Ansys Mechanical Linear and Nonlinear Dynamics

Module 04: Modal Cyclic Symmetry Analysis

Release 2022R2

Please note:
- These training materials were developed and tested in Ansys Release 2022 R2. Although they are expected to behave similarly in later releases, this has not been tested and is not guaranteed.
- The screen images included with these training materials may vary from the visual appearance of a local software session.

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Module 04 Learning Outcomes

- After completing this module, you will:
  - Understand the common terminology associated with Cyclic Symmetry analysis.
  - Be able to use Symmetry objects to allow dynamic characterization of a structure from a small sector of repeating geometry.
  - Know how to review results of Cyclic Symmetry analysis from either a harmonic index or frequency basis and to visualize traveling waves within a structure.

Module 04 Topics

A. Definition and Application of Cyclic Symmetry
B. Terminology
C. Solution Architecture
D. Specifying Symmetry
E. Analysis Settings
F. Postprocessing

A. Definition and Application of Cyclic Symmetry

- When a structure is said to be cyclically symmetric, it exhibits a repetitive geometric pattern in 360 degrees around an axis of symmetry.
- Common examples of this might be fan wheels, spur gears, turbine blades, pump impellers, milling cutters, etc.

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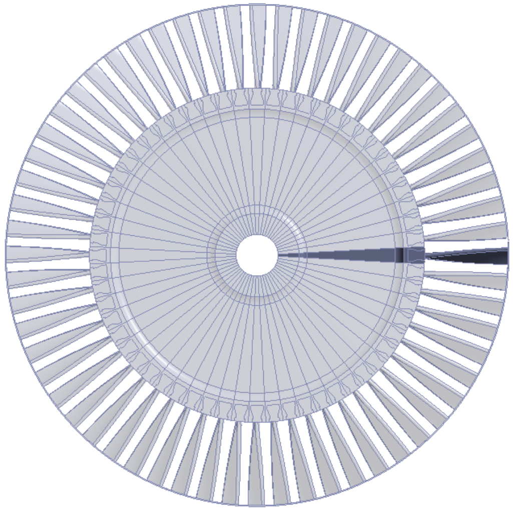
... Definition and Application of Cyclic Symmetry

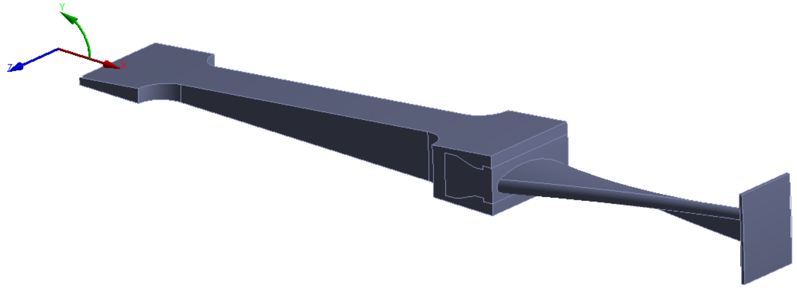
- As with other types of symmetry, the analyst can save considerable computation time by modeling a single repeating pattern (sector) of the geometry, instead of the full 360° structure.
- Ansys Mechanical currently supports Static, Modal, (Full) Harmonic, and Thermal Cyclic Symmetry analyses.
- Limitations include:
  - Use of Mesh Connections is not recommended
  - Layered Sections are not supported
  - 2D Analysis is not supported
  - Line bodies are not supported
  - Gasket bodies are not supported
  - Use of Joints is not supported
  - Damped modal analysis producing asymmetric matrices is not supported

B. Terminology

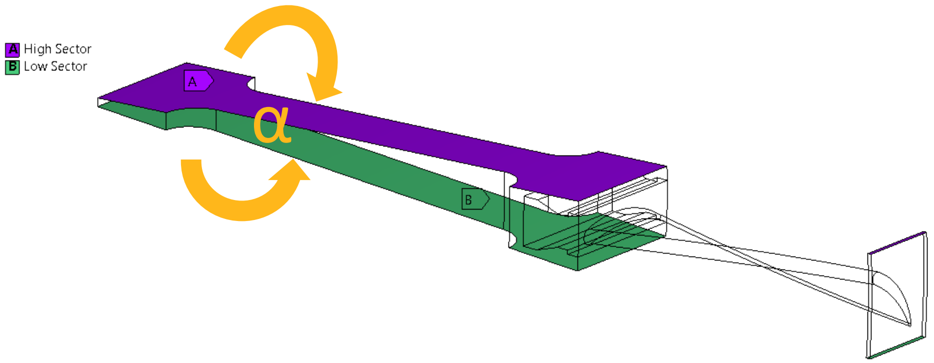
Common terms and their definitions include:

- Basic Sector – The single, repeating geometric pattern, chosen such that *N* repetitions in a cylindrical coordinate system constitutes the full structure.
- Sector Angle – Typically designated as α, the angle spanned by the basic sector, where *N* x α = 360°; *N* is an integer.
- Low and High Sector Boundaries – The faces of the model (and ultimately the nodes thereon) that encompass the sector angle and form the “boundaries” of the model.





Basic Sector



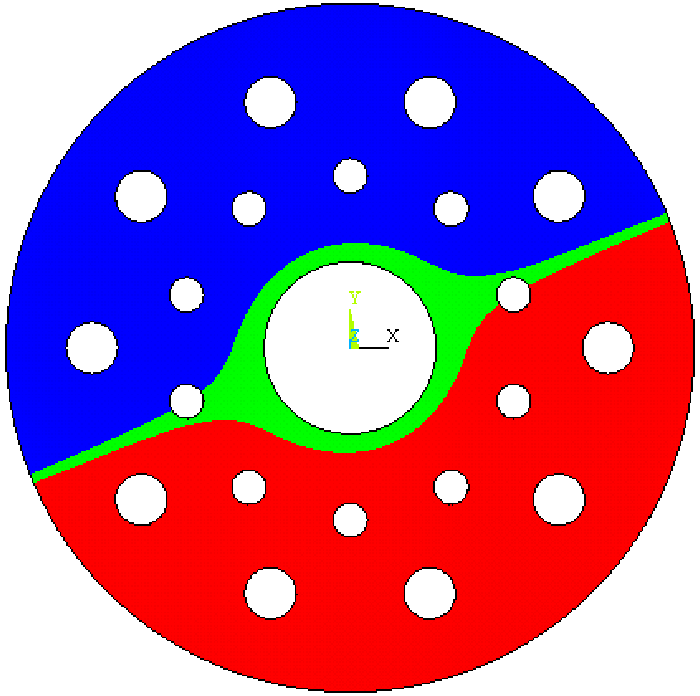
… Terminology

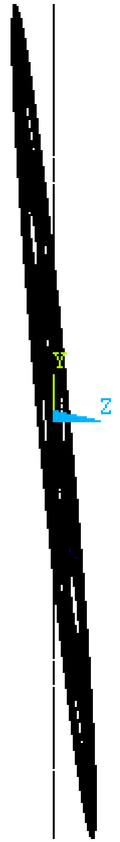
Common terms and their definitions include:

- Nodal Diameter – A diametric line of zero out of plane displacement during vibration of a circular disc (like a cosine wave)
  - 1 nodal diameter causes one wave around the circumference
  - 2 nodal diameters produces two waves, 3 nodal diameters produces 3 waves, etc.
  - The nodal diameter provides the relationship to enable calculating the mode shapes of the entire structure from the single sector.
  - Examples follow…..

… Terminology

- One Nodal Diameter





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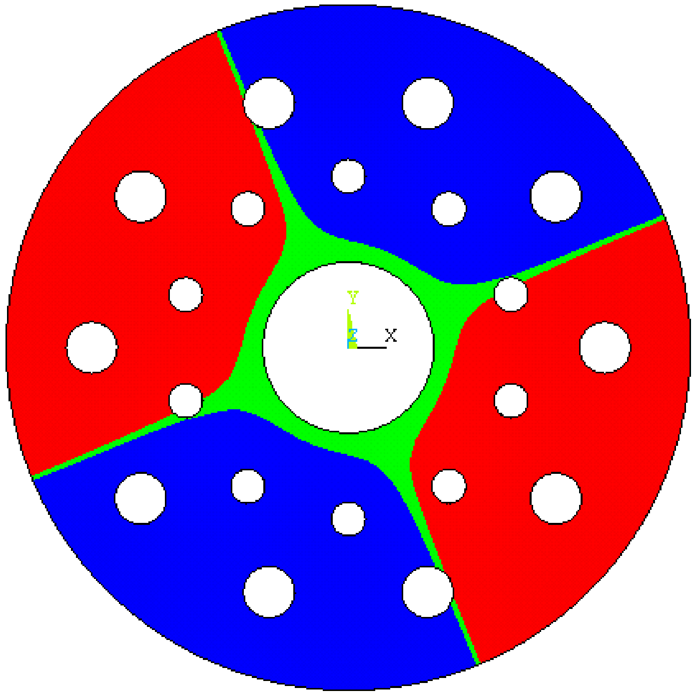
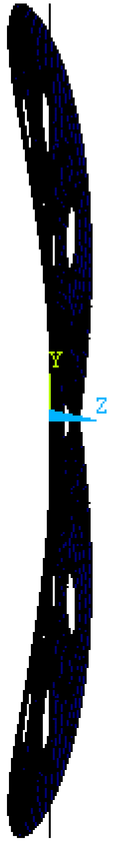
… Terminology

- Two Nodal Diameters





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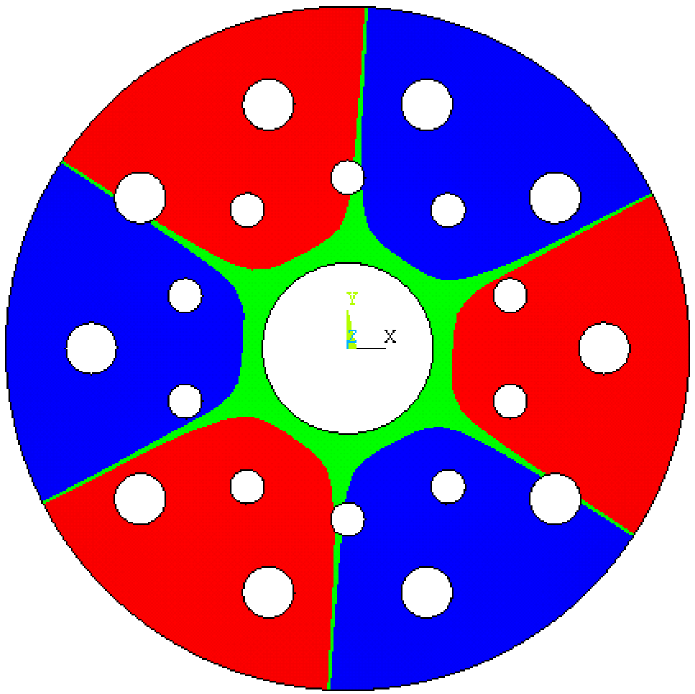
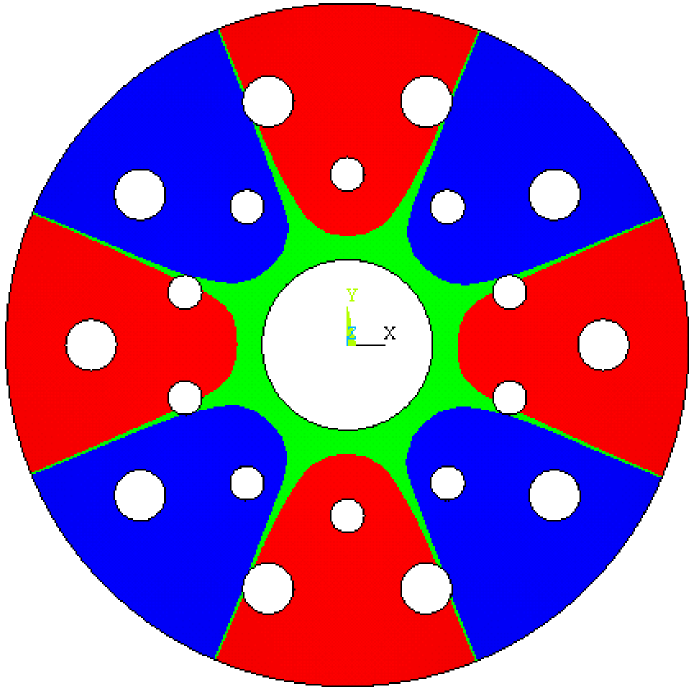
… Terminology

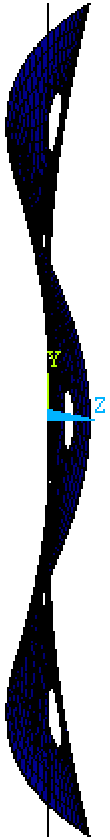
- Three Nodal Diameters



… Terminology

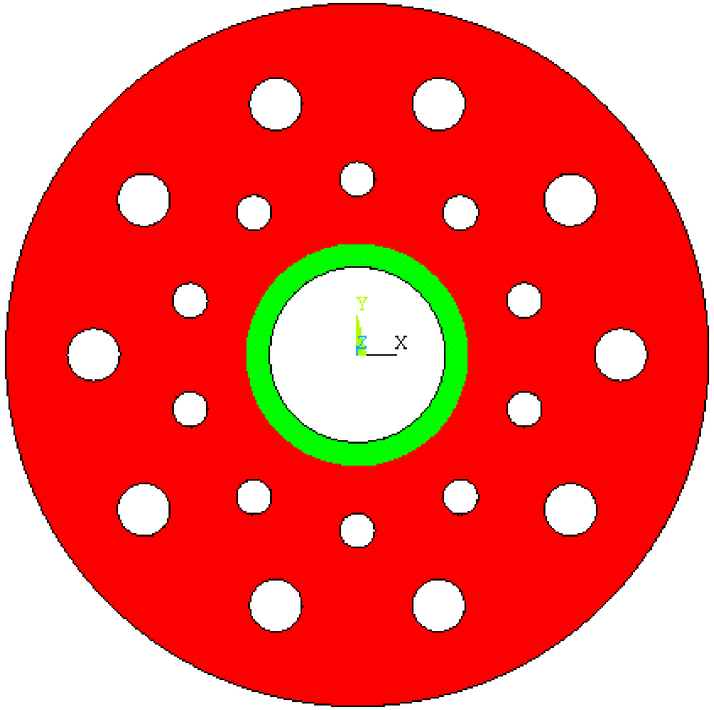
- Four Nodal Diameters

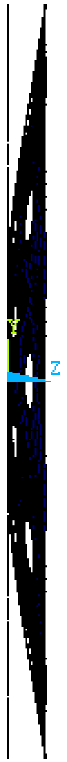




… Terminology

- Zero Nodal Diameters (referred to as the “breathing mode”)





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… Terminology

- *Harmonic Index* - an integer that determines the variation in the value of a single DOF at points spaced at a circumferential angle equal to the sector angle. Therefore, for a given harmonic index, a varying number of waves (i.e. nodal diameters) of vibration will exist around the circumference.
- The relationship between nodal diameter, d, and harmonic index, k, for a model consisting of N sectors is:

$$d = m * N \pm k$$

  Where number of modes, m = 0, 1, 2, 3, …, ∞
- Harmonic index, k, ranges from 0 to N/2 if N is even
- Harmonic index, k, ranges from 0 to (N-1)/2 if N is odd

… Terminology

- In tabular form the preceding equation looks like this:

$$d = m * N \pm k$$

Harmonic Index	Nodal Diameter(d)					
0	0	N	N	2N	2N	…
1	1	N-1	N+1	2N-1	2N+1	…
2	2	N-2	N+2	2N-2	2N+2	…
3	3	N-3	N+3	2N-3	2N+3	…
4	4	N-4	N+4	2N-4	2N+4	…
…	…	…	…	…	…	…
N/2 N is even	N/2	N/2	3N/2	3N/2	5N/2	…
(N-1)/2 nis odd	(N-1)/2	(N+1)/2	(3N-1)/2	(3N+1)/2	(5N-1)/2	…

- Using above equation and requesting 3 modes, calculating the nodal diameters based upon a sector angle of α=60°, through all possible harmonic indices, gives the following (see next slide):

… Terminology

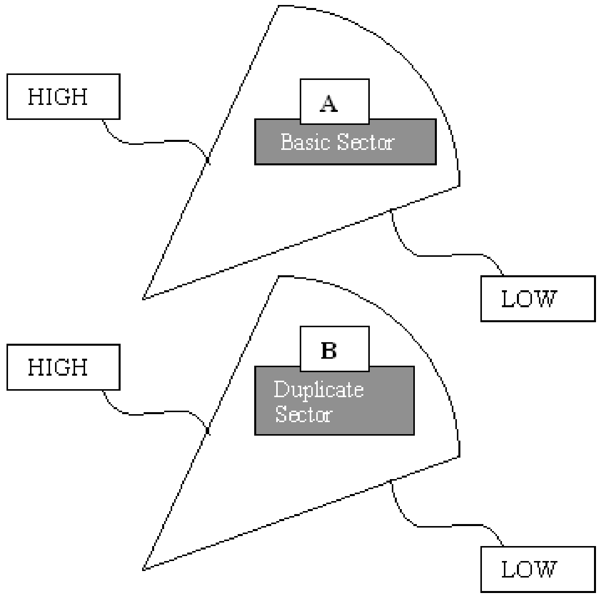
Harmonic Index	Nodal Diameter(d)					
0	0	N	N	2N	2N	…
1	1	N-1	N+1	2N-1	2N+1	…
2	2	N-2	N+2	2N-2	2N+2	…
3	3	N-3	N+3	2N-3	2N+3	…
4	4	N-4	N+4	2N-4	2N+4	…
…	…	…	…	…	…	…
N/2 N is even	N/2	N/2	3N/2	3N/2	5N/2	…
(N-1)/2 nis odd	(N-1)/2	(N+1)/2	(3N-1)/2	(3N+1)/2	(5N-1)/2	…

$d = m * N \pm k$

- N=360/60=6; k varies from 0 to (N)/2 or $0 \leq k \leq 3$, m $\longrightarrow$ 0,1,2,3,…,∞:
- For k=0: d=0, N, 2N, 3N, 4N, … = 0,6,12,18
- For k=1: d=1, N-1, N+1, 2N-1, 2N+1, 3N-1, 3N+1, … = 1,5,7,11,13,17,19
- For k=2: d=2, N-2, N+2, 2N-2, 2N+2, 3N-2, 3N+2, … = 2,4,8,10,14,16,20
- For k=3: d=3, N-3, N+3, 2N-3, 2N+3, 3N-3, 3N+3, … = 3,9,15,21,…,
- From above, given that $m \longrightarrow \infty$, there are also an infinite number of nodal diameters; we have accounted for all sequential nodal diameters from 0 through 21 using values of $0 \leq m \leq 3$.

C. Solution Architecture

- Mechanical provides a completely automated cyclic symmetry solution architecture, although it may be useful to understand how this is accomplished for us.
  - Automatic (and transparent) creation of a *duplicate sector* model from the basic sector model, residing in the same coordinate space as the basic sector.
  - All loads and boundary conditions applied to the basic sector are also applied to the duplicate sector.
  - Internal constraint equations connecting the lower and higher boundary nodes on the basic and the duplicate sectors, taking the form of:



$$\begin{Bmatrix} U^A_{High} \\ U^B_{High} \end{Bmatrix} = \begin{bmatrix} \cos k\alpha & -\sin k\alpha \\ \sin k\alpha & \cos k\alpha \end{bmatrix} \begin{Bmatrix} U^A_{Low} \\ U^B_{Low} \end{Bmatrix}$$

k = Harmonic index -- $(0,1,2,....,N/2)$ when N is even, $(0,1,2,..., (N\text{-}1)/2)$ when N is odd

α = Sector angle$\left(\frac{2\pi}{N}\right)$

U = Vector of displacement and rotational degrees of freedom

U^A_{Low} represents the *basic sector* low side edge

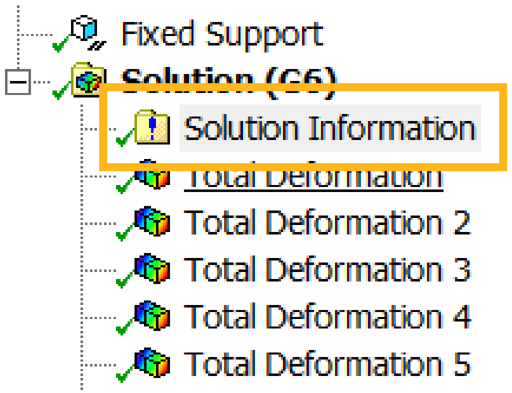
U^A_{High} represents the *basic sector* high side edge

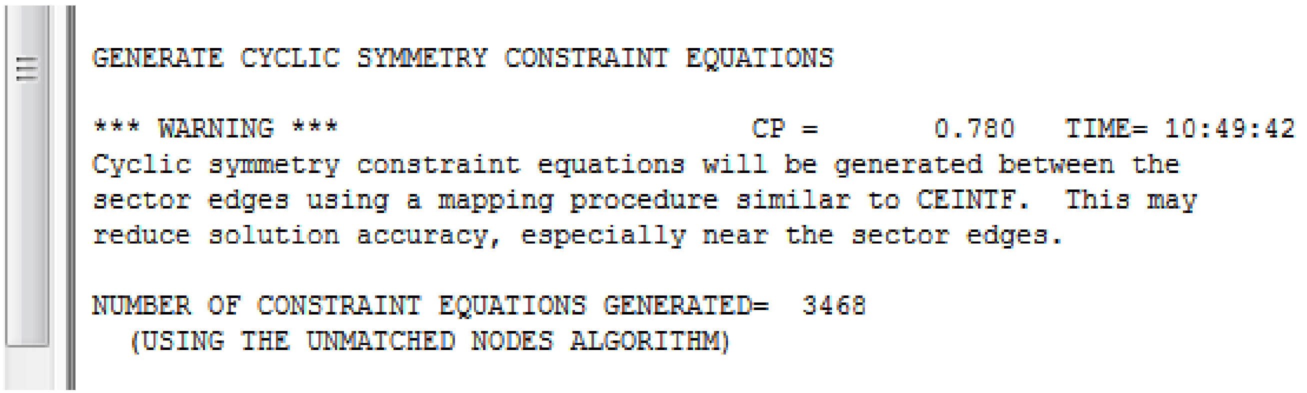
U^B_{Low} represents the *duplicate sector* low side edge

U^B_{High} represents the *duplicate sector* low side edge

… Solution Architecture

- For optimum accuracy, the mesh pattern on the low and high sector boundaries should be identical, such that a node on the low boundary with cylindrical coordinates (r, θ, z) has a corresponding node on the high boundary with cylindrical coordinates (r, θ+α, z).
- We will see in section D ways to accomplish the proper matched mesh on the low and high sector boundaries.
- If the mesh is not matched, the solution may proceed, however with the following warning found in Solution Information (solve.out file) …





```
GENERATE CYCLIC SYMMETRY CONSTRAINT EQUATIONS

*** WARNING ***                         CP =       0.780   TIME= 10:49:42
Cyclic symmetry constraint equations will be generated between the
sector edges using a mapping procedure similar to CEINTF.  This may
reduce solution accuracy, especially near the sector edges.

NUMBER OF CONSTRAINT EQUATIONS GENERATED=  3468
  (USING THE UNMATCHED NODES ALGORITHM)
```

… Solution Architecture

  - Solution conducted with respect to each harmonic index, given the sector angle of the input geometry, as a separate solution step for a user-defined number of modes (nodal diameters).
  - Internal constraint equations generated are unique to each solution step. The harmonic index is important for the creation of these constraint equations.
  - These are the automated aspects of cyclic symmetry solutions!

- Mode shapes in each sector are obtained from the eigenvector solution.
- Displacement components (x, y, z) at any node in the sector for each harmonic index k are given by:

$$u = u_A \cos(n-1)k\alpha - u_B \sin(n-1)k\alpha$$

Where :

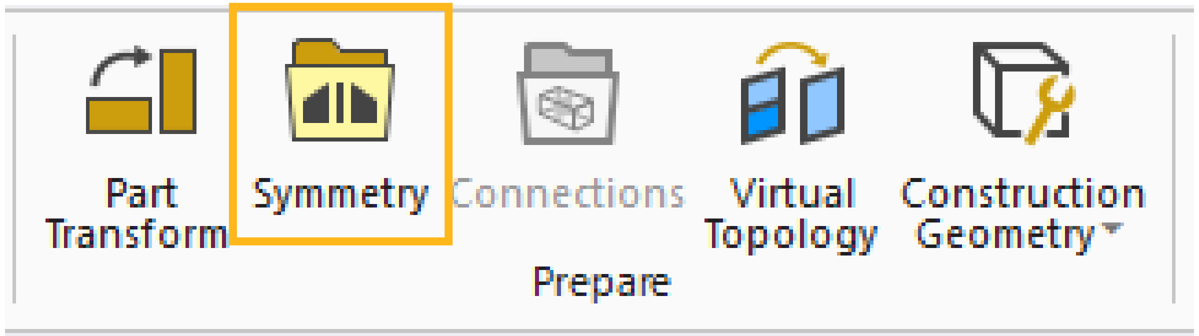
n = Sector Number, varies from 1 to N

u_A= Basic Sector displacement

u_B= Duplicate Sector displacement

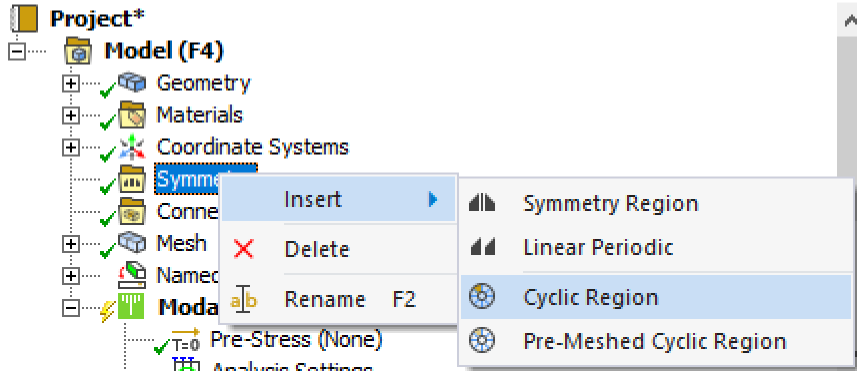
D. Specifying Symmetry

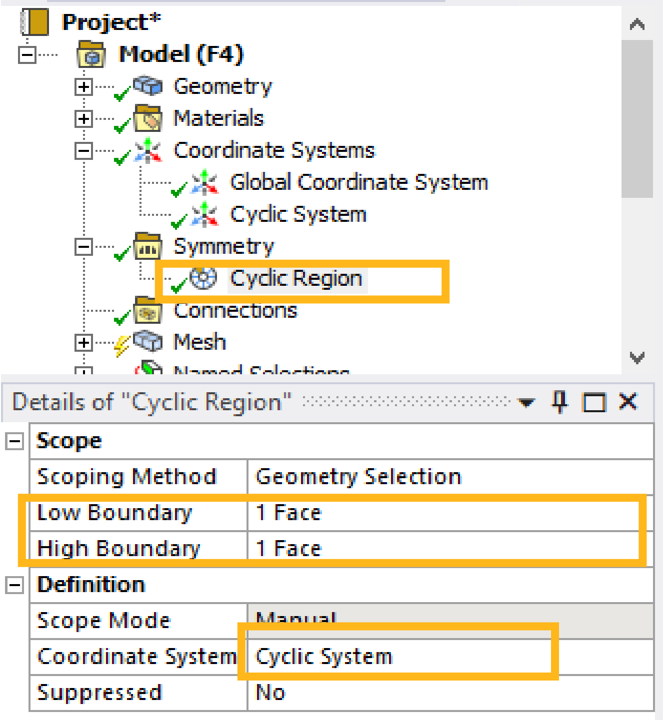
- Mechanical affords two ways of specifying the proper symmetry conditions.
- Symmetry boundary conditions are needed to ensure the constraint equations relating the low and high sector boundaries are constructed properly.
- Use a Symmetry Object for this:

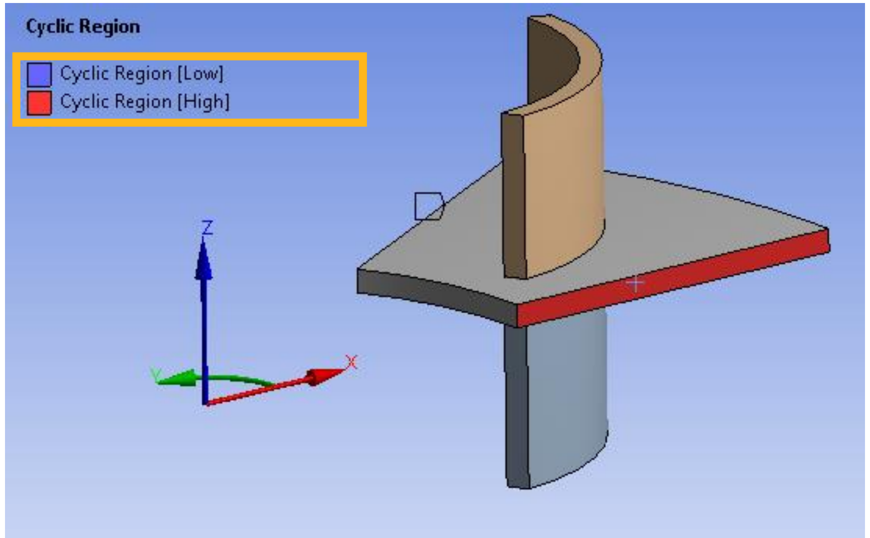


… Specifying Symmetry

- Method 1 – Cyclic Region
  - A Cyclic Region object applied to the low and high sector boundaries will ensure that the resulting mesh is matched on the boundaries, producing correct constraint equations.
  - No other mesh controls are required in order to achieve the matching mesh.
  - Cyclic Region requires a local cylindrical coordinate system at the symmetry axis.







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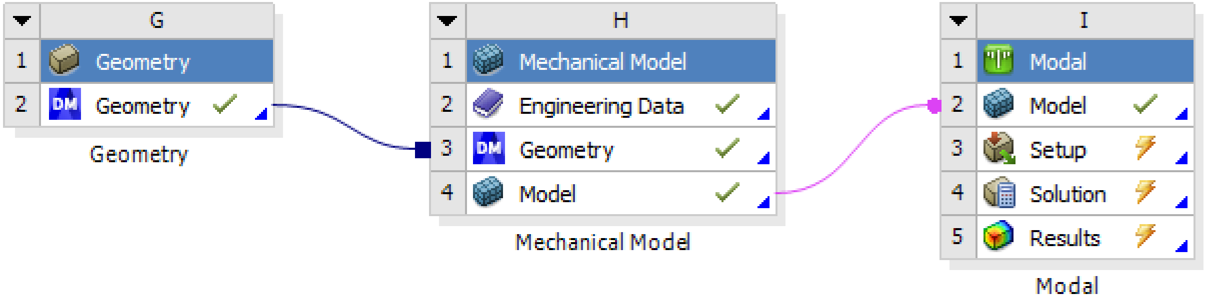
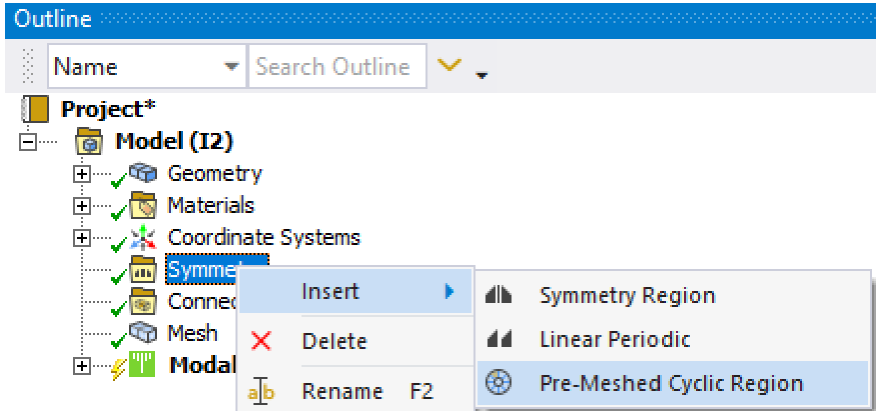
… Specifying Symmetry

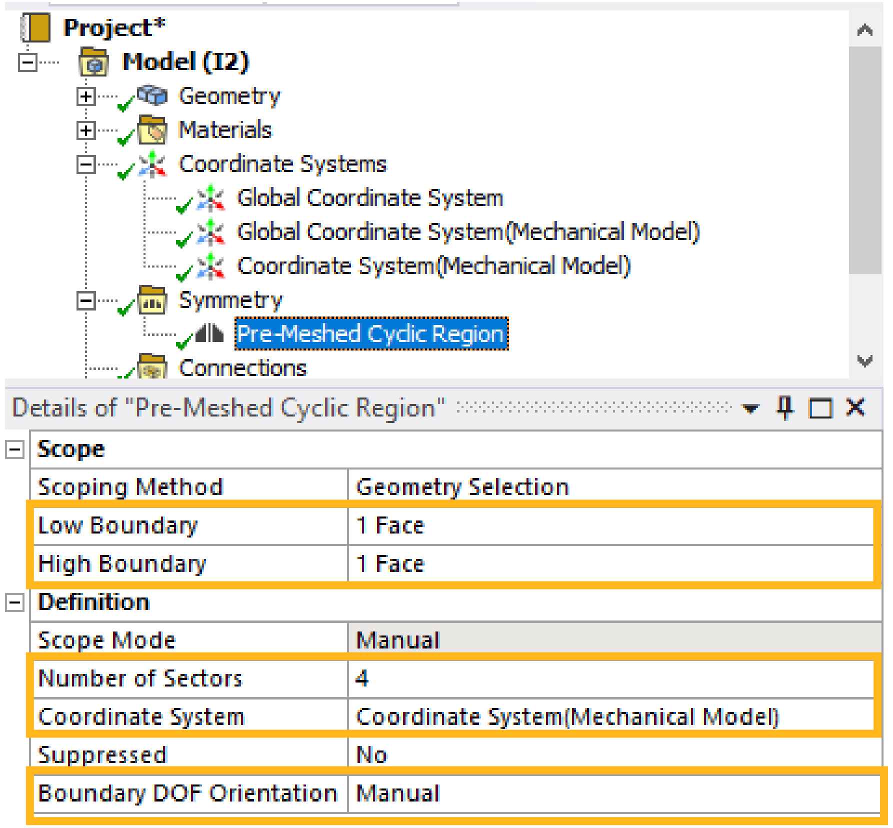
- Method 2 – Pre-Meshed Cyclic Region
  - Unlike a Cyclic Region, this method does not produce a matched mesh on the cyclic boundaries; instead, user will apply proper mesh controls (discussed later).
  - Required when the mesh is imported into Mechanical as read-only (i.e. it is linked to an Ansys Composite PrepPost, Mechanical Model, or External Model Component System in the Workbench Project application.

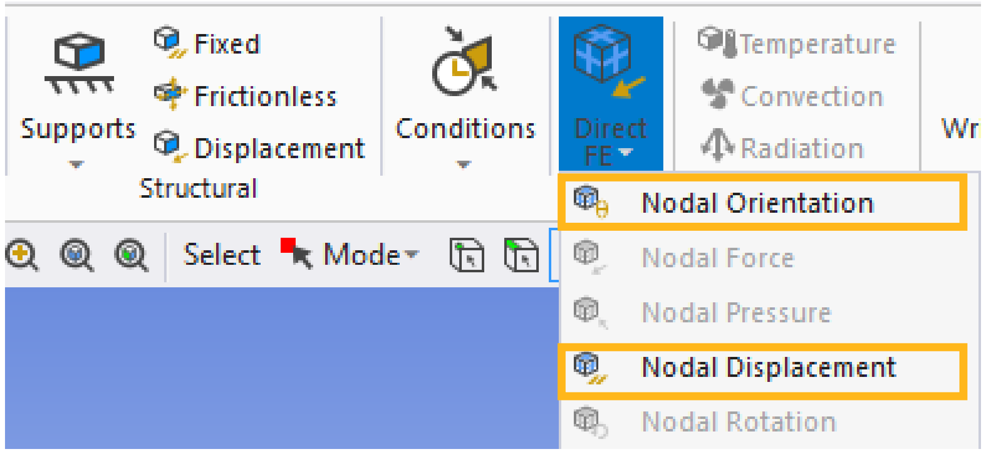




… Specifying Symmetry

- Method 2 – Pre-Meshed Cyclic Region
  - Select the low and high sector boundaries as with a Cyclic Region
  - Specify the Number of Sectors (360/α)
  - Specify the local cylindrical coordinate system
  - Specify Boundary DOF Orientation to Manual
  - Use Nodal Orientations and Nodal Displacements applied to the desired regions such that both cyclic symmetry and loading conditions are satisfied





See an example here:
https://ansyshelp.ansys.com/account/secured?returnurl=/Views/Secured/corp/v222/wb_sim/ds_pre_meshed_cyclic.html

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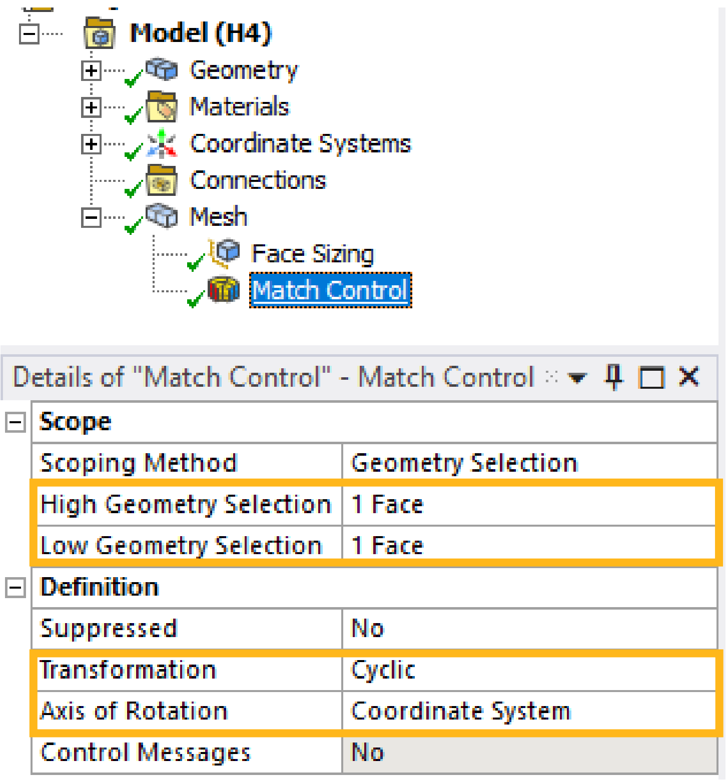
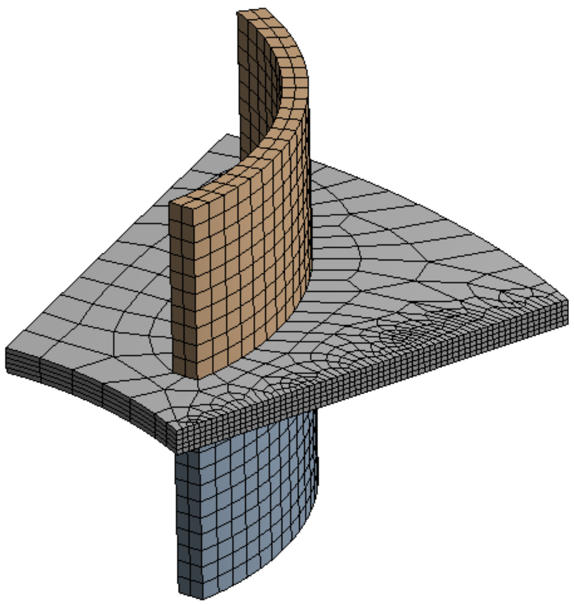
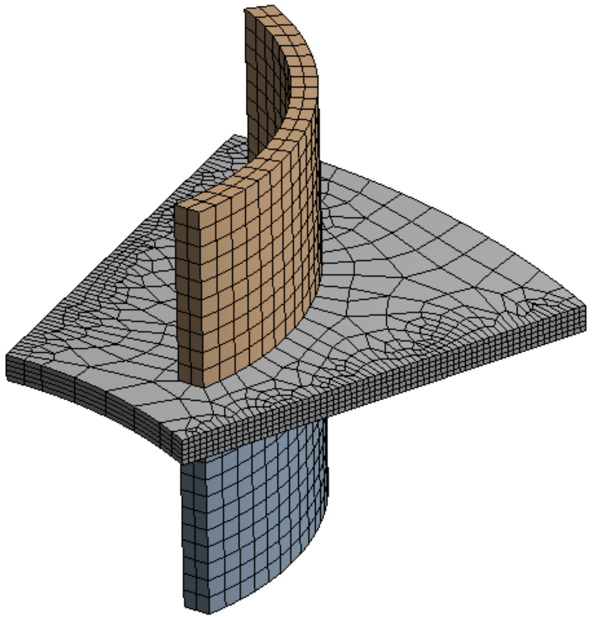
… Specifying Symmetry

- Method 2 – Pre-Meshed Cyclic Region
  - To achieve a matched mesh with this method, apply a Match Control mesh control in the upstream application, selecting the low and high sector boundaries and cyclic coordinate system as inputs.





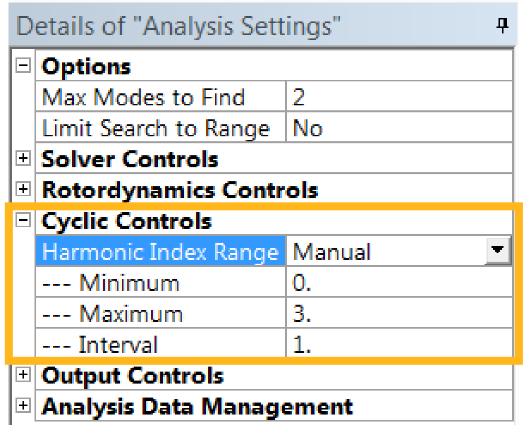
Without Match Control; Face sizing applied to lower boundary, higher boundary mesh doesn't match



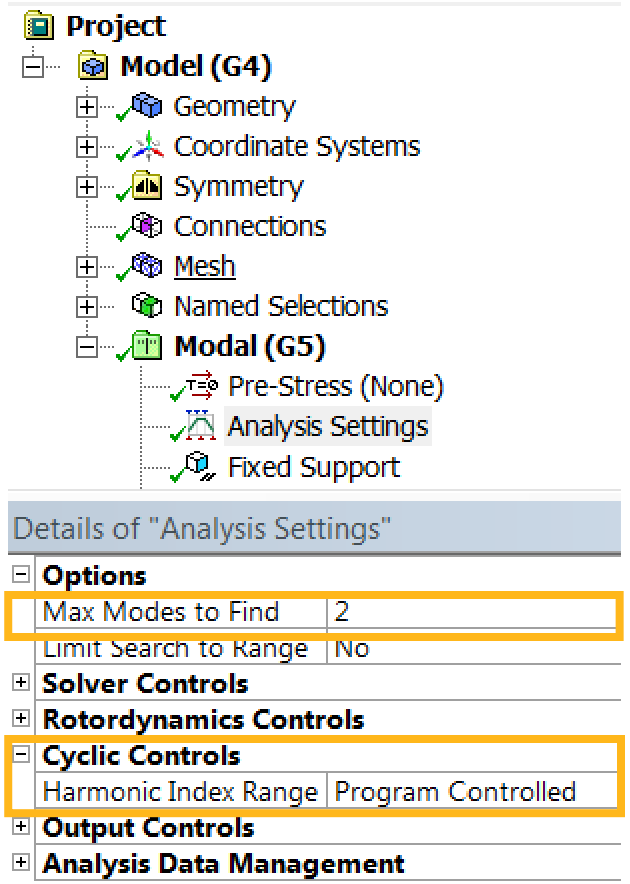
With Match Control; Face sizing applied to lower boundary, higher boundary mesh matches

E. Analysis Settings

- Analysis settings for modal cyclic symmetry are similar to those for modal analysis, with the following exceptions:
  - Max Modes to Find: This is on a *per harmonic index* basis
  - Harmonic Index Range: Defaults to Program Controlled (All Harmonic Indices are used; N/2 if N is even, (N-1)/2 if N is odd)
  - Alternatively, specify the desired range:



  - Damped mode extraction is not supported, nor is the unsymmetric solver option



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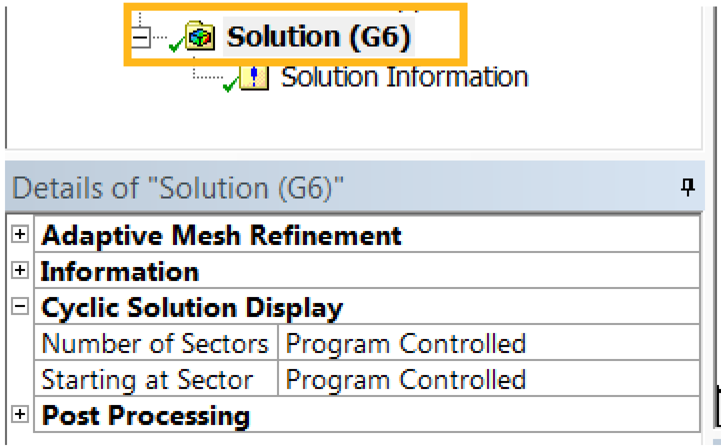
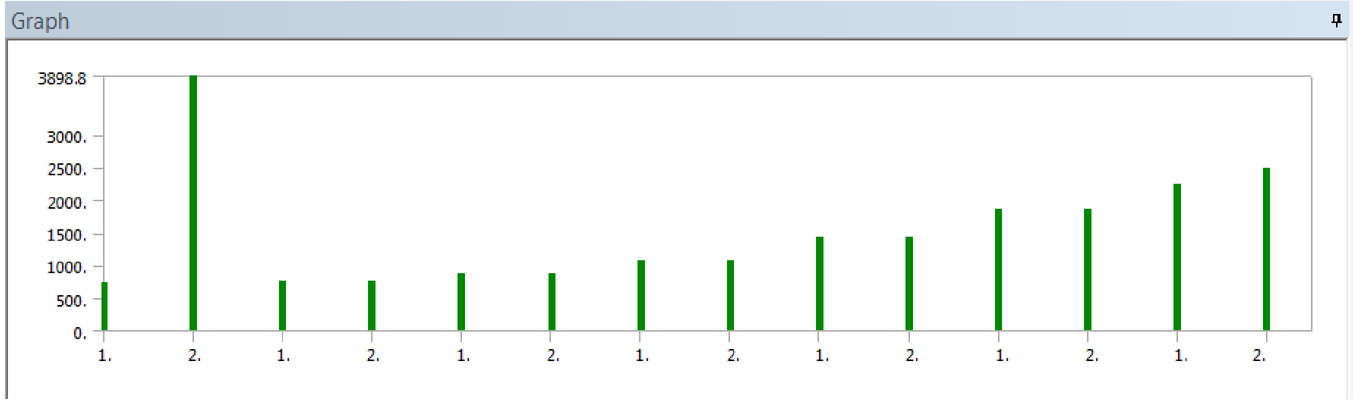
F. Postprocessing

- Requesting Results
  - When solution is finished, the modes will be displayed in the Graph and Tabular Data.
  - By default, the modes are *not* listed in ascending order as for typical modal analysis, but rather they are sorted by ascending *Harmonic Index*.
  - Select modes either from the Graph or Tabular Data, RMB > Create Mode Shape Results





Tabular Data

	Mode	Harmonic Index	Frequency [Hz]
1	1.	0.	749.71
2	2.	0.	3898.8
3	1.	1.	771.89
4	2.	1.	771.89
5	1.	2.	867.67
6	2.	2.	867.67
7	1.	3.	1085.7
8	2.	3.	1085.7
9	1.	4.	1431.6
10	2.	4.	1431.6

Frequencies are not sequential

… Postprocessing

- Requesting Results
  - Once a result is defined, the Details view of that result can be used to change the way the mode shapes are sorted in both the Graph and Tabular Data views
  - Select Graph Controls > X-Axis and choose between Mode and Frequency options

Solution (G6)
- Solution Information
- Total Deformation

Details of "Total Deformation"

Scope	
Scoping Method	Geometry Selection
Geometry	All Bodies
Definition	
Type	Total Deformation
Cyclic Mode	1.
Harmonic Index	0.
Identifier	
Suppressed	No
Results	
Minimum	0. in
Maximum	20.139 in
Minimum Occurs On	Solid
Maximum Occurs On	Solid
Graph Controls	
X-Axis	Mode
Information	Mode
Frequency	Frequency

Mode Option – *Sorted by Harmonic*

Tabular Data

	Mode	Harmonic Index	Frequency [Hz]
1	1.	0.	749.71
2	2.	0.	3898.8
3	1.	1.	771.89
4	2.	1.	771.89
5	1.	2.	867.67
6	2.	2.	867.67
7	1.	3.	1085.7
8	2.	3.	1085.7

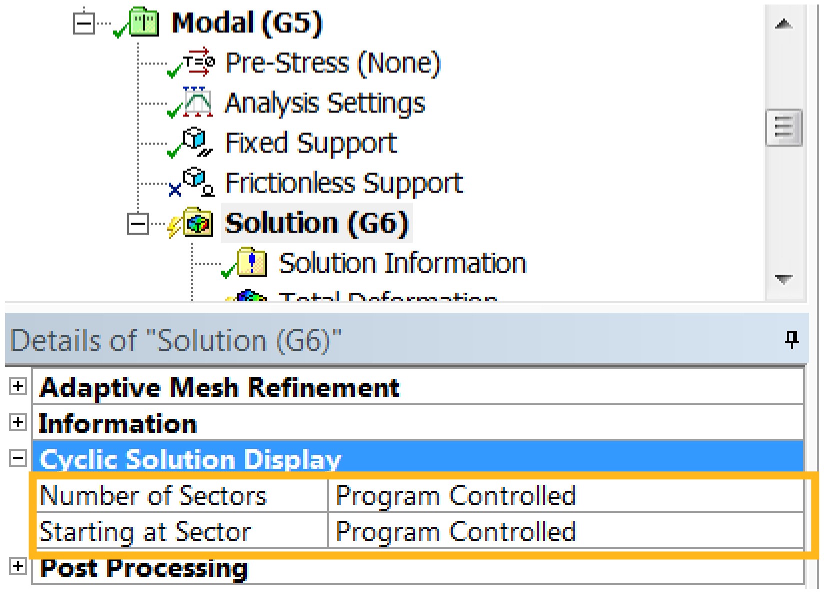
Frequency Option – *Sorted by Hz*

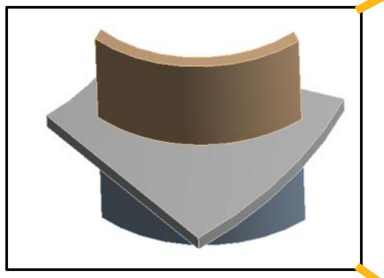
Tabular Data

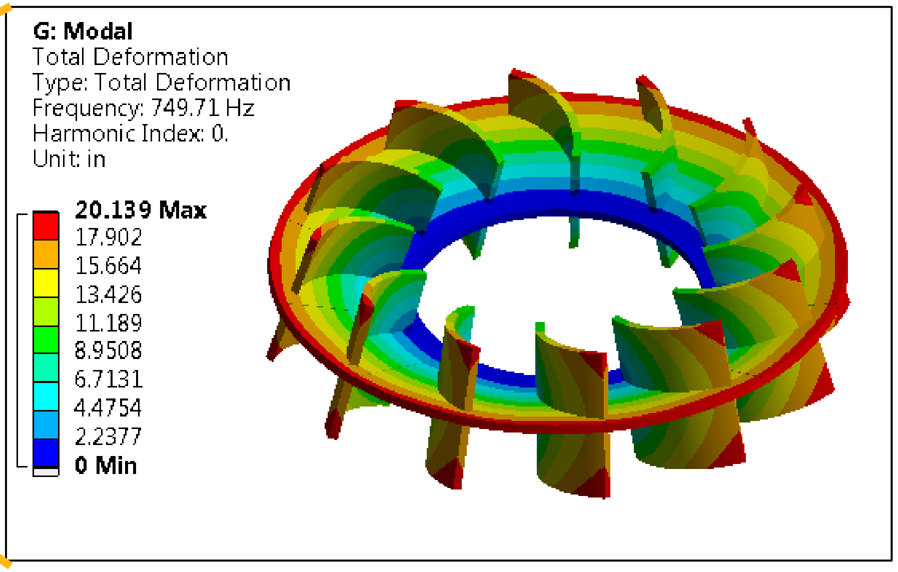
	Frequency [Hz]	Mode	Harmonic Index
1	749.71	1.	0.
2	771.89	1.	1.
3	771.89	2.	1.
4	867.67	1.	2.
5	867.67	2.	2.
6	1085.7	1.	3.
7	1085.7	2.	3.
8	1431.6	1.	4.

… Postprocessing

- Graphics Expansion
  - By Default, Mechanical will show mode shapes on the sector geometry expanded to a full 360°.

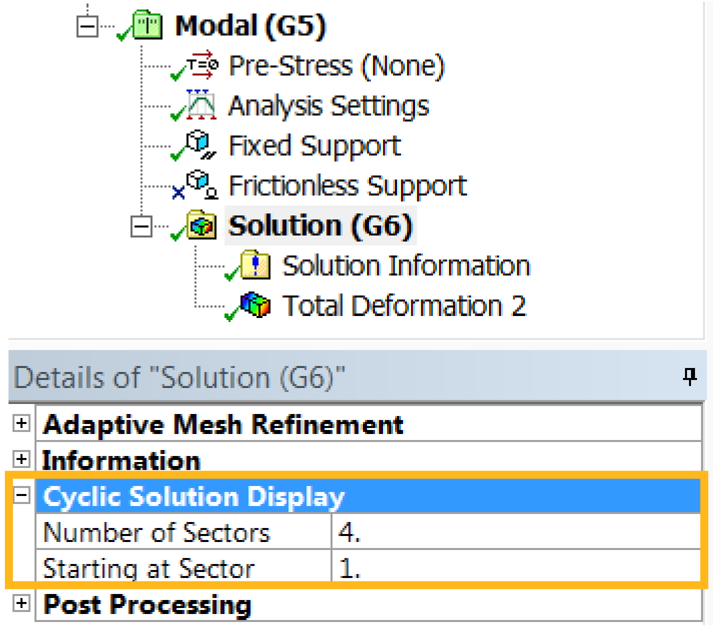


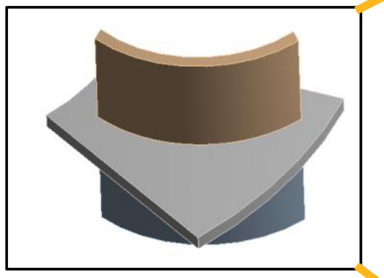


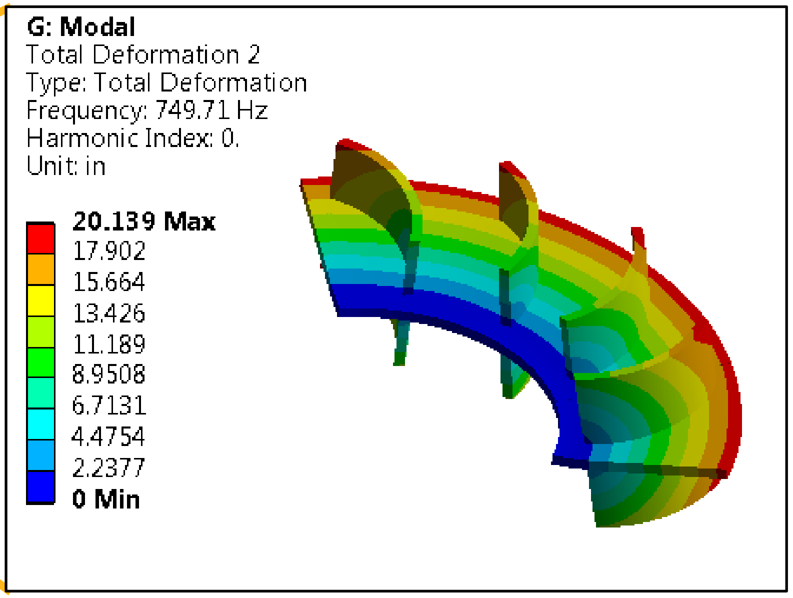


… Postprocessing

- Graphics Expansion
  - Alternatively, specify the desired number of sectors along with the starting sector number…







… Postprocessing

- Traveling Wave
  - Close inspection of the Results Tabular Data reveals that natural frequencies are reported in pairs (couplets) for harmonic indices other than 0 and N/2.
  - Recall earlier we said that the displacement of any node in the sector can be expressed by:

$$u = u_A \cos(n-1)k\alpha - u_B \sin(n-1)k\alpha$$

  - Thus the mode shapes in each couplet represent two waves, one based on a sine and the other a cosine of the same frequency, with a phase difference of 90°.
  - The mode shapes associated with these couplets are non-unique; they can also be combined linearly to result in a valid shape for that common frequency.
  - The orientation of the combined mode shape may be along a nodal diameter that differs from that of either mode shape, resulting in what is known as a *traveling wave*.

Tabular Data

	Mode	Harmonic Index	Frequency [Hz]
1	1.	0.	749.71
2	2.	0.	3898.8
3	1.	1.	771.89
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7	1.	3.	1085.7
8	2.	3.	1085.7
9	1.	4.	1431.6
10	2.	4.	1431.6
11	1.	5.	1865.6
12	2.	5.	1865.6
13	1.	6.	2241.5
14	2.	6.	2486.1

… Postprocessing

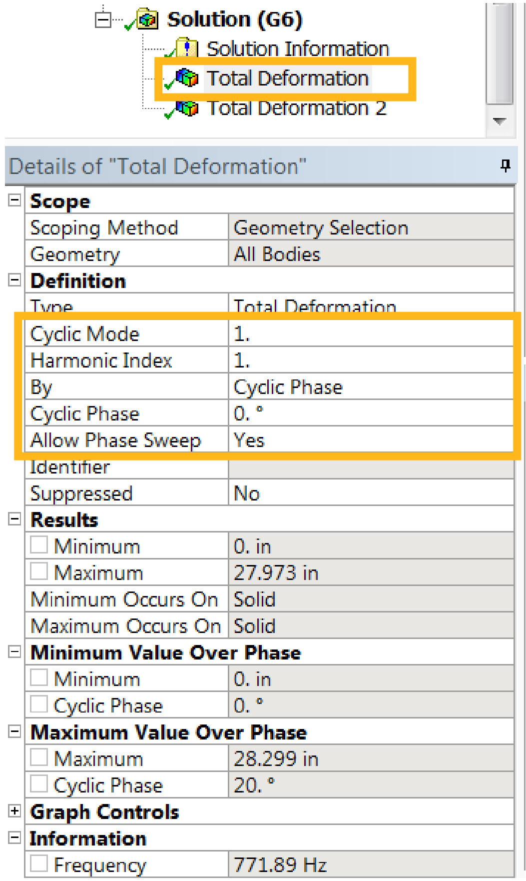
- Traveling Wave
  - Since the full structure may have stress-raising features, it is important to be able to determine results at all possible mode combinations (phase angels) between 0° and 360°.
  - To demonstrate this, consider a model with 12 sectors (30°), looking at the couplet for Harmonic Index 1:

Tabular Data

	Mode	Harmonic Index	Frequency [Hz]
1	1.	0.	749.71
2	2.	0.	3898.8
3	1.	1.	771.89
4	2.	1.	771.89
5	1.	2.	867.67
6	2.	2.	867.67
7	1.	3.	1085.7
8	2.	3.	1085.7
9	1.	4.	1431.6
10	2.	4.	1431.6
11	1.	5.	1865.6
12	2.	5.	1865.6
13	1.	6.	2241.5
14	2.	6.	2486.1

… Postprocessing

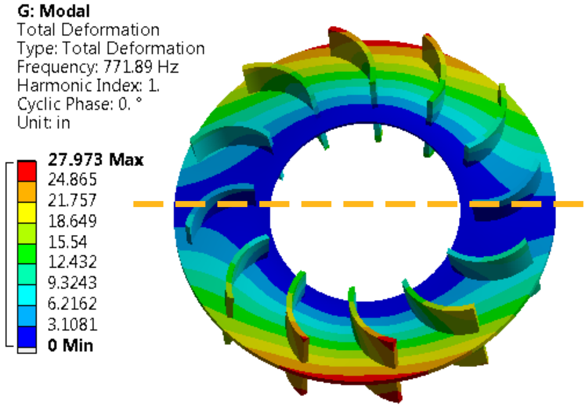
- Traveling Wave
  - When requesting results for the couplet at Harmonic Index 1, the Details view provides additional options related to phase.



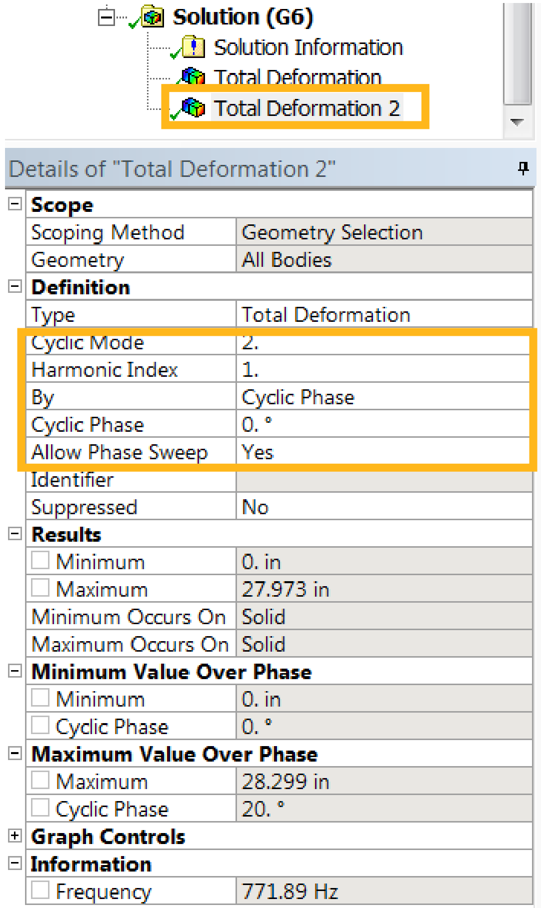
Solution (G6)
- Solution Information
- Total Deformation
- Total Deformation 2

Details of "Total Deformation"

Scope	
Scoping Method	Geometry Selection
Geometry	All Bodies
Definition	
Type	Total Deformation
Cyclic Mode	1.
Harmonic Index	1.
By	Cyclic Phase
Cyclic Phase	0. °
Allow Phase Sweep	Yes
Identifier	
Suppressed	No
Results	
Minimum	0. in
Maximum	27.973 in
Minimum Occurs On	Solid
Maximum Occurs On	Solid
Minimum Value Over Phase	
Minimum	0. in
Cyclic Phase	0. °
Maximum Value Over Phase	
Maximum	28.299 in
Cyclic Phase	20. °
Graph Controls	
Information	
Frequency	771.89 Hz



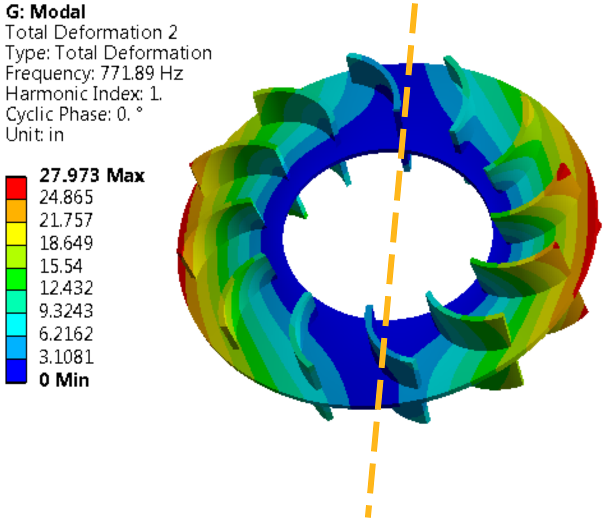
Notice presence of 1 nodal diameter, 90° out of phase with mode 2



Solution (G6)
- Solution Information
- Total Deformation
- Total Deformation 2

Details of "Total Deformation 2"

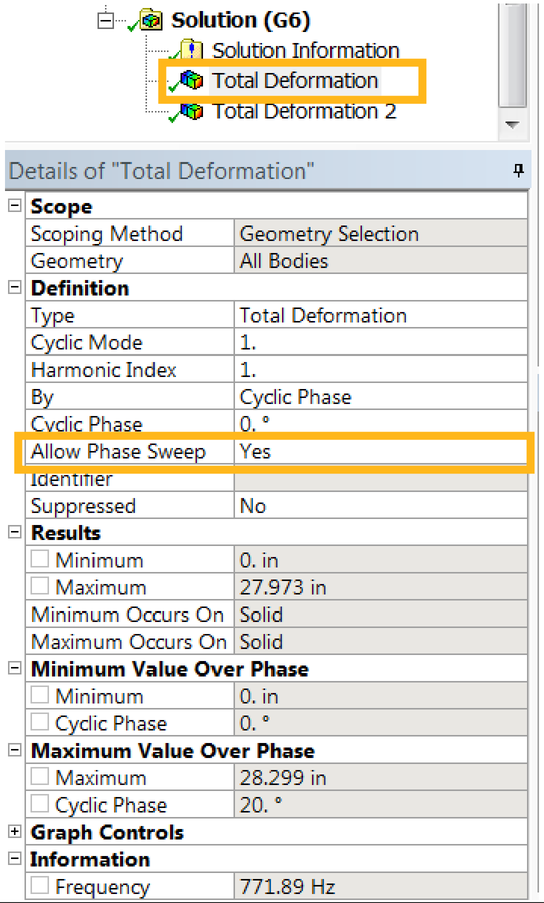
Scope	
Scoping Method	Geometry Selection
Geometry	All Bodies
Definition	
Type	Total Deformation
Cyclic Mode	2.
Harmonic Index	1.
By	Cyclic Phase
Cyclic Phase	0. °
Allow Phase Sweep	Yes
Identifier	
Suppressed	No
Results	
Minimum	0. in
Maximum	27.973 in
Minimum Occurs On	Solid
Maximum Occurs On	Solid
Minimum Value Over Phase	
Minimum	0. in
Cyclic Phase	0. °
Maximum Value Over Phase	
Maximum	28.299 in
Cyclic Phase	20. °
Graph Controls	
Information	
Frequency	771.89 Hz

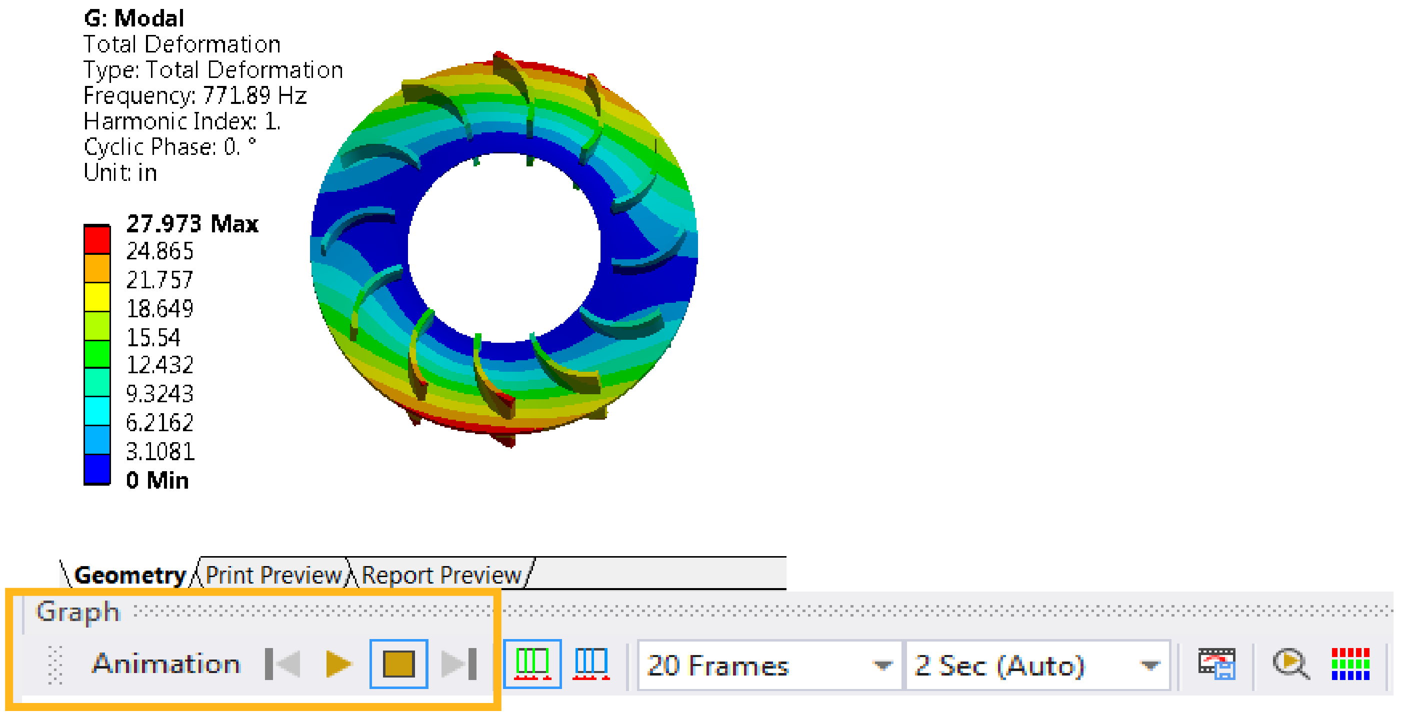


Ansys

… Postprocessing

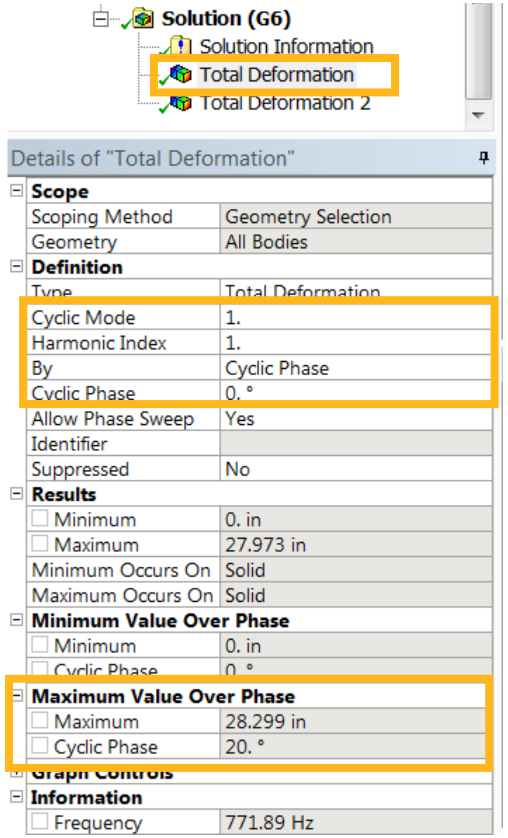
- Traveling Wave
  - The “Allow Phase Sweep” option enables mode shape calculations through the entire 360° geometry, thus enabling animation of the traveling wave through all phase angles.





… Postprocessing

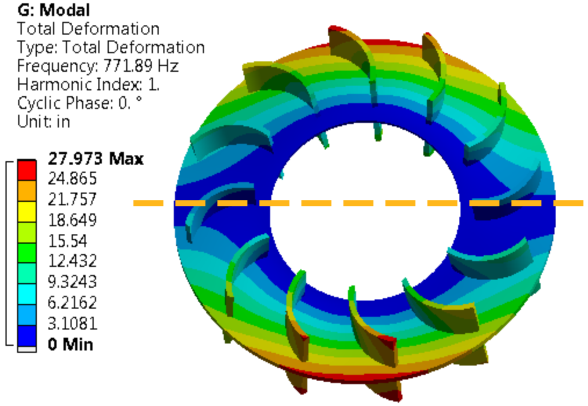
- Traveling Wave
  - The “Maximum Value Over Phase” output indicates the maximum deformation (normalized) occurs at a cyclic phase of 20°. Comparing results at 0° and 20°:

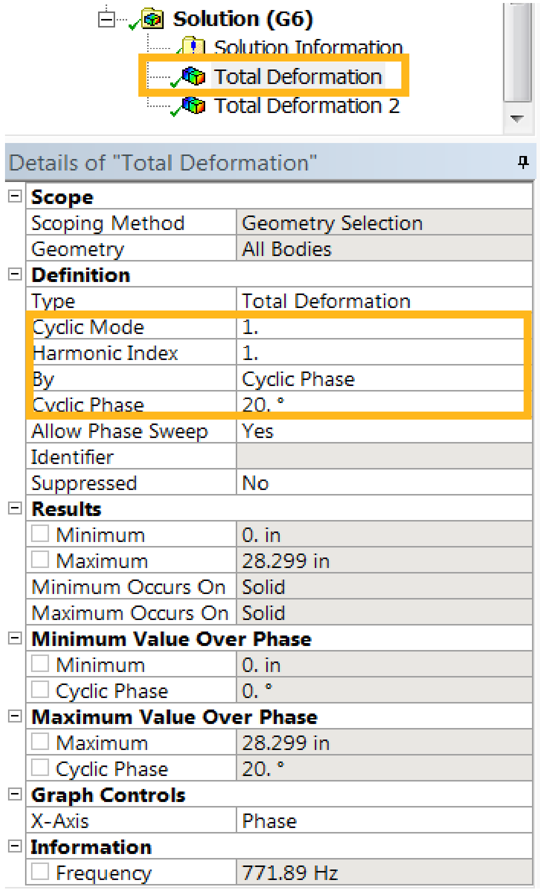


Solution (G6)
Solution Information
Total Deformation
Total Deformation 2

Details of "Total Deformation"

Scope	
Scoping Method	Geometry Selection
Geometry	All Bodies
Definition	
Type	Total Deformation
Cyclic Mode	1.
Harmonic Index	1.
By	Cyclic Phase
Cyclic Phase	0. °
Allow Phase Sweep	Yes
Identifier	
Suppressed	No
Results	
Minimum	0. in
Maximum	27.973 in
Minimum Occurs On	Solid
Maximum Occurs On	Solid
Minimum Value Over Phase	
Minimum	0. in
Cyclic Phase	0. °
Maximum Value Over Phase	
Maximum	28.299 in
Cyclic Phase	20. °
Graph Controls	
Information	
Frequency	771.89 Hz

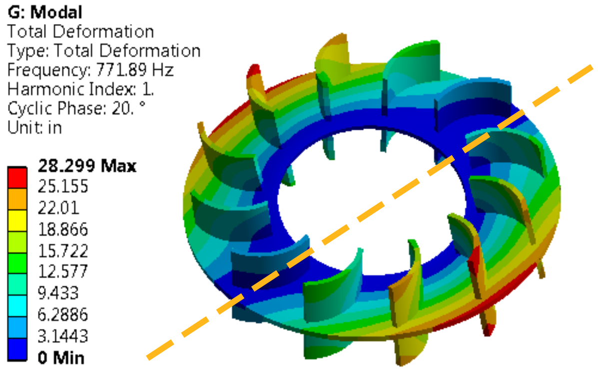




Solution (G6)
Solution Information
Total Deformation
Total Deformation 2

Details of "Total Deformation"

Scope	
Scoping Method	Geometry Selection
Geometry	All Bodies
Definition	
Type	Total Deformation
Cyclic Mode	1.
Harmonic Index	1.
By	Cyclic Phase
Cyclic Phase	20. °
Allow Phase Sweep	Yes
Identifier	
Suppressed	No
Results	
Minimum	0. in
Maximum	28.299 in
Minimum Occurs On	Solid
Maximum Occurs On	Solid
Minimum Value Over Phase	
Minimum	0. in
Cyclic Phase	0. °
Maximum Value Over Phase	
Maximum	28.299 in
Cyclic Phase	20. °
Graph Controls	
X-Axis	Phase
Information	
Frequency	771.89 Hz



… Postprocessing

- Standing Wave
  - Mode shapes at Harmonic Indices of 0 and N/2 have no phase dependency and are therefore commonly known as *standing waves.*

	Mode	Harmonic Index	Frequency [Hz]
1	1.	0.	749.71
2	2.	0.	3898.8
3	1.	1.	771.89
4	2.	1.	771.89
5	1.	2.	867.67
6	2.	2.	867.67
7	1.	3.	1085.7
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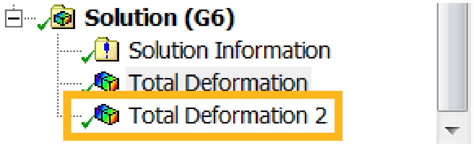
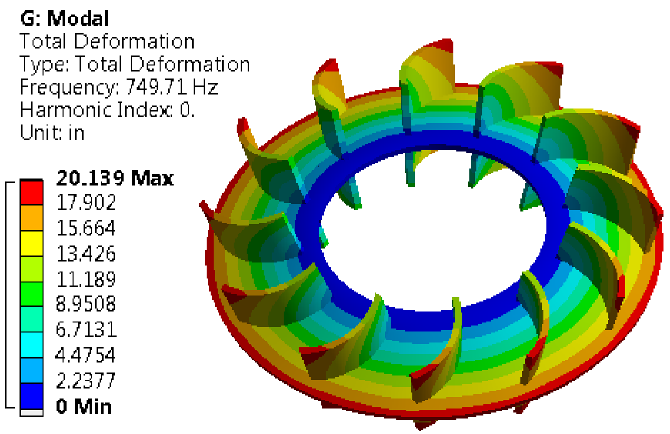
11	1.	5.	1865.6
12	2.	5.	1865.6
13	1.	6.	2241.5
14	2.	6.	2486.1

Tabular Data



Details of "Total Deformation"

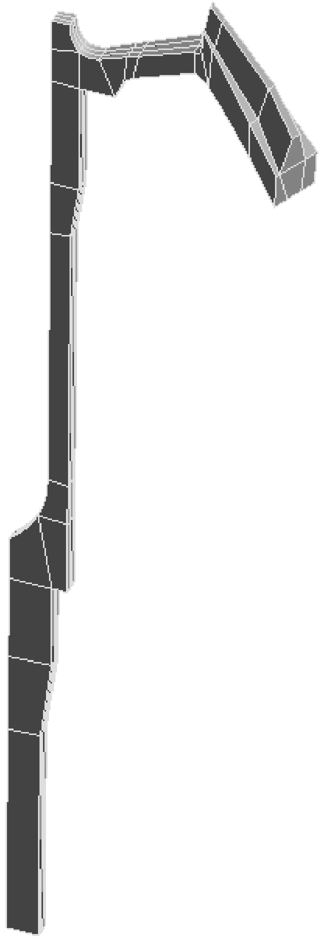
Scope	
Scoping Method	Geometry Selection
Geometry	All Bodies
Definition	
Type	Total Deformation
Cyclic Mode	1.
Harmonic Index	0.
Identifier	
Suppressed	No
Results	
Minimum	0. in
Maximum	20.139 in
Minimum Occurs On	Solid
Maximum Occurs On	Solid
Graph Controls	
X-Axis	Mode
Information	
Frequency	749.71 Hz

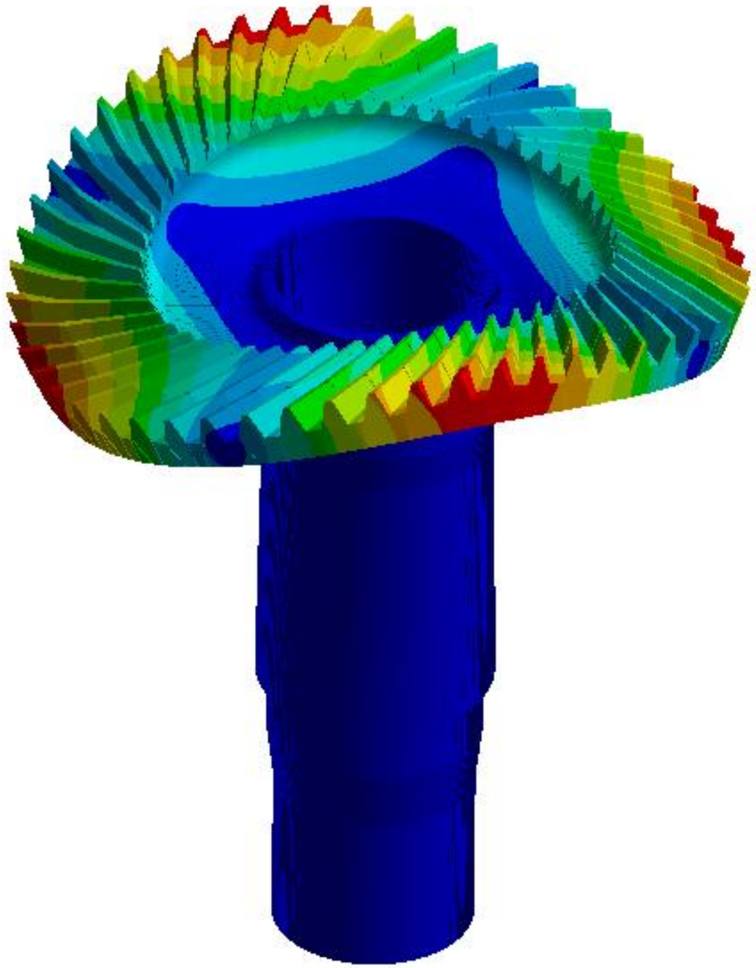


Since Harmonic Index = 0, this is a "breathing mode" and does not exhibit a nodal diameter.

Notice missing presence of Phase information in Details view

Workshop 04.1: Modal Cyclic Symmetry of Bevel Gear





End of presentation