Roads and Libraries

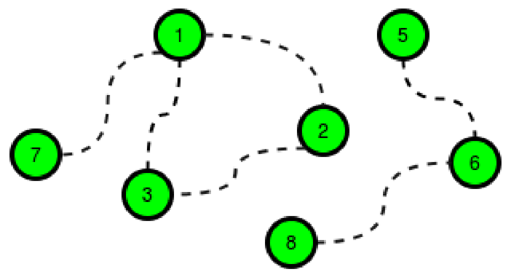
HackerRank

Determine the minimum cost to provide library access to all citizens of HackerLand. There are n cities numbered from 1 to n. Currently there are no libraries and the cities are not connected. Bidirectional roads may be built between any city pair listed in $cities$. A citizen has access to a library if:

- Their city contains a library.
- They can travel by road from their city to a city containing a library.

Example

The following figure is a sample map of HackerLand where the dotted lines denote possible roads:



$c\_road = 2$
$c\_lib = 3$
$cities = [[1,7],[1,3],[1,2],[2,3],[5,6],[6,8]]$

The cost of building any road is $cc\_road = 2$, and the cost to build a library in any city is $c\_lib = 3$. Build 5 roads at a cost of $5 \times 2 = 10$ and 2 libraries for a cost of 6. One of the available roads in the cycle $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$ is not necessary.

There are q queries, where each query consists of a map of HackerLand and value of c_lib and c_road. For each query, find the minimum cost to make libraries accessible to all the citizens.

Function Description

Complete the function *roadsAndLibraries* in the editor below.
roadsAndLibraries has the following parameters:

- *int n*: integer, the number of cities
- *int c_lib*: integer, the cost to build a library
- *int c_road*: integer, the cost to repair a road
- *int cities[m][2]*: each $cities[i]$ contains two integers that represent cities that can be connected by a new road

Returns
- *int*: the minimal cost

Input Format

The first line contains a single integer q, that denotes the number of queries.

The subsequent lines describe each query in the following format:
- The first line contains four space-separated integers that describe the respective values of n, m, c_lib and c_road, the number of cities, number of roads, cost of a library and cost of a road.
- Each of the next m lines contains two space-separated integers, $u[i]$ and $v[i]$, that describe a bidirectional road that can be built to connect cities $u[i]$ and $v[i]$.

Constraints

- $1 \leq q \leq 10$
- $1 \leq n \leq 10^5$
- $0 \leq m \leq min(10^5, \frac{n \cdot (n-1)}{2})$
- $1 \leq c\_road, c\_lib \leq 10^5$
- $1 \leq u[i], v[i] \leq n$
- Each road connects two distinct cities.

Sample Input

```
STDIN       Function
-----       --------
2           q = 2
3 3 2 1     n = 3, cities[] size m = 3, c_lib = 2, c_road = 1
1 2         cities = [[1, 2], [3, 1], [2, 3]]
3 1
2 3
6 6 2 5     n = 6, cities[] size m = 6, c_lib = 2, c_road = 5
1 3         cities = [[1, 3], [3, 4],...]
3 4
2 4
1 2
2 3
5 6
```

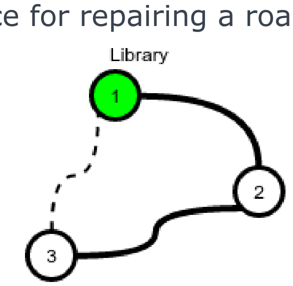
Sample Output

```
4
12
```

Explanation

Perform the following $q = 2$ queries:

1. HackerLand contains $n = 3$ cities and can be connected by $m = 3$ bidirectional roads. The price of building a library is $c_{lib} = 2$ and the price for repairing a road is $c_{road} = 1$.

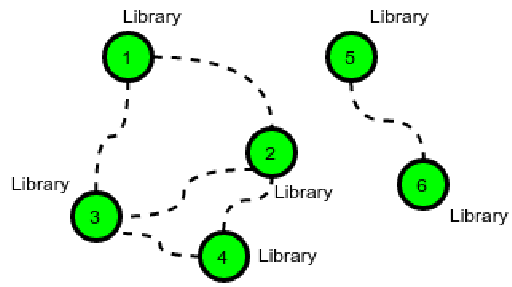


The cheapest way to make libraries accessible to all is to:

- Build a library in city 1 at a cost of $x = 2$.
- Build the road between cities 1 and 2 at a cost of $y = 1$.
- Build the road between cities 2 and 3 at a cost of $y = 1$.

This gives a total cost of $2 + 1 + 1 = 4$. Note that the road between cities 3 and 1 does not need to be built because each is connected to city 2.

2. In this scenario it is optimal to build a library in each city because the cost to build a library is less than the cost to build a road.



There are 6 cities, so the total cost is $6 \times 2 = 12$.