

TIMBER BEAM N. 001

Caratteristiche meccaniche legno GL24c←←		
$\gamma_M = 1.35$ $k_{mod} = 0.8$		
$f_{m,k} = 24 \text{ MPa}$	$f_{m,d} = 14.22 \text{ MPa}$	$E_{0,mean} = 11000 \text{ MPa}$
$f_{t,0,k} = 17 \text{ MPa}$	$f_{t,0,d} = 10.07 \text{ MPa}$	$E_{0,05} = 9100 \text{ MPa}$
$f_{t,90,k} = 0.5 \text{ MPa}$	$f_{t,90,d} = 0.296 \text{ MPa}$	$E_{90,mean} = 300 \text{ MPa}$
$f_{c,0,k} = 21.5 \text{ MPa}$	$f_{c,0,d} = 12.74 \text{ MPa}$	$G_{mean} = 650 \text{ MPa}$
$f_{c,90,k} = 2.5 \text{ MPa}$	$f_{c,90,d} = 1.48 \text{ MPa}$	$r_k = 365 \text{ kg/m}^3$
$f_{v,k} = 3.5 \text{ MPa}$	$f_{v,d} = 2.07 \text{ MPa}$	$r_{mean} = 400 \text{ kg/m}^3$

Caratteristiche sezione rettangolare	
$b = 160 \text{ mm}$ $h = 840 \text{ mm}$ $\gamma = 5 \frac{\text{kN}}{\text{m}^3}$	
$A = 1344 \text{ cm}^2$	$g_{1,k} = 0.672 \text{ kN/m}$
$I_y = 790272 \text{ cm}^4$	$I_z = 28672 \text{ cm}^4$
$W_y = 18816 \text{ cm}^3$	$W_z = 3584 \text{ cm}^3$
$i_y = 24.25 \text{ cm}$	$i_z = 4.62 \text{ cm}$

$$l = 8.79 \text{ m}$$

$$k_m = 0.7 \text{ legno massiccio, sezioni rettangolari}$$

$$\beta_c = 0.1 \text{ legno lammellare}$$

$$l_{0,y} = 8.79 \text{ m}$$

$$l_{0,z} = 8.79 \text{ m}$$

$$\beta = 1 \text{ coef. di vincolo}$$

Azioni agenti

$$M_{Ed,y} = 218.47 \text{ kNm}$$

$$M_{Ed,z} = 0 \text{ kNm}$$

$$N_{Ed} = 0 \text{ kN}$$

$$V_{Ed,y} = 0 \text{ kN}$$

$$V_{Ed,z} = 73.27 \text{ kN}$$

Tensioni agenti

$$\sigma_{c,0,d} = \frac{N_{Ed}}{A} = \frac{0 \text{ kN}}{1344 \text{ cm}^2} = 0 \text{ MPa}$$

$$\sigma_{m,y,d} = \frac{M_{Ed,y}}{W_y} = \frac{218.47 \text{ kNm}}{18816 \text{ cm}^3} = 11.61 \text{ MPa}$$

$$\sigma_{m,z,d} = \frac{M_{Ed,z}}{W_z} = \frac{0 \text{ kNm}}{3584 \text{ cm}^3} = 0 \text{ MPa}$$

$$k_{cr} = \frac{2.5 \text{ MPa}}{f_{v,k}} = \frac{2.5 \text{ MPa}}{3.5 \text{ MPa}} = 0.714$$

$$\tau_{y,d} = \frac{1.5 \cdot V_{Ed,y}}{k_{cr} \cdot A} = \frac{1.5 \cdot 0 \text{ kN}}{0.714 \cdot 1344 \text{ cm}^2} = 0 \text{ MPa}$$

$$\tau_{z,d} = \frac{1.5 \cdot V_{Ed,z}}{k_{cr} \cdot A} = \frac{1.5 \cdot 73.27 \text{ kN}}{0.714 \cdot 1344 \text{ cm}^2} = 1.14 \text{ MPa}$$

Verifica flessione

$$\frac{\sigma_{m,y,d} + k_m \cdot \sigma_{m,z,d}}{f_{m,d}} = \frac{11.61 \text{ MPa} + 0.7 \cdot 0 \text{ MPa}}{14.22 \text{ MPa}} = 81.64 \% \text{ Ok } \checkmark$$

$$k_m \cdot \frac{\sigma_{m,y,d}}{f_{m,d}} + \frac{\sigma_{m,z,d}}{f_{m,d}} = 0.7 \cdot \frac{11.61 \text{ MPa}}{14.22 \text{ MPa}} + \frac{0 \text{ MPa}}{14.22 \text{ MPa}} = 57.15 \% \text{Ok} \checkmark$$

Verifica flessione e compressione combinata

$$\left(\frac{\sigma_{c,0,d}}{f_{c,0,d}} \right)^2 + \frac{\sigma_{m,y,d}}{f_{m,d}} + k_m \cdot \frac{\sigma_{m,z,d}}{f_{m,d}} = \left(\frac{0 \text{ MPa}}{12.74 \text{ MPa}} \right)^2 + \frac{11.61 \text{ MPa}}{14.22 \text{ MPa}} + 0.7 \cdot \frac{0 \text{ MPa}}{14.22 \text{ MPa}} = 81.64 \% \text{Ok} \checkmark \quad [\text{EC5 §6.2.4}]$$

$$\left(\frac{\sigma_{c,0,d}}{f_{c,0,d}} \right)^2 + k_m \cdot \frac{\sigma_{m,y,d}}{f_{m,d}} + \frac{\sigma_{m,z,d}}{f_{m,d}} = \left(\frac{0 \text{ MPa}}{12.74 \text{ MPa}} \right)^2 + 0.7 \cdot \frac{11.61 \text{ MPa}}{14.22 \text{ MPa}} + \frac{0 \text{ MPa}}{14.22 \text{ MPa}} = 57.15 \% \text{Ok} \checkmark$$

Verifica a taglio

$$\frac{\tau_{y,d}}{f_{v,d}} = \frac{0 \text{ MPa}}{2.07 \text{ MPa}} = 0 \% \text{Ok} \checkmark$$

$$\frac{\tau_{z,d}}{f_{v,d}} = \frac{1.14 \text{ MPa}}{2.07 \text{ MPa}} = 55.2 \% \text{Ok} \checkmark$$

Verifica instabilità di colonna

$$\lambda_y = \frac{l_{0,y}}{i_y} = \frac{8.79 \text{ m}}{24.25 \text{ cm}} = 36.23 \quad [\text{EC5 §6.3.2}]$$

$$\lambda_{\text{rel},y} = \frac{\lambda_y}{\pi} \sqrt{\frac{f_{c,0,k}}{E_{0,05}}} = \frac{36.23}{3.14} \sqrt{\frac{21.5 \text{ MPa}}{9100 \text{ MPa}}} = 0.561$$

$$k_y = 0.5 \cdot (1 + \beta_c \cdot (\lambda_{\text{rel},y} - 0.3) + \lambda_{\text{rel},y}^2) = 0.5 \cdot (1 + 0.1 \cdot (0.561 - 0.3) + 0.561^2) = 0.67$$

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y^2 - \lambda_{\text{rel},y}^2}} = \frac{1}{0.67 + \sqrt{0.67^2 - 0.561^2}} = 0.964$$

$$\lambda_z = \frac{l_{0,z}}{i_z} = \frac{8.79 \text{ m}}{4.62 \text{ cm}} = 190.22$$

$$\lambda_{\text{rel},z} = \frac{\lambda_z}{\pi} \sqrt{\frac{f_{c,0,k}}{E_{0,05}}} = \frac{190.22}{3.14} \sqrt{\frac{21.5 \text{ MPa}}{9100 \text{ MPa}}} = 2.94$$

$$k_z = 0.5 \cdot (1 + \beta_c \cdot (\lambda_{\text{rel},z} - 0.3) + \lambda_{\text{rel},z}^2) = 0.5 \cdot (1 + 0.1 \cdot (2.94 - 0.3) + 2.94^2) = 4.96$$

$$k_{c,z} = \frac{1}{k_z + \sqrt{k_z^2 - \lambda_{\text{rel},z}^2}} = \frac{1}{4.96 + \sqrt{4.96^2 - 2.94^2}} = 0.112$$

$$\frac{\sigma_{c,0,d}}{k_{c,y} \cdot f_{c,0,d}} + \frac{\sigma_{m,y,d}}{f_{m,d}} + k_m \cdot \frac{\sigma_{m,z,d}}{f_{m,d}} = \frac{0 \text{ MPa}}{0.964 \cdot 12.74 \text{ MPa}} + \frac{11.61 \text{ MPa}}{14.22 \text{ MPa}} + 0.7 \cdot \frac{0 \text{ MPa}}{14.22 \text{ MPa}} = 81.64 \% \text{Ok} \checkmark$$

$$\frac{\sigma_{c,0,d}}{k_{c,z} \cdot f_{c,0,d}} + k_m \cdot \frac{\sigma_{m,y,d}}{f_{m,d}} + \frac{\sigma_{m,z,d}}{f_{m,d}} = \frac{0 \text{ MPa}}{0.112 \cdot 12.74 \text{ MPa}} + 0.7 \cdot \frac{11.61 \text{ MPa}}{14.22 \text{ MPa}} + \frac{0 \text{ MPa}}{14.22 \text{ MPa}} = 57.15 \% \text{Ok} \checkmark$$

Verifica instabilità di trave

$$l_{\text{ef}} = \beta \cdot l = 1 \cdot 8.79 \text{ m} = 8.79 \text{ m} \quad [\text{EC5 §6.3.3}]$$

$$\sigma_{m,\text{crit}} = \frac{0.78 \cdot b^2}{h \cdot l_{\text{ef}}} \cdot E_{0,05} = \frac{0.78 \cdot (160 \text{ mm})^2}{840 \text{ mm} \cdot 8.79 \text{ m}} \cdot 9100 \text{ MPa} = 24.62 \text{ MPa}$$

$$\lambda_{\text{rel},m} = \sqrt{\frac{f_{m,k}}{\sigma_{m,\text{crit}}}} = \sqrt{\frac{24 \text{ MPa}}{24.62 \text{ MPa}}} = 0.987$$

$$k_{\text{crit}} = \begin{cases} \text{if } \lambda_{\text{rel},m} < 0.75: 1 \\ \text{if } \lambda_{\text{rel},m} < 1.4: 1.56 - 0.75 \cdot \lambda_{\text{rel},m} \\ \text{else: } \frac{1}{\lambda_{\text{rel},m}^2} \end{cases} = \begin{cases} \text{if } 0.987 < 0.75: 1 \\ \text{if } 0.987 < 1.4: 1.56 - 0.75 \cdot 0.987 \\ \text{else: } \frac{1}{0.987^2} \end{cases} = 0.82$$

$$\frac{\sigma_{m,y,d}}{k_{\text{crit}} \cdot f_{m,d}} = \frac{11.61 \text{ MPa}}{0.82 \cdot 14.22 \text{ MPa}} = 99.62 \% \text{Ok} \checkmark$$

$$\left(\frac{\sigma_{m,y,d}}{k_{crit} \cdot f_{m,d}}\right)^2 + \frac{\sigma_{c,0,d}}{k_{c,z} \cdot f_{c,0,d}} = \left(\frac{11.61 \text{ MPa}}{0.82 \cdot 14.22 \text{ MPa}}\right)^2 + \frac{0 \text{ MPa}}{0.112 \cdot 12.74 \text{ MPa}} = 99.24 \% \text{ Ok } \checkmark$$

Freccia

$$q_G = 8.64 \frac{\text{kN}}{\text{m}}$$

$$q_{Q1} = 10.92 \frac{\text{kN}}{\text{m}}$$

$$k_{def} = 0.6$$

$$\psi_{21} = 0.2$$

$$w_{lim,inst} = \frac{l}{300} = \frac{8.79 \text{ m}}{300} = 29.29 \text{ mm}$$

$$w_{lim,fin} = \frac{l}{200} = \frac{8.79 \text{ m}}{200} = 43.93 \text{ mm}$$

$$w(q) = \frac{5}{384} \cdot \frac{q \cdot l^4}{E_{0,mean} \cdot I_y} + 1.2 \cdot \frac{1}{8} \cdot \frac{q \cdot l^2}{G_{mean} \cdot A}$$

$$w\left(1 \frac{\text{kN}}{\text{m}}\right) = 1.03 \text{ mm}$$

$$w_{inst,G} = w(q_G) = w(8.64 \text{ kN/m}) = 8.86 \text{ mm}$$

$$w_{inst,Q1} = w(q_{Q1}) = w(10.92 \text{ kN/m}) = 11.19 \text{ mm}$$

$$w_{inst,TOT} = w_{inst,G} + w_{inst,Q1} = 8.86 \text{ mm} + 11.19 \text{ mm} = 20.05 \text{ mm}$$

$$w_{inst,TOT} = 20.05 \text{ mm} \leq w_{lim,inst} = 29.29 \text{ mm} \text{ Ok } \checkmark$$

$$w_{fin,G1} = w_{inst,G} \cdot (1 + k_{def}) = 8.86 \text{ mm} \cdot (1 + 0.6) = 14.17 \text{ mm}$$

$$w_{fin,Q1} = w_{inst,Q1} \cdot (1 + \psi_{21} \cdot k_{def}) = 11.19 \text{ mm} \cdot (1 + 0.2 \cdot 0.6) = 12.54 \text{ mm}$$

$$w_{fin,TOT} = w_{fin,G1} + w_{fin,Q1} = 14.17 \text{ mm} + 12.54 \text{ mm} = 26.71 \text{ mm}$$

$$w_{fin,TOT} = 26.71 \text{ mm} \leq w_{lim,fin} = 43.93 \text{ mm} \text{ Ok } \checkmark$$