Caribbean Advanced Proficiency Examinations
Internal Moderation Test 2013
Pure Mathematics Unit 2
Module 1

School: Saint Stanislaus College
Guyana

Time: 1 hour 30 minutes
Tutor: Rudolph Deoraj

Instruction to Candidates

Answer **ALL** questions. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.
This paper consists **7** questions. The maximum mark on this paper is **75**.

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1. (a) Showing all your working clearly, solve the equation

$$\mathrm{i}\,w^2 = (2-2\mathrm{i})^2.$$ [3]

(b) (i) Sketch an Argand diagram showing the region R consisting of points representing the complex numbers z where

$$|z-4-4\mathrm{i}| \le 2.$$ [2]

(ii) For the complex numbers z represented in the region R, it is given that

$$p \le |z| \le q.$$

Find the value of p and the value of q. [3]

(c) (i) Given that $z = \cos\theta + \mathrm{i}\sin\theta$, show that

$$z + \frac{1}{z} = 2\cos\theta.$$ [2]

(ii) State the value of $z^n + \dfrac{1}{z^n}$ in terms of θ. [1]

(iii) Hence, show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. [4]

Total 15 marks

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2. The curve C has equation $16y^3 + 9x^2y - 54x = 0$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [5]

(b) Find the coordinates of the points on C where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. [6]

Total 11 marks

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3. A curve C is given by the parametric equations

$$x = \frac{t}{2t+3}, \quad y = \frac{1}{\mathrm{e}^{2t}}, \ t \in \mathbb{R}.$$

(a) Find, in terms of t, $\frac{\mathrm{d}y}{\mathrm{d}x}$. [5]

(b) Find an equation of the normal to C at the point where $x = 0$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.
[4]

Total 9 marks

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4. (a) Given that $y = x \sin^{-1}(x) - \sqrt{(1 - x^2)}$,

show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \sin^{-1}(x) + \frac{2x}{\sqrt{(1-x^2)}}$. [6]

(b) $\mathrm{f}(x, y) = w = \ln(x^3 + y^3)$. Find, in a simplified form, $\frac{\partial^2 w}{\partial x^2}$. [5]

Total 11 marks

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5. (a) $$I = \int_{2}^{5} \frac{5}{x + \sqrt{(6 - x)}}\,\mathrm{d}x.$$

Using the substitution $u = \sqrt{(6 - x)}$, show that $I = \displaystyle\int_{1}^{2} \frac{10u}{(3 - u)(2 + u)}\,\mathrm{d}u.$ [4]

(b) Hence show that $I = 2\ln\left(\frac{9}{2}\right)$. [6]

Total 10 marks

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6. $$I_n = \int_{0}^{\frac{\pi}{2}} x^n \cos x \,\mathrm{d}x, \quad n \geq 0.$$

(a) Prove that, for $n \geq 2$, $I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2}$, $n \geq 2$. [6]

(b) Find the exact value of I_2. [4]

(c) Show that $I_4 = \frac{1}{16}\left(\pi^4 - 48\pi^2 + 384\right)$. [2]

Total 12 marks

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7.

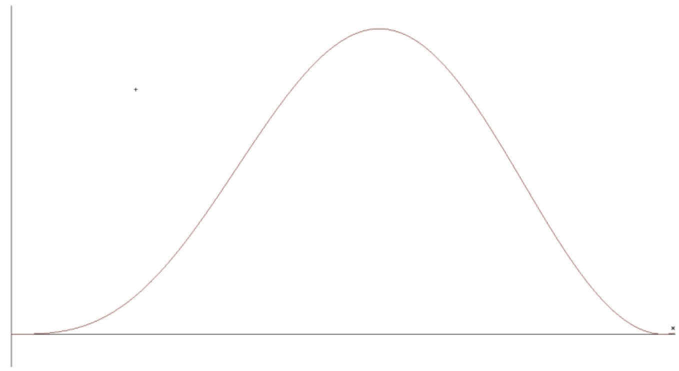

Figure 1

Figure 1 shows the curve C with equation

$$y = 5\sin^3 x\cos^2 x,\ 0 \le x \le \frac{\pi}{2}.$$

The finite region R is enclosed by C and the x-axis.

(a) Show that the area of R is given by

$$5\int_0^1 \left(u^2 - u^4\right)\mathrm{d}u.$$ [4]

(b) Hence find the exact area of R.

[3]

Total 7 marks

End of Test

Formulae

Trigonometric Identities:

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B) \neq (k + \tfrac{1}{2})\pi$$

Differentiation:

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}^1(x)\mathrm{g}(x) - \mathrm{g}'(x)\mathrm{f}(x)}{[\mathrm{g}(x)]^2}$
$\arcsin x$	$\dfrac{1}{\sqrt{(1 - x^2)}}$
$\arccos x$	$-\dfrac{1}{\sqrt{(1 - x^2)}}$
$\arctan x$	$\dfrac{1}{1 + x^2}$

Integration: (+ constant: $a > 0$ where relevant)

f (x)	$\int f(x)\, dx$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert = \ln\lvert\tan \tfrac{1}{2}x\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$\dfrac{1}{\sqrt{(a^2 - x^2)}}$	$\arcsin\left(\dfrac{x}{a}\right)$ $\quad \lvert x\rvert < a$
$\dfrac{1}{a^2 + x^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)$

Complex Numbers:

$e^{i\theta} = \cos\theta + i\sin\theta \qquad \left\{r(\cos\theta + i\sin\theta)^n\right\} = r^n \cos n\theta + i\sin n\theta$