CANDIDATE – PLEASE NOTE!

You must sign below and return this booklet with the Answer Sheet. Failure to do so may result in disqualification.

Signature

FORM TP 2011233

TEST CODE 02234010

MAY/JUNE 2011

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

ANALYSIS, MATRICES AND COMPLEX NUMBERS

Unit 2 – Paper 01

90 minutes

01 JUNE 2011 (a.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This test consists of 45 items. You will have 90 minutes to answer them.
2. In addition to this test booklet, you should have an answer sheet.
3. Do not be concerned that the answer sheet provides spaces for more answers than there are items in this test.
4. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.
5. On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

Sample Item

The expression $(1 + \sqrt{3})^2$ is equivalent to

(A) 4
(B) 10
(C) $1 + 3\sqrt{3}$
(D) $4 + 2\sqrt{3}$

Sample Answer

Ⓐ Ⓑ Ⓒ ●

The best answer to this item is "$4 + 2\sqrt{3}$", so answer space (D) has been shaded.

6. If you want to change your answer, be sure to erase it completely before you fill in your new choice.
7. When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, omit it and go on to the next one. You can return later to the item omitted. Your score will be the total number of correct answers.
8. You may do any rough work in this booklet.
9. The use of silent, non-programmable scientific calculators is allowed.

Examination Materials:

A list of mathematical formulae and tables. (Revised 2010)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

1. A simplified form of
$\ln 120 - \ln 3 - \ln 8 + \ln 2$ is

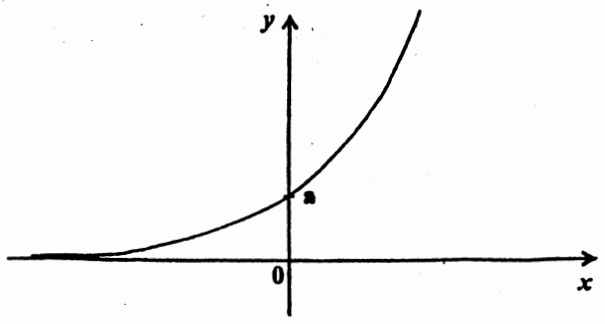
(A) $\ln 10$

(B) $\ln\left(\frac{15}{2}\right)$

(C) $\ln 111$

(D) $\ln 640$

Item 2 refers to the following graph.



2. The graph above represents

(A) $y = \log_x a$
(B) $y = \log_e x$
(C) $y = -e^x + a$
(D) $y = e^x + a - 1$

3. $\frac{d}{dx}(\ln x)^3$

(A) $\frac{3}{x}$

(B) $3x$

(C) $\frac{3}{x}(\ln x)^2$

(D) $3(\ln x)^2$

4. A curve is given parametrically by the equations $x = t^2 - 2t$, $y = t^2 + 2t$. The expression for $\frac{dy}{dx}$ is given by

(A) $\frac{t-1}{t+1}$

(B) $\frac{t+1}{t-1}$

(C) $\frac{2t-1}{2t+1}$

(D) $\frac{2t+1}{2t-1}$

5. $\int \frac{1}{x^2+4}\,dx =$

(A) $2\sec^{-1}\left(\frac{x}{2}\right)+c$

(B) $\frac{1}{2}\sec^{-1}\left(\frac{x}{2}\right)+c$

(C) $2\tan^{-1}\left(\frac{x}{2}\right)+c$

(D) $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$

6. The general solution for the second-order differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ is

(A) $y = Ae^{2x} + x\,Be^x$
(B) $y = Ae^{2x} + Be^x$
(C) $y = e^x(A + Bx)$
(D) $y = e^{2x}(A + Bx)$

7. If $\ln(2x-5) - \ln(x+3) = 0$ then x is equal to

(A) 0

(B) $\frac{5}{2}$

(C) 8

(D) 10

8. The derivative of $\ln x^{\frac{1}{3}}$ is

(A) $\frac{1}{3x}$

(B) $-\frac{1}{3x}$

(C) $-3x$

(D) $3x$

9. If $x^2y - xy^2 = 10$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{10}{2x-2y}$

(B) $\frac{y^2-2xy}{x^2}$

(C) $\frac{y^2-2xy}{2xy}$

(D) $\frac{y^2-2xy}{x^2-2xy}$

10. If $y = \tan^{-1}(3x)$, then $\frac{dy}{dx}$ is

(A) $\frac{1}{1+9x^2}$

(B) $\frac{1}{9+x^2}$

(C) $\frac{3}{1+9x^2}$

(D) $\frac{3x}{9+x^2}$

11. Given that $\cos 2x = 2\cos^2 x - 1$, $\int \cos^2\left(\frac{x}{4}\right)dx$ is

(A) $\frac{1}{2}x + \frac{1}{2}\int \cos 4x\, dx$

(B) $\frac{1}{2}x + \int \cos\frac{x}{4}\, dx$

(C) $\frac{1}{2}x + \int \cos\frac{x}{2}\, dx$

(D) $\frac{1}{2}x + \frac{1}{2}\int \cos\frac{x}{2}\, dx$

12. Given that $f(x) = \ln 3x^2$, then $f''(-2)$ is equal to

(A) -1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

13. A function f is defined by $f(x) = 2e^{3x} - 1$ for all real values of x. The inverse function, f^{-1} of f, is defined by

(A) $3\ln(x+1), x \in \mathbb{R}$

(B) $\frac{1}{3}\ln(x+1), x \in \mathbb{R}$

(C) $\frac{1}{3}\ln\left(\frac{x+1}{2}\right), x > -1$

(D) $\frac{1}{3}\ln\left(\frac{x+1}{2}\right), x \in \mathbb{R}$

GO ON TO THE NEXT PAGE

14. If $I_n = \int \tan^n(x)\,dx$ may be expressed as $I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$, then $\int \tan^3(x)\,dx$ is

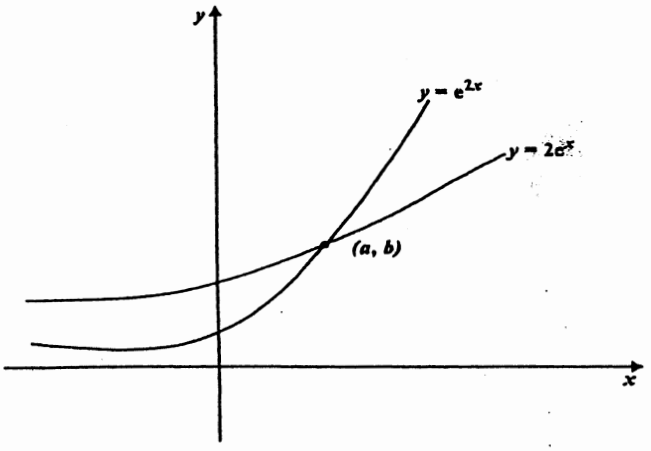
(A) $\frac{\tan^2 x}{2} - \int \tan^2 x\,dx$

(B) $\frac{\tan^2 x}{2} - \int \tan x\,dx$

(C) $\frac{\tan^2 x}{2} + \int \tan^2 x\,dx$

(D) $\frac{\tan^2 x}{2} + \int \tan x\,dx$

Item 15 refers to the diagram below.



15. The coordinate (a, b) is

(A) $(\ln 2, 4)$
(B) $(2, e^4)$
(C) $(4, \ln 2)$
(D) $(4, 2e^4)$

16. For $2n < 1$, $\sum_{r=0}^{\infty}(2n)^r =$

(A) $\frac{2}{1-n}$

(B) $\frac{1}{1-2n}$

(C) $\frac{2n}{1+2n}$

(D) $\frac{2n}{1-2n}$

17. Which of the following sequences is the first four terms of an arithmetic progression?

(A) n, n – 2, n – 4, n – 6

(B) n, –(n + 1), (n + 2), –(n + 3)

(C) n, 2n + 1, 2n + 3, 2n + 8, 2n + 14

(D) $n, \frac{n}{10}, \frac{n}{100}, \frac{n}{1000}$

18. For what values of x is the series $\sum_{r=0}^{\infty}(2x)^r$ convergent?

(A) $-4 < x < 4$

(B) $-2 < x < 2$

(C) $-1 < x < 1$

(D) $-\frac{1}{2} < x < \frac{1}{2}$

19. The sum to infinity of the geometric series $\frac{1}{2} - \frac{1}{3} + \frac{2}{9} - \frac{4}{27} + \ldots$ is

(A) $\frac{3}{10}$

(B) $\frac{3}{5}$

(C) $\frac{3}{2}$

(D) 3

20. The function $f(x) = x^3 - 3x - 1$ has NO real root in the open interval

(A) $(-2, -1)$
(B) $(-1, 0)$
(C) $(0, 1)$
(D) $(1, 2)$

GO ON TO THE NEXT PAGE

21. A sequence is defined as $u_1 = 2$, $u_2 = 4$, $u_n = u_{n-1} + u_{n-2}$ for $n \geq 3$. This sequence is

(A) convergent
(B) divergent
(C) oscillating
(D) periodic

22. The expression $\frac{n!}{(n-2)!}$ can be simplified and written as

(A) 2!
(B) 3!
(C) $n-3$
(D) $n(n-1)$

23. If $S \equiv 2^0 - 2^1 + 2^2 - 2^3 + 2^4 + \ldots$, then the $(r+1)$ term in S is

(A) $(-1)^r 2^r$
(B) $(-1)^r (-2)^r$
(C) $(-1)^{r-1} (2)^r$
(D) $(-1)^{r-1} (-2)^{r-1}$

24. The sum of the infinite geometric series $180 - 60 + 20 - , \ldots\ldots$ is

(A) 45
(B) 120
(C) 135
(D) 270

25. $\sum_{r=1}^{n} \left(\frac{1}{r+1} - \frac{1}{r}\right) =$

(A) $\frac{n+2}{n+1}$
(B) $-\frac{n}{n+1}$
(C) $-\frac{1}{n+1}$
(D) $\frac{n}{n+1}$

26. The value of the term independent of x in the expansion of $\left(x - \frac{3}{x}\right)^4$ is

(A) 6
(B) 16
(C) 54
(D) 81

27. The equation $e^x - x^4 = 0$ has a root between

(A) 0 and 1
(B) 1 and 2
(C) 2 and 3
(D) 3 and 4

28. The sum of the first n terms of a series is $1 - \left(\frac{1}{4}\right)^n$. The value of the SECOND term is

(A) $\frac{3}{16}$
(B) $\frac{3}{4}$
(C) $\frac{15}{16}$
(D) 1

GO ON TO THE NEXT PAGE

29. The binomial coefficient $\binom{n}{4}$ is equivalent to

(A) $\binom{n}{3}+\binom{n}{1}$

(B) $\binom{n}{n-4}$

(C) $\binom{n-4}{n}$

(D) $\binom{n}{n+4}$

30. A table of height 92.5 cm will have error bounds of

(A) ± 0.05 cm
(B) ± 0.10 cm
(C) ± 0.15 cm
(D) ± 5 cm

31. Two events A and B are such that $P(A) = 0.5$, $P(B) = 0.16$, $P(A \cup B) = 0.48$. $P(A \cap B) =$

(A) 0.14
(B) 0.18
(C) 0.26
(D) 0.82

32. Given that H is a non-singular, square matrix, the determinant $|H^2|$ of H^2 is

(A) $2\,|H|$

(B) $\frac{1}{2}|H|$

(C) $|H|^2$

(D) $\frac{1}{|H|^2}$

33. In how many ways can the letters PQRSTU be arranged so that the T and U are always together?

(A) $\frac{6!}{2}$
(B) $5!$
(C) $6!$
(D) $5! \times 2$

34. The number of distinct permutations of the letters of the word POSSIBILITY is

(A) ${}^{11}P_{11}$

(B) $({}^{11}P_3)\,({}^{11}P_2)$

(C) $\frac{11!}{3!2!}$

(D) ${}^{11}P_8$

Item 35 refers to the table below which shows the number of males and females and their preference for Drink A and Drink B.

	Male	Female	Total
Drink A	12	18	30
Drink B	20	10	30
Total	32	28	60

35. One person is randomly selected. What is the probability that this person is female or prefers Drink B?

(A) $\frac{4}{5}$

(B) $\frac{1}{6}$

(C) $\frac{7}{30}$

(D) $\frac{29}{30}$

GO ON TO THE NEXT PAGE

Items 36 - 39 refer to the matrix M below.

$$M = \begin{pmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 0 & 1 & 5 \end{pmatrix}$$

36. The determinant of M is

(A) -21
(B) -9
(C) 9
(D) 21

37. The co-factor of the element 3 in M above may be written as

(A) $\begin{vmatrix} 1 & 4 \\ 0 & 5 \end{vmatrix}$

(B) $-\begin{vmatrix} 1 & 4 \\ 0 & 5 \end{vmatrix}$

(C) $\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix}$

(D) $-\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix}$

38. M^T is

(A) $\begin{pmatrix} 4 & 2 & 1 \\ 0 & 3 & -1 \\ 5 & 1 & 0 \end{pmatrix}$

(B) $\begin{pmatrix} -1 & -2 & -4 \\ 1 & -3 & 0 \\ 0 & -1 & -5 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 1 \\ 4 & 0 & 5 \end{pmatrix}$

(D) $\begin{pmatrix} -1 & 1 & 0 \\ -2 & 3 & -1 \\ -4 & 0 & -5 \end{pmatrix}$

39. The second row of M^2 is equal to the vector

(A) $(1 \quad 9 \quad 0)$
(B) $(-2 \quad 6 \quad 0)$
(C) $(2 \quad -11 \quad -4)$
(D) $(-4 \quad 7 \quad -4)$

40. If H is a non-singular, square matrix, then the determinant of H^T is

(A) $\frac{-1}{\det H}$

(B) $\frac{1}{\det H}$

(C) $-\det H$

(D) $\det H$

41. $\bar{z}$ is the conjugate of z. Which of the following are always true?

I. $|\bar{z}| = |z|$
II. $\arg z = \arg \bar{z}$
III. $z\bar{z}$ is real
IV. $\frac{z}{\bar{z}}$ is real

(A) I and II only
(B) I and III only
(C) II and IV only
(D) III and IV only

42. The locus of the points described by a complex number z is given by $|z-1-2i|=3$. The locus describes a circle with

(A) centre $(-1,-2)$ and radius 3 units
(B) centre $(-1,-2)$ and radius 9 units
(C) centre $(1, 2)$ and radius 3 units
(D) centre $(1, 2)$ and radius 9 units

43. The complex number $z=\sqrt{3}+i$ can be expressed as

(A) $\sqrt{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

(B) $\sqrt{2}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

(C) $2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

(D) $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

44. A relay team of five teachers is to be chosen from a group of fifteen teachers.

In how many ways could this relay team be chosen?

(A) $\frac{15!}{10!\,5!}$

(B) $\frac{15!}{10!}$

(C) $\frac{15!}{5!}$

(D) $15!$

45. If marbles are chosen, without replacement, from a bag of 11 blue and 9 red marbles, then the probability of getting a red marble followed by 2 blue marbles is

(A) $\frac{9}{20}+\frac{11}{19}+\frac{10}{18}$

(B) $\frac{9}{20}+\frac{11}{20}+\frac{10}{20}$

(C) $\frac{9}{20}\times\frac{11}{19}\times\frac{10}{18}$

(D) $\frac{9}{20}\times\frac{11}{20}\times\frac{10}{20}$

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.