Ansys Mechanical Linear and Nonlinear Dynamics

Module 08: Random Vibration Analysis

Release 2022 R2

Please note:
- These training materials were developed and tested in Ansys Release 2022 R2. Although they are expected to behave similarly in later releases, this has not been tested and is not guaranteed.
- The screen images included with these training materials may vary from the visual appearance of a local software session.

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Module 08 Learning Outcomes

- After completing this module, you will:
  - Be able to recognize applications most suited to the Random Vibration analysis technique.
  - Understand the statistical basis of the Random Vibration solution technique and its implication on the interpretation of results.
  - Learn to recognize when an input PSD curve contains insufficient data and how to correct it.
  - Understand how damping is accounted for in a Random Vibration analysis.
  - Be able to extract RMS response information from discrete locations within a model for purposes of comparing against design allowables.

Module 08 Topics

A. Definition and Purpose of Random Vibration Analysis

B. Power Spectral Density (PSD)

C. Theory Overview
  - Excitation Distribution
  - Random Vibration

D. PSD Curve Fitting

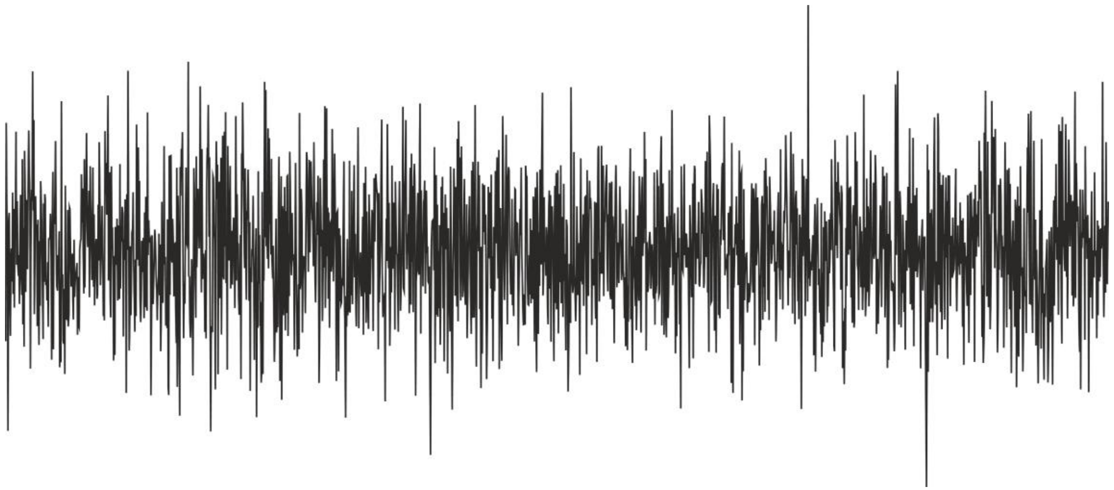
E. Analysis Settings

F. Loads and Supports

G. Results

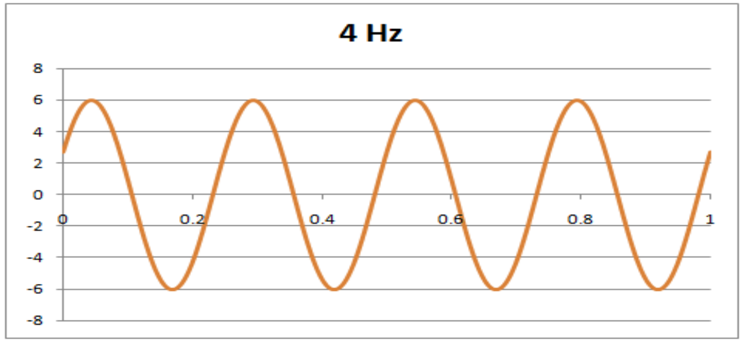
A. Definition and Purpose of Random Vibration Analysis

- Random vibration analysis is another spectral method
- The purpose of a random vibration analysis is to determine some *statistical properties of a structural response*, normally the standard deviation (1σ) of a displacement, force, or stress.
- (1σ) is used to determine fatigue life of a structure

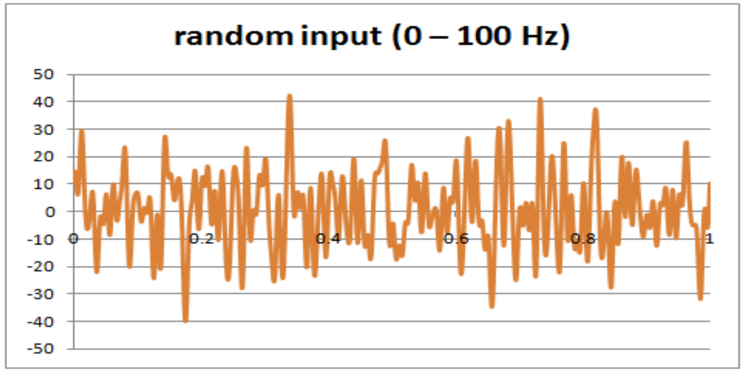


… Definition and Purpose of Random Vibration Analysis

- We have already seen sinusoidal vibration (free and forced)
  - This is vibration at one predominant frequency



- A more common type of vibration is random vibration
  - This is vibration at many frequencies simultaneously



… Definition and Purpose of Random Vibration Analysis

- Many common processes result in random vibration
  - Parts on a manufacturing line
  - Vehicles travelling on a roadway
  - Airplanes flying or taxiing
  - Spacecraft during launch

- These random vibrations *contain all frequencies at all times*
- The amplitudes at these frequencies vary randomly with time.
  - *We need some way of describing and quantifying this excitation*.

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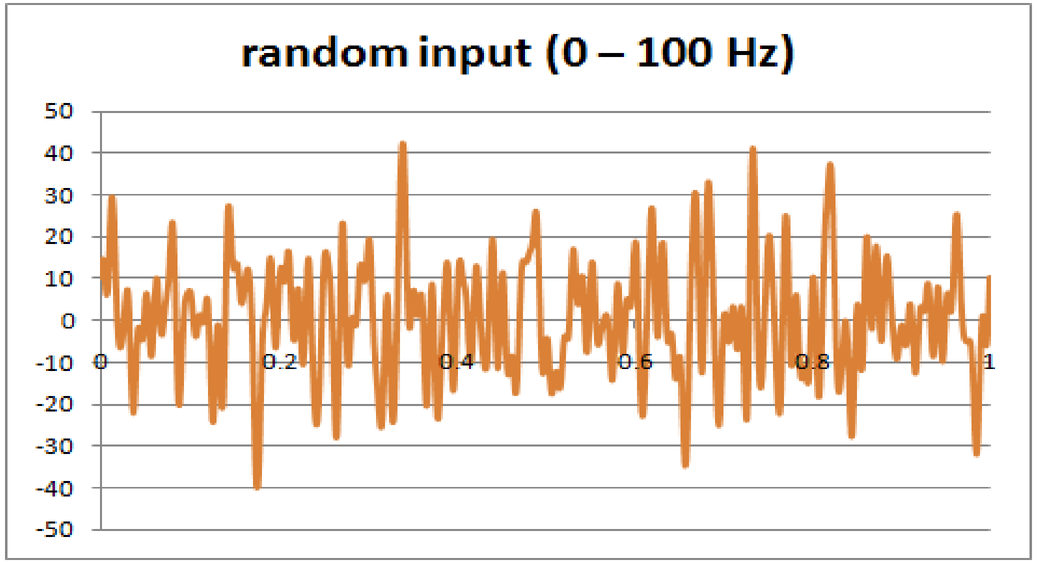
… Definition and Purpose of Random Vibration Analysis

- If the amplitude is constantly changing, how can a random excitation be evaluated?



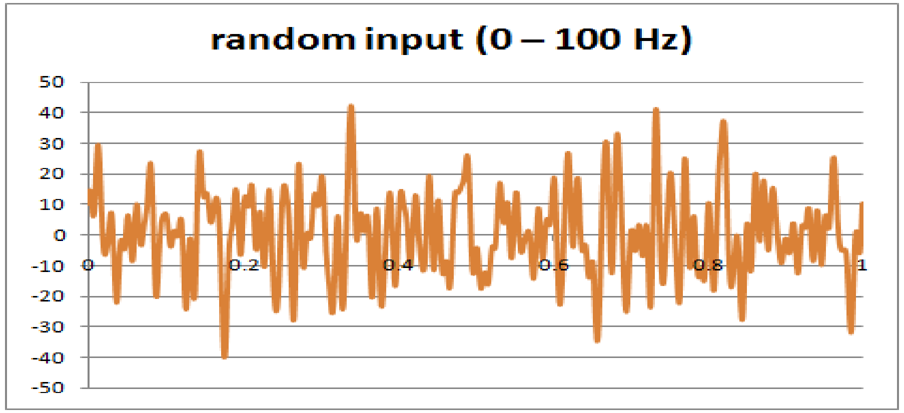
- *<u>Key observation:</u> at a given frequency*, the amplitude of the excitation does constantly change, but for many processes, its *average value* tends to remain *relatively constant*.
  - This gives us the ability to characterize random excitation as a statistical process.

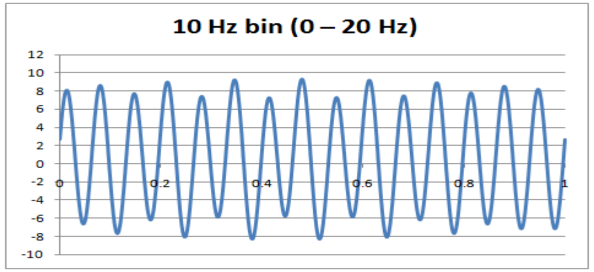
… Definition and Purpose of Random Vibration Analysis

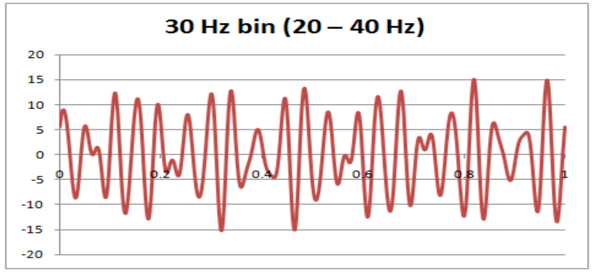
- Random excitation can be characterized statistically in terms of a Power Spectral Density plot
  - PSD amplitude versus frequency
- PSD spectra plots are generally supplied
  - Design spec, building code, etc.
- Ansys does not provide tools for generating PSD spectra plots, but a general approach will be described in next several slides

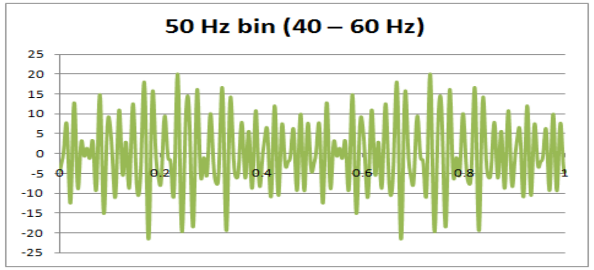
B. Power Spectral Density (PSD)

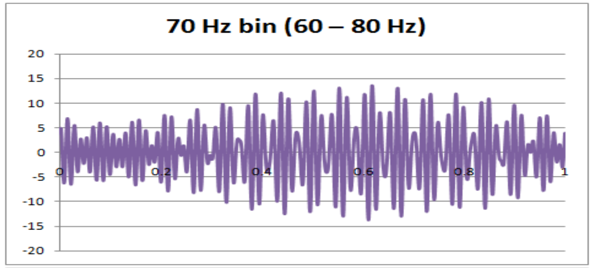
- The total frequency range is split into individual ranges (called bins).
  - this can be done using bandbass filters
  - real analyzers typically have hundreds of bins

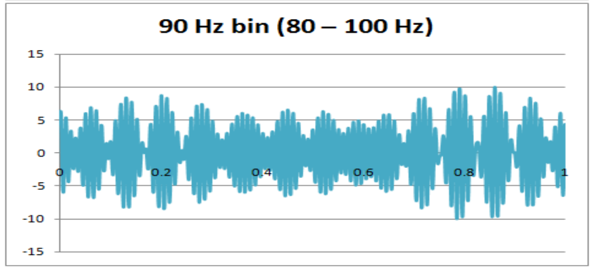






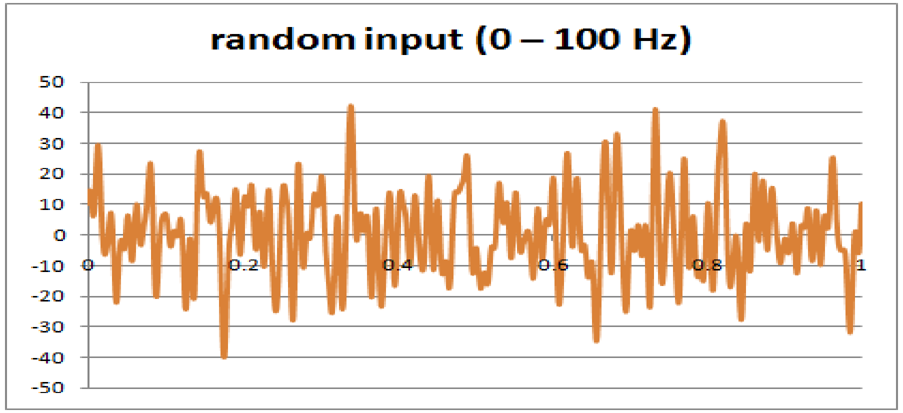


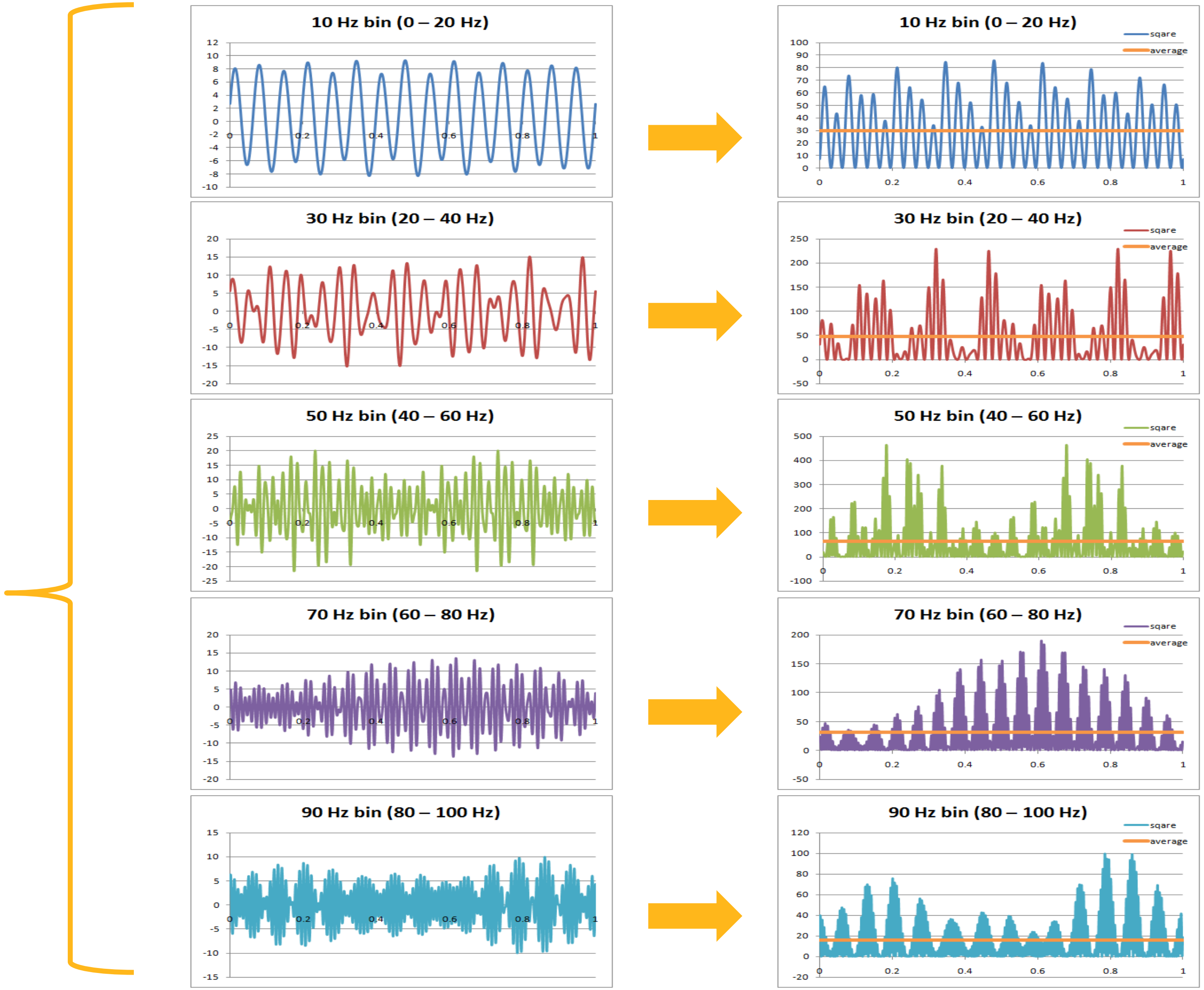




… Power Spectral Density (PSD)

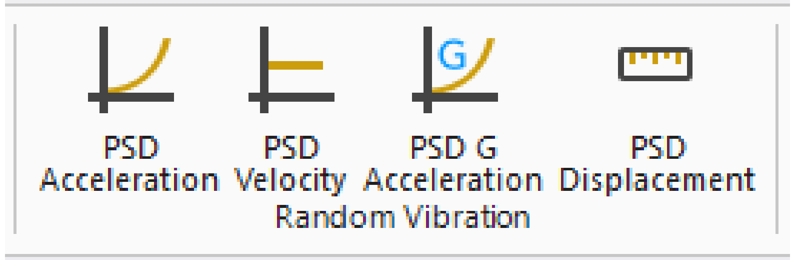
- The excitation is squared and the average is calculated for each bin.
  - called the mean square
  - *gives (units RMS)^2*





… Power Spectral Density (PSD)

- If a wider bin were used, the average value would be larger
  - a consistent definition is needed to account for different bin sizes
- Consequently, the average squared amplitudes are divided by the bin bandwidth
  - gives (units RMS)^2/Hz
- The "RMS" is generally dropped
  - leaves units^2/Hz
- For structural vibrations, the units may be
  - Acceleration e.g., [(mm/s^2)^2/Hz] or [G^2/Hz]
  - Velocity e.g., [(mm/s)^2/Hz]
  - Displacement e.g., [(mm)^2/Hz]
  - Force* e.g., [N^2/Hz]



***Currently not available as an excitation input in Mechanical**

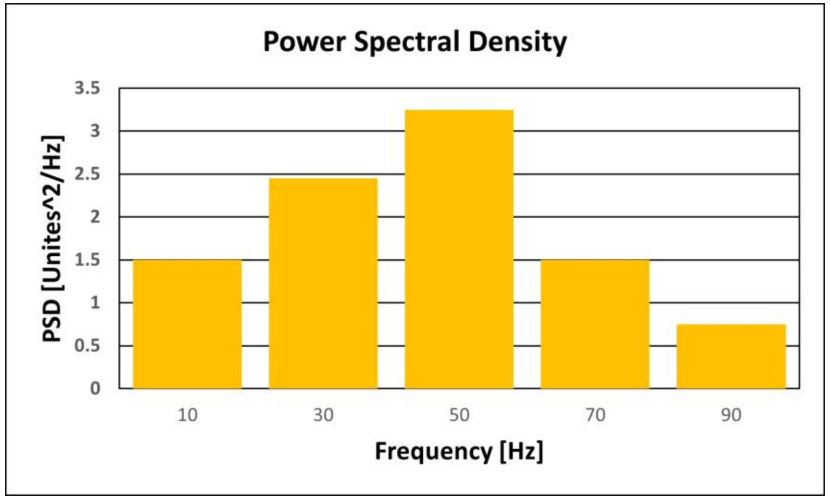
… Power Spectral Density (PSD)

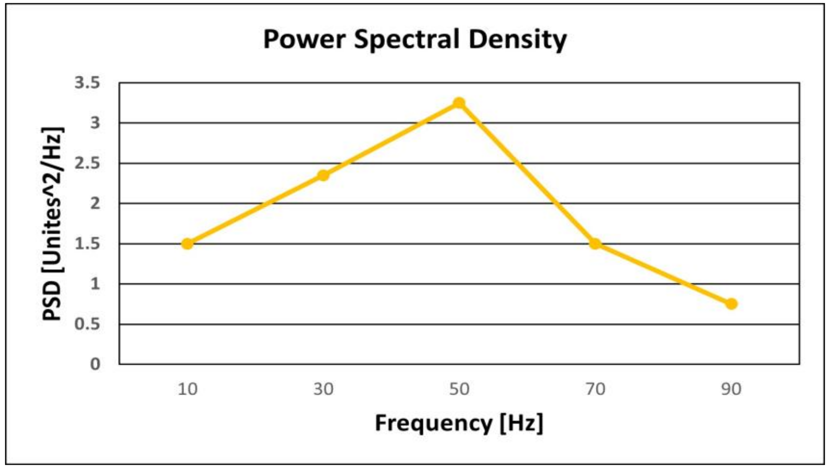
- The value (units^2/Hz) is plotted as a function of the bin frequency.
  - each bin is referred to by its center frequency.

- A line could be used to represent the same graph.

- The convention is to use a line graph in log-log plot.

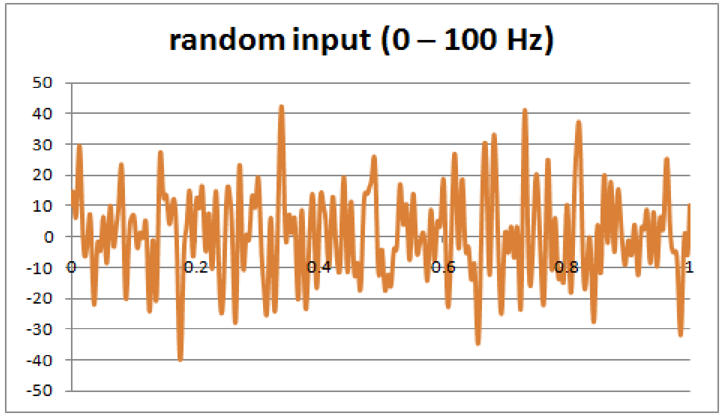
- Although the process is truly random, it obeys the limits defined in the plot.

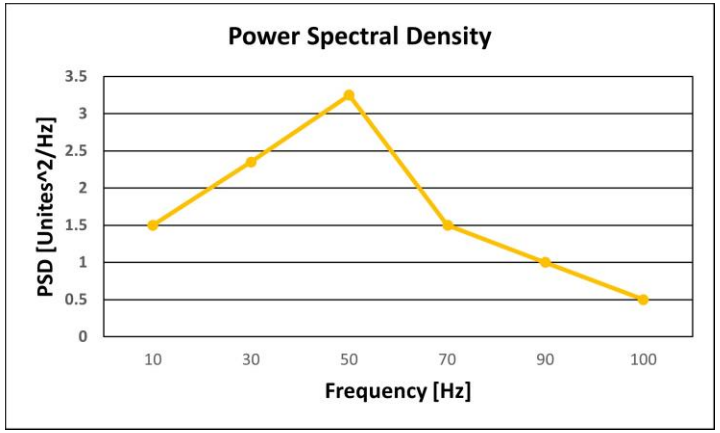




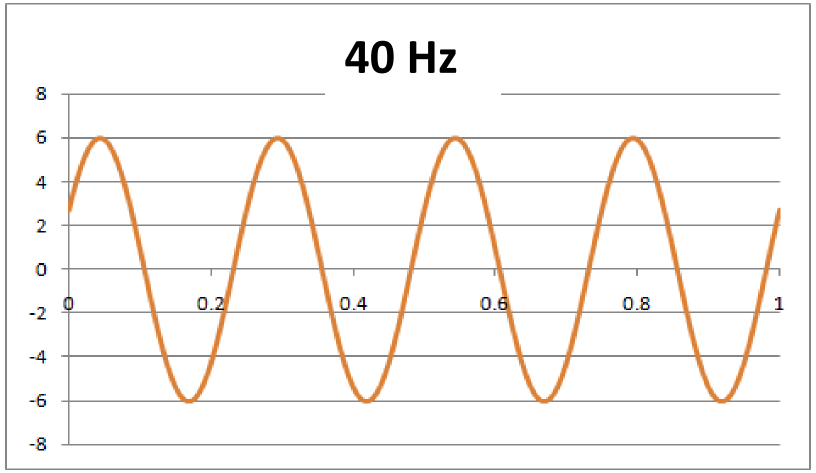
… Power Spectral Density (PSD)

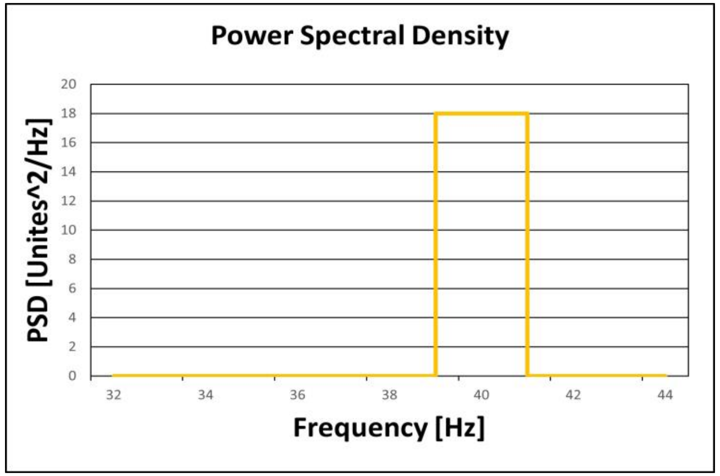
- The representation of the random excitation is called its Power Spectral Density.





- By comparison, a singe sinusoid would result in a narrow flat PSD.
  - For a bandwidth of 1 Hz, the PSD value would be the RMS amplitude squared. PSD=(6/sqrt(2))^2=18

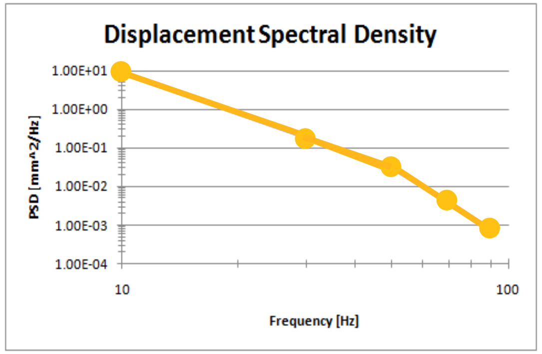




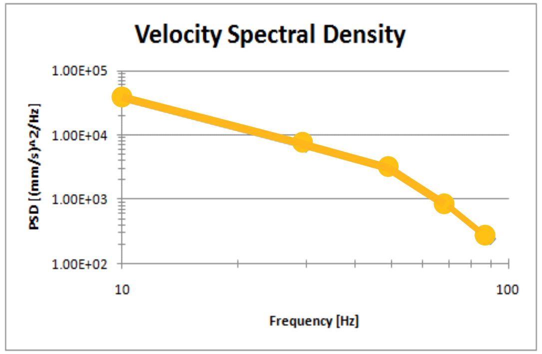
… Power Spectral Density (PSD)

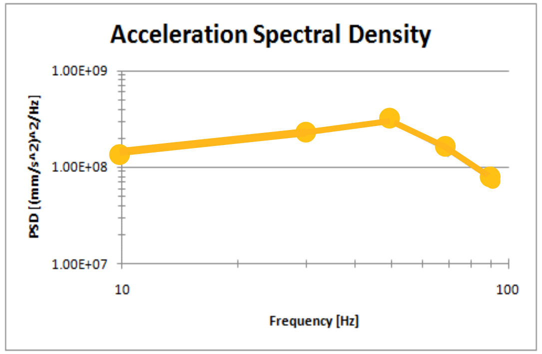
- We can easily convert between acceleration (including G acceleration), velocity, and displacement spectra by multiplying or dividing by the square of the frequency.
  - *remember to convert frequency units: ω rad/s = 2πf Hz*

$$S_d = S_v/(2\pi f)^2 = S_a/(2\pi f)^4$$

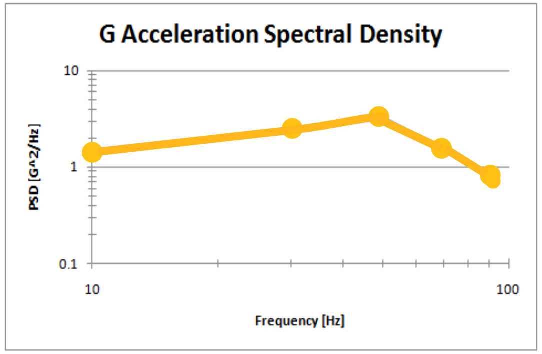


$$S_v = S_d(2\pi f)^2 = S_a/(2\pi f)^2$$





$$S_a = S_d(2\pi f)^4 = S_v(2\pi f)^2$$



$$S_G = S_a/g^2$$

C. Theory Overview

Assumptions & Restrictions

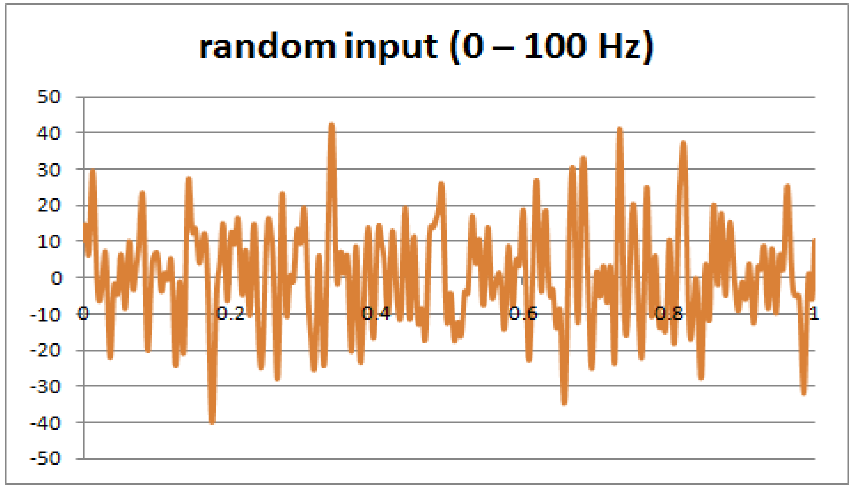
- The structure has
  - no random properties
  - no time varying stiffness, damping, or mass
  - no time varying forces, displacement, pressures, temperatures, etc. applied
  - light damping
    - damping forces are much smaller than inertial and elastic forces

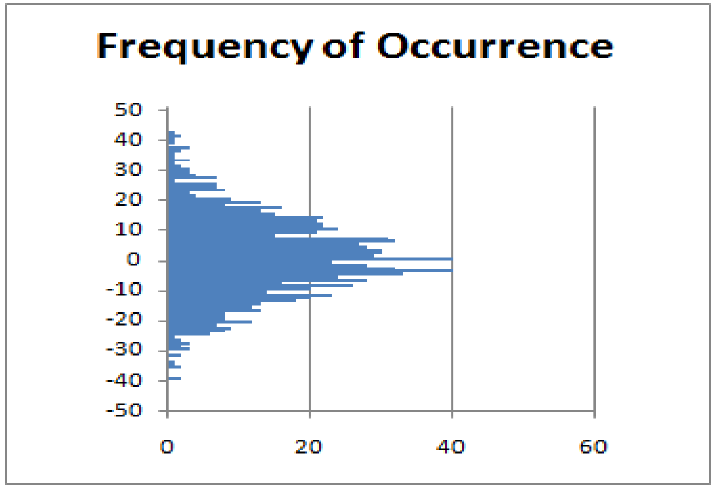
- The random process is
  - stationary (does not change with time)
    - the response will also be a stationary random process
  - ergodic (one sample tells us everything about the random process)

… Theory Overview

1. Excitation Distribution

- A key concept is the fact that many random processes follow a Gaussian distribution.

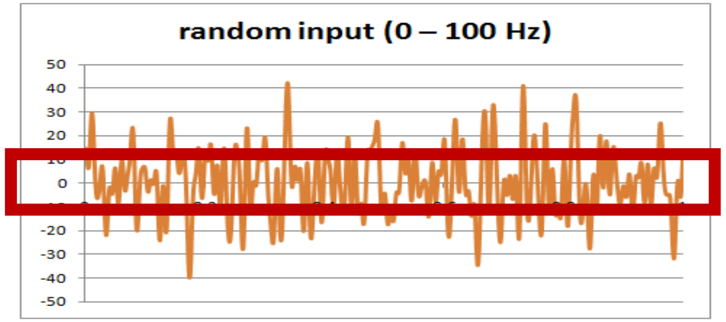


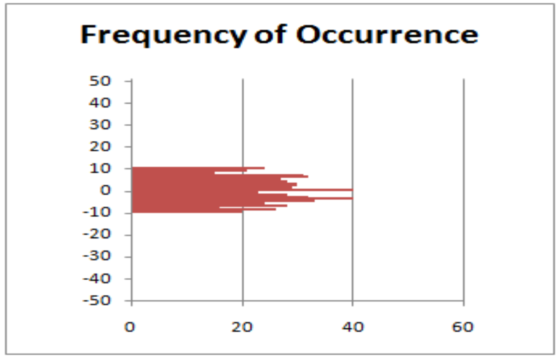


- The mean value of a Gaussian probability curve is defined as the standard deviation (or sigma value) of the distribution.
  - By taking multiples of sigma, we can account for a greater percentage of all possible excitations.

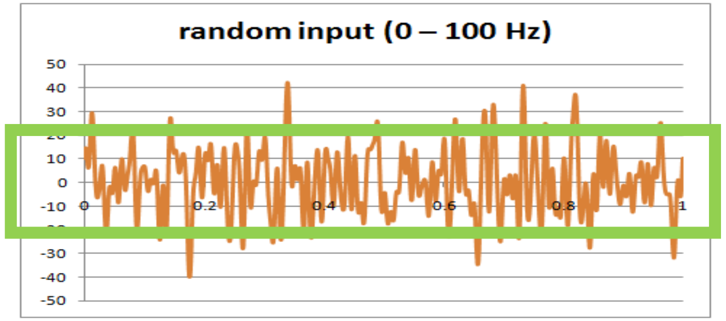
… Theory Overview

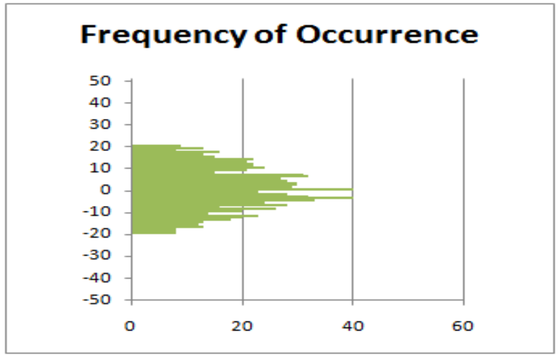
… Excitation Distribution



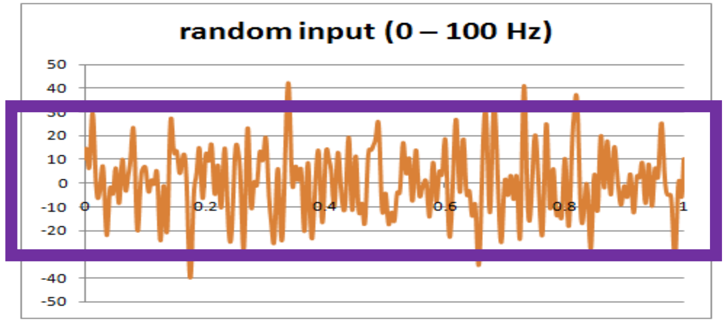


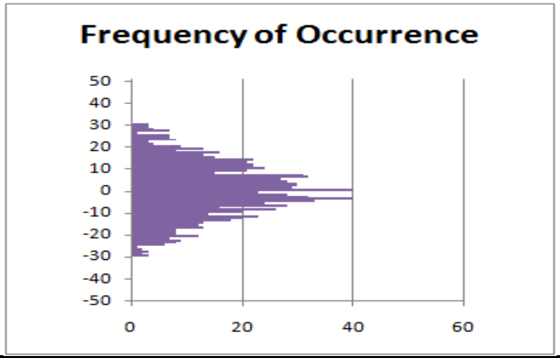
±1 sigma:
~ 68.27 %





±2 sigma:
~ 95.45 %



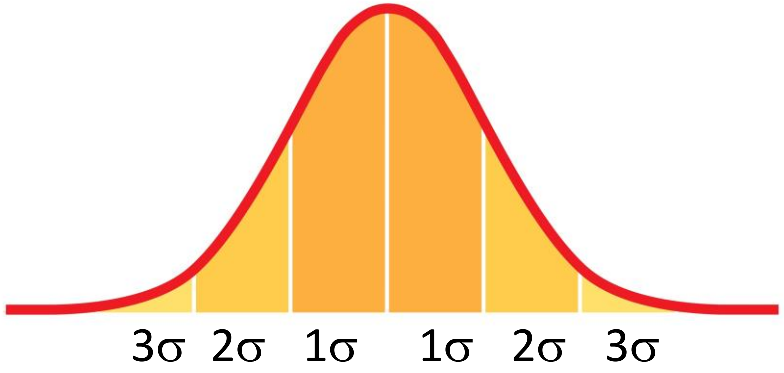


±3 sigma:
~ 99.73 %

… Theory Overview

… <u>Excitation Distribution</u>

- Because the distribution is assumed to be normal, we can never account for 100% of the possible excitations.
  - In reality, the distribution of excitations is more likely truncated.
  - Furthermore, high-sigma excitations occur very rarely.

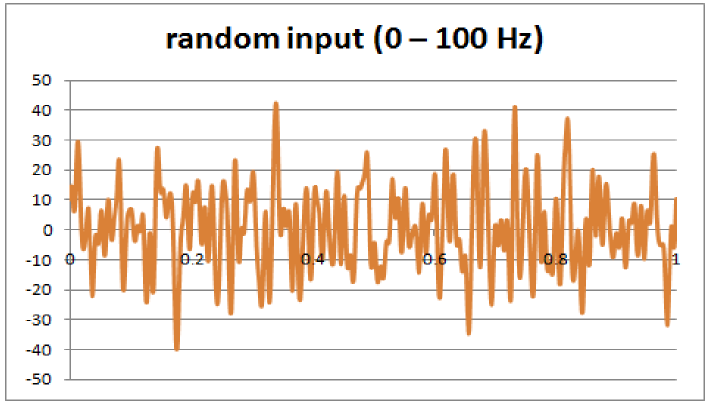


- For these reasons, it is common to use *3 sigma* as the upper limit.
- Important property of Gaussian distribution:
  - if the excitation of a *linear system* is a *Gaussian process*, then the response is generally a different random process, but *still* a *normal* one

… Theory Overview

2. Random Vibration

- We want to quantify the response of a system to random vibration.



- We must first quantify the response of a system to a deterministic excitation

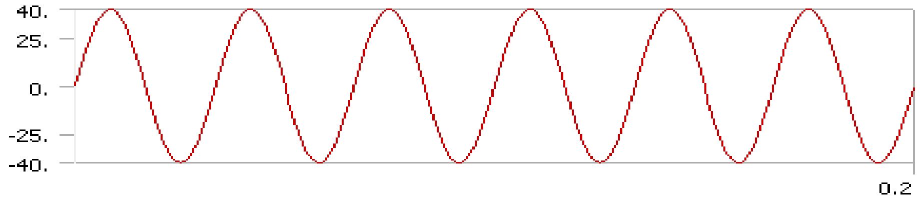
- We can describe the dynamic characteristics of a linear system by determining its steady-state response to a sinusoidal input

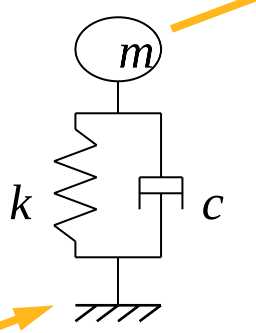
… Theory Overview

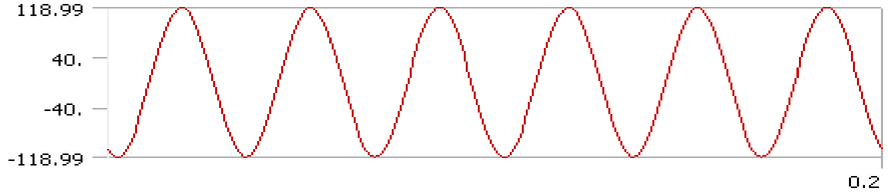
… Random Vibration

- Take a single DOF oscillator and subject it to a sinusoidal excitation:

example input motion:
ω = 30 Hz
a_{in} = 40 mm/s^2







corresponding output motion:
ω = 30 Hz
a_{out} = 119 mm/s^2, φ = 116°

- Information about the amplitude ratio (output/input) and the phase angle defines the dynamic characteristics of the system at this one frequency.
  - this is also called the transmission or transfer function
  - the input and output could be any quantity, not only acceleration

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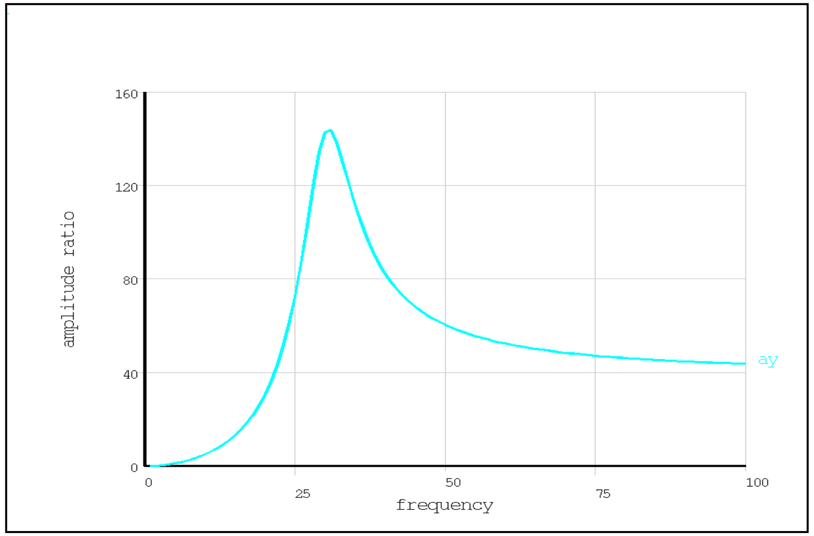
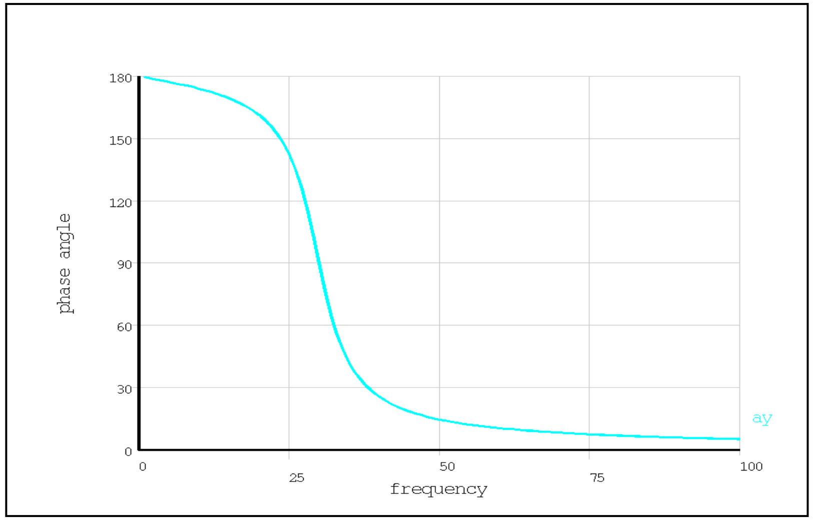
… Theory Overview

… <u>Random Vibration</u>

- We can sweep across a range of frequencies to determine how the response (amplitude and phase angle) changes with frequency.





- Theoretically, sweeping from a frequency of zero to infinity completely defines the dynamic characteristics.

… Theory Overview

… Random Vibration

- We have described amplitude and phase angle separately, but they can also be described as a single complex number, called the (complex) *frequency response function* $H(\omega)$ *(FRF).*

$$H(\omega) = A(\omega) - iB(\omega)$$

- By definition,
  - the magnitude of the FRF is equal to the amplitude ratio, and
  - the ratio of FRF imaginary part to its real part is equal to the tangent of the phase angle.

$$|H(\omega)| = \sqrt{A^2 + B^2} = \frac{a_{out}}{a_{in}}$$

$$\frac{\mathrm{Im}[H(\omega)]}{\mathrm{Re}[H(\omega)]} = \frac{B}{A} = \tan\phi$$

… Theory Overview

… Random Vibration

- According to the theory of random vibration, the response of the system to a single input PSD is

$$S_{out}(\omega) = \left|H(\omega)\right|^2 S_{in}(\omega) \qquad \text{or} \qquad S_{out}(\omega) = \left(\frac{a_{out}}{a_{in}}\right)^2 S_{in}(\omega)$$

- Where:
  - S_{out} = spectral density response (conventional terminology)
  - S_{in} = spectral density input (value from PSD curve)
  - a_{out} = calculated sinusoidal output
  - a_{in} = sinusoidal input

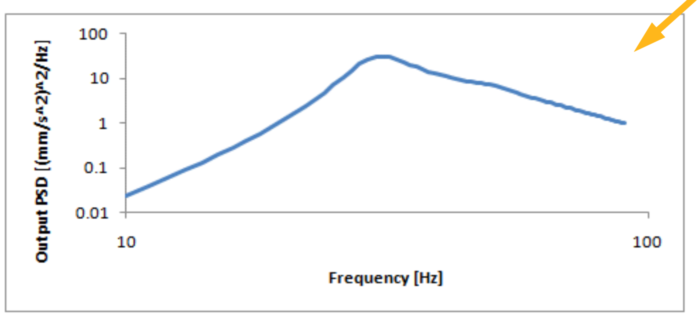
  - **Note: within Ansys the spectral density response is typically called the response PSD (RPSD) and the spectral density input is typically called the input PSD.**

… Theory Overview

… Random Vibration

- To calculate the response PSD (RPSD), multiply the input PSD by the response function

$$S_{out}(\omega)=\left(\frac{a_{out}}{a_{in}}\right)^2 S_{in}(\omega)$$



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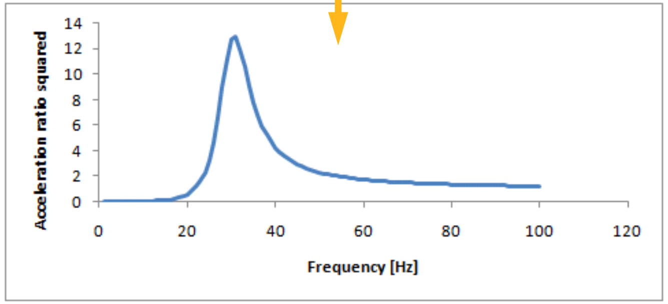

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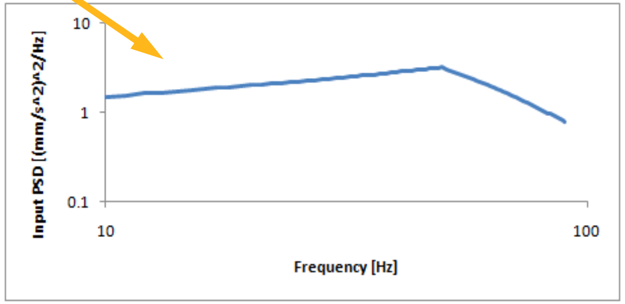


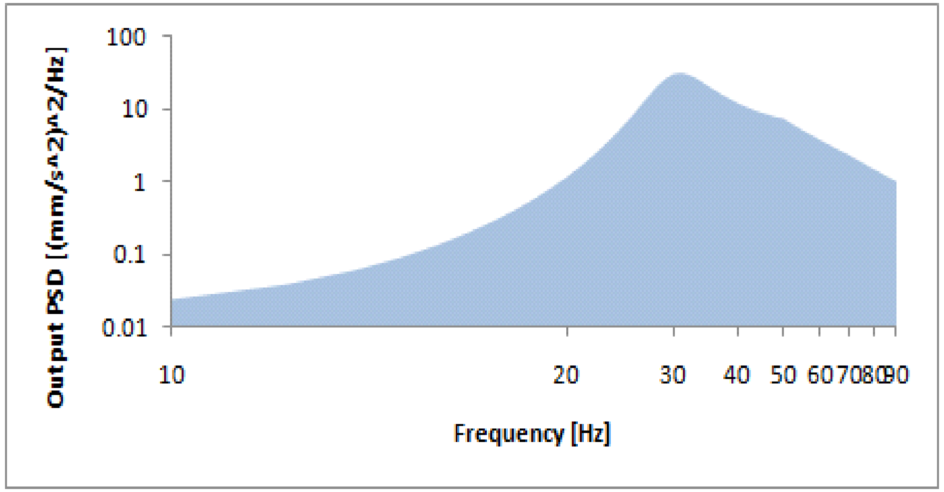
or

$$\text{RPSD}=\left(\frac{a_{out}}{a_{in}}\right)^2(\text{input PSD})$$

… Theory Overview

… Random Vibration

- As stated earlier, we are typically interested in the average response of the system.
- The area under the RPSD curve gives the “mean square” response.
  - the square root of the mean square is the “root mean square” (RMS)
  - the RMS is the average, or one standard deviation (1-sigma), response



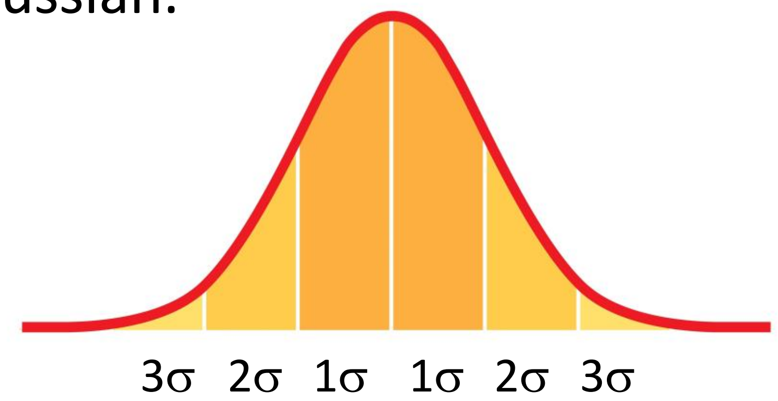
$$RMS = \sqrt{\int_0^\infty S(\omega)\,\mathrm{d}\omega}$$

integration in log-log space (requires special consideration)

… Theory Overview

… Random Vibration

- We don’t know exactly what the response will look like, but we do know that it will respond to the given input with the RMS response, on average.
- Given our assumptions that (1) the input is Gaussian and (2) the system is linear, then our output must also be Gaussian.

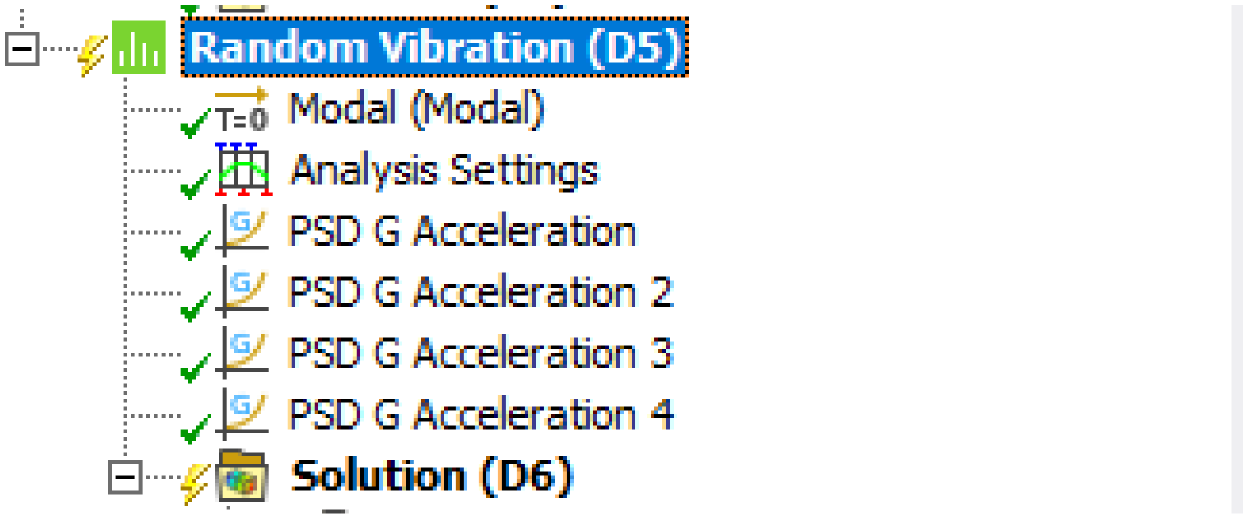


- 1 x RMS (*1-sigma*) accounts for ~ 68.27 % of the total response
- 2 x RMS (*2-sigma*) accounts for ~ 95.45 % of the total response
- 3 x RMS (*3-sigma*) accounts for ~ 99.73 % of the total response

… Theory Overview

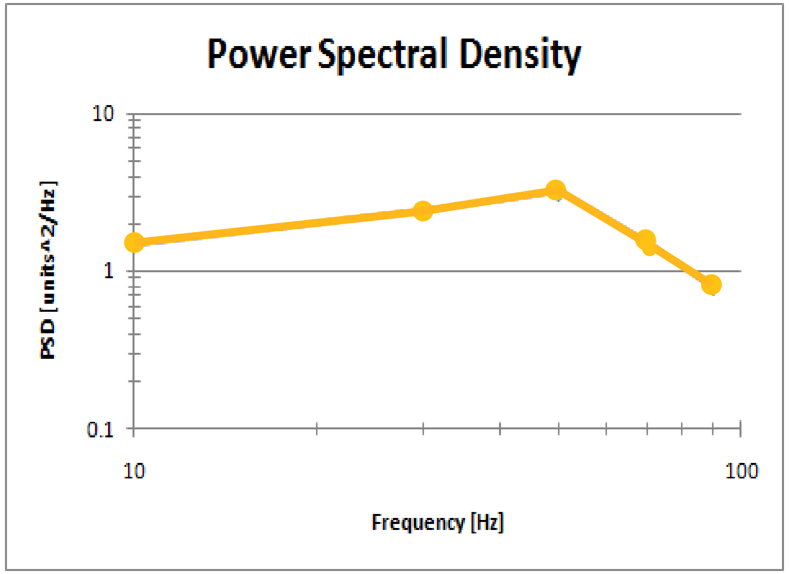
… Random Vibration

- For multiple PSDs in the same model, the results are combined using the SRSS method.
  - We could alternatively perform separate analyses and manually SRSS the results together.



D. PSD Curve Fitting

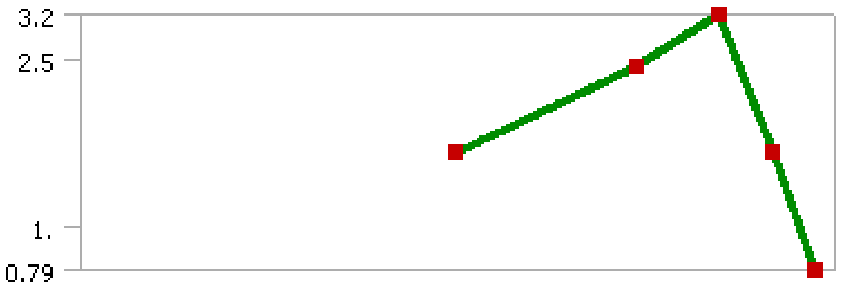
- The PSD is defined as a piecewise linear frequency table and plotted as such in log-log space:

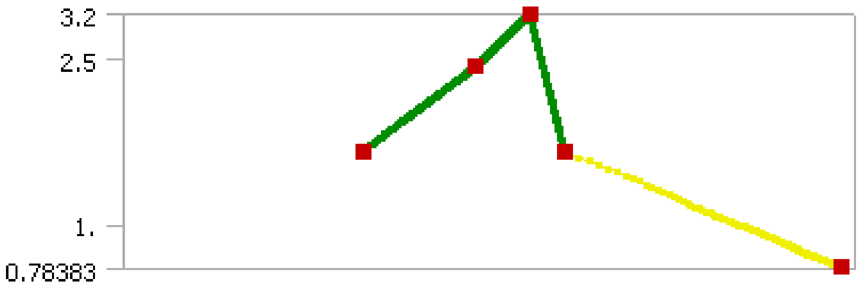


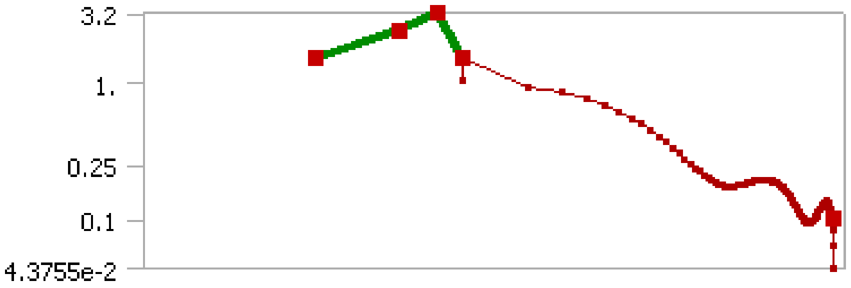
- A curve-fitting polynomial is used for the closed-form integration of the curve.
    - For a good fit, the PSD values between consecutive points should not change by more than an order of magnitude

… PSD Curve Fitting

- Once load entries are entered, the graph provides one of the following color-code indicators for each segment:
  - **Green**: Values are considered reliable and accurate.
  - **Yellow**: This is a warming indicator. Results produced are not considered to be reliable and accurate.
  - **Red**: Results produced are not considered trustworthy. It is recommended that you modify your input PSD loads prior to the solution process.







… PSD Curve Fitting

- To resolve goodness-of-fit issues:
  - Click the fly-out of the Load Data option and choose Improved Fit.
- Interpolated points are displayed if they are available from the goodness of fit approximation.

Details of "PSD G Acceleration 5"

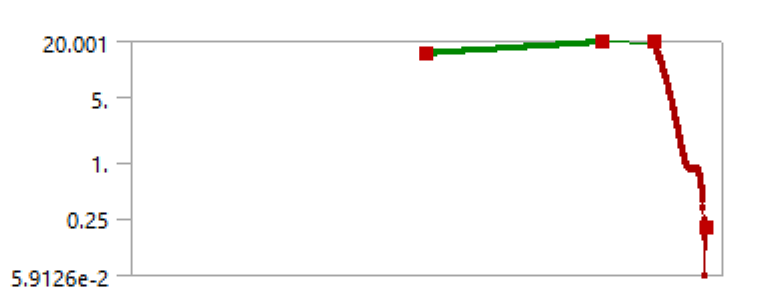
Scope	
Boundary Condition	All Supports
Definition	
Load Data	Tabular Data
Direction	Y Axis
Suppressed	No

- Add to Engineering Data
- Improved Fit
- Import...
- Export...

Tabular Data

	Frequency [Hz]	G Acceleration [G^2/Hz]
1	5.	15.
2	20.	20.
3	30.	20.
4	45.	0.2
*		

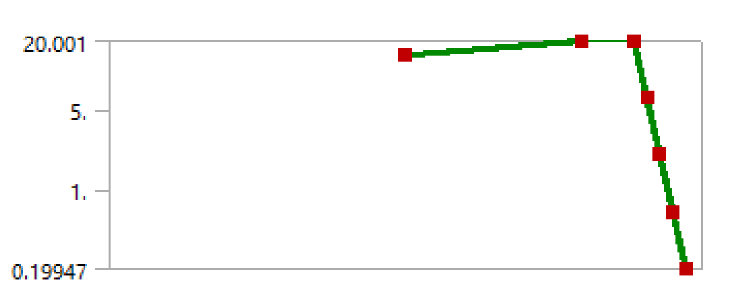
Graph



Tabular Data

	Frequency [Hz]	G Acceleration [G^2/Hz]
1	5.	15.
2	20.	20.
3	30.	20.
4	33.2	6.3246
5	36.742	2.
6	40.662	0.63246
7	45.	0.2
*		

Graph



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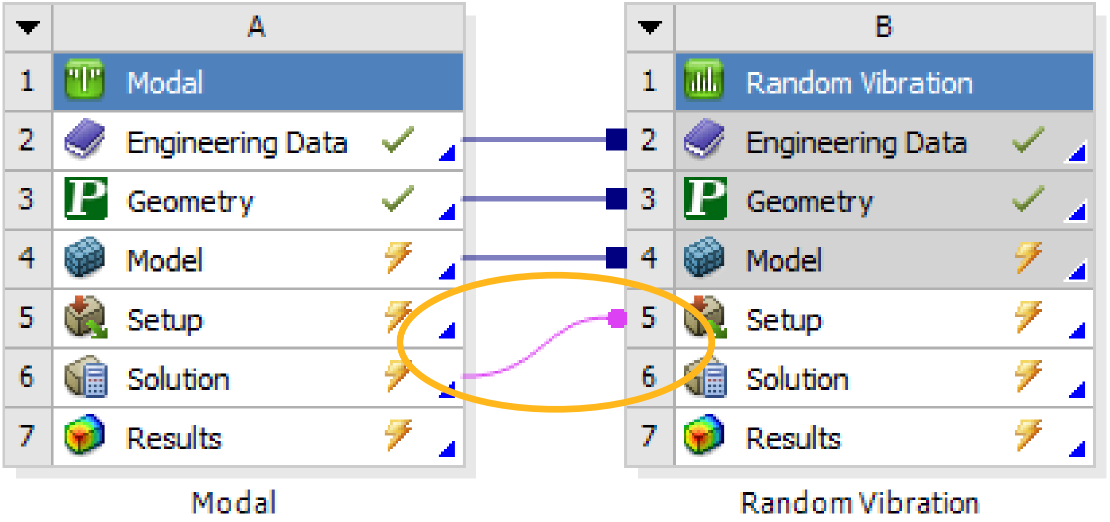
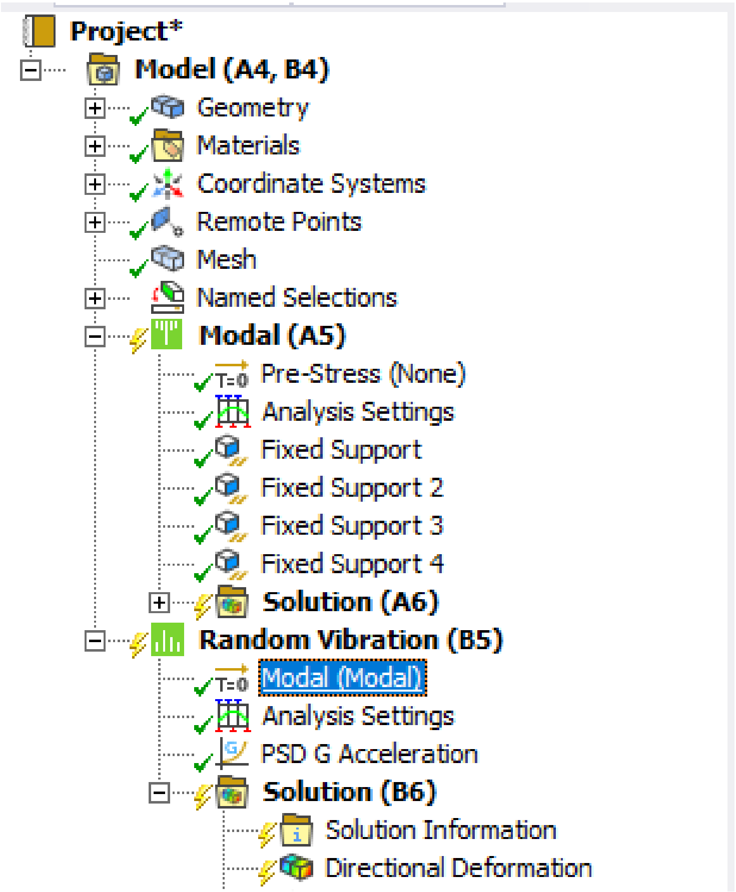
E. Analysis Settings

- Set up a random vibration analysis in the Project Schematic by linking a modal system to a random vibration system *at the solution level*:



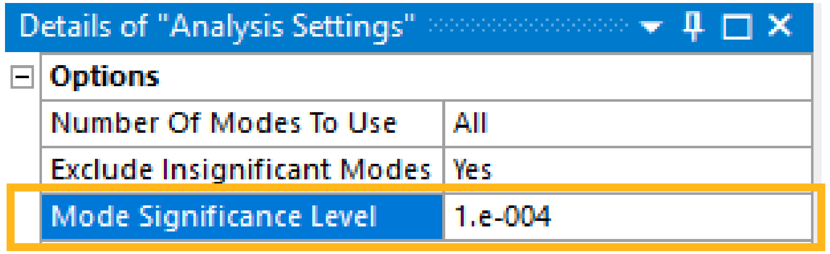


… Analysis Settings

- Analysis Settings > Options
  - Number of Modes To Use:
    - General guidance to consider when extracting modes from the *Modal analysis*
      - Extract all modes whose frequencies span the range from *zero to 1.5 the maximum frequency contained in the input PSD*
    - Then, under Analysis Settings in the Random Vibration analysis, simply set “Number Of Modes To Use” to “All.”
    - Ensure that PSD Curve Fitting (section D) has been performed on the PSD input
  - In addition to the above, consider the following as guidance:
    - Check the Ratio of Effective Mass to Total Mass from the modal analysis and strive for 90%
    - Check the sensitivity of PSD results to the number of modes extracted from the modal analysis

… Analysis Settings

- Analysis Settings > Options
  - Insignificant Modes: if “Exclude Insignificant Modes” is set to “Yes,” then
    - Mode Significance Level:
      - 0 (all modes selected), and
      - 1 (no modes selected).
      - For models with 100+ modes extracted, a Mode Significance Level = 1e-4 is recommended to avoid potential long solution time.



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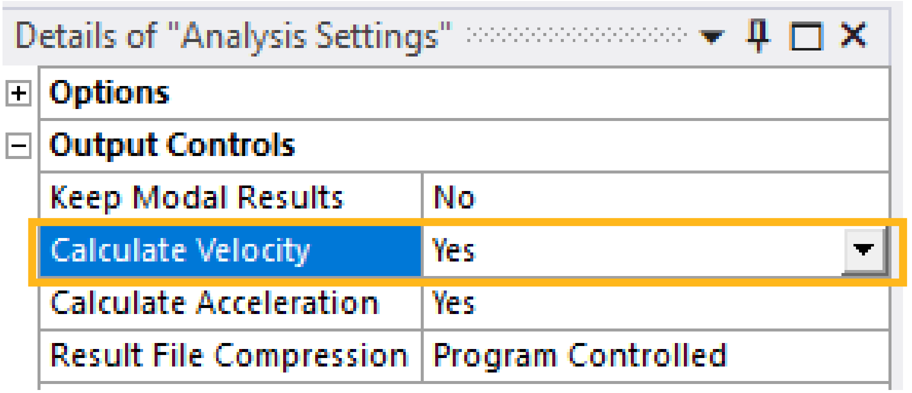
… Analysis Settings

- Analysis Settings > Output Controls
  - Displacement responses are calculated by default.
  - To include Velocity and/or Acceleration responses, set their respective Output Controls to "Yes."
  - Keep Modal Results is set to "No" by default. When set to "Yes", the prerequisite files from the upstream modal analysis will be copied within the project, thus possibly increasing overall solution time and disk usage.



… Analysis Settings

- Analysis Settings > Damping
  - Like other Mode Superposition (MSUP) analyses, the damping matrix [C] for Random Vibration is not calculated explicitly, but instead damping is defined directly in terms of a damping ratio ξ^d for mode i :

$$\xi_i^d = \xi + \xi_i^m + \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2}$$

ξ : constant modal damping ratio (DMPRAT)

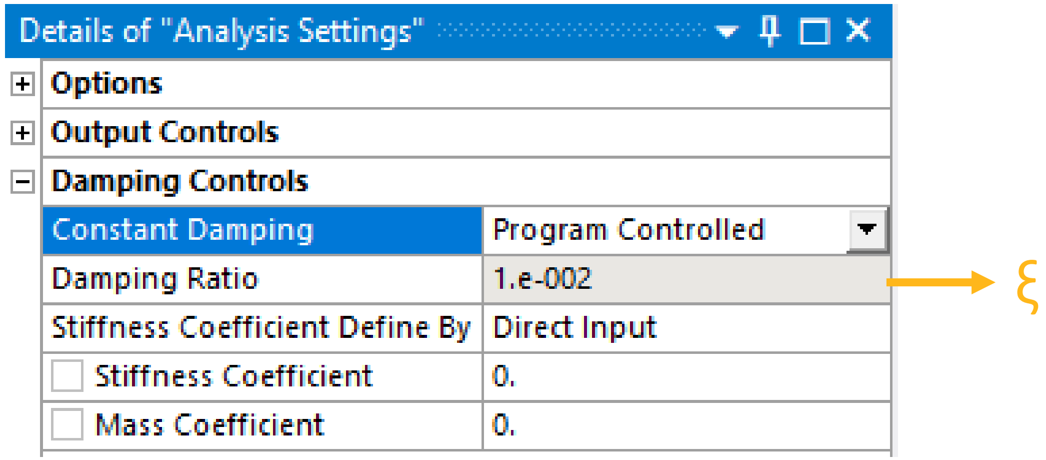
ξ_i^m: modal damping ratio for mode shape i (MP,DMPR during modal analysis)

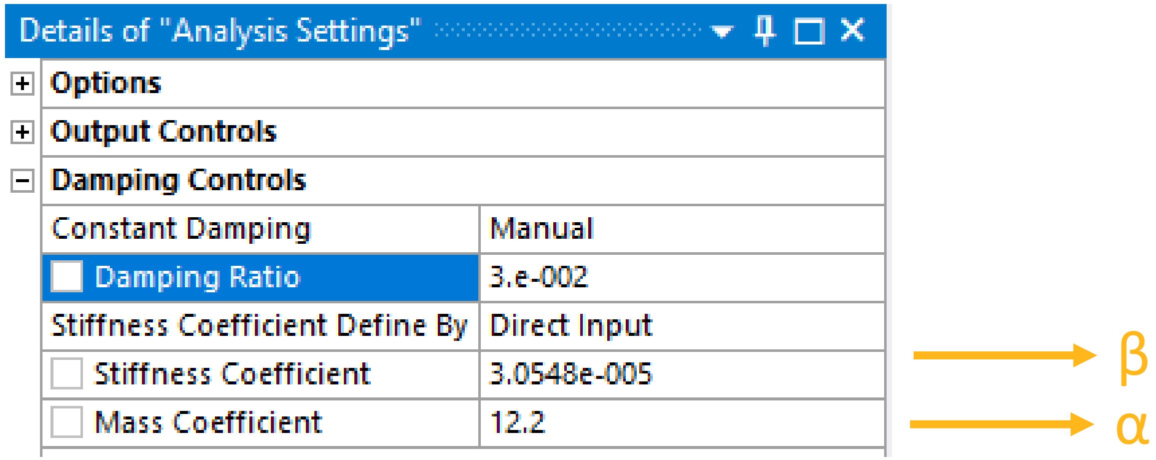
α: Global Mass matrix multiplier (alpha damping, ALPHAD)

β: Global k-Matrix Multiplier (beta damping, BETAD)

… Analysis Settings

- Analysis Settings > Damping Controls
  - By default (Constant Damping = “Program Controlled”), random vibration analyses have a small amount of constant damping applied, in the form of Constant Damping Ratio, ξ = 1%.
  - Otherwise, Constant Damping = “Manual” allows a user-defined Damping Ratio.
  - Rayleigh damping constants α and β are also available.





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… Analysis Settings

- Analysis Settings > Damping Controls
  - The value for ξ^m is entered on a material basis within Engineering Data as part of the undamped Modal Analysis:

Properties of Outline Row 3: Structural Steel

	A	B
1	Property	Value
2	Material Field Variables	Table
3	Density	7850
4	Material Dependent Damping	
5	Damping Ratio	0.01
6	Constant Structural Damping Coefficient	= 0.02
7	Isotropic Elasticity	

ξ^m

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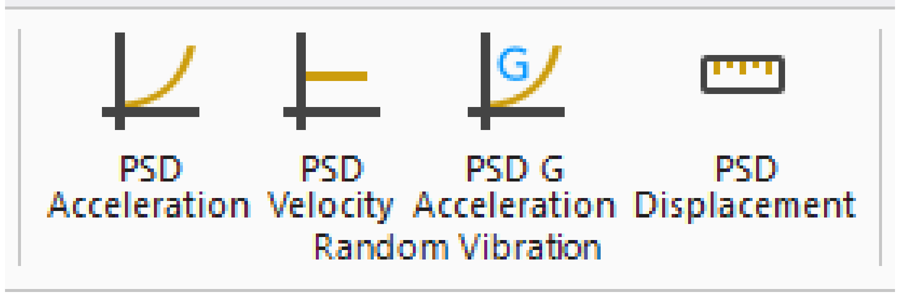
F. Loads and Supports

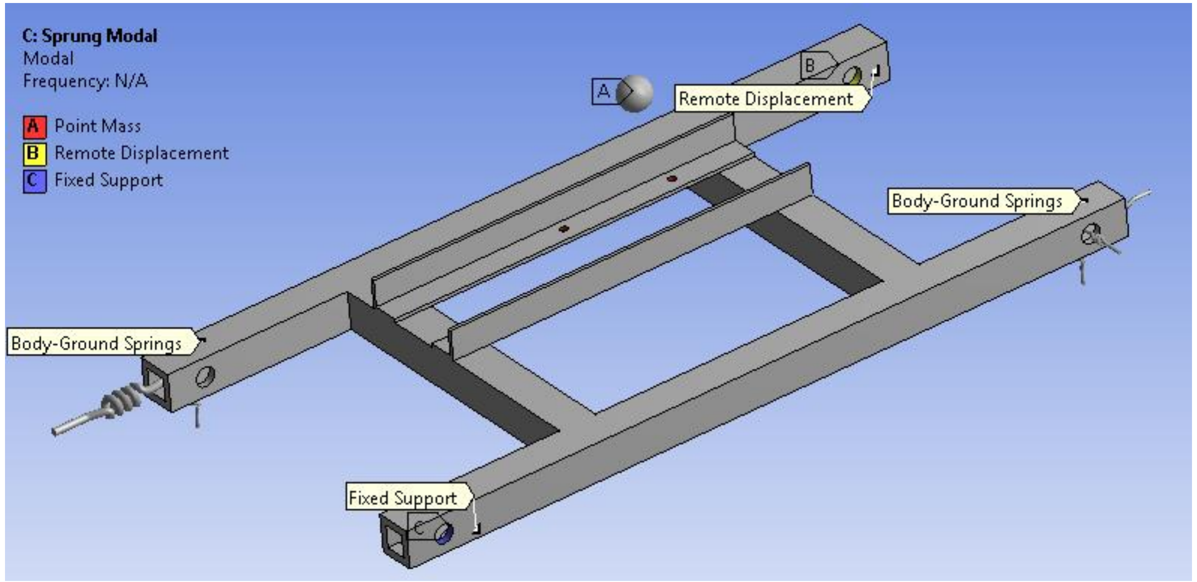
- Support boundary conditions must be defined in the modal analysis itself.
  - Fixed Supports
  - Displacements
  - Remote Displacements
  - Body-Ground Springs
- The only applicable load is a PSD Base Excitation of spectral value vs. frequency.
  - PSD Acceleration
  - PSD G Acceleration
  - PSD Velocity
  - PSD Displacement

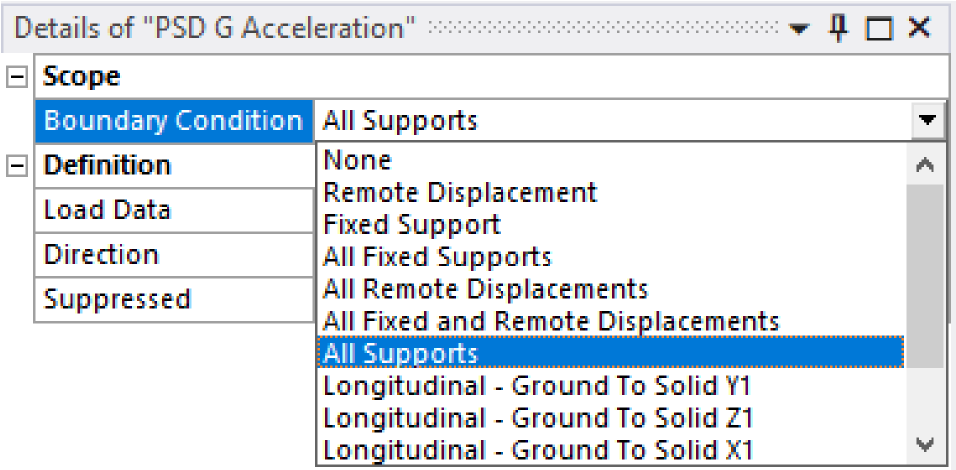


- Multiple PSD excitations (uncorrelated) can be applied; however, correlation between PSD excitations is not supported.

… Loads and Supports

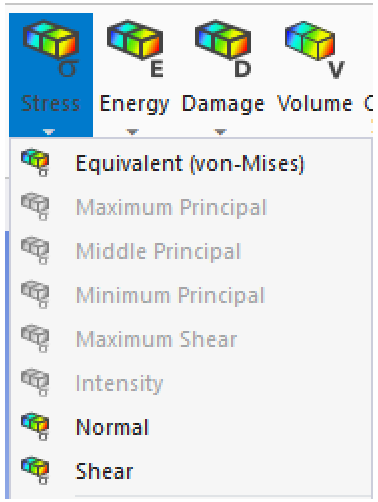
- Supports through which the Base Excitation is applied are chosen in the details of the PSD input.
  - Can be applied to All, or just individual supports of any of the supported types.
- Multiple PSD excitations (uncorrelated) can be applied, each to a unique support.

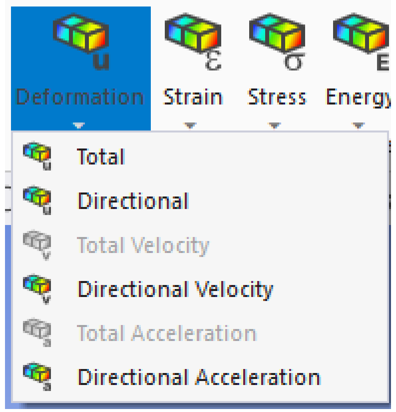


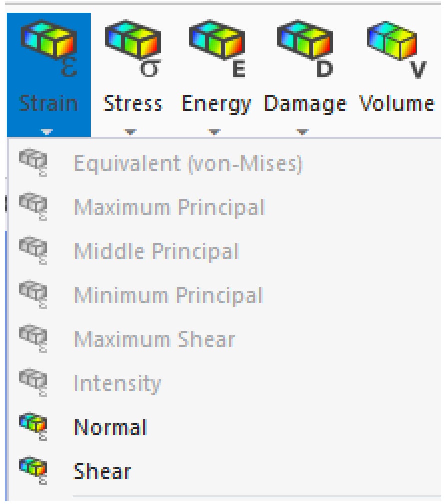


G. Results

- Applicable deformation results are:
  - directional (X/Y/Z) displacement
  - Directional velocity (provided it has been requested in Output Controls)
  - Directional acceleration (provided it has been requested in Output Controls)
- Since the directional results are statistical in nature, they cannot be combined in the usual way.
- If strain/stress are requested (during modal analysis), applicable results are normal strain and stress, shear strain and stress, and equivalent stress.







… Results

- Displacement results are:
  - *relative* to the base of the structure (the fixed supports).

- Velocity and acceleration results:
  - include base motion effects *(absolute).*

Details of "Directional Deformation"

Scope	
Scoping Method	Geometry Selection
Geometry	All Bodies
Definition	
Type	Directional Deformation
Orientation	Y Axis
Reference	Relative to base motion
Scale Factor	1 Sigma
Probability	68.269 %
Coordinate System	Solution Coordinate System
Identifier	
Suppressed	No

Details of "Directional Velocity"

Scope	
Scoping Method	Geometry Selection
Geometry	All Bodies
Definition	
Type	Directional Velocity
Orientation	Y Axis
Reference	Absolute (including base motion)
Scale Factor	1 Sigma
Probability	68.269 %
Coordinate System	Solution Coordinate System
Identifier	
Suppressed	No

… Results

- Force Reaction and Moment Reaction probes can be scoped to a Fixed Support, Displacement, Remote Displacement and Body-Ground Springs to view Reaction Results.
- Should be used carefully since reactions are summed by mode combinations and thus phase information will be partially lost.

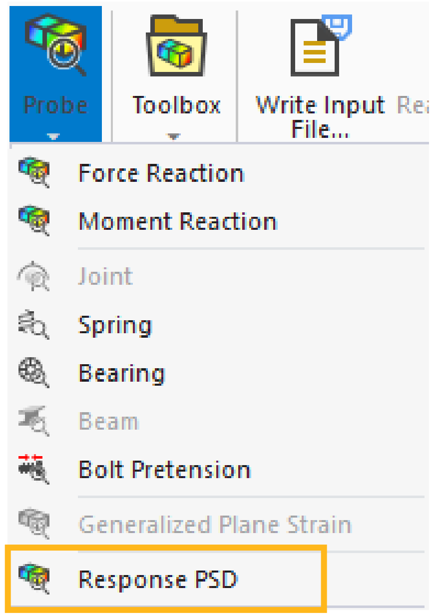
Details of "Force Reaction 5"

Definition	
Type	Force Reaction
Location Method	Boundary Condition
Boundary Condition	Remote Displacement
Orientation	Solution Coordinate System
Reference	Relative to base motion
Scale Factor	1 Sigma
Probability	68.269 %
Suppressed	No

Details of "Force Reaction 2"

Definition	
Type	Force Reaction
Location Method	Spring
Orientation	Solution Coordinate System
Reference	Relative to base motion
Spring	Longitudinal - Ground To Solid Y2
Scale Factor	1 Sigma
Probability	68.269 %
Suppressed	No

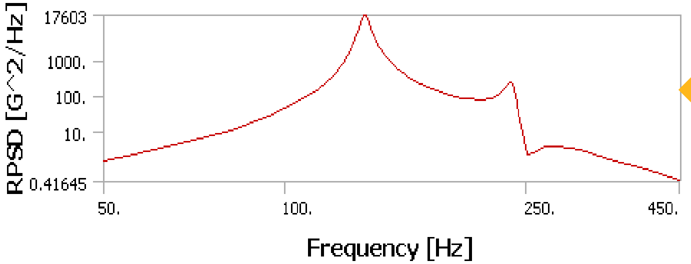
… Results

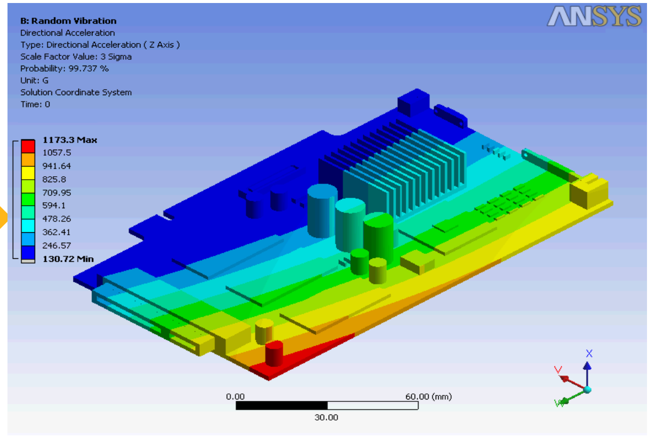


- Response PSDs can be requested for displacement, velocity, acceleration, stress, and strain.
  - RPSDs are calculated for every node in every free direction at each frequency
  - RPSDs can be plotted for each node in a specific direction versus frequency
- an RMS value (sigma value) for the entire frequency range is calculated for every node in every free direction
- Response PSDs can be scoped to geometry (vertex), coordinate systems, and remote points and can be relative to base or absolute.

Details of "Response PSD"

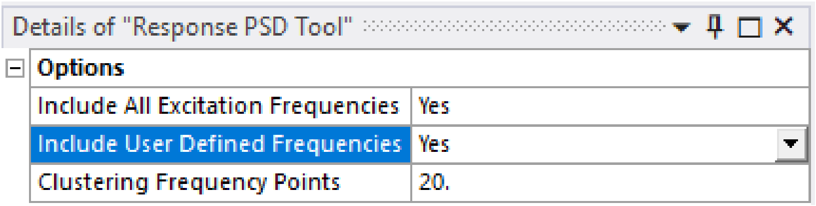
Definition	
Type	Response PSD
Location Method	Geometry Selection
Geometry	1 Vertex
Orientation	Solution Coordinate System
Reference	Relative to base motion
Suppressed	No
Options	
Result Type	Displacement
Result Selection	Y Axis
Selected Frequency Range	Full
Results	
RMS Value	9.5581e-004 in
RMS Percentage	100. %
Expected Frequency	6.7243 Hz

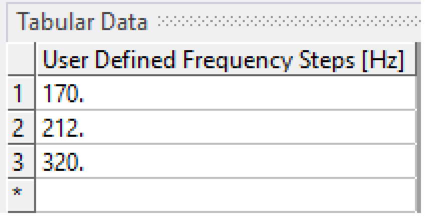


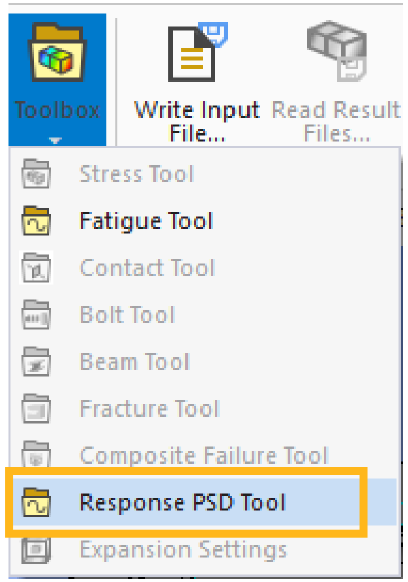


… Results

- For additional control of Response PSD frequency sampling, use a Response PSD Tool.
  - Include All Excitation Frequencies (Default = “Yes”)
    - When set to “No”, includes only min and max excitation frequencies and natural frequencies
  - Include User Defined Frequencies (Default = “No”)
    - When set to “Yes”, user manually enters additional user defined frequencies to be used in addition to the normal sampling frequencies
  - Clustering Frequency Points (Default, and Minimum = 20)
    - Defines the number of frequencies generated on either side of the natural frequencies, enabling more accuracy; useful when PSD input curve contains large number of spikes, but can increase evaluation time

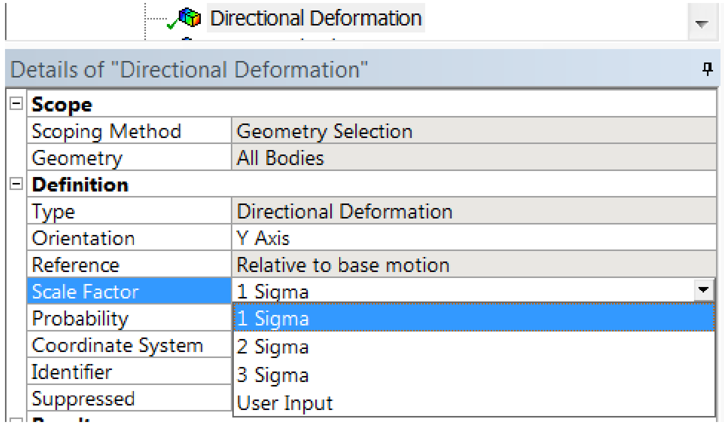




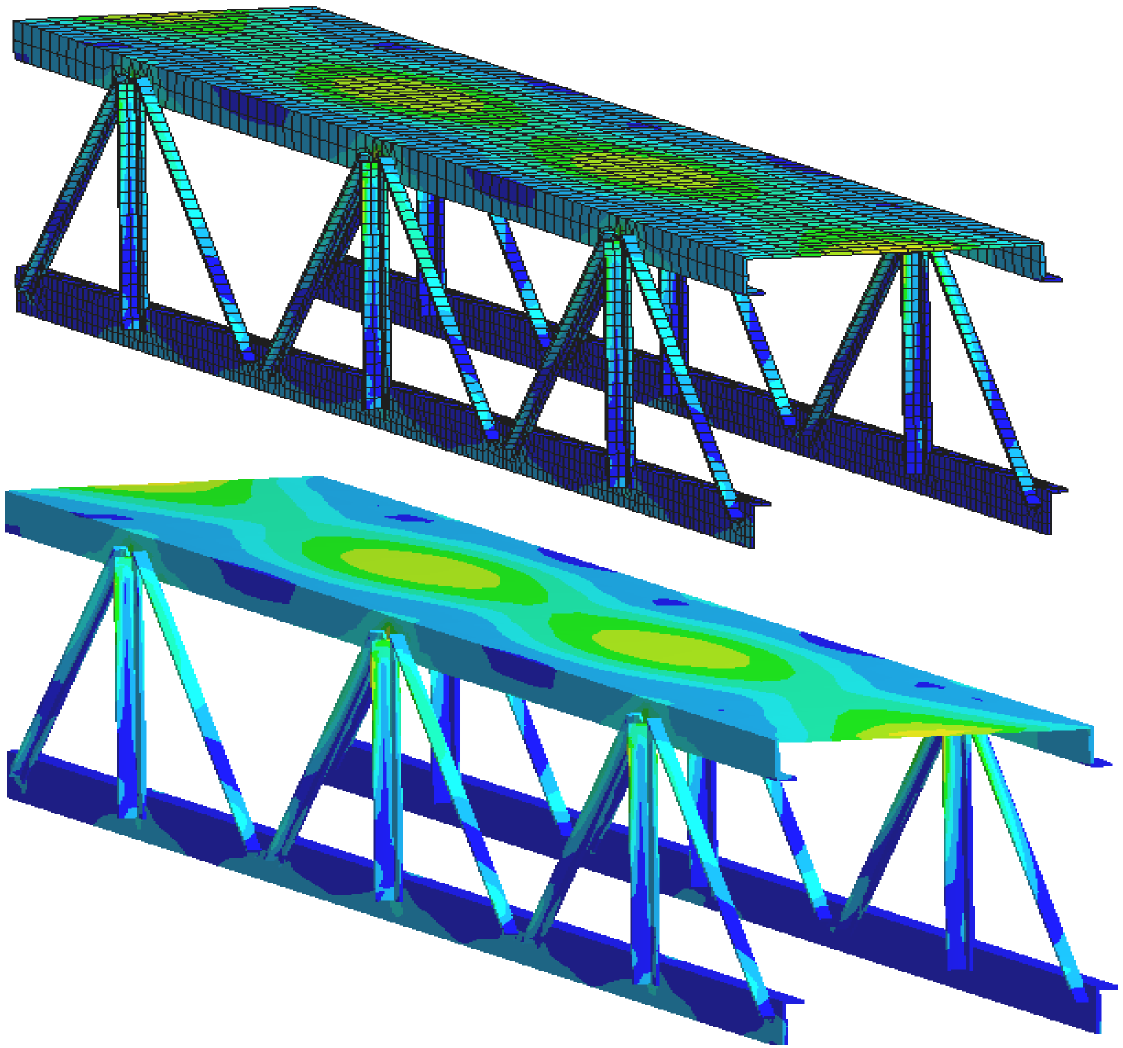


… Results

- The default output results are *one sigma* (1σ) or one standard deviation values (with zero mean value).
- These results follow a Gaussian distribution.
- The interpretation is that *68.3% of the time* the response will be *less than* the standard deviation value.
- You can scale the result by 2 times to get the *2 sigma* (2σ) values and by 3 to get the 3 sigma (3σ) values.
- The response will be less than the *2 sigma* values *95.45%* of the time and less than the *3 sigma* values *99.73%* of the time.



Workshop 08.1: Girder Assembly



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End of presentation