QUEEN'S COLLEGE - 090041

CARIBBEAN ADVANCED PROFICIENCY EXAMINATIONS

PURE MATHEMATICS

UNIT1 : 1 – TRIGONOMETRY, GEOMETRY & VECTORS

INTERNAL MODERATION TEST- 2015

MODULE 2

TIME: 1 hour 30 minutes

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This paper consists **6** questions.

The maximum mark for this examination is **75**.

INSTRUCTIONS TO CANDIDATES

1. Answer **ALL** questions.
2. Unless otherwise stated in the question, any numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Module 2 – Trigonometry, Geometry & Vectors

1. (a) Given $\tan\alpha = x + 1$ and $\tan\beta = x - 1$, show that $\cot(\alpha - \beta) = \frac{1}{2}x^2$. (5 marks)

   (b) Prove that $\sin^2\theta\left(\cot^2\theta + \text{cosec}^2\theta\right) \equiv 1 + \cos^2\theta$. (3 marks)

Total: 8 marks

2. (a) Find the general solution of $\cos\theta = \sin 2\theta$. (5 marks)

   (b) (i) Express $2\sin\theta + 4\cos\theta$ in the form $r\sin(\theta + \alpha)$,

   where $r > 0$ and $0 < \alpha < 90^0$. (3 marks)

   (ii) Determine the maximum value of $\dfrac{2}{2\sin\theta + 4\cos\theta}$. (3 marks)

   (iii) State the smallest positive value of θ for which $\dfrac{2}{2\sin\theta + 4\cos\theta}$

   is maximum. (3 marks)

Total: 14 marks

3. A circle C has centre A (2, 1) and passes through the point B (10, 7).

   (a) Find the equation of C in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are integers to be determined. (3 marks)

   The line l_1 is the tangent to C at the point B.

   (b) Find the equation for l_1. (4 marks)

   The line l_2 is parallel to l_1 and passes through the midpoint of AB. Given that l_2 intersects C at the points P and Q,

   (c) find the x-coordinate of P and the x-coordinate of Q in surd form. (8 marks)

Total: 15 marks

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4. A curve C is given by the parametric equations $x = 4\cos\left(t + \dfrac{\pi}{6}\right), \quad y = 2\sin t,$

where $0 \le t \le 2\pi$.

(a) Show that $x + y = 2\sqrt{3}\cos t$. (4 marks)

(b) Find the Cartesian equation of C, simplified as far as possible. (5 marks)

The point $P\,(x, y)$ varies such that it is 3 times the distance from the point $A\,(0, 6)$ as from the point $B\,(8, 0)$.

(c) Determine the locus of P, identifying its characteristics. (7 marks)

Total: 16 marks

5. With respect to a fixed origin O, the point A with position vector $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ lies on the line l_1 with equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$, where λ is a scalar parameter, and the point B with position vector $4\mathbf{i} + p\mathbf{j} + 3\mathbf{k}$, where p is a constant, lies on the line l_2 with equation

$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}$, where μ is a scalar parameter.

(a) Find the value of the constant p. (2 marks)

(b) Show that l_1 and l_2 intersect and find the position vector of their point of intersection, C. (4 marks)

(c) Show that the vector $\mathbf{n} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$ is mutually perpendicular to the lines l_1 and l_2.

(2 marks)

(d) Find the Cartesian equation of the plane containing the lines l_1 and l_2. (3 marks)

Total: 11 marks

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6. Relative to a fixed origin O, the point A has position vector $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$ and the point B has position vector $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$. The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{\mathbf{AB}}$. (2 marks)

(b) Hence find a vector equation of the line l_1. (1 mark)

The point P has position vector $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$. Given that $\angle PBA$ is θ,

(c) show that $\cos\theta = \dfrac{1}{3}$. (3 marks)

The line l_2 passes through the point P and is parallel to the line l_1.

(d) Find a vector equation for the line l_2. (2 marks)

The points C and D both lie on the line l_2. Given that $AB = PC = DP$ and that the x-coordinate of C is positive,

(e) find the coordinates of C and the coordinates of D. (3 marks)

Total: 11 marks

END OF TEST

Formulae

Trigonometric Identities:

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B) \neq (k + \tfrac{1}{2})\pi$$