Ansys Mechanical Linear and Nonlinear Dynamics

Module 06: Harmonic Analysis

Release 2022 R2

Please note:
- These training materials were developed and tested in Ansys Release 2022 R2. Although they are expected to behave similarly in later releases, this has not been tested and is not guaranteed.
- The screen images included with these training materials may vary from the visual appearance of a local software session.

Module 06 Learning Outcomes

- After completing this module, you will:
  - Know how to analyze a structure subjected to sinusoidally varying load conditions.
  - Understand the advantages and disadvantages of each harmonic solution method and be able to apply the one most-suited to your application.
  - Be able to choose the appropriate form(s) of damping for a given solution method and see their effect on the response of a structure.
  - Learn techniques to ensure peak harmonic responses are captured during the simulation.

Module 06 Topics

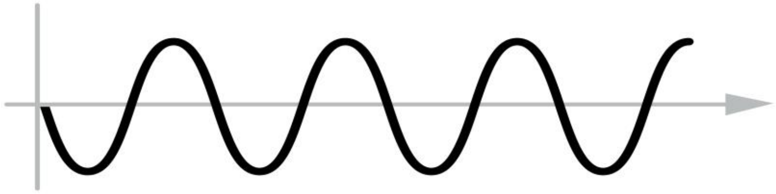
A. What is Harmonic Analysis?
B. Theory and Terminology
C. Contact in Harmonic Analysis
D. Full Harmonic Analysis
E. Damping in Full Harmonic Analysis
F. Loads and Boundary Conditions
G. Analysis Settings—Full Harmonic Analysis
H. Results – Full Harmonic Analysis
I. Mode-Superposition Harmonic Analysis
J. Damping in MSUP Harmonic Analysis
K. Analysis Settings—MSUP Harmonic Analysis
L. MSUP Harmonic Analysis Based on Linear Perturbation
M. Full Harmonic Analysis Based on Linear Perturbation

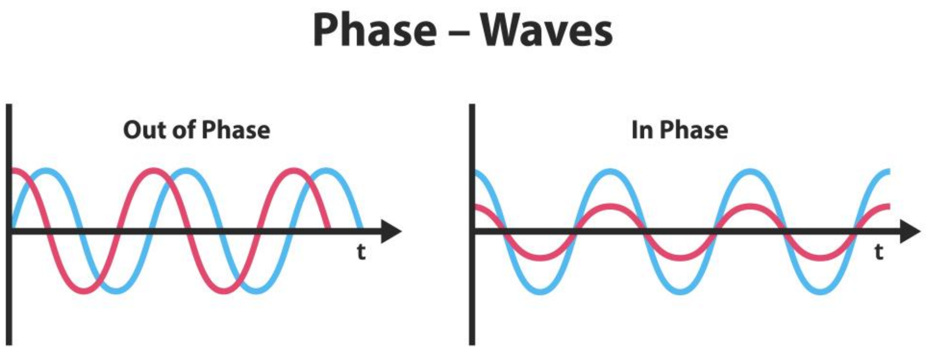
A. What is Harmonic Analysis?

- Used to determine steady-state response of a structure to loads that vary sinusoidally (harmonically) with time.
- Input:
  - Harmonic loads (forces, pressures, imposed displacements, etc.) of known magnitude and frequency.
  - May be multiple loads all at the same frequency.
  - Most types of applied loads can be in phase or out of phase.
  - Body loads can only be specified with a phase angle of zero.
- Output:
  - Harmonic displacements as a function of frequency at each DOF, usually out of phase with the applied loads.
  - Other derived quantities, such as stresses and strains, usually evaluated at “peak” response frequencies.

A. What is Harmonic Analysis?

- Used to determine steady-state response of a structure to loads that vary sinusoidally (harmonically) with time.
- Input:
  - Harmonic loads (forces, pressures, imposed displacements, etc.) of known magnitude and frequency.
  - May be multiple loads all at the same frequency.
  - Most types of applied loads can be in phase or out of phase.
  - Body loads can only be specified with a phase angle of zero.
- Output:
  - Harmonic displacements as a function of frequency at each DOF, usually out of phase with the applied loads.
  - Other derived quantities, such as stresses and strains, usually evaluated at "peak" response frequencies.





… What is Harmonic Analysis?

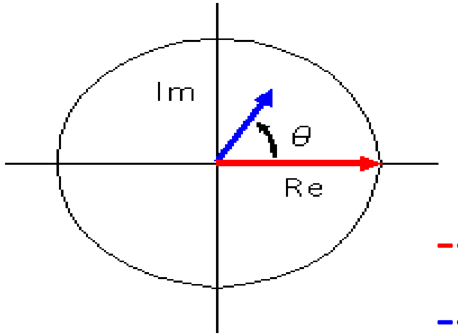
- Assumptions and Restrictions:
  - The entire structure has constant or frequency-dependent stiffness, damping, and mass effects.
  - No nonlinearities are permitted; nonlinear material properties, such as plasticity, are ignored even if defined.
  - Transient effects (vibrations that usually occur at the beginning of excitation) are not calculated.
  - Acceleration and bearing loads are assumed to be real (in-phase) only.

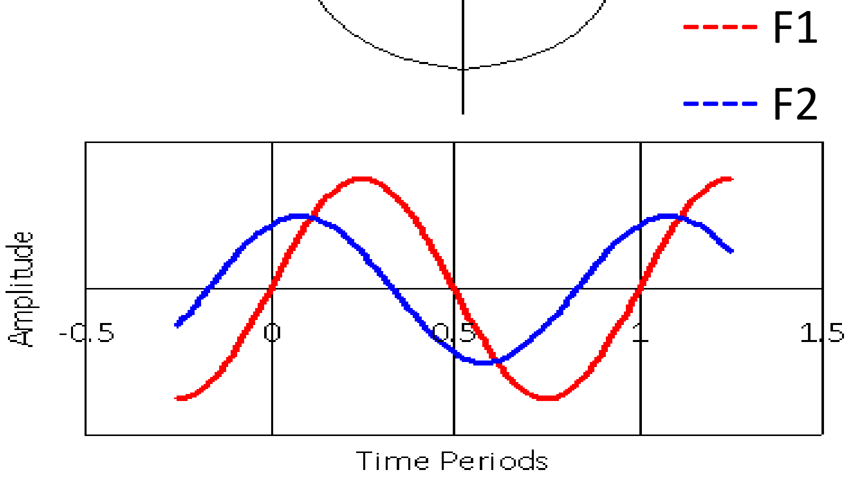
… What is Harmonic Analysis?

- Assumptions and Restrictions:
  - All loads and displacements vary sinusoidally at the same known frequency (although not necessarily in phase).
  - All outputs, likewise, are assumed to occur at the same frequency.
  - Calculated displacements are complex if:
    - damping is specified, or
    - applied load is complex (i.e. includes phase angle).

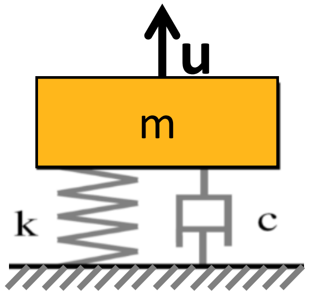
$$F_i = F_i \sin(\omega t + \theta_i)$$

$$\begin{aligned} \text{where } F &= \text{amplitude} \\ \omega &= \text{freqency} \\ \theta &= \text{phase angle} \end{aligned}$$





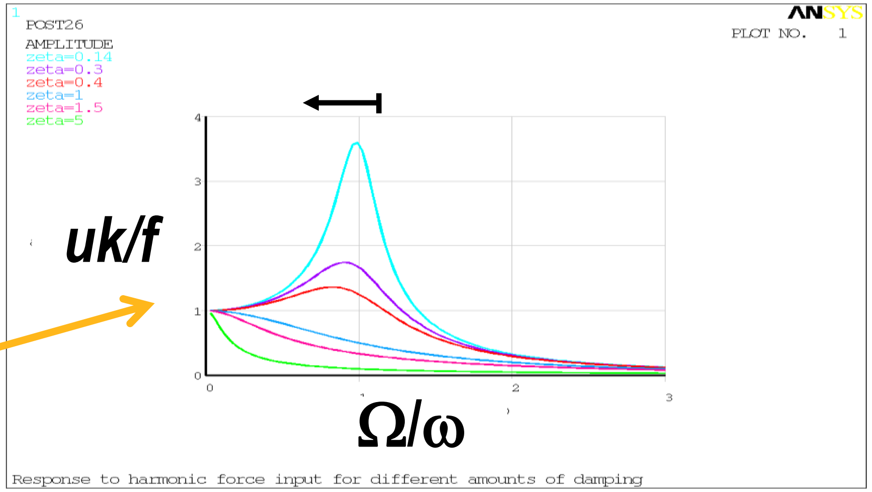
B. Theory and Terminology



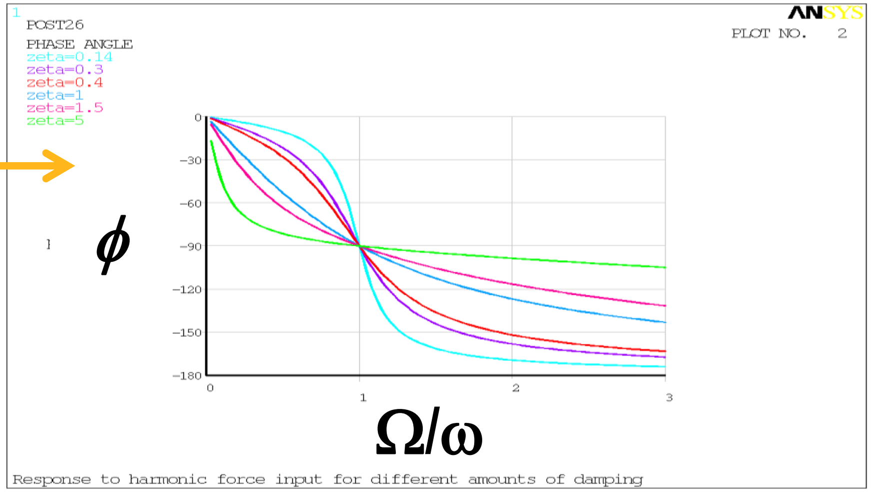
- Governing equation for a mass-spring-damper system subject to a sinusoidal force is

$$m\ddot{u} + c\dot{u} + ku = f\sin\Omega\, t$$

$$u = \frac{f/k}{\sqrt{(1-(\Omega/\omega_n)^2)^2 + \left(2\xi(\Omega/\omega_n)\right)^2}}$$

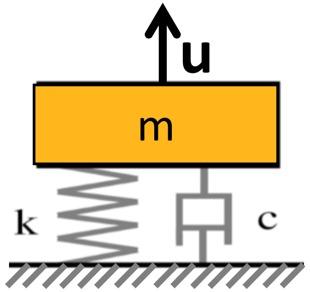


$$\varphi = \tan^{-1}\frac{2\xi(\Omega/\omega_n)}{1-(\Omega/\omega_n)^2}$$

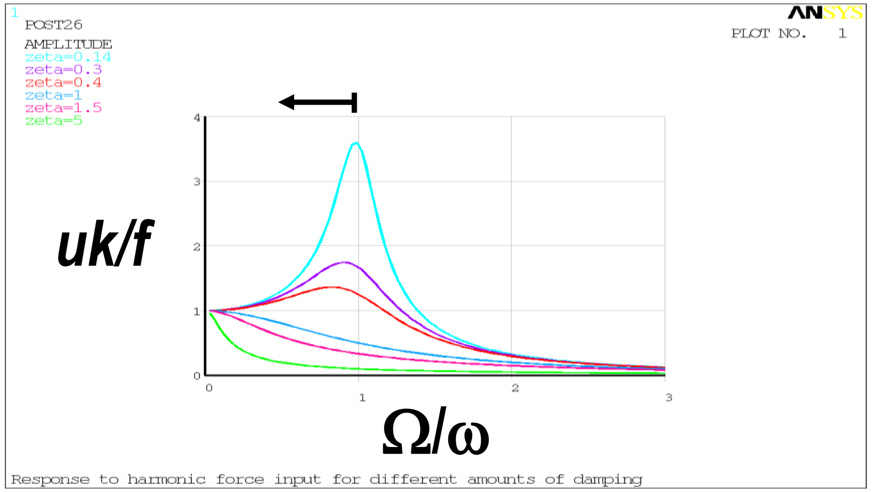


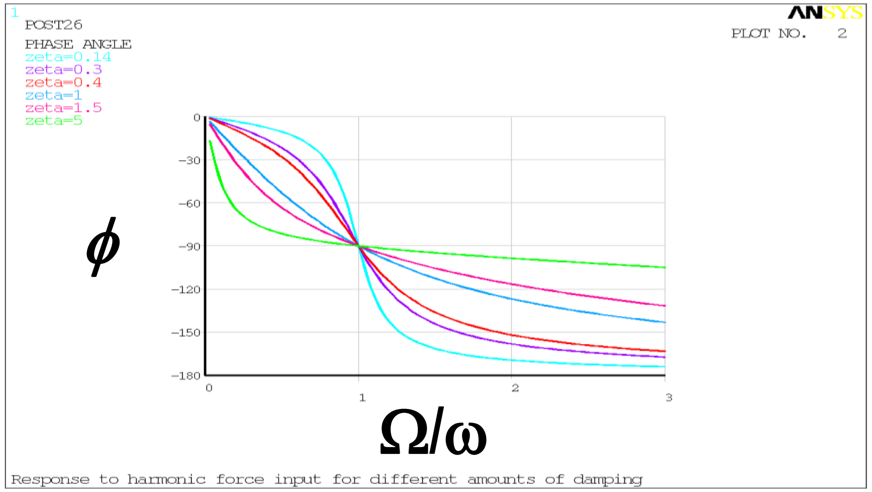
$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

... Theory and Terminology



- When the imposed frequency approaches a natural frequency in the direction of excitation, resonance occurs.
- An increase in damping decreases the amplitude of the response for all imposed frequencies,
- A small change in damping has a large effect on the response near resonance, and,
- The phase angle always passes through ±90° at resonance for any amount of damping.





Ansys

... Theory and Terminology

- The governing equation for a linear structure is:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F\}$$

- Assume $\{F\}$ and $\{u\}$ are harmonic with frequency Ω:

$$\{F\}=\{F_{\max}e^{i\psi}\}e^{i\Omega t}$$
$$\{u\}=\{u_{\max}e^{i\psi}\}e^{i\Omega t}$$

Note: The symbols Ω and ω differentiate the input from the output:

Ω = input (imposed) circular frequency

ω = output (natural) circular frequency

… Theory and Terminology

- Take two time derivatives:

$$
\begin{aligned}
\{u\} &= && (\{u_1\}+i\{u_2\})e^{i\Omega t} \\
\{\dot{u}\} &= && i\Omega\,(\{u_1\}+i\{u_2\})e^{i\Omega t} \\
\{\ddot{u}\} &= && -\Omega^2\,(\{u_1\}+i\{u_2\})e^{i\Omega t}
\end{aligned}
$$

- Substitute and simplify:

$$\left(-\Omega^2[M]+i\Omega[C]+[K]\right)(\{u_1\}+i\{u_2\})=(\{F_1\}+i\{F_2\})$$

- This can then be solved using one of two methods.

... Theory and Terminology

- Solution Techniques:
  - *Full Harmonic Response Analysis*
    - solves a system of simultaneous equations directly using a static solver designed for complex arithmetic:

$$\overbrace{\left(-\Omega^2[M]+i\Omega[C]+[K]\right)}^{[K_c]}\overbrace{\left(\{u_1\}+i\{u_2\}\right)}^{\{u_c\}}=\overbrace{\left(\{F_1\}+i\{F_2\}\right)}^{\{F_c\}}$$

$$[K_c]\{u_c\}=\{F_c\}$$

  - *Mode-Superposition (MSUP) Harmonic Response Analysis*
    - expresses the displacements as a linear combination of mode shapes:

$$\left(-\Omega^2[M]+i\Omega[C]+[K]\right)\left(\{u_1\}+i\{u_2\}\right)=\left(\{F_1\}+i\{F_2\}\right)$$

$$\vdots$$

$$\left(-\Omega^2+i2\omega_j\Omega\,\xi_j+\omega_j^2\right)y_{jc}=f_{jc}$$

C. Contact in Harmonic Analysis

- Contact regions are available in harmonic analysis; however, since this is a purely linear analysis, contact behavior will differ for the nonlinear contact types, as shown below:

Contact Type	Static Analysis	Linear Dynamic Analysis		
		Initially Touching	Inside Pinball Region	Outside Pinball Region
Bonded	Bonded	Bonded	Bonded	Free
No Separation	No Separation	No Separation	No Separation	Free
Rough	Rough	Bonded	Free	Free
Frictionless	Frictionless	No Separation	Free	Free
Frictional	Frictional	$\mu = 0$, No Separation $\mu > 0$, Bonded	Free	Free

- Contact behavior will reduce to its linear counterparts.

D. Full Harmonic Analysis

- Exact solution.
- Generally slower than MSUP.
- Supports all types of loads and boundary conditions.
- Solution points may be uniformly distributed across the frequency domain or non-uniformly distributed at user-defined locations.
- Solves the full system of simultaneous equations using the Sparse matrix solver for complex arithmetic.

$$\overbrace{\left(-\Omega^2[M]+i\Omega[C]+[K]\right)}^{[K_c]}\overbrace{\left(\{u_1\}+i\{u_2\}\right)}^{\{u_c\}}=\overbrace{\left(\{F_1\}+i\{F_2\}\right)}^{\{F_c\}}$$

$$[K_c]\{u_c\}=\{F_c\}$$

E. Damping In Full Harmonic Analysis

- The complete expression for the damping matrix [C] is given by:

α: Global Mass-Matrix Multiplier (alpha damping, ALPHAD)

β: Global k-Matrix Multiplier (beta damping, BETAD)

g: constant structural damping coefficient (DMPSTR)

α_i^m : Mass matrix multiplier for material *i* (alpha damping, MP,ALPD)

β_j^m : Stiffness matrix multiplier for material *j* (beta damping, MP,BETD)

m_j: constant structural damping coefficient for material *j* (MP,DMPS)

C_k: Element damping (via the various Connection elements, COMBIN14, MPC184, etc.)

Other terms not mentioned have not been exposed within Engineering Data or Mechanical. See the Ansys Theory manual for a complete description of the additional terms: https://ansyshelp.ansys.com/account/secured?returnurl=/Views/Secured/corp/v222/ans_thry/thy_tool3.html

$$[C]=\alpha[M]+\left(\beta+\frac{g}{\Omega}\right)[K]+\sum_{i=1}^{N_{ma}}\alpha_i^m[M_i]+\sum_{i=1}^{N_{ma}^{MD}}\sum_{k=1}^{N_{sa}}\alpha_p[M_k]_i$$

$$+\sum_{j=1}^{N_m}\left(\beta_j^m+\frac{m_j}{\Omega}+\frac{g_j^E}{\Omega}\right)[K_j]+\sum_{j=1}^{N_{mb}^{MD}}\sum_{n=1}^{N_{sb}}\beta_q[K_n]_j$$

$$+\sum_{k=1}^{N_e}[C_k]+\sum_{m=1}^{N_v}\frac{[K_m]}{\Omega}+\sum_{l=1}^{N_g}[G_l]+\frac{1}{\Omega}\sum_{k=1}^{N_e^*}[K_k^*]$$

… Damping in Full Harmonic Analysis

- Full Harmonic Analysis accepts Rayleigh, Element, and Hysteretic damping in the form of the Constant Structural Damping Coefficient.

1. Rayleigh Damping (Defined Globally):

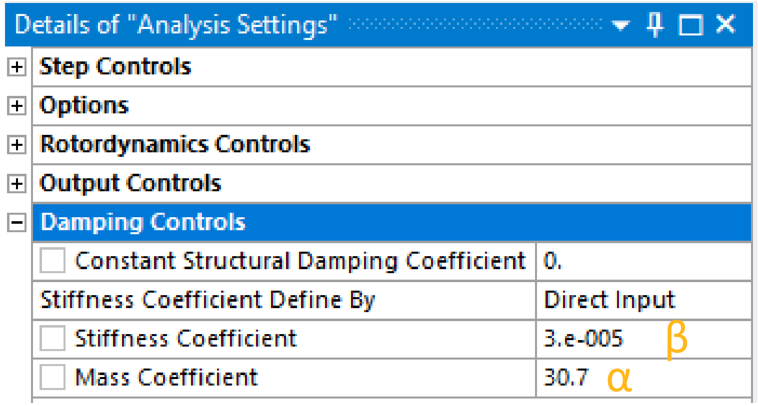
- Alpha damping and Beta damping are defined by Rayleigh damping constants α and β. The damping matrix [C] is calculated by using these constants to multiply the mass matrix [M] and stiffness matrix [K]:

$$[C] = \alpha[M] + \beta[K]$$

Equivalent damping

$$\xi = \frac{\alpha}{2\omega} + \frac{\beta\omega}{2}$$

- Values of α and β are not usually known directly but are calculated from damping ratio ξ as discussed in Module 02.

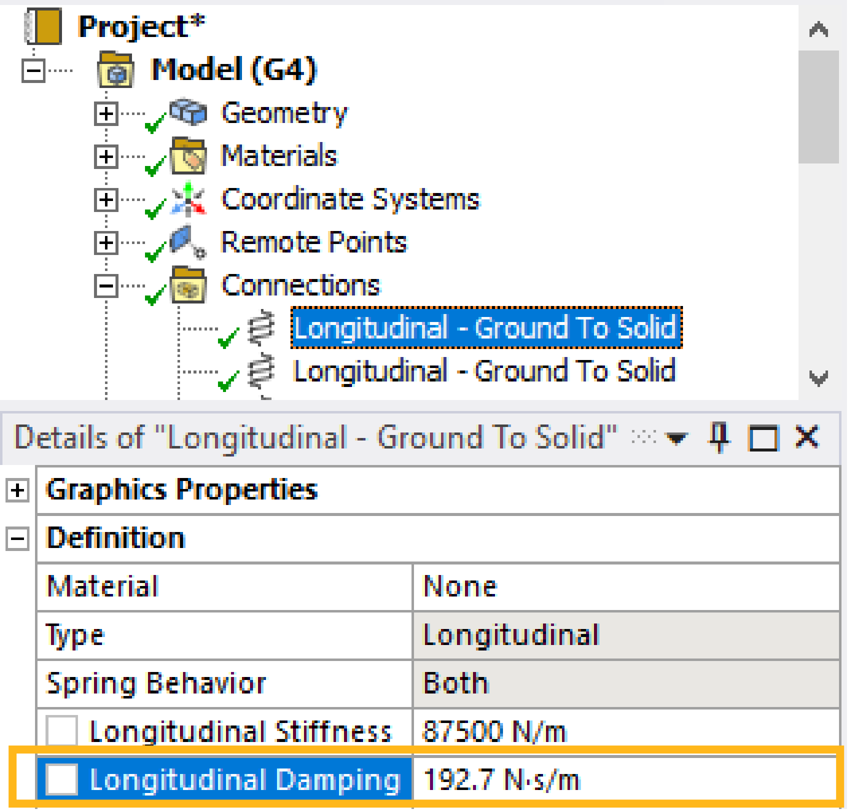


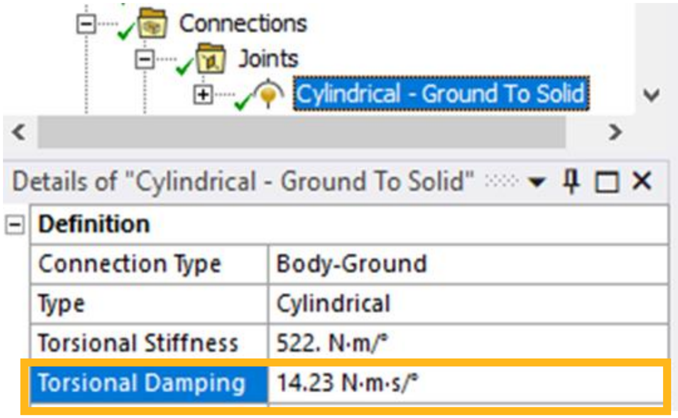
… Damping in Full Harmonic Analysis

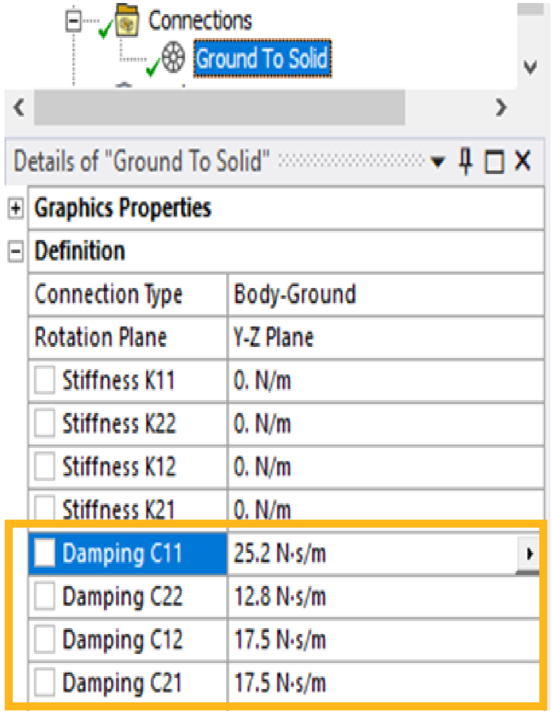
2. Element Damping:

- Element damping involves element types having viscous damping characteristics; Body-Body and/or Body-Ground longitudinal Spring Connections, Cylindrical Joints, and Bearings are some examples.

$$\sum_{k=1}^{N_e}[C_k]$$







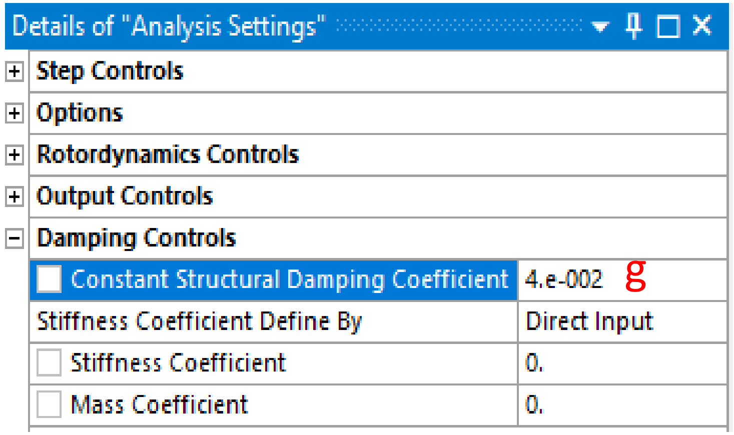
… Damping in Full Harmonic Analysis

3. Hysteretic Damping (Defined Globally):

- Material damping (solid, hysteretic) is inherently present in a material (energy is dissipated by internal friction), so it is typically considered in a dynamic analysis.
- Energy dissipated by internal friction in a real system does not depend on the cyclic frequency.
- The simplest device to represent it is to assume the damping force is proportional to displacement (strain).

$$[C] = \frac{g}{\Omega}[K] \quad \xrightarrow{\text{Equivalent damping}} \quad \xi = g/2$$

where g = constant structural damping coefficient



… Damping in Full Harmonic Analysis

The values of g (or m_j as depicted below), α^m and β^m can be defined on a material basis as well:

1. Rayleigh Damping and Hysteretic Damping (Defined per material):
   - Mass-Matrix Damping Multiplier, K-Matrix Damping Multiplier, Constant Structural Damping Coefficient

$$[C] = \sum_{i=1}^{N_{ma}} \alpha_i^m [M_i] + \sum_{j=1}^{N_{mb}} \left(\beta_j^m + \frac{1}{\Omega} m_j \right) [K_j]$$

Equivalent damping

$$\xi_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} + \frac{g}{2}$$

Define in Engineering Data

Properties of Outline Row 3: Structural Steel

	A	B
1	Property	Value
2	Material Field Variables	Table
3	Density	7850
4	⊟ Material Dependent Damping	
5	Damping Ratio	0.01
6	Constant Structural Damping Coefficient	= 0.02 m_j
7	⊟ Damping Factor (α)	
8	Mass-Matrix Damping Multiplier	12.56 α^m
9	⊟ Damping Factor (β)	
10	k-Matrix Damping Multiplier	0.003 β^m
11	⊞ Isotropic Elasticity	

F. Loads and Boundary Conditions

- Structural loads and supports may be used in harmonic analyses with the following exceptions:
  - Gravity Loads
  - Thermal Loads
  - Rotational Velocity and Rotational Acceleration
  - Pretension Bolt Load and Hydrostatic Pressure
  - Compression-Only Support (if present, it behaves similar to a Frictionless Support)
- Remember that all structural loads will vary sinusoidally at the same excitation frequency
- Loads can be out of phase with each other.
- Transient effects are not calculated.
- Acceleration loads may be defined, but will be assumed to act at a phase angle of zero.
- Bearing loads, also assumed to act at a phase angle of zero, may be defined, but are inappropriate in most cases and should be avoided.

... Loads and Boundary Conditions

- A list of supported loads is shown below:

Type of Load	Solution Method	Phase Input	Frequency Dependent
Acceleration	Full or MSUP	No	Yes
Pressure	Full or MSUP	Yes	Yes
Pipe Pressure	Full or MSUP	No	No
Force	Full or MSUP	Yes	Yes
Moment	Full or MSUP	Yes	Yes
Remote Force	Full or MSUP	Yes	Yes
Bearing	Full or MSUP	No	No
Line Pressure	Full or MSUP	Yes	No
Displacement	Full or MSUP*	Yes	Yes
Remote Displacement	Full Only	Yes	No
Direct FE Nodal Force	Full or MSUP	Yes	No
Direct FE Nodal Displacement	Full Only	Yes	No
Rotating Force	Full Only with Coriolis	Yes	No

*Displacement Loads in MSUP must be applied as Base Excitation applied to a Fixed Support(s) or Body-Ground Spring

- Acceleration and Bearing Loads do not support phase input. If other loads are present, try shifting the phase angle of the other loads such that the Acceleration and Bearing Loads appear at a phase angle of 0°.
- Any type of linear support is valid. The Compression Only (nonlinear) support may be defined, but should be avoided.

... Loads and Boundary Conditions

- Specifying harmonic loads requires:
  1. Amplitude $F_{i_{max}}$, or Real
  2. Phase angle θ, or Imaginary, and,
  3. Frequency ω (supplied via Frequency Spacing in Analysis Settings or via Tabular Data in Details of Load)

$$F_i = F_{i\,max} \sin(\omega t + \theta_i)$$

$$\text{where } F_{i\,max} = \text{amplitude}$$
$$\omega = \text{freqency}$$
$$\theta = \text{phase angle}$$

Details of "Analysis Settings"

Options	
Frequency Spacing	Linear
Range Minimum	0. Hz
Range Maximum	100. Hz
Solution Intervals	10

Details of "Force"

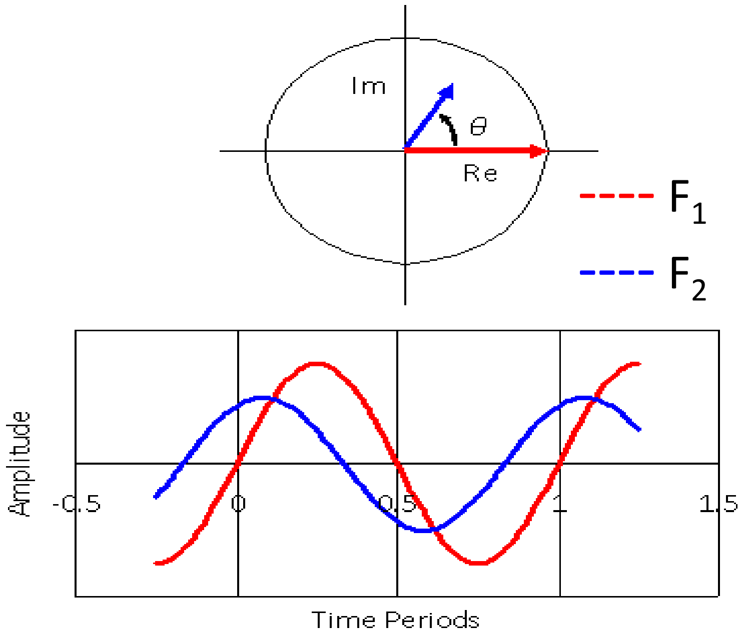
Scope	
Scoping Method	Geometry Selection
Geometry	1 Face
Definition	
Type	Force
Define By	Components
Coordinate System	Global Coordinate System
X Component	0. N
Y Component	0. N
Z Component	Tabular Data
X Phase Angle	0. °
Y Phase Angle	0. °
Z Phase Angle	Tabular Data
Suppressed	No

... Loads and Boundary Conditions

- Amplitude and phase angle
  - The load value (magnitude) represents the amplitude (F_1 and F_2).
  - Phase angle θ is the phase shift between two or more harmonic loads.
  - θ is not required if only one load is present.

Details of "Force"

Scope	
Scoping Method	Geometry Selection
Geometry	1 Face
Definition	
Type	Force
Define By	Components
Coordinate System	Global Coordinate System
X Component	0. N
Y Component	1. N
Z Component	0. N
X Phase Angle	0. °
Y Phase Angle	0. °
Z Phase Angle	0. °



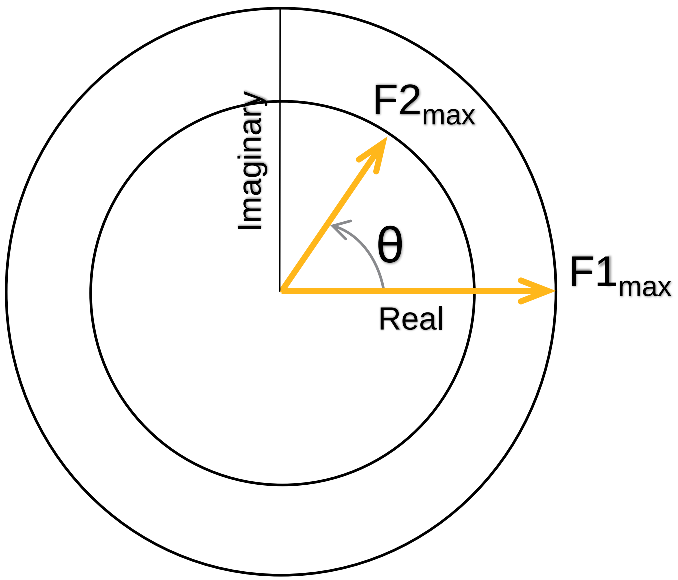
Ansys

... Loads and Boundary Conditions

- Real and Imaginary
  - $F1_{real} = F1_{max} \cos(0°) = F1_{max}$
  - $F1_{imag} = F1_{max} \sin(0°) = 0$
  - $F2_{real} = F2_{max} \cos(\theta)$
  - $F2_{imag} = F2_{max} \sin(\theta)$

Details of "Force"

Scope	
Scoping Method	Geometry Selection
Geometry	1 Face
Definition	
Type	Force
Define By	Components : Real – Imaginary
Coordinate System	Global Coordinate System
X Component - Real	20. N
X Component - Imag	14.4 N
Y Component - Real	0. N
Y Component - Imag	0. N
Z Component - Real	0. N
Z Component - Imag	0. N

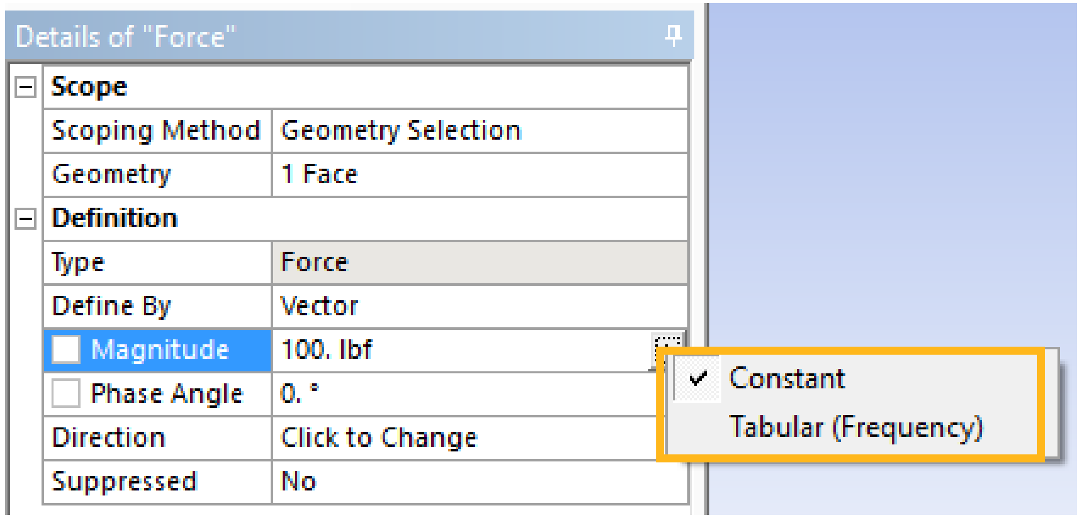


Ansys

... Loads and Boundary Conditions

Frequency-Dependent Load for Harmonic Analysis

- Applicable to Full Harmonic, MSUP Harmonic and MSUP Harmonic with Modal Restart.
- Force, Pressure, Moment, Remote Force, and Displacement are currently supported for frequency-dependent load definition for both Magnitude and Phase Angle.
- Acceleration is currently supported for frequency dependent Magnitude.



Ansys

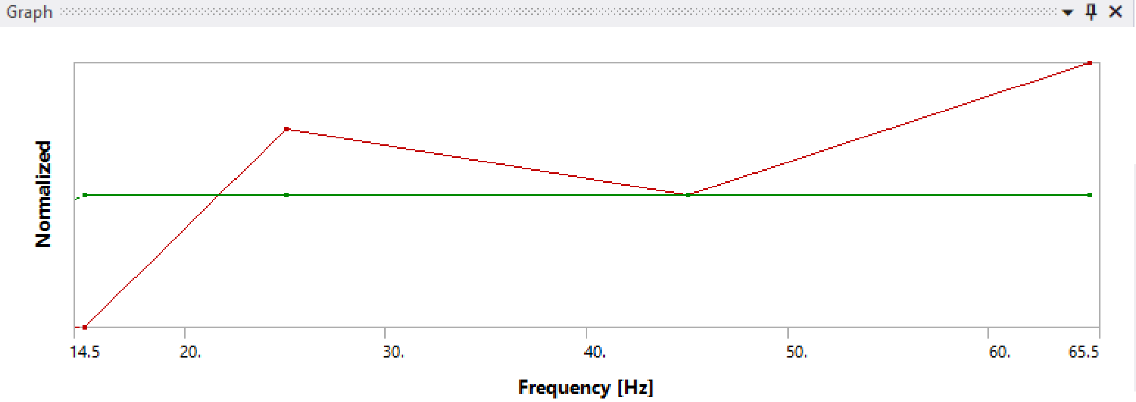
... Loads and Boundary Conditions

- Frequency-dependent Pressure:

Harmonic Response (A5)
- Pre-Stress/Modal (None)
- Analysis Settings
- Fixed Support
- Pressure
- Solution (A6)
  - Solution Information
  - Frequency Response

Details of "Pressure"

Scope	
Scoping Method	Geometry Selection
Geometry	1 Face
Definition	
Type	Pressure
Define By	Normal To
Magnitude	Tabular Data
Phase Angle	Tabular Data
Suppressed	No
Tabular Data	
Independent Variable	Frequency

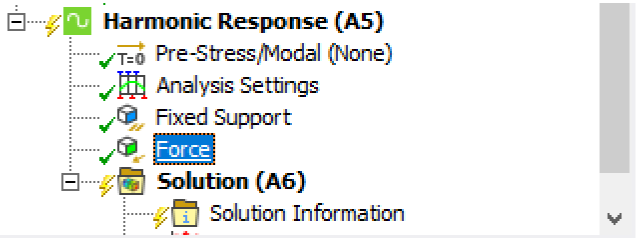


Tabular Data

	Frequency [Hz]	Magnitude [MPa]	Phase Angle [°]
1	15.	2.	0.
2	25.	8.	0.
3	45.	6.	0.
4	65.	10.	0.
*			

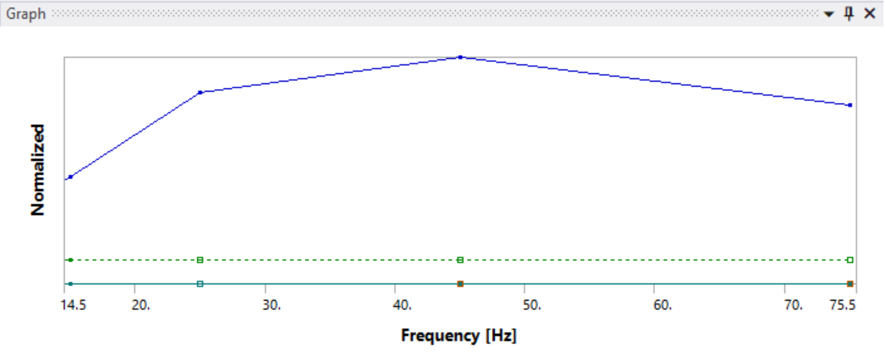
... Loads and Boundary Conditions

- Loads can have some components defined as constant magnitude, with other components defined as frequency-dependent:



Details of "Force"

Scope	
Scoping Method	Geometry Selection
Geometry	1 Face
Definition	
Type	Force
Define By	Components
Coordinate System	Global Coordinate System
X Component	0. N
Y Component	10. N
Z Component	Tabular Data
X Phase Angle	0. °
Y Phase Angle	0. °
Z Phase Angle	Tabular Data

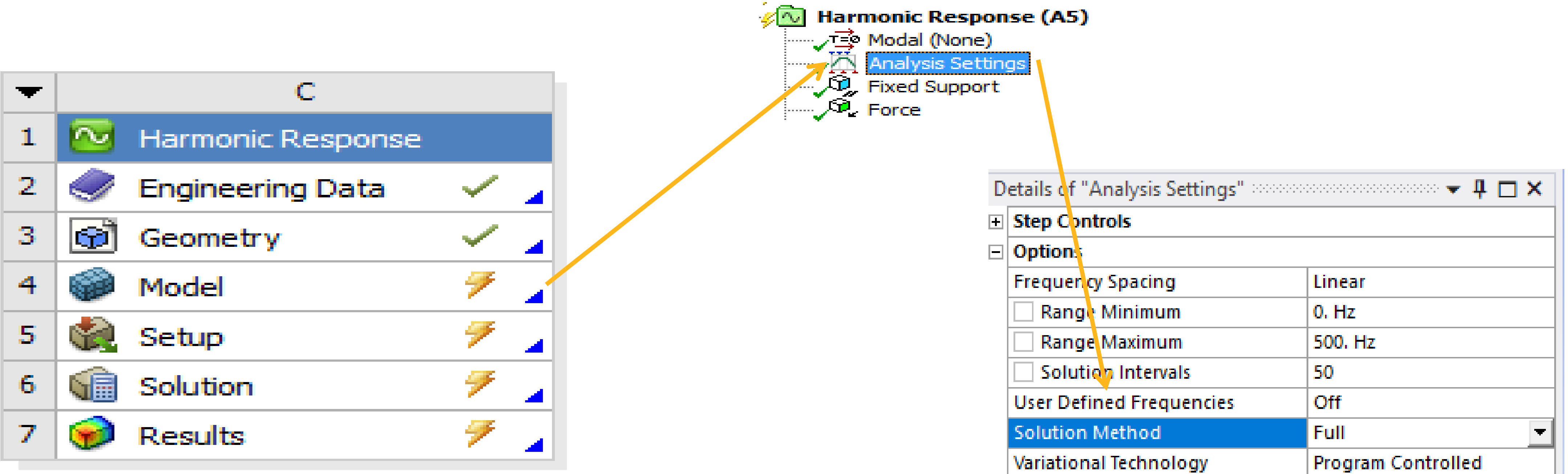


Tabular Data

	Frequency [Hz]	X Component [N]	Y Component [N]	Z Component [N]
1	15.	0.	10.	45.
2	25.	= 0.	= 10.	80.
3	45.	= 0.	= 10.	95.
4	75.	= 0.	= 10.	75.
*				

G. Analysis Settings—Full Harmonic Analysis

Analysis Settings > Solution Method > Full



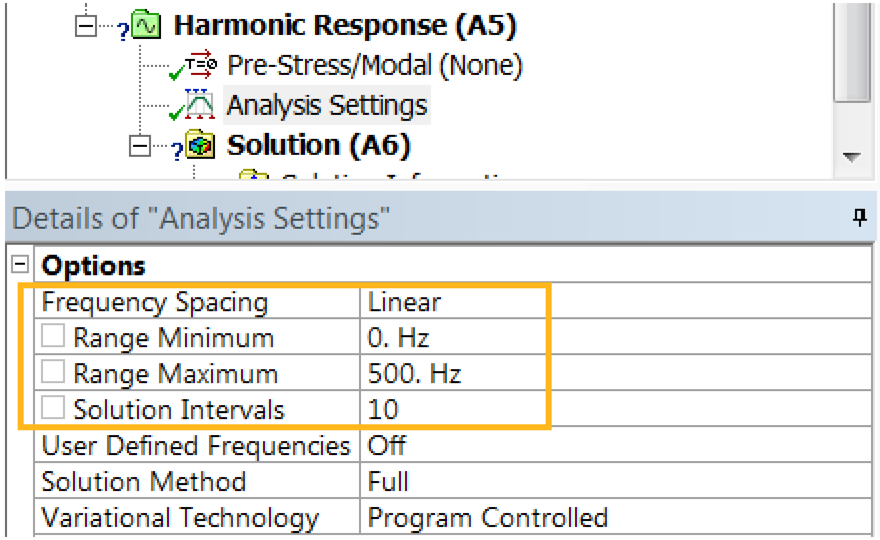
Full Harmonic Response

… Analysis Settings—Full Harmonic Analysis

- Analysis Settings > Options

- Frequency Spacing: Specified Linearly, Logarithmic, or by Octaves
  - Range Minimum >> Minimum Frequency (Hz)
  - Range Maximum >> Maximum Frequency (Hz)

- A linear spacing with range of 0-500 Hz and 10 solution intervals gives solutions at frequencies of 50, 100, 150, …, 450,and 500 Hz. Same range with 1 interval gives one solution at 500 Hz.

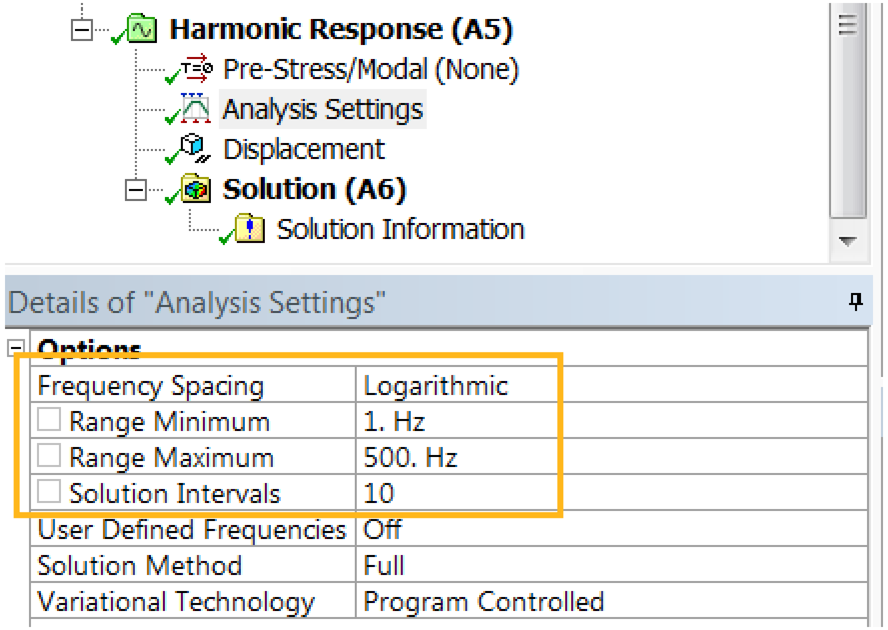


… Analysis Settings—Full Harmonic Analysis

- Analysis Settings > Options

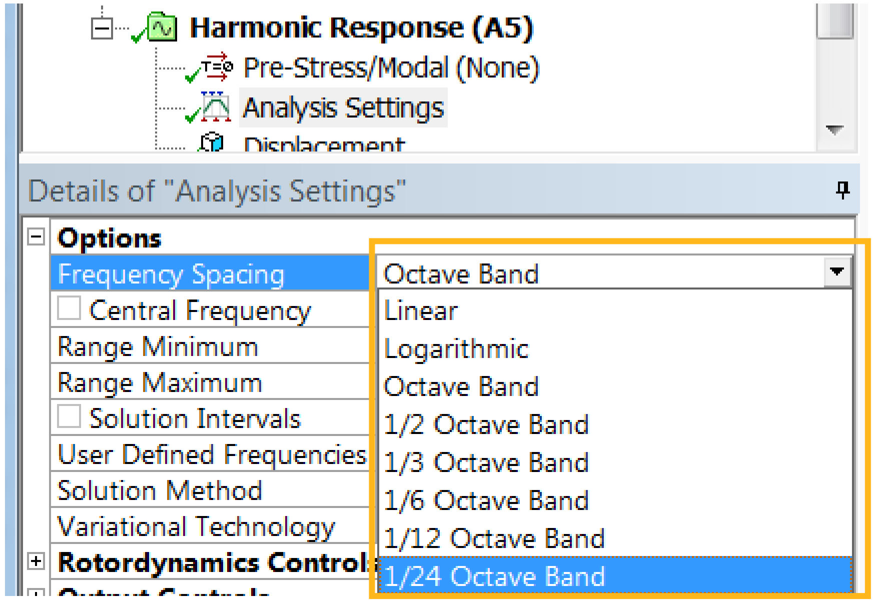
- Frequency Spacing: Specified Linearly, Logarithmic, or by Octaves
  - Range Minimum >> Minimum Frequency (Hz)
  - Range Maximum >> Maximum Frequency (Hz)

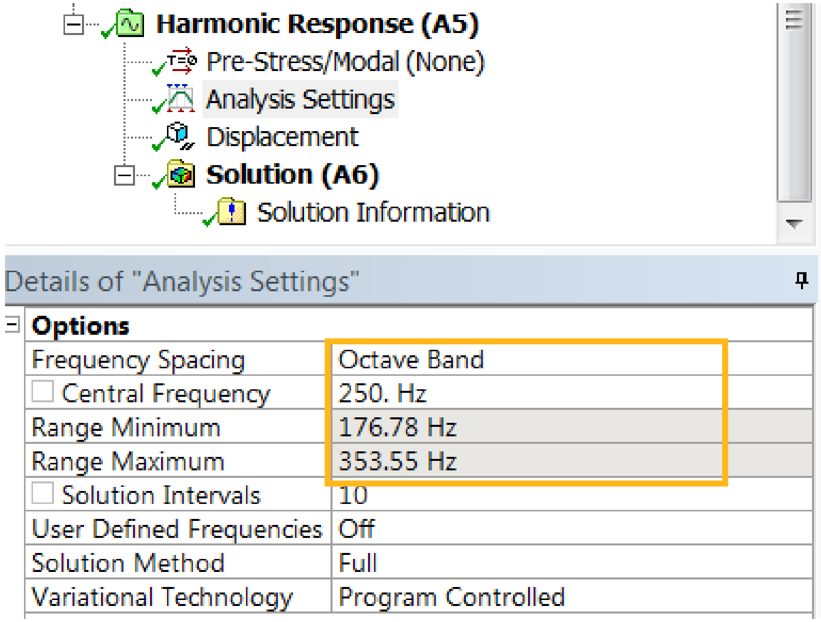
- A logarithmic spacing with range of 1-500 Hz and 10 solution intervals gives solutions at frequencies of 1, 1.9947, 3.979, 7.937, 15.83,31.58, 62.99, 125.66, 250.66, and 500 Hz.



… Analysis Settings—Full Harmonic Analysis

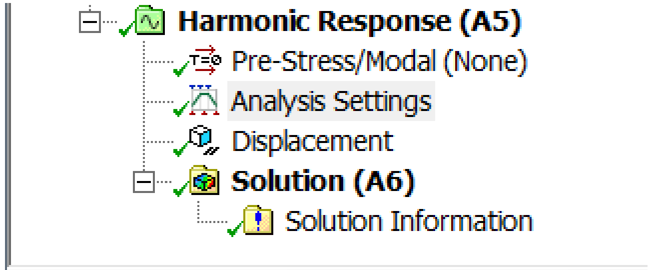
- Analysis Settings > Options
  - Frequency Spacing: Specified Linearly, Logarithmic, or by Octaves
    - Central Frequency (Hz)
    - Range Minimum >> calculated from central based upon type of Octave band
    - Range Maximum >> calculated from central based upon type of Octave band





… Analysis Settings—Full Harmonic Analysis

- Analysis Settings > Options
  - Solution Intervals: results in evenly spaced intervals between Range Min and Range Max, unless….
  - … User Defined Frequencies is enabled. Solutions are then conducted at both evenly spaced intervals and those entered in Tabular Data; useful for capturing peak amplitudes near resonance.



Details of "Analysis Settings"

Options	
Frequency Spacing	Linear
Range Minimum	0. Hz
Range Maximum	500. Hz
Solution Intervals	10
User Defined Frequencies	On
Solution Method	Full
Variational Technology	Program Controlled

Tabular Data

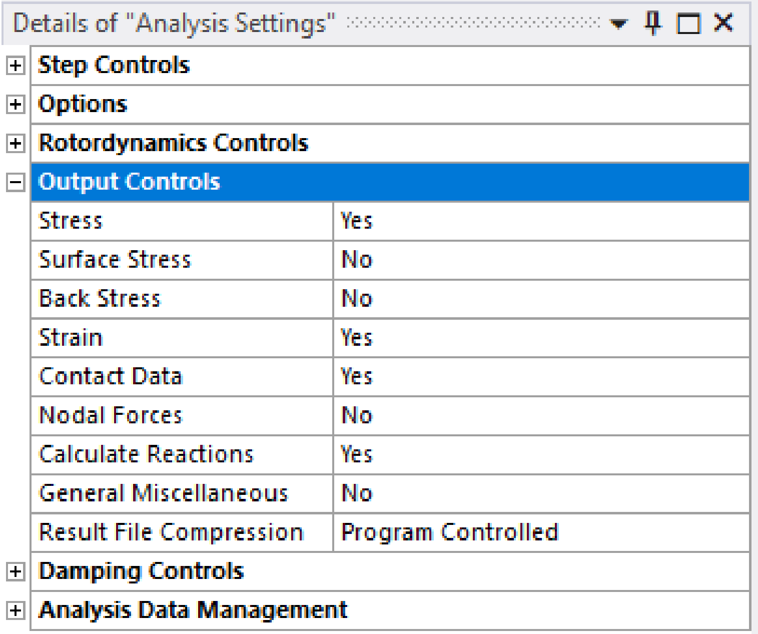
	User Defined Frequency Steps [Hz]
1	65.
2	130.

Tabular Data

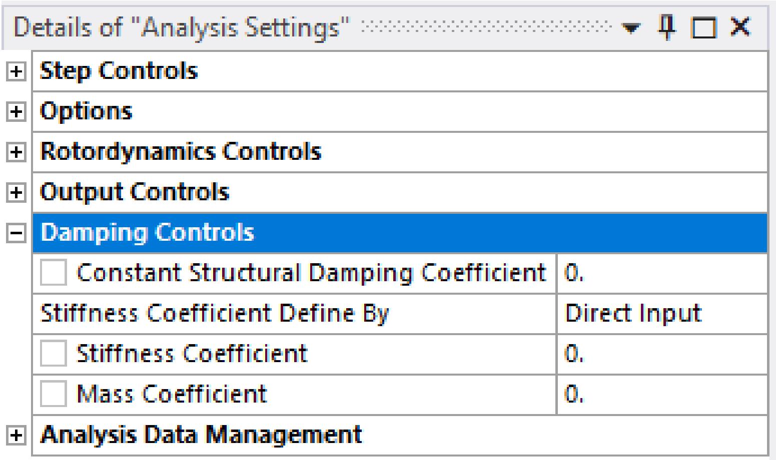
	Set	Frequency [Hz]
1	1.	50.
2	2.	65.
3	3.	100.
4	4.	130.
5	5.	150.
6	6.	200.
7	7.	250.
8	8.	300.
9	9.	350.
10	10.	400.
11	11.	450.
12	12.	500.

… Analysis Settings—Full Harmonic Analysis

- Analysis Settings > Output Controls
  - Adjust as desired



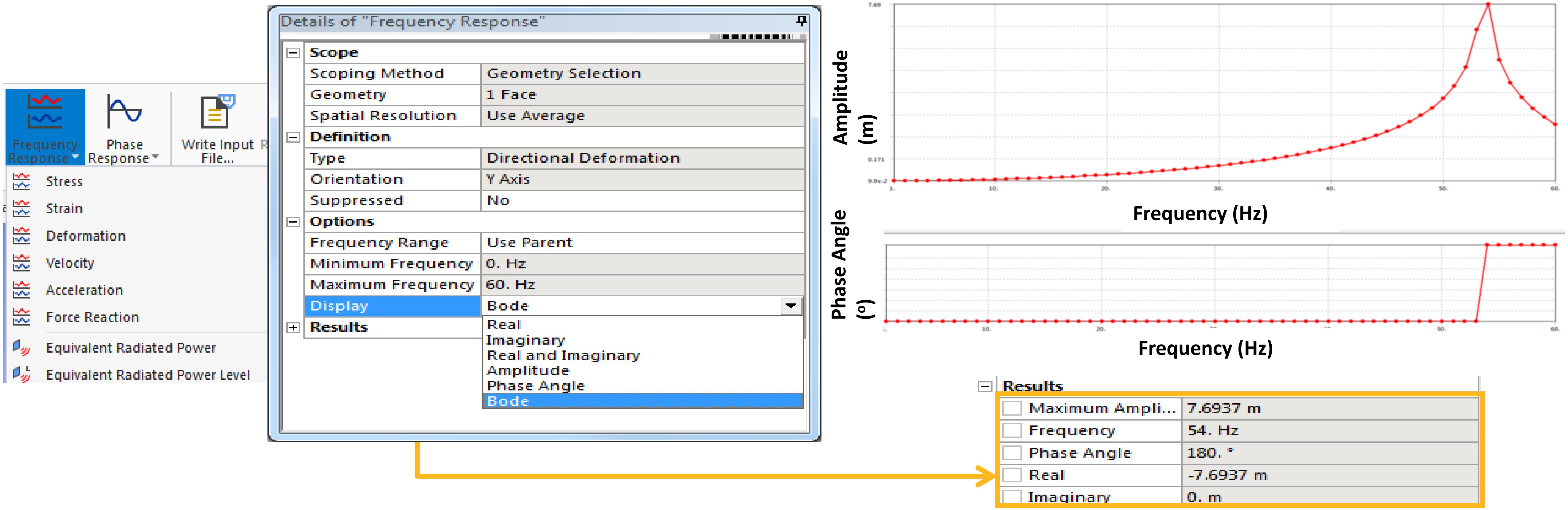
- Analysis Settings > Damping Controls
  - Damping is recommended to avoid excessive deformations at resonant frequencies



Ansys

H. Results—Full Harmonic Analysis

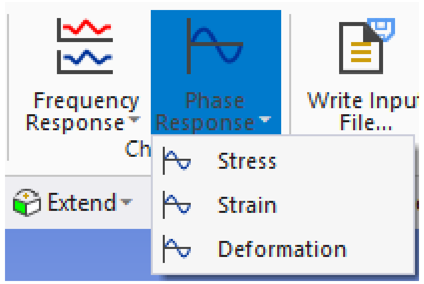
- Postprocessing often begins with a deformation Frequency Response:
  - displays how the response varies with frequency

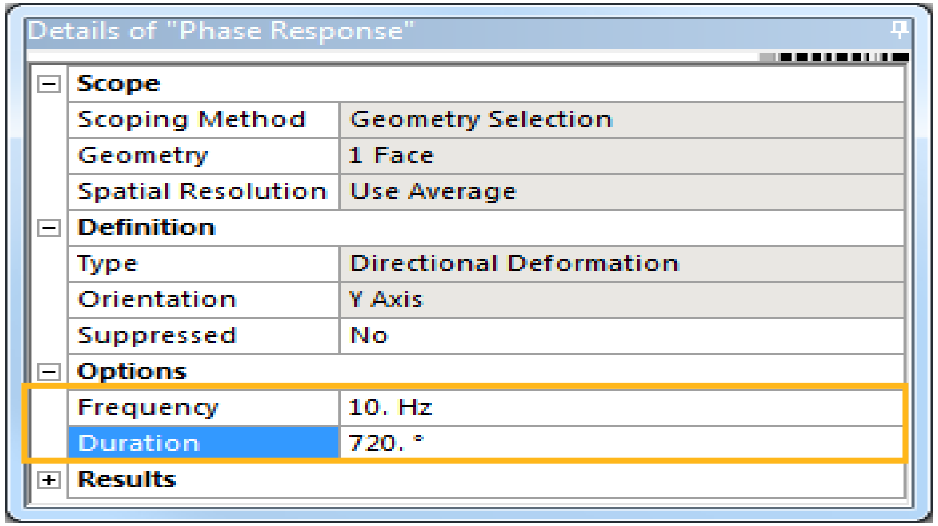


Ansys

… Results—Full Harmonic Analysis

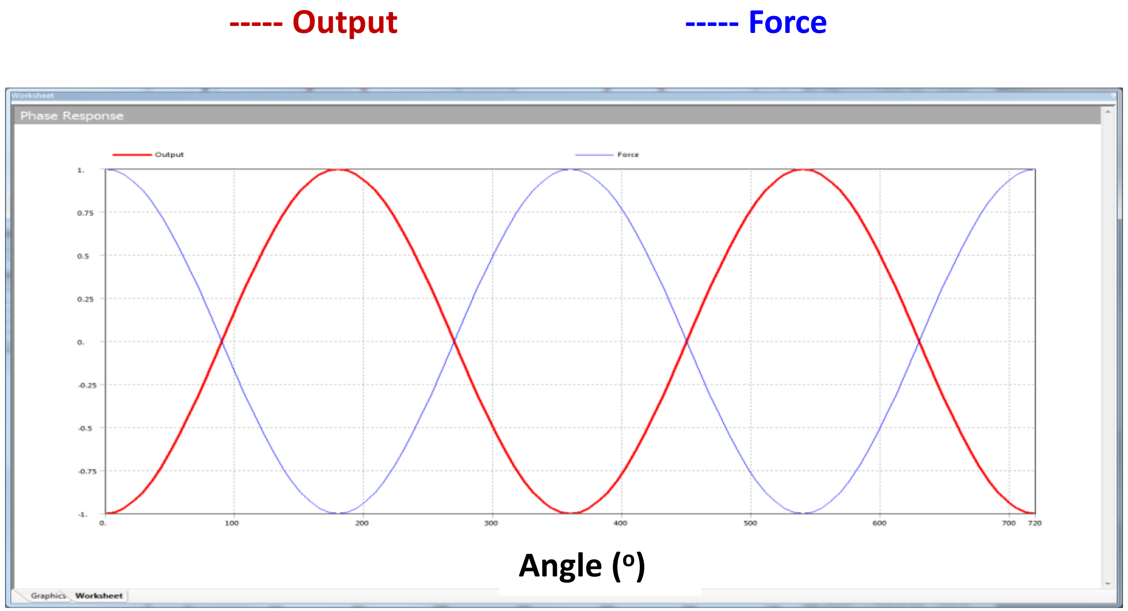
- Phase Response:
  - displays how much a response lags the applied loads.





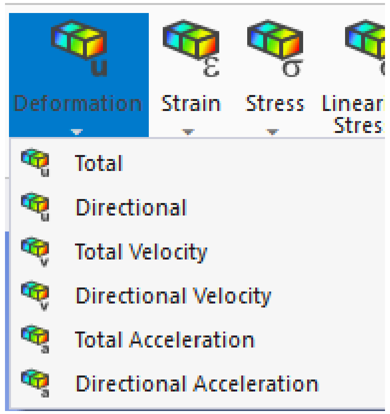
Details of "Phase Response"

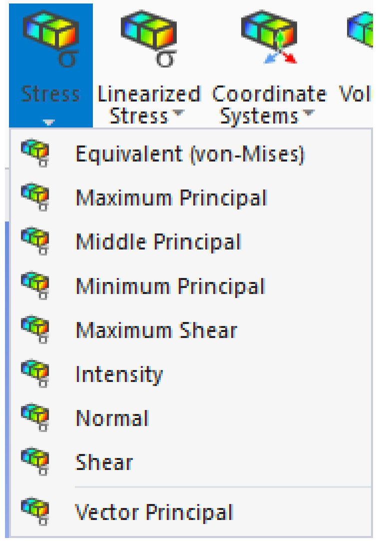
Scope	
Scoping Method	Geometry Selection
Geometry	1 Face
Spatial Resolution	Use Average
Definition	
Type	Directional Deformation
Orientation	Y Axis
Suppressed	No
Options	
Frequency	10. Hz
Duration	720. °
Results	

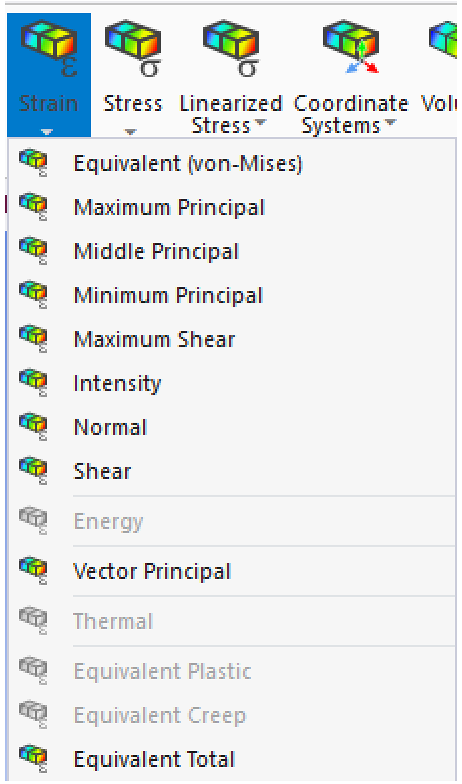


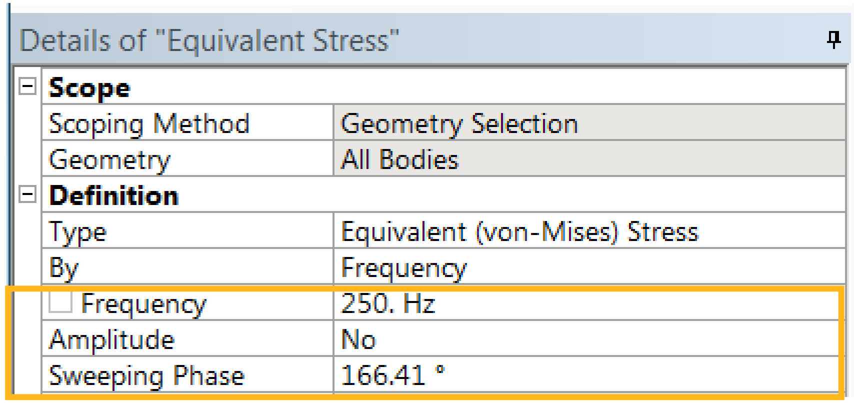
… Results—Full Harmonic Analysis

- Contour plots include:
  - Stress,
  - Elastic strain, and
  - Deformation, velocity, acceleration
- For these results, you must specify a frequency and sweeping phase:








Details of "Equivalent Stress"

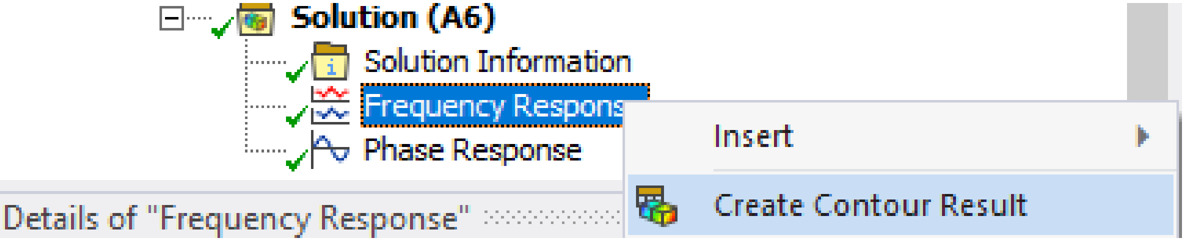
Scope	
Scoping Method	Geometry Selection
Geometry	All Bodies
Definition	
Type	Equivalent (von-Mises) Stress
By	Frequency
Frequency	250. Hz
Amplitude	No
Sweeping Phase	166.41 °

Frequency and Sweeping Phase are usually obtained by choosing the frequency and phase angle associated with the peak deformation frequency response amplitude…see next slide.

Ansys

… Results—Full Harmonic Analysis

- A deformation contour result can be created from a Frequency Response.
- The Phase Angle of the contour result has the same magnitude as the frequency result type but an opposite sign.
- The sign of the phase angle is reversed so that the response amplitude of the frequency response plot for that frequency and phase angle matches with the contour results.



Details of "Frequency Response"

Scope	
Definition	
Options	
Results	
Maximum Amplitude	1.6267 in
Frequency	56.5 Hz
Phase Angle	43.82 °
Real	1.1737 in
Imaginary	1.1263 in

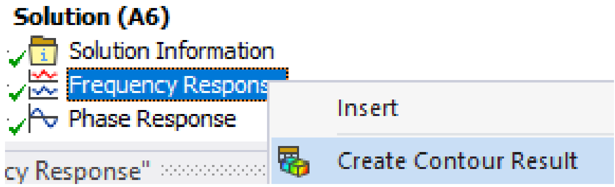
Details of "Directional Deformation"

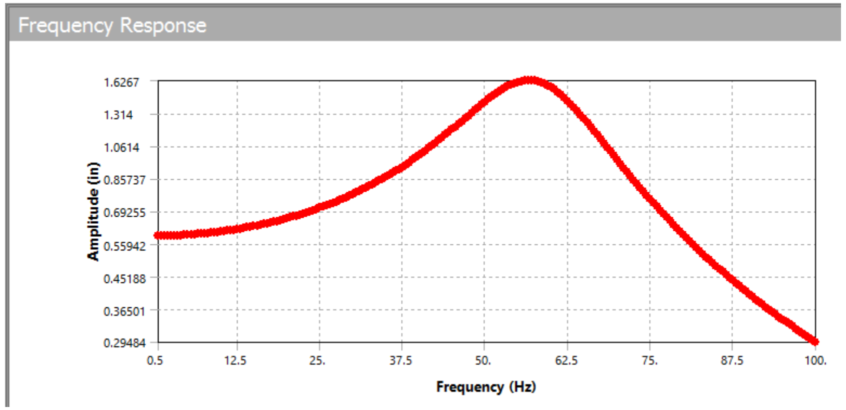
Scope	
Scoping Method	Geometry Selection
Geometry	All Bodies
Definition	
Type	Directional Deformation
Orientation	Y Axis
By	Frequency
Frequency	56.5 Hz
Amplitude	No
Sweeping Phase	-43.82 °
Coordinate System	Global Coordinate System
Identifier	
Suppressed	No
Results	
Minimum	1.6267 in
Maximum	1.6267 in
Minimum Occurs On	Solid
Maximum Occurs On	Solid
Information	

… Results—Full Harmonic Analysis

Contour Results can be viewed several different ways:

1. By Frequency





Details of "Directional Deformation"

Scope	
Scoping Method	Geometry Selection
Geometry	All Bodies
Definition	
Type	Directional Deformation
Orientation	Y Axis
By	Frequency
Frequency	56.5 Hz
Amplitude	No
Sweeping Phase	-43.82 °
Coordinate System	Global Coordinate System
Identifier	
Suppressed	No
Results	
Minimum	1.6267 in
Maximum	1.6267 in
Minimum Occurs On	Solid
Maximum Occurs On	Solid
Information	

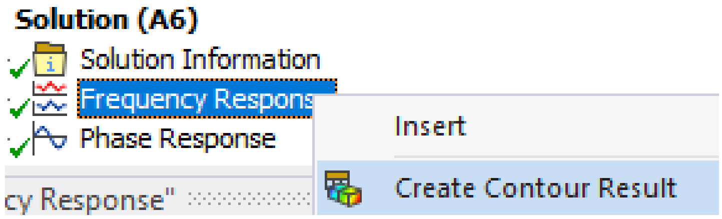
Tabular Data

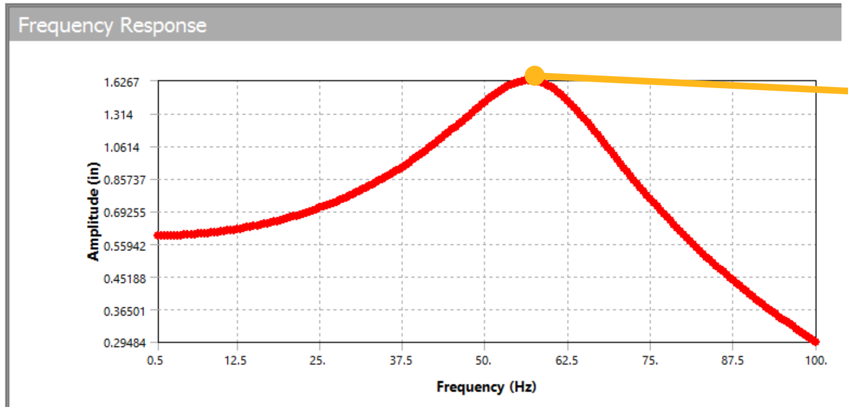
	Frequency [Hz]	Amplitude [in]	Phase Angle [°]
106	53.	1.5479	61.11
107	53.5	1.5663	58.825
108	54.	1.5827	56.469
109	54.5	1.597	54.045
110	55.	1.6088	51.561
111	55.5	1.6178	49.023
112	56.	1.6238	46.44
113	56.5	1.6267	43.82
114	57.	1.6263	41.173
115	57.5	1.6225	38.511
116	58.	1.6154	35.844

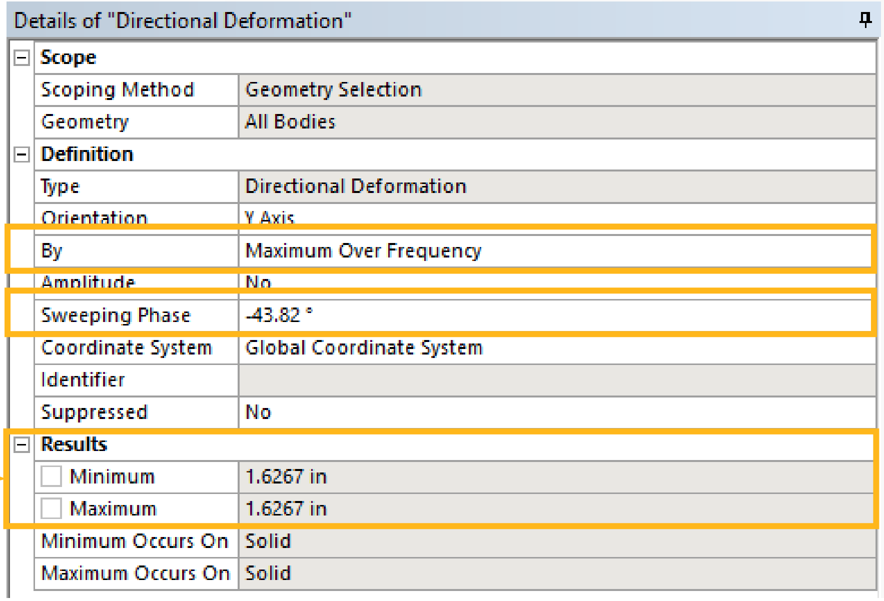
Note: The sign of the phase angle in the contour result is reversed so that the response amplitude of the frequency response plot for that frequency and phase angle matches with the contour results.

... Results—Full Harmonic Analysis

2. By Maximum Over Frequency





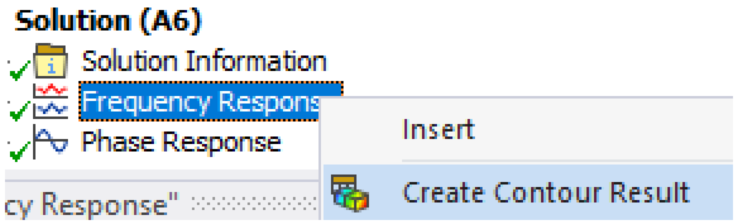


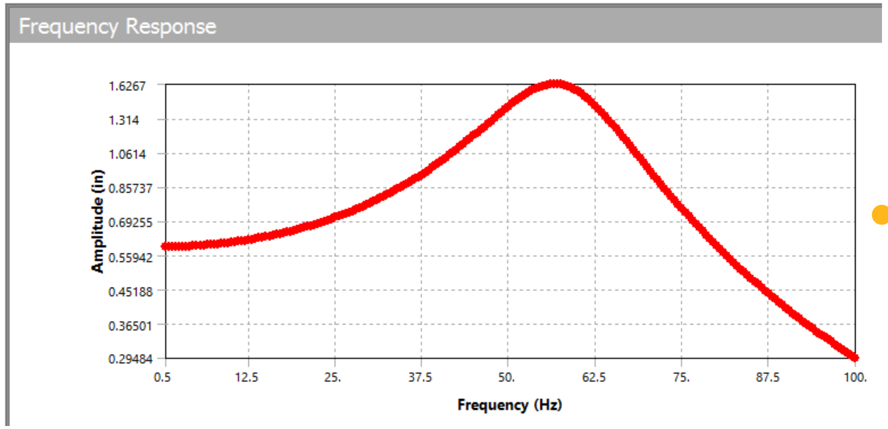
Details of "Directional Deformation"

Scope	
Scoping Method	Geometry Selection
Geometry	All Bodies
Definition	
Type	Directional Deformation
Orientation	Y Axis
By	Maximum Over Frequency
Amplitude	No
Sweeping Phase	-43.82 °
Coordinate System	Global Coordinate System
Identifier	
Suppressed	No
Results	
Minimum	1.6267 in
Maximum	1.6267 in
Minimum Occurs On	Solid
Maximum Occurs On	Solid

… Results—Full Harmonic Analysis

3. By Frequency of Maximum



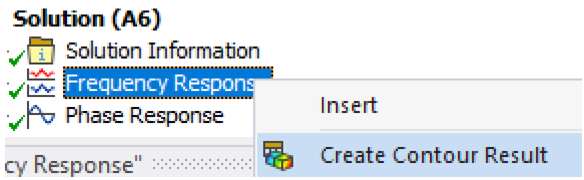


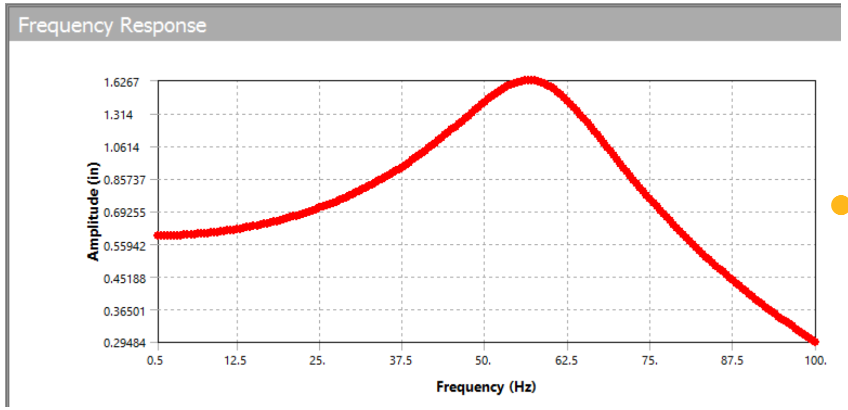
Details of "Directional Deformation"

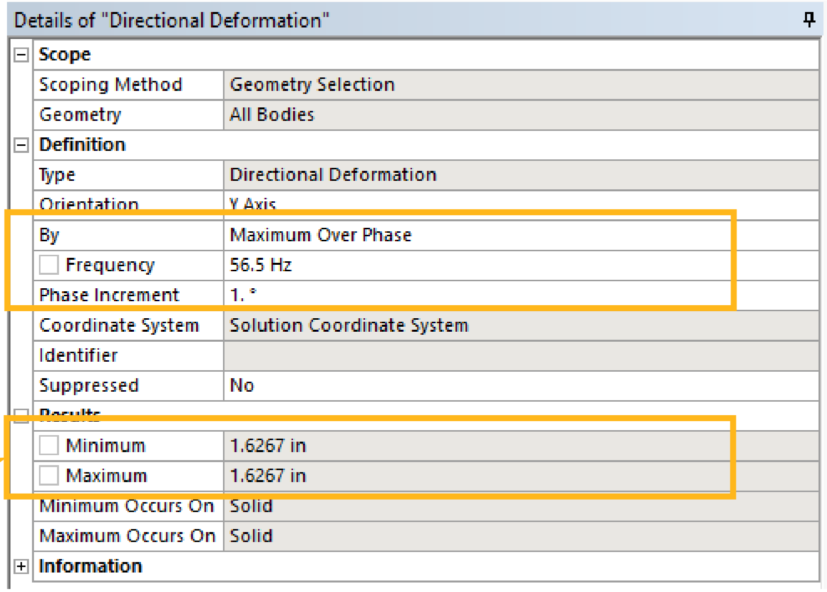
Scope	
Scoping Method	Geometry Selection
Geometry	All Bodies
Definition	
Type	Directional Deformation
Orientation	Y Axis
By	Frequency Of Maximum
Amplitude	No
Sweeping Phase	-43.82 °
Coordinate System	Global Coordinate System
Identifier	
Suppressed	No
Results	
Minimum	56.5 Hz
Maximum	56.5 Hz
Minimum Occurs On	Solid
Maximum Occurs On	Solid

… Results—Full Harmonic Analysis

4. By Maximum over Phase

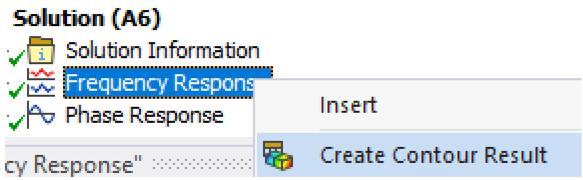






… Results—Full Harmonic Analysis

5. By Phase of Maximum



Tabular Data

	Frequency [Hz]	Amplitude [in]	Phase Angle [°]
106	53.	1.5479	61.11
107	53.5	1.5663	58.825
108	54.	1.5827	56.469
109	54.5	1.597	54.045
110	55.	1.6088	51.561
111	55.5	1.6178	49.023
112	56.	1.6238	46.44
113	56.5	1.6267	43.82
114	57.	1.6263	41.173
115	57.5	1.6225	38.511
116	58	1.6154	35.844

Details of "Directional Deformation"

Scope	
Scoping Method	Geometry Selection
Geometry	All Bodies
Definition	
Type	Directional Deformation
Orientation	Y Axis
By	Phase Of Maximum
Frequency	56.5 Hz
Phase Increment	1. °
Coordinate System	Solution Coordinate System
Identifier	
Suppressed	No
Results	
Minimum	316. °
Maximum	316. °
Minimum Occurs On	Solid
Maximum Occurs On	Solid
Information	

Note: The sign of the phase angle is reversed. (316° ≡ −44 °)

... Results—Full Harmonic Analysis

- Reaction Force and/or Moment expansions of the selected frequency point(s) are reported at the closest frequencies specified under “Solution Intervals”.

Details of "Analysis Settings"

Options	
Range Minimum	0. Hz
Range Maximum	100. Hz
Solution Intervals	10
Solution Method	Mode Superposition
Cluster Results	No
Modal Frequency Range	Program Controlled
Store Results At All Frequencies	Yes
Output Controls	Yes / No
Damping Controls	
Analysis Data Management	

Solution calculated at frequencies of 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.

Details of "Force Reaction"

Definition	
Type	Force Reaction
Location Method	Boundary Condition
Boundary Condition	Fixed Support
Orientation	Global Coordinate System
By	Frequency
Frequency	18. Hz
Phase Angle	0. °
Suppressed	No
Options	
Result Selection	All
Results	
Information	

Force Reaction requested at 18 Hz, but reported at available closet frequency point of 20 Hz.

Tabular Data

	Frequency [Hz]	Fx (Real) [N]	Fy (Real) [N]	Fz (Real) [N]	Ftotal (Real) [N]	Fx (Imaginary) [N]	Fy (Imaginary) [N]	Fz (Imaginary) [N]	Ftotal (Imaginary) [N]
1	20.	8.8296e-010	38.933	-5.149e-008	38.933	0.	0.	0.	0.

Ansys

… Results—Full Harmonic Analysis

Reaction Force and/or Moment

- User-defined phase increment allowed for a more accurate probing of force and moment reactions.
  - Option provided under Details of Force Reaction and Moment Reaction as Phase Increment when Defined By = Maximum Over Phase or Defined By = Phase Of Maximum.
  - Default set to 10 degrees, allowable between 0.01 and 10 degrees.
  - High resolution providing accurate results but longer evaluation time, and vice versa.

Details of "Force Reaction"

Definition	
Type	Force Reaction
Location Method	Boundary Condition
Boundary Condition	Fixed Support
Orientation	Global Coordinate System
By	Phase Of Maximum
Frequency	18. Hz
Phase Increment	10. °
Suppressed	No
Options	
Result Selection	All
Results	
Information	

Details of "Moment Reaction"

Definition	
Type	Moment Reaction
Location Method	Boundary Condition
Boundary Condition	Fixed Support
Orientation	Global Coordinate System
Summation	Centroid
By	Maximum Over Phase
Frequency	18. Hz
Phase Increment	10. °
Suppressed	No
Options	
Result Selection	All
Results	
Information	

I. Mode-Superposition Harmonic Analysis

- Approximate solution; accuracy depends on whether an adequate number of modes have been extracted, or whether clustering option was chosen.
- Generally faster than FULL method.
- Does not support non-zero imposed harmonic displacements, unless applied as Base Excitation through a Fixed Support(s) or Body-Ground Spring.
- Solution points may be equally distributed across the frequency domain, clustered about the natural frequencies of the structure or user-defined.
- Solves an uncoupled system of equations by performing a linear combination of orthogonal vectors (mode shapes).

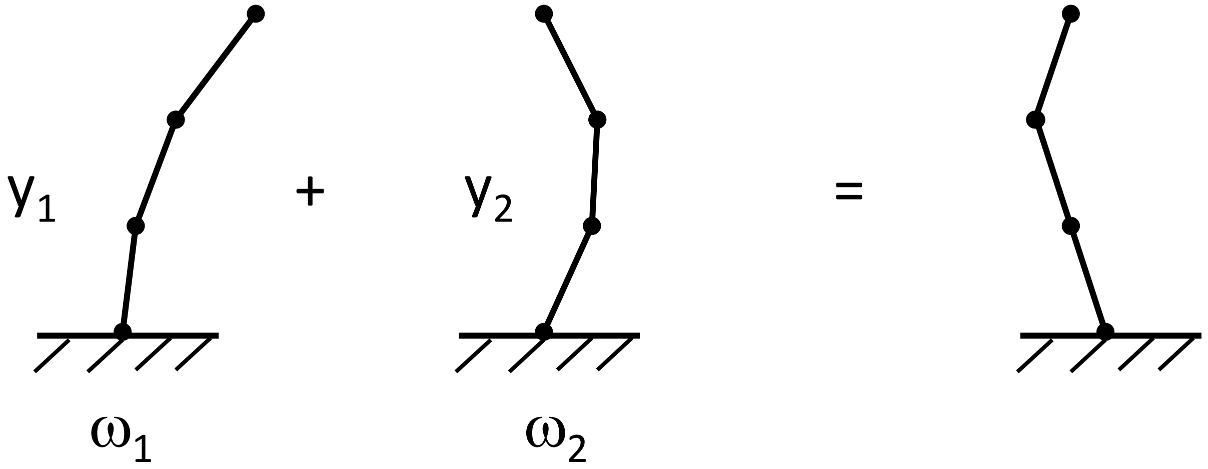
$$(-\Omega^2[M] + i\Omega[C] + [K])(\{u1\} + i\{u_2\}) = (\{F1\} + i\{F_2\})$$

$$\vdots$$

$$\left(-\Omega^2 + i2\omega_j\Omega\xi_j + \omega_j^2\right)y_{jc} = f_{jc}$$

… Mode-Superposition Harmonic Analysis

- <u>Example:</u>

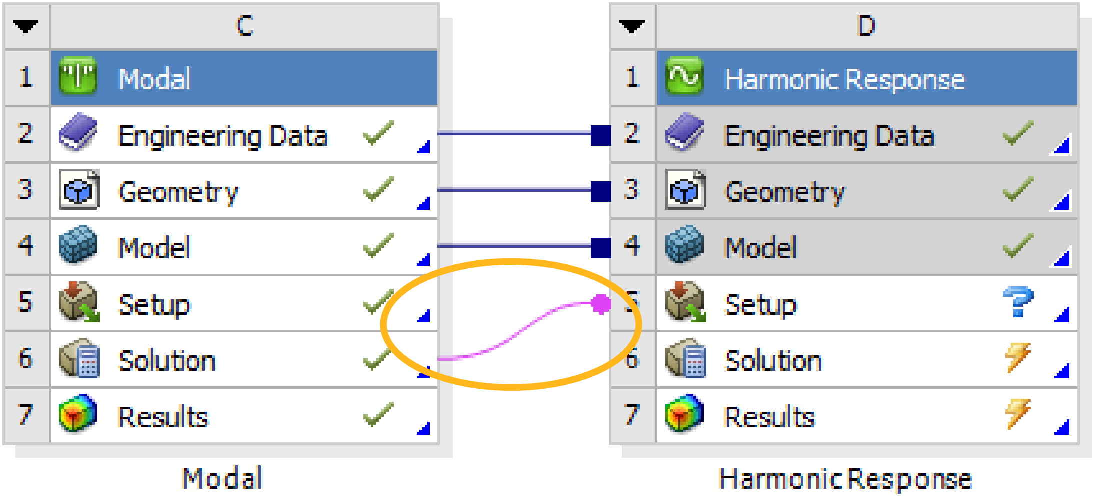


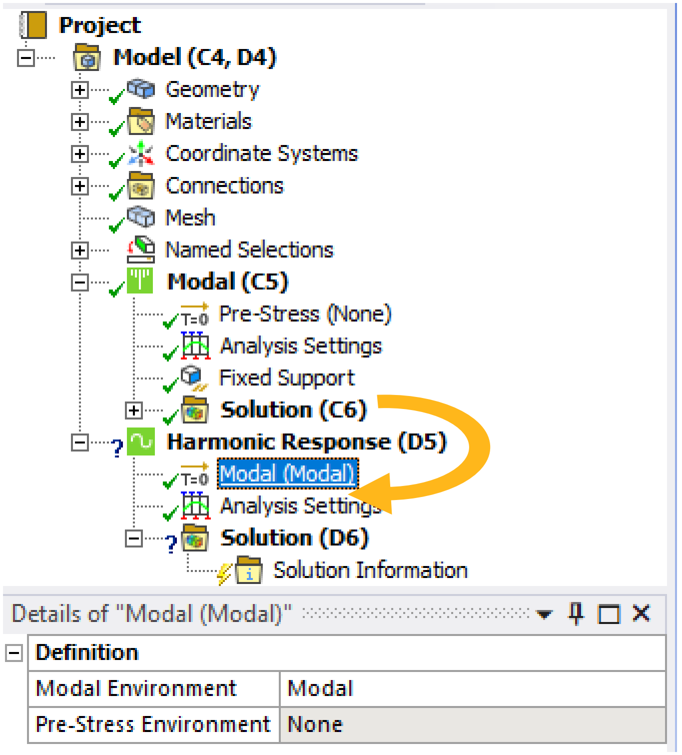
  - Here, the sum of mode shape 1 and mode shape 2 approximates the final response. Since mode shapes are relative, the coefficients y_1 and y_2 are required.
  - Mode shapes (eigenvectors) are also known as generalized coordinates, and in this case, coefficients y_1 and y_2 are the DOF.

… Mode-Superposition Harmonic Analysis

- Two ways to set up an MSUP harmonic response analysis in the schematic by:

1. Linking a Modal system to a Harmonic Response system at the solution level.

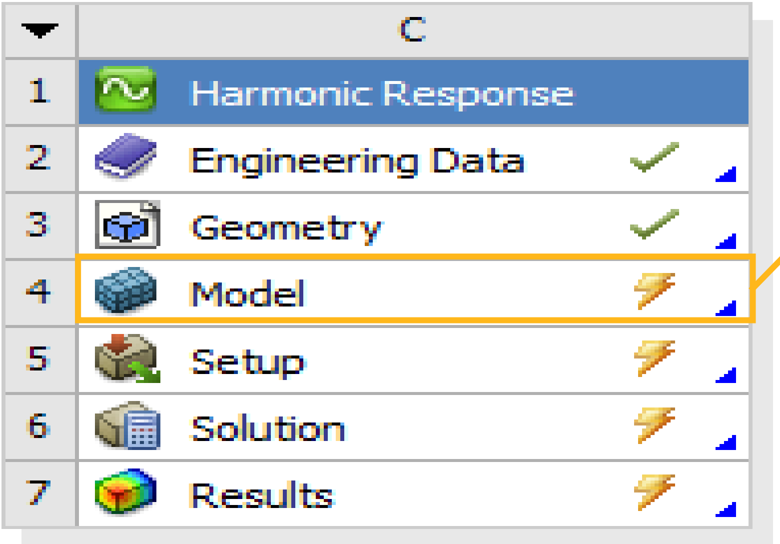




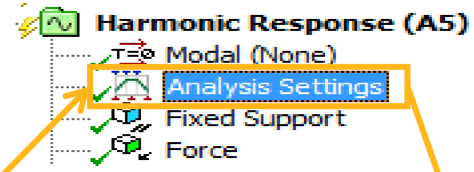
- Notice that, in the Harmonic Response branch, the modal analysis result becomes an initial condition. This is known also as a Modal Restart.

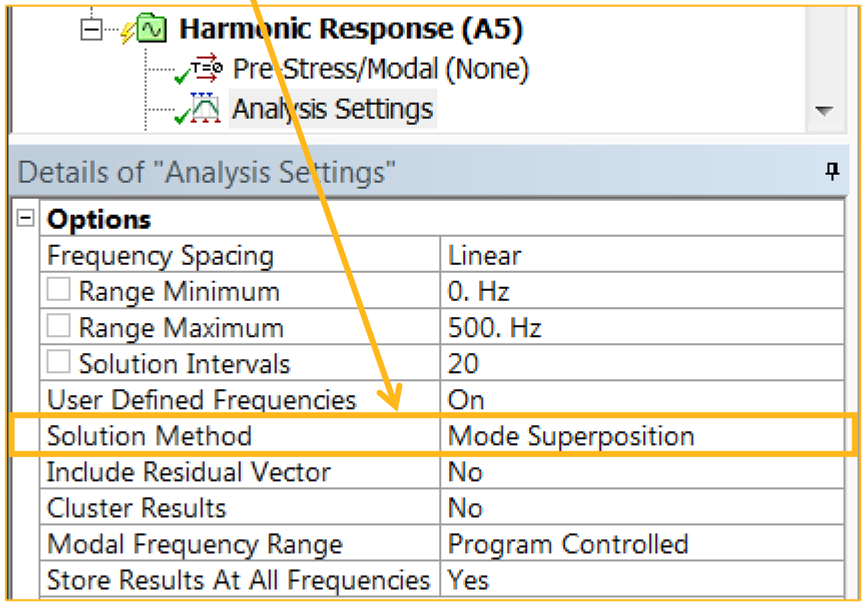
… Mode-Superposition Harmonic Analysis

2. Using Analysis Setting > Solution Method > Mode Superposition (Standalone Analysis)



MSUP Harmonic Response





J. Damping in MSUP Harmonic Analysis

- The damping matrix [C] is not calculated explicitly, but instead damping is defined directly in terms of a damping ratio ξ^d for mode i :

$$\xi_i^d = \xi + \xi_i^m + \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2}$$

ξ: constant modal damping ratio (DMPRAT)

ξ_i^m: modal damping ratio for mode shape i (MP,DMPR during modal analysis)

α: Global Mass matrix multiplier (alpha damping, ALPHAD)

β : Global k-Matrix Multiplier (beta damping, BETAD)

…Damping in MSUP Harmonic Analysis

- Values for ξ, α, and β can be entered on a Global basis via the Damping Controls section of Analysis Settings (top right) :
- The value for ξ^m is entered on a material basis within Engineering Data as part of the undamped Modal Analysis (for Harmonic Solutions using a linked Modal Analysis) or within Engineering Data during the standalone MSUP Harmonic analysis (below right):
  - In either case, set the "Eqv. Damping Ratio from Modal" = Yes in order to see the effects of damping in the Harmonic

Damping Controls	
Eqv. Damping Ratio From Modal	Yes
Damping Ratio	0.
Stiffness Coefficient Define By	Direct Input
Stiffness Coefficient	0.
Mass Coefficient	0.
Analysis Data Management	

Details of "Analysis Settings"

Step Controls		
Multiple Steps	No	
Options		
Frequency Spacing	Linear	
Range Minimum	0. Hz	
Range Maximum	50. Hz	
Cluster Number	20	
User Defined Frequencies	Off	
Solution Method	Mode Superposition	
Include Residual Vector	No	
Cluster Results	Yes	
On Demand Expansion	No	
Store Results At All Frequencies	Yes	
Rotordynamics Controls		
Output Controls		
Damping Controls		
Eqv. Damping Ratio From Modal	No	
Damping Define By	Damping Ratio	
Damping Ratio	2.e-002	ξ
Stiffness Coefficient Define By	Direct Input	
Stiffness Coefficient	2.65e-005	β
Mass Coefficient	8.72	α
Analysis Data Management		

Properties of Outline Row 3: Structural Steel

	A	B	
1	Property	Value	
2	Material Field Variables	Table	
3	Density	7850	
4	Material Dependent Damping		
5	Damping Ratio	0.01	ξ^m
6	Constant Structural Damping Coefficient	= 0.02	
7	Isotropic Elasticity		

…Damping in MSUP Harmonic Analysis

- In MSUP harmonic analysis, we now also have a choice of defining damping by constant structural damping coefficient (g).
- In this case, the harmonic equation of motion in modal coordinates is:

$$\left(-\Omega^2+i\left(2\omega_i\Omega\xi_i^d+\omega_i^2 g\right)+\omega_i^2\right)y_{ic}=f_{ic}$$

- where:

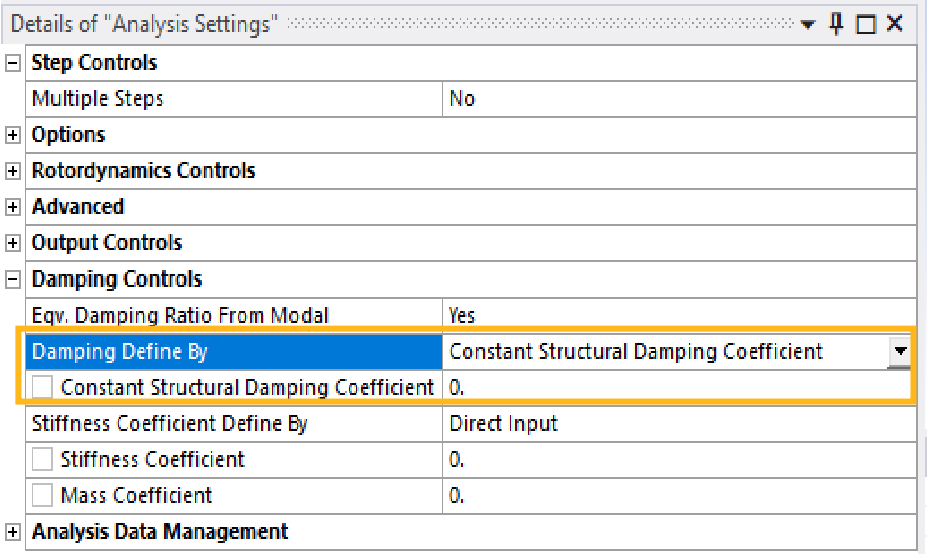
  y_{ic} = complex modal coordinate

  ω_i= natural circular frequency of mode i

  ξ_i^d= fraction of critical damping for mode i

  g= constant structural damping coefficient

  f_{ic}= complex force in modal coordinates



g

… Damping in MSUP Harmonic Analysis

- From Module 02, MSUP Harmonic also supports a special case of Element Damping provided that the upstream Modal analysis is conducted using the Reduced Damped solver.
  - In this case, the full damping matrix must be retained in the MSUP Harmonic analysis.

$$[C_m]=[\Phi^T][C][\Phi]+\frac{g}{2\pi\Omega}[\Phi^T][K][\Phi]+\sum_{j=1}^{N_m}\frac{m_j}{2\pi\Omega}[\Phi^T][K_j][\Phi]+[\Xi]$$

$[C_m]$ is the damping matrix in the modal basis

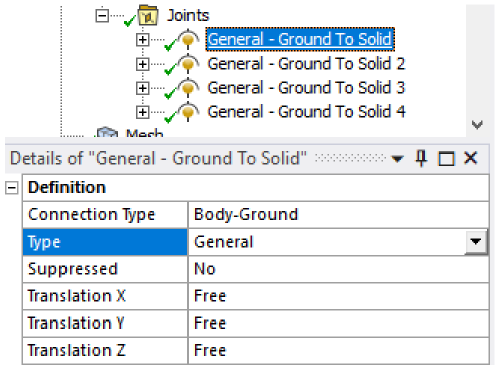
[C] is identical to that used within the Full Harmonic method (slide 13) and which contains the term

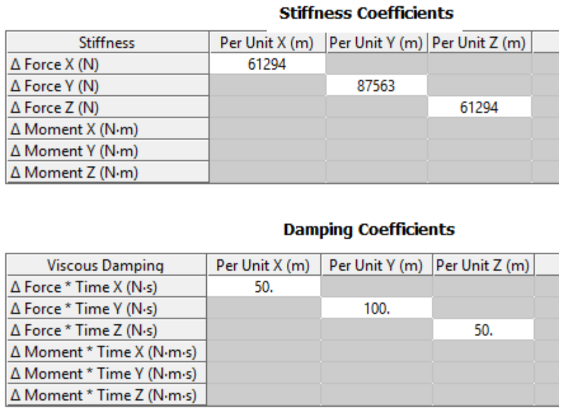
$$\sum_{k=1}^{N_e}[C_k]$$

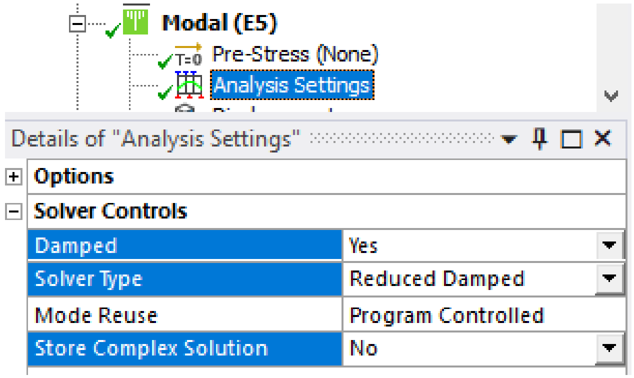
C_k: Element damping (via the various Connection elements, COMBIN14, MPC184, etc.)

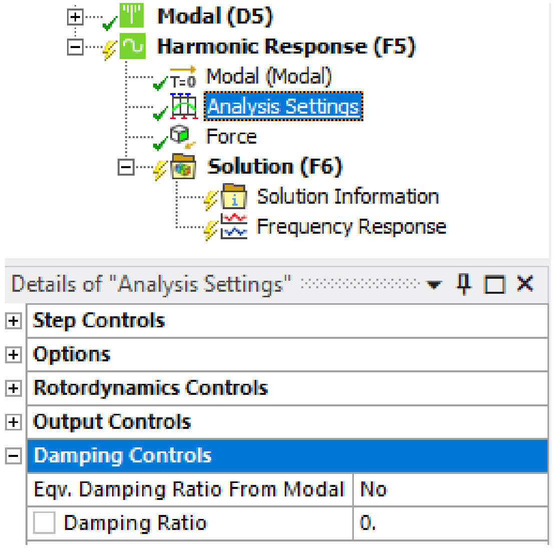
… Damping in MSUP Harmonic Analysis

- Below are the requirements to include Element Damping in an MSUP Harmonic analysis:
  - Modal analysis with Connections that include damping (Body/Ground Springs, General Joints, etc.)
  - Reduced Damped solver in Modal
    - Store Complex Solution = No
  - Damping need not be defined in the MSUP Harmonic, although it may be if desired.
    - “Eqv. Damping From Modal” is not needed in this scenario.





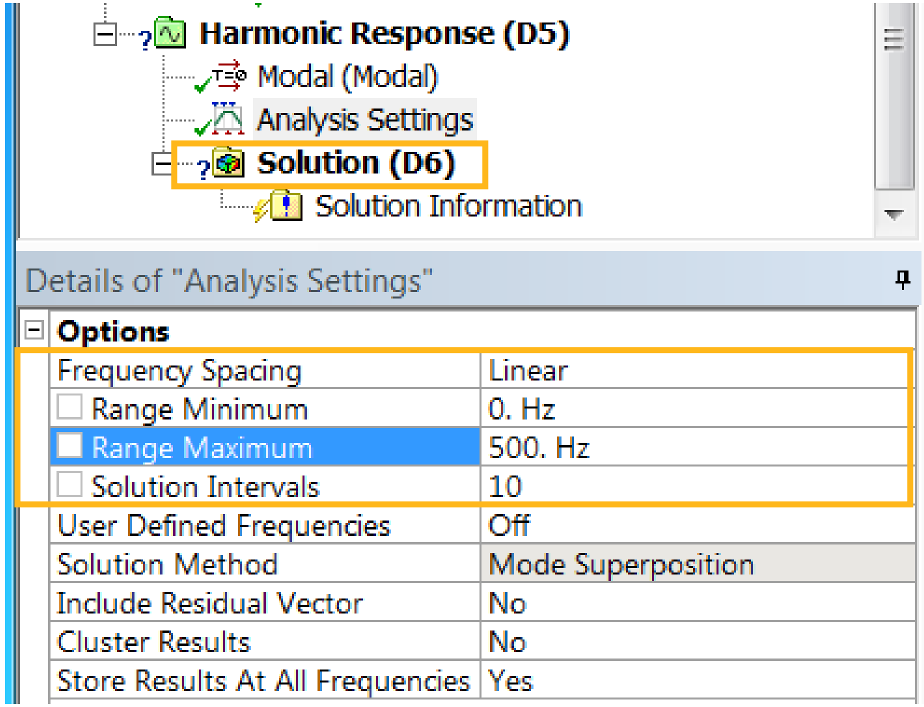




Ansys

K. Analysis Settings—MSUP Harmonic

- Analysis Settings > Options
- Frequency Range
  - Range Minimum >> Minimum Frequency
  - Range Maximum >> Maximum Frequency
- Solution Intervals
- Solution Method > Mode Superposition



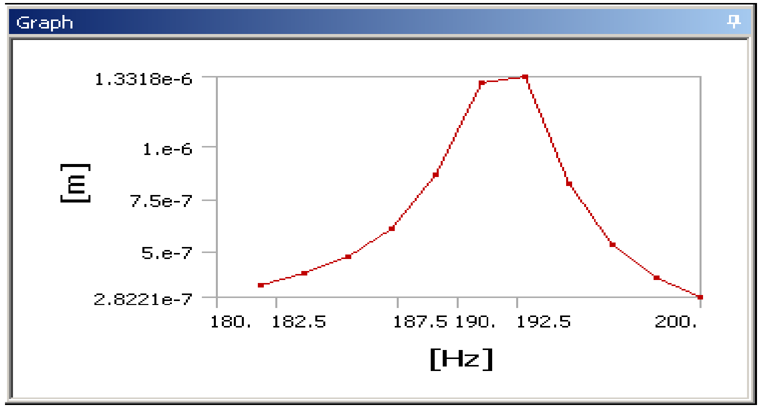
… Analysis Settings—MSUP Harmonic

- Analysis Settings > Options
  - Cluster Results > Yes
  - Cluster Number > 4 (default is usually sufficient)

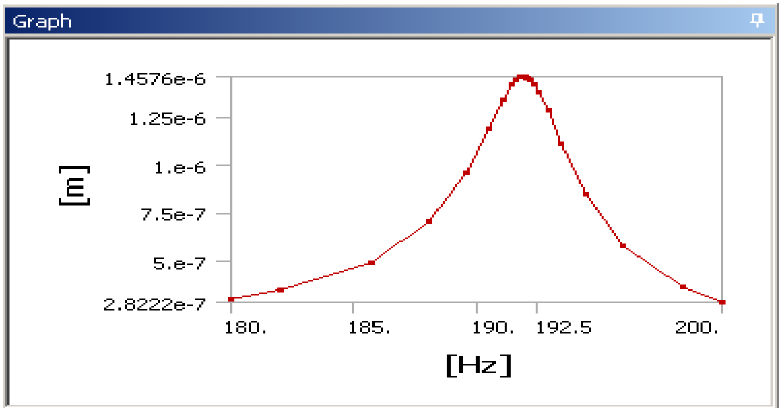
Options	
Frequency Spacing	Linear
Range Minimum	0. Hz
Range Maximum	500. Hz
Cluster Number	4
User Defined Frequencies	Off
Solution Method	Mode Superposition
Include Residual Vector	No
Cluster Results	Yes
Store Results At All Frequencies	Yes

- Note: It's good practice to add a realistic amount of damping to your models; this can serve not only to increase the physical fidelity of your model and results, but also to prevent unbounded amplitude solutions near resonances.

Without Cluster Option

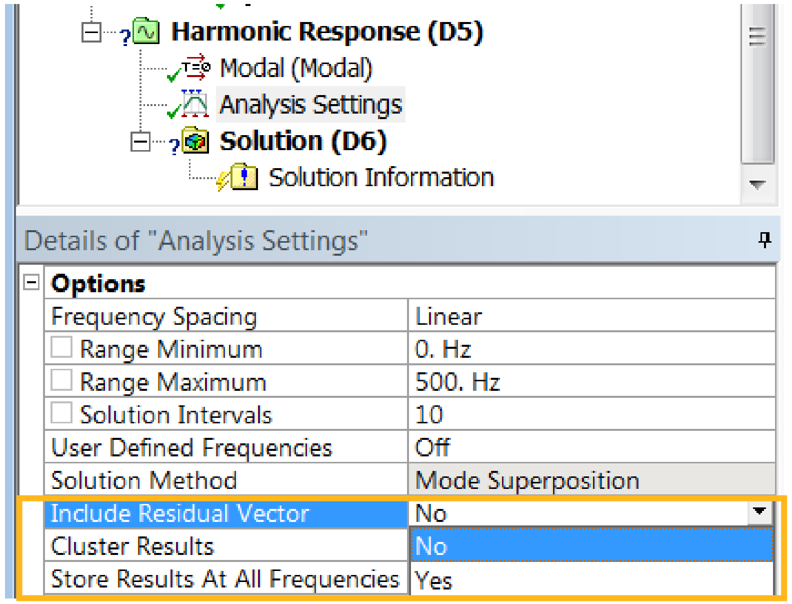


With Cluster Option



… Analysis Settings—MSUP Harmonic

- Analysis Settings > Options
  - Include Residual Vector



- In MSUP analysis, the dynamic response will be approximate when the applied loading excites the higher frequency modes of a structure.
  - <u>The residual vector method:</u>
    - employs additional modal transformation vectors in addition to the eigenvectors in the modal transformation .
    - accounts for high frequency dynamic responses with fewer eigenmodes.

… Analysis Settings—MSUP Harmonic

Details of "Analysis Settings"

Options	
Frequency Spacing	Linear
Range Minimum	0. Hz
Range Maximum	500. Hz
Solution Intervals	10
User Defined Frequencies	Off
Solution Method	Mode Superposition
Include Residual Vector	No
Cluster Results	No
Store Results At All Frequencies	Yes

Store Results at All Frequencies option!

- When set to "No," minimal data will be retained to supply just the results requested at the time of solution.
- The availability of the results is not determined by the settings in the Output Controls.

New solution is required if:

1. The addition of new frequency or phase responses to a solved environment requires a new solution.
2. New contour result of any type (stress or strain)
3. New probe result of any type for the first time on a solved environment

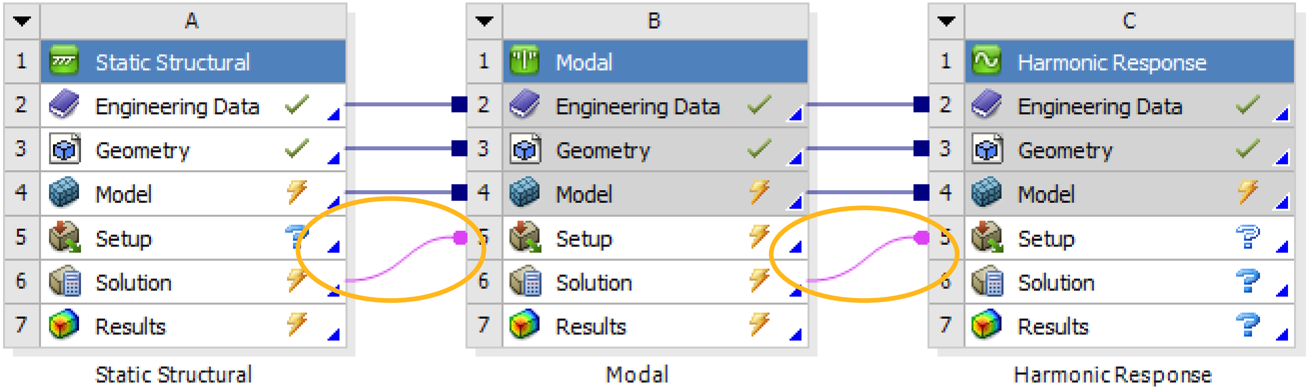
Available if:

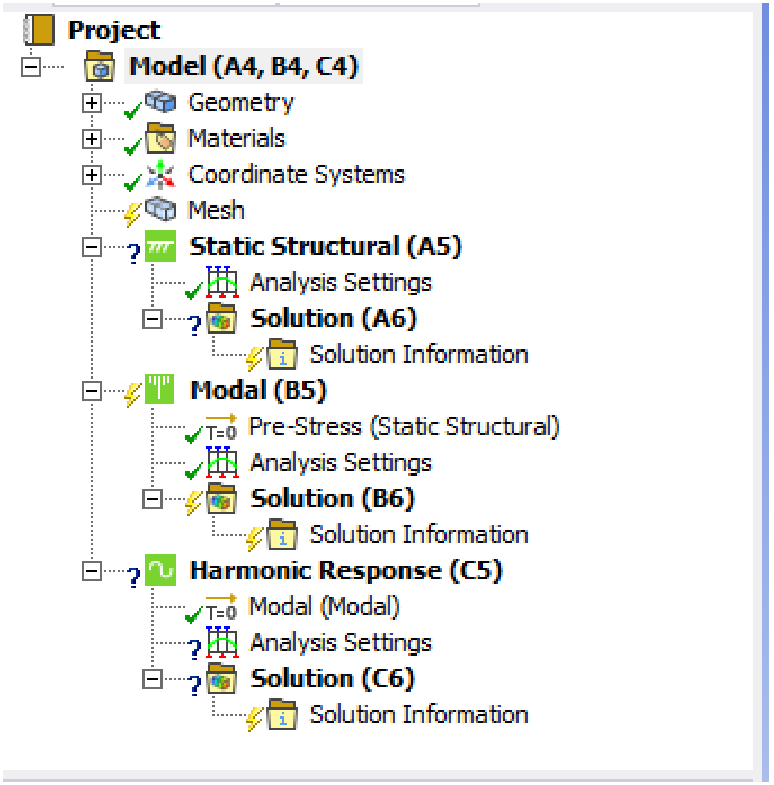
1. New contour or probe results of the same type; data from the closest available frequency is displayed.
2. New displacement contour results as well as bearing probe results.

Ansys

L. MSUP Harmonic Analysis Based on Linear Perturbation

- Pre-stressed Mode Superposition Harmonic Analysis.

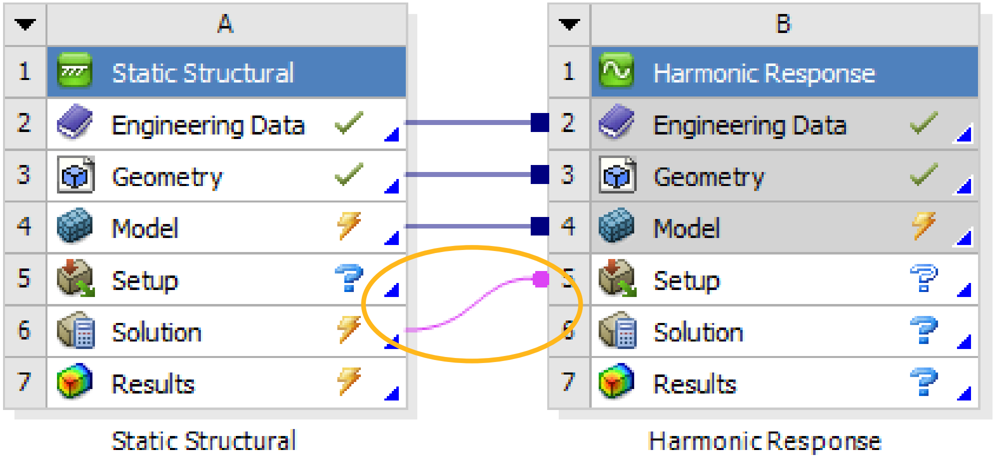


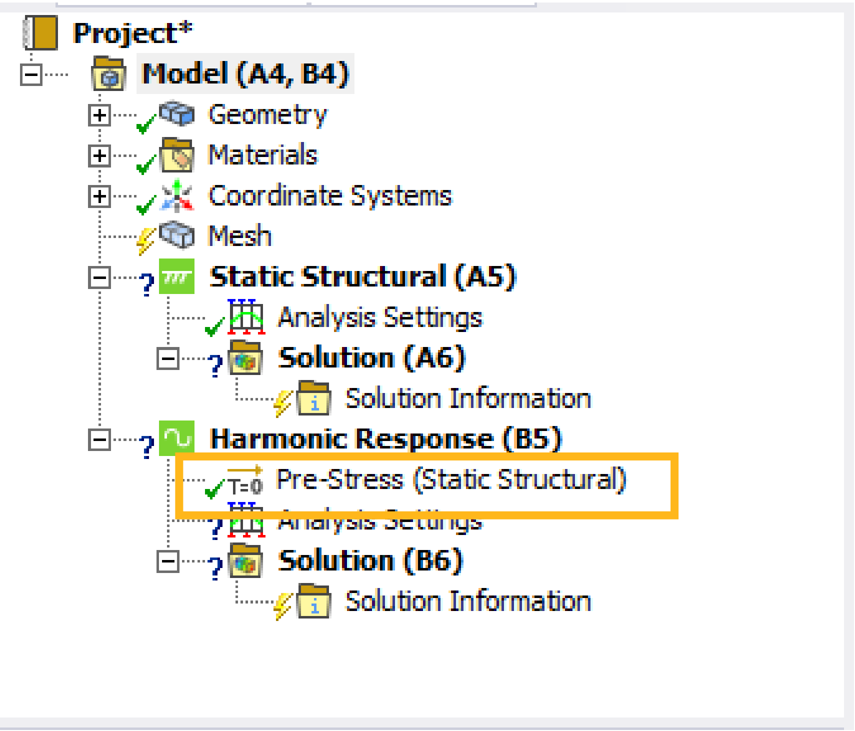


*See Module 05 for an overview of the Linear Perturbation Analysis Method

M. Full Harmonic Analysis Based on Linear Perturbation

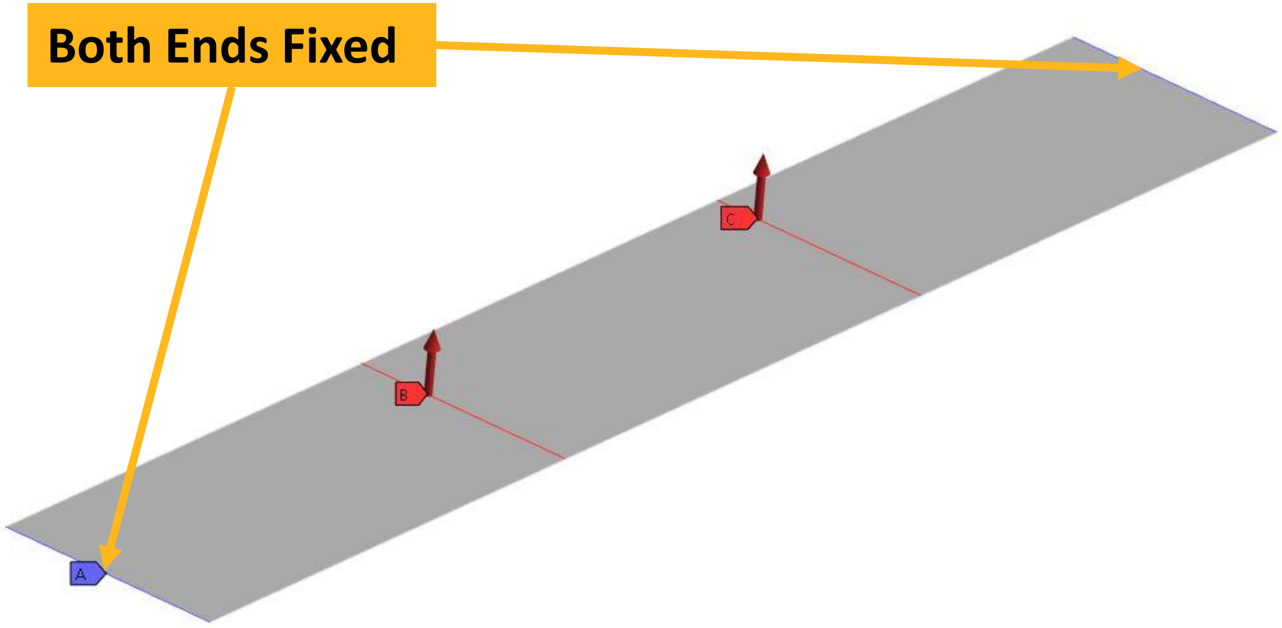
- Pre-Stressed Full Harmonic Analysis.





- *See Module 05 for an overview of the Linear Perturbation Analysis Method

Workshop 06.1: Fixed-Fixed Beam



End of presentation