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CANDIDATE – PLEASE NOTE!

You must sign below and return this booklet with the Answer Sheet. Failure to do so may result in disqualification.

Signature

FORM TP 2009236

TEST CODE **02234010**

MAY/JUNE 2009

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

ANALYSIS, MATRICES AND COMPLEX NUMBERS

Unit 2 – Paper 01

90 minutes

03 JUNE 2009 (a.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This test consists of 45 items. You will have 90 minutes to answer them.
2. In addition to this test booklet, you should have an answer sheet.
3. Do not be concerned that the answer sheet provides spaces for more answers than there are items in this test.
4. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.
5. On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

Sample Item

The expression $(1+\sqrt{3})^2$ is equivalent to

(A) 4
(B) 10
(C) $1+3\sqrt{3}$
(D) $4+2\sqrt{3}$

Sample Answer

(A) (B) (C) ●

The best answer to this item is "$4+2\sqrt{3}$", so answer space (D) has been shaded.

6. If you want to change your answer, be sure to erase your old answer completely and fill in your new choice.
7. When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, omit it and go on to the next one. Your score will be the total number of correct answers.
8. You may do any rough work in this booklet.
9. The use of silent, non-programmable scientific calculators is allowed.

Examination Materials:

A list of mathematical formulae and tables. **(Revised 2009)**

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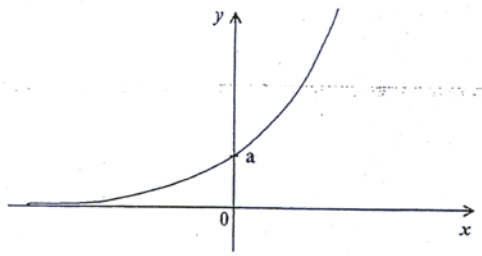
DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

Item 1 refers to the following graph.



1. The graph above represents

(A) $y = \log_x a$
(B) $y = \log_a x$
(C) $y = -e^x + a$
(D) $y = e^x + a - 1$

2. The expression $e^{3\ln x}$ may be written as

(A) $3x$
(B) x^3
(C) $\ln x^3$
(D) $\ln 3x$

3. $\frac{d}{dx}(\ln x)^3 =$

(A) $\frac{3}{x}$

(B) $3x$

(C) $\frac{3}{x}(\ln x)^2$

(D) $3\ (\ln x)^2$

4. $\int \frac{dx}{1+9x^2}$ is

(A) $\frac{1}{9}\tan^{-1} x + c$

(B) $\frac{1}{3}\tan^{-1} 3x + c$

(C) $\frac{1}{3}\tan^{-1} \frac{x}{3} + c$

(D) $\frac{1}{9}\tan^{-1} 3x + c$

5. $\int \frac{2x}{(x-1)(x+3)}\, dx =$

(A) $\int\left(\frac{2}{3(x-1)} + \frac{2}{x+3}\right)dx$

(B) $\int\left(\frac{2}{x-1} + \frac{2}{3(x+3)}\right)dx$

(C) $\int\left(\frac{3}{2(x-1)} + \frac{1}{2(x+3)}\right)dx$

(D) $\int\left(\frac{1}{2(x-1)} + \frac{3}{2(x+3)}\right)dx$

6. The population P of an insect colony at time t is given by $P(t) = 2000e^{kt}$.

If $P = 4000$ when $t = 6$, then k is

(A) $\frac{-\ln 6}{2}$

(B) $\frac{-\ln 2}{6}$

(C) $\frac{\ln 2}{6}$

(D) $\frac{\ln 6}{2}$

7. If $y = e^{(x^2)}$, then $\dfrac{dy}{dx} =$

(A) $e^{(x^2)}$

(B) $2e^{(x^2)}$

(C) $e^{(2x)}$

(D) $2\,xe^{(x^2)}$

8. The derivative of $\ln\left(\dfrac{1}{\sqrt[3]{x}}\right)$ is

(A) $-3x$

(B) $-\dfrac{1}{3x}$

(C) $\dfrac{1}{3x}$

(D) $3x$

9. Given $e^{x+y} - x = 0$ then $\dfrac{dy}{dx}$ is equal to

(A) $\dfrac{11}{e^x + y}$

(B) $\dfrac{1}{e^{x+y}}$

(C) $\dfrac{1-e^{x+y}}{e^{x+y}}$

(D) $\dfrac{1-e^{x+y}}{e^x}$

10. $\dfrac{d}{dx}\cos^{-1}\left(\dfrac{x}{2}\right)$ is equal to

(A) $-\dfrac{1}{\sqrt{4-x^2}}$

(B) $-\dfrac{2}{\sqrt{4-x^2}}$

(C) $-2\sqrt{4-x^2}$

(D) $-\sqrt{4-x^2}$

11. Given that $\cos 2x = 2\cos^2 x - 1$, $\int \cos^2\left(\dfrac{x}{4}\right) dx$ is

(A) $\dfrac{1}{2}x + \dfrac{1}{2}\int \cos 4x \, dx$

(B) $\dfrac{1}{2}x + \int \cos \dfrac{x}{4} \, dx$

(C) $\dfrac{1}{2}x + \dfrac{1}{2}\int \cos \dfrac{x}{2} \, dx$

(D) $\dfrac{1}{2}x + \int \cos \dfrac{x}{2} \, dx$

12. The solution to the differential equation $\dfrac{dy}{dx} + y = e^x$ is

(A) $y = e^x + c$

(B) $y = 2e^x + c$

(C) $y = \dfrac{1}{2}e^x + ce^{-x}$

(D) $y = 2e^x + ce^{-x}$

13. The population of a colony of bacteria is given by $A_0e^{0.2t}$, where A is the population after time t hours and A_0 is the inital population. In how many hours will the population triple?

(A) 5
(B) 15
(C) 3 ln 5
(D) 5 ln 3

14. If $I_n = \int \tan^n(x)dx$ may be expressed as $I_n = \dfrac{\tan^{n-1} x}{n-1} - I_{n-2}$, then $\int \tan^3(x)\, dx$ is

(A) $\dfrac{\tan^2 x}{2} - \int \tan^2 x\, dx$

(B) $\dfrac{\tan^2 x}{2} - \int \tan x\, dx$

(C) $\dfrac{\tan^2 x}{2} + \int \tan^2 x\, dx$

(D) $\dfrac{\tan^2 x}{2} + \int \tan x\, dx$

15. The solution of the differential equation $\dfrac{dy}{dx} + y\tan x = \cos x$ is given by the solution of

(A) $\int \dfrac{d}{dx}(y\sec x)dx = \int dx$

(B) $\int \dfrac{d}{dx}(y\sec x)dx = \int \cos x\, dx$

(C) $\int \dfrac{d}{dx}(\sec x)dx = \int \sec x\, dx$

(D) $\int \dfrac{d}{dx}(y\sec x)dx = \int \cos^2 x\, dx$

16. Which of the following sequences, $\{U_n\}$, converges?

(A) $\{(-1)^n\}$

(B) $\left\{\left(\dfrac{1}{n}\right)^{-n}\right\}$

(C) $\left\{\left(\dfrac{1}{2}\right)^{n}\right\}$

(D) $\{2^n\}$

17. The sum of the first n terms of a geometric series is $\left[1-\left(\dfrac{1}{2}\right)^n\right]$. The value of the SECOND term is

(A) $\dfrac{1}{4}$

(B) $\dfrac{1}{2}$

(C) $\dfrac{3}{4}$

(D) 1

18. For what values of x is the series $\sum_{r=0}^{\infty}(2x)^r$ convergent?

(A) $-4 < x < 4$

(B) $-2 < x < 2$

(C) $-1 < x < 1$

(D) $-\dfrac{1}{2} < x < \dfrac{1}{2}$

19. Given that the expansion of e^x, up to and including the term in x^3 is $1 + x + \frac{x^2}{2} + \frac{x^3}{6}$, the expansion of e^{x+h}, up to and including the term in x^3 is

(A) $e^h\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)$

(B) $h\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)$

(C) $\left(1 + (xh) + \frac{(xh)^2}{2} + \frac{(xh)^3}{6}\right)$

(D) $\left(\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) + \left(1 + h + \frac{h^2}{2} + \frac{h^3}{3}\right)\right)$

20. A sequence is defined as $u_1 = 2$, $u_2 = 4$, $u_n = u_{n-1} + u_{n-2}$, for $n \geq 3$. This sequence is

(A) convergent
(B) divergent
(C) oscillating
(D) periodic

21. Given that u_n represents the n[th] term of a sequence, which of the following converges?

(A) $u_n = \frac{5 - 3n}{2 - n}$

(B) $u_n = 3n - 4$

(C) $u_n = 5\left(\frac{4^{n-1}}{3^n}\right)$

(D) $u_n = 2(-1)^{n-1}$

22. The value of $\sum_{m=3}^{6} \frac{(-1)^m}{m}$ is

(A) $-\frac{17}{60}$

(B) $-\frac{7}{60}$

(C) $\frac{7}{60}$

(D) $\frac{17}{60}$

23. Which of the following series are arithmetic series?

I. $\sum_{r=1}^{n} (7 + 4r)$

II. $\sum_{r=1}^{n} 2(4^r)$

III. $\sum_{r=1}^{n} \ln 4^{(r+1)}$

IV. $\sum_{r=1}^{n} \ln(r + 1)$

(A) I and III only
(B) I and IV only
(C) III and IV only
(D) I, II and IV only

24. The sum of the infinite geometric series $180 - 60 + 20 - , \ldots\ldots$ is

(A) 45
(B) 120
(C) 135
(D) 270

25. The value of $\sum_{n=1}^{\infty} 2\left(\frac{1}{4}\right)^{n-1}$ is

(A) 0

(B) $\frac{3}{2}$

(C) 2

(D) $\frac{8}{3}$

26. The value of the term indepedent of x in the expansion of $\left(x - \frac{3}{x}\right)^4$ is

(A) 6
(B) 16
(C) 54
(D) 81

27. A suitable Newton-Raphson iteration for the equation $e^{2x} + 4x - 5 = 0$ is

(A) $x_{n+1} = \frac{e^{2x_n}(2x_n + 1) - 5}{2e^{2x_n} + 4}$

(B) $x_{n+1} = \frac{e^{2x_n}(2x_n - 1) - 5}{2e^{2x_n} + 4}$

(C) $x_{n+1} = \frac{e^{2x_n}(2x_n + 1) + 5}{2e^{2x_n} + 4}$

(D) $x_{n+1} = \frac{e^{2x_n}(2x_n - 1) + 5}{2e^{2x_n} + 4}$

28. The values of x for which the expansion of $\frac{1}{\sqrt{(100 - 50x)}}$ is valid are

(A) $-1 < x < 1$

(B) $-2 < x < 2$

(C) $-\frac{1}{2} < x < \frac{1}{2}$

(D) $x < -2$ and $x > 2$

29. The formula $T = 2\pi\sqrt{\frac{l}{g}}$ is used to calculate the period T seconds of a simple pendulum of length l. When the rounded values $\pi = 3.14$, $l = 0.8$ and $g = 9.81$ are used to calculate T, the upper bound of T is

(A) $2(3.145)\sqrt{\frac{0.85}{9.815}}$

(B) $2(3.15)\sqrt{\frac{0.9}{9.82}}$

(C) $2(3.145)\sqrt{\frac{0.85}{9.805}}$

(D) $2(3.14)\sqrt{\frac{0.85}{9.815}}$

30. The binomial coefficient $\binom{n}{4}$ is equivalent to

(A) $\binom{n}{3} + \binom{n}{1}$

(B) $\binom{n}{n-4}$

(C) $\binom{n-4}{n}$

(D) $\binom{n}{n+4}$

31. If $M = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 1 \\ 5 & 0 & 2 \end{pmatrix}$, then the determinant of M is

(A) $\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} + 5\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$

(B) $\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} + 5\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$

(C) $\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$

(D) $\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$

32. In how many ways can the letters PQRSTU be arranged so that the T and U are always together?

(A) $\frac{6!}{2}$
(B) 5!
(C) 6!
(D) $5! \times 2$

33. The number of possible values of x which satisfy the system of simultaneous equations,

$$2x + 3y + 2z = -5$$
$$4x + 6y + 4z = -10$$
$$6x + 9y + 6z = -16$$

is

(A) 0
(B) 1
(C) 2
(D) 3

34. If $P = \begin{pmatrix} 1 & -2 & 0 \\ 3 & 1 & 5 \\ -1 & 2 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} -7 & 6 & -10 \\ -14 & 3 & -5 \\ 7 & 0 & 7 \end{pmatrix}$

then $PQ = \begin{pmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{pmatrix}$. The matrix P^{-1} equals

(A) $(PQ)^{-1}$
(B) $(QP)^{-1}$
(C) $\frac{1}{21}P$
(D) $\frac{1}{21}Q$

35. The modulus of the complex number $\frac{1}{2} - \frac{1}{2}i$ is

(A) $\frac{1}{2\sqrt{2}}$
(B) $\frac{1}{\sqrt{2}}$
(C) $\sqrt{2}$
(D) $2\sqrt{2}$

36. Two coins and a die with faces numbered 1 to 6 are thrown together once. Assuming that the die and coins are fair, the probability of obtaining 2 heads and a number less than 4 is

(A) $\frac{2}{3}$
(B) $\frac{1}{24}$
(C) $\frac{1}{8}$
(D) $\frac{3}{4}$

37. The non-singular matrix A is given by $\begin{pmatrix} 3 & 4 & 1 \\ 2 & -1 & 2 \\ 3 & 5 & 1 \end{pmatrix}$. The determinant of A is

(A) -36
(B) -4
(C) -2
(D) 24

38. Which of the following systems are consistent?

I. $x + y = 5$
$x - y = 3$

II. $x - 3y - 2z = -11$
$-x + 5y + 3z = 15$

III. $3x - 2y = 7$
$9x - 6y = 12$

IV. $x + z = 5$
$3x + 3z = 15$
$x + y + z = 8$

(A) I and II only
(B) II and IV only
(C) I, II and IV only
(D) II, III and IV only

39. The roots of the equation $2x^2 + 6x + 17 = 0$ are

(A) $\dfrac{-3 \pm 5i}{2}$

(B) $\dfrac{3 \pm 5i}{2}$

(C) $\dfrac{-3}{2} \pm 5i$

(D) $3 \pm \dfrac{5i}{2}$

40. The complex number $z = \sqrt{3} + i$ can be expressed as

(A) $\sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(B) $\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(C) $2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(D) $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

41. The value of $\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^4$ is

(A) -4
(B) -1
(C) 1
(D) 4

42. Given that $z = -1 + \sqrt{3}i$, then the exponential form of the complex number z is

(A) $2e^{\frac{\pi}{6}i}$

(B) $2e^{\frac{\pi}{3}i}$

(C) $2e^{\frac{2\pi}{3}i}$

(D) $2e^{\frac{5\pi}{6}i}$

43. Chad, Matthew, Josh, Paul and Tifanny are travelling in a 5-seater car with 3 persons in the back and 2 persons in the front. Each person occupies a seat. The number of different ways they can sit in the car if Tifanny sits in the back and Josh drives is

(A) 18
(B) 24
(C) 114
(D) 120

44. A committe of 3 teachers, 3 doctors and 3 lawyers is to be chosen from 5 teachers, 4 doctors and 6 lawyers. The number of ways in which this committee can be chosen is

(A) 27
(B) 54
(C) 182
(D) 800

45. If one marble is chosen, without replacement, from a bag of 11 blue and 9 red marbles, then the probability of getting a red marble followed by 2 blue marbles is

(A) $\frac{9}{20}+\frac{11}{19}+\frac{10}{18}$

(B) $\frac{9}{20}+\frac{11}{20}+\frac{10}{20}$

(C) $\frac{9}{20}\times\frac{11}{19}\times\frac{10}{18}$

(D) $\frac{9}{20}\times\frac{11}{20}\times\frac{10}{20}$

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.