1. $\bar{z}$ is the conjugate of z. Which of the following are always true?

I. $|\bar{z}| = |z|$
II. $\arg z = \arg \bar{z}$
III. $z\,\bar{z}$ is real
IV. $\frac{z}{\bar{z}}$ is real

(A) I and II only
(B) I and III only
(C) II and IV only
(D) III and IV only

2. The complex number $z = \sqrt{3} + i$ can be expressed as

(A) $\sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(B) $\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(C) $2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(D) $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

3. $\frac{2-i}{3+2i} =$

(A) $\frac{8-i}{13}$

(B) $\frac{8+i}{13}$

(C) $\frac{7-4i}{13}$

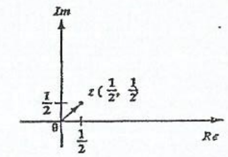
(D) $\frac{4-7i}{13}$

4. The expresssion $i\,[(1+i)^2 - (1-i)^2]$ is equal to

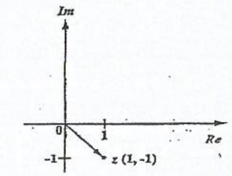
(A) –4
(B) –2
(C) 2
(D) 4

5. The complex number $z = \frac{1}{1-i}$ can be represented on an Argand diagram as

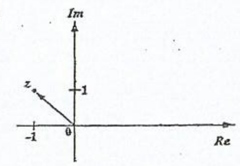
(A)



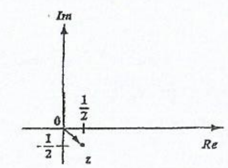
(B)



(C)



(D)



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6. If $f(x) = \ln 2x$, then $f'(x) =$

(A) $\frac{2}{x}$

(B) $\frac{1}{x^2}$

(C) $\frac{-2}{x}$

(D) $\frac{-1}{x^2}$

7. If $y = e^{(x^2)}$, then $\frac{dy}{dx}$

(A) $e^{(x^2)}$

(B) $2e^{(x^2)}$

(C) $e^{(2x)}$

(D) $2xe^{(x^2)}$

8. The derivative of $\ln\left(\frac{1}{\sqrt[3]{x}}\right)$ with respect to x is

(A) $-3x$

(B) $-\frac{1}{3x}$

(C) $\frac{1}{3x}$

(D) $3x$

9. If $x^2y - xy^2 = 10$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{10}{2x-2y}$

(B) $\frac{y^2-2xy}{x^2}$

(C) $\frac{y^2-2xy}{2xy}$

(D) $\frac{y^2-2xy}{x^2-2xy}$

10. If $y = \tan^{-1}(3x)$, then $\frac{dy}{dx}$ is

(A) $\frac{1}{1+9x^2}$

(B) $\frac{1}{9+x^2}$

(C) $\frac{3}{1+9x^2}$

(D) $\frac{3x}{9+x^2}$

11. The integral of $\frac{1}{1-\sin^2 x}$ with respect to x is

(A) $\tan x + c$
(B) $\tan^2 x + c$
(C) $\sec^2 x + c$
(D) $\sec x \tan x + c$

12. $\int \frac{\sec^2 x}{2\tan x}\,dx =$

(A) $\frac{1}{2}\ln|\sec^2 x| + c$

(B) $\frac{1}{2}\ln|\tan x| + c$

(C) $2\ln|\sec x| + c$

(D) $2\ln|\tan x| + c$

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13. A curve is given parametrically by the equations $x = t^2 - 2t$, $y = t^2 + 2t$. The simplest expression for the gradient of the tangent in terms of t is

(A) $\frac{t-1}{t+1}$
(B) $\frac{2t-2}{2t+2}$
(C) $\frac{t+1}{t-1}$
(D) $\frac{t+1}{2t-2}$

14. If $I_n = \int \tan^n(x)\,dx$ may be expressed as $I_n = \frac{\tan^{n-1}x}{n-1} - I_{n-2}$, then $\int \tan^3(x)\,dx$ is

(A) $\frac{\tan^2 x}{2} - \int \tan^2 x\,dx$
(B) $\frac{\tan^2 x}{2} - \int \tan x\,dx$
(C) $\frac{\tan^2 x}{2} + \int \tan^2 x\,dx$
(D) $\frac{\tan^2 x}{2} + \int \tan x\,dx$

15. $\frac{x+5}{x(x^2+6x+17)}$ may be expressed as

(A) $\frac{P}{x} + \frac{Q}{(x-1)} + \frac{R}{x+17}$
(B) $\frac{P}{x} + \frac{Qx+R}{(x-1)(x+17)}$
(C) $\frac{P}{x} + \frac{Qx+R}{x^2+6x+17}$
(D) $\frac{P}{x} + \frac{Qx}{x^2+6x+17}$

16. For $-1 < 2n < 1$, $\sum_{r=0}^{\infty}(2n)^r =$

(A) $\frac{2}{1-n}$
(B) $\frac{1}{1-2n}$
(C) $\frac{2n}{1+2n}$
(D) $\frac{2n}{1-2n}$

17. Which of the following sequences is the first four terms of an arithmetic progression?

(A) n, n – 2, n – 4, n – 6
(B) n, –(n + 1), (n + 2), –(n + 3)
(C) n, 2n + 1, 2n + 3, 2n + 8
(D) n, $\frac{n}{10}$, $\frac{n}{100}$, $\frac{n}{1000}$

18. The sum of the first n terms of a series is $1 - \left(\frac{1}{4}\right)^n$. The value of the SECOND term is

(A) $\frac{3}{16}$
(B) $\frac{3}{4}$
(C) $\frac{15}{16}$
(D) 1

19. If the terms of the sequence $u_1, u_2, u_3, \ldots, u_n \ldots$ satisfy the recurrence relation $u_{n+1} = u_n + 3$, $n \geq 1$, then the n^{th} term may be expressed as

(A) $u_1 + 6n$
(B) $u_1 + 3n$
(C) $u_1 + 6(n-1)$
(D) $u_1 + 3(n-1)$

20. The function $f(x) = x^3 - 3x - 1$ has NO real root in the open interval

(A) (–2, –1)
(B) (–1, 0)
(C) (0, 1)
(D) (1, 2)

21. The sum to infinity of a geometric series is $\frac{1}{1-2x}$. The range of x is

(A) $-1 < x < 1$
(B) $-2 < x < 2$
(C) $-\frac{1}{2} < x < \frac{1}{2}$
(D) $x > \frac{1}{2}$

22. Let a_n and S_n denote respectively, the value of the n^{th} term and the n^{th} partial sum of a series. The value of $S_{n+2} - S_n$ when calculated on the series is

(A) a_{n+1}
(B) a_{n+2}
(C) $a_{n+1} + a_{n+2}$
(D) $a_{n+1} - a_{n-2}$

23. Which of the following is an arithmetic series?

(A) $\sum_{r=1}^{n}(7+4r)$
(B) $\sum_{r=1}^{n}2(4^r)$
(C) $\sum_{r=1}^{n}4^{(r-1)}$
(D) $\sum_{r=1}^{n}\ln(r+1)$

24. In how many ways can a student council consisting of 8 students be formed from 40 students if 2 particular students must be on the council?

(A) ${}^{38}C_6$
(B) ${}^{38}C_8$
(C) ${}^{40}C_6$
(D) ${}^{40}C_8$

31. A team of five teachers is to be chosen from a group of fifteen teachers.

In how many ways could this team be chosen?

(A) $\frac{15!}{10!\,5!}$

(B) $\frac{15!}{10!}$

(C) $\frac{15!}{5!}$

(D) $15!$

32. Two events A and B are such that $P(A) = 0.5, P(B) = 0.16, P(A \cup B) = 0.48$. $P(A \cap B) =$

(A) 0.14
(B) 0.18
(C) 0.26
(D) 0.82

33. The number of distinct permutations of the letters of the word POSSIBILITY is

(A) $^{11}P_{11}$

(B) $(^{11}P_3)(^{11}P_2)$

(C) $\frac{11!}{3!2!}$

(D) $\frac{11!}{3!}$

34. X and Y are mutually exclusive events. If $P(X) = \frac{1}{4}$ and $P(Y) = \frac{1}{5}$ then $P(X \cup Y) =$

(A) $\frac{1}{9}$

(B) $\frac{2}{5}$

(C) $\frac{9}{20}$

(D) $\frac{1}{20}$

35. A bag contains six blue balls and four red balls. Terry chooses two balls at random from the bag without replacement. The probability that BOTH balls are red is

(A) $\frac{4}{10} \times \frac{3}{9}$

(B) $\frac{6}{10} \times \frac{3}{9}$

(C) $\left(\frac{6}{10} \times \frac{4}{9}\right) + \left(\frac{4}{10} \times \frac{3}{9}\right)$

(D) $\left(\frac{6}{10} \times \frac{4}{9}\right) \times \left(\frac{4}{10} \times \frac{3}{9}\right)$

36. Given that H is a non-singular, square matrix, the determinant $|H^2|$ of H^2 is

(A) $2\,|H|$

(B) $\frac{1}{2}|H|$

(C) $|H|^2$

(D) $\frac{1}{|H|^2}$

25. The binomial coefficient $\binom{n}{2}$ is equivalent to

(A) $\binom{n}{1} \div \binom{n}{1}$

(B) $\binom{n}{n-2}$

(C) $\binom{n-2}{n}$

(D) $\binom{n}{n+2}$

26. The value of the term independent of x in the expansion of $\left(x - \frac{3}{x}\right)^4$ is

(A) 6
(B) 16
(C) 54
(D) 81

27. By using the Newton-Raphson method with a first approximation x_n, the second approximation x_{n+1} for a root of the equation $x^5 = x^3 + 25$ may be expressed as

(A) $\frac{x_n - x_n^5 - x_n^3 + 25}{5x_n^4 - 3x_n^2}$

(B) $\frac{x_n - x_n^5 - x_n^3 - 25}{5x_n^4 - 3x_n^2}$

(C) $\frac{4x_n^5 - 4x_n^3 - 25}{5}$

(D) $\frac{4x_n^5 - 2x_n^3 + 25}{5x_n^4 - 3x_n^2}$

28. The Maclaurin series for $\sin x$, up to the term in x^3, is

(A) $x - \frac{x^3}{6}$

(B) $x + \frac{x^3}{6}$

(C) $1 + x + \frac{x^2}{2} - \frac{x^3}{6}$

(D) $1 + x - \frac{x^2}{2} - \frac{x^3}{6}$

29. A continuous function is defined by $f(0) = 1$ and $f(0.8) = -0.76$.

The first approximation to the root in $[0, 0.8]$, to 3 decimal places, using linear interpolation is

(A) 0.000
(B) 0.400
(C) 0.444
(D) 0.455

30. In an arithmetic progression (AP) the first term is -3 and the common difference is 5. The 6th term of the AP is

(A) -28
(B) 22
(C) 27
(D) 57

43. Given that $y = 0$ at $x = 0$, the general solution of the differential equation $y'' + 6y' + 9y = 0$ is

(A) $y = e^{3x} + Bx$
(B) $y = xBe^{-3x}$
(C) $y = e^{3x}(A + Bx)$
(D) $y = e^{-3x} + Bx$

44. The general solution for the second-order differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ is

(A) $y = Ae^{2x} + xBe^{x}$
(B) $y = Ae^{2x} + Be^{x}$
(C) $y = e^{x}(A + Bx)$
(D) $y = e^{2x}(A + Bx)$

45. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

(A) $y = e^x$
(B) $y = kx$
(C) $y = x + k$
(D) $y = \ln x + k$

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

37. If $M = \begin{pmatrix} 1 & 1 & 4 \\ 3 & 2 & -1 \\ 6 & 0 & 5 \end{pmatrix}$, then the co-factor of the element 3 in M above may be written as

(A) $\begin{vmatrix} 1 & 4 \\ 0 & 5 \end{vmatrix}$

(B) $-\begin{vmatrix} 1 & 4 \\ 0 & 5 \end{vmatrix}$

(C) $\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix}$

(D) $-\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix}$

38. The FIRST ROW of the product PQ of the two 3 × 3 matrices

$$P = \begin{pmatrix} 2 & 3 & 1 \\ 5 & -6 & 5 \\ -1 & 2 & 3 \end{pmatrix} \text{ and } Q = \begin{pmatrix} 2 & 1 & 3 \\ 5 & 0 & -1 \\ -3 & -2 & 4 \end{pmatrix}$$

is

(A) (22 −4 −5)
(B) (16 −5 7)
(C) (16 0 7)
(D) (22 −35 −1)

39. The letters of the word I R R E G U L A R are to be arranged in a line. The number of possible arrangements in which the 3 Rs are NOT together is

(A) 7!

(B) $\frac{9!}{3!} - 7!$

(C) 9! − 7!

(D) 9! − 7! × 3!

40. If $M = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 1 \\ 5 & 0 & 2 \end{pmatrix}$, then the determinant of M is

(A) $\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} + 5\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$

(B) $\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} + 5\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$

(C) $\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$

(D) $\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$

41. The general solution of a second order ordinary differential equation is $y(t) = c_1 \cos 2t + c_2 \sin 2t$. How many solutions result from applying the boundary conditions $y(0) = y(2\pi) = b$ where b is a constant?

(A) no solution
(B) one solution
(C) two solutions
(D) infinitely many solutions

42. If the auxiliary equation for a second order differential equation with real, constant coefficients is given by $\lambda^2 + 6\lambda + 50 = 0$, then the general solution of the differential equation may be given by

(A) $y = e^{\lambda x}(A + Bx)$
(B) $ye^{\int P(x)dx} = \int Q(x)e^{\int P(x)dx}\,dx + C$
(C) $y = Ae^{\lambda_1 x} + Be^{\lambda_2 x}$
(D) $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$