Caribbean Advanced Proficiency Examinations
Internal Moderation Test 2013
Pure Mathematics Unit 2
Module 1

School: Queen's College
Guyana

Time: 1 hour 30 minutes
Tutor: Rudolph Deoraj

Instruction to Candidates

Answer **ALL** questions. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.
This paper consists **7** questions. The maximum mark on this paper is **75**.

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1. (a) Draw on the same Argand diagram

(i) the locus of the points for which $|z - 2 - 5\text{i}| \leq 5$. [3]

(ii) the locus of the points for which $\arg(z + 2\text{i}) = \dfrac{\pi}{4}$. [2]

(b) Indicate on your diagram the set of points representing both

$$|z - 2 - 5\text{i}| \leq 5 \text{ and } \arg(z + 2\text{i}) = \frac{\pi}{4}. \quad [2]$$

(c) Use de'Moivre's Theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta. \quad [7]$$

Total 14 marks

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2. A curve is defined by the equation $2y + e^{2x} y^2 = x^2 + C$, where C is a constant. The point $P\left(1, \frac{1}{e}\right)$ lies on the curve.

(a) Find the exact value of C. [2]

(b) Find an expression for $\frac{dy}{dx}$ in terms of x and y. [6]

(c) Verify that $P\left(1, \frac{1}{e}\right)$ is a stationary point on this curve. [2]

Total 10 marks

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3. A curve C is given by the parametric equations

$$x = \sqrt{3}\sin 2t, \quad y = 4\cos^2 t, \quad 0 \le t \le \pi.$$

(a) Show that $\frac{dy}{dx} = k\sqrt{3}\tan 2t$, where k is a constant to be determined. [5]

(b) Find an equation of the normal to C at the point where $t = \frac{\pi}{3}$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[4]

Total 9 marks

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4. (a) Given that $y = x\cos^{-1}(2x) - \frac{1}{2}\sqrt{(1 - 4x^2)}$, find $\frac{dy}{dx}$, simplifying your answer as far as possible.

[8]

(b) $f(x, y) = w = \tan^{-1}\left(\frac{y}{x}\right)$. Find, in a simplified form, $\frac{\partial w}{\partial x}$. [3]

Total 11 marks

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5. (a) Express $\dfrac{1}{(3-2x)(1-x)^2}$ as a sum of partial fractions. [3]

(b) Solve the equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2\sqrt{y}}{(3-2x)(1-x)^2}$, where $y = 0$ when $x = 0$, expressing your answer in the form

$y^p = q \ln|\mathrm{f}(x)| + \dfrac{x}{1-x}$, where p and q are constants to be determined. [9]

Total 12 marks

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6. $$I_n = \int_0^{\frac{\pi}{4}} x^n \sin 2x \;\mathrm{d}x, \quad n \geq 0.$$

(*a*) Prove that, for $n \geq 2$, $I_n = \dfrac{1}{4} n \left(\dfrac{\pi}{4}\right)^{n-1} - \dfrac{1}{4} n(n-1)\, I_{n-2}$. [5]

(b) Find the exact value of I_2. [4]

(c) Show that $I_4 = \dfrac{1}{64}(\pi^3 - 24\pi + 48)$. [2]

Total 11 marks

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7.

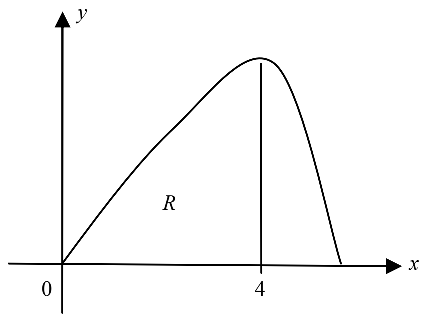

Figure 1

Figure 1 shows the curve C with parametric equations

$$x = 8\cos t, \quad y = 4\sin 2t, \quad 0 \le t \le \frac{\pi}{2}.$$

The finite region R s enclosed by C, the line $x = 4$ and the x-axis.

(a) Show that the area of R is given by

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(64\sin^2 t\cos t\right)\mathrm{d}t.$$ [5]

(b) Hence find the area of R in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

[3]

Total 8 marks

End of Test

Formulae

Trigonometric Identities:

$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) \equiv \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B) \neq (k + \tfrac{1}{2})\pi$

Differentiation:

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}^1(x)\mathrm{g}(x) - \mathrm{g}'(x)\mathrm{f}(x)}{[\mathrm{g}(x)]^2}$
$\arcsin x$	$\dfrac{1}{\sqrt{(1-x^2)}}$
$\operatorname{arcos} x$	$-\dfrac{1}{\sqrt{(1-x^2)}}$
$\arctan x$	$\dfrac{1}{1+x^2}$

<u>Integration</u>: (+ constant: $a > 0$ where relevant)

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert = \ln\lvert\tan \tfrac{1}{2}x\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

$\frac{1}{\sqrt{(a^2 - x^2)}}$	$\arcsin\left(\frac{x}{a}\right) \quad \lvert x\rvert < a$
$\frac{1}{a^2 + x^2}$	$\frac{1}{a}\arctan\left(\frac{x}{a}\right)$

<u>Complex Numbers</u>:

$\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta \qquad \{r(\cos\theta + \mathrm{i}\sin\theta)^n\} = r^n\cos n\theta + \mathrm{i}\sin n\theta$