Ansys Mechanical Linear and Nonlinear Dynamics

# Module 03: Modal Analysis

Release 2022 R2

Please note:
- These training materials were developed and tested in Ansys Release 2022 R2. Although they are expected to behave similarly in later releases, this has not been tested and is not guaranteed.
- The screen images included with these training materials may vary from the visual appearance of a local software session.

# Module 03 Learning Outcomes

- After completing this module, you will:
  - Understand the theoretical foundation of Modal analysis.
  - Know and understand the importance of performing Modal analysis and how to use participation factors and effective mass outputs to your advantage.
  - Be able to specify the relevant solution parameters required when performing Modal analysis, including when and how to apply damping.
  - Learn how to interpret modal analysis results and their relevance to other dynamic analysis types.

# Module 03 Topics

A. What is Modal Analysis?
B. Theory and Terminology
C. Eigenfrequencies and Mode Shapes
D. Participation Factors and Effective Mass
E. Mode Extraction Methods—Undamped
F. Contact in Modal Analysis
G. Analysis Settings
H. Damped Modal Analysis
I. Mode Extraction Methods—Damped

# A. What is Modal Analysis?

- The modal analysis technique is used to determine the vibration characteristics (i.e., natural frequencies and mode shapes) of linear elastic structures.
- The most fundamental of all dynamic analysis types.
- Allows the design to avoid resonant vibrations or to vibrate at a specified frequency.
- Gives engineers an idea of how the design will respond to different types of dynamic loads.
- Helps in calculating solution controls for other dynamic analyses.

# … What is Modal Analysis?

Assumptions and restrictions

- The structure is linear (i.e. $[M]$ and $[K]$ matrices are constant).
- Generally, loads (forces, non-zero displacements, pressures) are not allowed; i.e., free vibration.
  - Exceptions include:
    - Thermal Conditions (will affect Young’s Modulus, and therefore stiffness, if temperature dependent material properties are defined)
    - Rotational Velocity (commonly used in Rotordynamics applications; requires a damped modal solver and Coriolis effects enabled →refer to Ansys Mechanical Rotordynamics course)

**Recommendation: Because a structure’s vibration characteristics determine how it responds to any type of dynamic load, it is generally recommended to perform a modal analysis first before trying any other dynamic analysis.**

# B. Theory and Terminology

The linear equation of motion for free, un-damped vibration is

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\}$$

Assume harmonic motion:

$$\begin{aligned} \{u\} &= \{\varphi\}_i \sin(\omega_i t + \theta_i) \\ \{\ddot{u}\} &= -\omega_i^2\{\varphi\}_i \sin(\omega_i t + \theta_i) \end{aligned}$$

Substituting $\{u\}$ and $\{\ddot{u}\}$ In the governing equation gives an eigenvalue equation:

$$\left([K] - \omega_i^2[M]\right)\{\varphi_i\} = \{0\}$$

# ... Theory and Terminology

$$\left([K] - \omega_i^2 [M]\right)\{\varphi_i\} = \{0\}$$

This equality is satisfied if

1. $\{\varphi_i\}$ = 1

   → (trivial, implies no vibration)

2. or if $det\left([K] - \omega_i^2 [M]\right) = \{0\}$

   - This is an eigenvalue problem which may be solved for up to *n* roots ($\omega_1^2$, $\omega_2^2$, ....., $\omega_n^2$).
   - These roots are the ***eigenvalues*** of the equation
   - For each root (eigenvalue), there is a corresponding ***eigenvector*** ($\{\varphi\}_1$, $\{\varphi\}_2$, ......, $\{\varphi\}_n$)

# C. Eigenfrequencies and Mode Shapes

In Modal Analysis:

- The *eigenvalues* → the square of the *natural circular frequency* of the structure $\omega_i$
- The *eigenvectors* → the corresponding *mode shapes* $\{\varphi_i\}$
- Mode shapes can be normalized either to the mass matrix

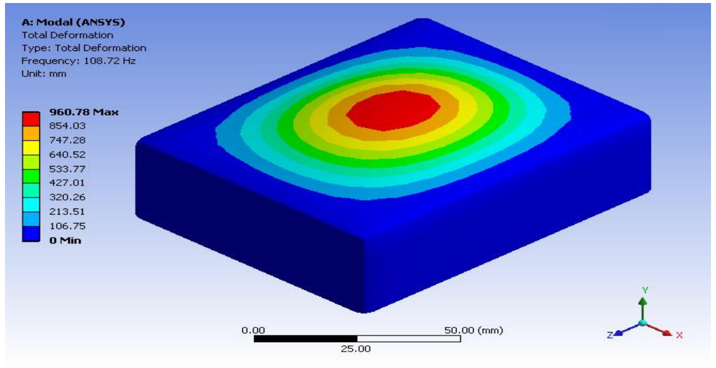
$$\{\varphi\}_i^T [M] \{\varphi\}_i = 1$$

  or to unity, where the largest component of the vector $\{\phi\}_i$ is set to 1.

- Mechanical displays results normalized to the mass matrix.
- Because of this normalization, only the shape of the DOF solution has real meaning.

# ... Eigenfrequencies and Mode Shapes

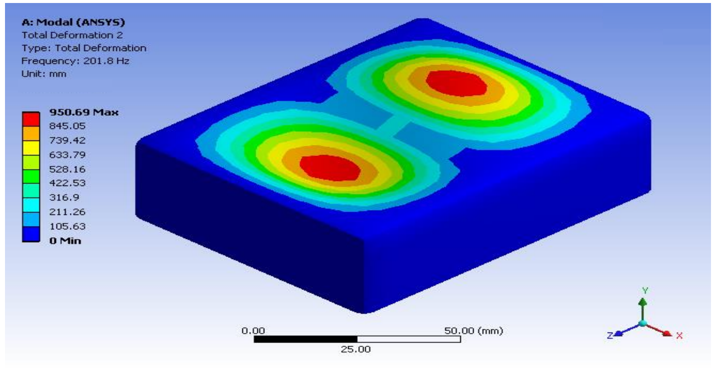
- The square roots of the eigenvalues are $\omega_i$, the structure's natural circular frequencies *(rad/s)*.
- Natural frequencies $f_i$ can then calculated as $f_i = \omega_i / 2\pi$ *(cycles/s)*.
  - It is the natural frequencies, $f_i$ in Hz, that are input by the user and output by Mechanical.
- The eigenvectors $\{\phi\}_i$ represent the mode shapes, *i.e.* the shape assumed by the structure when vibrating at frequency $f_i$.




mode 1

$\leftarrow \{\phi\}_1$

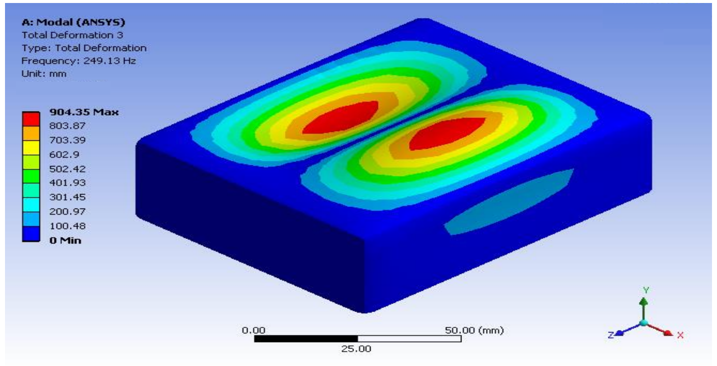
$f_1$ = 109 Hz




mode 2

$\leftarrow \{\phi\}_2$

$f_2$ = 202 Hz




mode 3

$\leftarrow \{\phi\}_3$

$f_3$ = 249 Hz



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# D. Participation Factors and Effective Mass

- The participation factors are calculated by

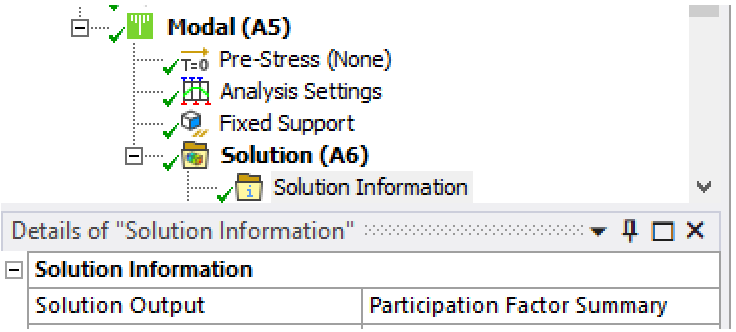
$$\gamma_i = \{\varphi\}_i^T [M]\{D\}$$

  where $\{D\}$ is an assumed unit displacement spectrum in each of the global Cartesian directions and rotation about each of these axes.

- Measures the amount of mass moving in each direction for each mode.
- A high value in a direction indicates that the mode will be excited by forces in that direction.
- *The concept of participation factors will be important in later chapters.*

# ... Participation Factors and Effective Mass

- The participation factors can be viewed within Solution Information.
  - Largest values in each direction indicate that the structure will likely respond at that frequency when excited in that direction.




**Participation Factor**

| Mode | Frequency [Hz] | X Direction | Y Direction | Z Direction | Rotation X | Rotation Y | Rotation Z |
|---|---|---|---|---|---|---|---|
| 1 | 72.609 | -2.1959e-018 | -6.0582e-019 | -7.1565e-006 | -0.11644 | 0.17344 | -4.1391e-018 |
| 2 | 72.649 | -1.8293e-018 | -6.785e-019 | 2.8654e-005 | 0.17343 | 0.11643 | 8.0774e-018 |
| 3 | 86.452 | 8.7338e-019 | 5.9756e-019 | 5.3485e-002 | -1.0795e-004 | -3.0281e-005 | 2.3352e-017 |
| 4 | 93.853 | -6.1553e-019 | 2.7778e-019 | 2.3053e-005 | -4.058e-005 | 4.9349e-005 | 5.1772e-017 |
| 5 | 139.47 | 1.7984e-018 | 2.4265e-018 | 3.0373e-006 | -4.5345e-005 | -4.3605e-005 | 1.2926e-017 |
| 6 | 247.54 | 2.8125e-018 | -2.9157e-018 | 2.2194e-006 | 5.8115e-004 | 2.1509e-002 | -2.3111e-017 |
| 7 | 247.56 | 4.3966e-018 | 8.252e-018 | 2.4087e-006 | 2.1513e-002 | -5.6428e-004 | 2.2871e-017 |
| 8 | 341.03 | -5.3614e-017 | 2.5886e-017 | -1.8662e-002 | 6.2826e-006 | 1.6353e-005 | -2.3191e-016 |
| 9 | 466.7 | -6.4701e-014 | 9.2235e-014 | -1.5481e-005 | 2.3082e-005 | 7.9242e-007 | -7.9124e-016 |
| 10 | 497.46 | -2.2235e-013 | 2.0395e-013 | -7.9127e-006 | -1.8341e-002 | -3.0883e-003 | 2.2977e-014 |

# ... Participation Factors and Effective Mass

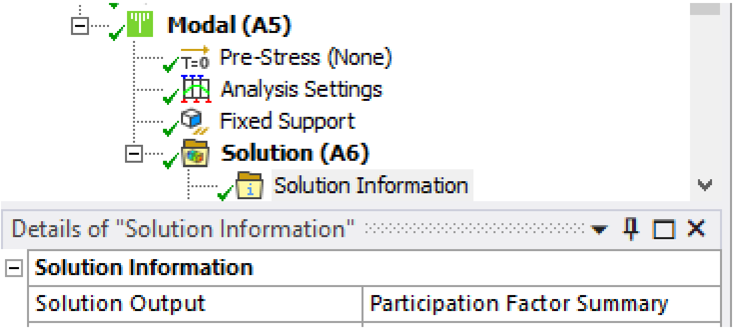
- The effective mass is calculated by

$$M_{eff,i} = \frac{\gamma_i^2}{\{\varphi\}_i^T [M] \{\varphi\}_i} = \gamma_i^2 \text{ , if } \{\varphi\}_i^T [M] \{\varphi\}_i = 1$$

- Ideally, the sum of the effective masses in each direction should equal the total mass of the structure but it will depend on the number of modes extracted.
- The ratio of effective mass to total mass can be useful for determining whether or not a sufficient number of modes have been extracted.
  - Values larger than 0.90 (90%) are desired when performing downstream analyses that use the modal results.

# ... Participation Factors and Effective Mass

- The effective mass can also be viewed within Solution Information.




**Effective Mass**

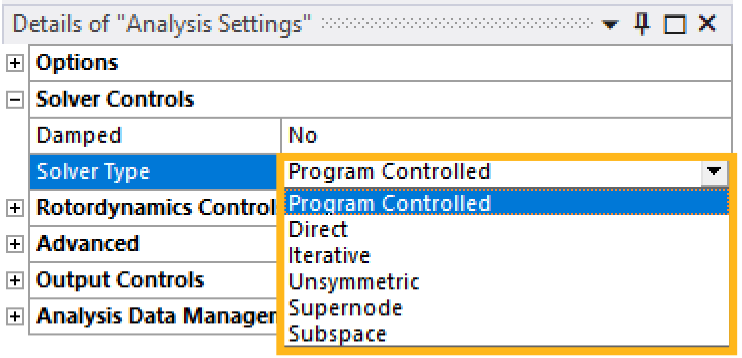
| Mode | Frequency [Hz] | X Direction [slinch] | Y Direction [slinch] | Z Direction [slinch] | Rotation X [slinch] | Rotation Y [slinch] | Rotation Z [slinch] |
|---|---|---|---|---|---|---|---|
| 1 | 72.609 | 4.8221e-036 | 3.6702e-037 | 5.1215e-011 | 1.3558e-002 | 3.008e-002 | 1.7132e-035 |
| 2 | 72.649 | 3.3463e-036 | 4.6036e-037 | 8.2103e-010 | 3.0079e-002 | 1.3557e-002 | 6.5245e-035 |
| 3 | 86.452 | 7.628e-037 | 3.5708e-037 | 2.8606e-003 | 1.1653e-008 | 9.1696e-010 | 5.453e-034 |
| 4 | 93.853 | 3.7887e-037 | 7.7163e-038 | 5.3145e-010 | 1.6467e-009 | 2.4354e-009 | 2.6803e-033 |
| 5 | 139.47 | 3.2341e-036 | 5.8879e-036 | 9.2249e-012 | 2.0562e-009 | 1.9014e-009 | 1.6709e-034 |
| 6 | 247.54 | 7.91e-036 | 8.5014e-036 | 4.9257e-012 | 3.3774e-007 | 4.6265e-004 | 5.3412e-034 |
| 7 | 247.56 | 1.933e-035 | 6.8096e-035 | 5.8017e-012 | 4.6282e-004 | 3.1841e-007 | 5.2309e-034 |
| 8 | 341.03 | 2.8744e-033 | 6.7007e-034 | 3.4829e-004 | 3.9471e-011 | 2.6741e-010 | 5.3781e-032 |
| 9 | 466.7 | 4.1863e-027 | 8.5072e-027 | 2.3967e-010 | 5.3278e-010 | 6.2792e-013 | 6.2606e-031 |
| 10 | 497.46 | 4.9438e-026 | 4.1595e-026 | 6.261e-011 | 3.3641e-004 | 9.5375e-006 | 5.2794e-028 |
| Sum | | 5.3624e-026 | 5.0102e-026 | 3.2089e-003 | 4.4436e-002 | 4.4109e-002 | 5.2862e-028 |

**Ratio of Effective Mass to Total Mass**

| Mode | Frequency [Hz] | X Direction | Y Direction | Z Direction | Rotation X | Rotation Y | Rotation Z |
|---|---|---|---|---|---|---|---|
| 1 | 72.609 | 1.3073e-033 | 9.95e-035 | 1.3884e-008 | 0.30272 | 0.67164 | 1.9128e-034 |
| 2 | 72.649 | 9.0717e-034 | 1.248e-034 | 2.2258e-007 | 0.6716 | 0.3027 | 7.2845e-034 |
| 3 | 86.452 | 2.0679e-034 | 9.6803e-035 | 0.77552 | 2.6019e-007 | 2.0474e-008 | 6.0882e-033 |
| 4 | 93.853 | 1.0271e-034 | 2.0919e-035 | 1.4408e-007 | 3.6769e-008 | 5.4378e-008 | 2.9926e-032 |
| 5 | 139.47 | 8.7676e-034 | 1.5962e-033 | 2.5009e-009 | 4.5911e-008 | 4.2454e-008 | 1.8655e-033 |
| 6 | 247.54 | 2.1444e-033 | 2.3047e-033 | 1.3354e-009 | 7.5412e-006 | 1.033e-002 | 5.9634e-033 |
| 7 | 247.56 | 5.2405e-033 | 1.8461e-032 | 1.5728e-009 | 1.0334e-002 | 7.1095e-006 | 5.8402e-033 |
| 8 | 341.03 | 7.7926e-031 | 1.8166e-031 | 9.442e-002 | 8.8133e-010 | 5.9708e-009 | 6.0046e-031 |
| 9 | 466.7 | 1.1349e-024 | 2.3063e-024 | 6.4974e-008 | 1.1896e-008 | 1.402e-011 | 6.9899e-030 |
| 10 | 497.46 | 1.3403e-023 | 1.1276e-023 | 1.6974e-008 | 7.5113e-003 | 2.1296e-004 | 5.8944e-027 |
| Sum | | 1.4538e-023 | 1.3583e-023 | 0.86994 | 0.99218 | 0.98489 | 5.902e-027 |

# E. Mode Extraction Methods—Undamped

In most cases, the Program Controlled option selects the optimal solver automatically.




| Procedure | Usages | Applications |
|---|---|---|
| **Block Lanczos (Direct)** | Symmetric | • Many modes (about 40+) of large models.<br>• Recommended when poorly shaped solid and shell elements exist.<br>• Shells or a combination of shells and solids. |
| **PCG Lanczos (Iterative)** | Symmetric<br>(but not applicable for buckling) | • Few modes (up to about 100) of very large models (1,000,000+ degrees of freedom).<br>• Well-shaped 3-D solid elements. |
| **Unsymmetric** | Unsymmetric matrices | • Uses the full [K] and [M] matrices.<br>• When K and M are unsymmetric (i.e., acoustic fluid-structure interaction problems). |
| **Supernode** | Symmetric<br>(but not applicable to buckling) | • Many modes (up to 10,000).<br>• Used for 2-D plane or shell/beam structures (100 modes or more) and for 3-D solid structures (250 modes or more). |
| **Subspace** | Symmetric<br>(Applicable to buckling) | • Similar applicability as Block Lanczos<br>• Advantage over Block Lanczos is that [K], [S] / [M] matrices can be indefinite. |



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# F. Contact in Modal Analysis

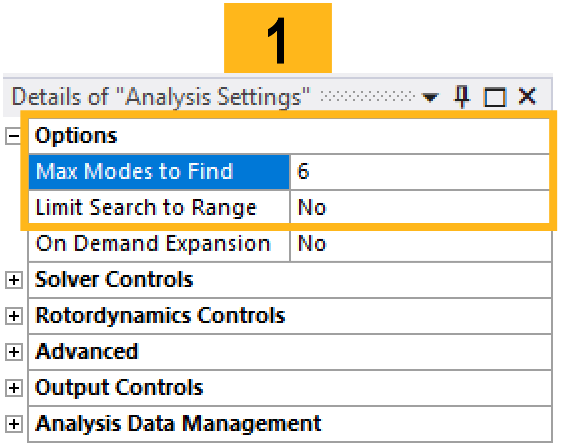
- Contact regions are available in modal analysis; however, since this is a purely linear analysis, contact behavior will differ for the nonlinear contact types, as shown below:

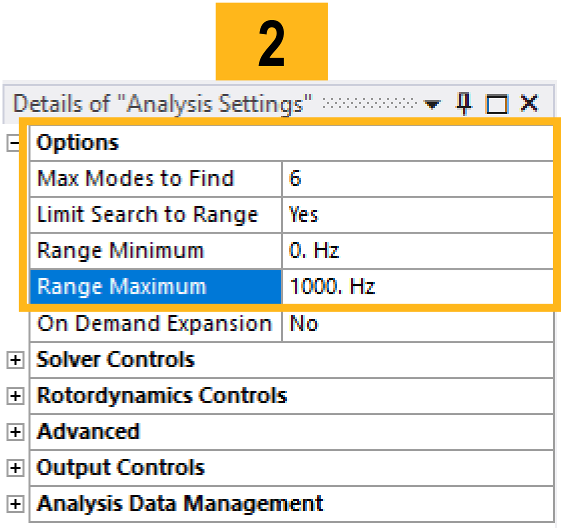
| Contact Type | Static Analysis | Linear Dynamic Analysis | | |
|---|---|---|---|---|
| | | Initially Touching | Inside Pinball Region | Outside Pinball Region |
| Bonded | Bonded | Bonded | Bonded | Free |
| No Separation | No Separation | No Separation | No Separation | Free |
| Rough | Rough | Bonded | Free | Free |
| Frictionless | Frictionless | No Separation | Free | Free |
| Frictional | Frictional | $\mu = 0$, No Separation<br>$\mu > 0$, Bonded | Free | Free |

- Contact behavior will reduce to its linear counterparts.

# G. Analysis Settings

- Number of modes
  - You need to specify the number of frequencies of interest. The default is to extract the *__first 6 natural frequencies__*.
  - The number of frequencies can be specified in two ways*:
    1. The first N frequencies (N > 0), or
    2. The first N frequencies in a selected range of frequencies.

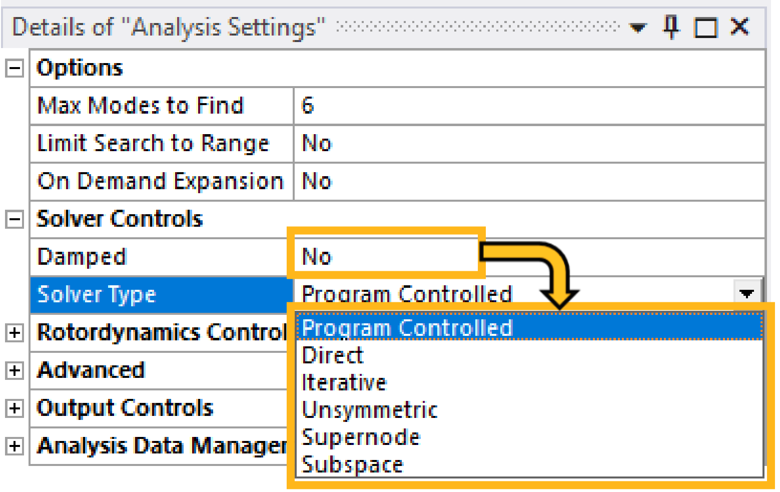


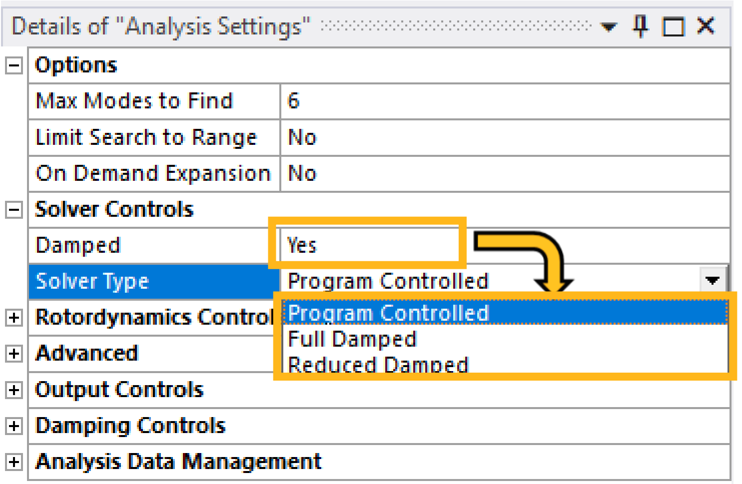




* Accuracy of the Supernode Extraction method is improved using method 2 above.

# ... Analysis Settings

- Solver Controls
  - Two settings are available in this control – Damped and Solver Type.
    - Choices for Damped are No (the default) and Yes
    - Depending on the selection made for Damped, different solver options are provided accordingly.



# ... Analysis Settings

- Output Controls
  - By default, only mode shapes are calculated.
  - Stress and Strain results can be requested to be calculated.
  - “stress” results only show the relative distribution of stress in the structure and are not real stress values. Linked systems on the Project page will affect the Output Controls according to what quantities will be needed in the downstream analysis.

Details of "Analysis Settings"

| | |
|---|---|
| **+ Options** | |
| **+ Solver Controls** | |
| **+ Rotordynamics Controls** | |
| **− Output Controls** | |
| Stress | No |
| Surface Stress | No |
| Back Stress | No |
| Strain | No |
| Contact Data | Yes |
| Nodal Forces | Yes |
| Calculate Reactions | No |
| Store Modal Results | Program Controlled |
| General Miscellaneous | Yes |
| Result File Compression | Program Controlled |
| **+ Damping Controls** | |
| **+ Analysis Data Management** | |

# H. Damped Modal Analysis

- Damped Modal analysis is most applicable to Rotordynamics applications and is covered in more detail in the Ansys Mechanical Rotordynamics course. A brief introduction is given here.
- The linear equation of motion for free, damped vibration is

$$[M]\{\ddot{u}\} + \boxed{[C]\{\dot{u}\}} + [K]\{u\} = \{0\}$$

- The eigenvalues are complex
- The imaginary part of the eigenvalue is the natural frequency
- The real part of eigenvalue is a measure of stability: positive = unstable, negative = stable
- Frequency is often plotted as a function of velocity using Campbell diagrams, as modes of rotating structures tend to become unstable at higher velocities

**Undamped**

Tabular Data

| | Mode | Frequency [Hz] |
|---|---|---|
| 1 | 1. | 417.48 |
| 2 | 2. | 1705.2 |
| 3 | 3. | 2545.7 |
| 4 | 4. | 3577.2 |
| 5 | 5. | 5326.1 |
| 6 | 6. | 6896.2 |

**Damped**

$$\omega_d = \omega_n\sqrt{1 - \xi^2}$$

Tabular Data

| | Mode | Damped Frequency [Hz] | Stability [Hz] | Modal Damping Ratio |
|---|---|---|---|---|
| 1 | 1. | 417.44 | -5.4755 | 1.3116e-002 |
| 2 | 2. | 1702.8 | -91.349 | 5.3571e-002 |
| 3 | 3. | 2537.5 | -203.59 | 7.9974e-002 |
| 4 | 4. | 3554.5 | -402. | 0.11238 |
| 5 | 5. | 5251. | -891.19 | 0.16732 |
| 6 | 6. | 6732.4 | -1494.1 | 0.21665 |

# ... Damped Modal Analysis

- For a damped modal analysis, the damping matrix $[C^*]$ is complex:

$$[C^*]=i\left(\omega[C]+g[K]+\sum_{j=1}^{N_m} m_j[K_j]+\sum_{k=1}^{N_e^*}[K_k^*]\right)$$

Where $$[C]=\alpha[M]+\beta[K]+\sum_{i=1}^{N_{ma}}\alpha_i^m[M_i] \quad +\sum_{j=1}^{N_{mb}}\beta_j^m[K_j]$$

$\omega$ : circular frequency

$g$ : constant structural damping coefficient (DMPSTR)

$m_j$ : constant structural damping coefficient for material *j* (MP,DMPS)

$K_k^*$ : imaginary stiffness element matrix (via the various Connection elements, COMBIN14, MPC184, etc.)

And within [C],

$\alpha$ : Global Mass-Matrix Multiplier (alpha damping, ALPHAD)

$\beta$ : Global k-Matrix Multiplier (beta damping, BETAD)

$\alpha_i^m$ : Mass matrix multiplier for material *i* (alpha damping, MP,ALPD)

$\beta_j^m$ : Stiffness matrix multiplier for material *j* (beta damping, MP,BETD)

# … Damped Modal Analysis

The values of $\alpha^m$ and $\beta^m$ and $m_j$ can be input using the following:

[1] Material-dependent damping value

(Mass-Matrix Damping Multiplier, and k-Matrix Damping Multiplier, and constant structural damping coefficient)

$$[C] = \sum_{i=1}^{N_{ma}} \alpha_i^m [M_i] + \sum_{j=1}^{N_{mb}} \beta_j^m [K_j]$$

Equivalent damping

$$\xi_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2}$$

Define in Engineering Data

Properties of Outline Row 3: Structural Steel

| | A | B |
|---|---|---|
| 1 | Property | Value |
| 2 | Material Field Variables | Table |
| 3 | Density | 7850 |
| 4 | Isotropic Secant Coefficient of Thermal Expansion | |
| 6 | Material Dependent Damping | |
| 7 | Damping Ratio | 0.04 |
| 8 | Constant Structural Damping Coefficient | = 0.08 $m_j$ |
| 9 | Damping Factor (α) | |
| 10 | Mass-Matrix Damping Multiplier | 12.56 $\alpha^m$ |
| 11 | Damping Factor (β) | |
| 12 | k-Matrix Damping Multiplier | 3.35E-05 $\beta^m$ |
| 13 | Isotropic Elasticity | |

$$m_j = 2\xi$$

# … Damped Modal Analysis

The values of $\alpha$ and $\beta$ and g can be input using the following:

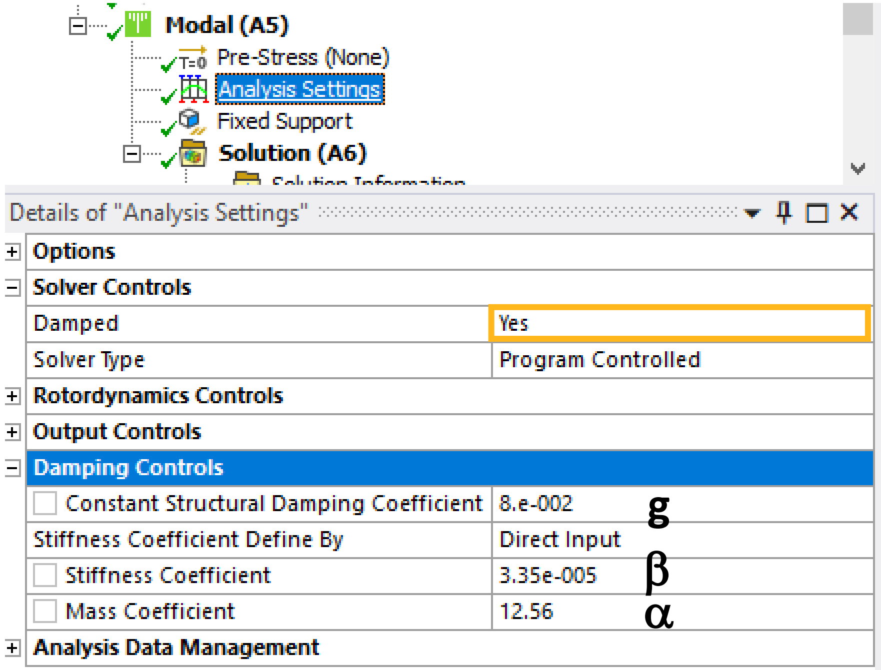
[2] <u>Directly as global damping value</u>

(Details section of Analysis Settings)

$$[C] = \alpha[M] + \beta[K]$$

Equivalent damping

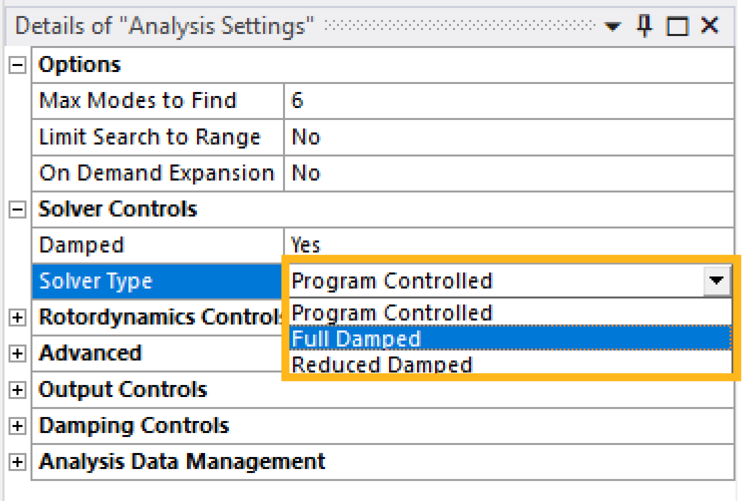
$$\xi_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2}$$




$$g = 2\xi$$

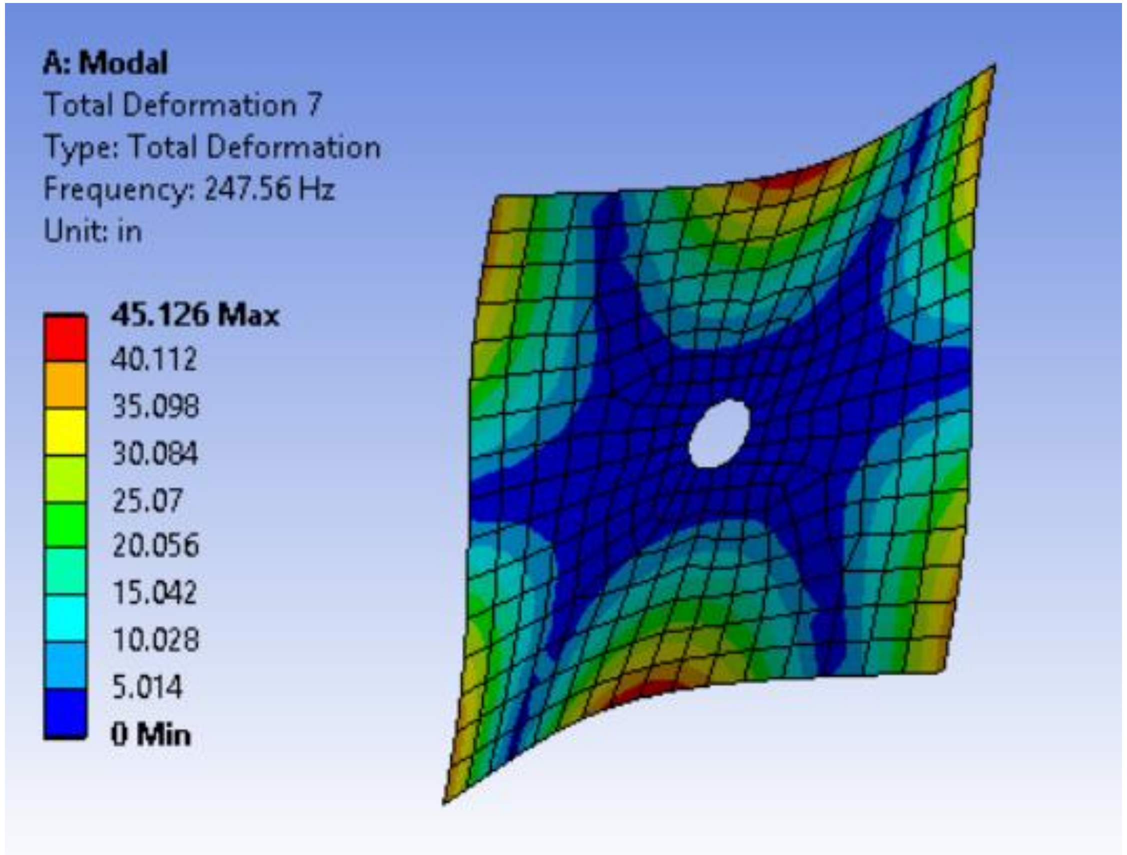
# I. Mode Extraction Methods—Damped

- In most cases, the Program Controlled option selects the optimal solver automatically.

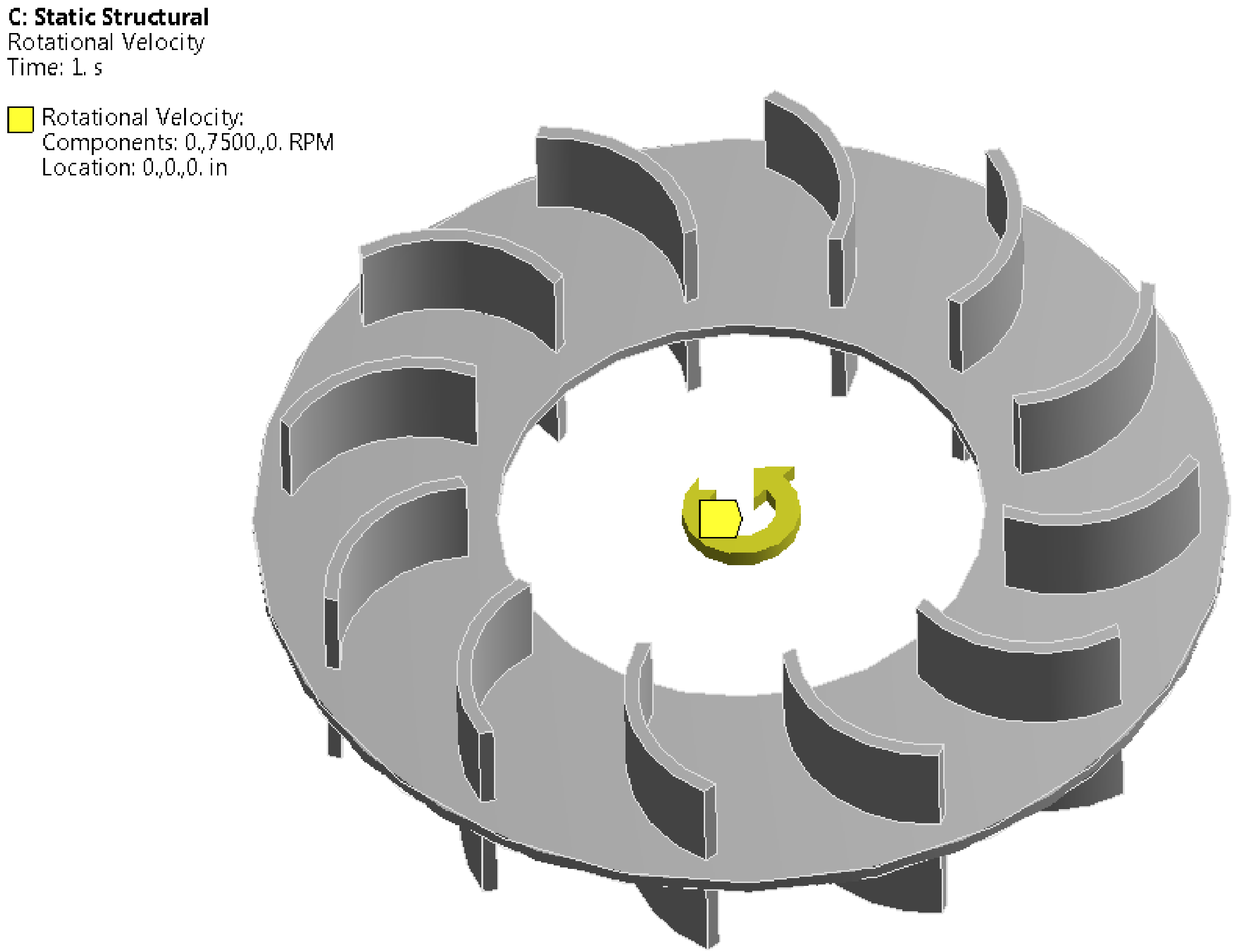



| Procedure | Usages | Applications |
|---|---|---|
| **Full Damped** | Symmetric or unsymmetric damped systems | • To find all modes of small to medium models (less than 10,000 degrees of freedom).<br>• Uses full matrices ([K], [M], and the damping matrix [C]) |
| **Reduced Damped** | Symmetric or unsymmetric damped systems | • Approximately represent the first few complex damped eigenvalues by modal transformation using a larger number of eigenvectors of the undamped system.<br>• After the undamped mode shapes are evaluated by using the real eigensolution (Block Lanczos method), the equations of motion are transformed to these modal coordinates. |

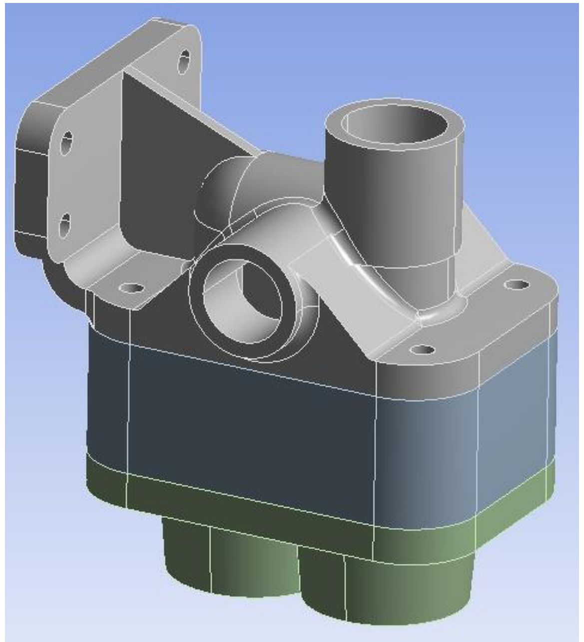
# Workshop 03.1: Plate with Hole

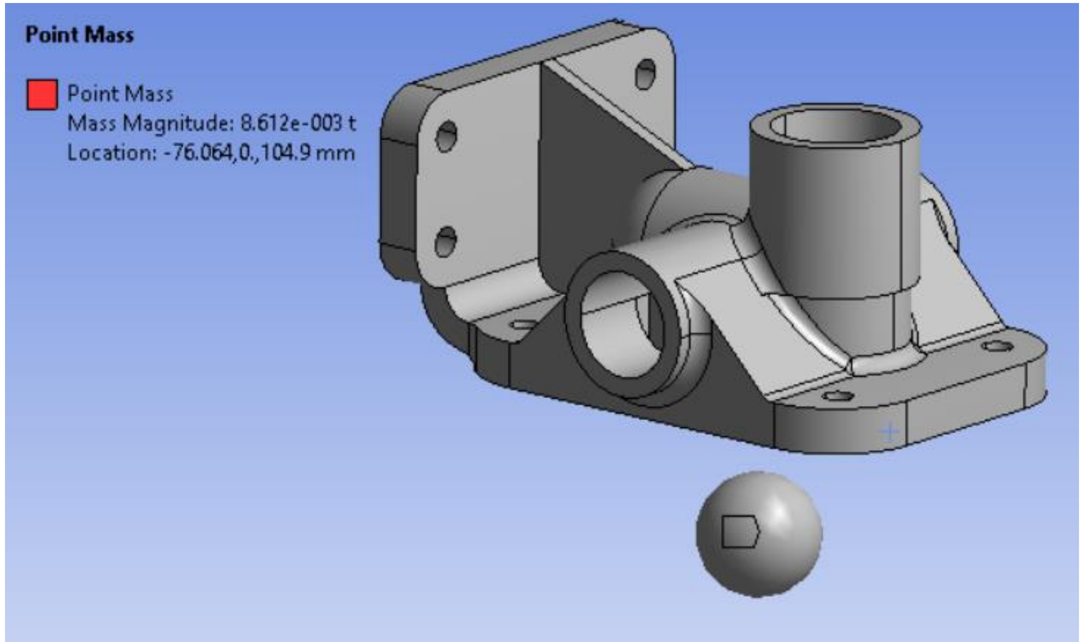


# Workshop 03.2: Bladed Disk



# Workshop 03.3: Valve Body





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End of presentation