 9

This question examined geometric series and mathematical modelling.

The general performance on this question was poor with the majority of candidates earning between 0 and 5 marks.

- (a) Several of the candidates were unable to recall that the common ratio r of a convergent geometric series must satisfy the condition $|r| < 1$.
- (b) (i) Some candidates were unable to express the answers to 2 significant figures.
(ii) Most candidates did not recognise that maximum output meant sum to infinity.

Answer(s): (a) $x < 6, \quad -1 < x < 1, \quad x > 6$

(b) (i) a) 1 300 000 to 2 sig. fig. b) 920 000 to 2 sig. fig.
(ii) 29 to 2 sig. fig.

SECTION C
(Module 3: Counting, Matrices and Modelling)

Question 3

Specific Objective(s): (a) 1, 2, 6, 7, 9, 10

The question covered arrangements of objects, probability, and solutions of system of equations. Mathematical modelling is also evident.

- (a) Candidates were able to find the probability with no restrictions in (i), but had severe difficulties in obtaining the correct answer to Part (ii).
- (b) Generally, this part of the question was well done with 95 per cent of the candidates obtaining the correct answers.

Answer(s): (a) (i) 10! (ii) 0.8

(b) (i)
$$\begin{aligned} 2x + 2y + z &= 5950 \\ 4x + y + z &= 11\ 450 \\ 5x + 3y + 2z &= 14\ 600 \end{aligned}$$

(ii)
$$\begin{pmatrix} 2 & 2 & 1 \\ 4 & 1 & 1 \\ 5 & 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5\ 950 \\ 11\ 450 \\ 14\ 600 \end{pmatrix}$$

(iii)
$$\begin{aligned} x &= 2800 \\ y &= 100 \\ z &= 150 \end{aligned}$$

PAPER 03
INTERNAL ASSESSMENT
Module Tests

The main features assessed are:

- The mapping of the items tested to the specific objectives in the syllabus;
- Coverage of content of each Module test;
- The appropriateness of the items tested for the CAPE level;
- The presentation of the sample (Module Tests and students' scripts);
- The quality of the teachers' solutions and mark schemes;
- The quality of the teachers' assessment – consistency of marking using the mark scheme.

In general, the question papers, solutions and detailed mark schemes met the CXC CAPE Internal Assessment requirements for both Unit 1 and Unit 2.

Too many of the Module tests comprised items from CAPE past examination papers. Untidy ‘cut and paste’ presentations with varying font size were common place. Teachers are encouraged to use the past CAPE examination papers ONLY as a guide and to include original and creative items in the Module tests.

The stipulated time for Module Tests ($1 - 1\frac{1}{2}$ hours) must be strictly adhered to as students may be at an undue disadvantage when Module Tests are too extensive or insufficient.

The specific objectives tested in a particular module test must be from that Module.

In most cases, the Internal Assessments that were based on collective testing of Modules were inadequate in terms of the coverage of the Unit.

Improvement is needed in the presentation of samples for moderation. (See Recommendations below).

The moderation process relies on the validity of teacher assessment. In a few instances, the students’ scripts reflected evidence of rewriting of some solutions after scripts were formally assessed by the teacher. Also, there were a few cases where students’ solutions were replicas of the teacher’s solutions – some contained identical errors and full marks were awarded for incorrect solutions.

To enhance the quality of the design of Module tests, the validity and accuracy of teacher assessment and the validity of the moderation process, the following Internal Assessment guidelines are recommended.

Recommendations (Module Tests):

- (i) Design a separate test for each Module.
- (ii) In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered, and the same marking scheme used. ONE sample of FIVE students will form the sample for the centre.
- (iii) **In 2008, the format of the Internal Assessment remains unchanged. [Multiple Choice Examinations will not be accepted].**

Please note Recommendations for Module Tests and Presentation of Sample.

Recommendations for Module Tests and Presentation of Samples

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required at cover for each Module test.

- Name of School and Territory; Name of Teacher;
- Unit Number and Module Number;
- Date and duration ($1 - 1\frac{1}{2}$ hours) of Module Test;
- Clear Instructions to candidates;
- Total Marks allotted for Module Test;
- Sub-marks and total marks for each question must be clearly indicated.

2. COVERAGE OF SYLLABUS CONTENT

- The number of questions in each Module Test must be appropriate for the stipulated time of 1 - 1 $\frac{1}{2}$ hours;
- CAPE Past Examination papers should be used as a guide ONLY;
- Duplication of specific objectives and questions must be avoided;
- Specific objectives tested must be within the syllabus.

3. MARK SCHEME

- Detailed mark schemes must be submitted; holistic scoring is not recommended;
- FRACTIONAL MARKS MUST NOT BE AWARDED;
- The total mark for Module tests must be clearly stated on the teacher's solution sheet;
- The student's mark must be entered on the FRONT page of the student's script.

4. PRESENTATION OF SAMPLE

- Students' responses must be written on normal size paper, preferably 8 $\frac{1}{2}$ " x 11";
- Question numbers are to be clearly written in the left margin;
- The total score for each question marked on students' scripts must be clearly written in the right margin;
- ONLY original students' scripts must be sent for moderation.
Photocopied scripts will not be accepted;
- Module Tests must be typed using a legible font size, (or if handwritten must be neat and legible)
- The following are required for EACH Module test:
 - A question paper
 - Detailed solutions with detailed mark schemes
 - The scripts of the candidates comprising the sample
(Students' scripts in the sample are to be organized by Modules)
- Marks recorded on PMaths 1 - 3 and PMath 2 - 3 must be rounded off to the NEAREST WHOLE NUMBER;
- In cases where there are five or more registered candidates, FIVE samples must be sent.

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2008

PURE MATHEMATICS
(Trinidad & Tobago)

**CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2008**

INTRODUCTION

This is the first year that the revised syllabus for Pure Mathematics is examined. The new format of Paper 01 is multiple choice (MC) and Papers 02 and 03 have retained the format with extended-response questions. Circumstances dictated that examination papers for the Trinidad and Tobago (T&T) candidates were not the same as those for the Rest of the Region (ROR), nevertheless, the Internal Assessment (IA) of candidates from Trinidad and Tobago was included in the overall IA for the entire region and a common report has been written for that aspect of the examination process. A copy of that report is appended under the heading PAPER 03.

Generally, the performance of candidates was very satisfactory with a number of excellent to very good grades. There still remained, however, too large a number of weak candidates who seemed unprepared for the examination. Approximately 3 500 scripts were marked.

GENERAL COMMENTS

The new topics in the revised Unit I syllabus are Cubic Equations, Indices and Logarithms, and L'Hopital's rule, with Complex Numbers moved to Unit 2. Of these new topics, candidates showed reasonable competence in Cubic Equations and Logarithms, but some seemed not to have been exposed to L'Hopital's rule. Among the old topics comprising Unit 1, candidates continue to experience difficulties with Indices, Mathematical Induction and Summation Notation (Σ). General skills at algebraic manipulation including substitution at all levels continue to pose challenges. A new area of difficulty has emerged, the topic of Trigonometric Identities. Strong performances were recorded in Differentiation, the Plotting of Graphs, Vectors, and Coordinate Geometry. This was encouraging. Some effort should be made in providing students with practice in connecting parts of the same question in order to facilitate efficient solutions.

DETAILED COMMENTS

**UNIT 1
PAPER 01**

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily. The mean score was 61.0 per cent and standard deviation was 8.6.

**PAPER 02
SECTION A
(Module 1: Basic Algebra and Functions)**

Question 1

Specific Objective(s): (d) 3, 8, 10; (b) 4; (f) 3, 5; g (2)

This question tested properties of the roots of quadratic equations, cubic equations, the modulus function and graphs.

- (a) (i) There were several attempts at this part of the question. Many candidates wrote the condition for real roots without the 'greater than' sign in $b^2 - 4ac \geq 0$ and so did not obtain full marks. Others made errors in simplifying $(-2h)^2$.
- (ii) This was one of the new topics and presented some challenges, but there were many encouraging attempts. Very few candidates used the fact that $5(5 - k)(5 + k) = 105$ to obtain the values of k . More practice is recommended.

- (b) (i) (iii) This part of the question was very well done. Most candidates did not use the graph to find the values of x in Part (iii), but simply read the values from the table.

Answer(s): (a) (i) $h \geq 4$ or $h \leq -8$

(ii) $p = 71$, $k = \pm 2$

(b) (iii) $f(x) = g(x)$ when $x = 0$ or $x = 4$

Question 2

Specific Objective(s): (a) 5, 6, 8; (c) 1, 2, 3, 5

This question tested knowledge about indices, logarithms and the principle of mathematical induction.

- (a) Several candidates attempted this part of the question, many of whom realized that each term could be expressed in the form 3^x , $x \in \mathbb{N}$. Due to errors in the algebraic manipulation, only about 50 per cent of those attempting the question obtained the correct answer.
- (b) (i) Not many candidates knew how to derive this result although several of them knew how to use it as they demonstrated in Part (ii).
- (ii) This was very well done by the many candidates who attempted it, although they found the underlying principle at (i) hard to derive.
- (c) There were many good attempts at this question with several candidates obtaining at least 90 per cent of the marks. The step from $n = k$ to $n = k + 1$, which is the main task in the principle of mathematical induction, still eludes many. More practice is recommended.

Answer(s): (a) $3^4 = 81$

(b) (ii) $y = 5$

SECTION B **(Module 2: Trigonometry and Plane Geometry)**

Question 3

Specific Objective(s): (a) 9, 10, 12, 13; (c) 8, 9, 10

This question tested properties of vectors, trigonometric identities and solutions of trigonometric equations.

- (a) This part of the question was quite well done although Part (iii) did pose a challenge to a few candidates.
- (b) Several successful attempts were made in solving this part, however, some candidates had difficulty expanding $\cos 2A$.
- (c) The manipulation of the trigonometric identity troubled some candidates in this part of the question. It was also noted that not many candidates used the 'otherwise' route in Part (iii) to solve $\sin 3\theta = \sin \theta$.
More practice of this type of question and better use of the formula sheet are recommended.

Answer(s): (a) (i) $\lambda = -2$

(ii) $\lambda = 2$

(iii) $\lambda = 4 \pm 2\sqrt{3}$

(c) (iii) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

Question 4

Specific Objective(s): (b) 5, 7, 9

The question dealt with tangents to circles and basic properties of circles in the context of coordinate geometry.

- (a) (i) Most candidates were able to find the coordinates of P, A and B. Approximately 90 per cent of attempts were successful.
- (ii) Several candidates found it difficult to obtain the value of λ . Those who were successful substituted the coordinates of P but made simple errors in extracting the correct value of λ . Many obtained $\lambda = \frac{10}{3}$ instead of $-\frac{10}{3}$.

For those candidates who had a value of λ to carry forward, further marks were obtained from Parts (a) to (d).

- (b) (i) Many candidates did not relate the result required to the trigonometric relationship $\sin^2 t + \cos^2 t = 1$ and hence missed out on the simplicity of the process in obtaining the required equation.
- (ii) This part of the question was more of a challenge for the candidates. Few obtained full marks.

Answer(s): (a) (i) $P \equiv (1, 10)$, $A \equiv (2, 3)$, $B \equiv (6, 5)$
(The coordinates of A, B may be interchanged)

(ii) a) $\lambda = -\frac{10}{3}$

b) $3x^2 + 3y^2 - 16x - 40y + 113 = 0$

c) $|PQ| = \frac{5\sqrt{5}}{3}$

d) $|PM| = 3\sqrt{5}$

SECTION C
(Module 3: Calculus I)

Question 5

Specific Objective(s): (a) 4, 7; (b) 8, 9(i), 10, 16; (c) 13, 14, 15

The question examined knowledge about limits, L'Hopital's rule for limits, differentiation of rational functions and maxima in mensuration.

- (a) The topic of L'Hopital's rule for finding limits is a new topic in the revised syllabus and some candidates did not seem to be familiar with it. As a consequence, candidates were not penalized for using other methods to solve the particular problem posed.
- (b) (i) This part of the question was generally well done. Several candidates obtained full marks for both a) and b).
- (ii) Most candidates who attempted this part of the question obtained full marks. Several of them found $\frac{d^2y}{dx^2}$ by differentiating $\frac{dy}{dx}$ as a quotient. A small number of candidates used implicit differentiation to find $\frac{d^2y}{dx^2}$.
- (c) (i) This part was very well done with the majority of candidates who attempted it gaining full marks.
- (ii) Several candidates succeeded in doing this part correctly although some had difficulty in substituting h in the expression for V . A few did not find the second derivative in order to obtain the value of h for V a maximum.

Answer(s): (a) $\frac{4}{5}$

(b) (i) a) $\frac{dy}{dx} = \frac{1}{(1 - 4x)^2}$

(c) (ii) $h = 4$

Question 6

Specific Objective(s): (c) 1, 3, 4, 5, 6, 7, 8 (i), 9

The question tested knowledge and skill in differentiation and integration. Generally the question was well done, with approximately 70 per cent of candidates obtaining at least 16 of the 25 marks allocated.

- (a) This part of the question was the most challenging for the candidates. Many seemed not to be familiar with integrating functions using 'substitution', which should be a regular procedure for problems whenever the integrand is not straightforward. Many candidates found difficulty manipulating $x dx$ to change from the variable x to the variable u . More practice is recommended.
- (b) Too many candidates were unable to relate the gradient of the curve with the need for integrating to obtain the equation for the curve. Several candidates treated $x^2 - 4x + 3$ as the function $f(x)$ rather than $f'(x)$ and proceeded in the wrong direction.

- (c) (i) Candidates found this part of the question easy, however, some struggled with the algebra involved.
- (ii) There were some excellent responses to this part of the question. Common errors included:
- Incorrect choice of limits
 - Attempting to combine the equations of the line and curve into a single function to integrate
 - Attempting to use approximation to find the area despite the stipulation to obtain the exact value

Answer(s): (a) $\frac{1}{3} \sqrt{3x^2 + 1} + \text{a constant}$

(b) Equation of C is $\frac{x^3}{3} - 2x^2 + 3x - 1$

(c) (i) $A \equiv (1, 3)$, $B \equiv (0, 5)$, $C \equiv (4, 0)$

(ii) Exact value of area = 13 units²

UNIT 1
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (a) 7; (c) 2, 3; (d) 7, 9; (f) 5(ii)

This question tested candidates' abilities in solving logarithmic equations, and their knowledge of the factor theorem, concept of a decreasing function, and the sigma notation relating to an arithmetic progression.

- (a) (i) This part of the question was satisfactorily done.
- (ii) This part of the question was not well done. Candidates found it difficult to express the given logarithmic equation in index form thus allowing for the solution of a simple linear equation.
- (b) (i) A significant number of candidates were unable to use the intercepts of the curve to determine the constants required. Preparation for specific topics seem to be stereotype. Apparently, the use of the factor theorem is studied without any reference to the relationship of a curve and its intercepts.
- (ii) Most of the candidates stated the range as seen on the graph but included the point where $x = -1$. Instances were seen where candidates attempted differentiation of $f(x)$ to find the required range. More practice on graphs and how to use graphs to determine some features of a function should be done.
- (iii) This part of the question was satisfactorily done.

- Answer(s): (a) (i) p
(ii) $x = 9$
(b) (i) $h = 4, k = -1, m = -2$
(ii) $-1 < x \leq 0$
(c) 15 350

SECTION B
(Module 2: Trigonometry and Plane Geometry)

Question 2

Specific Objective(s): (b) 1, 2, 6, 7, 9.

This question tested candidates' abilities to determine a Cartesian curve from given parametric equations, finding a tangent and a normal to a Cartesian curve in linear form and in terms of its parameter, intersection of a line and a curve, and the distance between two points on a curve.

The majority of candidates performed poorly with approximately 10 per cent of them giving no responses.

Symbolic representation and application of the given data as required was a big challenge to most candidates. The algebraic skills demonstrated were very weak.

Such candidates require more preparation and practice to perform satisfactorily at these examinations.

- Answer(s): (b) (ii) $y + t_1x - at_1^3 - 2at_1 = 0$
(iv) $2a(1 + t_1^2)$

SECTION C
(Module 2: Calculus1)

Question 3

Specific Objective(s): (a) 4, 5; (c) 2, 4, 5, 6; (b) 11.

This question tested the concept of limits and of definite integration, as well as the use of a simple model involving rate of change.

- (a) (i) This part of the question was well done since candidates were given a useful hint which simplified the rational function.
(ii) Most candidates were able to follow through with the result from (i) to perform well on this part of the question.
- (b) (i) This part of the question was satisfactorily done by most of the candidates.

(ii) Some candidates had difficulty separating the integral and using the result given for $\int_1^4 f(x) dx = 7$. In addition candidates could not use the fact that for a continuous function $\int_1^2 f(x) dx + \int_2^4 f(x) dx = \int_1^4 f(x) dx$. Candidates therefore could not obtain the correct answer. A number of candidates attempted integration of the problem in the form given with obvious difficulties.

- (c) (i) The majority of candidates merely found $\frac{dV}{dh}$, apparently not aware that finding $\frac{dV}{dt}$ required multiplication of $\frac{dV}{dh}$ by $\frac{dh}{dt}$.
- (ii) The candidates were required to find $\frac{dh}{dt}$ but many of them failed to do so since their result at (i) was incorrect. There were no correct responses to this part of the question.

Answer(s): (a) (i) $\frac{1}{6}$

(ii) $\frac{1}{48}$

(b) (i) $u = 2$

(ii) 4

(c) (i) $\frac{dV}{dt} = \frac{1}{3}\pi(48h - 3h^2) \times \frac{dh}{dt}$

(ii) $\frac{25}{7\pi} \text{ cms}^{-1}$

$$= \pi h(16 - h) \times \frac{dh}{dt}$$

GENERAL COMMENTS

UNIT 2

Topics satisfactorily covered were those relating to solution of Exponential Equations, Calculus of Composite Functions, (including Inverse Trigonometric Functions), First-order Differential Equations, Solution of Second-order Differential Equations, Series, Mathematical Induction, Permutations and Simple Probability, Approximations to Roots of Equations, Series, Complex Numbers (including De Moivre's theorem), and Matrix Algebra.

This examination tested the new topics which included Calculus of Inverse Trigonometrical Functions and the Second Derivative, the use of an Integrating Factor for First-order Differential Equations, Second-order Differential Equations, Maclaurin's Theorem for Series Expansions, Binomial Expansion Series, Reduction to Row-Echelon Form, and Row Reduction of an Augmented Matrix, Complex Numbers with application of Demoivre's Theorem for integral n .

The majority of candidates continue to display weaknesses in tasks requiring algebraic manipulation or involving substitution. It is imperative that more emphases be placed on these areas of weaknesses. Extensive practice in the use of substitution and algebraic manipulation is demanded if candidates are to be well prepared to show improved performances in these areas. Candidates continue to demonstrate a lack of appreciation for questions which allow for “hence or otherwise”. They fail to see existing links from previous parts of the questions and never seem disposed to using “otherwise” thus employing any other suitable method for solving the particular problem.

**UNIT 2
PAPER 01**

Paper 01 comprised 45 multiple-choice items. The candidates performed satisfactorily. The mean score on this paper was 68.4 per cent and the standard deviation was 8.6.

**UNIT 2
PAPER 02
SECTION A
(Module 1: Calculus II)**

Question 1

Specific Objective(s): (a) 7, 9; (b) 1, 2, 3, 5, 6, 7.

This question tested differentiation of various functions, exponential, logarithmic, and inverse trigonometrical, as well as parametric equations, and distinguishing a point of inflexion.

This question was generally well done by the majority of candidates. The average marks obtained were within the range 17 – 20 from a maximum of 25 marks.

- (a) Most of the candidates gained full marks for this part of the question. A small number of candidates found it difficult to solve the quadratic equation obtained in terms of e^x . Some candidates attempted to take the natural log of each term with the obvious difficulties experienced.
- (b) This part of the question was well done. All the candidates used second differentiation to prove the result. No candidate attempted to use implicit differentiation.
- (c)
 - (i) Generally, responses to this part of the question were good. Some candidates had difficulties with the Multiple Composite Functions. An application of the product rule over three terms is not a regular feature and more practice would be needed in this regard.
 $\frac{d}{dx} \sin^{-1}(2x)$ was not well done. Most candidates used the result of $\frac{d}{dx} \sin^{-1}(x)$ without paying attention to the composite $2x$. With the testing of this new topic it was not unexpected that lack of adequate practice would be evident.
 - (ii)
 - a) This part of the question was well done. A few candidates attempted to set t in terms of x and y before differentiation. Clearly, they had an idea but failed to develop it successfully.
 - b) Very few candidates obtained full marks for this part of the question. The majority of candidates set $\frac{dy}{dx} = 0$ to find the point of inflexion. Those candidates who attempted to find $\frac{d^2y}{dx^2}$ merely found $\frac{d}{dx} \left(\frac{dy}{dx} \right)$ and not multiplying by $\frac{dt}{dx}$. In

fact distinguishing a point of inflexion appeared to be new to most of the candidates. A lot of practice is required in this regard.

Answer(s): (a) $x = \ln 7, x = 0$

(c) (i) $\ln x \left(\frac{2x}{\sqrt{1-4x^2}} + \sin^{-1} 2x \right) + \sin^{-1} 2x$

Question 2

Specific Objective(s): (c) 1, 3, 6, 8, 11, 12 (ii)

This question required candidates to evaluate an indefinite integral using integration by parts, solving a first-order differential equation using an integrating factor, the general solution of a second-order differential equation with the principal integral being a trigonometric function, resolving partial fractions, and integration involving $\int \frac{f'(x)}{f(x)} dx$.

The majority of candidates attempted this question. The average range of marks obtained was 15 – 20 with a satisfactory number of candidates earning marks in the range 21 – 25 out of a maximum of 25 marks.

- (a) (i) Approximately 30 per cent of the candidates who attempted this question obtained full marks. The substitution $u = \ln x$ was widely used. The practice of not stating the constant of integration continues to be a source of concern. Emphasis must be placed on this aspect for candidates to appreciate the importance of this constant and to earn full marks.
- (ii) Some candidates had problems finding the correct integrating factor. They also failed to write the equation in the form $I \frac{dy}{dx} + Iy = I \ln x$. However, the majority of candidates seemed to have grasped the concept of using an integrating factor.
- (b) (i) Most candidates successfully found the first and second derivatives of $m \cos x + n \sin x$. However, too many of these candidates made simple errors in calculating the values of m and n .
- (ii) Candidates had no difficulties finding the complementary function correctly. Due to errors made in (i) some marks were lost overall.
- (c) (i) The majority of candidates performed well in this part of the question.
- (ii) Generally most of the candidates who were successful in Part (i) were able to integrate correctly. A very small number of candidates mistakenly found $\dots \int \frac{3x}{(x^2 + 1)} dx$ as $\arctan x$. Candidates are guilty of omitting the constant of integration.

Answer(s): (a) (i) $\frac{1}{2} \{\ln(x)\}^2 + C$
(ii) $xy = \frac{1}{2} \{\ln(x)\}^2 + C$

(b) (i) $m = 2, n = 1$

(ii) $y = Ae^{3x} + Be^x + 2 \cos x + \sin x$

(c) (i) $\frac{2}{x-1} - \frac{3x}{x^2+1}$

(ii) $2 \ln|x-1| - \frac{3}{2} \ln|x^2+1| + C$

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 3

Specific Objectives: (b) 1, 3, 6, (e) 1, 2, 4

This question tested the candidates' abilities with respect to arithmetic progressions, the principle of mathematical induction, the intermediate value theorem for the existence of a real root, and the Newton-Raphson method.

The majority of candidates attempted this question. The range of marks obtained for this question was between 10 and 20 out of a maximum of 25 marks.

- (a) (i) a) Most candidates used the approach of evaluating the sums for S and T by using formulae stated in the Formulae Booklet. However, they did not understand the concept tested and failed to gain marks for this part of the question.
- b) Candidates, having failed to answer (a) correctly, proceeded to find the sum of the arithmetic series, S , using the formula stated in the Formulae Booklet. This type of candidate needs adequate practice to be proficient with algebraic manipulation and the deductions made from these manipulations.
- (ii) Most candidates demonstrated a sound understanding of the principle of mathematical induction. However, some candidates are still unclear of the inductive process and failed at the step where the assumption that P_k is true is used to show that P_{k+1} is true for $n = \text{some } k$. More work on the principle of mathematical induction is required. Some candidates simply substituted $k+1$ for k in the statement for P_k . Candidates also failed to express $(k+2)(2k+3)$ in the form $[(k+1)+1][2(k+1)+1]$ to show that the statement P_{k+1} is of the form P_k for $n = k+1$. It is clear, however, that candidates are becoming more adept at applying the principle of mathematical induction.
- (iii) This part of the question required the use of the substitution of the formulae for $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r$, and simplifying to get the answer. A number of candidates mistakenly applied the principle of mathematical induction to prove the result. Those candidates who used substitution of the formulae obtained full marks.
- (b) (i) Most candidates found $f(0)$ and $f(1)$, concluding that since there was a sign change that condition was sufficient for the existence of a real root in the interval $(0, 1)$. Candidates were not aware that the function must be continuous in that interval to use the intermediate value theorem. Many candidates failed to obtain full marks due to this omission.

- (ii) The majority of candidates were not able to use the concept of differentiation to show that the function was continuously increasing, hence the existence of only one real root. Some candidates very logically showed by way of two graphs that $f(x) = x^3$ and $g(x) = 3 - 6x - 3x^2$ had only one point of intersection. A few candidates used the roots of a cubic equation to show that the cubic equation had one real root and two complex roots.
- (iii) More than 60 per cent of the candidates were able to obtain maximum marks for this part of the question. Very few candidates showed weaknesses in applying the Newton-Raphson method.

Answer(s): (b) (iii) 0.41

Question 4

Specific Objective(s): (a) 1, 2; (c) 1, 3; (e) 3, 4

This question tested the use of sequences defined by recurrence relations, simple algebraic proofs, and the binomial theorem. The average range of marks for this question was between 10 and 15 from a maximum of 25 marks. Generally the responses to this question were unsatisfactory.

- (a) (i) Because of arithmetical errors, a few candidates did not earn full marks for this part of the question.
 - (ii) A number of candidates showed weaknesses in basic algebraic simplification and correct numerical answers.
 - (iii) a) b) Most candidates were unable to answer this part of the question logically. Simple algebraic proofs continue to be problematic and much more is required in this regard.
- (b) The majority of candidates did well on this topic. Generally, most of them used an inspection method to determine the term independent of x . In fact, few candidates used the binomial expansion to determine the term required. Emphasis must be placed on the binomial theorem.
 - (c) A significant number of candidates found this part of the question difficult. Many of them simply used the calculator to find the difference. The use of the binomial expansion series for approximations seemed unfamiliar to most of the candidates. Practice in this regard is necessary.

Answer(s): (a) (i) $\frac{20}{11}, \frac{31}{16}$

(ii) $\frac{2(a_n - 2)}{4 + a_n}$

(b) $\frac{15!}{6! 9!} (6^6)$

(c) 10.28620

SECTION C
(Module 3: Counting, Matrices and Complex Numbers)

Question 5

Specific Objective(s): (a) 1, 2, 7, 8, 9; (b) 2; (c) 1, 2, 4, 5, 11

This question tested selections using permutations, classical probability, complex numbers including De Moivre's theorem, and matrix algebra. The overall performance by candidates was satisfactory. The average range of marks was between 10 and 20 from a maximum 25 marks.

- (a) (i) a) Most candidates responded well to this part of the question. A few candidates found distinguishing between combinations and permutations rather challenging.
b) The majority of candidates gave satisfactory responses to this part of the question.
- (ii) This part of the question was well done.
- (b) Candidates demonstrated a good understanding of probability theory, including the use of Venn diagrams and laws of probability.
- (c) (i) A majority of candidates substituted $3 + 4i$ into the equation but had difficulties comparing coefficients since many of them made arithmetic errors in the expansions. Very few candidates used the principles of complex conjugate and the sum and product of roots of a quadratic equation. Not many candidates were able to obtain full marks.
- (ii) The majority of candidates demonstrated an understanding of De Moivre's theorem. However, many of them made errors in the expansion, particularly the terms involving i^2 . A number of candidates were unaware that they had to consider the real part of the expansion for $\cos 3\theta$. Very rarely did candidates define the complex number $\cos \theta + i \sin \theta$ as z , $\cos \theta - i \sin \theta$ as $\frac{1}{z}$ and used the principle of $\left(z + \frac{1}{z}\right) = 2\cos\theta$. More practice in this topic will improve candidates' understanding and performance.

Answer(s): (a) (i) a) $6^4 = 1296$

b) 360

(ii) $\frac{1}{3}$

(b) $\frac{13}{28}$

(c) (i) $h = -6, k = 25$

Question 6

Specific Objective(s): (b) 1, 2, 7, 8.

This question tested matrix algebra including solution of a variable for a singular matrix, multiplication of conformable matrices, finding the inverse of a non-singular matrix, and solution of a system of equations using matrix algebra. Approximately 20 per cent of the candidates gained full marks on this question. The average range of marks for this question was between 10 and 25 from a maximum of 25 marks.

- (a) The majority of candidates performed well in this part of the question. Some candidates made errors in the cubic expansion for x and subsequently found it difficult to factorize the cubic equation correctly.
- (b) (i) This part of the question was well done.
- (ii) a) This part of the question was well done. Very few candidates made some arithmetic errors in multiplication.
- b) A number of candidates failed to deduce the inverse of A from the result at a). Many of them went on to calculate A^{-1} as $\frac{1}{|A|} \text{adj } A$. This showed a weakness in understanding the concepts involved. Much practice in this topic will improve performance.
- (iii) This part of the question was well done by candidates who deduced or otherwise found A^{-1} correctly. However, some candidates failed to obtain full marks because they included the number of coaches in their answers.

Answer(s): (a) $x = 1, 2, -3$

$$(b) (i) \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 34 \\ 49 \\ 71 \end{pmatrix}$$

$$(ii) a) AB = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad b) \frac{1}{2} B = \begin{pmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{pmatrix}$$

(iii) 24

UNIT 2
SECTION A
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)
(Module 1: Calculus II)

Question 1

Specific Objective(s): (a) 5, 6, 7, 8; (c) 5

This question examined an exponential and a logarithmic expression, integration by parts of a trigonometric function, and an exponential model. Overall this question was poorly done.

- (a) Most of the candidates who attempted this part of the question did not demonstrate an understanding of exponential functions and of natural logarithms. This part of the question was poorly done.
- (b) Candidates could not separate $\cos^3 x$ as $\cos x (1 - \sin^2 x)$ in order to substitute $\cos x dx$ for du . Many of the candidates only found $du = \cos x dx$. No candidate got marks beyond this point.
- (c) (i) Many candidates failed to use the fact of $t = 0$ to find the answer to this part of the question.

- (ii) Candidates were required to find the value of the constant k before proceeding to find the answer to this part of the question. However, substitution and subsequent solution proved beyond the ability of most of the candidates.

Answer(s): (a) $y = \frac{1}{2} (e^x + e^{-x})$

(b) $\sin x - \frac{1}{3} \sin^3 x + C$

(d) (i) 70°C

(ii) 7.5 minutes

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 2

Specific Objective(s): (b) 3, 11, 12

This question tested the sum of a convergent series using the method of differences, and a model involving a geometric progression. The overall response was poor. Candidates seemed generally unprepared.

- (a) (i) Candidates found it difficult to express the general term of the series. They appeared unfamiliar with patterns and sequences.
- (ii) Follow through from (i) was not possible since the majority of candidates did not get the correct partial fractions to work with.
- (iii) Most of the candidates did not respond to this part of the question. The few who did could not determine the nature of the series since answers to (i) and (ii) were either non-existent or wrong.
- (b) (i) (ii) Overall they were very few and very poor responses to this part of the question. Analysis of the problem proved to be challenging for the candidates.

Answer(s): (a) (i) $\frac{1}{(2r-1)(2r+1)}$

(ii) $\frac{1}{2} - \frac{1}{2(2n+1)}$

(iii) $\frac{1}{2}$

(b) (i) $\$ \left(100 + \frac{1}{10} r^2 \right)$ (ii) \$6 205

SECTION C
(Module 3: Counting, Matrices and Complex Numbers)

Question 3

Specific Objective(s): (b) 7, 8

This question tested complex numbers and matrix algebra. Candidates showed a fair understanding of these topics. However, some candidates had difficulty finding the inverse of the invertible matrix.

- (a) (i) This part of the question was well done by the majority of candidates.
(ii) Very few candidates made arithmetical errors in this part of the question. Generally, this part of the question was well done.
- (b) (i) A significant number of candidates did not demonstrate a clear understanding of the process required to find the inverse of an invertible matrix. No candidate attempted the row reduction of an augmented matrix of the identity matrix.
(ii) There were no correct responses to this part of the question.

Answer(s): (a) (i) $\frac{-13}{2} - \frac{9}{2} i$

(ii) $5\sqrt{\left(\frac{5}{2}\right)}$

(b) (i) $\frac{1}{5} \begin{pmatrix} 8 & 7 & -6 \\ -2 & 2 & -1 \\ -5 & -5 & 5 \end{pmatrix}$

(ii) $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

PAPER 03 – INTERNAL ASSESSMENT

The Internal Assessment comprises three Module tests.

The main features assessed are the:

- Mapping of the items on the Module tests to the specific objectives in the syllabus for the relevant Unit
- Content coverage of each Module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (Module tests and students' scripts)
- Quality of the teachers' solutions and mark schemes
- Quality of the teachers' assessments – consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one Module test for each Unit

GENERAL COMMENTS

1. Too many of the Module tests comprised items from CAPE past examination papers.
2. Untidy 'cut and paste' presentations with varying font size were common place.
3. This year there was a general improvement in the creativity of the items, especially with regards to mathematical modelling. Teachers are reminded that the CAPE past examination papers should be used ONLY as a guide.
4. The stipulated time for Module tests ($1-1\frac{1}{2}$ hours) must be strictly adhered to as students may be at an undue disadvantage when Module tests are too extensive or insufficient. The following guide can be used: 1 minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling **MUST** be included.
5. Multiple-choice Questions will **NOT** be accepted in the Module tests.
6. Cases were noted where teachers were unfamiliar with recent syllabus changes, for example, complex numbers, 3-dimensional vectors, dividing a line segment internally or externally.
7. The moderation process relies on the validity of teacher assessment. There were a few cases where students' solutions were replicas of the teachers' solutions – some containing identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on the students' scripts did not correspond to the marks on the Moderation sheet.
8. Teachers **MUST** present evidence of having marked each individual question on the students' scripts before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of the students' scripts.
9. To enhance the quality of the design of the Module tests, the validity of teacher assessment and the validity of the moderation process, the Internal Assessment guidelines are listed below for emphasis.

Module Tests

- (i) Design a separate test for each Module. The Module test **MUST** focus on objectives from that module.
- (ii) In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered, and a common marking scheme used. One sample of FIVE students will form the sample for the centre.

(iii) In 2009, the format of the Internal Assessment remains unchanged.

[Multiple Choice Examinations will NOT be accepted].

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required on the cover of each Module test.

- Name of School and Territory, Name of Teacher, Centre Number.
- Unit Number and Module Number.
- Date and duration (1-1½ hours) of Module Test.
- Clear instruction to candidates.
- Total marks allotted for the Module Test.
- Sub – marks and total marks for each question MUST be clearly indicated.

2. COVERAGE OF THE SYLLABUS CONTENT

- The number of questions in each Module test must be appropriate for the stipulated time of 1-1½ hours.
- **CAPE past examination papers should be used as a guide ONLY.**
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant Unit of the syllabus.

3. MARK SCHEME

- Detailed mark schemes MUST be submitted, holistic scoring is not recommended that is, **one mark per skill should be allocated.**
- FRACTIONAL DECIMAL MARKS MUST NOT BE AWARDED.
- The total marks for Module tests MUST be clearly stated on the teacher's solution sheets.
- A student's marks MUST be entered on the FRONT page of the student's script.
- Hand written mark schemes MUST be NEAT and LEGIBLE. The marks MUST be presented in the right hand side of the page.
- Diagrams MUST be neatly drawn with geometrical/mathematical instruments.

4. PRESENTATION OF THE SAMPLE

- Student's responses MUST be written on normal sized paper, preferably 8 ½ ×11.
- Question numbers are to be written clearly in the left margin.
- The total marks for each question on students' scripts MUST be clearly written in the right margin.
- ONLY original students' scripts MUST be sent for moderation. **Photocopied scripts will not be accepted.**
- Typed Module tests MUST be in a legible font size (for example, size 12). Hand written tests MUST be NEAT and LEGIBLE.
- The following are required for each Module test:
 - A question paper.
 - Detailed solutions with detailed mark schemes.
 - The scripts (for each Module) of the candidates comprising the sample. The scripts MUST be collated by Modules.

- Marks recorded on the PMath 1 – 3 and PMath 2 – 3 forms must be rounded off to the nearest whole number.
- The guidelines at the bottom of these form should be observed. (see page 57 of the syllabus, no.6).
- In cases where there are five or more candidates, FIVE samples MUST be sent.
- In cases where there are five or less registered candidates, ALL samples MUST be sent.

CARIBBEAN EXAMINATIONS COUNCIL

**REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION**

MAY/JUNE 2008

**PURE MATHEMATICS
(Rest of the Region)**

**CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2008**

INTRODUCTION

This is the first year that the revised syllabus for Pure Mathematics is examined. The new format of Paper 01 is multiple choice (MC) and Papers 02 and 03 have retained the format with extended-response questions.

Generally, the performance of candidates was very satisfactory with a small number of excellent to very good grades. There still remained, however, too large a number of weak candidates who seemed unprepared for the examination. Approximately 3700 scripts were marked.

GENERAL COMMENTS

UNIT 1

The new topics in the revised Unit 1 syllabus are Cubic Equations, Indices and Logarithms, and L'Hopital's rule, with Complex Numbers moved to Unit 2. Of these new topics, candidates showed reasonable competence in Cubic Equations and Logarithms, but some seemed not to have been exposed to L'Hopital's rule. Among the old topics comprising Unit 1, candidates continue to experience difficulties with Indices, Mathematical Induction and Summation Notation (Σ). General skills at algebraic manipulation including substitution at all levels continue to pose challenges. A new area of difficulty has emerged, the topic of Trigonometric Identities. Strong performances were recorded in Differentiation, the Plotting of Graphs, Vectors, and Coordinate Geometry. This was encouraging. Some effort should be made in providing students with practice in connecting parts of the same question in order to facilitate efficient solutions.

DETAILED COMMENTS

**UNIT 1
PAPER 01**

Paper 01 comprised 45 multiple-choice items. The candidates performed satisfactorily. The mean score was 58 per cent with a standard deviation of 7.7.

**UNIT 1
PAPER 02**

Question 1

Specific Objectives(s): (a) 1, 2, 3, 5, 7, 8; (b) 1, 3, 4, 6; (f) 5 (ii)

The question tested properties of cubic equations, surds and the process of summation.

Overall, the question was attempted by over 90 per cent of the candidates. Various approaches were used in each part with a high degree of success.

- (a) Few mistakes were made in this part; however, incorrect expansions of $(x + 1)(x - 1)(x - 3)$ were mainly responsible for those that occurred. Incorrect equating of coefficients was also a main source of error.
- (b) This part of the question was generally well done, however, some candidates had some difficulties in rationalizing the surds. Many gained full marks in (ii) by using the result in (i). Errors occurred in (ii) by incorrectly evaluating $\sqrt{12}$.

- (c) (i) Many candidates used the principle of mathematical induction to obtain the result. Some candidates, in using this method, had difficulty manipulating the algebra involved. Others, who observed that results on the formula sheet were appropriate, had an easier passage to the final result.
- (ii) Candidates were not as successful in this part as in (i). The most frequent error occurred in the separation of the summation

$$\sum_{r=31}^{50} r(r+1) = \sum_{r=1}^{50} r(r+1) - \sum_{r=1}^{30} r(r+1).$$

A common error was subtracting

$$\sum_{r=1}^{31} r(r+1) \text{ instead of } \sum_{r=1}^{30} r(r+1).$$

Answer(s): (a) $p = -1, q = -1, r = 3$

(c) (ii)

$$\sum_{r=31}^{50} r(r+1) = 34\,280$$

Question 2

Specific Objective(s): (f) 1, 5 (i); (c) 1, 3 (i), (ii).

This question tested quadratic equations, the sums and products of the roots of such equations, the solutions of quadratic equations and logarithms.

- (a) This part of the question dealt with quadratic equations. Approximately 99 per cent of the candidates attempted this question and several obtained at least 10 of the 12 marks allocated to this part.
- (b) There was mixed success with this part of this question which examined logarithms in (ii) and (iii). Many candidates did (i) successfully although there were some who did not obtain the correct value for x from $x^{1/3} = -1$. In Part (ii), some candidates did not discard the negative value of x while in (iii) some candidates unwisely used calculators although the question stated that calculators should not have been used.

Answer(s): (a) (i) $\alpha + \beta = -2, \alpha \beta = 5/2$

$$(ii) a) \alpha^2 + \beta^2 = -1$$

$$b) \alpha^3 + \beta^3 = 7$$

$$(iii) 8x^2 - 56x + 125 = 0$$

$$(b) (i) x = 64, -1$$

$$(ii) x = 2$$

$$(iii) -1$$

UNIT 1
PAPER 02
SECTION B
(Module 2: Trigonometry and Plane Geometry)

Question 3

Specific Objective(s): (a) 4, 5, 9, 12; (b) 1.

This question tested the candidates' ability to use the gradients of line segments and to develop and apply trigonometric identities.

This question was not popular with most of the candidates. Most candidates attempting this question scored in the range of (0 - 4) marks, which was very disappointing.

- (a) The majority of the candidates did not recognize the relationship between the gradient of the straight line and the tangent of the angle between the line and the positive direction of the x -axis. These candidates did not link the word "tangent" with "tan", finding instead points and lines (in many variations). The few who recognized "tangent of an angle", frequently found " $\tan \alpha - \tan \beta$ " instead of " $\tan(\alpha - \beta)$ " where $\alpha > \beta$. Some of those who started well seemed not to be aware that $\tan(\alpha \pm \beta) \neq \tan \alpha \pm \tan \beta$, which spoiled the work thereafter.
- (b) The majority of candidates attempted (i) and were capable of finding the correct identities to replace $\sin 2\theta$, $\cos 2\theta$ and $\tan \theta$, but some had difficulty manipulating the identity to get to $\tan \theta$ in terms of $\sin 2\theta$ and $\cos 2\theta$.

In (ii) a significant number of candidates did not realize that "Express $\tan \theta$ in terms of $\sin 2\theta$ and $\cos 2\theta$ " meant change the subject (or transpose) the formula given in (i). The word "hence" was not understood by the candidates to use previous work and thus in most cases this was not done.

Several **errors** were made in (iii) such as:

$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta} \quad \cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$
$$\tan \theta = \frac{\sin 2\theta}{\cos 2\theta} \quad \cos \theta = 1 - \sin \theta$$

- (c) A large number of candidates did not use the fact that the angles of a triangle add up to 180° . Instead, it was stated that $A + B = C$ or in some cases, particular values of angles were used. Therefore, they did not recognize that the sine of an angle is equal to the cosine of its complement. In (c) (i) b), those candidates who actually attempted the question chose the correct factor formula but were unable to follow through for the second mark. Very few of the candidates recognised the link between (c) (i) b) and (ii). They did not recognise that A could be used as a double angle so they failed to use $\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2}$.

Answer(s): (a) (i) $\tan \alpha = 3$; $\tan \beta = \frac{3}{4}$

(ii) $\tan(\alpha - \beta) = \frac{9}{13}$

(b) (ii) $\tan \theta = \frac{\sin 2\theta}{1 + \cos 2\theta}$

Question 4

Specific Objective(s): (b) 1, 2, 3, 5, 7, 8.

This question examined the application of coordinate geometry to the properties of a circle, straight lines, tangents, normal, and intersections between straight lines and curves. For this question, Part (b) proved more challenging to the candidates than Part (a).

Part 4 (a) (i) was fairly well done. Seventy per cent of the candidates attempted the question and were able to get most of the eight marks allocated to it. The remaining 30 per cent of the candidates had difficulties with

- (a) finding the midpoint of a line
 - (b) calculating the gradient of a line using the formula
 - (c) knowing that the perpendicular bisector passes through the midpoint
 - (d) knowing that the gradient of the perpendicular bisector is the negative reciprocal of the gradient of the line.
- (ii) Of the few candidates that actually attempted this question, most were unable to recognize that the centre of the circle was the point of the intersection of the perpendicular bisector of any two chords of the circle. The alternative solution of substituting the points into the equation of the circle was also challenging for some candidates. Candidates substituted incorrect values and could not solve simultaneously the equations generated.

The main areas of concern in the students' approach were as follows:

- (a) Some candidates could not write coordinates properly as (x, y) .
- (b) Some candidates were using the formula for finding the length of a line to calculate the midpoint of the line.
- (c) When calculating the equation of the perpendicular line, several candidates erroneously submitted either the point P (-2, 0) or Q (8, 8), even when the midpoint was correctly found.

Most of the candidates solved Part 4 (b) (i) correctly. However, many of them did not recognise the significance of the repeated roots in relation to the tangent of the circle.

Some candidates also correctly used alternative solutions such as

- (a) finding the perpendicular distance from the centre of the circle by means of the formula $\frac{|ax+by+c|}{\sqrt{a^2+b^2}}$ and showing that this is equal to the radius of the circle
- (b) showing that the gradient of the line from the centre of the circle to the tangent is the negative reciprocal of the gradient of the tangent and therefore the line and tangent are perpendicular to each other.

Part 4 (b) (ii) was very well done. In fact, most candidates were able to recover from (b)(i) as above.

Answer(s): (a) (i) $4y + 5x = 31$

(ii) $x = -1, y = 9$

(b) (ii) $x = 0, y = 1$

SECTION C
(Module 3: Calculus 1)

Question 5

Specific Objective(s): (a) 5, (b) 5, 6, 11-19

The question tested knowledge of limits, differentiation and integration. Curve sketching and the nature of turning points of a curve were also investigated.

- (a) This part of the question dealt with the limit of a rational function in which both numerator and denominator are polynomials.

Candidates showed knowledge of the methods involved in finding the required limit but fell down on the mechanics of factorizing the polynomials correctly. The main difficulty occurred in the factorization of $x^3 - 27$. Many candidates stated that $x^3 - 27 = (x - 3)(x^2 - 9)$.

- (b) Very few candidates performed well on this part of the question. The main area of weakness was in differentiating the term $\frac{u}{t}$. As a consequence, many candidates had difficulty in deriving the appropriate equations required for the correct solutions.
- (c) There were mixed performances on this part of the question. In finding the equation for y , many candidates omitted the constant of integration when integrating $\frac{dy}{dx}$, and this led to an incorrect equation for the curve C and an incorrect sketch. In spite of this, Part (ii) was reasonably well done.

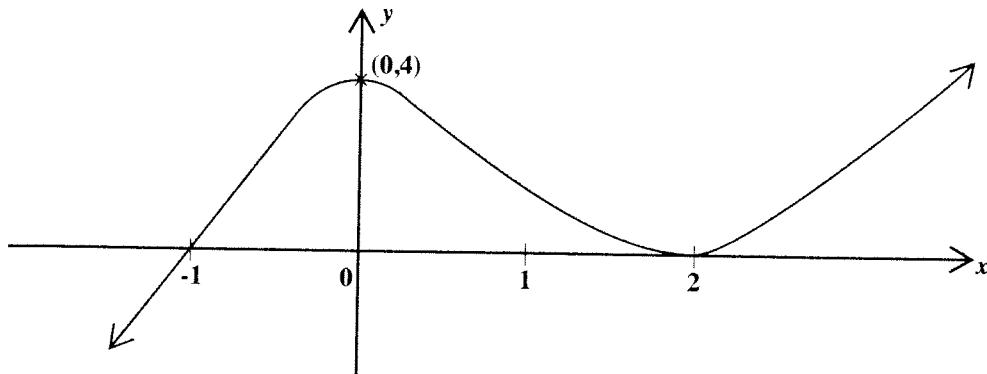
Answer(s): (a) $\frac{27}{7}$

(b) $u = \frac{4}{5}, v = \frac{-9}{5}$

(c) (i) $y = x^3 - 3x^2 + 4$

- (ii) Stationary points are (0,4) and (2,0).
(0,4) is a maximum and (2,0) is a minimum.

(iii)



Question 6

Specific Objective(s): (b) 5, 6, 8, 9, 10 (c) 4, 6, 8 (i)

The question tested aspects of differentiation, integration and some applications to mensuration. The question was poorly done.

- (a) This part of the question covers basic differentiation. In both parts, many candidates found difficulty in applying the chain rule to the process of differentiation. In too many cases, the function in (i) was replaced with $(2x^2 - x)^{1/2}$ or $x(2x^{1/2} - 1^{1/2})$. In Part (ii), $\sin^2(x^3 + 4)$ was not interpreted correctly as a function requiring the chain rule for differentiation and terms were lost in the process.
- (b) (i) Generally, candidates did not appear to recognize the need to apply the linearity property of integrals.
- (ii) For those candidates who recognized the need to integrate the given function between limits 1 and 3, a few were unable to follow through to obtain the solution of $k = 4$. Many were unable to manipulate the fractions after substitution of the limits. Several candidates simply substituted the limits into the given function without integrating to find the required area.
- (c) (i) Many candidates mistakenly used the volume of the sphere rather than that of the hemisphere.
- (ii) Several candidates neglected to include the base area when finding the total surface area.
- (iii) Of those candidates who answered this part of the question, many did not verify that A was indeed a minimum when $r = 3$.

Answer(s): (a) (i) $\frac{3x-1}{\sqrt{(2x-1)}}$

(ii) $6x^2 \sin(x^3 + 4) \cos(x^3 + 4)$

(b) (i) 3

(ii) $k = 4$

(c) (ii) $r = 3$

UNIT 1
PAPER 03/B (ALTERNATE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (c) 1, 2, 3, 4; (d) 7; (f) 5 (i)

The question tested properties of quadratic equations and their roots, functions, indices and logarithms.

- (a) There were several good attempts at this part of the question but few candidates obtained complete solutions.
- (b) (i) This part was poorly done. Many candidates did not observe that a critical approach to the solution was to start with $f(0) = 6$. Others did not discern that substituting $x = 3$ in the given equation would provide a lead to obtaining $f(9)$.

- (ii) This part was fairly well done.
(c) Both Parts (i) and (ii) were well done.

Answers: (a) (i) Roots are -3, -9

(ii) $k = 27$

(b) (i) $f(3) = 15$ and $f(9) = 33$

(ii) $x = 6$

(c) (i) 30

(ii) 78

**UNIT 1
PAPER 3 B
SECTION B**

(Module 2: Sequences, Series and Approximations)

Question 2

Specific Objective(s): (a) 5, 13; (c) 7, 8, 9, 10.

This question tested candidates' ability to solve trigonometric equations for a given range; to apply the properties of vectors; to find the scalar (dot) product and the angle between the given vectors as well as the magnitude and direction of a vector.

- (a) Candidates did not display sufficient knowledge of the trigonometric identities. The solutions presented were incomplete. Overall, this part of the question was poorly done.
- (b) In most cases, candidates recognized the need to use the dot product, but some of them did not know that $\mathbf{a} \cdot \mathbf{b} = 0$ for perpendicularity.
- (c) While a number of candidates knew the formula to find the acute angle between the two vectors, few knew how to manipulate it correctly.
- (d) Candidates were able to evaluate correctly the magnitude of F in most of the cases. Few correctly attempted to evaluate the angle of inclination requested in Part (b).

Answer(s): 2 (a) $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

(b) (i) $t = \frac{12}{5}$

(ii) $\theta = 22.3^\circ$

(iii) a) $|F| = 3.60$

b) $\phi = 56.3^\circ$

SECTION C
(Module 3: Calculus I)

Question 3

Specific Objective(s): (b) 5, 6, 7(i), 16, 21; (c) 1, 2, 3, 4, 5(i), (ii), 9

3. The question tested knowledge of the differential and integral calculus, the equation of a normal to a curve and rate of change in calculus.
- (a) This part of the question was reasonably well done although a small number of the candidates experienced difficulties in differentiating the term $\frac{1}{x}$ in $y = x + \frac{1}{x}$.
 - (b) Some candidates had problems with expressing the integrand as a sum of three separate terms. Very few candidates obtained full marks for this part.
 - (c) Most candidates obtained a differential equation in V without the negative sign. Nevertheless a good understanding of the concept involved was exhibited in the solutions.

Answer(s): (a) (ii) $3y + 4x = 31$

(b) $-\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} + a \text{ constant}$

(c) (i) $\frac{dV}{dt} = -30t$

(ii) Liquid lost is 75 cm^3 .

GENERAL COMMENTS

UNIT 2

In general, the performance of candidates in Unit 2 may be regarded as satisfactory. A small number of candidates reached an outstanding level of proficiency. A number of candidates were inadequately prepared for the examinations.

Topics in Calculus, Simple Probability, Approximations to Roots of Equations, and Series were satisfactorily covered. The examination tested new topics which included Calculus of Inverse Trigonometrical Functions and the Second Derivative, the use of an Integrating Factor for First-order Differential Equations, Second-order Differential Equations, Maclaurin's Theorem for Series Expansions, Binomial Expansion Series for Rational and Negative Indices, Complex Numbers, De Moivre's Theorem for integral n , and the Locus of a Complex Number.

Weaknesses in algebraic manipulation and tasks involving substitution were manifestly evident. Candidates obviously found it difficult to solve problems which required these applications. It is imperative that more emphases be placed on these areas of weakness. Extensive practice in the use of substitution and algebraic manipulation is necessary if candidates are to be well-prepared to show improved performances in these topics. Candidates continue to demonstrate a lack of appreciation for questions which allow for "hence or otherwise". They fail to see existing links from previous parts of a question, and rarely seem disposed to using "otherwise," thus employing any other suitable method for solving the particular problem.

PAPER 01

Paper 01 comprised 45 multiple-choice items. The candidates performed fairly well with a mean score of 64 per cent and standard deviation of 8.6.

DETAILED COMMENTS

UNIT 2
PAPER 02
SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (a) 6, (b) 2, 5 (c) 1, 5, 6

This question tested the differentiation of exponential, trigonometric and logarithmic functions, resolution into partial fractions, and integration involving inverse trigonometric functions.

All candidates attempted this question with varying degrees of success.

- (a) (i) Most of the candidates omitted the constant π in the derivative. Answers included in part, $e^{4x} \sin \pi x \dots$
A majority of the candidates treated e^{4x} as a constant rather than as a function of x . Consequently, some candidates did not obtain $4e^{4x}$ as the derivative of e^{4x} . Emphases must be placed on recognizing functions of a stated variable as against constants.
- (ii) The majority of candidates attempting this question opted to use the chain rule
$$\frac{d}{du} \ln(u) \times \frac{d}{dx} \left[\frac{x^2+1}{\sqrt{x}} \right].$$
 However, they failed to apply the quotient rule correctly for $\frac{d}{dx} \left[\frac{x^2+1}{\sqrt{x}} \right]$, and were unable to secure full marks for this part of the question.
- (b) Many candidates expressed y as $\frac{1}{3^x}$ and attempted to use the quotient rule which, for some, ran into difficulties. The majority of candidates used \log_{10} or \log_3 and attempted to differentiate with respect to x . Some candidates **erroneously** expressed $y = \ln_3 x$ and attempted to differentiate with respect to x . A small number of candidates **erroneously** stated $\ln y = \ln 3^{-x}$ giving $-x = \frac{\ln y}{\ln 3}$.

It is clear that this method of differentiation was new to many candidates. Logarithmic and implicit differentiation were not part of many of the candidates' skills.

- (c) (i) The majority of candidates demonstrated competence in this topic. Some errors in simple arithmetic were common. A few candidates had problems simplifying the terms of the fractions, thus making it difficult to answer Part (ii) successfully.
- (ii) The majority of candidates were successful in obtaining the correct partial fractions from Part (i). However, the evaluation of

$$\dots \int \frac{-5}{2(x^2 + 1)} dx$$

proved challenging for almost all the candidates. It seems that insufficient tutorials and practice in inverse trigonometrical calculus contributed to candidates' inability to complete the integration to this part of the question.

Candidates continue to omit the constant of integration from indefinite integrals. This results in loss of marks, and impacts negatively on their overall performances. The concept of the constant of integration must be fully explained so that candidates can be aware of the importance of including it in their resulting integrals.

Answer(s): (a) (i) $e^{4x}(4 \cos \pi x - \pi \sin \pi x)$

$$(ii) \frac{2x}{x^2+1} - \frac{1}{2x} = \frac{3x^2-1}{2x(x^2+1)}$$

$$(c) (i) \frac{3}{2(x-1)} + \frac{x-5}{2(x^2+1)}$$

$$(ii) \frac{3}{2} \ln|x-1| + \frac{1}{4} \ln|x^2+1| - \frac{5}{2} \arctan x + \text{const}$$

Question 2

Specific Objective(s): (c) 5, 7, 8, 11.

This question required candidates to use an integrating factor, integration of exponential functions, and integration by parts for a definite integral.

The majority of candidates attempted this question. However, many of them were unable to secure maximum marks on all parts of the question for various reasons.

- (a) The majority of candidates recognized the use of an integrating factor. However, many of them failed to evaluate this factor correctly. In addition, having found the integrating factor, many of them did not multiply the equation by the integrating factor. As a result, these candidates were not able to solve the differential equation completely.
- (b) This question was satisfactorily done by most of the candidates who earned the maximum mark.
- (c) The responses to this question revealed some weaknesses in identifying which of the terms to take as v and which as $\frac{du}{dx}$. This resulted in many candidates having to evaluate $\int \ln x \, dx$, since x^2 was taken as v . Those candidates who integrated correctly had some difficulty evaluating the integral using the stated limits. In general, candidates demonstrated a satisfactory understanding of integration by parts.
- (d) (i) Many candidates continue to show weakness in working with given substitutions. Many of them failed to find $-dv = du$. They, in fact, substituted

$$v = 1 - u \text{ to get } \int \frac{1}{\sqrt{v}} \, du$$

Common errors included $\int -v^{\frac{-1}{2}} \, dv$, omitting dv , not stating the constant of integration, and not replacing $v^{\frac{1}{2}}$ with $\sqrt{1-u}$ in the final answer. **The use of substitution for integration must be extensively practised in order for candidates to show improved performances in this topic.**

- (ii) The majority of candidates were able to replace $\cos x \, dx$ with du . However, the substitution $u = \sin x$ was not correctly used to transform the original integral into a manageable form using the given substitution. Candidates were unable to express $\sqrt{(1+\sin x)}$ in terms of u . As a result they were unable to obtain the cancellation of the term $\sqrt{(1+u)}$ to obtain $\frac{1}{\sqrt{1-u}}$ in order to make use of the answer at (i). Candidates were unable to obtain maximum marks for this part of the question.

Answer(s): (a) $y = \frac{1}{3} e^{2x} + \frac{c}{e^x}$

$$(b) 4y = e^{4x} + 3$$

$$(c) \frac{1}{9}(2e^3 + 1)$$

(d) (i) $I = -2\sqrt{(1-u)} + C$

(ii) 2

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 3

Specific Objectives: (a) 2, 5, 12, (b) 5, 7, 9, 10, (c) 13.

This question tested the candidates' abilities to use the recurrence relation of a sequence, apply the principle of mathematical induction, geometric progression, and the application of Maclaurin's expansion series.

- (a) (i) This part of the question was well done by the majority of candidates. Simple substitution was required.
- (ii) Few candidates demonstrated a sound understanding of the principle of mathematical induction. Moreover, the majority of candidates are still unclear about the inductive process and failed to show proper proof that the statement P_k and P_{k+1} were true for $n = \text{some } k$. More work on the principle of mathematical induction is required. Some candidates simply substituted $k + 1$ for k in the statement for P_k .
- (b) The majority of candidates were able to obtain the equations in terms of a and r for the given conditions. However, weaknesses in algebra continue to be evident and many students failed to solve for a and r correctly. Few candidates gained full marks for this part of the question.
- (c) (i), (ii), (iii) This part of the question was poorly done. Insufficient exposure and practice in using Maclaurin's theorem for expansions were evident. The Maclaurin's theorem is given in the Formulae Booklet issued to candidates at examinations and should have made it easier for them to answer this part of the question. Candidates also omitted the range of values of x for which the expansions are valid. This is one of the additional topics tested for the first time and it is evident that much more needs to be done by way of tutorials and practice.

Answer(s): (a) (i) 3, 4, 6, 9

(b) $a = 27, r = \frac{2}{3}$

(c) (i) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots \quad -1 < x \leq 1$

(ii) (a) $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \dots \quad -1 \leq x < 1$

Question 4

Specific Objective(s): (b) 13, (c) 3, 4, (e) 1, 2, 4.

This question tested the concept of existence of roots for a continuous function, the Newton-Raphson method of approximation, the binomial expansion series for positive and negative rational indices, Maclaurin's theorem, and use of expansion series for calculating the fractional value of a surd.

- (a) (i) The majority of candidates knew the principle of the Intermediate Value Theorem. However, very few of them stated that the function was a polynomial and more importantly that it was continuous.

- (ii) This part of the question was well done. Some students misinterpreted the rubric and proceeded to find a numerical value for the root α .
- (b) (i) All of the candidates opted to use the binomial expansion series for this part of the question. Arithmetical errors in calculating the coefficients of terms resulted in candidates losing marks. Very few students stated the range of values of x for which the expansion is valid. Emphasis should be placed on this aspect.
- (ii) No candidate was able to deduce this expansion from (i) and proceeded to use the binomial expansion. As in (i), arithmetical errors, particularly signs of the coefficients, resulted in loss of marks. Candidates omitted the range of values of x .
- (iii) Candidates who were able to complete (i) and (ii) correctly gained full marks on this part of the question.
- (iv) Most candidates had difficulties using the given substitution and in reducing the value of $\sqrt{2}$ to the required answer. Surds continue to be challenging for many candidates. They have difficulties working numerical problems without the use of calculators or tables.

Answer(s): (b) (i) $1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \dots, \quad -1 < x < 1$

(ii) $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16} + \dots, \quad -1 < x < 1$

SECTION C

(Module 3: Counting, Matrices and Complex Numbers)

Question 5

Specific Objective(s): (a) 2, 3, 4, 6, (b) 1, 2, 7, 8.

This question tested the principle of combinations, matrix algebra, and complex numbers.

- (a) (i) Responses to this part of the question were very satisfactory. Some candidates had difficulties distinguishing between combinations and permutations.
- (ii) The majority of candidates gained full marks for this part of the question, having obtained (i) correctly.
- (b) (a) Generally, this part of the question was well done. Errors were mostly due to incorrect arithmetic.
- (b) Candidates demonstrated good techniques for multiplying conformable matrices.
- (ii) A significant number of candidates were not able to deduce A^{-1} from the previous results. Many of them attempted to find the inverse of A by the process $\frac{1}{|A|} \text{adj } A$. Arithmetic errors did not allow some of them to obtain the correct answer. Some candidates also attempted to use row reduction of the augmented matrix
$$\left(\begin{array}{ccc|ccc} 3 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{array} \right)$$
 but had difficulties completing the process. A number of candidates were able to deduce A^{-1} from (i) b).

(iii) Most candidates demonstrated weaknesses in matrix algebra for this part of the question. Instances were seen where candidates merely stated $X = \frac{A-B}{A}$.

Answer(s): (a) (i) 70

(ii) 65

$$(b) \quad (i) \quad a) \quad \begin{pmatrix} 2 & 2 & -1 \\ 1 & 0 & 2 \\ 1 & -4 & 1 \end{pmatrix}$$

$$b) \quad \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$(ii) \quad A^{-1} = \frac{1}{3} M = \begin{pmatrix} \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & -1 \\ \frac{-1}{3} & 1 & \frac{-1}{3} \end{pmatrix}$$

$$(iii) \quad \begin{pmatrix} 1 & \frac{-2}{3} & 0 \\ -1 & 4 & -1 \\ 0 & \frac{2}{3} & 2 \end{pmatrix}$$

Question 6

Specific Objective(s): (c) 4, 7, 8, 9, 10.

This question tested rationalization of a complex number, determining the value of a multiple of a complex number, use of a conjugate, and determining the loci of a complex number.

(a) (i) This part of the question was well done.

(ii) Arithmetical errors resulted in a few candidates stating an incorrect value for λ .

(iii) Candidates used more than one method to answer this part of the question. With (i) and (ii) some candidates used the binomial expansion. Some candidates used De Moivre's theorem. Generally this part of the question was well done.

(b) (i) Apart from some difficulties with algebra, candidates performed well in this part of the question.

(ii) Approximately 95 per cent of the candidates did not attempt this question. Candidates failed to see the link with (i) and were unable to determine the technique to be used. Seemingly, the area relating to loci of complex numbers was not extensively dealt with. Candidates should be exposed to much more of this, with adequate practice.

Answer(s): (a) (i) $\frac{1}{2}(1-i)$

(ii) $\frac{1}{2}$

(iii) $\frac{-1}{4}$

(b) (ii) $C\left(\frac{1}{6}, 0\right)$

UNIT 2
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Calculus II)

Question 1

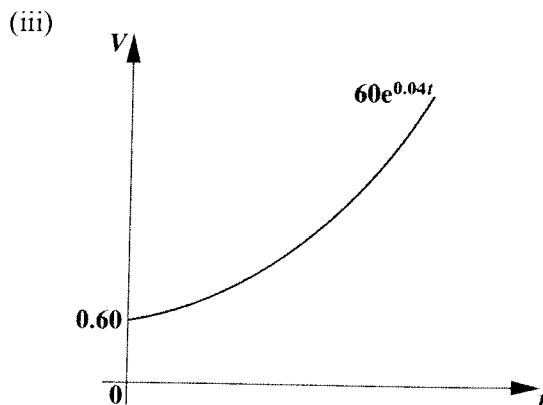
Specific Objective(s): (b) 2, 3, 5.

This question examined parametric differentiation and its application to a normal to a curve, and a modelling based on exponentials, differential equations, rate of change, and a graph of the model.

- (a) (i) Most of the candidates who attempted this part of the question were unable to determine $\frac{dy}{dt}$ and $\frac{dx}{dt}$, hence could not evaluate $\frac{dy}{dx}$. In addition the value for x at $y = 18$ was not found.
- (ii) Consequently, the correct equation of the normal was not determined. A majority of candidates did not apply the gradient of the normal as $-\frac{dx}{dy}$.
- (b) (i) A majority of the candidates did not attempt this part of the question. The few who did demonstrated a lack of understanding of differentiation of natural logarithms.
- (ii) a) This part of the question was poorly done since there was no follow-through from (i) to work with.
- b) Candidates who attempted this part of the question substituted $180 = \frac{dV}{dt}$ instead of $V = 180$.
- (iii) Only a few candidates attempted this part of the question. No candidate obtained full marks. Errors included no labels on axes and failing to use the fact that $t \geq 0$.

Overall this question was poorly done.

- Answer(s):**
- (a) (i) $\frac{1}{3}$
- (ii) $3x + y - 99 = 0$
- (b) (i) $2.4e^{0.04t}$
- (ii) (a) 3.58
- (b) 7.2



Question 2

Specific Objective(s): (a) 2, 3, (b) 5, 8, 9, (c) 3, 4.

This question tested the arithmetic and the geometric progressions, the binomial expansion series for a negative index, and the Maclaurin's theorem for expansion of a trigonometric function.

- (a) (i) a) Generally, this part of the question was poorly done. Candidates could not determine the geometric progression.
b) There was no follow-through after being unable to obtain a).
(ii) There was no follow-through after being unable to obtain the previous results.
- (b) (i) (ii) Responses to these parts of the question were poor.
- (c) (i) Some candidates attempted this part of the question but obtained the wrong answer due to arithmetical errors.
(ii) Candidates could not use the expansion given to express $\sec x$ as $\frac{1}{\cos x}$ algebraically. No candidate attempted differentiation to obtain the coefficients for the expansion of $\sec x$.

Answer(s): (a) (i) a) $5 \times 2^{r-1}$

b) $5(2^n - 1)$

(ii) $n = 8$

(b) (i) $S = (1 + 3 + 5 + 7 + \dots) + \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots\right)$

(ii) $n^2 + 1 - \left(\frac{1}{2}\right)^n$

(c) (i) $1 + y + y^2 + y^3 + y^4 + \dots$

(d) $1 + \frac{x^2}{2} + \frac{5x^4}{24} + \dots$

Question 3

Specific Objective(s): (a) 8, 9, 11, 12, 13; (b) 1, 2, 5, 6.

This question tested matrix algebra, and classical probability.

- (a) (i) Most candidates were able to state the augmented matrix. However, beyond this point there were serious challenges. They were unable to perform the necessary row reduction.
(ii) No candidate answered this part of the question.
- (b) (i) Most of the candidates re-stated the probabilities given. No evidence of the use of the Venn diagram or the formula for solving the required probability was seen.
(ii) a) b) Responses to these parts of the questions were poor. Candidates seemed not to know the definitions of independent and mutually exclusive events.

Answer(s): (a) (i) -1

(ii) $x = 5 - 23z, y = 6z - 1$

(b) (i) 0.05

(ii) a) $P(A \cup B) \neq P(A) + P(B)$; not independent

b) $P(A \cup B) \neq 0$; not mutually exclusive

PAPER 03 – INTERNAL ASSESSMENT

The Internal Assessment comprises these Module tests.

The main features assessed are the:

- Mapping of the items tested to the specific objectives in the syllabus for the relevant Unit
- Content coverage of each Module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (Module tests and students' scripts)
- Quality of the teachers' solutions and mark schemes
- Quality of the teachers' assessments – consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one Module test for each Unit

GENERAL COMMENTS

1. Too many of the Module tests comprised items from CAPE past examination papers.
2. Untidy 'cut and paste' presentations with varying font sizes were common place.
3. This year there was a general improvement in the creativity of the items, especially with regards to mathematical modelling. Teachers are reminded that the CAPE past examination papers should be used ONLY as a guide.
4. The stipulated time for Module tests ($1-1\frac{1}{2}$ hours) must be strictly adhered to as students may be at an undue disadvantage when Module tests are too extensive or insufficient. The following guide can be used: 1 minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling **MUST** be included.
5. Multiple-choice Questions will **NOT** be accepted in the Module tests.
6. Cases were noted where teachers were unfamiliar with recent syllabus changes, for example, complex numbers, 3-dimensional vectors, dividing a line segment internally or externally.
7. The moderation process relies on the validity of teacher assessment. There were a few cases where students' solutions were replicas of the teachers' solutions – some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on the students' scripts did not correspond to the marks on the Moderation sheet.
8. Teachers **MUST** present evidence of having marked each individual question on the students' scripts before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of the students' scripts.

9. To enhance the quality of the design of the Module tests, the validity of teacher assessment and the validity of the moderation process, the Internal Assessment guidelines are listed below for emphasis.

Module Tests

- (i) Design a separate test for each Module. The Module test MUST focus on objectives from that module.
- (ii) In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered, and a common marking scheme used. One sample of FIVE students will form the sample for the centre.
- (iii) In 2009, the format of the Internal Assessment remains unchanged.
[Multiple Choice Examinations will NOT be accepted.]

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required on the cover of each Module test.

- Name of School and Territory, Name of Teacher, Centre Number.
- Unit Number and Module Number.
- Date and duration (1-1½ hours) of Module Test.
- Clear instruction to candidates.
- Total marks allotted for the Module Test.
- Sub – marks and total marks for each question MUST be clearly indicated.

2. COVERAGE OF THE SYLLABUS CONTENT

- The number of questions in each Module test must be appropriate for the stipulated time of 1-1½ hours.
- **CAPE past examination papers should be used as a guide ONLY.**
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant Unit of the syllabus.

3. MARK SCHEME

- Detailed mark schemes MUST be submitted, holistic scoring is not recommended, that is, **one mark per skill should be allocated.**
- **FRACTIONAL DECIMAL MARKS MUST NOT BE AWARDED.**
- The total marks for Module tests MUST be clearly stated on the teacher's solution sheets.
- A student's marks MUST be entered on the FRONT page of the student's script.
- Hand written mark schemes MUST be NEAT and LEGIBLE. The marks MUST be presented in the right hand side of the page.
- Diagrams MUST be neatly drawn with geometrical/mathematical instruments.

4. PRESENTATION OF THE SAMPLE

- Student's responses MUST be written on normal sized paper, preferably 8 1/2 ×11.
- Question numbers are to be written clearly in the left margin.
- The total marks for each question on students' scripts MUST be clearly written in the right margin.
- **ONLY original students' scripts MUST be sent for moderation. Photocopied scripts will not be accepted.**

- Typed Module tests MUST be in a legible font size (for example, size 12). Hand written texts MUST be NEAT and LEGIBLE.
- The following are required for each Module test:
 - A question paper.
 - Detailed solutions with detailed mark schemes.
 - The scripts (for each Module) of the candidates comprising the sample. The scripts MUST be collated by Modules.
- Marks recorded on the PMath 1 – 3 and PMath 2 – 3 forms must be rounded off to the nearest whole number.
- The guidelines at the bottom of these forms should be observed. (See page 57 of the syllabus, no.6).
- In cases where there are five or more candidates, FIVE samples MUST be sent.
- In cases where there are less than five registered candidates, ALL samples MUST be sent.

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

**REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2009**

PURE MATHEMATICS

PURE MATHEMATICS

MAY/JUNE 2009

GENERAL COMMENTS

This is the second year that the current syllabus has been examined in the new format of Paper 01 as Multiple Choice (MC) and Papers 02 and 03 in the typical essay-type questions. The syllabus is arranged into two Units, each consisting of three Modules:

Unit 1

- Module 1 - Basic Algebra and Functions
- Module 2 - Trigonometry and Plane Geometry
- Module 3 - Calculus I

Unit 2

- Module 1 - Calculus II
- Module 2 - Sequences, Series and Approximations
- Module 3 - Counting, Matrices and Complex Numbers

There were 5579 candidates who wrote the examinations for Unit 1 in 2009 compared to 4 995 in 2008 and for Unit 2, 2 701 compared to 2 690 in 2008. Performances varied across the entire spectrum of candidates with a significant number obtaining excellent grades. Nevertheless, there continues to be a number of candidates who seem unprepared to write the examinations, particularly for Unit 1. A more effective screening process needs to be instituted to reduce the number of poorly prepared candidates.

DETAILED COMMENTS

UNIT 1

The overall performance in this Unit was satisfactory with a number of candidates excelling in such topics as Trigonometric Identities, Coordinate Geometry, Basic Differential and Integral Calculus and Surds. However, many candidates continue to find Indices, Limits, Continuity/Discontinuity and Algebraic Manipulation challenging. These topics and techniques should be given special attention if improvement in performance is to be achieved. Other areas that need consolidation are general algebraic manipulation of simple terms, expressions and equations, substitution, either as a substantive topic in the syllabus or as a tool for problem solving.

Paper 01 comprised 45 multiple-choice items, with 15 items based on each Module. The candidates performed satisfactorily with a mean score of 21 out of a possible 45. Paper 02 comprised six compulsory questions, two testing each Module. The mean mark on this paper was 51 out of a possible 150.

UNIT 1**PAPER 02****SECTION A****Module 1: Basic Algebra and Functions****Question 1**

Specific Objective(s): (a)5; (b)5; (c)1, 3(iii), 5;(g).

This question tested knowledge of surds factors for expressions of the form $a^n - b^n$, simple skills in equalities and logarithms. Many candidates had difficulty with the algebraic manipulation of $x^4 - y^4$ and the change of base in Part (c).

- (a) There were several good and complete answers to this part of the question. The mistakes most frequently encountered related to $\sqrt{4 \times 7} = 4\sqrt{7}$ and/or $\sqrt{7^2 \times 7} = 7^2\sqrt{7}$.
- (b)(i) The observation that $x^4 - y^4 = (x^2 + y^2)(x^2 - y^2) = (x^2 + y^2)(x^2 - y^2)$ presented most difficulty for many candidates who tried the method factorization. Many of those who used long division succeeded in gaining full marks for this part of the question.
- (ii) The substitution of $x = y + 1$ was poorly done by many of the candidates.
- (iii) There were not many good attempts at this part.

The change of base concepts presented enormous difficulties for candidates. More practice in this area is recommended.

Answer(s):

- (a) $k = 9$,
- (b)(i) $x^3 + x^2y + xy^2 + y^3$,
- (c) $x = \frac{1}{16}$

Question 2

Specific Objective(s): (d) 1, 2, 7; (f) 3, 5(i).

This question examined properties of the roots of quadratic equations of functions and evaluation of a given function defined on intervals of the real numbers.

Part (a) of this question was very well done. However, in Part (b), candidates had difficulty applying the basic definition of a function, while in Part (c) the main weakness arose in recognizing the piecewise nature of the function f . More practice in these topics is recommended.

Answer(s):

(a) $5x^2 + 8x + 8 = 0$

(b)(i) $f = \{(u, 1), (v, 2), (v, 3), (x, 1), (y, 3), (z, 4)\}$

(ii) a) $v \in A$ has two images in B and $w \in A$ has no image in B .

b) For $g: A \rightarrow B$, remove from $f: A \rightarrow B$ one of the ordered pairs $(v, 2)$ or $(v, 3)$ and map $w \in A$ to some $b \in B$.
 eg. $g = \{(u, 1), (v, 2), (x, 1), (w, 1), (y, 3), (z, 4)\}$

c) No. of functions $g = 4 \times 2 = 8$

(c)(i) $f(f(20)) = f(5) = \frac{5}{4},$

(ii) $f(f(8)) = f(2) = -1,$

(iii) $f(f(3)) = f(0) = -3,$

SECTION B**Module 2: Trigonometry and Plane Geometry****Question 3**

Specific Objective(s): (b) 1, 2, 3, 5, 7, 8.

This question examined in Part (a), the application of coordinate geometry to the properties of a circle, straight lines, tangents, normal and intersections between straight lines and curves. Additionally, Part (b) of the question examined the concept of finding the angle between two given vectors, position vectors and displacement vectors, as well as finding the area of the triangle.

The majority of the candidates attempted this question, and while a few of them would have attained full marks, a number of candidates had difficulties working out the coordinate geometry, especially the vector questions.

- (a) Finding the radius and coordinates of the centre were easily answered. However, many candidates unnecessarily expanded the equation of the circle to find the coordinates of the centre. A number of candidates did not recognize that the gradient of the radius is actually the gradient of the normal at the point and hence did not get the equation of the tangent correct. Many students recognized they had to solve simultaneous equations for Part (iii) but used the equation of the tangent from Part (ii) instead of the equation of the circle, since they did not read the definition of C carefully.
- (b) This part was attempted by the majority of the candidates who successfully used various methods to calculate the size of the angle between the vectors \mathbf{p} and \mathbf{q} . A number of students used $A = \frac{1}{2}bh$ to find the area of the triangle without checking to see if it was a right-angled triangle. However, some candidates used the correct formula $A = \frac{1}{2}pq \sin\theta$ but some used $\mathbf{p} \cdot \mathbf{q}$ rather than $|p||q|$. The majority of candidates found the vector \mathbf{PQ} , but had difficulty

finding the midpoint of **PQ** since they took OM as $\frac{1}{2}$ PQ instead of $OM = OP + PM$ or $\frac{1}{2}(OP + OQ)$. Candidates did not recognize that OR was equal and parallel to PQ. Even those candidates who did well in the majority of the question fell down at this point.

It was evident that aspects of the syllabus needed to be reinforced. More emphasis must be placed on the equation $(x - a)^2 + (y + b)^2 = r^2$, where (a, B) represents the centre of the circle. The use of diagrams in the teaching and answering of exercises on coordinate geometry and vectors should be encouraged in order to strengthen the responses in this area.

Answer(s):

(a) (i) 5 units; (3, 4)

(b) (i) a) 30° ;

(ii) $y = -\frac{3}{4}x + \frac{25}{2}$

b) 13 square units

(iii) (-1,1); (3,9)

(ii) a) $i + 7j$ b) $4i + 2j$

Question 4

Specific Objectives(s): (a) 4, 5, 9, 12; (b) 1.

This question tested the candidates' ability to use and apply trigonometric functions, identities and equations.

A significant number of students attempted this question. A number of the candidates who attempted Part (a) attempted Part (ii) only. Part (b) and (c) proved to be quite popular with the candidates, with a significant number of candidates scoring the majority of marks in Part (b).

In Part (a) (i), there were few candidates who drew lines parallel to AD and CD respectively, to create the two right-angled triangles. Those who did were then able to use these two triangles to prove the result. Some candidates attempted methods such as sine and cosine rules without success. Most of the candidates who attempted Part (ii) of this question were able to obtain the correct values for r and α .

Some of the errors observed included:

- $r = \sqrt{(4+9)}$
- $r^2 = \sqrt{(4^2 + 9^2)} \Rightarrow r = 13$
- $\tan \alpha = \frac{4}{9}$
- Maximum value is $\theta = \alpha$ rather than the x-value

Part (b) was successfully completed by a significant number of candidates.

Some candidates, however, obtained incorrect solutions mainly due to

- Obtaining incorrect values for $\cos A$ and $\sin B$
- Improper use of the relevant identities
- Incorrect substitution

Most candidates attempted Part (c) of the question. Many were able to successfully complete the first two steps of the proof, that is the expansion of $\tan(A + B)$, as well as recognizing $\tan \frac{\pi}{4} = 1$.

Many of the candidates failed to realize that some form of rationalization (use of $(a + b)(a - b) = a^2 - b^2$) had to be invoked to successfully complete the process.

Some candidates were successful using the t-approach. It was also observed that those candidates who were successful were adept at manipulating trigonometric identities.

Answer(s):

(a) (ii) $\sqrt{97}$

(b) (i) $\frac{63}{65}$, (ii) $\frac{56}{65}$, (iii) $\frac{7}{25}$

SECTION C

Module 3: Calculus 1

Question 5

Specific Objectives(s): (a) 1, 3, 4, 5, 8, 10; (b) 4; (c) 3, 4, 5 (ii), 6

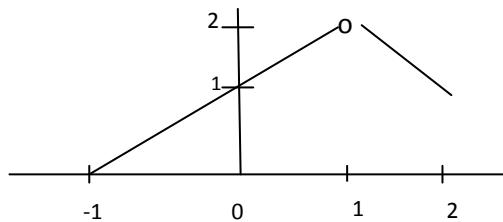
This question covered topics on limits, continuity, differentiation from first principles and integration. The question was attempted by most of the candidates. The general performance was below average with only a limited number of candidates scoring more than 20 marks.

- (a) In this part, several errors were made in factorizing $x^3 - 8$ which suggests that more practice is required on exercises of this sort.
- (b) The graph of the function was done correctly by many candidates. A few recognized that there was a ‘break’ somewhere in the graph but did not know where it should be placed.
Many candidates substituted -1 into $f(x) = 1 + x$ to find $\lim_{x \rightarrow -1^-} f(x)$. Very few candidates seemed to know the definition of ‘continuity’ and as a consequence did not find $f(1)$.
- (c) Many candidates did not seem to know what ‘differentiation from first principles’ meant and some who knew were not able to complete the process successfully.
- (d) Many candidates did not include kx and a constant of integration after integrating. Others attempted to find k before integrating.

Answer(s):

(a) -6

(b)(i)



$$\text{(ii) a) } \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (3 - x) = 3 - 1 = 2$$

$$\text{b) } \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (1 + x) = 1 + 1 = 2$$

$$\text{(iii) } f(1) = 3 - 1 = 2 \Rightarrow f(x) \text{ is continuous at } x = 1$$

$$\text{(c) } \frac{dy}{dx} = -\frac{2}{x^3}$$

$$\text{(d) } f(x) = x^3 + 3x^2 - x - 6$$

Question 6

Specific Objective(s): (b) 7(ii), 10, 14, 15; (c) 4, 5, 6 (i)

This question tested areas of the differential and integral calculus related to the definite integral and maximum/minimum problems.

- (a) This part of the question required finding the first and second derivatives of a trigonometric function and the formation of a differential equation from such derivatives.

The question was very popular with an excellent success rate.

- (b) Many candidates obtained parts of the integrals correctly but were unable to complete the question successfully because of errors in the algebraic manipulation of the terms.

- (c) A few candidates found difficulty in obtaining the correct expression for V in (i). Others lost their way in solving $\frac{dv}{dx} = 0$ and using $\frac{d^2v}{dx^2}$ correctly.

Despite the weaknesses identified above there were several candidates who obtained full marks for this question.

Answer(s):

$$\text{(b) } a = 4$$

$$\text{(c) (ii) } x = 2$$

UNIT 1**PAPER 03/B - ALTERNATE TO INTERNAL ASSESSMENT****SECTION A****Module 1: Basic Algebra and Functions****Question 1**

Specific Objective(s): (c) 1, 2, 3(ii), (iii), 5; (f) 3; (g) 1, 4

This question tested inequalities, the modulus of real numbers, algebraic expressions involving substitution, logarithms and mathematical modeling.

- (a) Although a number of the candidates attempted this part of the question, many of them had difficulty manipulating the modulus sign and this weakness translated into the formation of inappropriate inequalities. There were, however, a few good answers to the problem.
- (b) Many candidates did not use the substitution to its full advantage. Some others equated the expression in y to 3 000 and not to 3; nevertheless, there were some encouraging attempts presented by a few candidates.
- (c) Some candidates found difficulty in establishing Part (i), while other candidates did not see the relevance of Part (i) to Part (ii). Outside of these instances, there were a few candidates who completed this part of the question successfully.

Answer(s):

(a) $\{x \in \mathbb{R} : -2 < x < 0\}$

(b) $x = 1, 3, 2 \pm \sqrt{5}$

(c)(ii) 10

SECTION B**Module 2: Trigonometry and Plane Geometry****Question 2**

Specific Objective(s): (a) 6, 14; (b) 6, 7, 9

This question tested tangents to circles and properties of the locus of a point with coordinates described in parametric form.

- (a) Many candidates showed the correct methodology in solving the problem but seemed unprepared to cope with the general point (p, q) on the circle. As a consequence, there were several unfinished solutions to this part of the question.
- (b) Several candidates did not appeal to the basic properties of $\sin x$ and $\cos x$, namely, $0 \leq |\sin x| \leq 1$, $0 \leq |\cos x| \leq 1$ and $\sin^2 x + \cos^2 x = 1$ to solve the problems posed in this part of the question, and hence missed the simple approach to the solutions. Attempts at using calculus were made.

Answer(s):

(a)(iii) $p = -5, q = 1$ or $p = -2\frac{3}{5}, q = \frac{1}{5}$

(b)(i) $\max x = 5, \min x = -1$
 $\max y = 8, \min y = 0$

(ii) $\left(\frac{x-2}{3}\right)^2 + \left(\frac{y-4}{4}\right)^2 = 1$

SECTION C**Module 3: Calculus 1****Question 3**

Specific Objective(s): (b) 7 to 10, 13 to 16, (c) 1 to 4

This question covered indefinite integrals and point of inflexion of, and normal to curves, as well as the notion of mathematical modeling.

- (a) This part was not well done. The main hindrance to obtaining the correct solution stemmed from the candidates' failure to resolve the integrand into separate terms before attempting to integrate.
- (b) The concept of a 'point of inflexion' seemed unfamiliar to many candidates. This resulted in several candidates not being able to find the values of b and c in (i), without which it was impossible to solve Part (ii) explicitly.
- (c) Not many candidates attempted this part of the question, which depended on the notion of small increments, which is knowledge applied to the standard approach to the introduction of differentiation from first principles in calculus.

Answer(s):

(a) $\frac{t^2}{2} - \frac{1}{t^3} + \frac{1}{4t^4} + \text{constant of integration}$

(b)(i) $b = 3, c = 3$
(ii) $3y = x + 16$

(c)(i) when $r = 3, \frac{dV}{dt} = 0.72\pi$
(ii) $p=2$

DETAILED COMMENTS

UNIT 2

In general, the performance of candidates on Unit 2 was very satisfactory. Although an increased number of candidates reached an outstanding level of proficiency, some candidates were inadequately prepared for the examinations.

The examination tested some of the newer topics in the revised syllabus and included Calculus of Inverse Trigonometrical Functions and Second Derivative, the use of an Integrating Factor for First-order Differential Equations, Second-order Differential Equations, Maclaurin's Theorem for Series Expansions, Binomial Expansion Series for Rational and Negative Indices, Complex Numbers and the Locus of a Complex Number.

Weaknesses in algebraic manipulation and tasks involving substitution were again evident and candidates found it difficult to solve problems which required these skills. It is imperative that more emphasis be placed on these areas of weakness. Extensive practice in the use of substitution and algebraic manipulation is necessary if candidates are to be well-prepared to show improved performances in these areas.

Paper 01 comprised 45 multiple choice items. The candidates performed fairly well with a mean score of 25 out of a possible 45. Paper 02 comprised six compulsory questions, two testing each Module. The mean mark on this paper was 54 out of a possible 150.

UNIT 2

PAPER 02

SECTION A

Module 1: Calculus II

Question 1

Specific Objective(s): (b) 2, 3, 4, 5, 6, 7

This question examined concepts in differentiation as they apply to trigonometric functions, inverse trigonometric functions, implicit functions and rational functions. Second derivatives emerged in the process leading to the formation of differential equations.

- (a)(i) This part of the question was well done although many candidates did not use the identity $\sin^2 3x + \cos^2 3x = 1$ to simplify the given expression for y . As a consequence, many answers were not given in the simplest form. No penalty was applied for non-simplification.
- (ii) Many candidates found difficulty in differentiating $\cos x^2$. Several interpreted $\cos x^2$ as $(\cos x)^2$ or $(\cos x)x$.
- (iii) This part of the question was not well done. Too many candidates did not know how to cope with the implicit nature of the expression for y .
- (b)(i) This part of the question was generally well done. However, amongst the candidates who did not perform satisfactorily, many equated $\cos^{-1} x$ with $\frac{1}{\cos x}$.

- (ii) Candidates found this exercise manageable and readily recognized the relevance of the chain rule to the results. Mistakes were made in a), in differentiating $\sqrt{1-t}$ while in b) the second derivative $\frac{d^2y}{dx^2}$ proved to be a major challenge for many. A common mistake made in this case was $\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} \times \frac{d^2t}{dx^2}$.

Answer(s):

(a) (i) $\frac{dy}{dx} = 10 \sin 5x \cos 5x$ (in its simplest form)

(ii) $\frac{dy}{dx} = -\frac{x \sin x^2}{\sqrt{\cos x^2}}$

(iii) $\frac{dy}{dx} = x^x(1 + \ln x)$

(b)(ii) $\frac{d^2y}{dx^2} = -\frac{\sqrt{1-t}}{4}$

Question 2

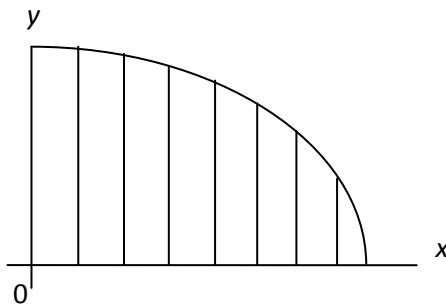
Specific Objective(s) (c) 7, 8, 13

This question examined knowledge of the trapezium rule and integration by parts in the internal calculus. Inverse trigonometric functions and approximations were also involved.

- (a) This part of the question asked for the sketch of the particular function $\sqrt{1-x^2}$ on the interval $0 \leq x \leq 1$. Some candidates were unable to determine the correct quadrant or the circular property of the function.
- (b) Several candidates could not find the width of the strips, but nevertheless, showed competent knowledge of the trapezium rule.
- (c) (i) The majority of candidates were able to obtain the first 5 of the 9 marks allocated to this integration but could not proceed to completion of this part.
- (ii) Several candidates did not see the link between (c)(i) above and failed to obtain the result for I .
- (iii) Many answers for this part were stated in terms of degrees and not radians as expected.
- (iv) The majority of candidates were unable to combine (c)(i) to (iii) to obtain the approximation to π . However, there were a few good answers to this question.

Answer(s):

(a)



The width of each strip = 0.2

$$(c)(iii) \int_0^1 \sqrt{1-x^2} dx = \frac{\pi}{4}$$

$$(iv) \pi \approx 0.759 \times 4 = 3.036$$

SECTION B

Module 2: Sequences, Series and Approximations

Question 3

Specific Objective(s): (a) 1, 2; (b) 1, 7, 11, 12

This question tested candidates' abilities to use the recurrence relation of a sequence to obtain values of the common ratio of a convergent geometric series, the method of differences for the summation of series and the sum to infinity.

- (a) The majority of candidates answered Part (i) correctly but had serious difficulties in obtaining t_n in Part (ii).
- (b) Few candidates coped well with this part of the question and among those attempting the question, some had severe challenges resolving the inequality produced.
- (c) Part (i) was easily obtained by many of the candidates several of whom fell down as they proceeded through to Part (ii) in order to find S_n . Many resorted to partial fractions in order to cope with Part (ii)

It is recommended that extended practice in questions of this nature be undertaken to consolidate the fundamental concepts portrayed in this question.

Answer(s):

$$(a) (i) t_2 = 16, t_3 = 21, t_4 = 26$$

$$(ii) t_n = 5n + 6$$

$$(b) -\frac{1}{3} < x < 7$$

$$(c)(i) f(r) - f(r+1) = \frac{1}{(r+1)(r+2)} \quad (ii) S_n = 2 - \frac{4}{n+2} \quad (iii) S_{\infty} = 2$$

Question 4

Specific Objective(s): (b) 13; (c) 1, 3

The topics examined in this question were the binomial theorem, Maclaurin's theorem and power series expansions.

Overall, the candidates' performances in this question were below the expected level. Only approximately one-third of the candidates obtained more than 13 marks out of a possible 25. Areas of good performances involved Parts (a) (ii) and (b) (ii).

- (a)(i) Candidates experienced difficulty in using the general binomial coefficient nC_r in problems of this kind.
- (ii) Some good performances were registered in this section. Some of the candidates who answered poorly ignored the fact that terms in the separate expansions of $(1+2x)^5$ and $(1+px)^4$ should have been multiplied instead of added to obtain the correct results.
- (b)(i) The expansion of $\ln(1+x)$ appeared to be unfamiliar to many candidates. Without this basic expansion, $\ln(1+2x)$ became much harder to obtain.
- (ii) Several errors were made in deriving the various derivatives of $\sin 2x$.
- (iii) Many candidates did not link this part to the earlier results obtained in the question and hence lost direction in trying to proceed. More practice is recommended.

Answer(s):

$$\begin{aligned} \text{(a) (i)} \quad n &= 4, \\ \text{(ii)} \quad p &= -3 \text{ or } \frac{-11}{3} \end{aligned}$$

$$\text{(b)(i)} \quad \ln(1+2x) = 2x - 2x^2 + \frac{8}{3x^3} - 4x^4 + \dots$$

$$\text{(ii)} \quad \sin 2x = 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 - \dots$$

$$\text{(iii)} \quad \ln(1+\sin 2x) = 2x - 2x^2 + \frac{4}{3}x^3 \dots$$

SECTION C**Module 3: Counting, Matrices and Complex Numbers****Question 5**

Specific Objective(s): (a) 1, 2, 3, 7, 10; (c) 1, 3, 4, 5

This question tested simple counting techniques, elements of probability and properties of complex numbers.

- (a) There were several attempts at this part of the question with a high degree of success. The majority of candidates who attempted the question obtained full marks.

- (b) A large number of the candidates who did this part of the question obtained full credit for their efforts. Some candidates, however, had difficulty in writing down the correct combinations. Many candidates tried to capitalize on the result in (b)(i) above but experienced challenges.
- (c) Most candidates who attempted this part of the question were able to substitute and expand correctly. Some failed to achieve this end because of faulty algebraic manipulation of the expressions. Many candidates did not appreciate that the theory of quadratic equations applied and hence did not obtain the discriminant. Others who obtained the discriminant did not observe that the result of (c) (i) was relevant.

Answer(s):

(a) 50

(b)(i) $\frac{5}{22}$,
(ii) $\frac{6}{11}$

(c)(i) $u = 1 + 4i$ or $-1 - 4i$
(ii) $z = 2 + 3i$ or $1 - i$

Question 6

Specific Objective(s): (b) 1, 2, 6, 7

This question examined properties of determinants and matrices and solutions of simultaneous linear equations in three variables.

- (a) There were many attempts at this part of the question with a high degree of success. However, poor algebraic manipulation was the cause of many errors in the solutions.
- (b)(i) Several candidates who attempted this part of the question gained full marks.
- (ii) Most candidates were able to express the system of equations in the required form. Many candidates saw the relevance of Part (ii) a) to the solutions of the system. Several others used the ‘otherwise’ path and employed different approaches to solving the system of equations.

Answer(s):

(a) $x = 2, 3$ or 6

(b)(i) a) $AB = 20I$, **b)** $A^{-1} = \frac{1}{20}B$

(ii) a)
$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 3 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 25 \end{pmatrix}$$

b) $x = 1, y = 2, z = 12$

UNIT 2**PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)****SECTION A****Module 1: Calculus II****Question 1**

Specific Objective(s): (c) 11, 12, and MM

One part of the question posed a mathematical problem against the background of a differential equation. Both parts examined the candidates' skills in solving such equations.

The success rate in this question was not very high, although many candidates attempted it. More exposure to such problems is required at the instructional level.

Answer(s):

- (a) $xy = (x + 1)(c - \frac{1}{2}e^{-x^2})$, c is constant
- (b) $y = e^{-x} - e^{4x} - \sin x$

SECTION B**Module 2: Sequences, Series and Approximations****Question 2**

Specific Objective(s) 6, 8, 9 and MM

The question tested the principle of mathematical induction as well as arithmetic and geometric progressions.

The majority of candidates who attempted this question performed satisfactorily.

- (a) Candidates knew how to verify the initial step in the proof for $n=1$, but some had difficulty with the induction step in proceeding from $n = k$ to $n = k + 1$.
- (b) Most candidates used the formula for S_n to find the sum of all the terms, however, a few went the route of trying to calculate the value of the common difference and were unsuccessful. Despite the incidence of such cases, this part of the question was very well done.
- (c) Many candidates failed to realize that the problem posed involved a geometric progression, nevertheless, some used the simple interest formula for each year and were able to reach the correct result. Most candidates did not approximate the answers to the nearest dollar.

Answer(s):

- (b) Sum = -300;
- (c)(i) \$29 877, (ii) \$ 273 743

SECTION C

Module 3: Counting, Matrices and Complex Numbers

Question 3

Specific Objective(s): (a) 1, 2, 4, 7, 9, 11; (c) 5, 7, 10

This question examined basic principles in counting methods, probability and the locus of complex numbers.

In general, candidates exhibited familiarity with the topics examined. At the level of detail, however, weaknesses were evident.

- (a) Some candidates demonstrated an understanding of combinations. However, interpretation of exclusiveness was weak and led to incorrect solutions.
- (b) Many candidates used a Venn diagram to reason out what was required, while some tried to use formulae but failed to obtain the correct answers.
- (c) This part of the question presented an enormous amount of difficulty. Many were unable to obtain the circle in (i) and hence (ii) was not manageable in such circumstances.

Answer(s):

(a) 251

(b)(i) $P(A \cup B) = 0.7$, (ii) $P(A \cap B') = 0.5$, (iii) 0.6

(c)(i) locus is the circle $(x - 1)^2 + (y + 1)^2 = 5^2$

(ii) radius = 5, centre $\equiv (1, -1)$

PAPER 03 – INTERNAL ASSESSMENT

This year 170 Unit 1 and 158 Unit 2 Internal Assessments were moderated. Far too many teachers continue to submit solutions without unitary mark schemes. In some cases neither question papers with solutions nor mark schemes, were submitted. The majority of the samples submitted were not of the required standard. Teachers MUST pay particular attention to the following guidelines and comments to ensure effective and reliable submission of the Internal Assessments.

The Internal Assessment is comprised of three Module tests. The main features assessed are:

- Mapping of the items tested to the specific objectives of the syllabus for the relevant Unit
- Content coverage of each Module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (Module test and students' scripts)
- Quality of the teachers' solutions and mark schemes
- Quality of the teachers' assessments-consistency of marking using the mark schemes
- Inclusion of mathematical modeling in at least one Module test for each Unit

GENERAL COMMENTS

1. Too many of the Module tests comprised of items from CAPE past examination papers.
2. Untidy “cut and paste” presentations with varying font sizes were common place.
3. Teachers are reminded that the CAPE past examination papers should be used ONLY as a guide.
4. The stipulated time for Module tests (1 to $1\frac{1}{2}$ hours) must be strictly adhered to as students may be at an undue disadvantage when Module test are too extensive or are inadequate.
5. The following guide can be used: 1 minute per mark. About 75% of the syllabus should be tested and mathematical modeling MUST be included.
6. Multiple choice questions will NOT be accepted in the Module tests.
7. Cases were noted where teachers were unfamiliar with recent syllabus changes i.e.
 - Complex numbers and the Intermediate Value Theorem are now tested in Unit 2.
 - Three dimensional vectors, dividing a line segment internally and externally, systems of linear equations have been REMOVED for the CAPE syllabus (2008).
8. The moderation process relies on the validity of the teachers’ assessment. There were few cases where students’ solutions were replicas of the teachers’ solutions – some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on the students’ scripts did not correspond to the marks on the Moderation sheet.
9. Teachers MUST present evidence of having marked each individual question on the students’ script before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of the students’ scripts. To enhance the quality of the design of the Module tests, the validity of the teacher assessment and validity of the moderation process, the Internal Assessment guidelines are listed below for emphasis.

Module Tests

- I. Design a separate test for each Module. The Module test MUST focus on objectives from that Module.
- II. In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered and a common marking scheme used.
- III. One sample of FIVE students will form the sample for the centre. If there are less than five students ALL scripts will form the sample for the centre.
- IV. In 2009, the format of the Internal Assessment remains unchanged.

MULTIPLE CHOICE EXAMINATIONS WILL NOT BE ACCEPTED AS INTERNAL ASSESSMENTS.

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required on the cover of each Module test.

- Name of the school and territory, Name of teacher and the Centre number.
- Unit Number and Module Number
- Date and Duration ($1-1\frac{1}{2}$ hours) of Module test.
- Clear instructions to candidates
- Total marks allotted for Module test.
- Sub-marks and total marks for each question MUST be clearly indicated.

2. COVERAGE OF THE SYLLABUS CONTENT

- The number of questions in each Module test must be appropriate for the stipulated time of ($1-1\frac{1}{2}$ hours).
- **CAPE past examination papers should be used as a guide ONLY.**
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant Unit of the syllabus.

3. MARK SCHEME

- Detailed mark schemes **MUST** be submitted, that is, **one mark should be allocated per skill.** (not 2, 3, 4, etc marks per skill)
- **FRACTIONAL/DECIMAL MARKS MUST NOT BE AWARDED**
(i.e. DO NOT ALLOCATE $(\frac{1}{2})$ MARKS).
- The total marks for Module test **MUST** be clearly stated on the teachers' solutions sheets.
- A student's marks **MUST** be entered on the front page of the student's scripts.
- Handwritten marks schemes **MUST** be NEAT and LEGIBLE. The unitary marks **MUST** be written on the right side of the page.
- **Diagrams MUST be neatly drawn with geometrical/mathematical instruments.**

4. PRESENTATION OF SAMPLE

- Student's responses **MUST** be written on letter sized paper ($8\frac{1}{2} \times 11$).
- Question numbers **MUST** be written clearly in the left hand margin.
- The total marks for EACH QUESTION on student's scripts **MUST** be clearly written in the left or right margin.
- **ONLY ORIGINAL** students' scripts **MUST** be sent for moderation.
- Photocopied scripts **WILL NOT BE ACCEPTED.**
- Typed Module tests **MUST** be NEAT and LEGIBLE.
- The following are required for each Module test:
 - ❖ A question paper.
 - ❖ Detailed solutions with detailed Mark Scheme.
 - ❖ The question paper, detailed solutions, Marks Schemes and 5 students' samples should be batched together for each Module.

- Marks are recorded on PMath1 – 3 and PMath2 – 3 forms and must be rounded off to the nearest whole number. If a student scored zero, then zero must be recorded. If a student was absent, then absent must be recorded.
- The guidelines at the bottom of these forms should be observed. (See Page 57 of the syllabus, No. 6).

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

**REPORT ON CANDIDATES' WORK IN THE
ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2010**

PURE MATHEMATICS

GENERAL COMMENTS

This is the third year that the current syllabus has been examined in the format of Paper 01 as Multiple Choice (MC) and Papers 02 and 03 structured questions. Approximately 5 600 candidates wrote the Unit 1 and 2 800 Unit 2 examinations in 2010. Performances continued in the usual pattern across the total range of candidates with some obtaining excellent grades while some candidates seemed unprepared to write the examinations at this level, particularly in Unit 1.

UNIT 1

The overall performance in this unit was satisfactory, with several candidates displaying a sound grasp of the subject matter. Excellent scores were registered with specific topics such as Trigonometry, Coordinate Geometry and Calculus. Nevertheless, candidates continue to show weaknesses in areas such as Limits and Continuity, Indices and Logarithms. Other aspects that need attention are summation as part of general algebraic manipulation of simple expressions, substitution and pattern recognition as effective tools in problem solving.

UNIT 2

In general, the performance of candidates on Unit 2 was satisfactory. It was heartening to note the increasing number of candidates who reached an outstanding level of proficiency in the topics examined. However, there was evidence of significant unpreparedness by many candidates.

Candidates continue to show marked weakness in algebraic manipulation. Much more emphasis must be placed on improving these skills. In addition, reasoning skills must be sharpened and analytical approaches to problem solving must be emphasized. Too many candidates demonstrate a favour for problem solving by memorized formulae.

DETAILED COMMENTS

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily with a mean score of 46.27 and a standard deviation of 17.02.

Paper 02 – Structured Questions

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (b) 1, 3, 4; (c) 1–5; (f) 3; (g) 3.

The topics examined in this question covered the Remainder and Factor Theorems, simultaneous equations and logarithms, inequalities, quadratic equations and indices.

- (a) The majority of candidates correctly applied the Factor Theorem to $f(x)$. Several opted to find the remainder by division but were unable to follow through successfully because of weaknesses in the algebraic manipulation required. Some were unable to factorize the equation for $f(p) = 0$ using the quadratic formula.
- (b) Several candidates misinterpreted the basic laws of logarithms, replacing $\log(x - 1) + 2 \log y$ with expressions such as $\frac{\log x}{\log 1} + \log 2y$. Such equivalences produced erroneous results and, in some cases, equations which the candidates could not solve. Elimination was tried in some cases without success.
- (c) Many candidates multiplied through by $x + 1$ instead of $(x + 1)^2$ and had difficulty afterwards in solving the resulting inequality.
- (d) The substitution $y = 2^x$ proved problematic for some candidates, many of whom interpreted 4^x as 2×2^x which led to incorrect answers. Other candidates did not complete the question, giving the answers for y only and not for x as required. More practice is needed to consolidate the areas of weakness identified above.

Solutions:

- (a) $p = \frac{3}{2}$ or -1
 (b) $x = 2, y = 3$
 (c) $-\frac{8}{3} < x < -1$
 (d) $x = 1$ or 2

Question 2

Specific Objectives: (a) 7; (d) 5–8.

The topics covered in this question related to summation notation and functions, together with the interpretation of the graphs of simple polynomial functions and the existence or non-existence of inverses.

- (a) The majority of candidates answered (i) correctly. A common mistake made was to equate S_{2n} with $2 \times S_n$, while some candidates misinterpreted the question as a problem solving mathematical induction. Several candidates encountered simplification problems in (ii). A common error involved writing $-Sn$ as $-n^2/2 + n/2$ instead of $-n^2/2 - n/2$. In (iii), the correct quadratic equation in n , namely $3n^2 + n - 520 = 0$, escaped many candidates and some who derived it failed to solve it correctly to obtain $n = 13$.
- (b) (i) Approximately 80 per cent of the candidates gave $x \geq 3$ as a possible solution. Some obtained the complete solution but failed to write in set notation. Several used the graph and gave the solution as $x \leq 0, x \leq 3$.
- (ii) Less than 10 per cent of the candidates gave the correct solution. Many assigned values to k and attempted to solve the equation $x^2(3-x) - k = 0$, thereby completely ignoring the graph as a guide to the solution.

- (iii) Many candidates showed familiarity with the terms injective, surjective and bijective as they applied to functions but seemed to struggle to separate the terms in respect of the given graphical representations for f and g . Approximately 80 per cent of the candidates used the horizontal line test (a) and (b).

Solutions:

(a) (i) $S_{2n} = n(2n + 1)$

(ii) $p = \frac{3}{2}$, $q = \frac{1}{2}$

(iii) $n = 13$

(b) (i) $\{0\} \cup \{x \geq 3\}$ (ii) $\{k : 0 \leq k \leq 4\}$

Section B

Module 2: Trigonometry and Plane Geometry

Question 3

Specific Objectives: (b) 4, 5; (c) 3, 9, 10

Overall, candidates performed poorly on this question with approximately 70 per cent scoring less than 12 of the maximum marks (25) and approximately 40 per cent scoring less than 4 marks. Generally, candidates scored higher in Part (a) than in Part (b) of the question. Attempts at Part (a) (i) by the candidates who achieved an acceptable score were usually fruitful. Many of the candidates gained full marks for finding the angle. In addition, many of them used the form $\mathbf{p} \cdot \mathbf{q} = |\mathbf{p}||\mathbf{q}| \cos \theta$. However, a large percentage of candidates did not score maximum marks as they had a problem adding directed numbers accurately or they simply forgot the negative sign in front of the 84. Many candidates used the alternative method which included the graph and trigonometrical ratios. However, a few of them forgot to add the 90° which was required to find the entire angle.

A significant number of candidates experienced difficulty in the relatively easier form of finding the x co-ordinate of the vector in (a) (ii). Many started at $(6\mathbf{i} + 4\mathbf{j}) \cdot (x\mathbf{i} + y\mathbf{j}) = 6x + 4y = 0$ and then stopped. Astoundingly, many candidates declared that $\mathbf{p} \cdot \mathbf{v} = 0$ was either parallel or inverse.

Overall, performance on (b) (i) was also disappointing. Many candidates started at the given equation for C_1 and ignored the given end points completely. Others simply substituted the given points into equation C_1 and then faltered. Some worked backwards and did not use the end points at all.

The responses to (b) (ii) were similarly disappointing. Only a small percentage of candidates attempted to eliminate the x^2 and y^2 hence obtaining $y = x + 5$ which could easily be substituted into C_1 or C_2 . The majority who opted to obtain x or y as the subject ended up with an awkward round of algebra, often with limited success. Some of the candidates simply could not appreciate what to do with the equation $y = x + 5$ and proceeded with a completely different and inappropriate strategy.

General recommendations to teachers include:

1. Constant revision of algebraic simplification
2. The provision of opportunities for students to enhance their problem-solving skills

Solutions:

- (a) (i) 165°
 (ii) $(a) 2ki - 3 kj, k \in \mathbb{R}$
 (iii) \mathbf{p}, \mathbf{v} are perpendicular
- (b) (ii) Points of intersection are $(0, 5)$ and $(-3, 2)$

Question 4

Specific Objectives: (a) 4, 5, 13, 14

Most candidates attempted this question but performed poorly overall. Approximately 40 per cent of candidates scored less than 4 marks and approximately 65 per cent scored less than 12 marks.

- (a) (i) Many candidates were able to gain marks for stating $3A = \cos^{-1}0.5$ and proceeding from there. However, only a small number of candidates arrived at the three solutions in the given range. The majority ended with the answer $A = \frac{\pi}{9}$ or 20° . In some cases, candidates attempted to find the solutions by using trigonometric identities. Most of them simply stalled thereafter.
- (ii) Responses to this part of the question were surprisingly very good. Even some of the relatively lower-scoring candidates completed the exercise with a flourish, gaining the full six marks. Some showed great competence, utilizing standard identities, to prove this more difficult identity.
- (iii) This part of the question challenged many candidates including many with high scores. They could not make the link to the earlier parts of the question, even when directed to do so. It was also noted that answers were not given to three significant figures (as indicated at the front of the question paper).
- (b) (i) Many candidates were able to make substantial progress on this part of the question. Many of them knew the compound angle formulae and also derived the correct ratios for $\tan \alpha$ and $\tan \beta$, but some lacked the algebraic techniques needed to get the required answer.
- (ii) Many candidates were clueless and had no idea how to solve this question. Indeed, there were many ‘no responses’ seen for this part of the question.

Solutions:

- (a) (i) $A = \frac{1}{9}\pi, \frac{5}{9}\pi, \frac{7}{9}\pi$
 (iii) $b = \cos \frac{\pi}{9}, \cos \frac{5\pi}{9}, \cos \frac{7\pi}{9} \approx 0.940, -0.174, -0.766$
- (b) (ii) $\max(\alpha - \beta) = \tan 0.125 \approx 0.124 \text{ rad.}$

Section C

Module 3: Calculus 1

Question 5

Specific Objectives: (a) 1, 3–7, 9; (c) 1, 4, 6 (ii)

This question tested knowledge of limits, continuity and discontinuity, and integration.

There were some very good attempts, many of which gained full marks on the question. Nevertheless, some weaknesses in the preparation of the candidates were evident and there were cases of carelessness in pursuing routine procedures.

- (a) (i) Many candidates experienced difficulty factorizing the denominator in this part and this curtailed success in obtaining the limit. Some used L'Hôpital's rule with great success.
- (ii) Few candidates were successful in obtaining full marks on this part. Those who did used L'Hôpital's to great advantage.
- (b) Some candidates misunderstood the symbols for left hand and right hand limits while others had difficulty with the concept of discontinuity. The word 'deduce' was largely ignored by many.
- (c) (i) There were several good returns on this part of the question for the candidates who expanded the expression in the integrand correctly. Many of these candidates were able to follow through with the integration, termwise, of the expanded expression and the evaluation at each of the limit points $x = 1$ and $x = -1$, to obtain full marks for this part.
- (ii) Several candidates performed well on this part of the questions. Many candidates who did not, failed to change the integrand completely from x 's to u 's.

Solutions:

- (a) (i) $\frac{2}{9}$ (ii) 2
- (b) (i) a) 5, b) 9
- (c) (i) at $x = 1$, $-2\frac{2}{3}$; at $x = -1$, $2\frac{2}{3}$
- (ii) $\frac{1}{3}(x^2 + 4)^{\frac{3}{2}} + k$ (constant)

Question 6

Specific Objectives: (b) 4, 7, 8, 9 (i), 10

This question covered differentiation, integration and calculus.

Several candidates attempted the question with varying degrees of success. The principles involved seemed to be familiar to most, yet some candidates did not receive maximum reward because of weaknesses in simple algebraic manipulations. More practice is recommended to reduce the incidence of such pitfalls.

- (a) (i) Many candidates were strong on differentiating the basic trigonometric functions of sin and tan but some were unable to cope with the composite nature of the functions involved.
- (ii) Several candidates had difficulty using the quotient rule often interchanging numerator with denominator in applying it. Nevertheless, there were some excellent answers returned on this question.
- (b) (i) The use of the relevant integration theorem seemed not to be recognized by some candidates. In such cases, the candidates performed poorly to the extent that they did not seem to be aware that $\int_1^4 4 dx$ had to be obtained.
- (ii) It seemed that some candidates had little practice in using substitution to perform integration. However, several of those who knew the technique were able to obtain full marks on this part of the question. Greater attention should be paid to Specific Objectives 6 and 7 of Unit 1 Module 3, in preparing candidates.
- (c) (i) This part of the question was well done. The majority of candidates obtained full marks.
- (ii) The majority of candidates knew the methods to be used but many encountered problems with the algebra involved. Several methods were used to obtain the correct answer.

Solutions:

$$\begin{array}{ll}
 \text{(a) (i)} & 3 \cos(3x+2) + 5 \sec^2(5x) \\
 & \text{(ii)} \quad -\frac{(x^4 + 3x^2 + 2x)}{(x^3 - 1)^2} \\
 \text{(b) (i)} & 33 \\
 \text{(c) (i)} & P \equiv (-2, 4), \quad Q \equiv (1, 1); \quad \text{(ii)} \quad \text{Area} = 4\frac{1}{2} \text{ units}^2
 \end{array}$$

Paper 03/B (Alternative Internal Assessment)

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (a) 8, (d) 1, 7, (f) 4, 5 (ii)

This question examined the theory of cubic equations, functions and mathematical induction.

- (a) (i) This part of the question was poorly done. There was little evidence of candidates demonstrating the theory of cubic equations. Some candidates merely stated the concepts of the sum of roots, product pair-wise and the product of roots of the cubic equations. However, they were unable to complete the calculations by substituting the correct values.
- (ii) Without a correct follow through from (i) candidates could not find the required values of p and q .

- (b) (i) Candidates understood the meaning of ‘one-to-one’ but were unable to show the required result mathematically. Instead, they substituted some values for x and concluded that f is one-to-one. No candidate used the horizontal line test.
- (ii) A few candidates made correct expansions but failed to make the correct deductions. Generally, this part of the question was satisfactorily done.
- (c) The majority of candidates knew that an assumption was necessary, followed by the inductive process. However, the inductive process and the algebra involved proved beyond their abilities. They merely stated conclusions without proof.

Solutions:

- (a) (i) $\alpha = -2$
- (ii) $p = 12; q = 44$
- (b) $x = \frac{5}{2}$

Section B

Module 2: Trigonometry and Plane Geometry

Question 2

Specific Objectives: (b) 1, 2, 3; Content: (a) (ii), (b) (iii)

This question examined the intersection of lines, equations of straight lines and the area of a triangle using basic concepts of coordinate geometry.

- (a) (i) This part of the question was well done.
- (ii) This part of the question was well done. Some exceptions included the correct calculations for the gradient of a straight line given two points on the line.
- (b) This part of the question was well done.
- (c) Candidates had problems calculating the lengths of AC and DC which were required to find the area. The majority of candidates obtained partial marks for some related attempts to find the area.

Solutions:

- (a) (i) A (0, 2), B (3, 0), C (6, -2)
- (ii) CD: $3x - 2y - 22 = 0$ AD: $5x + y - 2 = 0$
- (b) D (2, -8)
- (c) Area = 26 square units

Section C

Module 3: Calculus 1

Question 3

Specific Objectives: (b) 6, 15, (c) 2, 3, 4, 5, (ii), (iii), 8 (i)

This question examined indefinite integration of composite trigonometric functions, area under the curve and applications of differentiation to maxima/minima situations. There was also some mathematical modelling included.

- (a) This part of the question was poorly done. Approximately three per cent of the candidates obtained maximum marks. A common error among candidates was their inability to use the identity $\tan^2 x \equiv \sec^2 x - 1$ in order to integrate $\tan^2 x$ correctly. A significant number of candidates evaluated $\int (\cos 5x) dx$ as $5\sin 5x$ or $-\frac{1}{5} \sin 5x$. Candidates continue to neglect the arbitrary constant of integration for indefinite integrals.
- (b) (i) This part of the question was well done.
- (ii) The majority of candidates used $\int_0^2 y dx$ to find the area of the enclosed region. A few candidates recognized symmetry of the two regions and used $2 \int_0^1 y dx$ to find the area.
- (c) (i) This part of the question was poorly done. Difficulties included being unable to state the perimeter in terms of r and x , and failing to equate the expression for the perimeter to 60.
- (ii) Without the correct follow through from Part (i), many candidates could not obtain the area in terms of r only.
- (iii) Differentiation of the expression for the area was poorly done. Candidates could not interpret the term $\left(1 + \frac{\pi}{4}\right)$ as a constant. In addition, most of the candidates did not follow the instruction to find the **exact** answer.

Solutions:

(a) $\frac{1}{5} \sin 5x + \tan x - x + C$

(b) (i) $p = 1 ; q = 2$ (ii) Area = $\frac{1}{2}$ square units

(c) (i) $x = 30 - r - \frac{\pi r}{2}$

(iii) $r = \frac{60}{4 + \pi}$

UNIT 2**Paper 01 – Multiple Choice**

Paper 01 comprised 45 multiple-choice items. Candidates performed fairly well with a mean score of 50.39 per cent and a standard deviation of 17.93.

Paper 02 – Structured Questions**Section A****Module 1: Calculus II**Question 1

Specific Objectives: (a) 9, (b) 1, 4, 5, 6, 7, (c) 1 (i), 5, 6, 7

This question examined a real-world situation involving an exponential equation, differentiation of an exponential function, differentiation of an inverse trigonometrical function, implicit differentiation including second derivatives and integration using partial fractions with suitable substitutions.

- (a) (i) Approximately 85 per cent of the candidates who responded to this part of the question were able to deduce that the initial temperature occurred when $t = 0$. Full marks were obtained by all of the candidates who interpreted this question correctly. Some candidates demonstrated a lack of understanding of the term ‘initial temperature’.
- (ii) Generally, a significant number of candidates attempting this part of the question found it difficult to interpret the term ‘stabilise’. The general response was below average. Approximately ten per cent of the candidates obtained the correct answer, deducing that the limiting value of the temperature for large increasing t was $65^0 C$. It may be useful at the level of instruction to consider terms that may be easily interpreted by candidates who may not be science oriented.
- (iii) This part of the question required candidates to solve the equation to find a value for t . Common errors seen were
 - a) $0 = 65 + 8e^{-0.02t}$
 - b) $\ln 70 = \ln 65 + \ln 8e^{-0.02t}$
 - c) $73 = 65 + 8e^{-0.02t}$, using the value 73 obtained in (a) (i)
 - d) $\ln 5 = 8 \ln e^{-0.02t}$, treating 8 as a power
 - e) $t = \frac{\ln 70 - \ln 65}{8(-0.02)}$

Approximately half of the candidates demonstrated a marked weakness in algebraic manipulation, particularly regarding logarithms. A small percentage of candidates who recognized that logarithms had to be applied used common logarithms instead of natural logarithms. Emphasis must be placed on the properties of exponential and logarithmic equations. This will allow candidates to gain confidence to solve these equations correctly.

- (b) (i) For this part of the question approximately 30 per cent of the candidates expressed the equation $y = e^{\tan^{-1}(2x)}$ as $\ln y = \tan^{-1}(2x)$. However, these candidates were not able to carry out the required implicit differentiation to obtain the correct answer. Common errors made included failing to apply the chain rule when differentiating the composite function $\tan^{-1}(2x)$. Some answers given were

$$\frac{dy}{dx} = \frac{1}{1+4x^2} x e^{\tan^{-1}(2x)} \times 2 \text{ and } \frac{dy}{dx} = \frac{1}{1+2x^2} x e^{\tan^{-1}(2x)} \times 2.$$

Very few candidates obtained full marks on this part of the question.

- (ii) Approximately 15 per cent of the candidates attempted to use the result given in Part (b) (i) to show the result required in Part (b) (ii). However, they were unable to carry out the implicit differentiation required when using the form given in Part (b) (i). They failed to show that $\frac{d}{dx} 2y = 2 \frac{dy}{dx}$, but instead showed $\frac{d}{dx} 2y = 2$.

Approximately 80 per cent of the candidates stated the equation

$(1+4x^2) \frac{dy}{dx} = 2y$ as $\frac{dy}{dx} = 2e^{\tan^{-1}(2x)} x (1+4x^2)^{-1}$ and proceeded to use the product rule correctly. These candidates obtained full marks with the correct algebraic manipulation to show the required result.

- (c) (i) Approximately 50 per cent of the candidates used the substitution given, $u = e^x$, to express $\int \frac{4}{e^x + 1} dx$ as $\int \frac{4}{u+1} du$ in this part of the question and obtained the result $4 \ln(u+1) + C$. They failed to replace dx with $\frac{1}{u} du$. A number of candidates obtained the form $\int \frac{4}{u(u+1)} du$ correctly. However, they failed to carry out the correct integration since they did not recognize that partial fractions were necessary at this point. A few candidates who carried out the correct procedure for the integration left their answer as $4 \ln u - 4 \ln(u+1) + C$.

- (ii) Approximately 70 per cent of the candidates were able to obtain the expression $\int \frac{4e^{-x}}{1+e^{-x}} dx$. However, these candidates could not proceed with the correct integration.

They failed to recognise the systematic integration of the form $\int \frac{f(x)}{f'(x)} dx$. A significant number of those candidates who carried out the integration incorrectly obtained $4 \ln(1 + e^{-x}) + C$.

Solutions:

(a) (i) 73°C

(ii) 65°C

(iii) 23.5 hours

(c) (i) $4 \ln(e^x) - 4 \ln(e^x + 1) + C$ or $4 \ln\left(\frac{e^x}{e^x + 1}\right) + C$

(ii) $-4 \ln(1 + e^{-x}) + C$ or $4 \ln\left(\frac{e^x}{1 + e^x}\right) + C$

Question 2

Specific Objectives: (b) 2, 5, (c) 8, 10, 11

This question required differentiation of $\ln x$ using product and chain rules, reduction formula and the solution of a differential equation using an integrating factor.

- (a) (i) Most candidates recognized that this part of the question required the use of the product and chain rules. However, application of the chain rule to differentiate the logarithmic function $(\ln x)^n$ proved to be difficult in many cases. Approximately 50 per cent of the candidates obtained full marks.
- (ii) Generally, most of the candidates used integration by parts using the approach $\int (1)(\ln x)^n dx = x \ln n - \int x \frac{d}{dx} (\ln x)^n dx$. No candidate used the result from (a) (i) to derive the reduction formula. This part of the question was well done by the majority of candidates.
- (iii) This part of the question was poorly done. Many candidates demonstrated a lack of simple use of the derived reduction formula. No candidate successfully managed $I_1 = \int (\ln x) dx$. In addition, the weakness in algebraic manipulation was again evident in substituting successive values of n in the reduction formula, making correct use of brackets. Approximately ten per cent of the candidates obtained full marks.
- (b) (i) A small percentage of candidates demonstrated the ability to find a suitable integrating factor and proceeded to show the required general solution of the differential equation.

Approximately ten per cent of candidates were able to state $I = e^{\int \frac{2}{t+10} dt}$ but were unable to complete this integration correctly. The remaining candidates did not demonstrate any knowledgeable attempt to find the integrating factor.

- (ii) Using the general solution to the differential equation in (b) (i), some candidates were able to obtain the correct answer to this part of the question. However, a significant number of candidates did not understand the term ‘initially’ and substituted random values.

Solutions:

- (a) (i) $n(\ln x)^{n-1} + (\ln x)^n$
- (iii) $x(\ln x)^3 - 3x(\ln x)^2 + 6x(\ln x) - 6x + C$
- (b) (ii) 24.2 kg

Section B

Module 2: Sequences, Series and Approximation

Question 3

Specific Objectives: (b) 2, 4, (c) 3

This question examined candidates’ ability to define the r^{th} term of a sequence, obtain the n^{th} partial sum of a finite series, find the first term and common difference of an arithmetic progression and their application of the binomial theorem.

- (a) (i) Approximately 70 per cent of the candidates had difficulty obtaining the r^{th} term of the sequence.
- (ii) In this part of the question most candidates attempted to sum the given series as an A. P. or a G. P. Those candidates who were able to express the r^{th} term correctly used the standard formulae for $\sum_{r=1}^n r^2$, $\sum_{r=1}^n r$ and $\sum_{r=1}^n c$ to find the required sum. Evidence of a weakness in algebraic manipulation resulted in some candidates not being able to simplify the answer.
- (b) This part of the question was well done by approximately 75 per cent of the candidates. Some candidates incorrectly defined the equation required to express the 9^{th} term as 3 times the 3^{rd} term. As a result, incorrect values were obtained for the first term a , and the common difference d .

- (c) (i) This part of the question was generally well done although some errors were made in simplifying the coefficients. Too many candidates did not state the correct range of values of x for which the expansion is valid.
- (ii) A significant number of candidates did not attempt to rationalize the denominator by recognizing the resulting difference of squares. Many of those candidates who attempted to rationalize the denominator used $(1-x) - \sqrt{(1+2x)}$ as the ‘conjugate’ of $(1+x) + \sqrt{(1+2x)}$. Other candidates used the result from Part (i) to expand the denominator and could not show the required expression. Only a few candidates obtained full marks.
- (iii) Using the result from Part (ii), the majority of candidates expanded the expression $\frac{1}{x}(1+x - \sqrt{(1+2x)})$ up to and including the term in x^2 . Consequently, the resulting expression could not be $\frac{1}{2}x(1-x)$ as required. It was clear that candidates did not take into account that the expansion was divided by x , thus requiring the expansion up to and including the term in x^3 .

Solutions:

(a) (i) $(3r-1)(2r+1)$

(ii) $S_n = \frac{n}{2}(4n^2 + 7n + 1)$

(b) $a = d = 2$

(c) (i) $1 + x - \frac{x^2}{2} + \frac{x^3}{2} - \dots - \frac{1}{2} < x < \frac{1}{2}; |x| < 1$

Question 4

Specific Objectives: (b) 1, 11, 12, (c) 1, (e) 1, 2, 4

This question examined candidates’ ability to manipulate and prove expressions involving nC_r , the sum of finite series using the method of differences and application of the Newton-Raphson procedure.

- (a) (i) A small percentage of candidates obtained full marks for this part of the question. A significant number of candidates expressed ${}^nC_{r-1}$ as $\frac{n!}{(r-1)!(n-r-1)!}$. Many candidates stated the correct expression for ${}^nC_r + {}^nC_{r-1}$ but were unable to complete the algebraic manipulation to obtain the required result. Some candidates substituted numbers as a means of proof.

- (ii) (a) Many candidates merely stated $f(r) - f(r+1) = \frac{1}{r!} - \frac{1}{(r+1)!}$. The algebraic manipulation and understanding of factorials required for showing the result were beyond the ability of these candidates.
- (b) Most candidates recognized that the method of differences was needed to find the sum required. This part of the question was generally well done.
- (c) A few candidates demonstrated an understanding of deducing the limiting value of this type of sum. Many of the candidates attempted to use the formula for the sum to infinity of a geometric progression but found it impossible to apply. A significant number of candidates did not respond to this part of the question.
- (b) (i) Most of the candidates used the Intermediate Value Theorem but did not state the continuous property of the polynomial. Very few candidates obtained full marks.
- (ii) Generally, this part of the question was well done. Some errors were made in substitution and there were incorrect calculations.

Solutions:

(a) (ii) b) $1 - \frac{1}{(n+1)!}$

c) $\lim_{n \rightarrow \infty} S = 1$

(b) (ii) 0.725

Section C

Module 3: Counting, Matrices and Complex Numbers

Question 5

Specific Objectives: (a) 1, 3, 11, 12, 13 (c) 1, 2, 4, 5

This question examined the concepts of counting, using permutations and combinations, basic probability theory and complex numbers.

- (a) (i) Approximately half of the candidates wrongly applied the concept of all the letters of the word SYLLABUS as being unique, thus giving the solution as $8!$. Some candidates subtracted the repeated letters from the total number of letters and gave the solution as $\frac{6!}{2!2!}$.

- (ii) This part of the question was poorly done and was not attempted by the majority of the candidates. Among the responses seen were 6C_5 , 8C_5 and $\frac{5!}{2!2!}$. Some candidates were awarded partial marks for applying the concept but not accounting for all of the combinations.
- (b) (i) This part of the question was well done. Approximately 75 per cent of the candidates obtained full marks.
- (ii) This part of the question was fairly well done although some candidates did not demonstrate an understanding of independence and mutual exclusivity of events. Many of the candidates attempted to define the terms ‘independence’ and ‘mutually exclusive’ and related the concepts to arbitrary sets of events rather than to the given set of events.
- (c) (i) Generally, this question was fairly well done. Many candidates were unable to rationalize the denominator correctly, failing with the algebra involved.
- (ii) A significant number of candidates deduced the root $1 + i$ but were unable to find by division, or otherwise, the third root. Some candidates used the theory of quadratic equations to deduce the third root by inspection.

Solutions:

- (a) (i) $\frac{8!}{2!2!} = 10\ 080$
- (ii) 30
- (b) (i) 0.17
- (ii) a) not mutually exclusive
b) not independent
- (c) (i) $2 + 4i$
- (ii) $1 + i, -3$

Question 6

Specific Objectives: (b) 1, 3, 4, 5, 6, 7, 8

This question examined the augmented matrix, row-echelon reduction, consistency of a system of linear equations and inverting a matrix.

- (a) (i) This part of the question was well done although some candidates merely wrote down the matrix for the system of equations.
- (ii) In this part of the question, a common error seen was obtaining a row of zeroes in rows other than the last row. However, the question was generally well done.
- (iii) A number of candidates did not understand the term ‘consistent’.
- (iv) This part of the question proved to be the most challenging to the majority of candidates. Most of them attempted to find a unique solution. Others expressed the variables x and y in terms of z but did not choose an arbitrary constant.
- (b) (i) a) This part of the question was well done although some arithmetic errors were made. A number of candidates squared the elements of the matrix A instead of finding $A \times A$.
- b) This part of the question was well done.
- (ii) This part of the question was well done.
- (iii) A significant number of candidates used the cofactor method to find the inverse of A . However, they did not recognize the product AB was equal to $3I$.

Solutions:

(a) (i)
$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 1 & -1 & -1 \\ 1 & 2 & 4 & k \end{array} \right)$$
 (iii) $k = 1$

(ii)
$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & -3 & -1 \\ 0 & 0 & 4 & k-1 \end{array} \right)$$
 (iv)
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2\lambda-1 \\ 1-3\lambda \\ \lambda \end{pmatrix}$$

(b) (i) a)
$$\left(\begin{array}{ccc} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{array} \right)$$
 (iii) $\frac{1}{3} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

b)
$$\begin{pmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$(ii) \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Paper 03/B – Alternative to Internal Assessment

Section A

Module 1: Calculus II

Question 1

Specific Objectives: (c) 1 (iii), 5, 11

This question examined partial fractions, solution of a logarithmic differential equation and proportional increase.

- (a) This part of the question was poorly done. Candidates showed a marked weakness in manipulating fractions of the form $\frac{A}{x} + \frac{Bx+C}{x^2+1}$. Invariably, candidates expressed $\frac{1-x^2}{x(x^2+1)}$ as $\frac{A}{x} + \frac{Bx}{x^2+1} + C$ and $\frac{A}{x} + \frac{Bx}{x^2+1} + \frac{C}{x^2+1}$.
- (b) (i) (a) This part of the question was poorly done in its entirety. A small number of candidates separated the variable successfully but could not complete the integration.
- (b) Approximately 40 per cent of the candidates obtained full marks on this part of the question, using the follow through from (i) and making the correct substitution.
- (ii) Candidates merely substituted $t = 2$ in the equation. This part of the question was poorly done.

Solutions:

$$(a) \frac{1}{x} - \frac{2x}{x^2+1}$$

$$(b) (ii) 2^{2/3} - 1$$

Section B

Module 2: Sequences, Series and Approximations

Question 2

Specific Objectives: (b) 1, 10, 11, (c) 1, MM

This question examined the geometric progression, the limiting sum of an infinite series using Maclaurin's expansion and the applications of an exponential series.

- (a) This part of the question was well done.
- (b) This part of the question was hardly attempted. In fact, candidates seemed unfamiliar with the form of such series.
- (c)
 - (i) This part of the question was well done.
 - (ii) Only a small percentage of the candidates were able to state the correct value in terms of t .
 - (iii) A few candidates solved this part of the question using the index approach. A significant number of candidates used an arithmetic approach, calculating the depreciated value for successive years.

Solutions:

(a) $r = \frac{5}{6}$

(b) $3e - 1$

(c) (i) \$13 125

(ii) $\$15\ 000 \left(\frac{7}{8}\right)^t$

(iii) \$4 510

Section C

Module 3: Counting, Matrices And Complex Numbers

Question 3

Specific Objectives: (a) 1, 2, 4, 7 (b) 1, 2, 6, 8, MM

This question examined selections with and without restrictions, classic probability and the solution of a system of linear equations using a matrix approach.

- (a) (i) This part of the question was poorly done. Candidates applied combinations for calculations instead of reasoning.

- (ii) The approach in Part (i) to finding a solution was repeated in this part of the question.
- (b) This part of the question was poorly done since incorrect methods in Part (ii) resulted in incorrect answers being obtained.
- (c) (i) This part of the question was well done.
 (ii) This part of the question was well done.
- (iii) Responses to this part of the question were poor. Candidates appeared to have no knowledge of the row-reduction method, the inverse method, or the solution of 3 linear equations with 3 unknowns.

Solutions:

(a) (i) 48

(ii) 100

(b) $\frac{1}{5}$

(c) (i) $20x + 40y + 60z = 1120$
 $40x + 60y + 80z = 1720$
 $60x + 80y + 120z = 2480$

(ii)
$$\begin{pmatrix} 20 & 40 & 60 \\ 40 & 60 & 80 \\ 60 & 80 & 120 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1120 \\ 1720 \\ 2480 \end{pmatrix}$$

(iii) $x = 12; y = 10; z = 8$

Paper 03/1 – Internal Assessment

This year, 171 Unit 1 and 141 Unit 2 internal assessments were moderated. Far too many teachers continue to submit solutions without unitary mark schemes. In some cases, neither question papers with solutions nor mark schemes were submitted. In an increasing number of cases, marks awarded were either too few or far too many. For example: an entire IA module test was worth 20 marks and in another case a simple polynomial division was awarded 78 marks.

The majority of samples submitted were not of the required standard. Teachers must pay particular attention to the following guidelines and comments to ensure effective and reliable submission of the internal assessments.

The internal assessment is comprised of three module tests.

The main features assessed are:

- Mapping of the items tested to the specific objectives of the syllabus for the relevant Unit
- Content coverage of each module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (module test and students' scripts)
- Quality of teachers' solutions and mark schemes
- Quality of teachers' assessments, that is, consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one module test for each unit

GENERAL COMMENTS

1. Too many of the module tests comprised items from CAPE past examination papers.
2. Untidy 'cut and paste' presentations with varying font sizes were common place.
3. Teachers are reminded that the CAPE past examination papers should be used **only** as a guide.
4. The stipulated time for module tests must be strictly adhered to as students may be at an undue disadvantage when Module tests are too extensive or insufficient.
5. The following guide can be used: one minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling **must** be included.
6. Multiple-choice questions will **not** be accepted as the entire module test but the test may include some multiple-choice items.
7. Cases were noted where teachers were unfamiliar with recent syllabus changes. For example,
 - Complex numbers and the Intermediate Value Theorem are now tested in Unit 2.
 - Three dimensional vectors, dividing a line segment internally and externally, systems of linear equations have been removed from the CAPE syllabus (2008).
8. The moderation process relies on the validity of teachers' assessment. There were a few cases where students' solutions were replicas of the teachers' solutions – some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on students' scripts did not correspond to the marks on the moderation sheet.
9. Teachers must present evidence of having marked each individual question on students' scripts before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of students' scripts. To enhance the quality of the design of the module tests, the validity of the teacher assessment and the validity of the moderation process, the internal assessment guidelines are listed below for emphasis.

Module Tests

- (i) Design a separate test for each module. The module test **must** focus on objectives from that module.
- (ii) In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered and a common marking scheme used.
- (iii) One sample of **five** students will form the sample for the centre. If there are less than five students **all** scripts will form the sample for the centre.
- (iv) In 2010, the format of the internal assessment remains unchanged.

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required on the cover of each module test.

- Name of school and territory, name of teacher, centre number
- Unit number and module number
- Date and duration of module test
- Clear instructions to candidates
- Total marks allocated for module test
- Sub-marks and total marks for each question **must** be clearly indicated

2. COVERAGE OF THE SYLLABUS CONTENT

- The number of questions in each module test must be appropriate for the stipulated time.
- CAPE past examination papers should be used as a guide **ONLY**.
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant unit of the syllabus.

3. MARK SCHEME

- Detailed mark schemes **MUST** be submitted, that is, one mark should be allocated per skill (not 2, 3, 4 marks per skill)
- Fractional or decimal marks **MUST NOT** be awarded. (that is, Do not allocate $\frac{1}{2}$ marks).
- A student's marks **MUST** be entered on the front page of the student's script.
- Hand written mark schemes **MUST** be NEAT and LEGIBLE. The **unitary** marks **MUST** be written on the right side of the page.
- **Diagrams MUST be neatly drawn with geometrical/mathematical instruments.**

PRESENTATION OF SAMPLE

- Students' responses MUST be written on letter sized paper ($8\frac{1}{2} \times 11$).
- Question numbers MUST be written clearly in the left hand margin.
- The total marks for EACH QUESTION on students' scripts MUST be clearly written in the left or right margin.
- ONLY ORIGINAL students' scripts MUST be sent for moderation.
- Photocopied scripts WILL NOT BE ACCEPTED.
- Typed module tests MUST be NEAT and LEGIBLE.
- The following are required for each Module test:
 - ❖ A question paper
 - ❖ Detailed solutions with detailed unitary mark schemes.
 - ❖ The question paper, detailed solutions, mark schemes and five students' samples should be batched together for each module.
- Marks recorded on PMath-3 and PMath2-3 forms must be rounded off to the nearest whole number. If a student scored zero, then zero must be recorded. If a student was absent, then absent must be recorded. The guidelines at the bottom of these forms should be observed. (See page 57 of the syllabus, no. 6.)

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

**REPORT ON CANDIDATES' WORK IN THE
ADVANCED PROFICIENCY EXAMINATION**

MAY/JUNE 2011

PURE MATHEMATICS

GENERAL COMMENTS

In 2011, approximately 5,855 and 2,970 candidates wrote the Units 1 and 2 examinations respectively. Performances continued in the usual pattern across the total range of candidates with some candidates obtaining excellent grades while some candidates seemed unprepared to write the examinations at this level, particularly in Unit 1.

The overall performance in Unit 1 was satisfactory, with several candidates displaying a sound grasp of the subject matter. Excellent scores were registered with specific topics such as Trigonometry, Functions and Calculus. Nevertheless, candidates continue to show weaknesses in areas such as Modulus, Indices and Logarithms. Other aspects that need attention are manipulation of simple algebraic expressions, substitution and pattern recognition as effective tools in problem solving.

In general, the performance of candidates on Unit 2 was satisfactory. It was heartening to note the increasing number of candidates who reached an outstanding level of proficiency in the topics examined. However, there was evidence of significant unpreparedness by some candidates.

Candidates continue to show marked weakness in algebraic manipulation. In addition, reasoning skills must be sharpened and analytical approaches to problem solving must be emphasized. Too many candidates demonstrated a favour for problem solving by using memorized formulae.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily, with a mean score of 55.10 and a standard deviation of 20.02.

Paper 02 – Structured Questions

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (a) 5; (b) 1, 3, 4; (c) 1–3(iii); (d) 5; (e) 2; (f) 4; (g) 1

The topics examined in this question covered the Remainder and Factor Theorems, simultaneous equations, logarithms, inequalities, cubic functions and indices.

In general, both sections of the problem in Part (a) were handled well by the candidates. However, there were many cases in which the requisite skills for manipulation of the expressions were lacking.

Part (a) was attempted by almost all candidates and approximately 30 per cent scored between zero and five marks. Approximately 10 per cent of candidates scored between 21 and 25 marks.

In Part (a) (i) where manipulation of surds was the focus, some candidates did not recognize $(\sqrt{75} + \sqrt{12})^2 - (\sqrt{75} - \sqrt{12})^2$ as a difference of two squares and therefore missed out on a more efficient solution. However, those who saw this expression as a difference of two squares were also able to complete the subsequent manipulation required to arrive at the correct answers. Most candidates expanded the two bracketed terms then proceeded to further manipulation.

For Part (a) (ii), the manipulation of indices and the bases of the indices were the foci. Many candidates recognized that 3 was the smallest common base of the terms in the expression $27^{\frac{1}{4}} \times 9^{\frac{1}{8}} \times 81^{\frac{1}{8}}$, but several could not, complete the solution.

Approximately 90 per cent of the candidates responded well to Part (b) and were able to arrive at the correct answer. Identification from the graph was well done.

Many candidates were able to form the two simultaneous equations in Part (b) (ii). However, some of them were not able to solve the simultaneous equations.

Approximately 80 per cent of candidates who attempted Part (b) (iii) got the correct solution. Although most candidates were not able to factorize the polynomial, they were able to identify the x -value from the given graph.

Part (c) (i) focused on the solution of a quadratic equation involving logarithms. The equation which the candidates were required to solve was $\sqrt{\log_2 x} = \log_2 \sqrt{x}$ and the substitution $y = \log_2 x$ was provided as a possible means of solving the equation. In general, performance on this item was poor and candidates' attempt at solving the problem faltered at the substitution phase due to improper application of the laws of logarithms to transform the right hand side of the equation into a form that allowed the appropriate substitution. These candidates were unable to recognize that $\log_2 \sqrt{x} = \frac{1}{2} \log_2 x$ and hence failed to substitute $\frac{1}{2}y$ in its place. Candidates who completed the substitution correctly divided throughout by y instead of factorizing, thus losing one of the solutions.

Candidates found Part (c) (ii) the most difficult phase of the entire question. This item required solutions to the quadratic inequality $x^2 - |x| - 2 < 0$. Candidates performed poorly on this item due to a lack of understanding of how to deal with the modulus (absolute value) in such a context. In many cases, candidates just dropped both the modulus and the inequality signs and proceeded to solve $x^2 - x - 12 = 0$. In other cases, the inequality sign was retained after the removal of the modulus but only one resulting inequality was recognized ($x^2 - x - 12 < 0$). The candidates did not realize that another valid inequality was $x^2 + x - 12 < 0$, which also contributed to the overall solution.

Solutions:

- | | | | | | | |
|-----|-----|-----------------|------|--------------------------------|-------|--------------|
| (a) | (i) | 120 | (ii) | $3^2 = 9$ | | |
| (b) | (i) | $p = 4$ | (ii) | $m = -1, n = -4$ | (iii) | $x = -2at Q$ |
| (c) | (i) | $x = 1, x = 16$ | (ii) | $1 < x < 4 \quad (-4 < x < 4)$ | | |

Question 2

Specific Objectives: (a) 6, 8; (d) 7; (f) 3, 5 (i)

This question tested knowledge of the roots of quadratic equations, the evaluation of a function at discrete points and mathematical induction.

Apart from Part (a), performance on this question was weak due, in a large number of cases, to faulty basic algebraic manipulation.

Part (a) dealt with the sum and the product of roots of a quadratic equation. The equation given was $x^2 - px + 24 = 0$ for $p \in \mathbf{R}$ and the problem was divided into two major parts.

Part (a)(i) required that candidates express

a) $\alpha + \beta$ and b) $\alpha^2 + \beta^2$ in terms of p .

Most candidates were able to solve these problems which indicated that they understood the roots of quadratic equations. However, some candidates encountered difficulties in expressing $\alpha^2 + \beta^2$ in terms of sums and products of the roots of quadratic equations. Specifically these candidates did not recognize that $\alpha\beta$ had to be subtracted from $(\alpha + \beta)^2$ to give the desired $\alpha^2 + \beta^2$ and instead attempted to use only $(\alpha + \beta)^2$.

In Part (a)(ii) candidates were generally able to solve the equation $\alpha^2 + \beta^2 = 33$ to obtain the value for p .

For this problem, the candidates were given the equation $f(2x + 3) = 2f(x) + 3$ along with a stipulation that $f(0) = 6$ and they were then asked to evaluate $f(x)$ at three specific points $f(3)$, $f(9)$ and $f(-3)$.

Although some candidates were able to provide correct solutions, this item was very poorly done in general. The main difficulty was non-recognition of the need to first solve $2x + 3 = a$, where a is the given point at which $f(x)$ was to be evaluated in each of the three cases, then substitute the value of a obtained into the right hand side of the equation as the value of the variable x . Generally, candidates tended to substitute the point at which the function was to be evaluated into $2x + 3$, then substituted the result into the right hand side of the equation. The values of a required to calculate (i) $f(3)$, (ii) $f(-3)$ were respectively $a = 0, a = -3$ emphasizing the power of substitution in this question.

The third part of this problem required that candidates solve $f(-3) = 2f(-3) + 3$. Candidates who were able to solve the first two parts of the problem were also able to solve this final part.

Part (c) of this question posed significant difficulties for candidates. Candidates needed to prove that the product of any two consecutive integers k and $k + 1$ is an even integer, which merely required candidates to

state that for k and $k + 1$ as consecutive integers, one is even the other is odd so that the product $k(k + 1)$ must be even.

Part (d) was also poorly done by candidates primarily because they did not know, or did not understand, how to apply the steps required for a proof by induction. A small percentage of candidates who performed well on this part of the question also recognized the relevance of Part (c) to the solution of Part (d).

Solutions:

(a) (i) $\alpha + \beta = p$ (ii) $\alpha^2 + \beta^2 = p^2 - 48$

(b) (i) $f(3) = 15$ (ii) $f(9) = 33$ (iii) $f(-3) = -3$.

Section B

Module 2: Trigonometry and Plane Geometry

Question 3

Specific Objectives: (b) 1–7; C 1, 2, 9

This question examined vectors and the properties of the circle, the intersection of a straight line and a circle, and the parametric representation.

This question was generally not well done. The main errors encountered in 3 (a) (i) as highlighted below;

- Candidates substituted $|a|$ and $|b|$ into the vector expression $(a + b) \cdot (a - b)$ rather than the vectors $a_1\mathbf{i} + a_2\mathbf{j}$ and $b_1\mathbf{i} + b_2\mathbf{j}$.
- Candidates who were able to find the dot product $a_1^2 + a_2^2 - (b_1^2 + b_2^2)$ correctly were in many cases unable to make the final substitution of $a_1^2 + a_2^2 = 169$ and $b_1^2 + b_2^2 = 100$ to obtain the final answer. In fact, $a^2 = 169$ and $b^2 = 100$ rather than $a_1^2 + a_2^2 = 169$ and $b_1^2 + b_2^2 = 100$ were frequently seen.
- Candidates who found the dot product by expanding $(a_1\mathbf{i} + a_2\mathbf{j} + b_1\mathbf{i} + b_2\mathbf{j}) \cdot (a_1\mathbf{i} + a_2\mathbf{j} - b_1\mathbf{i} - b_2\mathbf{j})$ were generally unable to simplify the resulting expression by using the fact that $\mathbf{i} \cdot \mathbf{i} = 1$ and $\mathbf{i} \cdot \mathbf{j} = 0$.

Part (a)(ii) was poorly done. Many candidates were able to correctly equate the coefficients of \mathbf{i}, \mathbf{j} to obtain $2b_1 - a_1 = 11$ and $2b_2 - a_2 = 0$. They, however, did not recognize the need to use the previous results $a_1^2 + a_2^2 = 169$ and $b_1^2 + b_2^2 = 100$ to solve for a_1, a_2, b_1 and b_2 .

Part (b) was not generally well done.

For Part (b)(i), many instances, candidates were unable to correctly identify the centre of the circle.

In Part (b)(ii), though the majority of candidates realized that a substitution was required to find the points of intersection of the line and the circle, many of them were unable to follow through to the correct answers. Errors frequently seen were

- Incorrect transposition of the linear equation resulting in an invalid substitution.
 - Incorrect simplification after substitution leading to an invalid quadratic equation.
 - Inability to correctly evaluate the roots using the quadratic formula.
 - $x^2 = 8 \Rightarrow x = \sqrt{8}$ thereby omitting $x = -\sqrt{8}$.

For Part (b)(iii), though many candidates were able to put the Parametric equations in a valid Cartesian form $\left(\frac{x-b}{a}\right)^2 + \left(\frac{y-c}{a}\right)^2 = 1$, not all of them were able to follow through by comparing coefficients with the original equation given to determine a , b and c .

In Part (b)(iv), the majority of candidates were unable to determine the equations of C_2 . The main error seen was that candidates misinterpreted the question because they did not appreciate the difference between the *line intersecting* the circle and the *line touching* the circle. As a result, many candidates used P and Q from Part (b)(ii) as the centres of possible equations of C_2 . Very few candidates were able to recognize the need to use the equation of the perpendicular line through $(0, 1)$, $y = -x + 1$, rather than the original line $y = x + 1$. Even in cases where candidates acknowledged the new line $y = -x + 1$, they often could not follow through to the final equation required. In some cases, candidates correctly identified the equation of the new circle C_2 as of the form $(x - a)^2 + (y - b)^2 = 16$. However, this information was rarely used to complete the question.

Solutions:

$$(a) \quad (i) \quad 69 \quad (ii) \quad \mathbf{a} = 5\mathbf{i} + 12\mathbf{j}, \mathbf{b} = 8\mathbf{i} \pm 6\mathbf{j}$$

$$(b) \quad (ii) \quad (2\sqrt{2}, 1+2\sqrt{2}), (-2\sqrt{2}, 1-2\sqrt{2})$$

$$(iii) \quad a = 4, b = 0, c = 1 \quad (iv) \quad [(x + 2\sqrt{2})]^2 + [y - (1 + 2\sqrt{2})]^2 = 16$$

Question 4

Specific Objectives: (a) 4, 5, 10, 11

This question tested candidates' ability to use and apply Trigonometric Functions, Identities and Equations.

In most cases for Part (a), candidates were able to correctly deduce the correct quadratic equation $8x^2 - 10x + 3 = 0$. However, very few of were able to follow through to obtain full marks because they

- Incorrect factorization leading to invalid roots.
- Did not recognize that these roots represented values of $\cos^2 \theta$ and therefore it was required to find the square root to determine $\cos \theta$.
- Candidates worked in degrees rather than in radians as specified.
- Candidates neglected to find the second quadrant angle corresponding to the negative value of $\cos \theta$.
- Candidates changed $8 \cos^4 \theta$ to $8x^4$ instead of $8x^2$.

Part b (i) was generally well done. Candidates were able to obtain $BC = 8 \sin \theta + 6 \sin \theta$. However, there were many instances of candidates incorrectly giving BR as $6 \sin \theta$ or $6 \sin(90^\circ + \theta)$.

In Part (b)(ii) the majority of candidates were able to correctly equate the answer from Part (i) to 7 to obtain $8 \sin \theta + 6 \sin \theta = 7$. However, many candidates were unable to follow through to the correct value of $\theta = 7.6^\circ$. Common errors seen included;

- taking $\frac{1-\cos 4\theta}{\sin 4\theta} = \tan 2\theta$ to mean $2 \times \frac{1-\cos 2\theta}{\sin 2\theta} = 2 \tan \theta = \tan \theta$
- using $\frac{1-\cos 6\theta}{\sin 6\theta} = \tan 3\theta$ to mean $3 \times \frac{1-\cos 2\theta}{\sin 2\theta} = 3 \tan \theta = \tan \theta$
- $\frac{1-\cos 4\theta}{\sin 4\theta} = \frac{1-1+4\sin^2 2\theta}{4\sin 2\theta \cos 2\theta}$
- $\frac{1-\cos 6\theta}{\sin 6\theta} = \frac{1-1+6\sin^2 3\theta}{6\sin 3\theta \cos 3\theta}$
- squaring both sides of the equation in an attempt to solve rather than using the form $R \cos(\theta - \alpha)$ or $R \sin(\theta + \alpha)$
- in some cases choosing to use the form $10 \cos(\theta - 53.13) = 7$, candidates failed to realize that they had to use -45.6 rather than 45.6 as the value of $\cos^{-1}(0.7)$ obtaining $\theta = 53.13 + 45.57 = 98.7$ rather than $\theta = 53.13 - 45.57 = 7.56$

For Part (b)(iii), the majority of candidates were able to identify that 15 was *not* a possible value for $|BC|$. However, candidates did not always justify their answers with a valid reason.

Although in most cases candidates were able to substitute the correct identities $\cos 2\theta = 1 - 2\sin^2 \theta$ or $\cos 2\theta = 2\cos^2 \theta - 1$ and $\sin 2\theta = 2\sin \theta \cos \theta$, failure to use brackets in substituting resulted in incorrect simplification of the numerator in Part (c)(i).

The majority of candidates did Part (ii) as a ‘hence or otherwise’ rather than as a ‘hence’ opting to basically redo Part (c) (i) rather than deduce the correct results.

For Part (c), very few candidates were able to follow through to get the correct answer ‘ n ’. Common errors seen included substituting $r = 1$ before using the previous results to reduce the summation to $\sum_{r=1}^n 1$. In fact, many candidates using this method, gave the final answer as 1 not recognizing that they had in fact done the summation.

Solutions:

(a) (ii) $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}$

(b) (i) $|BC| = 8\sin\theta + 6\cos\theta$ or $|BC| = 8\sin\theta + 6\sin(90 - \theta)$, (ii) $\theta \approx 8^\circ$,

(iii) No

(c) (iii) n

Section C

Module 3: Calculus 1

Question 5

Specific Objectives: (a) 3, 5, 7, 9; (b) 7, 21

This question tested candidates’ knowledge of limits, continuity and basic elements of calculus.

In Part (a), both the L’Hopital and factorization methods were seen. Quite a significant number of candidates substituted positive (+ve) 2 rather than negative (–ve) –2, however, and were penalized for this.

Some candidates divided both the numerator and the denominator by x^2 and therefore lost direction.

The majority of candidates were able to score at least 5 out of 11 for Part (b).

For Part (b)(i), a significant number of candidates seemed not to know how to use a piece-wise function and determine the relevant part of the function for the given domain value. Many candidates substituted into both parts of the function.

For Part (b)(ii), elementary approaches to limits were seen where candidates used a table of values rather than direct substitution. Again some candidates simply substituted in both parts of the function.

Again in Part (b)(iii), some candidates substituted in both parts of the function. Several candidates gave the answer as $-2b + 1$.

For Part (b)(iv), few candidates were able to use the condition for continuity at a point to correctly find the value of b . Many candidates did not respond to this part of the question.

Teachers must reinforce, that

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$

is the necessary condition for continuity at the point a .

In Part(c)(i), some candidates had difficulty translating the given information into mathematical statements. Many of the more successful candidates were not able to solve both simultaneous equations to find the solutions for p and q .

For Part (c)(ii), the incorrect gradient of the normal was seen. A few candidates applied the formula $y - y_1 = m(x - x_1)$ incorrectly.

Candidates who were able to do Parts (c)(i) and (ii), in most cases calculated the length of MN correctly for Part (c)(iii).

Solutions:

(a) $-\frac{1}{5}$ (b) (i) 5 (ii) 5 (iii) $2b + 1$ (iv) $b = 2$

(c) (i) $p = 10, q = -13$ (ii) $7y + x = 15$ (iii) $MN = 14$

Question 6

Specific Objectives: (b) 4, 7, 8, 9; (i), 10

This question tested differentiation, integration and calculus.

Several candidates attempted the question with varying degrees of success. The principles involved seemed to be familiar to most, yet some candidates did not receive maximum reward because of weaknesses in simple algebraic manipulations. More practice is recommended in applying the principle involved in part (b) of this question.

In Part (a)(i), some candidates did not know that they should have differentiated to find the stationary points, while a few candidates were unable to differentiate correctly. Some candidates were unable to solve the equation $x^2 = 4$, although many got only '2' as the solution and others got ± 4 . A number of candidates were unable to substitute correctly.

For Part (a) (ii), to find the gradient, many candidates treated the function as a straight line.

In Part (a)(iii), a number of candidates chose the wrong function to integrate.

Many candidates did not use the correct limits of integration, several of them used 2 or 4 as the upper limit. Some candidates failed to recognize that area cannot be negative.

Some candidates did not understand the concept of proving, hence, they simply rewrote the question in Part (b)(i). This was also done for Part (b)(ii). Some candidates chose to integrate the product $x \sin x$ in the same manner you would integrate $x + \sin x$ would be integrated.

A few candidates chose to use the method of integration by parts.

Solutions:

- (a) (i) $A \equiv (-2, 16)$, $B \equiv (2, -16)$
(ii) $12y = x$ is the equation of the normal
(iii) Area = 36 sq. units

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (c) 1–4; (d) 1, 7

This question examined the theory of logarithms, functions and exponentials.

Among the small number of candidates there were a few good attempts at Part (a). However, the term 2^{2-x} was not correctly interpreted in the majority of cases.

In Part (b)(i), the notion of one-on-one functions was not properly understood.

There were a few encouraging attempts in Part (b)(ii) but poor algebraic manipulation spoiled some of the efforts at completing the solutions correctly.

Poor or inappropriate substitution was evidenced in the few attempts at Part (c). More practice is recommended in this area.

Solutions:

- (a) $x = 0, x = 2$ (b) (ii) $x = -4$ (c) (i) \$35 \text{ million}\$ (ii) \$4 \text{ million}\$

Section B

Module 2: Trigonometry and Plane Geometry

Question 2

Specific Objectives: (b) 1, 2, 3; Content: (a) (ii), (b) (iii)

This question examined the intersection of lines, equations of straight lines, circles and tangents to circles, using basic concepts of coordinate geometry. Vectors and trigonometric identities were also included.

There were some good attempts at Part (a) although some weaknesses were evident in finding the coordinates of the point P in Part (a)(i) and the point Q in Part (a)(ii).

Part (b) was well done by using established formulae for $2A$.

Not many candidates who attempted Part (b)(ii) completed it correctly. The main source of difficulty was the incorrect manipulation of trigonometric formulae.

Solutions:

- (a) (i) $P \equiv (3, 1)$ (ii) $Q \equiv (1, -3)$ (iii) $4y = 3x - 5$
(b) (ii) $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \pi.$

Section C

Module 3: Calculus 1

Question 3

Specific Objectives: (a) 3, 5, 7; (b) 8, 9 (i), 15, 16, 18; (c) 1, 5 (i), 3

This question examined limits, differentiation and the reverse process of integration and applications of differentiation to maxima/minima situations. There was also some mathematical modeling included.

In Part (a) several of the few candidates who made an attempt did not factorize $x^3 - 4x$ correctly, the consequence of which was an incorrect limit.

Some candidates had difficulty differentiating $\frac{x}{3x+4}$ as a quotient in Part (b)(i). However, a few candidates did it quite competently as the product $x(3+4x)^{-1}$.

A few candidates saw the connection between Part (b)(ii) and Part (b)(i). Most of those who were able to make the connection were able to complete the solution competently.

There were some good attempts at Part (c)(i).

Not many of the candidates who did Part (c)(i) were able to complete part (c)(ii).

Solutions:

- (a) 8
(b) (i) $\frac{4}{(3x+4)^2}$ (ii) $\frac{4x}{3x+4} + \text{constant}$
(c) (ii) S_{\min} at $r = \left(\frac{5}{\pi}\right)^{\frac{1}{3}}$.

UNIT 2

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed fairly well with a mean score of 60.73 per cent and a standard deviation of 9.29.

Paper 02 – Structured Questions

Section A

Module 1: Calculus II

Question 1

Specific Objectives: (b) 1, 3, 4, 5

This question examined implicit differentiation, differentiation of combinations of polynomials, exponentials, trigonometric functions, application of the chain rule to obtain the tangent of a curve given by its parametric equations, and the second derivative.

Part (a)(i), was well done by the majority of candidates. Full marks were obtained by almost all candidates.

Common errors included incorrect transposition of $\frac{dy}{dx} = \dots$

Full marks were obtained for Part (a)(ii) by approximately 99 per cent of the candidates.

In Part (a)(iii), few instances of errors in using the concept of differentiation of composite functions were seen. Common errors included $\frac{d}{dx} \cos x = \sin x$. Generally, most candidates obtained full marks for this part of the question.

For Part (b)(i), a small percentage of candidates had difficulty applying the concept of differentiation of the composite function $\sin \frac{1}{x}$, particularly $\frac{d}{dx} \left(\frac{1}{x} \right)$, although, the correct application of the product rule was applied. However, only a small percentage of candidates were unable to obtain full marks for this part of the question.

Generally, Part (b)(ii) was well done. A very small percentage of candidates was unable to apply the concept of implicit differentiation correctly, with the result that they were unable to show the final answer as required.

Part (c)(i) was well done. Some arithmetic errors were made in substituting for $t = 4$ resulting in the incorrect value of the gradient of the tangent. However, candidates were able to get follow through marks for Part (c)(ii).

Part (c)(ii) was well done. Candidates who made errors calculating the correct gradient in Part (c)(i) were not penalized having earned follow through marks.

Solutions:

(a) (i) $\frac{dy}{dx} = \frac{1-x}{1+y}$.

(ii) $\frac{dy}{dx} = (-\sin x)e^{\cos x}$

(iii) $\frac{dy}{dx} = 8 \sin 16x - 6 \sin 12x$

(c) (i) $\left(\frac{dy}{dx}\right)_{x=4} = \frac{15}{4}$

(ii) $15x - 4y = 12$

Question 2

Specific Objectives: (c) 1, 3, 4, 8, 10

This question required candidates to derive a reduction formula and use it for a partial expansion of the product of an exponential function of x and the derived reduction formula; derive partial fractions and the integration of a rational function involving a trigonometric substitution and an inverse trigonometric function.

Approximately 20 per cent of the candidates were unable to obtain the correct answers to Part (a)(i). Many candidates could not deduce that $\int_0^0 F_n(0) dx = 0$. Some difficulty was also experienced in evaluating $\int_0^x F_0(x)$ dx correctly, particularly using the limits of integration, obtaining $e^{-x} - 1$ instead of $1 - e^{-x}$.

In Part (a)(ii), poor algebraic skills resulted in many candidates being unable to complete integration by parts and to show the correct reduction formula for $F_n(x)$. In particular, some candidates were unable to simplify $\frac{n}{n!} \equiv \frac{n}{n(n-1)!} = \frac{1}{(n-1)!}$ to enable the expression of $F_{n-1}(x)$.

Very few candidates were able to complete Part (a)(iii), having failed to determine $F_0(x)$ and $F_n(0)$ correctly. Partially correct answers were facilitated using the given result in Part (a)(ii).

In Parts (b)(i)(ii), there was evidence of candidates applying the concepts of repeated factors and the form of a linear numerator for a quadratic factor to find the partial fractions required. Some candidates were able to find the required partial fractions and proceeded to integrate the resulting rational functions. A few candidates successfully completed that part of the integration which involved an inverse trigonometric function. *This part of the question was zero-weighted and adjustments were made to the final marks so that candidates were not disadvantaged.*

Solutions:

(a) (i) $F_n(0) = 0, \quad F_0(x) = 1 - e^{-x}$

(b) (i) $\frac{2x^2 + 3}{(x^2 + 1)} \equiv \frac{2}{x^2 + 1} + \frac{1}{(x^2 + 1)^2}$

(ii)
$$\int \frac{2x^2 + 3}{(x^2 + 1)^2} dx = \frac{5}{2} \tan^{-1}(x) + \frac{x}{2(x^2 + 1)} + \text{constant}$$

Section B

Module 2: Sequences, Series and Approximation

Question 3

Specific Objectives: (a) 5; (b) 2, 4, 5

This question examined candidates' abilities to establish the properties of a sequence by applying Mathematical Induction; expand $(1 + ax)^n$, for $n = -1$; identify that a given series follows an arithmetic progression. Overall performance on this question was unsatisfactory.

A small percentage of candidates obtained full marks for Part (a)(ii). Generally, candidates seemed unfamiliar with proof by induction of sequences. Many of them knew that they had to prove the assertion for $n = 1$ and to proceed to taking arbitrary $k + 1$ for $n + 1$. However, having obtained $x_{k+2} = x_{k+1}^2 x + \frac{1}{4}$, they could not proceed further. Some candidates lost marks by using strict equality signs, ignoring the restriction $x_n < \frac{1}{2}$.

Only a few candidates were successful in obtaining full marks on Part (a)(ii). Those who were able to express $x_{n+1} - x_n$ as a perfect square made progress to completely solve the problem.

Part (b)(i) was well done. The majority of candidates demonstrated a sound knowledge of partial fractions and were able to obtain a correct solution. Those candidates who did not secure the full three marks were mainly faulted by arithmetic and algebraic errors.

Part (b)(ii) was well done by approximately half of the candidates. Generally, candidates used two different methods to solve this problem, namely, the binomial and Maclaurin's expansions. Those who failed to secure the full four marks committed a range of arithmetic and algebraic errors.

Part (b)(ii)a) was poorly done. More than 50 per cent of the candidates merely stated the ranges $|x| < 1$ and $|x| < \frac{1}{2}$ without proceeding to the correct answer $\left(-\frac{1}{2} < x < \frac{1}{2}\right)$.

Part (b)(iii)b) challenged the majority of the candidates' including those who successfully completed the previous parts of the question. They could not make the link to the earlier parts of the question. A significant number of candidates did not respond to this part of the question.

Candidates responded well to Part (b)(iv). The concepts of the difference between S_{n+1} and S_n resulting in the n^{th} term and subsequently $T_n - T_{n-1} = d$ were known to most candidates. However, poor algebraic manipulations resulted in candidates' unsuccessful efforts to prove the required solution

Solutions:

- (b) (i) $A = B = 1$
- (ii) $1 + x + x^2 + x^3; \quad 1 + 2x + 4x^2 + 8x^3$
- (iii) a) $-\frac{1}{2} < x < \frac{1}{2}$
- b) $1 + 2^n$
- (iv) $u_n = S_n - S_{n-1} = 6n - 7; d = u_n - u_{n-1} = 6$

Question 4

Specific Objectives: (b) 9, 11, 12, 13

This question examined candidates' ability to manipulate a geometric progression and determine the first term and common ratio; obtain a series expansion of a fraction involving a denominator of $e^x + e^{-x}$; find the sum and limit of a finite series using the method of differences. Overall, candidates' performance on this question was very unsatisfactory. Approximately 40 per cent of the candidates either offered no responses or scored no marks.

For Part (a)(i) a large percentage of the candidates obtained 2 of the 4 marks available by establishing the equations $a + ar + ar^2 = \frac{26}{3}$ and $a^3r^3 = 8$. Some candidates used the equations $\frac{a(1-r^3)}{1-r} = \frac{26}{3}$ and $a^3r^3 = 8$.

Poor algebraic skills prevented the majority of these candidates from eliminating a and thus finding the required equation in terms of r .

For Part (a)(ii) a), a significant number of candidates could not simplify the equation given in Part (a)(i) to solve for r . A few of the candidates who found two values for r , ($r = 3$) or ($r = \frac{1}{3}$), did not follow the constraint $0 < r < 1$.

Candidates who did not use the constraint for r abandoned Part (a)(ii)b).

A small percentage of candidates obtained full marks for this Part (a)(ii)c).

Part (b) required candidates to find a series expansion for a fraction involving the denominator $e^x + e^{-x}$. Although a few candidates were able to recall and use the expansion for e^x and e^{-x} , the majority of them employed Maclaurin's theorem for the expansion of the denominator without success. Successive differentiation proved problematic and the exercise was abandoned. Approximately 20 per cent of the

candidates were able to obtain the result $\frac{2}{e^x + e^{-x}} = \frac{1}{1 + \frac{x^2}{2} + \frac{x^4}{24}} + \dots$. The required expansion of the

denominator, using the binomial expansion with $\left(\frac{x^2}{2} + \frac{x^4}{24}\right) = X$ in the expansion of $(1 + X)^{-1}$ was beyond the ability of a majority of the candidates.

Part (c)(i) was well done.

In Part (c)(ii), a large percentage of the candidates incorrectly found

$$3 \sum_{r=1}^n \left(\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} \right)$$

giving the

$$\text{resulting incorrect sum of } 3 \left(\frac{1}{2} - \frac{1}{(n+1)(n+2)} \right).$$

Following the error made in Part (c) (ii), those candidates obtained the wrong limiting sum of the series in Part (c)(iii).

Solutions:

(a) (ii) a) $r = \frac{1}{3}$

b) $a = 6$

c) $S_\infty = 9$

(b) $1 - \frac{x^2}{2} + \frac{5x^4}{24} + \dots$

(c) (i) $\frac{2}{r(r+1)(r+2)}$

(ii) $\frac{3}{2} \left(\frac{1}{2} - \frac{1}{(n+1)(n+2)} \right)$

(iii) $S_\infty = \frac{3}{4}$

Section C

Module 3: Counting, Matrices and Complex Numbers

Question 5

Specific Objectives: (a) 2, 4, 7; (c) 2, 3, 5

This question examined the concepts of arrangements of n distinct objects; the selection of r distinct objects from n distinct objects; the probability of an event occurring; the complex roots of a quadratic equation and the square roots of a complex number.

The overall performance by most of the candidates was satisfactory in parts of the question. As evident in previous problems which required algebraic manipulation, most candidates were at a severe disadvantage in using algebra to show required results. Algebraic simplification continues to prove problematic to most candidates.

Approximately 75 per cent of the candidates was able to obtain full marks for Part (a)(i). Some candidates substituted numbers to show the required result.

For Part (a)(ii), the algebra required to show the required result was beyond most of the candidates. Half-hearted attempts were made to simplify the initial definitions of the left hand and right hand sides of the equations. Many candidates resorted to substituting numbers to balance the equation.

In Part (a)(iii), candidates simply used the numbers given in the equations and calculated the arithmetic results. No attempts were made to use the results of Parts (a)(i) and (ii).

Part (b)(i) was well done. Some arithmetic errors resulted in some candidates being unable to obtain full marks.

Part (b)(ii) was well done by approximately half of the candidates. Arithmetic errors and some loss of reasoning resulted in many candidates not obtaining full marks.

In Part (c)(i) a), a significant number of candidates found the square roots of $-2i$ using the approach $(x + iy)^2 = -2i$. This resulted in some of these candidates making algebraic errors and subsequently obtaining incorrect roots. Some candidates misunderstood the question and attempted to show that $(1 - i) \times (1 + i) = -2i$.

For Part (c) (i) candidates who found the square roots of $-2i$ using the method described in Part (c)(i) a) were able to get the correct answer. There was no evidence that candidates simply applied the concept that the square root of a complex number $(x + iy)$ is $\pm(a + ib)$.

Part (c)(ii), most candidates who used the quadratic formula to solve this equation could not establish the link with $b^2 - 4ac = -2i$ and use the results of Part (c)(i). Very few candidates were able to obtain full marks for this part of the question.

Solutions:

(b) (i) 96

(ii) $\frac{3}{8}$

(c) (i) b) $-(1 - i)$

(ii) $2 + 2i, 1 + 3i$

Question 6

Specific Objectives: (b) 1, 2, 6, 8

This question examined matrices and systems of linear equations. Particularly tested were operations with conformable matrices and manipulation of matrices using their properties; evaluation of determinants for 3×3 matrices; solutions of a consistent system; solution of a 3×3 system of linear equations.

Overall this question was well done. A notable number of candidates obtained marks ranging from 15 to 20.

Part (a)(i) was well done and, arithmetic errors apart, candidates obtained full marks. Candidates generally answered Parts (a), (b) and (c) of the question by making the required changes and using the algorithmic approach. No evidence was seen that any candidate used the properties of matrices to obtain their answers.

All parts of Part (b) were well done and full marks were obtained by almost all candidates.

All parts of Part (c) were well done and full marks were obtained by almost all candidates. In Part (c)(iv), some candidates having shown that $(1, 1, 1)$ was a solution for the system of equations, were unable to find the general solution for the system of equations, despite some attempts. It was not recognized that the system represented parallel planes thus resulting in infinitely many solutions.

Solutions:

(a) (i) $|A| = 5$

(ii) a) $|B| = |A| = 5$, The value of a determinant is unaltered when the columns and rows are completely interchanged.

b) $|C| = |A| = 5$, The value of the determinant is not changed if any row (column) is added or subtracted from any other row (column).

c) $|D| = 5^3 |A| = 625$. If all rows are multiplied by λ , the determinant is multiplied by λ^3 .

(b) (i) $AM = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} = 5I$

$$(ii) \quad \mathbf{A}^{-1} = \frac{1}{5} \quad \mathbf{M} = \frac{1}{5} \begin{pmatrix} 12 & -1 & 5 \\ 2 & -1 & 0 \\ -9 & 2 & -5 \end{pmatrix}$$

$$(c) \quad (i) \quad \begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 2 \\ -1 & -3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ -10 \\ -11 \end{pmatrix}$$

$$\mathbf{A} \quad \mathbf{x} \quad \mathbf{b}$$

$$(ii) \quad \mathbf{A}^{-1} \mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} 5 \\ -10 \\ -11 \end{pmatrix} \Rightarrow \mathbf{x} = \mathbf{A}^{-1} \mathbf{b}$$

$$(iii) \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$$

$$(iv) \quad b) \quad (x, y, z) = \lambda(1, 0, -1) + \mu(0, 1, -1) + (1, 1, 1)$$

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Calculus II

Question 1

Specific Objectives: (b) 3; (c) 12

This question examined differentiation of parametric equations, rate(s) of increase/decrease and the general solution of a second order differential equation.

Performance, overall, was generally poor. The majority of candidates seemed to be unprepared for this question.

In Part (a)(i), some measure of successful differentiation of y and x with respect to t was seen. However, candidates could not determine $\frac{dy}{dx}$ in terms of t and to proceeded to equate $\frac{dy}{dx} = \tan \theta$.

Candidates did not respond to Part (a)(ii), having not completed Part (a)(i).

Some attempts were made to answer Part (a)(iii). Problems encountered by candidates involved incorrect transpositions of x and y and identifying with the correct trigonometric identities.

Part (b) did not elicit many responses. Those candidates who attempted to solve the auxiliary equation used the wrong roots to express the complementary function. The solution for the particular integral was beyond the abilities of almost all the candidates.

Solutions:

(a) (i) rate of decrease = $\frac{24}{31}$

(ii) radians per second

(iii) $\left(\frac{x-4}{3}\right)^2 + \left(\frac{y-5}{2}\right)^2 = 1$

(b) $y = Ae^{-x} + Be^{4x} - 2x^2 + 3x - \frac{13}{4}$

Section B

Module 2: Sequences, Series and Approximations

Question 2

Specific Objectives: (b) 3, 13; (e) 1, 2

This question examined the existence of a real root in a given interval, finding an approximation using a given iterative method, expansion of a logarithmic and exponential function using Maclaurin's theorem and determining the n^{th} term of a sequence of terms.

It was evident that candidates were underprepared for most of this question. Overall, performance was poor.

For Part (a)(i), most candidates were able to establish a change of sign over the given interval. Without stating continuity of the function over this interval, candidates concluded that a real root existed over the interval.

Part (a)(ii) was done satisfactorily.

Some candidates showed some understanding of Maclaurin's theorem and were able to obtain full marks for Part (b)(i).

There were no favourable responses to Part (b)(ii).

Part (c)(i) was well done.

In Part (c)(ii), candidates were not able to make a deduction to obtain the equation.

There were no meaningful responses to Part (i)(iii). Candidates appeared to be guessing about a suitable approach to this part of the question.

Solutions:

(a) (ii) 0.904

(b) (i) $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots \quad (-1 \leq x < 1)$

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \dots \quad \text{for all real } x$$

(c) (i) $p_1 = 1000(1.20) - 100 \quad p_2 = 1.20[1000(1.20) - 100] - 100$

(ii) $p_{n+1} = (1.20)p_n - 100$

Section C

Module 3: Counting, Matrices and Complex Numbers

Question 3

Specific Objectives: (b) 1, 8; (c) 4, 6, 7

This question examined simple operations on a conformable matrix, solutions of a system of equations and operations on a complex number. Overall, performance was poor.

Most candidates were able to obtain marks for Part (a)(i) of the question. The common problems evidenced were arithmetic and in some cases failing to identify I (identity matrix).

In Part (a)(ii), candidates were unable to deduce A^{-1} in the given form since they could not identify the identity matrix.

For Part (a)(iii), those candidates who attempted to find the solution of the system of equations completely ignored the link from Part (a)(ii). As a result, arithmetic errors inhibited their ability to obtain the correct solutions.

In Part (b), candidates demonstrated an understanding of the method(s) to be used. However, poor algebra resulted in incorrect answers.

Most candidates did not attempt Part (c). The few candidates who attempted it did not show a fair understanding of the modulus and argument of a complex number.

Solutions:

(a) (iii)
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

(b)
$$z + \frac{1}{z} = \frac{1}{10} (7 + 9i)$$

(c)
$$r = \sqrt{\frac{13}{10}}, \tan \theta = \frac{9}{7}$$

Paper 031 – School-Based Assessment (SBA)

This year, 174 Unit 1 and 145 Unit 2 SBAs were moderated. Far too many teachers continue to submit solutions without unitary mark schemes. In some cases, neither question papers with solutions nor mark schemes were submitted. Mark schemes for questions and their subsequent parts were not broken down into unitary marks. In an increasing number of cases, the marks awarded were either too few or far too many for the skills tested. (Example: an entire SBA module test was worth 20 marks and in another case on one test paper a simple probability question was awarded 27 marks and a matrix question was awarded 24 marks).

In Unit 1, the majority of the samples submitted were not of the required standard. Teachers **must** pay particular attention to the following guidelines and comments to ensure effective and reliable submission of SBAs.

The SBA is comprised of three module tests. The main features assessed are:

- Mapping of the items tested to the specific objectives of the syllabus for the relevant Unit
- Content coverage of each module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (module test and students' scripts)
- Quality of the teachers' solutions and mark schemes
- Quality of teachers' assessments — consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one module test for each unit

FURTHER COMMENTS

1. Too many of the module tests comprised items from CAPE past examination papers.
2. Untidy ‘cut and paste’ presentations with varying font sizes were commonplace.
3. Teachers are reminded that the CAPE past examination papers should be used *only* as a guide.
4. The stipulated time for module tests (1–1 hour 30 minutes) must be strictly adhered to as students may be at an undue disadvantage when module tests are too extensive or too insufficient.
5. The following guide can be used: 1 minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling *must* be included.
6. Cases were noted where teachers were unfamiliar with recent syllabus changes that is,
 - Complex numbers and the Intermediate Value Theorem are now tested in Unit 2.
 - Three dimensional vectors, dividing a line segment internally and externally, systems of linear equations were removed from the Unit 1 CAPE syllabus (2008).
7. The moderation process relies on validity of the teachers’ assessments. There were few cases where students’ solutions were *replicas* of the teachers’ solutions — some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on students’ scripts did not correspond to the marks on the moderation sheet. The SBA must be administered under examination conditions at the school. It is not to be done as a homework assignment or research project.
8. Teachers must present evidence of having marked each individual question on students’ scripts before the marks scored out of the possible total is calculated at the top of the script. The corresponding whole number score out of 20 must be placed at the front of students’ scripts.

Module Tests

- Design a separate test for each module. The module test must focus on objectives from that module.
- In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered and a common marking scheme used.
- A sample of five students will form the sample for the centre. If there are less than five students, *all* scripts will form the sample for the centre.
- In 2011, the format of the SBA remains unchanged.

To enhance the quality of the design of module tests, the validity of teachers’ assessments and the validity of the moderation process, the SBA guidelines are listed below for emphasis.

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required on the cover of each module test.

- Name of school and territory, name of teacher, centre number
- Unit number and module number
- Date and duration of module test
- Clear instructions to candidates
- Total marks allocated for module test
- Sub-marks and total marks for each question **must** be clearly indicated

2. COVERAGE OF THE SYLLABUS CONTENT

- The number of questions in each module test must be appropriate for the stipulated time.
- CAPE past examination papers should be used as a guide **ONLY**.
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant unit of the syllabus.

3. MARK SCHEME

- Detailed mark schemes **MUST** be submitted, that is, one mark should be allocated per skill (not 2, 3, 4 marks per skill)
- Fractional or decimal marks **MUST NOT** be awarded. (that is, do not allocate $\frac{1}{2}$ marks).
- A student's marks **MUST** be entered on the front page of the student's script.
- Hand written mark schemes **MUST** be NEAT and LEGIBLE. The **unitary** marks **MUST** be written on the right side of the page.
- **Diagrams MUST be neatly drawn with geometrical/mathematical instruments.**

PRESENTATION OF SAMPLE

- Students' responses **MUST** be written on letter sized paper ($8 \frac{1}{2} \times 11$) or A4 (8.27×11.69).
- Question numbers **MUST** be written clearly in the left hand margin.
- The total marks for **EACH QUESTION** on students' scripts **MUST** be clearly written in the left or right margin.
- **ONLY ORIGINAL** students' scripts **MUST** be sent for moderation.
- Photocopied scripts **WILL NOT BE ACCEPTED**.
- Typed module tests **MUST** be NEAT and LEGIBLE.
- The following are required for each Module test:
 - ❖ A question paper
 - ❖ Detailed solutions with detailed unitary mark schemes.
 - ❖ The question paper, detailed solutions, mark schemes and five students' samples should be batched together for each module.
- Marks recorded on PMath-3 and PMath2-3 forms must be rounded off to the nearest whole number. If a student scored zero, then zero must be recorded. If a student was absent, then absent must be recorded. The guidelines at the bottom of these forms should be observed. (See page 57 of the syllabus, no. 6.)

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

**REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®**

MAY/JUNE 2012

PURE MATHEMATICS

**Copyright © 2012 Caribbean Examinations Council
St Michael, Barbados
All rights reserved.**

GENERAL COMMENTS

In 2012, approximately 5500 and 2800 candidates wrote the Units 1 and 2 examinations respectively. Performance continued in the usual pattern across the total range of candidates with some candidates obtaining excellent grades, while some candidates seemed unprepared to write the examinations at this level, particularly in Unit 1.

The overall performance in Unit 1 was satisfactory, with several candidates displaying a sound grasp of the subject matter. Excellent scores were registered with specific topics such as Trigonometry, Functions and Calculus. Nevertheless, candidates continue to show weaknesses in areas such as Modulus, Indices and Logarithms. Other aspects that need attention are manipulation of simple algebraic expressions, substitution and pattern recognition as effective tools in problem solving.

In general, the performance of candidates on Unit 2 was satisfactory. It was heartening to note the increasing number of candidates who reached an outstanding level of proficiency in the topics examined. However, there was evidence of significant unpreparedness by some candidates.

Candidates continue to show marked weaknesses in algebraic manipulation. In addition, reasoning skills must be sharpened and analytical approaches to problem solving must be emphasized. Too many candidates demonstrated a favour for problem solving by using memorized formulae.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily, with a mean score of 29.5 and a standard deviation of 20.55.

Paper 02 – Structured Questions

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (a) 5; (b) 1, 2, 3, 4; (g) 1, 4

The topics covered in this question were the Remainder and Factor Theorems, Operations on Surds and Modulus Inequalities.

The majority of candidates showed good performance in Parts (a) (i) and (ii). Some instances of errors in substitution were observed. These included $f(1) = -6$, $f(-1) = 6$ and $f(-1) = 0$. A small number of candidates failed to complete the factorization of $f(x)$.

Part (b) was generally well done. Some computational errors included incorrectly expanding $(\sqrt{x} + \sqrt{y})^2$ which resulted in the loss of marks by some candidates, failure to form two equations in x and y and failure to equate the terms to solve x and y . General algebraic weaknesses were also evident.

The majority of candidates removed the modulus in Part (c) (i) by squaring both sides of the inequality obtaining a quadratic inequality in x . Some candidates were able to use a graphical approach to identify the range of values of x for which the inequality was satisfied. Common errors included, (i) stating the critical values of $|3x - 7| = 5$ and $(3x - 2)(x - 4) \leq 0 \Rightarrow x \leq \frac{2}{3}, x \leq 4$.

In Part (c) (ii), a few candidates used the quadratic inequality obtained by squaring both sides of the inequality $|3x - 7| + 5 \leq 0$ and used the characteristic of the discriminant, $b^2 - 4ac < 0$, to prove the desired result. A number of candidates deduced that $|3x - 7| \geq 0$ for all real values of x .

Solutions

(a) (i) $p = -7, q = 1$ (ii) $f(x) = (x - 1)(x + 2)(2x + 5)$

(b) $x = 10, y = 6$ or $x = 6, y = 10$

(c) (i) $\frac{2}{3} \leq x \leq 4$

Question 2

Specific Objectives: (c) 1, 3; (d) 1, 7; (f) 3

This question tested applications of a composite quadratic function, solution of the resulting quartic equation equal to a linear equation, the relationships between the sum and product of the roots of a quadratic equation, applications of the laws of logarithms in base 10 to simplify a sum of quotients without the use of calculators or tables and the finite sum of a quotient of logarithms in base 10.

Part (a) (i) required a composite function $ff(x)$. This was generally well done. However, many candidates failed to simplify the resulting expression. This resulted in a number of candidates being unable to successfully complete Part (a) (ii). Poor algebraic skills did not allow for the correct solution of $x^4 - 7x^2 - 6x = 0$. Some candidates seemed perplexed that the equation did not contain a term in x^3 .

Parts (b) (i) and (ii) were generally well done. A small number of candidates were unable to state $\alpha^2 + \beta^2$ correctly in terms of $\alpha + \beta$ and $\alpha\beta$. Performance on Part (b) (iii) was generally good with the exception of a small number of candidates who found the algebra for $\frac{2}{\alpha^2} + \frac{2}{\beta^2}$ expressed in terms of $\alpha + \beta$ and $\alpha\beta$ beyond their capabilities. Too many candidates did not write the required quadratic equation but instead stated the expression $x^2 - 2x + 64$.

Part (c) (i) was generally well done. Candidates recognized the required laws of logarithms to use and were able to simplify the logarithmic expression.

In Part (c) (ii), a significant number of candidates demonstrated their inability to use the sigma notation.

The answer $\log_{10} \left(\frac{99}{100} \right)$ was seen in many cases. Some candidates were able to express $\sum_{r=1}^{99} \left(\frac{r}{r+1} \right)$

as $\sum_{r=1}^{99} (\log_{10} r - \log_{10} (r+1))$ but could not apply the concept of the sigma notation. Generally this

part of the question had limited successes.

Solutions

(a) (i) $ff(x) = x^4 - 6x^2 + 6$ (ii) $x = -2, -1, 0, 3$

(b) (i) $\alpha + \beta = \frac{3}{4}$, $\alpha\beta = \frac{1}{4}$ (ii) $\alpha^2 + \beta^2 = \frac{1}{16}$

(iii) $x^2 - 2x + 64 = 0$

(c) (i) -1 (ii) -2

Section B

Module 2: Trigonometry and Plane Geometry

Question 3

Specific Objectives: (a) 4, 5, 9, 10, 12, 13

This question tested trigonometric identities, compound and multiple angle formulae, the factor formulae and solutions of trigonometric equations, use of trigonometric identities, compound and multiple angles formulae and the factor formulae.

Most candidates attempted Part (a) (i). Many of them failed to show the desired result due to poor manipulation of the identities given and subsequent simplification. A number of candidates failed to deduce that the factor formula was required to show the desired result in Part (a) (ii). Those who attempted other approaches were not able to complete the result. In Part (a) (iii), many candidates did not heed the directive ‘Hence...’ thus enabling them to use the result at Part (a) (ii). The factor formula could have been easily used heeding the directive ‘or otherwise’. A small number of candidates attempted the expansions of $\sin 6\theta$ and $\sin 2\theta$ in terms of $\sin \theta$ but experienced algebraic difficulties to complete the solution.

In Part (b), the majority of candidates recognized the need to substitute $\frac{\cos^2 \theta}{\sin^2 \theta}$ for $\cot^2 \theta$. In some cases candidates divided the equation $2 \cos^2 \theta + \cos \theta \sin^2 \theta = 0$ by $\cos \theta$ thus losing the result $\cos \theta = 0$. A number of candidates solved the quadratic equation in $\cos \theta$ to obtain $\cos \theta = 1 \pm \sqrt{2}$. However, candidates were penalized for stating $\cos \theta = 1 + \sqrt{2}$. Generally, only a small number of candidates were able to successfully complete this part of the question.

Solutions

(a) (iii) $\theta = \frac{\pi}{8}, \frac{3\pi}{8}$

(b) $\cos \theta = 0 \quad \cos \theta = 1 - \sqrt{2}.$

Question 4

Specific Objectives: (b) 8, 9; (c) 1, 3, 7, 9, 10

This question tested candidates' ability to give the Cartesian equation of a curve defined in trigonometric parameters, the intersection of a curve with a straight line, expressions of coordinate points in vector form and finding the angle between two vectors using the dot product method.

Part (a) (i) was generally well done by the majority of candidates. There were a few cases of candidates being unable to use the appropriate trigonometric identities to eliminate the parameter. Algebraic errors were evident in Part (a) (ii) where many candidates wrote $(\sqrt{10x})^2 = 10x^2$ and $(\sqrt{10x})^2 = \sqrt{10x}$.

Attempts at subsequent solution of $\frac{y^2}{9} - \frac{x^2}{9} - 1 = \sqrt{10x}$ resulted in confusion and were often abandoned. The question required candidates to find the points of intersection. Many candidates solved the quadratic equation in x and stopped short of finding the corresponding values of y . In addition, a number of candidates did not state the coordinates of the points of intersection.

The majority of candidates successfully completed Parts (b) (i) to (iv). Some arithmetic errors were seen in computing the angle in degrees.

Solutions

- | | | | |
|---------|---|------|-----------------------------------|
| (a) (i) | $\frac{y^2}{9} - \frac{x^2}{9} = 1 \Rightarrow y^2 = x^2 + 9$ | (ii) | $(1, \sqrt{10}), (9, 3\sqrt{10})$ |
| (b) (i) | $p = -3i + 4j, q = -i + 6j$ | (ii) | $-2i - 2j$ |
| (iii) | 27 | (iv) | 27.41^0 |

Section C**Module 3: Calculus 1**Question 5

Specific Objectives: (a) 3–5, 7, 10; (b) 5, 11

This question tested the concepts of limit of a function, limit theorems, differentiation of simple functions, continuity and discontinuity and rate of change.

For Part (a) (i), the majority of candidates understood that it was necessary to show that for discontinuity the denominator must be zero. A few candidates solved $x^2 - 4 = 0$ as $x = 2$ only. Generally this part of the question was well done. The majority of candidates had no difficulties successfully completing

Part (a) (ii) by using the result at Part (a) (i) and making the relevant cancellation. A few candidates were unable to factorize $x^3 + 8$ as a sum of cubes. The composite function $\frac{2x^3 + 4x}{\sin 2x}$ in Part (a) (iii) seemed

to have left candidates in a state of confusion, apparently being drilled in limits involving either rational functions involving factors that cancel or trigonometric functions that are variations of $\frac{\sin x}{x}$. Poor

algebraic skills prevented many candidates from simplifying the composite function and using the theorems of limits of sums, differences and quotients. A small number of candidates used L'Hopital's rule and successfully found the correct limit. In Part (b) (i) a), the majority of candidates found the correct limit as 2. It was relatively simple to find the value of p for continuity having found the correct limit for $x > 1$ in b). The answer to Part (b) (ii) was merely deduced from Part (b) (i) a).

The majority of candidates was able to substitute $t = 1$ in the equation for M in order to find the first equation in terms of u and v . A number of candidates made errors in differentiating $\frac{v}{t^2}$ correctly and as a result found incorrect values for u and v . Some candidates did not deduce that a rate of change meant to find $\frac{dM}{dt}$. Generally, a significant number of candidates failed to find the correct values of u and v .

Solutions

- (a) (i) $x = \pm 2$ (ii) -3
 (iii) 2
- (b) (i) a) 2 b) $p = -2$
 (ii) $f(1) = 2$
- (c) $u = 2$ $v = -3$

Question 6

Specific Objectives: (b) 5, 10, 13 – 16; (c) 8 (i)

This question tested first and second derivatives using the chain rule and the product/quotient rules, evaluation of a definite integral, location of stationary points, nature of stationary points and sketching a cubic curve.

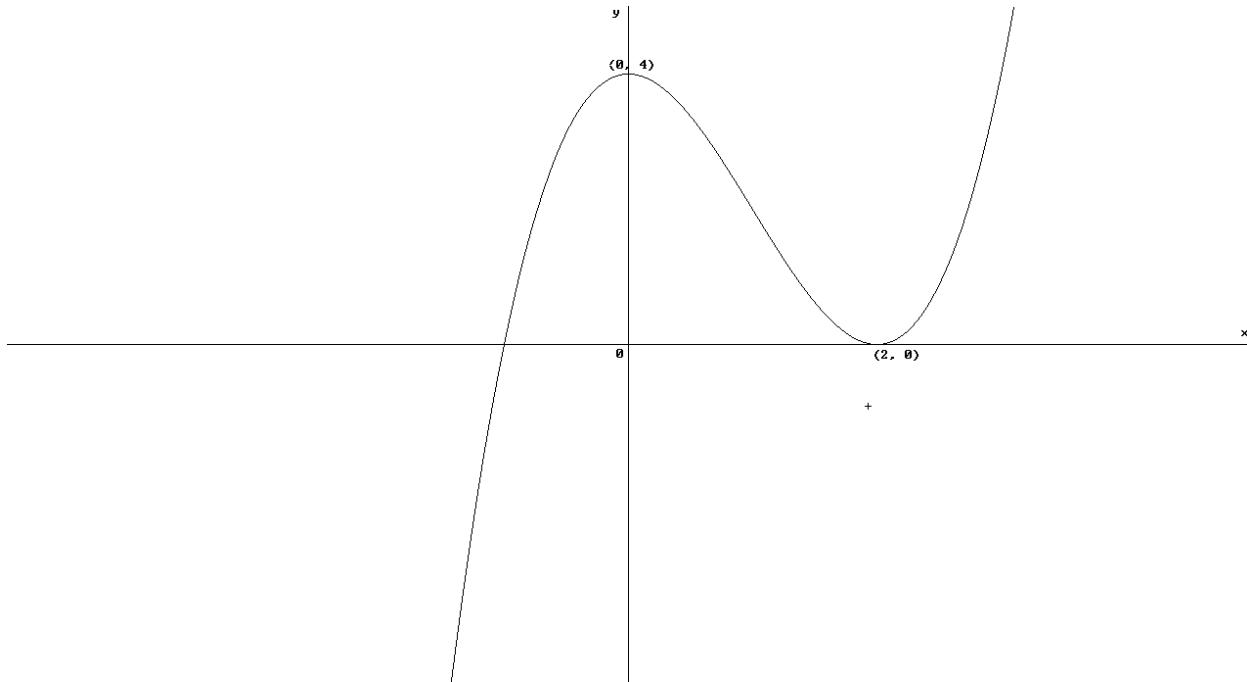
Part (a) (i) was satisfactorily done. However, some candidates were not able to show the desired result because of poor algebraic skills and subsequent simplification. Due to poor use of the unsimplified result of Part (a) (i), several candidates were not able to show the required result for Part (b) (ii). The continued applications of the chain rule and the product/quotient were beyond the majority of candidates. A very small number of knowledgeable candidates used implicit differentiation and successfully completed this part of the question.

Part (b) (i) required a definite integration. The majority of candidates performed well. Successful solution of $\frac{dy}{dx} = 0$ in (ii) assisted the majority of candidates in being awarded maximum marks. Too many

candidates substituted the values of x found for $\frac{dy}{dx} = 0$ in the equation for $\frac{dy}{dx} = 3x^2 - 6x$ to find the y -ordinates of the stationary points. Most candidates knew the concept of using the sign of the second derivative to determine the nature of the stationary points in Part (b) (iii). A number of candidates failed to factorize $x^3 - 3x^2 + 4 = 0$ correctly. Consequently, those candidates were unable to find the correct x -intercepts of the curve in Part (b) (iv). Following the failures at Parts (b) (ii) and (iv), a number of candidates were unable to sketch the curve C showing the critical points correctly.

Solutions

- (b) (i) $y = x^3 - 3x^2 + 4$ (ii) $(0, 4)$ and $(2, 0)$
 (iii) $(0, 4)_{\text{maximum}}$ $(2, 0)_{\text{minimum}}$ (iv) $(-1, 0), (2, 0)$
 (v) see plot



Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (a) 8; (c) 1, 3, 5; (f) 4, 5 (ii)

This question tested the roots of a cubic equation, mathematical induction for divisibility, applications of the laws of indices and the laws of logarithms and the solution of a logarithmic equation involving a change of base.

Parts (a) (i) and (ii) were satisfactorily done. Candidates were vague on Part (b). Apart from proving that the statement is true for $n = 1$ and hence the assumption that the statement is also true for $n = k$, $k > 1$, poor algebraic skills prevented a significant number of candidates from proving that the statement is true for $n = k + 1$. In the majority of cases, this part of the question was badly done. The majority of candidates was unable to express a logarithm in index form and use that form to change the base of a logarithm, thus (c) (i) was poorly done. Consequently, Part (c) (ii) was not done by the majority of candidates.

Solutions:

- | | | |
|----------|--------------------|---------------|
| (a) (i) | $\alpha = -2$ | |
| | | (ii) $p = 28$ |
| (c) (ii) | $x = 2$ or $x = 4$ | |

Section B**Module 2: Trigonometry and Plane Geometry**Question 2

Specific Objectives: (a) 11, 14; (b) 2, 5, 6, 7; (c) 4, 5, 8

This question tested the properties of a circle, a tangent to a circle at a given point, intersection of a curve with a straight line, expression for $a \cos \theta + b \sin \theta = r \cos(\theta - \alpha)$, maximum value of $a \cos \theta + b \sin \theta$, unit vector and a displacement vector.

Candidates performed with limited successes in Parts (a) (i) to (iii). Algebraic and arithmetic errors resulted in loss of marks by some candidates.

Most candidates demonstrated a lack of knowledge of the trigonometric forms in Parts (b) (i). Consequently, performance on Parts (b) (ii) was poor.

Most candidates in attempting Part (c) seemed unaware that it was required to find the unit vector to \overrightarrow{PQ} before multiplying \overrightarrow{OR} by $\sqrt{5}$. Generally this part of the question was done poorly.

Solutions

- | | | |
|---------|---|---|
| (a) (i) | $\text{centre } (3, -1)$ radius = 5 units | (ii) $\text{tangent}_{(7, 2)} 4x + 3y - 34 = 0$ |
| | | (iii) $Q(-1, -4)$ |
| (b) (i) | $f(\theta) = 6 \cos(\theta + 30^\circ)$ | (ii) $f(\theta)_{\text{maximum}} = 6$ |
| (c) | $a = \frac{1}{\sqrt{2}}$ | $b = \frac{3}{\sqrt{2}}$ |

Section C

Module 3: Calculus 1

Question 3

Specific Objectives: (a) 3 to 6; (b) 1, 5, 7, 11

This question tested limits using limit theorems, gradient at a point and rate of change.

Overall, candidates performed satisfactorily on this question. More than 50 per cent of the candidates scored at least seven of the 20 available marks and more than 30 per cent scored at least 10 of the 20 available marks. Most candidates were able to use the algebraic expression given and to correctly substitute the given value of x to obtain the limit in Part (a) (i). Using the result from Part (a) (i) the majority of candidates was able to obtain the correct limit in Part (a) (ii).

Candidates demonstrated a fair level of understanding of a gradient function at a point. Good performance was shown in Part (b).

Part (c) posed severe challenges for many candidates. The algebraic expressions for V in terms of t only and V in terms of x only in Parts (b) (i) and (ii) respectively were poorly done by many candidates. The majority of them could not see a relationship between the height of the water and the radius at that instant.

Solutions

$$(a)(i) \quad \frac{1}{4}$$

$$(ii) \quad \frac{1}{12}$$

$$(b) \quad 24$$

$$(c) \quad (i) \quad V = 10t$$

$$(ii) \quad V = \frac{1}{3}\pi x^3$$

$$(iii) \quad \approx 0.24 \text{ cm/s}$$

UNIT 2

Paper 01 – Multiple Choice

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily, with a mean score of 32.11 and a standard deviation of 18.27.

Paper 02 – Structured Questions

Section A

Module 1: Calculus II

Question 1

Specific Objectives: (b) 1, 3 to 7

This question tested first and second derivatives of a product of a polynomial and an exponential function, stationary points, differentiation of parametric equations including an inverse trigonometric function, gradient at a point and equation of a tangent at a point to a curve defined by parametric equations.

In Part (a) (i) a), candidates demonstrated a sound understanding of the first and second derivatives using the product rule although many of the answers were left unsimplified.

Candidates were able to determine the values of x -coordinates correctly for which $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ respectively in Parts (a) (i) b) and c). Using the second derivative and the significance of the sign change, the majority of candidates was able to distinguish the maxima and minima points as required in Part (a) (ii). However, a significant number of candidates seemed not to know the conditions for points of inflection, using the properties of the second derivative to identify these points. This part of the question had very limited successes.

Candidates demonstrated a sound understanding of the chain rule to obtain the first derivative of parametric equations required in Part (b) (i). However, the majority of candidates opted to find $\frac{dx}{dt}$ which involved differentiating an inverse trigonometric function along with differentiating a variable involving a fractional index. Many errors in the evaluation of this differential resulted in the incorrect expression for $\frac{dy}{dx}$ in terms of t . A number of candidates used the fact that $\sin^{-1}\left(\frac{1}{2}\right) = 30^0$ instead of

$$\frac{\pi}{6}.$$

As a result of the inability by many candidates to complete Part (b) (i) successfully, it was not possible to derive the correct equation of the tangent as required in Part (b) (ii).

Solutions

(a) (i) a) $\frac{dy}{dx} = xe^x(2+x)$ and $\frac{d^2y}{dx^2} = e^x(2+4x+x^2)$

b) $x = 0$ or $x = -2$ c) $x = -2 \pm \sqrt{2}$

(ii) $x = 0$ gives a relative minimum point, $x = -2$ gives a relative maximum point,

$x = -2 \pm \sqrt{2}$ gives relative inflection points

(b) (i) $\frac{dy}{dx} = 4(t-1)\sqrt{t(1-t)}$

(ii) $4y + 4x = \pi - 3$

Question 2

Specific Objectives: (c) 1 (iii), 3, 4, 6, 8, 10

This question tested partial fractions, indefinite integral using partial fractions, the reduction formula and definite integration of a trigonometric function.

Some candidates did not recognize the partial fractions of the form $\frac{A}{x-1} + \frac{Bx+C}{x^2+1}$ in Part (a) (i).

However, this part of the question was generally satisfactorily done despite some arithmetic errors which resulted in the wrong values for the constants A , B and C .

In Part (a) (ii) many candidates did not express

$$\int \left(-\frac{1}{x-1} + \frac{2x-1}{x^2+1} \right) dx \text{ as } \int -\frac{1}{x-1} dx + \int \frac{2x}{x^2+1} dx - \int \frac{1}{x^2+1} dx \text{ which would have}$$

facilitated simple integration. Approximately 20 per cent of the candidates evaluated $\int \frac{f'(x)}{f(x)} dx$

correctly and approximately 10 per cent of the candidates evaluated $\int \frac{1}{x^2+1} dx$ correctly. A number of

candidates did not show evidence of good integration skills. A common error was the omission of the constant of integration for indefinite integrals.

There were no difficulties with Part (b) (i). However, Part (b) (ii) was very challenging to most candidates. Most candidates understood that application of integration by parts was required to find the required result. However, the sequence of the integration techniques and the resulting algebra were

beyond the capabilities of most candidates. The candidates who attempted this part of the question failed to recognize and use the link at Part (b) (i). This part of the question was poorly done.

Given the substitution $m = 1$ allowed approximately 10 per cent of the candidates to show the desired result in Part (b) (iii). A number of candidates did not include the limits of integration for $-\cos x \cos 3x$.

Approximately 60 per cent of the candidates evaluated Part (b) (iv) correctly, without any reference to the preceding results.

Solutions

$$(a) \quad (i) \quad -\frac{1}{x-1} + \frac{2x-1}{x^2+1} \quad (ii) \quad \ln\left(\frac{x^2+1}{x-1}\right) - \tan^{-1}(x) + C$$

$$(b) \quad (iv) \quad \frac{1}{2}$$

Section B

Module 2: Sequences, Series and Approximation

Question 3

Specific Objectives: (a) 2, (b) 1, 4, 6, 9, 13

This question tested geometric progression, sequences, proof by mathematical induction and Maclaurin's series expansion.

The majority of candidates did not find it difficult to complete Part (a) (i) correctly. A few candidates made errors evaluating indices.

Successful completion of Part (a) (ii) by those candidates who found the correct values for a and r followed easily from Part (a) (i). However, a number of candidates made errors in evaluating

$$177146 = \frac{2(3^n - 1)}{3 - 1}. \text{ Common errors included } 2(3^n - 1) = 6^n - 2 \text{ or } 2 \times 3^{n-1}.$$

A number of candidates were unable to express the r th term of the sequence in Part (b) (i). Proof by mathematical induction was poorly done in Part (b) (ii) by the majority of candidates. A number of candidates showed some evidence of knowing how to begin the proof but lacked the algebraic skills and the sophistication necessary to complete the proof. Apart from proving the statement true for $n = 1$ and attempting to show that it is true for $n = k + 1$, the resulting algebraic substitutions were poorly done. As a result, a number of candidates were not able to complete the proof.

Some candidates who probably memorized Maclaurin's expansion for $\cos x$ simply substituted $2x$ for x and obtained the result. A number of candidates however used differentiation and Maclaurin's theorem to obtain the result for Part (c) (i).

Candidates who attempted to express $\sin^2 x$ in terms of $\cos 2x$ for Part (c) (ii) invariably used the wrong identity and could not obtain the expansion of $\sin^2 x$ correctly.

Solutions

- | | | | | |
|-----|-----|-------------------------------------|------|------------------------|
| (a) | (i) | $a = 2, r = 3$ | (ii) | $n = 11$ |
| (b) | (i) | $u_r = r(r + 2), r \in \mathbb{N}$ | | |
| (c) | (i) | $1 - 2x^2 + \frac{2}{3}x^4 + \dots$ | (ii) | $x^2 - \frac{1}{3}x^4$ |

Question 4

Specific Objectives: (c) 1, 2, 3, 4; (e) 1, 4

This question tested the concepts of factorials, binomial expansion, location of roots and the Newton-Raphson iterative method.

Parts (a) (i) and (ii) were well done by the majority of candidates.

In Part (a) (iii), a small number of candidates used the complete expansion of $\left(x^2 - \frac{3}{x}\right)^8$ before extracting the term in x^4 . Some arithmetic errors with the negative sign in the terms involving $\left(-\frac{3}{x}\right)^n$ were evident. Generally, this part of the question was well done, with the majority of candidates demonstrating a good understanding of the binomial theorem.

The majority of candidates who attempted Part (a) (iv) stopped at the expansion of $(1 + x)^{2n}$ up to and including the 4th term. No candidate recognized that ${}^{2n}C_n$ is the coefficient of the term in x^n . Consequently, candidates could not proceed to make the link with Part (a) (ii) and hence were unable to show the desired result. Beyond the expansion of $(1 + x)^{2n}$ no candidate obtained marks other than those awarded for the partial expansion. The analysis and algebra beyond this point was beyond the grasp of all the candidates who attempted this part of the question.

For Part (b) (i), the majority of candidates concluded that a root exists in the given interval, using the concept of a sign change. Only a very small number of candidates stated that the function is continuous over the given interval. This failure resulted in the majority of candidates being penalized.

The majority of candidates who attempted Part (b) (ii) carried out the required number of iterations with some cases of arithmetic errors. In the absence of the final answer specified to a given number of decimal places or significant figures, most candidates used varying approximate values for x_2, x_3, x_4 and x_5 . In many cases the value of the approximation required was not consistent.

Solutions

(a) (i) $\frac{n!}{(n-r)! r!}$ (iii) 5670

(b) (ii) $T \approx 1.002$

Section C**Module 3: Counting, Matrices and Complex Numbers****Question 5**

Specific Objectives: (a) 2, 4, 7; (b) 1, 7, 8

This question tested arrangements with and without repetitions, selections with restrictions, probability, matrix multiplication and subtraction and the solution of a system of linear equations.

Many candidates demonstrated a penchant for using the formula for permutations in Parts (a) (i) and (ii) rather than analysing the problem with particular regard to the restrictions. Satisfactory performance was duly awarded.

In Parts (b) (i) and (ii), candidates demonstrated a sound grasp of determining selections with or without restrictions. This part of the question was well done.

Apart from some arithmetic errors candidates performed well in Part (c) (i). Those candidates who were successful in Part (c) (i) were able to show the desired result in Part (c) (ii).

Candidates who failed to show the result in Part (c) (ii) made attempts to find the inverse of A as required in Part (c) (iii) using the cofactor method. A small number of candidates attempted the row-reduction method. Poor algebraic skills prevented a number of candidates from obtaining the specified result using the result at Part (c) (ii). Generally, this part of the question had limited successes.

Except for those candidates who were successful in Parts (c) (ii) and (iii), a number of candidates attempted to solve the system of linear equations with 3 unknowns by the elimination method. Arithmetic errors apart, some correct solutions were obtained for Part (c) (iv). Candidates who failed to show the result at Part (c) (i) did not reason that they could obtain the solutions at Part (c) (iv) using B^{-1} in terms of A from the equation at Part (c) (ii).

Solutions

(a) (i) 480 ways

(ii) 1372 ways

(b) (i) $\frac{1}{77}$

(ii) 144 ways

$$(c) \text{(i)} \quad B = \begin{pmatrix} 0 & -3 & -3 \\ -3 & -2 & 7 \\ -3 & 1 & 1 \end{pmatrix} \quad \text{(iii)} \quad A^{-1} = -\frac{1}{9} \begin{pmatrix} 0 & -3 & -3 \\ -3 & -2 & 7 \\ -3 & 1 & 1 \end{pmatrix}$$

$$\text{(iv)} \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ -\frac{1}{3} \\ -\frac{2}{3} \end{pmatrix}$$

Question 6

Specific Objectives: (c) 1, 2, 3, 5, 6, 9, 11

This question tested representation of complex numbers on the Argand diagram, the principal argument of a complex number, the square roots of a complex number, finding the complex roots of a quadratic equation and application of de Moivre's theorem to prove a trigonometric identity.

Part (a) (i) was well done by the majority of candidates. Severe constraints due to poor algebraic skills resulted in poor performance on Part (a) (ii). Candidates failed to use the relationship between the complex numbers A and B and the representation of these points on the Argand diagram to find the

argument of $A + B$. A number of candidates attempted to rationalize $\frac{1 + \sqrt{2} + i}{1 - i}$ and find $\tan^{-1}(1 + \sqrt{2})$

approximating the answer to $\frac{3\pi}{8}$.

The majority of candidates who attempted Part (b) (i) obtained full marks. However, a number of candidates could not link the result at Part (b) (i) to the solutions of the quadratic equation in Part (b) (ii). Nevertheless, most candidates demonstrated a sound grasp of the techniques involved to find complex roots of a quadratic equation.

The majority of candidates who attempted Part (c) understood de Moivre's theorem and its application to proof of trigonometric identities. This part of the question was well done.

Solutions

(a) (i) Points $(0, 1)$ and $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(b) (i) $z = \pm \frac{1}{\sqrt{2}}(1 + i)$ (ii) $z = 2 + 3i, 1 + 2i$

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Calculus II

Question 1

Specific Objectives: (b) 2, 4, 5; (c) 8, 13

This question tested logarithmic and implicit differentiation involving polynomials and trigonometric functions, the trapezium rule and definite integration by parts to find the area under a given curve.

The majority of candidates showed a marked weakness in the applications of logarithmic differentiation required in Part (a). A few candidates attempted to complete the differentiation by using the product and quotient rules. However, they failed to apply the chain rule for the composite functions correctly. This part of the question was poorly done.

Some weak attempts were made at sketching the curve in Part (b) (i). Most candidates showed a straight line between the limits stated.

Weak performances were also recorded in Part (b) (ii). A number of candidates failed to use the trapezium rule correctly. In addition, some arithmetic errors resulted in wrong values of the required approximation.

Most candidates demonstrated a fair understanding of integration by parts required in Part (c) (i).

However, many of them appeared at a loss when required to repeat the process for $\int 2x \sin x \, dx$.

Limited successes were recorded for this part of the question.

Solutions

(a) $\frac{dy}{dx} = \frac{x(x-1)^{\frac{1}{3}}}{1+\sin^3 x} \left[\frac{1}{x} + \frac{1}{3(x-1)} - \frac{3\sin^2 x \cos x}{1+\sin^3 x} \right]$

(b) (ii) 1.115 square units

(c) (i) $\sin^2 \sin x + 2x \cos x - 2 \sin x + C$

(ii) $\frac{\pi^2}{4} - 2$

Section B**Module 2: Sequences, Series and Approximation****Question 2**

Specific Objectives: (c) 2, 3, 4; (d) 1; (e) 3, 4

This question tested the binomial theorem, the Newton-Raphson iteration method and errors of measurement.

Most candidates performed well in Part (a) (i). In Part (a) (ii), a number of candidates could not determine that it was necessary to substitute $x = 0.1$ in the expansion found in Part (a) (i). Arithmetic errors resulted in some incorrect answers.

A number of candidates failed to observe the range of values of x for which the graphs of $f(x)$ and $g(x)$ were to be sketched in Part (b) (i). However, most candidates knew the shapes of the curves to be sketched.

Part (b) (ii) was generally well done, except for some arithmetic errors and failure to observe that the approximation was required to *four decimal places*.

The majority of candidates who attempted Part (b) (iii) knew the concept of errors of measurements and generally this part of the question was done satisfactorily.

Solutions

(a) (i) $1 + \frac{5}{4}x + \frac{5}{8}x^2 + \frac{5}{32}x^3 + \frac{5}{256}x^4 + \frac{1}{1024}x^5$

(ii) 1.131

(b) (ii) 0.3419

(iii) 0.1226

Section C

Module 3: Counting, Matrices and Complex Numbers

Question 3

Specific Objectives: (a) 2, 7; (b) 1, 2, 7

This question tested the number of arrangements with and without repetitions, selections with restrictions, multiplication of conformable matrices and inverting a 3×3 matrix.

Candidates attempting Part (a) (i) did not follow the requirement of repetitions. Rather, most of the candidates obtained distinct permutations of the letters given. Part (a) (ii) was poorly done by the few candidates who attempted it.

Candidates demonstrated a sound understanding of the cofactor method for inverting a matrix required in Part (b) (i). Arithmetic errors resulted in some candidates losing marks due to inaccuracy. Part (b) (ii) was generally well done.

Part (b) (iii) was poorly done. Candidates could not make the links with Parts (b) (i) and (ii) to determine the matrix B. Poor algebraic skills prevented candidates from manipulating the links in Parts (b) (i) and (ii) to solve Part (b) (iii). This part of the question was poorly done.

Solutions

$$(a) \quad (i) \quad 24 \text{ times} \quad (ii) \quad 576$$

$$(b) \quad (i) \quad A^{-1} = \begin{pmatrix} 3 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{pmatrix} \quad (ii) \quad \begin{pmatrix} 4 \\ 7 \\ 12 \end{pmatrix}$$

$$(iii) \quad B = \begin{pmatrix} 6 & -6 & 2 \\ -6 & 10 & -4 \\ 2 & -4 & 2 \end{pmatrix}$$

Paper 031 – School Based Assessment (SBA)

For the year 2012, 191 Unit 1 and 149 Unit 2 SBAs were moderated. It is increasingly difficult to successfully complete the moderation for samples for the SBAs submitted by some centres since many teachers continue to submit solutions *without unitary mark schemes*. In a number of cases, *neither question papers nor detailed worked solutions with unitary mark schemes* were submitted. Mark schemes for questions and their subsequent parts were not broken down into unitary marks. In an increasing number of cases, marks awarded were either too few or far too many. (Example: an entire SBA module test was 20 marks in total and in another case only one syllabus objective assessed within a module test was allocated 40 marks).

The majority of the samples submitted were not of the required standard. Teachers must pay particular attention to the following guidelines and comments to ensure effective and reliable submission of the SBAs.

The SBA is comprised of three module tests.

The main features assessed are:

- Mapping of the items tested to the specific objectives of the syllabus for the relevant unit.
- Content coverage of each module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (module test questions, detailed mark schemes and students' scripts)
- Quality of the teachers' solutions and mark scheme
- Quality of teachers' assessments – consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one module test for each unit

FURTHER COMMENTS

1. Too many of the module tests comprised items from CAPE past examination papers.
2. Teachers are reminded that CAPE past examination papers should be used *only* as a guide. They should not constitute any part of or an entire module test.
3. Untidy 'cut and paste' presentations with varying font sizes were common and as a result very unsatisfactory. Module tests must be neatly handwritten or typed.

4. The stipulated time for module tests (1hr – 1hr 30mins) must be strictly adhered to as students may be at an undue disadvantage when the times allotted for module tests are too extensive or insufficient.
5. The following guide can be used: 1 minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling *must* be included.
6. Multiple choice questions will **NOT** be accepted in the module tests.
7. Cases were noted where teachers were unfamiliar with recent syllabus changes, that is,
 - Complex numbers and the Intermediate Value Theorem are tested in Unit 2.
 - Three dimensional vectors, dividing a line segment internally and externally, systems of linear equations have been *removed* from the CAPE syllabus (2008).
8. The moderation process relies on the validity of teachers' assessment. There were a few cases where tampering of the scripts and subsequent questionable mark changes occurred. There were also instances where the marks on students' scripts did not correspond to the marks on the moderation sheet. There were a few cases where students' solutions were replicas of teachers' solutions — some contained identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on students' scripts did not correspond to the marks on the moderation sheet. The SBA must be administered under examination conditions at the centre. It is not to be done as a homework assignment or research project.
9. Teachers *must* present evidence of having marked each question on students' scripts before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of students' scripts.
10. Teachers *must* indicate any changes/omissions that were made to question papers, solutions and mark schemes and scripts. Teachers should also inform the examiner about the circumstances regarding missing script(s).
11. Students' names on the computer generated form must correspond to the names on PMATH 1-3 and PMATH 2-3 forms and students' scripts.
12. The maximum number of marks for each assessment should be the same for all students.

To enhance the quality of the design of the module tests, the validity of teachers' assessments and the validity of the moderation process, the SBA guidelines are listed below for emphasis.

Module Tests

- Design a separate test for each module. The module test *must* focus on objectives from that module.
- In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered and a common marking scheme used.
- One sample of *five* students will form the sample for the centre. If there are fewer than five students, *all* scripts will form the sample for the centre.
- In 2012, the format of the SBA remains unchanged.

Guidelines for Module Tests and Presentation of Samples

1. Cover Page To Accompany Each Module Test

The following information is required on the cover of each module test:

- Name of school and territory, name of teacher, centre number
- Unit number and module number
- Date and duration (1hr – 1hr 30 mins) of module test
- Clear instructions to students
- Total marks allotted for module test
- Sub-marks and total marks for each question *must* be clearly indicated.

2. Coverage Of The Syllabus Content

- The number of questions in each module test must be appropriate for the stipulated time of (1hr – 1hr 30 mins).
- *CAPE past examination papers should be used as a guide ONLY.*
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant unit of the syllabus.

3. Mark Scheme

- Unitary mark schemes *must* be done on the detailed worked solution. (that is, one mark should be allocated per skill assessed, not 2, 3, 4 etc marks per skill)
- Fractional/decimal marks must not be awarded (that is, do not allocate $\left(\frac{1}{2}\right)$ marks
- The total marks for module tests *must* be clearly stated on teachers' solution sheets.

- A student's final mark out of 20 *must* be entered on the front page of the student's script.
- Hand written mark schemes *must* be *neat* and *legible*. The unitary marks *must* be written on the right side of the page.
- Diagrams *must* be neatly drawn with geometrical/mathematical instruments.

4. Presentation Of Sample

- Students' responses *must* be written on letter-sized $\left(8\frac{1}{2} \times 11\right)$ or A4 $\left(8\frac{1}{2} \times 11.69\right)$ paper.
- Question numbers *must* be written clearly in the left hand margin.
- The total marks for *each* question on students' scripts MUST be clearly written in the left or right margin.
- Only original students' scripts *must* be sent for moderation.
- Photocopied scripts *will not be accepted*.
- Typed module tests *must* be *neat* and *legible*.
- The following are required for each module test:
 - ❖ A question paper
 - ❖ Detailed solutions with detailed unitary mark schemes
 - ❖ The question paper, detailed solutions, unitary mark schemes and five students' samples should be batched together for each module.
- Marks recorded on PMATH 1-3 and PMATH 2-3 forms must be rounded off to the nearest whole number. If a student scored zero, then zero must be recorded. If a student was absent, then absent must be recorded.
- Form PMATH 2-4 is for official use *only* and should not be completed by the teacher. However, teachers may complete the relevant information: Centre Code, Name of Centre, Territory, Year of Examination and Name of Teacher(s).
- The guidelines at the bottom of the PMath forms should be observed. (See page 57 of the syllabus, no. (6)).

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

**REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®**

MAY/JUNE 2013

PURE MATHEMATICS

GENERAL COMMENTS

In 2013, approximately 4,800 and 2,750 candidates wrote the Unit 1 and 2 examinations respectively. Overall, the performance of candidates in both units was consistent with performance in 2012. In Unit 1, 72 per cent of the candidates achieved acceptable grades compared with 70 per cent in 2012; while in Unit 2, 81 per cent of the candidates achieved acceptable grades compared with 83 per cent in 2012. Candidates continue to experience challenges with algebraic manipulation, reasoning skills and analytic approaches to problem solving.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

The paper comprised 45 items, 15 items per module. Most candidates performed satisfactorily. Marks on this paper ranged from a minimum of 6 to a maximum of 45. The mean mark for the paper was 64.58 per cent.

Paper 02 – Structured Questions

The paper consisted of six compulsory questions, two questions per module. The maximum score was 149 out of 150. The mean score was 52.26.

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (a) 2, 4; (b) 1, 3, 5; (c) 2, 3, 4; (d) 3, 8

The topics tested in this question included the use of truth tables, binary operations, proof by mathematical induction and the factor theorem. Overall, candidates demonstrated competence in this question with approximately 90 per cent of them attempting it and obtaining at least 16 marks. A number of candidates were also able to obtain the maximum score.

In answering Part (a), candidates used a variety of styles to represent the inputs and outputs such as ‘1 and 0’, ‘True and False’, ‘x and \vee ’. The majority of candidates attempted Part (a) (i) and was successful. In Part (a) (ii), some candidates misinterpreted $\sim(p \wedge q)$ and used it as if it were $\sim p \wedge \sim q$.

Part (b) was misinterpreted by many candidates. They substituted $x = 2, -2$ instead of $y = 2$ in the given function. Other candidates treated $y \oplus x$ as $y + x$, solved for y by replacing x and hence substituted y as -2 . They also had difficulty factorizing and solving the quadratic equation.

For Part (c), a majority of candidates were able to achieve the first four marks allocated. However, most candidates did not apply the induction steps correctly. Some were more familiar with questions involving the sigma notation and incorporated the sigma notation in their solution. A few candidates did not use the smallest natural number, 1, to begin the proof by induction and instead used 0 and 2. Other candidates were able to recognize the $k + 1^{\text{th}}$ term but they were unable to simplify the term 5^{k+1} because they did not realize that it could be expressed as $5^k \times 5^1$. The conclusion also posed a challenge to many candidates. Candidates should be reminded that the conclusion should relate to the hypothesis. The general format for the conclusion could be as follows: Since $P(1)$ is true and $P(k) \rightarrow P(k+1)$, the proposition $P(k)$ is true for all positive integers ' k '.

Most candidates obtained full marks in Part (d). Some candidates substituted the value of ' p ' into the function to prove that $(x+1)$ is a factor as opposed to using the factor to find ' p '. In general, long division was used to show that the remainder is zero under division by $(x+1)$. This method could also have been used for Part (d) (ii) to obtain the quadratic equation, followed by factorizing the quadratic equation to obtain the other two factors and then equating each factor to zero to solve the cubic function. In some cases, candidates were able to factorize the cubic function correctly but were unable to identify the roots.

Solutions

(a) (i) (ii)

p	q	$p \rightarrow q$	$p \wedge q$	$\sim(p \wedge q)$
T	T	T	T	F
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(b) $x = 8$ or $x = 1$

(d) (ii) $f(x) = (x - 8)(x - 2)(x + 1)$ (iii) $x = 8, x = 2, x = -2$

Question 2

Specific Objectives: (e) 2, 3, 4; (f) 2, 3; (g)

This question tested the concept of a one-to-one function on the domain of real numbers; the inverse of a linear and an exponential function; the inverse of the composite of a linear and exponential function; quadratic inequalities and the modulus function.

Part (a) failed to attract answers from the majority of candidates. Among the methods used for the proof of a one-to-one function was (i) the graphical approach, (ii) a deductive approach, (iii) differentiation and (iv) proof by induction. Candidates who attempted to use the graphical test used a vertical line test instead of the horizontal line test, not indicating the domain clearly on the graph used and in some cases graphed the quadratic function without indicating the specific domain over which the given function is one-to-one. Candidates who attempted to show that $f(a) = f(b) \Rightarrow a = b$ could not show the correct algebraic simplification and subsequent deductive proof. Those candidates who attempted differentiation to show the required result simply could not proceed beyond merely differentiating $x^2 - x$. Proof by induction was beyond the ability of those candidates who attempted to use this approach.

In Part (b), the results of $f^{-1}(x)$ and $fg(x)$ were easily shown. However, a significant number of candidates failed to find the expression for $g^{-1}(x)$. Common errors included the inability to take \log_e for the change of variable and in cases where \log_e was taken candidates incorrectly cancelled the logs on each side of the equation. In some cases candidates used $\log_e(x - 2)$ as $\log_e x - \log_e 2$. Evidence was seen where some candidates mistakenly interpreted $f^{-1}(x)$ and $g^{-1}(x)$ as $\frac{d}{dx} f(x)$ and $\frac{d}{dx} g(x)$.

In Part (c) (i), candidates used the quadratic graph to find the correct range of values of x .

Candidates who used the results $(x + 2)(3x - 2) \leq 0$ incorrectly reasoned that

$$x + 2 \leq 0 \Rightarrow x \leq -2 \text{ and } 3x - 2 \leq 0 \Rightarrow x \leq \frac{2}{3}.$$

Some candidates used methods including squaring both sides of the equation and thus finding values of x for which the resulting quadratic equation was equal to zero. However, they failed to test the values of x found and were not able to obtain the mark given for showing or stating

that $x = -\frac{7}{4}$ was inadmissible. Candidates who used the concept of $x + 2 = 3x + 5$

and $-(x + 2) = 3x + 5$ could not reason the correct value of x for which the equation was true.

Solutions

(b) (i) a) $f^{-1}(x) = \frac{x-2}{3}$ $g^{-1}(x) = \frac{1}{2} \ln x$ b) $fg(x) = 3e^{2x} + 2$

(c) (i) $-2 \leq x \leq \frac{2}{3}$ (ii) $x = -\frac{3}{2}$ only

Section B

Module 2: Trigonometry, Geometry and Vectors

Question 3

Specific Objectives: (a) 2, 3, 4, 5, 6

This question tested trigonometric identities of multiple angles; solving trigonometric equations involving multiple angles; expressing $a \cos x + b \sin x$ in the form $r \cos(x + \alpha)$; determining maximum and minimum values of trigonometric expressions; and proof of simple trigonometric equations.

In Part (a) (i), most candidates performed satisfactorily. Challenges encountered included the inability to use the identity given to obtain a quadratic equation in terms of $\tan \theta$. Candidates who obtained the correct equation $\tan \theta(1 - \tan^2 \theta) = 0$ wrongly divided the equation by $\tan \theta$ thus losing the roots of the equation $\tan \theta = 0$. Candidates also deduced $\tan^2 \theta = 1 \Rightarrow \tan \theta = 1$. This error resulted in candidates being unable to get the solutions for $\tan \theta = -1$.

Part (b) (i) was generally well done. Some errors included solving $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$ and $\alpha = \tan^{-1}\left(-\frac{4}{3}\right)$. Part (b) (ii) a) was satisfactorily done. Errors made by candidates included stating that the maximum value of $5 \cos(\theta + \alpha)$ is 1.

However, Part (b) (ii) b) was not satisfactorily done. Most candidates seemed unable to deduce the maximum value of a reciprocal function, particularly when another term is added to the denominator.

Responses to Parts (b) (iii) a) and (b) were very poor. Candidates did not demonstrate knowledge of the fact that the sum of the interior angles, A , B and C of the triangle given, was π radians. Evidence of candidates expanding $\sin(B + C)$ and being unable to link that expansion with $\sin A$ was observed. Some candidates attempted to substitute numerical values for angles A , B and C with no success.

Solutions

(a) (ii) $\theta = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$

(b) (i) $5 \cos(\theta + 0.927^\circ)$ (ii) a) $f(\alpha)_{\max} = 5$ (ii) b) $\left(\frac{1}{8 + f(\theta)} \right)_{\min} = \frac{1}{13}$

Question 4

Specific Objectives: (b) 1, 2, 5, 6; (c) 4, 5, 6, 8, 9

This question tested geometry of the circle; parametric equations to a Cartesian equation; three dimensional vectors; and equations of lines in the Cartesian of a plane.

Part (a) (i) was generally well done. Some candidates expressed the equation of the circle using completion of the square to deduce the coordinates of the centre and the radius. A number of candidates used the equation $x^2 + y^2 + 2fx + 2gy + c = 0$ to deduce that the coordinates of the centre were $(-f, -g)$ and the radius $\sqrt{f^2 + g^2 - c}$. Some errors were made when expressing the coordinates of the centre as (f, g) .

In Part (b) (ii) a), many candidates deduced that the normal to the circle lies along the diameter while other candidates worked through the equation $y - y_p = \frac{y_p - y_c}{x_p - x_c} (x - x_c)$.

In Part (b) (ii) (b), some candidates used the gradient of the normal to find the equation of the tangent. Those candidates who used the gradient of the tangent, $-\left(\frac{x_p - x_c}{y_p - y_c} \right)$ could not interpret the meaning of the resulting gradient $-\frac{3}{0}$. It was not unusual to see candidates stating the gradient as 0. The fact of the tangent being parallel to the y -axis was not understood by a significant number of candidates. Further, a number of candidates drew a graph of the circle and indicated the tangent parallel to the y -axis but were unable to state the equation as $x = 6$.

In Part (b), various methods were used by candidates. Apart from expressing $t = \frac{y + 4}{2}$ and substituting for $x = \left(\frac{y + 4}{2} \right)^2 + \frac{y + 4}{2}$, some candidates substituted the parametric equations given into the equation required to be shown. Common errors resulted from poor algebraic simplification. For example $\left(\frac{y + 4}{2} \right)^2 = \frac{y^2 + 16}{4}$.

Generally Part (c) (i) was done satisfactorily. Some candidates made the basic error that the vector $\overline{AB} = \overline{OA} + \overline{OB}$ and $\overline{BC} = \overline{OB} + \overline{BC}$. Part (c) (ii) appeared challenging to many candidates. It was generally understood that the dot product was required. However, most candidates used the \overline{OA} and \overline{OB} with the vector $\mathbf{r} = -16\mathbf{j} - 8\mathbf{k}$ to attempt to show the perpendicular property without success. A small number of candidates understood that they were required to show perpendicularity between the vectors \overline{AB} and $\mathbf{r} = -16\mathbf{j} - 8\mathbf{k}$ and between the vectors \overline{BC} and $\mathbf{r} = -16\mathbf{j} - 8\mathbf{k}$. It was interesting to observe some candidates using the vector cross product to show the perpendicular vector $\mathbf{r} = -16\mathbf{j} - 8\mathbf{k}$.

In Part (c) (iii), candidates quoted the vector equation of the plane, but were not able to use the correct resulting point and the normal vector to the plane to complete the equation $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$. Those candidates who used the correct values were unable to express the Cartesian equation of the plane as required.

Solutions

$$\begin{array}{lll} \text{(a) (ii) a) normal}_{(6, 2)}: & y = 2 & \text{b) tangent}_{(6, 2)}: & x = 6 \\ \text{(c) (i) } \overline{AB} = -2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k} & \overline{BC} = -2\mathbf{i} - \mathbf{j} + 2\mathbf{k} & \text{(iii) } 2y + z = 0 \end{array}$$

Section C

Module 3: Calculus I

Question 5

Specific Objectives: (a) 3, 7, 9, 10; (b) 3–8, 12

This question tested limits; continuity; differentiation using the quotient rule; parametric differentiation; and finding the area enclosed by two curves using integration.

In Part (a) (i), the majority of candidates gave satisfactory performances although only a small number of candidates presented their statements in acceptable mathematical language. However, Part (a) (ii) was poorly done since a number of candidates made guesses on the question of continuity and were unable to justify their responses mathematically.

In Part (b), most candidates showed a good understanding of the quotient rule for differentiation. However, there were cases in which candidates used the product rule by expressing the denominator as a multiplying term. Some candidates committed errors in their differentiation of the term $(x^2 + 2)^3$. They mainly differentiated the cubic term and neglected the differential of x^2 . Weaknesses in algebraic manipulation prohibited many candidates from obtaining the required simplified result.

Almost half of the candidates who responded to Part (c) demonstrated a lack of knowledge of differentiation of parametric terms. Some candidates opted to convert the equation to Cartesian form and to proceed with the differentiation. However, the term in y^2 made it difficult for them to successfully complete the correct expression for $\frac{dy}{dx}$. Only a small number of candidates were able to obtain the correct result.

Generally, the responses to Part (d) were satisfactory. Most candidates were familiar with the concept tested in this part of the question. Some errors included candidates subtracting the area enclosed by the curve $y = 4x$ from the area enclosed by the curve $y = x^2 + 3$ incorrectly. Very few candidates used the integral of $4x - (x^3 + 3)$ but preferred to find the area using the difference of two areas.

Solutions

(a) (i) $\lim = 4$ (ii) $f(x)$ is not continuous since $f(x)$ is not defined at $x = 2$

(c) $\frac{2}{3} \cot \theta$ (d) (i) P (1, 4) Q (3, 12) (ii) $\frac{4}{3}$ units²

Question 6

Specific Objectives: (c) 3, 4, 6, 8, 9 (b)

This question tested indefinite integration using substitution; the theorem of the integral of sums being equal to the sum of integrals, determining maxima using differentiation; and determining the constants of integration given initial conditions.

Part (a) (i) tested integration using substitution. The substitution $x = 1 - u$ was given and the majority of candidates demonstrated a good understanding of having to express dx in terms of du . However, in proceeding to complete the substitution of $x(1-x)^2$ in terms of u , the majority of candidates stated the expression as $x u^2$. Hence, they could not continue integration in this form since the variable x was not expressed as $1-u$. Many candidates who made the correct substitution and successfully integrated in terms of u failed to express their answer in terms of x .

Part (a) (ii) was generally well done. Some errors seen included:

$$\int 4 \sin 5t \, dt = -4 \cos 5t, \quad 4 \cos 5t \text{ and } 20 \cos 5t.$$

Part (b) was generally well done. However, in Part (b) (i), a significant number of candidates were unable to find the correct formula for the area of a simple plane figure. Candidates who used the correct formula for the area were not able to simplify the expression thus allowing for easy differentiation. As a result, a majority of the candidates failed to obtain the correct

differential to proceed to find the value of x . However, candidates demonstrated the knowledge that it was necessary to solve $\frac{dA}{dx} = 0$.

Part (c) (i) required candidates to find the first and second differentials of y , explicitly given in terms of x . The terms to be differentiated involved a product of x and a trigonometric term in x . The majority of candidates failed to apply the product rule in differentiating $-x \sin x$. The common results shown were $-x \cos x$, $x \cos x$ and $\sin x$. This error was compounded by adopting the same approach for the second differential. Consequently, candidates were unable to show the required answer. Overall, candidates performed poorly on this part of the question.

Part (c) (ii) required that candidates find the values of two constants of integration for an explicit function of y in terms of x given the boundary conditions. This required substitution of the values of y for given values of x . A significant number of candidates failed to recognize this simple procedure and appeared to think integration was required. Those candidates who recognized the methods required for solving this part of the question made algebraic and arithmetic errors in their substitutions.

Solutions

(a) (i) $-\frac{1}{12}(1-x)^3(1+3x) + C$ (ii) $5 \sin t - \frac{4}{5} \cos 5t + C$

(b) (ii) 84 metres approx.

(c) (ii) $y = -x \sin x - 2 \cos x + \frac{1}{\pi}x + 3$

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (a) 1, 3; (b) 2, 4; (c) 1, 2; (d) 1, 2, 4, 6; (f) 3

This question tested the converse, inverse and contrapositive of a conditional statement; solving logarithmic and exponential equations; and graphing a modulus and a linear function of x .

For Part (a), candidates generally demonstrated a lack of knowledge in these topics. A small number of candidates attempted to use truth tables to show the required results but performed poorly.

In Part (b), candidates demonstrated understanding of expressing a sum of two logs as a single log. However, they failed to express 3 as $\log_2 2^3$ which would have allowed them to proceed with the solution of the correct quadratic equation $(x + 3)(x + 2) = 8$.

Very few candidates attempted Part (c). Those who did substitute $A = 0$ could not proceed to express the resulting equation $3e^{4t} - 7e^{2t} - 6 = 0$ in a convenient quadratic form to solve for t .

In Part (d), the majority of candidates graphed the line $f(x) = 2x + 3$ correctly but did not graph $g(x) = |2x + 3|$ correctly.

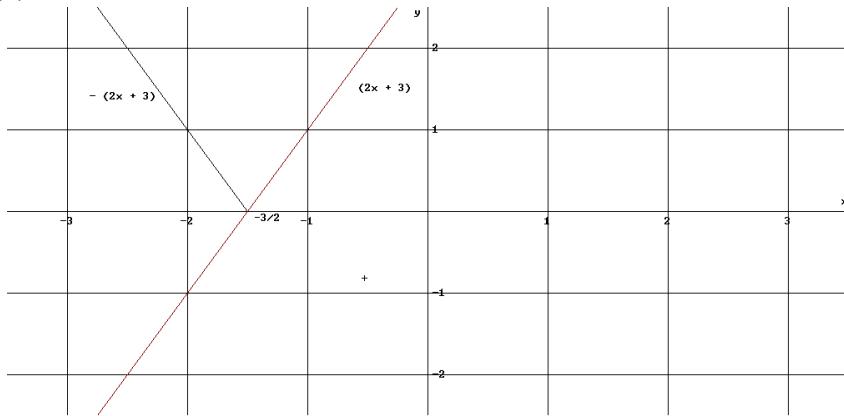
Solutions

(a) (i) $(q \vee \sim p) \rightarrow (p \wedge q)$

(b) $x = 0.37$ (2 d.p.)

(c) $t = \frac{1}{2} \ln 3$

(d)



Section B

Module 2: Trigonometry, Geometry and Vectors

Question 2

Specific Objectives: (a) 1, 3, 4, 7, 8; (b) 3, 4, 6; (c) 1, 7, 10

This question tested the exact value of $\cos 3A$ given exact values of $\sin A$ and $\cos B$; the solution of a trigonometric equation involving a mix of $\cos 2\theta$ and $\sin \theta$ in a given range; and coordinate geometry involving the intersection of two circles.

In general, the responses to Part (a) were unsatisfactory. Candidates attempted to convert $\sin A = \frac{4}{5}$ and $\cos B = -\frac{3}{5}$ into degrees and substitute for $\cos 3A = 3 \cos A - 4 \cos^3 A$ (their value of A).

It was clear that candidates did not have the required knowledge of this topic.

The responses to Parts (b) and (c) were also unsatisfactory. Candidates could not correctly express $\cos 2\theta$ in terms of $\sin \theta$ and the arithmetic errors made by candidates resulted in incorrect values for x and y in Part (c). However, most candidates demonstrated an understanding of using the equation two curves to find point(s) of intersection.

Solutions

(a) $-\frac{117}{125}$ (b) 6.031 radians (c) (1.46, -1) and (-5.46, -1)

Module 3: Calculus I

Question 3

Specific Objectives: (a) 2, 4, 5; (b) 1, 3, 9–12; (c) 2, 5, 7, 8 (b)

This question tested limits, differentiation from first principles; finding minimum and maximum stationary points; and integration to find the volume of revolution about the x -axis. Overall, the majority of candidates gave no response to this question. A few unsuccessful attempts were made for Part (c) with some candidates managing to find $\frac{d}{dx} f(x)$.

Solutions

(a) (i) a) 1 (b) 2 (ii) not continuous since it is not defined at $x = 2$

(b) $-\frac{1}{2\sqrt{2}} \sqrt{(x)^3}$

(c) $\frac{1}{3}$ and $-\frac{3}{2}$

(d) $\frac{100}{3}$ units³

UNIT 2

Paper 01 – Multiple Choice

The paper comprised 45 items, 15 items per module. Most candidates performed satisfactorily. Marks on this paper ranged from a minimum of 6 to a maximum of 44. The mean mark for the paper was 69.18 per cent.

Paper 02 – Structured Questions

The paper consisted of six compulsory questions, two questions per module. The maximum score was 150 out of 150. The mean score was 47.52.

Section A

Module 1: Complex Numbers and Calculus II

Question 1

Objectives: (a) 7, 8, 12, 13; (b) 1–5, 8

In Part (a), the majority of candidates completed the differentiation satisfactorily. Some candidates who completed the differentiation were unable to differentiate the natural log of the term $\ln(x^2y)$ correctly. Some common results for differentiating this term include $\frac{1}{x^2y}$ and

$2x\frac{dy}{dx}$. A few candidates applied the natural log laws to separate the terms before differentiating and many were successful with the differentiation using this approach.

In general, the candidates showed a lack of understanding of partial derivatives in Part (b). Many inserted additional terms. Common responses included:

$$\frac{\partial f}{\partial y} = 3z^2 - e^{4x} \cos 4z - 6y - y \quad \text{and} \quad \frac{\partial f}{\partial z} = 6yz + 4\sin 4ze^{4x} - 4\cos 4ze^{4x}.$$

Part (c) was generally well done, with candidates gaining the majority of marks. Almost all candidates were able to identify that only the real part of the complex number was needed for the solution.

In Part (d), many candidates recognized that $\tan^{\square 1}$ was needed to find the argument. However, most obtained:

$$\begin{aligned}\arg(z) &= \tan^{\square 1}(\square 1) \\ &= \square \frac{\square}{4}\end{aligned}$$

and completely ignored the use of the Argand Diagram. In addition, a majority of candidates obtained $|z| = \sqrt{2}$ but left out the exponent, 7, when doing their final calculation and hence they did not write the modulus as $(\sqrt{2})^7$.

Solutions

- (a) Undefined but marks awarded for finding the derivative and explaining why the gradient could not be found.

(b)
$$\frac{\partial z}{\partial y} = \frac{3(z^2 - 2y)}{2(3yz - 2e^{4x} \sin 4z)}$$

(d) (i) $z = \sqrt{2^7} e^{i7\left(\frac{3\pi}{4}\right)}$

Question 2

Objectives: (c) 1–3, 6, 8, 9, 11

Most candidates attempted Part (a) (i) using integration by parts. However, some candidates either differentiated or integrated the parts incorrectly while others had difficulty manipulating the signs. Some candidates used the identity $\cos 2\theta = 1 - 2\sin^2 \theta$ to simplify the integral to $\int \sin x \, dx - 2 \int \sin^3 x \, dx$, but most of them could not manipulate $\int \sin^3 x \, dx$.

In Part (a) (ii), the majority of candidates knew how to substitute the values into their answer from Part (a) (i) above and easily obtained full marks.

In Part (b), it was evident that many candidates did not understand what was meant by *four intervals*. Several candidates interpreted four intervals as four ordinates instead of four trapezia. Hence, they used $n = 3$. In other cases, candidates used $n = 5$ instead of $n = 4$. This resulted in a variety of incorrect responses. Further, most candidates did know what to do when the curve went below the x -axis. They did not take the modulus of the part of the curve that was below the x -axis, that is, $|f(-0.75)| = |-0.5625| = 0.5625$. Consequently, their responses to the problem were often smaller than the expected area between the curve and the x -axis.

The two common problems which arose in Part (c) were candidates not recognizing that partial fractions was not required and not competently dealing with irreducible quadratic factors and repeated roots. The vast majority therefore missed out on the opportunity to solve the problem easily by working on the right-hand side. Many candidates did not get the basic form of the expansion correct. Some did not recognize that $(x^2 + 4)$ is a factor of $(x^2 + 4)^2$. They made their denominator $(x^2 + 4)(x^2 + 4)^2$ which made the question more complicated and left them unable to complete the solution.

In Part (c) (ii), candidates experienced several difficulties. Many did not know how to use the substitution $x = 2\tan\theta$. Some candidates determined $\frac{1}{2} \int \frac{1}{\sec^2\theta} d\theta$, but could not go any further. Those who were able to manipulate the trigonometric function and integrate it to obtain $-\frac{1}{8}\sin 2\theta + \frac{3}{4}\theta + C$ experienced great difficulty changing the variable in $\sin 2\theta$ back to x . They did not realize that they could have used the identity $\sin 2\theta = 2 \sin \theta \cos \theta$ and a right-angled triangle to get their final answer in terms of x .

Solutions

- (a) (i) $-\frac{2\cos^3 x}{3} + \cos x + c$ or $-\frac{1}{6}\cos 3x + \frac{1}{2}\cos x + c$ (ii) $-\frac{1}{3}$
 (b) 4.22 square units
 (c) (ii) $\frac{3}{4}\tan^{-1}\left(\frac{x}{2}\right) - \frac{1}{2}\left(\frac{x}{x^2+4}\right) + c$

Section B

Module 2: Sequences, Series and Approximation

Question 3

Specific Objective(s): (a) 2–4; (b) 3, 6, 7, 9

This question tested the concepts of mathematical induction, telescoping and the Taylor series.

In Part (a), it was very clear that most candidates lacked understanding of the process of induction since they were unable to deduce what was to be proved. Many candidates attempted to prove the recurrence relation via mathematical induction, but they ignored the inequality. For those who recognized the inequality as the induction hypothesis and proved the base case, many found it difficult to carry out the inductive procedure.

Part (b) was generally well done. In Part (i) (b), most candidates applied the method of differences in the summation of a series although a few candidates opted to use mathematical induction to prove the equality with limited success. In Part (b) (ii), some candidates saw the

connection to Part (b) (i) (b) and were able to complete the solution competently. However, in many cases the correct limit notation was not used.

Overall, candidates performed below expectations in Part (c) (i). Several candidates started correctly but experienced difficulty completing the series. Some candidates did not differentiate sine and cosine functions correctly while others confused Taylor with Maclaurin's series. In Part (c) (ii), candidates saw the connection with the solution of Part (c) (i) and attempted to substitute $\frac{\pi}{16}$ into their solution. However, many equated $(x - \frac{\pi}{4})$ to $\frac{\pi}{16}$ and used $x = \frac{5\pi}{16}$ instead. Others failed to arrive at the correct solution because of faulty algebraic manipulation and arithmetic errors.

Solutions

(c) (i) $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)^2 + \frac{\sqrt{2}}{12}\left(x - \frac{\pi}{4}\right)^3$
 (ii) 0.977

Question 4

Specific Objective(s): (c) 1–4; (d) 1, 2, 6

This question examined the use of the binomial theorem to approximate a surd and to compute a coefficient of a term in an expansion; the Intermediate Value Theorem in testing for the existence of a root in an equation; and the ‘interval bisection’ method in finding successive approximations to a root in an equation.

Candidates generally found Part (a) (i) manageable and showed good understanding of the binomial expansion. However, a significant number of candidates simplified $\sqrt[4]{(1+x)} + \sqrt[4]{(1-x)}$ incorrectly as $(1+x)^2 + (1-x)^2$. Some candidates also had difficulty obtaining the correct binomial coefficients due to arithmetical errors, particularly the signs of coefficients.

In Part (a) (ii), most candidates substituted $x = \frac{1}{16}$ into the binomial expansion rather than first substituting $x = \frac{1}{16}$ into $\sqrt[4]{(1+x)} + \sqrt[4]{(1-x)}$ in order to determine how to modify the expansion to give the desired result. Simplifying surds also continues to be an area of difficulty for candidates. For example, several candidates wrote $\sqrt[4]{\frac{17}{16}} = \frac{\sqrt[4]{17}}{\sqrt[4]{16}} = \frac{\sqrt[4]{17}}{4}$, which is incorrect.

Part (b) examined candidates' ability to extract the coefficient of x^5 from a binomial expansion. Most candidates opted to expand both expressions first then multiply them. Some deviated from this and only wrote down the terms that had the desired power of x . On the

other hand, certain candidates used more advanced approaches. They were able to apply the binomial expansion to the product of the two binomials and obtain the correct coefficient of x^5 . In Part (b) (ii), most candidates appeared to be unfamiliar with interval bisection and continued to calculate midpoints using $b_n = 3$ or found an approximation using the Newton–Raphson method. Most candidates who demonstrated some knowledge of the topic produced at least two successive iterations. However, one commonly observed problem was the incorrect application of the stopping criterion. Interval bisection is a geometrical approach to finding a root; therefore, the use of diagrams in the teaching this topic should be encouraged in order to strengthen the responses in this area.

Solutions

- (a) (i) $2 - \frac{3}{16}x^2$ (ii) 3.9986
- (c) (i) $f(2) < 0; f(3) > 0$ hence continuity (ii) 2.92

Section C

Module 3: Counting, Matrices and Differential Equations

Question 5

Objectives: (a) 2, 4, 6, 16, 17, (b) 4–6.

In Part (a) (i), although the majority of the candidates was able to provide the required solution, some candidates did not know how to construct a tree diagram. In many cases, candidates used the actual letter of the word BRIDGE to show the outcomes rather than classifying the outcomes into vowel and consonant before recording the outcome on the tree. They were then unable to determine the required probability. In some cases although candidates correctly determined the number of outcomes, they were unable to calculate the associated probabilities.

In Part (b), a few candidates did not understand the use of the word *system* or what was meant by a system of equations having a "unique" solution. This question did not restrict candidates to using the row reduction approach and as a result, some candidates used the determinant method to arrive at their conclusion.

Performance on Part (c) (i) was unsatisfactory. Most candidates did not seem to understand how to move from the conditional probabilities given to the total probability required. Several candidates gave values greater than 1 which indicated that they lacked understanding of the basic concept of a probability. Some also used the values of 45 per cent, 30 per cent and 25

per cent and not the fractional form in their calculations. Consequently, they obtained incorrect responses.

Solutions

- (b) (i) The system is not consistent since we have $0x + 0y + 0z = 9$. (*The justification will depend on how the reduction is executed.*)

(ii) The solution is unique. The matrix can be reduced to

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{4}{3} \\ 0 & 1 & 0 & \frac{25}{6} \\ 0 & 0 & 1 & \frac{9}{2} \end{array} \right).$$

Since only the leading diagonal elements are non-zero, the solution is unique.

Alternatively, $|A|=12$ which is not 0. The system of equations can therefore be solved.

- (c) (i) 0.7975 (ii) 0.2665

Question 6

Objectives: (c) 2, 3 (c)

The first part of this question appeared to be the most difficult for candidates who were unable to determine the correct integrating factor and thus failed to arrive at the correct integral. In Part (a) (ii), several candidates used degrees instead of radians in their solutions. They were, therefore, unable to use the cancellation to assist in finding the answer. The given value of $y = \frac{15\sqrt{2}\pi}{32}$ was also not interpreted correctly by some candidates resulting in incorrect constants of integration.

In Part (b), most candidates appeared to know the appropriate form of the complementary function given the auxiliary equation. However, several candidates did not use the quadratic formula correctly to evaluate and many were unable to evaluate $\sqrt{-16}$. Most candidates who attempted Part (b) (iii) were able to find the particular solution correctly although some provided the general solution. Candidates were awarded the marks for either of these solutions.

Solutions

(a) (i) $y = x^2 \cos x + C \cos x$ (ii) $C = \frac{7\pi^2}{8}$

(b) (i) (a) $\lambda = -1 \pm 2i$ (b) $u_p(t) = C_1 e^{-t} \sin 2t + C_2 e^{-t} \cos 2t$

$$(iii) \quad y(t) = 0.981e^{-t} \sin 2t + 0.255e^{-t} \cos 2t - \frac{16}{17} \cos 2t + \frac{4}{17} \sin 2t$$

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Complex Numbers and Calculus II

Question 1

Objectives: (a) 4–6, 9; (b) 3, 4, 8; (c) 4, 5, 7, 8, 10

In Part (a), it was evident that candidates were unable to interpret the partial derivatives required. Similarly, in Part (b), most candidates were unable to choose appropriate expressions for the integration by parts. Consequently, they were unable to solve the problem. Those who were able to begin the integration by parts ended up with expressions containing incorrect signs. More exposure to the reduction formula is recommended.

In Part (c), most candidates were able to generate the simultaneous equations needed to solve for the square root. However, they experienced difficulty recognizing that the resulting equation was a quadratic equation in x^2 .

Solutions

- (a) 0.16% approximately
- (c) $z^{\sqrt{2}} = 1.453 + 0.344i$ and $z^{\sqrt{2}} = -1.453 - 0.344i$

Section B

Module 2: Sequences, Series and Approximation

Question 2

Specific Objective(s): (b) 1, 2, 4, 6, 8; (c) 4

This question examined the use of binomial theorem to approximate a decimal number; Maclaurin and geometric series.

In Part (a) (i), a significant number of candidates did not express $\frac{1}{2}x$ in brackets. As a result, they raised only x to the various powers instead of the entire expression $\frac{1}{2}x$. In Part (a) (ii), some candidates substituted $x = 1.377$ into their expansion rather than stating that $1 + \frac{1}{2}x = 1.377$ to find x as 0.754.

In Part (b), quite a few candidates simply copied the Maclaurin expansion from the formula booklet instead of deriving it as the question required and were unable to find the derivatives of the logarithmic function given. Of the few who recognized the general term, some did not use the sigma notation and others could not derive the appropriate sign for the terms of the sum.

In Part (c) (ii), most candidates were unable to show that $S_2 < 4$. Some found it difficult to work with the algebraic expressions as the terms of the series while others could not manage the reasoning required to complete the proof.

Solutions

- (a) (ii) 3.60
- (b) (i) $x - \frac{1}{2}x^2 + \frac{1}{3}x^3$ (ii) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k}$
- (c) (i) $-2 < x < 2$

Section C

Module 3: Counting, Matrices and Differential Equations

Question 3

Objectives: (a) 2, 3, 6; (b) 2, 7, 8

In general, candidates knew how to find the determinant of the matrix in Part (a) and demonstrated the ability to multiply matrices. However, many did not write out the 3×3 zero matrix. Instead, they simply wrote 0.

Most candidates did not recognize the link between the equation in Part (a) (ii) and the inverse of the matrix and instead attempted to use row reduction to find the inverse. Similarly, row reduction was used to solve the simultaneous equations. This approach required much more work and many computational errors were made.

In Part (b), the majority of candidates were unable to recognize that the problem involved permutations and used the $\binom{n}{r}$ format, instead of 6P_3 . Further, the concept of probability was not understood.

Solutions

$$(a) \quad (i) 18 \quad (ii) b) \quad A^{-1} = \frac{1}{18} C^T = \frac{1}{18} \begin{bmatrix} 8 & 4 & 2 \\ 7 & -1 & -5 \\ -3 & 3 & -3 \end{bmatrix}$$

$$(ii) (c) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

$$(b) \quad (i) 120 \quad (ii) 0.8$$

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

**REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®**

MAY/JUNE 2014

PURE MATHEMATICS

GENERAL COMMENTS

In 2014, approximately 5 312 and 2 909 candidates wrote the Unit 1 and 2 examinations respectively. Overall, the performance of candidates in both units was consistent with performance in 2013. In Unit 1, 70 per cent of the candidates achieved acceptable grades compared with 72 per cent in 2013; while in Unit 2, 85 per cent of the candidates achieved acceptable grades compared with 81 per cent in 2013. Candidates continue to experience challenges with algebraic manipulation, reasoning skills and analytic approaches to problem solving.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

The paper comprised 45 items, 15 items per module. Most candidates performed satisfactorily. Marks on this paper ranged from a minimum of 6 to a maximum of 90. The mean mark for the paper was 55.46 per cent.

Paper 02 – Structured Questions

The paper consisted of six compulsory questions, two questions per module. The maximum score was 149 out of 150. The mean score was 50.52.

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (a) (A) 1, 2, (B) 1, 2, 6, (C) 2

The topics tested in this question included the use of truth tables, binary operations, proof by mathematical induction and the factor theorem. Overall, candidates demonstrated competence in this question with approximately 98 per cent of them attempting it and obtaining a range of 9 to 12 marks. A few candidates were also able to obtain the maximum score.

In answering Part (a), candidates used a variety of styles to represent the inputs and outputs such as *1 and 0* and *T and F*. A number of candidates used only four propositions for p and q with no illustration of the proposition r in constructing the truth table. Generally, the results of *implication* were not well done. However, based on the candidates' responses, marks were awarded for the *conjunction*.

Part (b) (i) required candidates to give a reason for determining whether a binary operation is commutative. Generally, candidates stated a correct or incorrect result without giving a reason. Responses varied with candidates explaining the properties of commutative law, reasoning that since $y \oplus x = x \oplus y$ then \oplus is commutative or that since $-5y - 5x \neq -5x - 5y$ then \oplus is not commutative. No response of $-5(y + x) = -5(x + y)$ was given. A few candidates substituted real numbers for x and y to state a result.

Part (b) (ii) a) was well done by most candidates. A few candidates used the operation as $2 \oplus x = 2 \oplus 1$. However, poor algebraic substitution resulted in obtaining an incorrect cubic equation. Either by long division, or otherwise, many candidates were unable to obtain the other two linear factors correctly. Some candidates left their answers in the form $f(x) = (x - 1)Q(x)$ where $Q(x)$ is a quadratic in x .

Mathematical induction, as tested in Part (c), continues to be challenging for many candidates although the first phase of proving the statement P_n for $n = 1$ and $n = 2$ was generally well done. Many candidates do not state the assumption for the statement P_k following the proof for $n = 1$ and $n = 2$. The inductive steps required to show the algebraic expression for the $(k + 1)^{th}$ term, thus establishing the proof for P_k and subsequently for P_n , was generally poorly done.

Solutions

(a) (i) and (ii)

p	q	r	$p \rightarrow q$	$r \rightarrow q$	$(p \rightarrow q) \wedge (r \rightarrow q)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

(b) (i) $(y^3 + x^3) + a(y^2 + x^2) - 5(y + x) + 16 = (x^3 + y^3) + a(x^2 + y^2) - 5(x + y) + 16$

Since addition is commutative for x, y , \oplus is commutative

(ii) a) $a = -2$

b) $f(x) = (x - 1)(x - 3)(x + 2)$

Recommendations

Teachers are advised to use truth tables extensively for more than two propositions. The correct number of rows, 2^n , (where n is the number of propositions) should be understood. The correct layout of the rows of the truth tables is important for subsequent connectives and conclusion of truth values. Rules on propositional calculus should be understood.

Students must understand the concepts of *identity*, *closure*, *inverse*, *commutativity*, *associativity*, *distributivity* and other simple binary operations. Reasons for conclusions must be based on these concepts. Tabular presentations of binary operations and algebraic expressions must be given equal attention in teaching this topic.

The steps required for proof by Mathematical Induction are not followed with the sophistry required. The concepts of *the inductive process* are not fully understood by most students. Rigid teaching of these processes must be employed. The algebra required to simplify the $P_{(k+1)}$ form of the statement $P_{(k)}$ is generally quoted and not shown by algebraic manipulation. Students must be able to demonstrate their abilities to simplify the algebra that results in the required form. Please see Example 2: Page 14 of the syllabus.

Question 2

Specific Objectives: (B) 3, (D) 6, 7, (E) 1, 4

This question tested the concepts of *composite functions*, the use of the law of logarithms to simplify algebraic expressions, the solution of equations involving \log_e and \log_a and the simplification of surds. The question was attempted by approximately 90 per cent of the candidates with varied levels of responses.

Part (a) (i) a) required determining $f^2(x)$ where f is a polynomial in x . A significant number of candidates interpreted $f^2(x)$ as $[f(x)]^2$ and $f(x^2)$ instead of $f[f(x)]$.

In Part (a) (i) b), some candidates had difficulties simplifying $2\left[\sqrt{\frac{x-1}{2}}\right]^2 + 1 = x$. In a

few cases candidates found $gf(x)$. In stating the relationship between $f(x)$ and $g(x)$, candidates used descriptions such as injective, surjective, bijective and directly or inversely proportional.

Candidates' responses to Part (b) was generally poor. In some cases candidates attempted to use the law of logs to simplify the right hand side and found it challenging to express the left hand side in terms of logarithms. Conversely, those candidates who attempted to show the expression given found the algebra beyond their capacity. It was clear that most of the

candidates were unable to use the expression given as an aid to simplifying the problem and to show the required solution.

Approximately 99 per cent of candidates attempted Part (c) (i). Common errors included incorrectly factorizing the quadratic equation in terms of the variable substituted for e^x and not stating the value of $x = 0$ but instead $x = \ln 1$.

Part (c) (ii) was challenging for many candidates. Lack of knowledge of the laws of logarithms was evident. Errors included:

$$\log_2(x+1) - \log_2(3x+1) = 2 \Rightarrow \frac{x-1}{3x+1} = 2,$$

$$\log_2 x + \log_2 1 - \log_2 3x - \log_2 1 = 2$$

and other variations.

Approximately 98 per cent of candidates attempted Part (d). Generally, candidates demonstrated knowledge of the concept of *rationalizing surds*. Some candidates were unable to show the required answer due to poor multiplication of terms including surds.

Solutions

$$(a) (i) a) f^2(x) = f[f(x)] = 2(2x^2 + 1) + 1 = 8x^4 + 8x^2 + 3$$

$$b) f[g(x)] = 2\left[\sqrt{\left(\frac{x-1}{2}\right)}\right]^2 + 1 = x$$

(ii) f and g are inverse functions

(b)

$$3 \log\left(\frac{a+b}{2}\right) = \log\left(\frac{a+b}{2}\right)^3 = \log\left(\frac{a^3 + 3a^2b + 3ab^2 + b^3}{8}\right) = \log(ab^2) = \log a + 2 \log b$$

$$(c) (i) e^x + \frac{1}{e^x} - 2 = 0 \Rightarrow e^{2x} - 2e^2 + 1 = 0 \Rightarrow (e^x - 1)^2 = 0 \Rightarrow x = 0$$

$$(ii) \log_2\left(\frac{x+1}{3x+1}\right) = 2 \log_2 2 \Rightarrow \frac{x+1}{3x+1} = 4 \Rightarrow x = -\frac{3}{11}$$

Recommendations

It is recommended that teachers use different notations for composite functions. These may include $f^2(x) = ff(x) = f[f(x)] = f \circ f = f(x) \circ f(x)$. Further, it is important to emphasize that it is not necessarily true that $fg(x) = g(f(x))$.

More application of the laws of logarithms should be done. Properties of $f(x) = e^x$ and $\ln(x)$ must be fully understood, particularly, if given $f(x) = e^x$ then $f^{-1}(x) = \ln(x)$.

Section B

Module 2: Trigonometry, Geometry and Vectors

Question 3

Specific Objectives: (A) 1, 2, 5, 7, 8

This question tested trigonometric identities of reciprocal angles; the compound angle formulae; the solution of trigonometric equations involving reciprocal and compound angles; expressing $a \cos 2\theta + b \sin 2\theta$ in the form $r \sin(2\theta + \alpha)$; and determining maximum and minimum values of trigonometric expressions.

In Part (a) (i), approximately 80 per cent of candidates recognized the need to express $\cot x$ and $\cot y$ in terms of $\frac{\cos x}{\sin x}$ and $\frac{\cos y}{\sin y}$ respectively. However, a large number of candidates failed to expand $\frac{\sin(x-y)}{\sin(x+y)}$ and could not readily link or simplify the two sides of the equation, thus proving the identity. Part (a) (ii) was generally poorly done. The majority of candidates who attempted this part of the question merely stated $\frac{\sin(x-y)}{\sin(x+y)} = 1$ and could not follow through with the simplification and subsequent solution. Many of those candidates who followed through beyond this step did not make use of the given condition $\sin x = \frac{1}{2}$. In some cases candidates expressed $\sin(x \pm y)$ as $\sin x \pm \sin y$.

The majority of candidates attempted Part (b) (i) with a few making the error of calculating $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$ instead of $\tan^{-1}\left(\frac{4}{3}\right)$. Further, some candidates gave α as degrees and not radians. Part (b) (ii) a) was poorly done. Approximately half of the candidates who attempted this part of the question knew that $-1 \leq \sin \theta \leq 1$ but could not deduce that

$-r \leq r \sin(2\theta + \alpha) \leq r$. Many candidates failed to use the range given for θ and this resulted in their value of θ being incorrect.

Candidates who attempted Part (b) (ii) b) found the values $\frac{1}{12}$ and $\frac{1}{2}$ but could not distinguish the minimum and maximum values.

Solutions

(a) (ii) $y = 0, \pi, 2\pi$

(b) (i) $r \sin(2\theta + \alpha) = 5 \sin(2\theta + 0.927^\circ)$

(b) (ii) a) $\theta = 1.89^\circ$

b) minimum = $\frac{1}{12}$ and maximum = $\frac{1}{2}$

Recommendations

Proofs of trigonometric identities should involve expressing reciprocal ratios in terms of sine and cosine. Generally, either the left hand side (LHS) or the right hand side (RHS) of an equation is simplified but in some cases both sides require simplification. Candidates must be able to choose the most suitable formulae or ratios to simplify expressions. This is best achieved by repeated practice.

The concepts of *minimum and maximum of reciprocal functions* are related to inequalities. Candidates will better understand these concepts using the properties of reciprocals.

Question 4

Specific Objectives: Content (B) 1, (B) 4, 5, 6, (C) 1, 6, 7

This question tested geometry of the circle; the locus of a point satisfying given properties; the Cartesian equation of a curve given its parametric representation; three-dimensional vectors; the use of the modulus and the scalar product of three-dimensional vectors.

Part (a) (i) was generally well done. Some candidates substituted the coordinates given to show that the required answer is correct. In Part (a) (ii), approximately half of the candidates used the formula for the midpoint of a line between two points to deduce the correct coordinates. Some candidates determined the equation of the circle and found the point of intersection of the circle with L_1 . This entailed much more work and time.

A significant number of candidates recognized that the locus of p in Part (a) (iii) is a circle since the distance from the given point is fixed. This part of the question was well done.

Part (b) was poorly done. Most candidates who expressed t in terms of x were able to substitute correctly for t in y . However, poor algebraic manipulation by the majority of candidates resulted in challenges simplifying the expression of y in terms of x .

Part (c) (i) was generally well done. A small number of candidates calculated $\overrightarrow{PQ} = \vec{P} + \vec{Q}$ and $\overrightarrow{P} - \vec{Q}$.

Most of the candidates who attempted Part (c) (ii) used the scalar product since the property of perpendicular vectors was given in the problem. Some candidates used the Pythagorean method for a right-angled triangle.

Solutions

(a) (ii) B (3, 4) (iii) $(x - 2)^2 + (y - 3)^2 = 2$ circle: centre (2, 3) radius = $\sqrt{2}$ units.

(b) $x^2 + 2xy - x - y = 0$

(c) (i) $\overrightarrow{PQ} = -4\mathbf{i} + (\lambda + 2)\mathbf{j} + 4\mathbf{k}$ $\overrightarrow{QR} = 3\mathbf{i} + (1 - \lambda)\mathbf{j} - 9\mathbf{k}$ $\overrightarrow{RP} = \mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$

(ii) $\lambda = 15$

Recommendations

Expressing a curve given by parametric equations into Cartesian form and vice versa requires good algebraic skills and knowledge of trigonometric identities. Eliminating the given parameter will require proper algebraic substitution and simplification. Teachers are advised to use a wide variety of expressions in x and y given by parametric equations and which can be expressed in Cartesian form using algebraic manipulation. The same applies for determining the parametric equations for curves expressed in Cartesian form.

The concepts of an *angle between vectors*, *modulus of vectors* and *applications of vectors in geometry* are basic fundamentals for use in problem solving. These concepts must be thoroughly understood by students. Teachers are advised to ensure that these topics are covered comprehensively.

Section C

Module 3: Calculus I

Question 5

Specific Objectives: (A) 4, 8, 10, (B) 2 (b), 4 (a), 5 (b),

This question tested the use of simple limits theorems; identification of a point for which a function is continuous; differentiation using first principles, the quotient rule and parametric differentiation.

The majority of candidates attempted the question with fairly satisfactory responses. A small number of candidates obtained full marks and simple arithmetical errors were responsible for a few candidates not obtaining full marks.

Part (a) (ii) was generally understood by most candidates. However, poor substitution and algebraic skills continue to pose difficulties for a number of candidates.

Responses to Part (b) (i) were generally poor. Candidates demonstrated understanding of the concepts of *differentiation from first principles* and used the correct identity to simplify the expression $\frac{f(x+h) - f(x)}{h}$. However, poor algebraic skills particularly rationalizing surds,

hindered further work in this regard. In other cases, the steps required were poorly set out giving rise to very untidy presentations. Candidates used either the quotient rule or the product rule for completing Part (b) (ii). The concepts were fully understood but poor algebraic skills resulted in incorrect answers. Some candidates did not simplify the answer fully.

The major challenge seen in Part (c) was candidates' inability to apply the chain rule for parametric differentiation. Some candidates obtained the Cartesian equation of the curve but could not use it effectively to find $\frac{dy}{dx}$.

Solutions

$$(a) \quad (i) \quad a = \frac{1}{3} \quad (ii) \quad b = 7$$

$$(b) \quad (i) \quad \frac{dy}{dx} = \frac{1}{2x\sqrt{x}} \quad (ii) \quad \frac{dy}{dx} = \frac{x+2}{2\sqrt{(x+1)^3}}$$

$$(c) \quad \frac{dy}{dx} = -\cot \theta$$

Recommendations

Candidates must fully understand the concepts of *left-handed limits*, *right-handed limits* and *limit at a point*. Further, the concepts of *continuity* and *discontinuity* are very important when determining limits. It must be emphasized that a derivative at a point is the limit of the rate of change as the change approaches that point. Teachers are advised to allow students to determine the limits at a point using an intuitive approach and to confirm the results using a graphical method or the calculator.

Students should be exposed to differentiation of simple expressions from first principles

noting that the derivative of $f(x) = f'(x) = \lim_{\delta x \rightarrow 0} \frac{f(x + \delta x) - f(x)}{(x + \delta x) - x}$. The use of the chain rule

must be applied when differentiating composite functions and parametric equations.

Question 6

Specific Objectives: (B) 8, 11, 12, (C) 1, 2, 4, 5 (b), 6, 7 (c), 8 (c)

This question tested definite integration given boundary conditions; the theorem of the integral of sums being equal to the sum of integrals; the minimum and maximum stationary points of a cubic curve; definite integration using substitution; and the volume of a solid generated by rotating part of a polynomial curve about the x -axis.

Part (a) (i) a required candidates to find the equation of a curve given boundary conditions. Most candidates performed satisfactorily although a few made arithmetical errors in the calculation of the constant of integration. In Part (a) (i) b), common errors included using the wrong substitution to find the coordinates of y and incorrectly distinguishing between minimum and maximum stationary points. The use of the second derivative to distinguish the nature of stationary points was amply demonstrated. Part (a) (ii) was generally well done. Minor errors included not showing clearly the coordinates of the stationary points and the intercepts of the coordinate axes.

Many candidates demonstrated a good understanding of integration using substitution as required to answer Part (b) (i). Poor algebraic manipulation resulted in some candidates not obtaining the correct function to complete the integration. A few candidates did not effect a change of the limits given for x to the limits for the variable taken for substitution.

Part (b) (ii) required candidates to find the volume of the solid formed when the curve defined was rotated completely about the x -axis. A significant number of candidates quoted an incorrect formula for the volume thus being unable to integrate the function easily.

Solutions

(a) (i) a) $y = x^3 - 2x^2 + x$

b) $\left(\frac{1}{3}, \frac{4}{27}\right)_{\max} \quad (1, 0)_{\min}$

(b) (i) $\frac{2}{3}(\sqrt{10^3} - 1)$

(ii) $\text{Vol} = \frac{544\pi}{15} \text{ units}^3$

Recommendations

Teachers must emphasize the importance of including the constant of integration when completing indefinite integration. Integration by substitution requires extensive reinforcement to ensure that students fully understand the processes involved. Instances are seen where students merely substitute for the variable without expressing dx in terms of du if u is the substitution given for x . The resulting function to be integrated becomes a mix of the variable x and the variable u . In addition, no changes are made to the limits given in terms of x .

Students also require practice in finding areas and volumes of regions enclosed by curves and the coordinate axes, curves enclosed by $x = a$ and $x = b$ and two curves.

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (B) 2, F 3, G

This question required candidates to use a given table of a binary operation to determine whether the operation is commutative; name the identity element of a binary operation and determine the inverse of two elements of a set under a binary operation.

Generally, most candidates did not successfully answer this question. They appeared to be unfamiliar with binary operations, as well as the concepts of the *identity* and the *inverse*.

Candidates knew the concepts of *the sum of roots*, *the sum of products of roots pairwise* and *the product of roots of a cubic equation*. However, the level of algebra required to complete Parts (b) (i) a) and b) was beyond their capacity. Consequently, they were unable to complete Part (b) (ii) successfully.

In Part (c) (i), candidates experienced difficulties sketching a modulus graph given a rational function in x . In Part (c) (ii), failure to sketch the correct graphs in (i) resulted in many candidates being unable to solve the equation. The ‘otherwise’ approach seemed out of their scope.

Solutions

(a) (i) * is not commutative

(ii) identity element is e

(iii) a) inverse of d is f

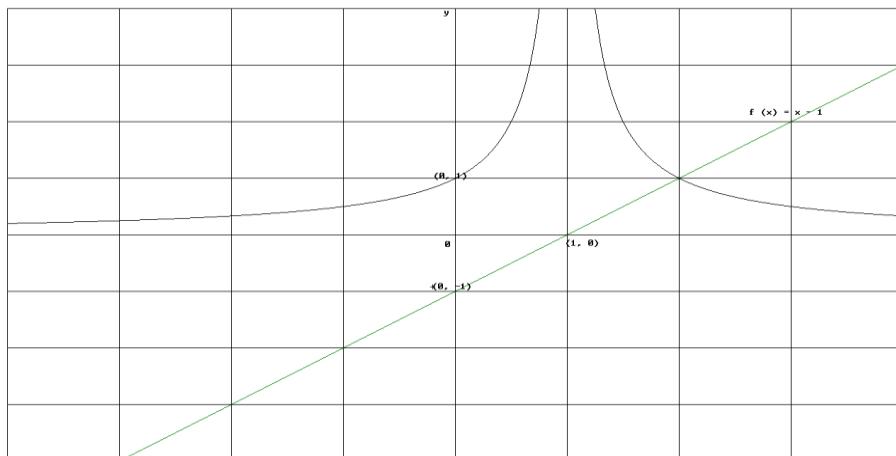
b) inverse of c is c

(b) (i) a) 17

b) 4

$$(ii) \quad x^3 - 17x^2 + 4x - 4 = 0$$

(c) (i)



$$(ii) \quad x = 2$$

Section B

Module 2: Trigonometry, Geometry and Vectors

Question 2

Specific Objectives: (A) 1, (C) 3, 4, 5, 6, 7

This question tested the derivation of displacement vectors, the modulus of vectors, the angle between two vectors and the use of the compound angle to verify the exact value of a trigonometric ratio.

Parts (a) (i) and (ii) were generally well done. However, candidates appeared unsure how to apply the previous results to find the area of the triangle. They experienced challenges determining the height to use in the formula $A = \frac{1}{2}bh$ or could not apply the formula

$$A = \frac{1}{2}ab \sin C .$$

In Part (b), despite being given the compound angle to use for the required answer, many candidates were not able to reason and use the compound formula for $\tan(A - B)$.

Solutions

(a) (i) $\overrightarrow{PQ} = -2\mathbf{i} - \mathbf{j} + \mathbf{k}$ $\overrightarrow{PR} = 3\mathbf{k}$

(ii) a) $|\overrightarrow{PQ}| = \sqrt{6}$ $|\overrightarrow{PR}| = 3$

b) $\cos \theta = \frac{1}{\sqrt{6}}$

c) area = $\frac{3}{2}\sqrt{5}$ units²

Section C

Module 3: Calculus I

Question 3

Specific Objectives: (A) 4, 5, 6, (B) 1, 14, (C) 8 (b)

This question tested the evaluation of a limit using simple limit theorems; the gradient of a curve at a given point; the equation of a normal to a curve at a given point and the area of a finite region enclosed by two curves.

Part (a) required candidates to use simple limit theorems and the fact that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ to find $\lim_{x \rightarrow 0} \frac{\sin 8x}{2x}$. Very few candidates gave any evidence of the requisite knowledge of this topic and performance was generally unsatisfactory. Except for minor algebraic and arithmetic errors, Parts (b) (i) and (ii) were done satisfactorily. The majority of candidates was unable to respond to Part (c).

Solutions

(a) 4

(b) (i) $x = \frac{1}{3}$ $y = \frac{58}{27}$ and $x = -1$ $y = 2$

(ii) $x + y - 1 = 0$

(c) $\frac{1}{3}$ units²

UNIT 2

Paper 01 – Multiple Choice

The paper comprised 45 items, 15 items per module. Most candidates performed satisfactorily. Marks on this paper ranged from a minimum of 10 to a maximum of 90. The mean mark for the paper was 61.48 per cent.

Paper 02 – Structured Questions

The paper consisted of six compulsory questions, two questions per module. The maximum score was 145 out of 150. The mean score was 51.33.

Section A

Module 1: Complex Numbers and Calculus II

Question 1

Specific Objectives: (A) 1, 2, 3, 12, 13, (B) 2, 3, 6

This question tested differentiation of $\ln f(x)$ and inverse trigonometric functions, using parametric differentiation where the parameter was given as a trigonometric ratio to find a tangent to the curve; the existence of complex roots of a quadratic equation; the use of de Moivre's theorem and the exponential form of a complex number.

Overall, performance on this question was unsatisfactory with approximately two-thirds of candidates earning less than 10 of the 25 marks available and a significant number scoring no marks. However, Part (a) (i) was generally well done by almost all candidates. The use of the chain rule for differentiation and subsequent simplification was successfully done by a small number of candidates. Some candidates found it difficult to work with the coordinates (x, y) to find the equation of the tangent. Limited skills in performing algebraic manipulations continue to be a challenge for candidates.

Almost all candidates seemed well prepared for Part (b) (i) and this part was generally well done. Most candidates successfully used the discriminant to determine the nature of the roots but a few failed to state the nature of the roots.

Part (b) (ii) was well done by the majority of candidates. Some candidates had difficulties stating the correct argument within the required range. In Part (b) (iii), less than half of the candidates demonstrated their ability to use de Moivre's theorem. Those candidates who got the wrong values for the sum required made arithmetic errors. A number of candidates opted

to use the expansion of $(\alpha + \beta)^3$ in terms of $\alpha + \beta$ and $\alpha\beta$ with $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$. This was done with varying success. In Part (b) (iv), the majority of candidates was familiar with the procedures involved and their responses were satisfactory.

Solutions

(a) (i) $\frac{dy}{dx} = -\tan^{-1}\left(\frac{x}{2}\right)$

(b) (i) Roots are complex

(ii) $\alpha = 3e^{i(2\pi/3)}$ $\beta = 3e^{i(-2\pi/3)}$

(iii) 54

(iv) $x^2 - 54x + 729 = 0$

Recommendations

More emphasis must be placed on the differentiation of composite functions using the chain rule. Teachers are advised to demonstrate as many applications of de Moivre's theorem as possible. These applications should include simplifying $(a + bi)^n$ where $a + bi$ is a complex number and n is real and proof of trigonometric identities.

Question 2

Specific Objectives: (C) 1 (d), 3, 5, 7, 10

This question tested the decomposition into partial fractions of a rational function with a denominator of repeated quadratic factors; definite integration of the rational function and the derivation and use of a reduction formulae to obtain a definite integral.

The formula derived in Part (a) (i) was effectively used by some of the candidates who cascaded the formula to find $F_{(3)}(2)$. However, a number of arithmetical errors were made by candidates who attempted to use the derived formula linearly. In many cases, the succeeding terms were incorrectly factored and arithmetically calculated. A few candidates attempted integration by parts after failing to obtain the reduction formula and not being able to use it effectively. However, the successive integration involved was beyond their capacity.

Part (b) (i) required candidates to decompose into partial fractions a rational function whose denominator included repeated quadratic factors. Approximately 50 per cent of candidates

obtained full marks. Generally, candidates appeared to know the concept of *decomposition*. However, poor algebraic manipulation hindered their efforts to find the correct values of the coefficients and constants in the numerators. Incorrect forms of the numerators were some of the errors made.

Candidates followed through with Part (b) (ii) using the partial fractions given in Part (b) (i).

The majority of the candidates successfully integrated $\int \frac{1}{y^2 + 1} dy$ but found $\int \frac{2y}{(y^2 + 1)^2} dy$ challenging. In addition, candidates were unable to use the substitution $u = y^2 + 1$.

Solutions

$$(b) \text{ (ii)} \quad \frac{1}{4}(\pi + 2)$$

Recommendation

Teachers should assist students with developing the skills of integration by substitution by integrating a variety of functions using simple polynomial, exponential and trigonometric substitutions.

Section B

Module 2: Sequences, Series and Approximation

Question 3

Specific Objectives: (B) 5, 6, 8,

This question tested the concepts of *mathematical induction for the sum of a series*, the *convergent sum of an infinite series* and the *use of the Maclaurin series for the expansion of a given series*.

Candidates' performance in Part (a) (i) was generally unsatisfactory. Candidates continue to demonstrate poor techniques when setting out proofs. Apart from the initial steps of proving the assumption of the statement made, the concept of the statement being true for the $(k + 1)^{\text{th}}$ term is not well understood. As a consequence, the algebra to establish the inductive process is poorly handled. In Part (a) (ii), a number of candidates simply stated that the convergent sum of the series was 2 without supporting working. Also, showing $\lim_{n \rightarrow \infty} \frac{1}{2^n} = 0$ was neglected thus denying candidates full marks.

Candidates recorded mixed success in Part (b). Those candidates who attempted to use differentiation for evaluating the coefficients of the required terms found that successive differentiation using the product rule was tedious and fraught with arithmetic and algebraic errors. The product was expressed in forms such as $(1 + 2x + x^2)\sin(x)$ and $\sin(x) + 2x \sin(x) + x^2 \sin(x)$. Candidates who opted to use the formula sheet provided used the expansion $\left(x - \frac{x^3}{6}\right) + x\left(x - \frac{x^3}{6}\right) + x^2\left(x - \frac{x^3}{6}\right)$ and found the series expansion required.

Solutions

(a) (ii) 2

(b) $x + 2x^2 + \frac{5x^3}{6}$

Recommendations

Proof by induction is a sophisticated mathematical process and teachers are encouraged to emphasize the required steps and the algebra needed while teaching the concept.

Maclaurin's series expansion may involve differentiation to determine the coefficients of the polynomial or the use of derived formulae for those functions listed in the Formulae Booklet. As such, candidates should be familiar with all the differentiation skills required for determining coefficients.

Question 4

Specific Objectives: (C) 1, 2, 3, (D) 1, 5

This question tested the use of the binomial theorem to determine specific terms of an expansion; a partial expansion to approximate a numerical value; the use of simple properties of the ${}^n C_r$ notation; the determination of real roots in a given interval and use of the Newton-Raphson method for approximation.

Part (a) (i) was satisfactorily done by the majority of candidates who demonstrated a sound understanding of the binomial theorem in spite of some arithmetic errors. Part (a) (ii) was also well done by the majority of candidates although a significant number of candidates used an incorrect substitution for x in Part (a) (ii) b).

Part (b) required candidates to use the ${}^n C_r$ notation to simplify a sum and prove a given result. Most of the candidates exhibited very limited skills in decomposing $n!$ and $r!$ to simplify a common denominator.

Candidates generally experienced challenges in responding to Part (c) (i). Invariably the Intermediate Value Theorem was not quoted and, more importantly, the fact that $f(x)$ must be continuous in the interval $[a, b]$. Part (c) (ii) was well done with the exception of arithmetic errors made by some candidates.

Solutions

(a) (ii) a $1 + 20x + 180x^2$

b) 1.1045

(c)) (ii) 2.20

Recommendations

Students must understand the definition of ${}^n C_r$. For proofs involving the ${}^n C_r$ notation the definitions and algebraic expressions should be used. Substitution of numbers for n and r are not accepted for proofs. Teachers are recommended to have extensive demonstrations of this strategy.

Section C

Module 3: Counting, Matrices and Differential Equations

Question 5

Objectives: (A) 2, 6, 13, (B) 1, 2, 7

This question tested the number of possible arrangements at a round table; simple probability theory; operations with conformable matrices and inversion of a 3×3 matrix.

In Part (a) (i), candidates were required to determine the number of possible arrangements of persons seated at a round table. From the responses it was clear that the majority of candidates had no knowledge of circular permutation. Further, determining the total number of ways of seating teams of three with the leader in the middle was challenging to most candidates. This part of the question was poorly done. However, Part (a) (ii) was generally well done.

The responses to Part (b) (i) were satisfactory. A number of candidates failed to state the range of values of x . In some cases an inequality was not solved hence distinct values of x were given. In other cases the range was stated incorrectly.

In Part (b) (ii), a few candidates demonstrated poor understanding of what is required when asked to show a result. Those candidates substituted the value of x asked to be shown and worked backwards. Marks were not awarded in those cases.

Part (b) (iii) was fairly well done. There were a few cases where arithmetic errors in calculating the cofactors were seen and there was little evidence of candidates applying the row reduction method to find the inverse of the matrix.

Solutions

(a) (i) $(5 - 1)! \times 2!^5 = 768$

(a) (ii) a) 0.1

(a) (ii) b) 3000

(b) (i) $x \neq \frac{5}{4}$ or $x < \frac{5}{4}$ and $x > \frac{5}{4}$

(b) (iii) $\frac{1}{7} \begin{pmatrix} -2 & -1 & 6 \\ 4 & 2 & -5 \\ 3 & 5 & -9 \end{pmatrix}$

Recommendations

Teachers are advised to expose students to all forms of permutations and combinations. Arrangements involving circular arrangements and beads on a circular ring are particularly important. It is also suggested that complex arrangements be illustrated using a diagrammatic approach for simplicity.

Question 6

Objectives: (C) 1, 2, 3 (b) (a)

Part (a) (i) was poorly done. Most candidates recognized that an integrating factor was required. However, an incorrectly calculated integrating factor was used and the required answer could not be found.

Candidates used the follow through from Part (a) (i) to complete Part (a) (ii). However, poor algebraic substitution and incorrect values for the constant C resulted in many candidates being unable to obtain full marks.

The majority of candidates successfully found the complementary function in Part (b). It was apparent that many candidates were not familiar with second order differential equations structured in this manner. For the small number of candidates who used the particular solution given, errors in differentiation and poor algebraic skills resulted in incorrect values for A and B .

Solutions

(a) (ii) $y = \sin(x) + \cos(x)$

(b) $y = C_1 + C_2 e^{5x} + \frac{1}{10} x^2 e^{5x} - \frac{1}{25} x e^{5x}$

Recommendations

Differential equations of the form $y' + f(x)y = g(x)$ must be recognized as a first order differential equation which can be solved using the integrating factor $e^{\int f(x) dx}$. Extensive tutorials using differentiation of polynomials and trigonometric functions must be done to prepare students for these differential equations.

In cases where the principal integral may be quoted as a general solution, it is recommended that candidates carry out first and second differentials in order to solve the unknown constants. The processes of finding the complementary function and the principal will then be combined for solution of the second order differential equation.

Paper 032 – Alternative to School-Based Assessment (SBA)

Section A

Module 1: Complex Numbers and Calculus II

Question 1

Objectives: (B) 8, (C) 6, 7, 11

This question tested use of the trapezium rule, definite integration of an exponential function using a given substitution and first order partial derivatives.

The majority of candidates who attempted this question did not follow the instruction to use two trapezia and instead used three ordinates to calculate the width, h . In Part (a) (ii), candidates used the substitution $u = e^{-x}$ instead of $u = e^x$ as suggested. Further, the limits were not changed in terms of u and in some cases candidates used $dx = e^{-x}du$ resulting in

trying to evaluate $\int_{-1}^1 \frac{1}{e^x(1+u^{-1})} du$. Poor algebraic manipulation resulted in many candidates

being unable to simplify the fraction in terms of u to allow for simple integration.

In Part (b) (i), many candidates were unable to determine the separate areas and the resulting total area of the box. Partial derivatives seemed unfamiliar to most of the candidates who attempted Part (b) (ii) a). Instead, a few candidates attempted to use implicit differentiation.

In Part (b) (ii) b), since there were no follow through of $\frac{\partial A}{\partial x}$ and $\frac{\partial A}{\partial y}$, candidates were not able

to solve for $\frac{\partial A}{\partial x} = 0$ and $\frac{\partial A}{\partial y} = 0$. Candidates who obtained the partial derivatives correctly substituted the values of x and y to obtain the desired result.

Solutions

(a) (i) 1

(a) (ii) 1

(b) (ii) $\frac{\partial A}{\partial x} = y - \frac{1152}{x^2}$ and $\frac{\partial A}{\partial y} = x - \frac{768}{y^2}$

Section B

Module 2: Sequences, Series and Approximation

Question 2

Specific Objectives: (B) 2, 3, 4, 7, (D) 1, 3

This question tested the sum of a finite series using the *method of differences*; use of the partial sums of an arithmetic series to find a particular term; the existence of a root in a given interval using the Intermediate Value Theorem; and linear interpolation.

Part (a) was attempted by all candidates and was generally well done. Simple algebraic manipulation was sufficient to achieve the required result but some candidates used partial fractions to decompose the sum given to n terms to show the left hand side. A few candidates attempted to use mathematical induction to answer Part (a) (ii) and errors in the procedure resulted in lost marks.

Parts (b) (i) and (ii) were well done. A few arithmetic errors were evident. A few candidates attempted to use the formula for the n^{th} term in Part (b) (i) with no success.

In Part (c) (i), candidates did not make reference to the Intermediate Value Theorem and continuity of the function in the given interval in establishing the existence of a root. Too many candidates merely showed a change of sign without further information.

Many candidates seemed not to know the term *linear interpolation* as required in Part (c) (ii) but those who used it obtained the correct results. Some candidates opted to use the Newton-Raphson method.

Solutions

$$(b) (i) \quad a = \frac{136}{25} \quad d = \frac{82}{25}$$

$$(b) (ii) \quad u_{15} = \frac{1284}{25}$$

$$(c) (ii) \quad 1.614$$

Section C

Module 3: Counting, Matrices and Differential Equations

Question 3

Objectives: (A) 6, 15, 17, (C) 1, 2

This question tested the use of a tree diagram to determine probabilities; the solution of a first order differential equation using an integrating factor; and determining the value of this equation given boundary conditions.

The most satisfactory responses for this question were given for Part (a). However, some candidates appeared unsure when using the tree diagram to determine the probability of the required outcome. They were divided on knowledge of the addition and multiplication rules of probability.

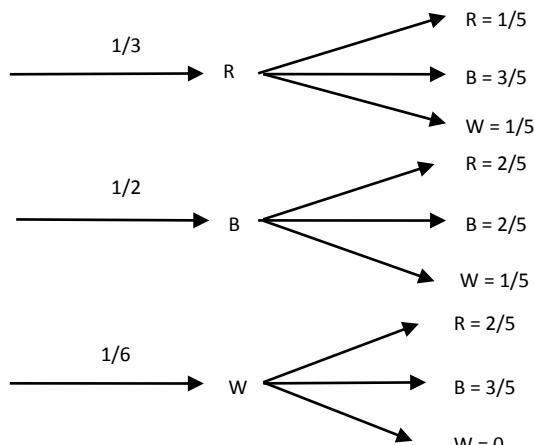
In Part (b) (i), most candidates were able to use the appropriate integrating factor to solve the differential equation. Common errors included incorrectly determining the constant of integration and in some cases omitting the constant of integration. Consequently, the required result could not be shown.

Most candidates did not attempt Part (b) (ii). However, those who attempted to find the limit

$$\text{failed to determine } \lim_{x \rightarrow \infty} e^{-\frac{R}{L}t} = \lim_{x \rightarrow \infty} \frac{1}{e^{\frac{-R}{L}t}} = 0.$$

Solutions

(a) (i)



(a) (ii)

$$\frac{1}{2}$$

(b) (ii)

$$\frac{V}{R}$$

Paper 031 – School-Based Assessment (SBA)

A total of 206 Unit 1 and 157 Unit 2 SBAs were moderated this year. Overall, there were notable improvements in the quality of the SBA packages submitted and it was evident that teachers have implemented the suggestions made in the past. Notably, multiple choice tests were not part of any of the SBAs and greater care was made to ensure that the content tested was relevant to the syllabus.

However, teachers are reminded that they should:

- Submit solutions with unitary mark schemes, that is, mark schemes for questions and their subsequent parts which are not broken down to show all single mark allocations
- Submit packages in which the materials are organized according to modules
- Create tests which are neat and professionally done instead of untidy ‘cut and paste’ additions with varying font styles and sizes, scrappily written or missing information
- Write subtotals per question, test totals, instructions, dates and examination duration or time allotted on the question papers
- Include at least one mathematical modelling question in the module test
- Award marks appropriately based on the skills assessed. (Fractional marks are not allowed.)

Teachers *must* pay particular attention to the following guidelines and comments to ensure appropriate and reliable SBAs.

The SBA comprises three separate module tests which *must be administered at school, under examination conditions* with the level of difficulty similar to that of the actual CAPE examination.

The main features assessed are:

- Mapping of the items tested to the specific objectives of the syllabus for the relevant unit.
- Content coverage of each module test
- Appropriateness of the items tested for the CAPE level.
- Presentation of the sample (Question paper, solutions with unitary mark scheme and students’ scripts should be batched per module).
- Quality of the teachers’ solutions and mark schemes.
- Quality of teachers’ assessments — consistency of marking using the mark schemes.
- Inclusion of mathematical modelling in at least one module test for each unit.

General Comments

- Module tests must be neatly hand written or typed in at least a size 12 font.
- The stipulated time for module tests is 1 to $1\frac{1}{2}$ hours and with a range of 60 to 90 marks awarded. This must be strictly adhered to as candidates may be at an undue disadvantage when module tests are too extensive or insufficient. The following guide can be used: 1 to $1\frac{1}{2}$ minutes per single, skill mark allocation. Approximately 75 per cent of the syllabus should be tested and mathematical modelling *must* be included.
- Multiple choice questions will *not* be accepted in the module tests.
- The moderation process relies on the validity of teachers' assessments. There were instances where the marks on students' scripts did not correspond to the marks on the moderation sheet. There were still situations where the integrity of the assessment was brought into question based on the presentation of the sample submitted. Teachers are reminded that the SBA must be administered under examination conditions at the school. *It is not to be done as a homework assignment or research project.*
- Teachers *must* present evidence of having marked each individual question on students' scripts before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be written on the front of the students' scripts.
- Teachers *must* indicate any changes/omissions that were made to question papers, solutions and marking schemes and scripts. Students' names on the computer generated form must correspond to the names on the PMATH 1-3 and PMATH 2-3 forms and students' scripts.
- The maximum number of marks for each assessment should be the same for *all* students.
- The number of tests used for the SBA should be the same for *all* students.
- If a student scores zero in an SBA, the script must be sent if that student's name is in the generated sample. Teachers should also inform the examiner about the circumstances regarding missing script(s). A letter *must* be submitted with a full explanation from the school.
- If a student was absent for an assessment then an official letter explaining this absence *must* be sent with the other samples submitted.

To enhance the quality of the design of the module tests, the validity of teachers' assessments and the validity of the moderation process, the following SBA guidelines are listed below for emphasis.

Guidelines for Module Tests and Presentation of Samples

1. Sample Package Considerations

- Design a separate test for each module. The module test *must* focus on objectives from that module.
- In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered and a common marking scheme used.
- One sample of *five* students will form the sample for the centre. If there are less than five students, *all* scripts will form the sample for the centre.
- In 2014, the format of the SBA remained unchanged.

2. Cover Page to Accompany Each Module Test

The following information is required on the cover of each module test.

- Name of school and territory, Name of teacher, Centre number.
- Unit Number and Module Number.
- Date and duration (1 to $1\frac{1}{2}$ hours) of module test.
- Clear instructions to students.
- Total marks allotted for module test.
- Sub-marks and total marks for each question *must* be clearly indicated.

3. Coverage of the Syllabus Content

- The number of questions in each module test must be appropriate for the stipulated time of (1 to $1\frac{1}{2}$ hours).
- *CAPE past examination papers should be used as a guide only* and should never appear in an SBA.
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant unit of the syllabus.

4. Mark Scheme

- Unitary mark schemes *must* be done on the detailed worked solution, that is, *one mark should be allocated per skill assessed and not 2, 3, 4 etc. marks per skill.*
- *Fractional / decimal marks must not be awarded, that is, do not allocate $\frac{1}{2}$ marks on the mark schemes or students' scripts.*

- The total marks for module tests *must* be clearly stated on teachers' solution sheets.
- A student's mark, that is, the final mark out of 20 *must* be entered on the front page of the student's script.
- Hand-written mark schemes *must be neat and legible*. The *unitary* marks *must* be written on the right side of the page.
- *Diagrams must be neatly drawn with geometrical / mathematical instruments.*

5. Presentation of Sample

- Candidates' responses *must* be written on A4 (210 x 297 mm) or letter-sized paper ($8\frac{1}{2} \times 11$) ins.
- Question numbers *must* be written clearly in the left hand margin.
- The total marks for *each question* on students' scripts **MUST** be clearly written in the left or right margin.
- *Only original* students' scripts *must* be sent for moderation.
- Photocopied scripts *will not be accepted*.
- Typed Module tests *must be neat and legible*.
- The following are required for each module test:
 - ❖ A question paper.
 - ❖ Detailed solutions with detailed unitary mark schemes.
 - ❖ The question paper, detailed solutions, unitary mark schemes and five students' samples should be batched together for each module.
- Marks recorded on the PMath1–3 and PMath2–3 forms *must* be rounded off to the nearest whole number. If a student scored zero, then zero must be recorded. If a student was absent, then absent must be recorded.
- PMaths 1–4 and PMaths 2–4 forms are for official use *only* and should not be completed by the teacher. However, teachers may complete the relevant information: Centre Code, Name of Centre, Territory, Year of Examination and Name of Teacher(s).
- The guidelines at the bottom of these forms should be observed. (See page 53 of the syllabus, no. 3, Part b).