

Data Structures Homework for ch. 2 (2):

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$$\begin{aligned} 1) \quad a. \sum_{i=0}^{n-1} (i^2 + 1)^2 &= \sum_{i=0}^{n-1} (i^4 + 1 + 2i^2) = \sum_{i=0}^{n-1} i^4 + \sum_{i=0}^{n-1} 1 + \sum_{i=0}^{n-1} 2i^2 \\ &= \sum_{i=0}^{n-1} i^4 + ((n-1) - 0 + 1) + 2 \sum_{i=0}^{n-1} i^2 \\ &= \sum_{i=0}^{n-1} i^4 + n + 2 \left(\frac{n(n+1)(2n+1)}{6} \right) = \sum_{i=0}^{n-1} i^4 + n + 1/3n^3 \\ &= 1/4n^4 + n + 1/3n^3 = 1/5n^5 + n + 1/3n^3 \approx \underline{\Theta(n^5)} \end{aligned}$$

$$\begin{aligned} b. \sum_{i=2}^{n-1} \lg(i^2) &= \sum_{i=2}^{n-1} 2 \lg(i) = 2 \sum_{i=2}^{n-1} \lg(i) = 2 \lg(n) \\ &= 2\Theta(n \log n) - \Theta(\log n) \end{aligned}$$

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- 2) a. Computes the difference between Max and Min of Array
b. Basic Operation = Comparison
c. Executed $n-1$
d. Efficiency class is $\Theta(n)$
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$$\begin{aligned} 3) \quad a. \underline{S(n) = S(n-1) + 2} &= (S(n-2) + 2) + 2 = S(n-2) + 4 \\ &= (S(n-3) + 2) + 4 = S(n-3) + 6 \\ &\Rightarrow \underline{S(n-1)} = S(n-2) + 2x \\ &= S(n - (n-1)) + 2(n-1) \\ &= S(1) + S(n-1) \end{aligned}$$

b. Straight forward non-recursive has the same complexity $\Theta(n)$ as this recursive one.

$$\begin{aligned} 4) \quad a. R_n &= P_{n-1} \Rightarrow B_n = R_{n-1} = P_{n-2} \quad TR_1 = 1 \\ &\Rightarrow TR_n = R_n + B_n = P_{n-1} + P_{n-2} \quad TR_2 = 1 \quad 12 \\ \text{The Sequence } &\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144\} \end{aligned}$$

$$\underline{12 \text{ Months} = 144 \text{ pairs}}$$

$$\begin{aligned} 5) \quad \text{int} &= 2^{31} - 1 = 2147483647 \quad \text{long} = 2^{63} - 1 = 9223372036854775807 \\ \underline{f(46) = 1936311903} \quad &\underline{f(92) = 7540113809746346429} \end{aligned}$$