Ansys Mechanical Linear and Nonlinear Dynamics

Module 07: Response Spectrum Analysis

Release 2022 R2

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Module 07 Learning Outcomes

- After completing this module, you will:
  - Be able to recognize applications most suited to the Response Spectrum analysis technique.
  - See the importance of participation factors and how they contribute to the response of a structure when subjected to a response spectrum.
  - Gain the knowledge and confidence needed to choose the appropriate solution combination method given the results of a Modal analysis and the input spectrum.
  - Learn techniques to account for an insufficient number of modes and/or modal mass that was not accounted for by a Modal analysis.
  - Understand how damping is accounted for in a Response Spectrum analysis.

Module 07 Topics

A. What is Response Spectrum Analysis?

B. Generating the Response Spectrum

C. Types of Analyses

D. Single Point Response Spectrum (SPRS) Analysis
   1. Participation Factor, γ
   2. Spectrum Values, S
   3. Mode Coefficients, A
   4. Response, R

E. Mode Combination Methods
   1. Square Root of the Sum of the Squares (SRSS) Method
   2. Sufficiently Spaced Modes
   3. Closely Spaced Modes (Correlated)

F. Rigid Response
   1. Low Frequency (below f_{SP})
   2. High Frequency (above f_{ZPA})
   3. Mid Frequency (between f_{SP} and f_{ZPA})
   4. Rigid Response Calculation

G. Missing Mass Response

H. Multi-Point Response Spectrum (MPRS) Analysis

I. Recommendations
   1. Setup
   2. Solution Procedure

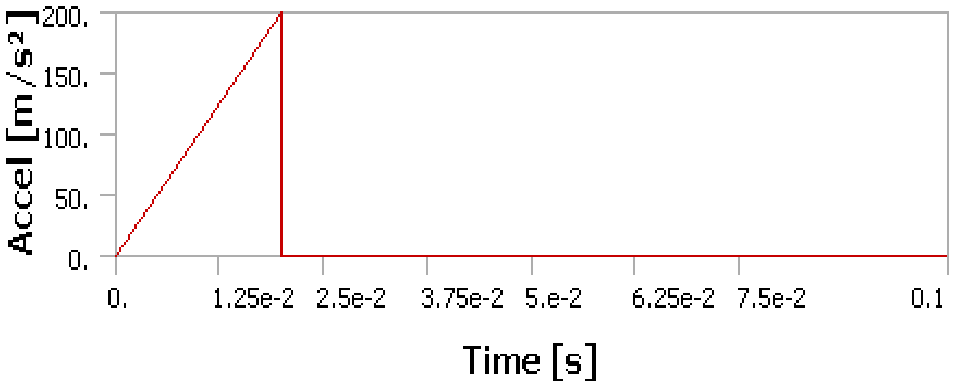
J. Analysis Settings

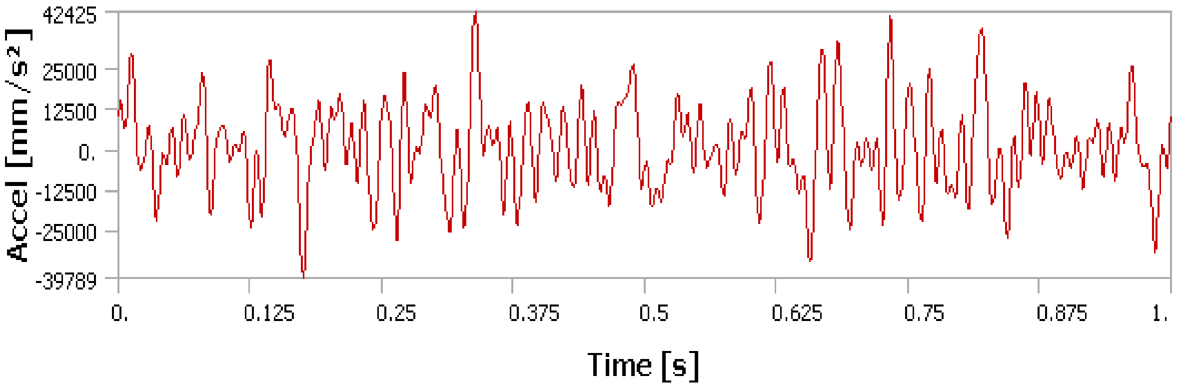
K. Loads and Supports

L. Results

A. What is Response Spectrum Analysis?

- A response spectrum analysis is mainly used in place of a time-history analysis to determine the response of structures to random or time-dependent loading conditions such as:
  - earthquakes,
  - wind loads,
  - ocean wave loads,
  - jet engine thrust,
  - rocket motor vibrations, and so on.



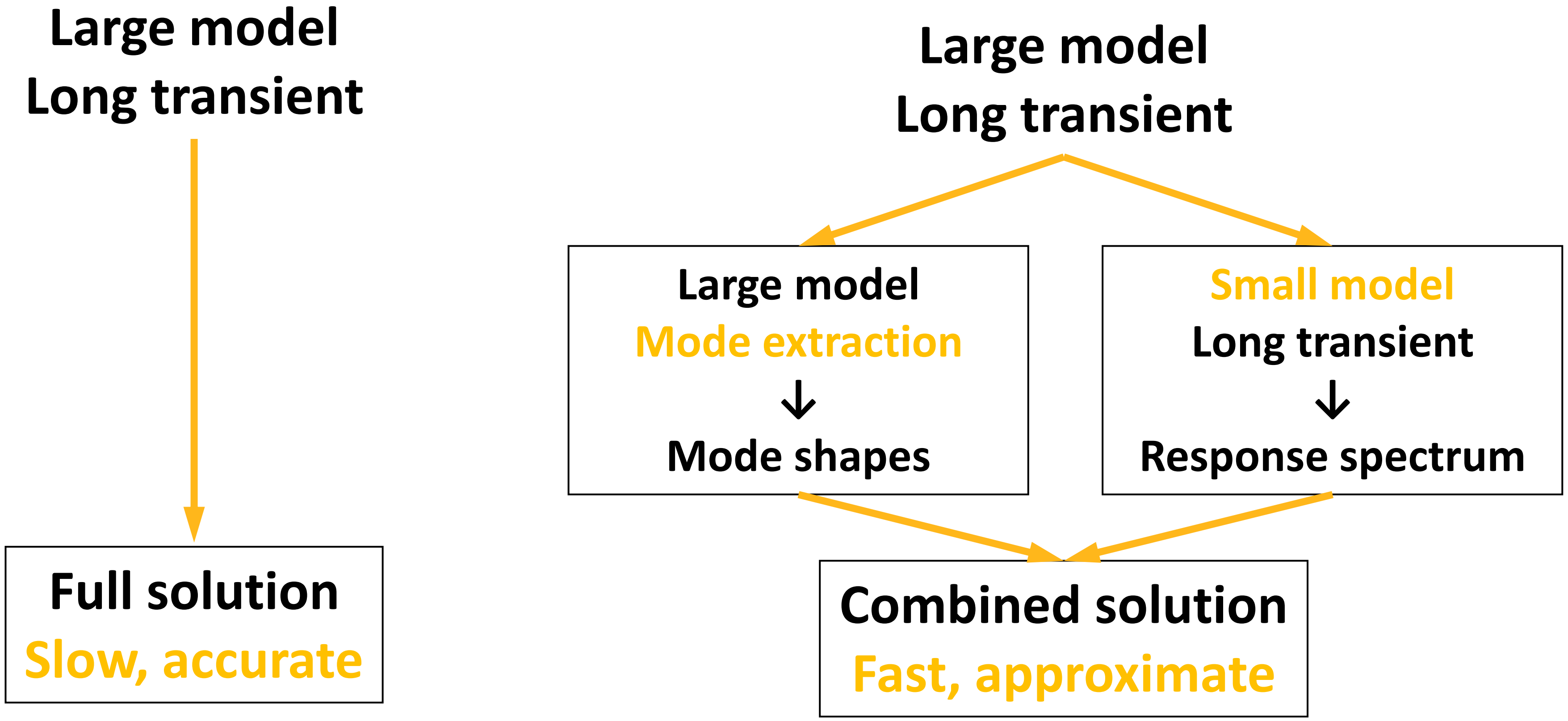


... What is Response Spectrum Analysis?

- The most accurate solution would be to run a large, long transient analysis.
  - Here, “large” means many DOF, and “long” means many time points.
  - In most cases, this approach would take too much time and compute resources.
- Instead of solving the (1) large model and (2) long transient together, it can be desirable to approximate the maximum response quickly by using the results of a modal analysis and a known input spectrum to calculate displacements and stresses in the model.
- The input spectrum is a graph of “spectral value” versus “frequency” that captures the intensity and frequency content of the input time-history loads.

... What is Response Spectrum Analysis?

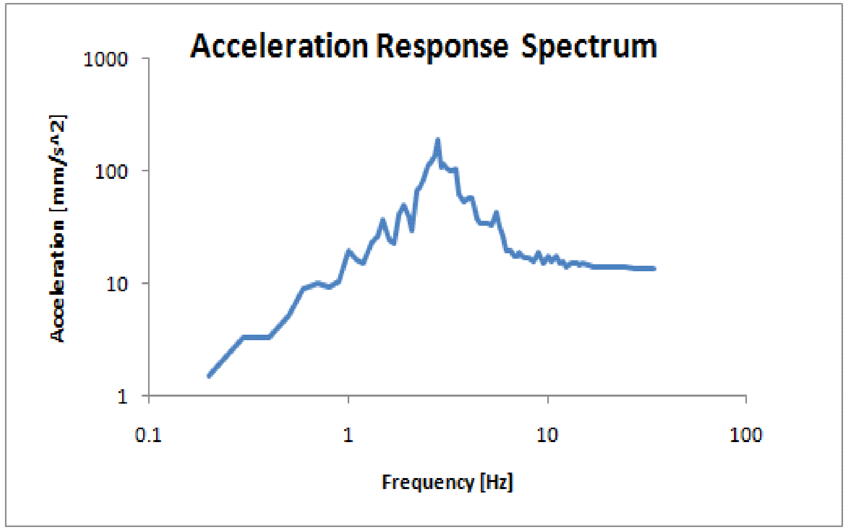
- Idea: solve the (1) large model and (2) long transient separately and combine the results.



B. Generating the Response Spectrum

- Response Spectrum:
  - A response spectrum is a plot of the *maximum response* of linear one-DOF systems to a given time-history input.
  - The abscissa of the plot is the *natural frequencies* of the systems, the ordinate is the *maximum response*:

    - Displacement
    - Velocity
    - Acceleration



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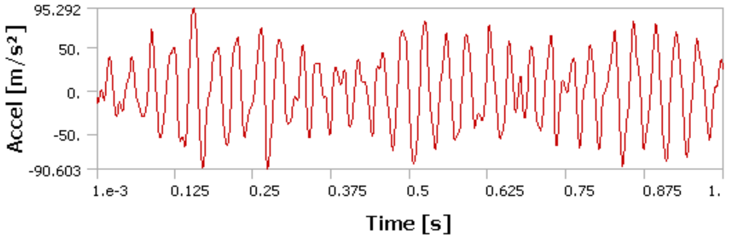
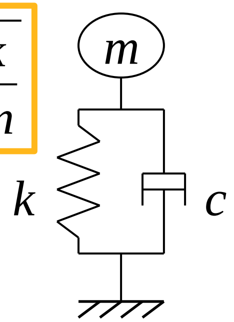
… Generating the Response Spectrum

- The generation of the response spectrum will usually be done for you, but the process can be described as follows:
  1. Subject a small model to the transient loading.
     - smallest is 1 DOF oscillator (*k, c, m*)
     - Note the presence of damping in this system
  2. Track the response over time (disp, velo, or accel).
     - note the maximum absolute amplitude over time



$$\boxed{\omega \approx \sqrt{\frac{k}{m}}}$$

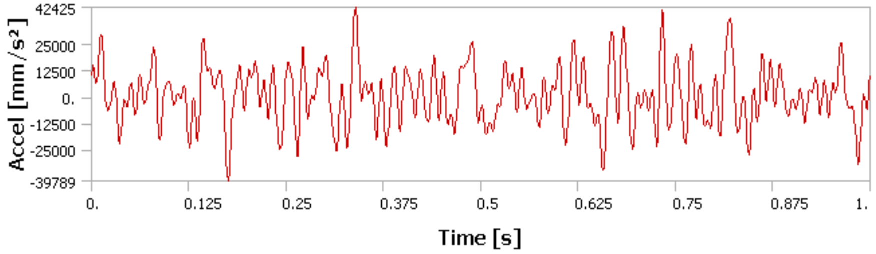


e.g., for oscillator with frequency

ω = 30 Hz

max absolute value over time

S = 95 m/s^2



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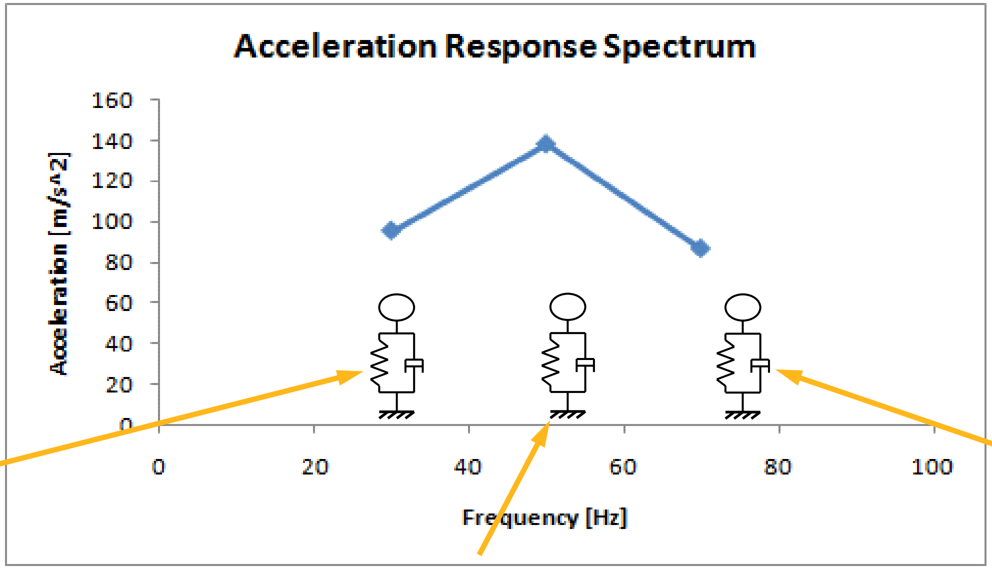
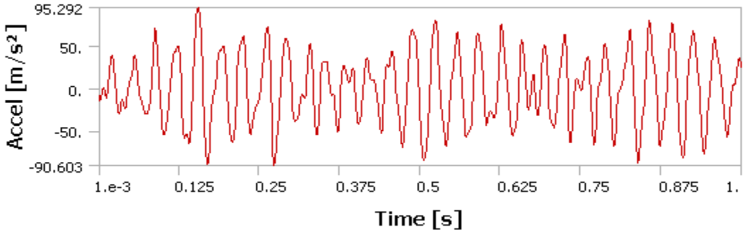
... Generating the Response Spectrum

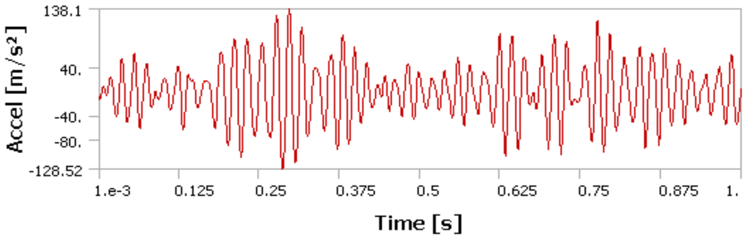
3. Repeat for oscillator with different frequency (same damping).
4. Plot the max response over time as a function of frequency.



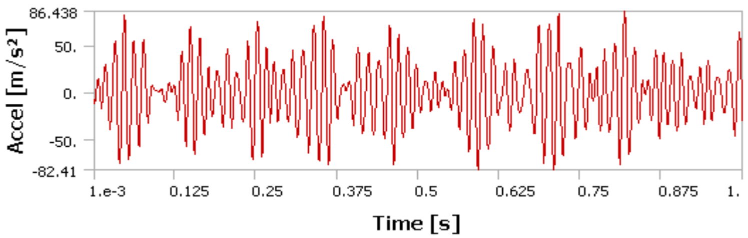
ω = 30 Hz, S = 95 m/s^2



ω = 50 Hz, S = 138 m/s^2

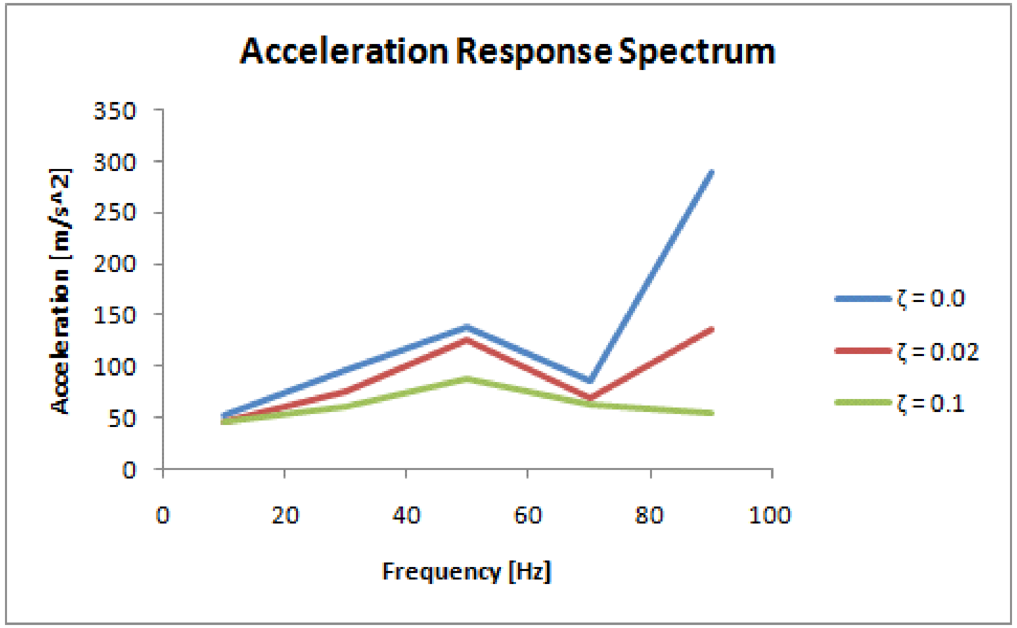


ω = 70 Hz, S = 86 m/s^2



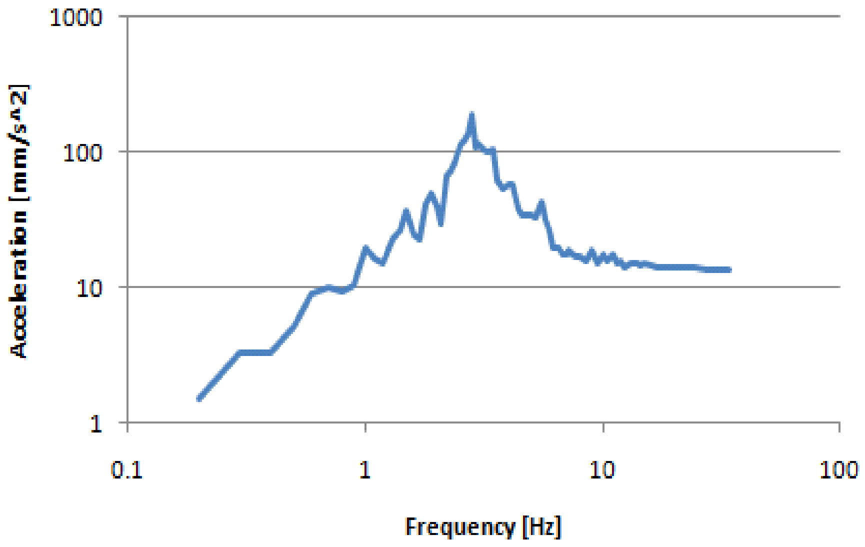
... Generating the Response Spectrum

- Note that the damping is included in the response spectrum.
- Additional spectra can be generated for other damping values.



... Generating the Response Spectrum

- Using many oscillators results in a more detailed curve.
  - The graph is typically plotted in log-log scale.
  - This also allows us to generate the spectrum only once and reuse it for any model.
- For efficiency, one analysis is done on an array of many oscillators.
  - The same damping is used for all oscillators.



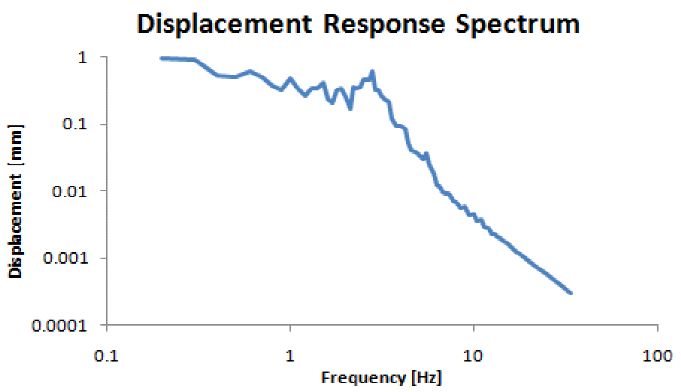
... Generating the Response Spectrum

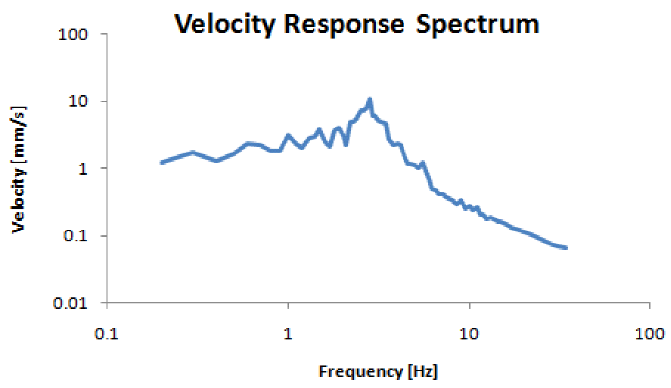
- We can easily convert between acceleration, velocity, and displacement spectra by multiplying or dividing by the frequency.
  - remember to convert frequency units; ω rad/s = 2πf Hz

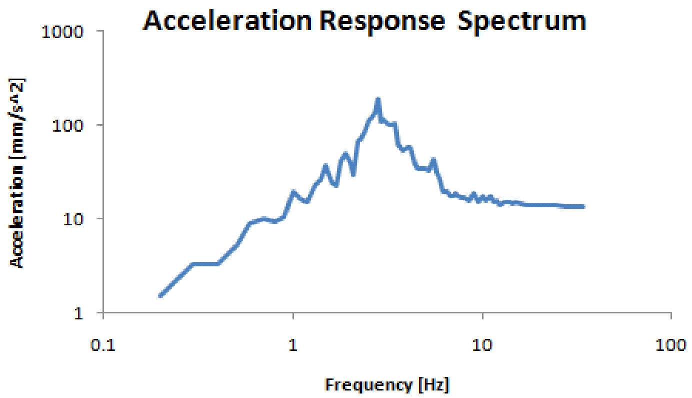
$$S_d = S_v/(2\pi f) = S_a/(2\pi f)^2$$

$$S_v = S_d(2\pi f) = S_a/(2\pi f)$$

$$S_a = S_d(2\pi f)^2 = S_v(2\pi f)$$







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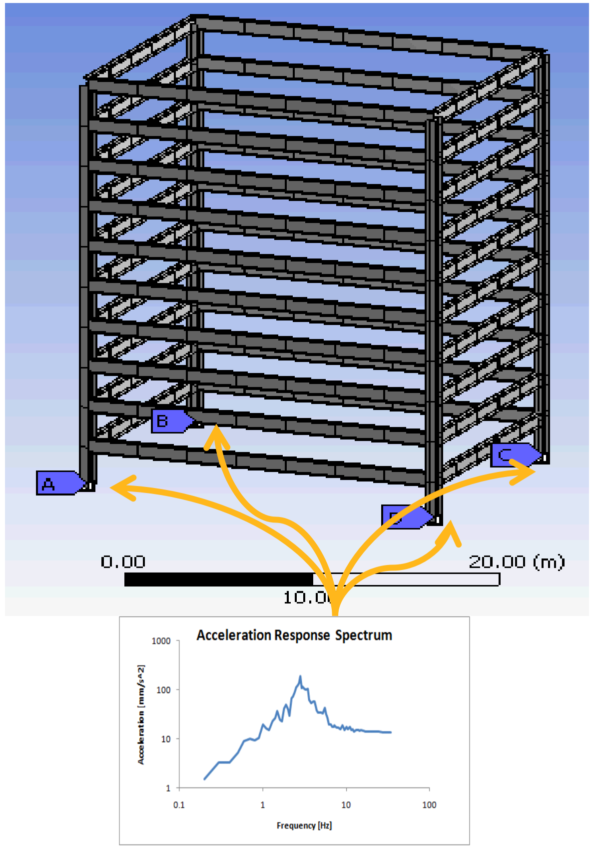
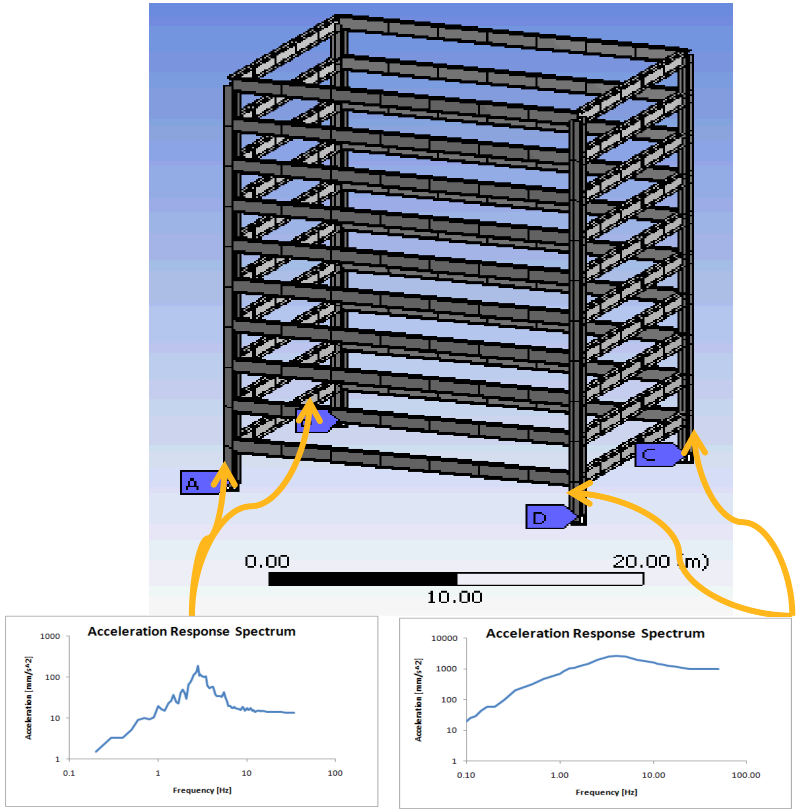
C. Types of Analyses

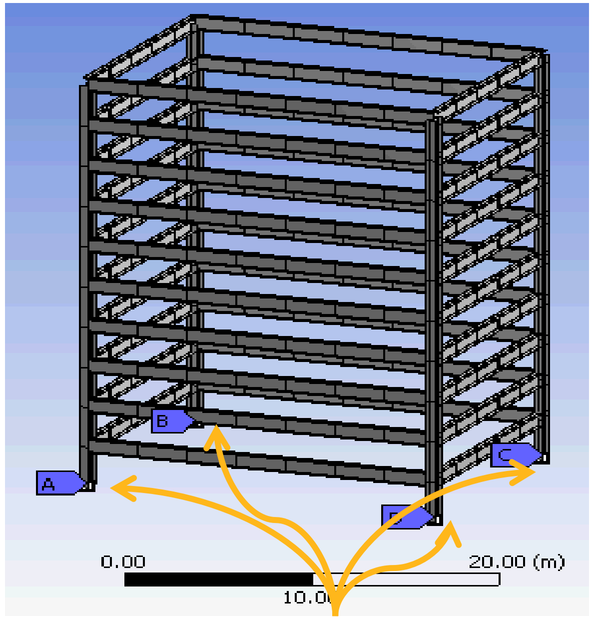
- There are two types of Response Spectrum Analysis available:
  - <u>Single-Point Response Spectrum (SPRS)</u>
  - <u>Multi-Point Response Spectrum (MPRS)</u>

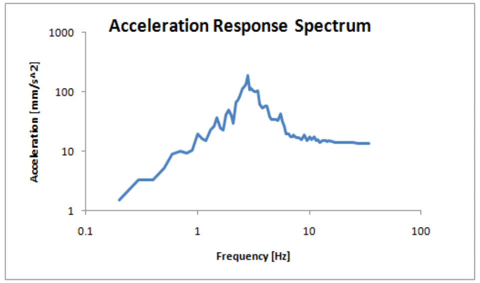




D. Single Point Response Spectrum (SPRS) Analysis

- The structure is excited by a spectrum of:
  - known direction and frequency components,
  - acting uniformly on all support points.
- Application examples:
  - Nuclear power plant buildings and components, for seismic loading
  - Airborne Electronic equipment for shock loading
  - Commercial buildings in earthquake zones
- The structure is linear (i.e., constant stiffness and mass).





… Single Point Response Spectrum (SPRS) Analysis

1. Participation Factor, γ

- Recall from the Modal Analysis Chapter:
  - The participation factor γ is a measure of the response of the structure at a given natural frequency.
  - γ represents how much each mode will contribute to the deflections and stresses in a particular direction.

Modal Analysis

$$\left(-\omega_i^2[M]+[K]\right)\{\varphi\}_i=\{0\}$$

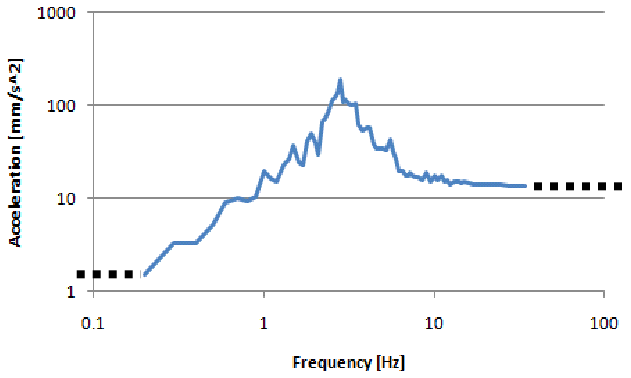
$$\gamma_i=\{\varphi\}_i^T[M]\{D\}$$

mode	frequency	mode shape		participation factor		
1	ω_1	$\{\phi\}_1$		γ_1		
2	ω_2	$\{\phi\}_2$		γ_2		
3	ω_3	$\{\phi\}_3$		γ_3		
⋮	⋮	⋮		⋮		

… Single Point Response Spectrum (SPRS) Analysis

2. Spectrum Values, S

- For each ω, the spectrum value S can be determined by a simple look-up from the response-spectrum table.
- Log-log interpolation is done for modal frequencies between spectrum points.
- No extrapolation is done for frequencies outside of the spectrum range; i.e. the value at the closest point is used.



mode	frequency	mode shape	spectrum value	participation factor		
1	ω_1	$\{\phi\}_1$	S_1	γ_1		
2	ω_2	$\{\phi\}_2$	S_2	γ_2		
3	ω_3	$\{\phi\}_3$	S_3	γ_3		
⋮	⋮	⋮	⋮	⋮		

… Single Point Response Spectrum (SPRS) Analysis

3. Mode Coefficients, A

- The mode coefficient A_i is defined as the *amplification factor* that is multiplied by the *eigenvector* to give the *actual displacement* in each mode.
- A_i can be determined from the *participation factors* and the *spectrum values*, depending on the type of spectrum input.

displacement	velocity	acceleration
$A_i = S_i\gamma_i$	$A_i = \dfrac{S_i\gamma_i}{\omega_i}$	$A_i = \dfrac{S_i\gamma_i}{\omega_i^2}$

- *Recall: participation factors measure the amount of mass moving in each direction for a unit displacement.*

mode	frequency	mode shape	spectrum value	participation factor	mode coefficient	
1	ω_1	$\{\phi\}_1$	S_1	γ_1	A_1	
2	ω_2	$\{\phi\}_2$	S_2	γ_2	A_2	
3	ω_3	$\{\phi\}_3$	S_3	γ_3	A_3	
⋮	⋮	⋮	⋮	⋮	⋮	

… Single Point Response Spectrum (SPRS) Analysis

4. Response, R

- The response (displacement, velocity or acceleration) for each mode can then be computed from the frequency, mode coefficient, and mode shape.

$$\{R\}_i = A_i\{\varphi\}_i \qquad \text{for displacement response}$$
$$\{R\}_i = \omega_i A_i\{\varphi\}_i \qquad \text{for velocity response}$$
$$\{R\}_i = \omega_i^2 A_i\{\varphi\}_i \qquad \text{for acceleration response}$$

- If there is more than one significant mode, the response for each mode must be combined using some method.

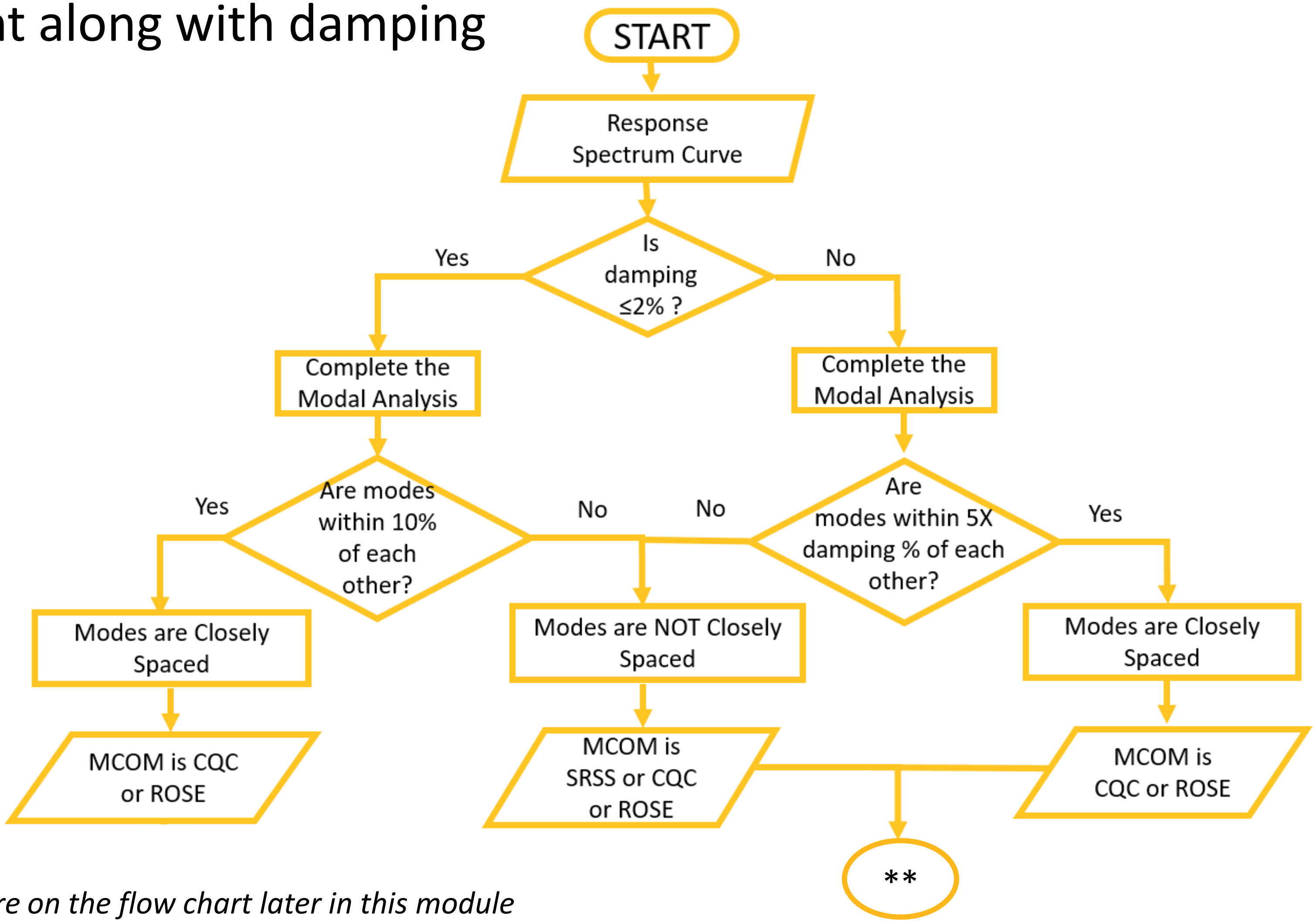
mode	+	mode shape	spectrum value	participation factor	mode coefficient	response
1	ω_1	$\{\phi\}_1$	S_1	γ_1	A_1	$\{R\}_1$
2	ω_2	$\{\phi\}_2$	S_2	γ_2	A_2	$\{R\}_2$
3	ω_3	$\{\phi\}_3$	S_3	γ_3	A_3	$\{R\}_3$
⋮	⋮	⋮	⋮	⋮	⋮	⋮

E. Mode Combination Methods

- The response spectrum analysis calculates the maximum displacement/stress response in the structure (i.e., individual maximum modal responses of the system).
- It is not directly known how much of these maxima will combine to give an actual total response (i.e., phasing of the modes).
- It is unlikely that all the modal maxima will act simultaneously and be of the same sign.
- To account for these unknowns, various mode combination methods are used…

… Mode Combination Methods

- While choosing a mode combination method, the relative spacing of one mode to another is important along with damping



** MCOM is Mode Combination*

*** More on the flow chart later in this module*

… Mode Combination Methods

1. Square Root of the Sum of the Squares (SRSS) Method
2. Complete Quadratic Combination (CQC) method
3. Rosenblueth (ROSE) method

The SRSS Method is the default, but there are certain scenarios in which it must be modified. That method is discussed, and those scenarios will be covered in the following slides.

… Mode Combination Methods

1. Square Root of the Sum of the Squares (SRSS) Method

- One straightforward method for calculating the combined response is to find the square root of the sum of the squares (SRSS).

$$\{R\} = \sqrt{\{R\}_1^2 + \{R\}_2^2 + \cdots + \{R\}_N^2} = \sqrt{\sum_{i=1}^{N} \{R\}_i^2}$$

- For modes with sufficiently separated frequencies, the SRSS is approximately the probable maximum value of response
- SRSS is also known as the Goodman-Rosenblueth-Newmark rule

mode	frequency	mode shape	spectrum value	participation factor	mode coefficient	response
1	ω_1	$\{\phi\}_1$	S_1	γ_1	A_1	$\{R\}_1$
2	ω_2	$\{\phi\}_2$	S_2	γ_2	A_2	$\{R\}_2$
3	ω_3	$\{\phi\}_3$	S_3	γ_3	A_3	$\{R\}_3$
⋮	⋮	⋮	⋮	⋮	⋮	⋮

… Mode Combination Methods

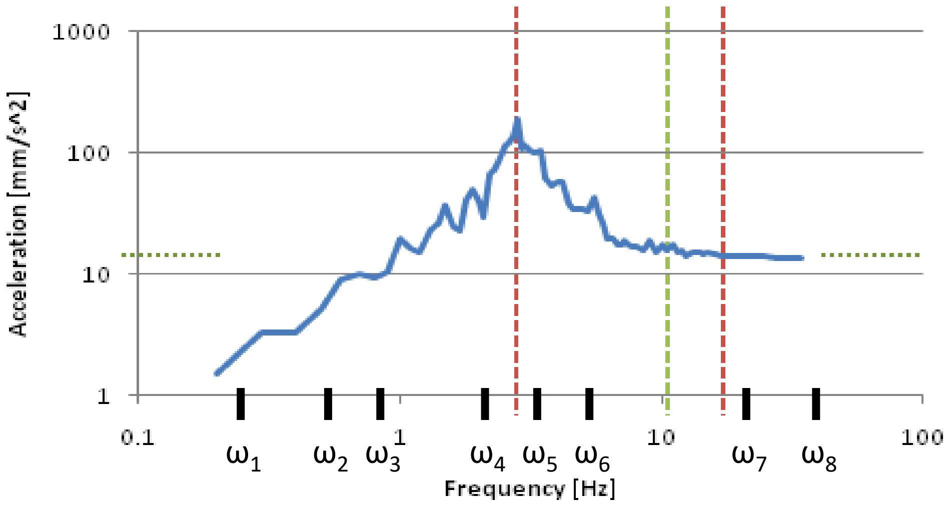
… Square Root of the Sum of the Squares (SRSS) Method

- There are circumstances in which this SRSS rule must be modified:
  - Accounting for modes with *closely-spaced* natural frequencies.
  - Adjusting for modes with partially- or fully-*rigid* responses.
  - Including *high-frequency* mode effects without extracting all modes.

… Mode Combination Methods

2. Sufficiently-Spaced Modes

- If modes are *sufficiently spaced*, the modes are uncorrelated.



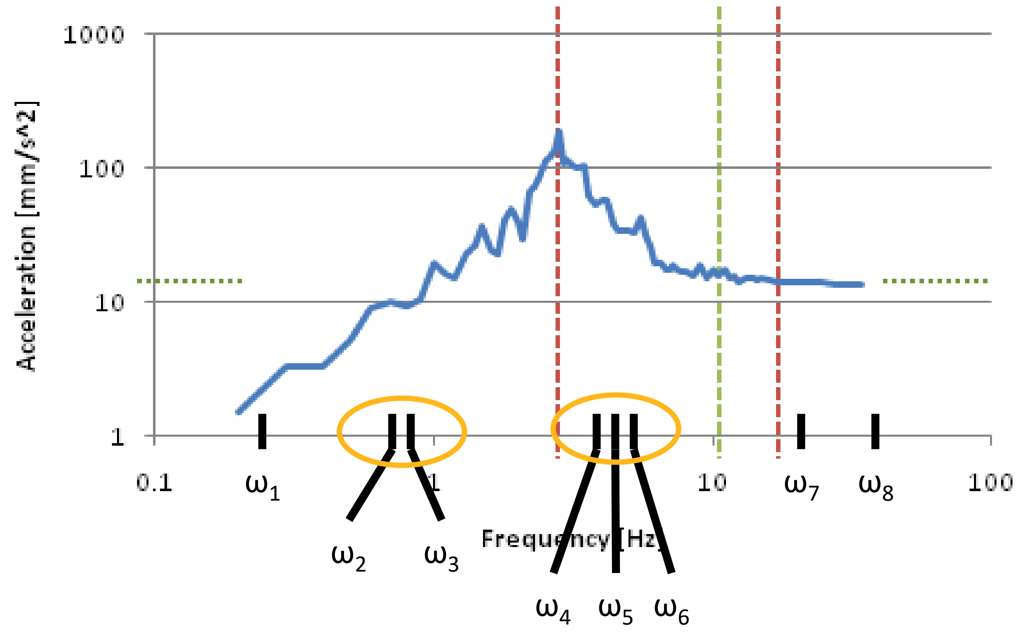
- It is valid to combine the responses using the SRSS method.

$$\{R\} = \sqrt{\sum_{i=1}^{N} \{R\}_i^2}$$

… Mode Combination Methods

3. <u>Closely-Spaced Modes (Correlated)</u>

- If modes with *closely-spaced* frequencies exist, the SRSS method is not applicable.
- Modes with closely spaced frequencies become *correlated*.



… Mode Combination Methods

… Closely-Spaced Modes (Correlated)

- The definition of modes with closely spaced frequencies is a function of the critical damping ratio:
- For critical damping ratios ≤ 2%
  - modes are considered closely spaced if the frequencies are within 10% of each other
    - Example, for $f_i < f_j$, f_i and f_j are closely-spaced if $f_j \le 1.1\ f_i$
- For critical damping ratios > 2%
  - modes are considered closely spaced if the frequencies are within five times the critical damping ratio of each other
    - Example: for $f_i < f_j$ and 5% damping, f_i and f_j are closely-spaced if $f_j \le 1.25\ f_i$
    - Example: for $f_i < f_j$ and 10% damping, f_i and f_j are closely-spaced if $f_j \le 1.5\ f_i$

… Mode Combination Methods

… Closely-Spaced Modes (Correlated)

- Idea: come up with a coefficient (ε) to determine the amount of correlation between modes.

$$0.0 \le \varepsilon \le 1.0$$

$\varepsilon = 0$:	uncorrelated
$\varepsilon = 1$:	fully correlated
$0.0 < \varepsilon < 1.0$:	partially correlated

- Two methods of combination can be used:
  1. Complete Quadratic Combination (CQC) method
  2. Rosenblueth (ROSE) method
- Each method has a formula for the correlation coefficient, ε, which
  1. is based on the frequency and damping of modes i and j
  2. is designed to vary between 1 (fully correlated) and 0 (uncorrelated)

… Mode Combination Methods

1. Square Root of the Sum of Squares (SRSS) Method

$$\{R\} = \left(\sum_{i=1}^{N} \{R\}_i^2 \right)^{\frac{1}{2}}$$

2. Complete Quadratic Combination (CQC) method

$$\{R\} = \left(\left| \sum_{i=1}^{N} \sum_{j=i}^{N} k\varepsilon_{ij} \{R\}_i \{R\}_j \right| \right)^{\frac{1}{2}}$$

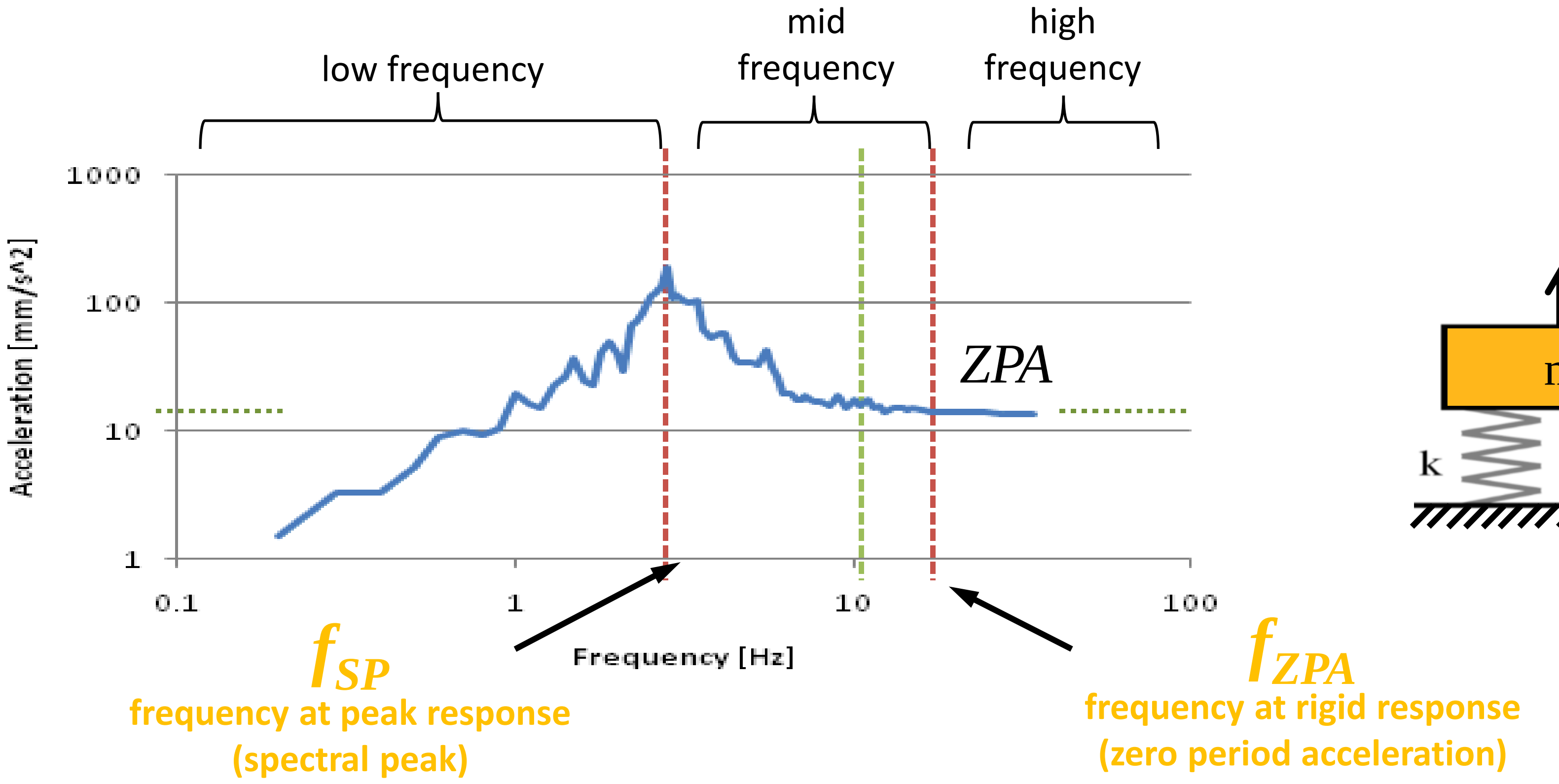
Additional guidance on choosing a mode combination method will be provided in the "Recommendations" section.

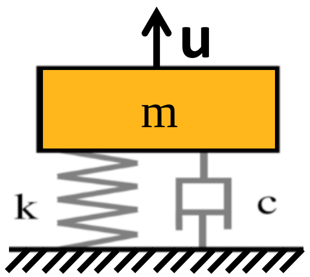
3. Rosenblueth (ROSE) method

$$\{R\} = \left(\sum_{i=1}^{N} \sum_{j=1}^{N} \varepsilon_{ij} \{R\}_i \{R\}_j \right)^{\frac{1}{2}}$$

F. Rigid Response

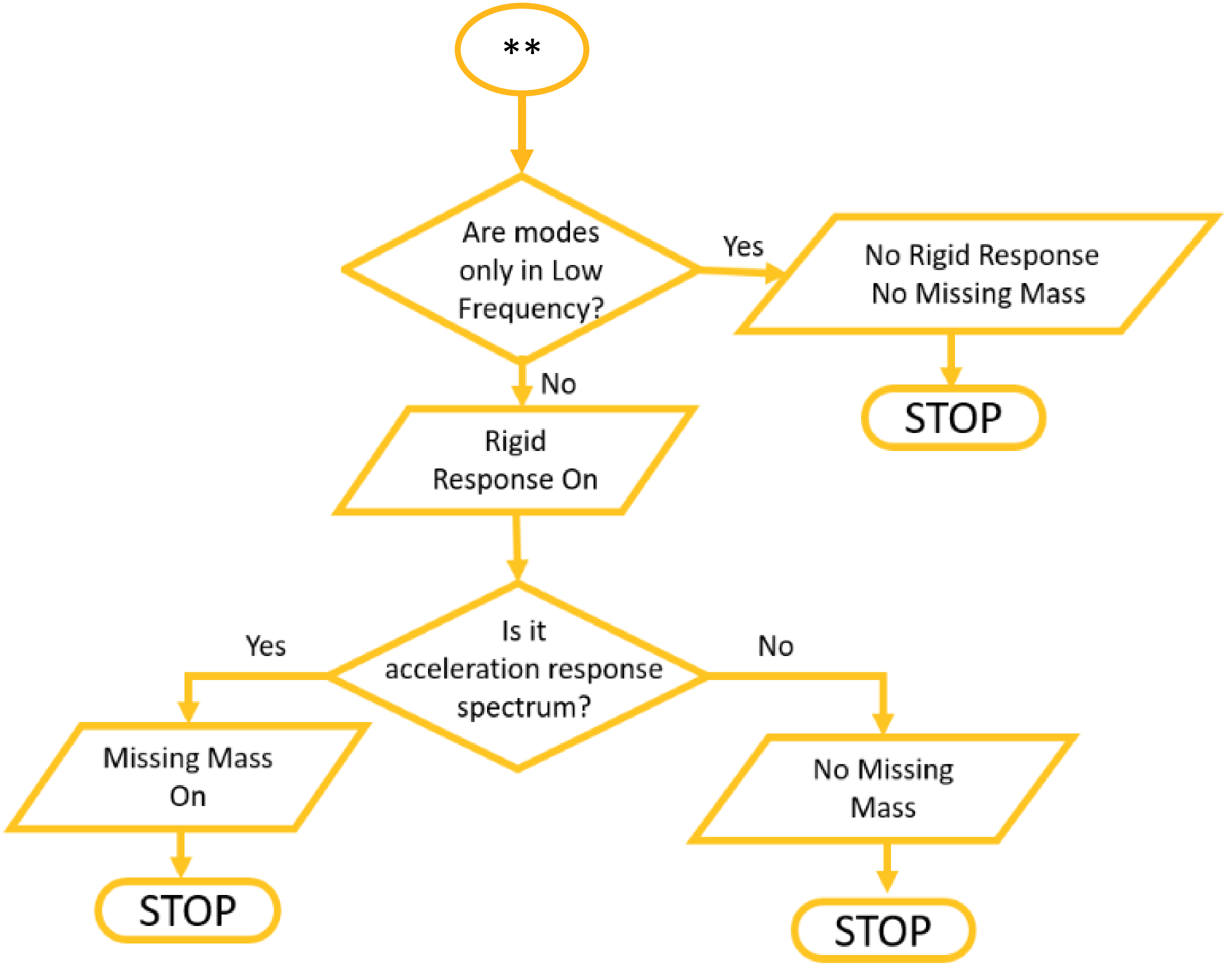
- A typical response spectrum often exhibits different spectral regions, identifiable by two frequencies:
  - f_{SP} (frequency at peak response: Spectral Peak)
  - f_{ZPA} (frequency at rigid response: Zero Period Acceleration, i.e., the horizontal portion of the spectra graph)





… Rigid Response

- This flow chart explains the conditions to choose a rigid response or the missing mass effect.



*** In continuation with the earlier flowchart*

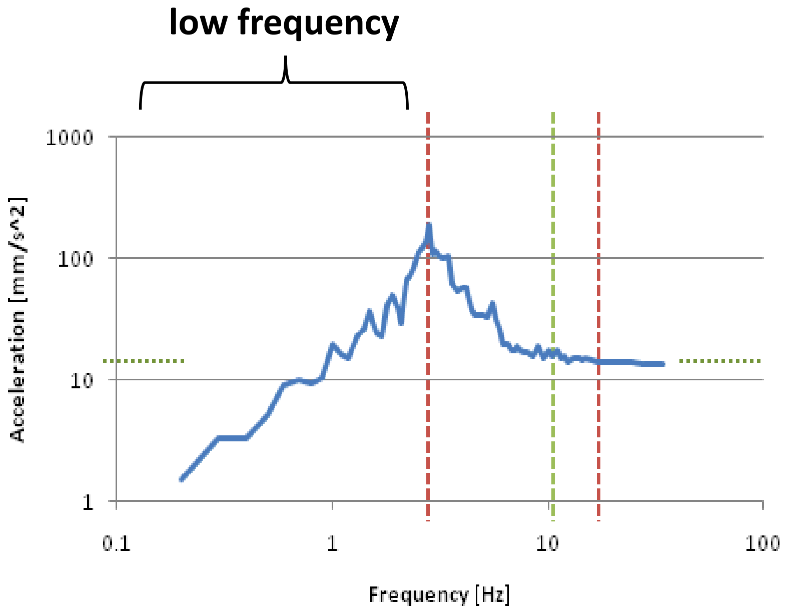
… Rigid Response

1. Low frequency (below f_{SP})
   - periodic region
   - modes generally uncorrelated (periodic) unless closely spaced
2. High Frequency (above f_{ZPA})
   - rigid region
   - modes correlated with input frequency and, therefore, also with themselves
3. Mid frequency (between f_{SP} and f_{ZPA})
   - transition from periodic to rigid
   - modes have periodic component and rigid component

… Rigid Response

1. <u>Low frequency (below f_{SP})</u>

- In the low-frequency range, the periodic responses predominate.



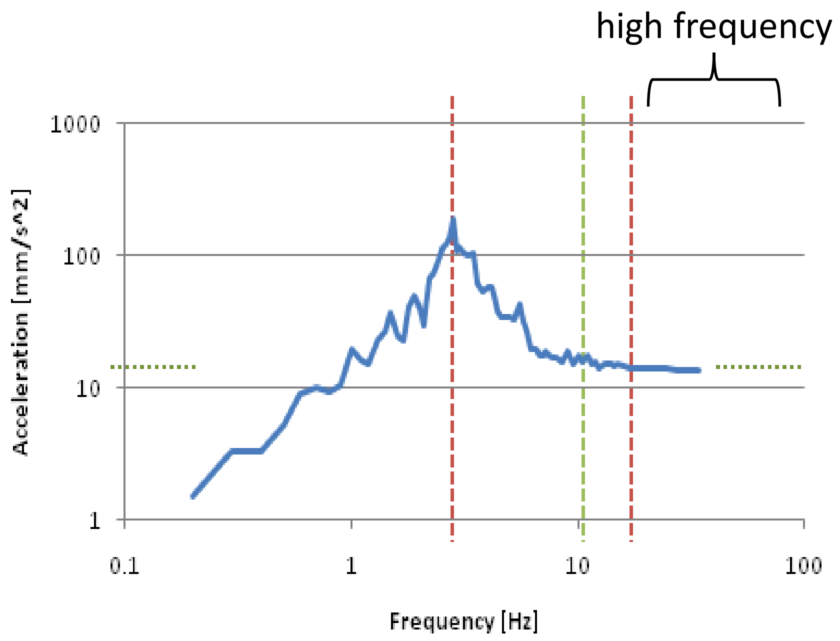
- The responses are uncorrelated (unless closely spaced) and can be combined using the SRSS, CQC, or ROSE methods.

$$\{R_p\} = \left(\sum_{i=1}^{N} \sum_{j=1}^{N} \varepsilon_{ij} \{R_p\}_i \{R_p\}_j \right)^{\frac{1}{2}}$$

… Rigid Response

2. High Frequency (above f_{ZPA})

- In the high-frequency range, the rigid responses predominate.



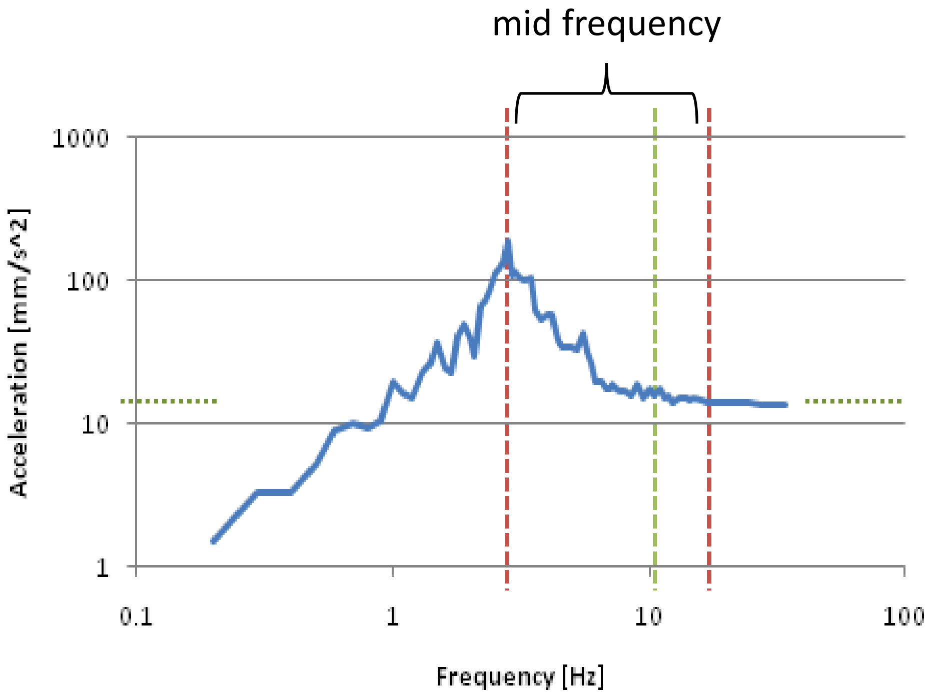
- The responses are fully correlated (with the input frequency and consequently with themselves) and can therefore be combined algebraically, as follows:

$$\{R_r\} = \sum_{i=1}^{N} \{R_r\}_i$$

… Rigid Response

3. <u>Mid frequency (between f_{SP} and f_{ZPA})</u>

- In the mid-frequency range, the modal responses consist of
  - periodic components, and
  - rigid components.



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… Rigid Response

… Mid frequency (between f_{SP} and f_{ZPA})

- Idea: come up with a coefficient (α) to split the response into a periodic component and a rigid component.

0.0 ≤ ε ≤ 1.0

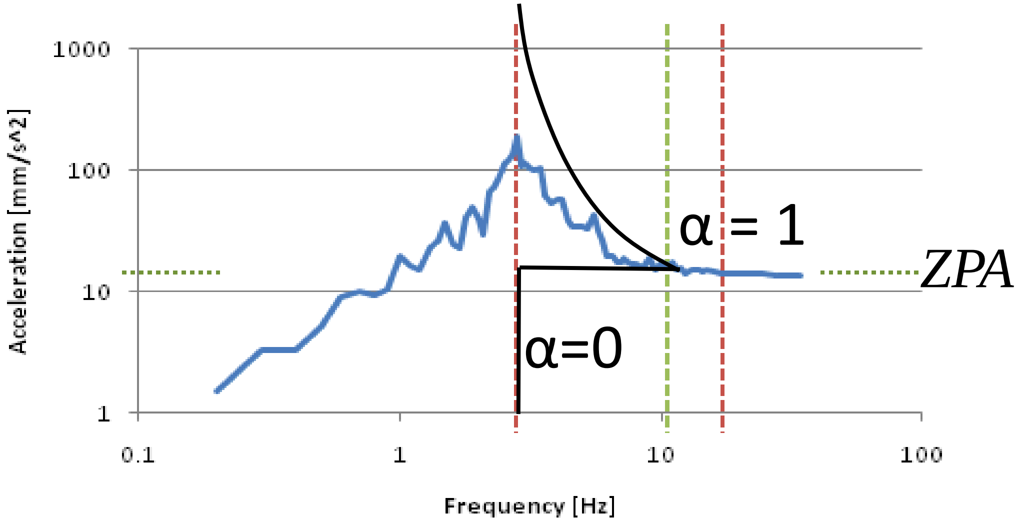
α = 0:	periodic
α = 1:	rigid
0.0 < α < 1.0:	part periodic, part rigid

- Methods to Calculate α:
  1. Lindley-Yow method
  2. Gupta method

… Rigid Response

… Mid frequency (between f_{SP} and f_{ZPA})

- Lindley-Yow method, α = α(S_a)



$$\alpha_i = \frac{ZPA}{S_{a_i}}$$

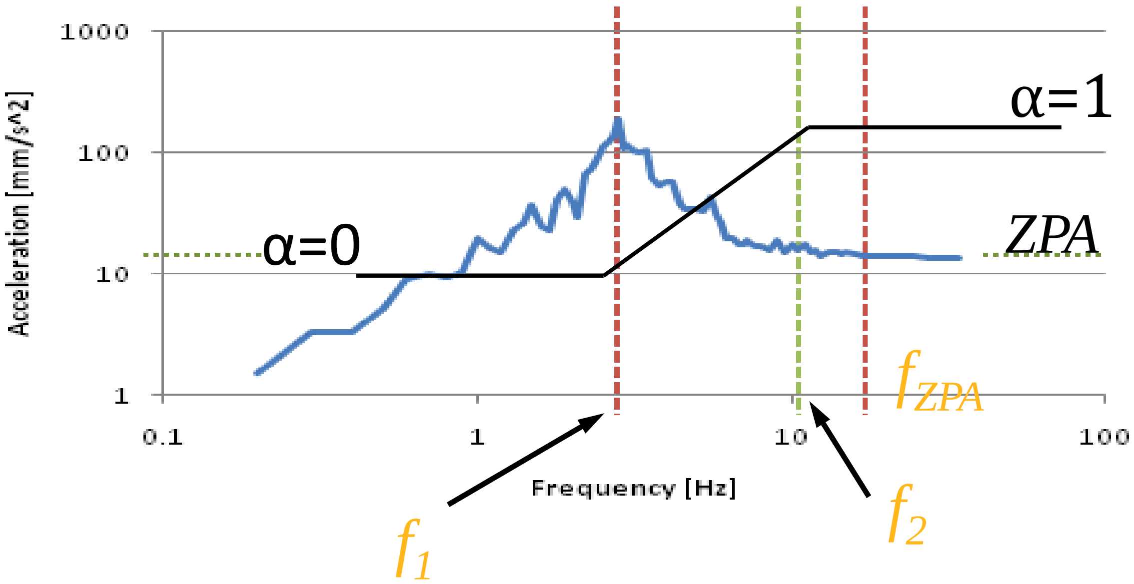
ZPA: acceleration at zero period
S_{ai}: acceleration the i^{th} frequency

- The rigid response coefficient, α,
  - attains a minimum value at $S_{a,max}$
  - increases for decreasing S_a
  - attains a maximum value of 1 at $S_a = ZPA$
  - is set to zero where $S_a < ZPA$

… Rigid Response

… Mid frequency (between f_{SP} and f_{ZPA})

- Gupta method, α = α(f)



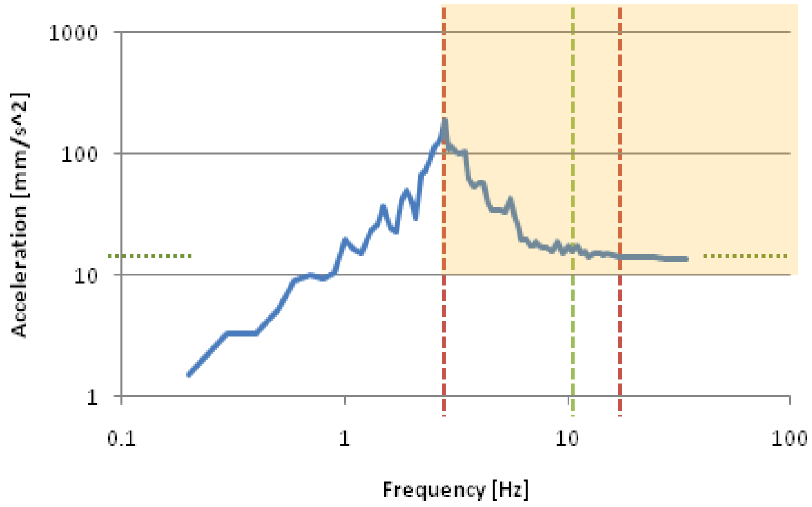
$$f_1 = \frac{S_{a,\max}}{2\pi S_{v,\max}}$$

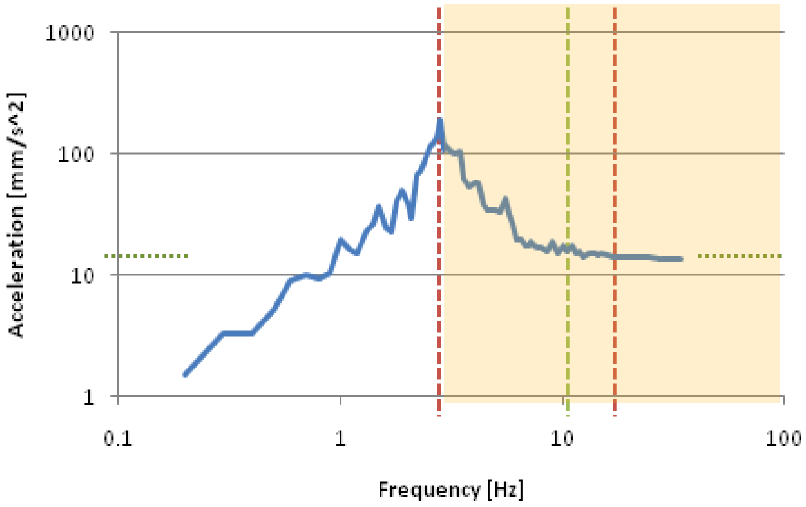
$$f_2 = \frac{\left(f_1 + 2 f_{ZPA}\right)}{3}$$

$$\alpha_i = \begin{cases} 0 & \text{for} \quad f_i \le f_1 \\ \dfrac{\ln\left(f_i / f_1\right)}{\ln\left(f_2 / f_1\right)} & \text{for} \quad f_1 \le f_i \le f_2 \\ 1 & \text{for} \quad f_i \ge f_2 \end{cases}$$

- The rigid response coefficient, α,
  - attains a minimum value of 0 at $f \le f_1$
  - increases linearly in log-log space between f_1 and f_2
  - attains a maximum value of 1 at $f \ge f_2$

… Rigid Response



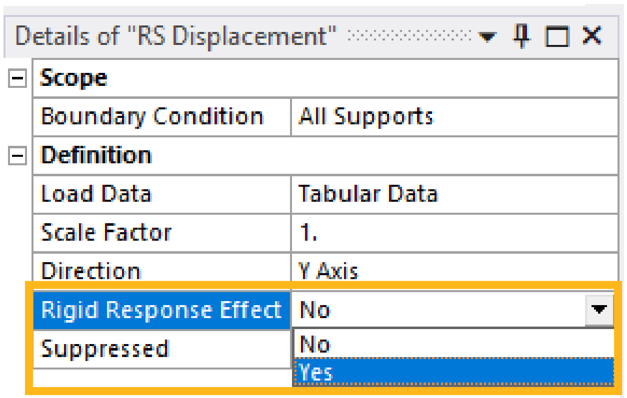


- Lindley-Yow method
  - Affects all modes with $S_a \geq ZPA$.
  - Should not be used on modes with $f < f_{SP}$.
- Gupta method
  - Affects all modes with $f > f_1$.

… Rigid Response

4. Rigid Response Calculations can be activated within the Details view of the RS Displacement/Velocity/Accel.

- The individual modal responses are calculated as before.
  - With rigid response turned on, these modes are not combined directly.
  - The responses will be split into periodic and rigid components.



mode	frequency	spectrum value	response			
1	ω_1	S_1	$\{R\}_1$			
2	ω_2	S_2	$\{R\}_2$			
3	ω_3	S_3	$\{R\}_3$			
⋮	⋮	⋮	⋮			

... Rigid Response

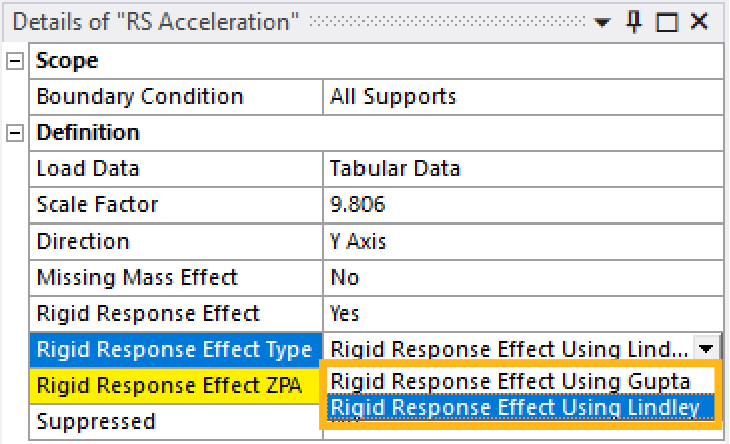
... Rigid Response Calculations

- The rigid response coefficient is calculated according to the method selected.

$$\text{Lindley–Yow: } \alpha_i = \frac{ZPA}{S_{a_i}}$$

$$\text{Gupta: } \alpha_i = \frac{\ln(f_i/f_1)}{\ln(f_2/f_1)}$$

$$0.0 \leq \alpha_i \leq 1.0$$



mode	frequency	spectrum value	response	rigid response coefficient		
1	ω_1	S_1	$\{R\}_1$	α_1		
2	ω_2	S_2	$\{R\}_2$	α_2		
3	ω_3	S_3	$\{R\}_3$	α_3		
⋮	⋮	⋮	⋮	⋮		

… Rigid Response

… Rigid Response Calculations

- The periodic and rigid components are calculated from the rigid response coefficient.

$$\{R_p\}_i = \sqrt{1-\alpha_i^2}\{R\}_i \qquad \text{periodic component}$$

$$\{R_r\}_i = \alpha_i\{R\}_i \qquad \text{rigid component}$$

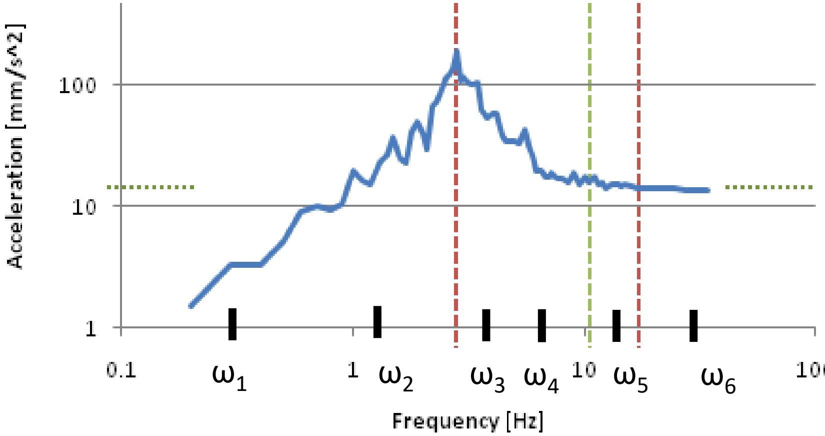
- As before, the response for each mode must be combined using some method. Now, the periodic modes and rigid modes will be treated separately.

mode	frequency	spectrum value	response	rigid response coefficient	periodic component	rigid component
1	ω_1	S_1	$\{R\}_1$	α_1	$\{R_p\}_1$	$\{R_r\}_1$
2	ω_2	S_2	$\{R\}_2$	α_2	$\{R_p\}_2$	$\{R_r\}_2$
3	ω_3	S_3	$\{R\}_3$	α_3	$\{R_p\}_3$	$\{R_r\}_3$
⋮	⋮	⋮	⋮	⋮	⋮	⋮

… Rigid Response

… Rigid Response Calculations

- Example: rigid response components using Gupta method.



mode	frequency	response	rigid response coefficient	periodic component	rigid component
1	0.5	$\{R\}_1$	0.0	$\{R\}_1$	{0}
2	1.4	$\{R\}_2$	0.0	$\{R\}_2$	{0}
3	3.6	$\{R\}_3$	0.18	0.98 $\{R\}_3$	0.18 $\{R\}_3$
4	6.4	$\{R\}_4$	0.61	0.79 $\{R\}_4$	0.61 $\{R\}_4$
5	12	$\{R\}_5$	1.0	{0}	$\{R\}_5$
6	25	$\{R\}_6$	1.0	{0}	$\{R\}_6$

periodic (modes 1–2)
transition (modes 3–4)
rigid (modes 5–6)

… Rigid Response

… Rigid Response Calculations

- The periodic modes are combined according to the desired combination rule (SRSS, CQC, or ROSE).
  - *Recall: CQC or ROSE are used if closely spaced modes are present.*

SRSS

$$\{R_p\} = \left(\sum_{i=1}^{N} \{R_p\}_i^2\right)^{\frac{1}{2}}$$

CQC

$$\{R_p\} = \left(\left|\sum_{i=1}^{N}\sum_{j=1}^{N} k\varepsilon_{ij}\{R_p\}_i\{R_p\}_j\right|\right)^{\frac{1}{2}}$$

ROSE

$$\{R_p\} = \left(\sum_{i=1}^{N}\sum_{j=1}^{N} \varepsilon_{ij}\{R_p\}_i\{R_p\}_j\right)^{\frac{1}{2}}$$

mode	frequency	spectrum value	response	rigid response coefficient	periodic component	rigid component
1	ω_1	S_1	$\cancel{\{R\}_1}$	α_1	$\{R_p\}_1$	$\{R_r\}_1$
2	ω_2	S_2	$\cancel{\{R\}_2}$	α_2	$\{R_p\}_2$	$\{R_r\}_2$
3	ω_3	S_3	$\cancel{\{R\}_3}$	α_3	$\{R_p\}_3$	$\{R_r\}_3$
⋮	⋮	⋮	⋮	⋮	⋮	⋮

… Rigid Response

… Rigid Response Calculations

- The ***rigid modes*** are summed algebraically.

$$\{R_r\} = \sum_{i=1}^{N} \{R_r\}_i$$

mode	frequency	spectrum value	response	rigid response coefficient	periodic component	rigid component
1	ω_1	S_1	$\cancel{\{R\}_1}$	α_1	$\{R_p\}_1$	$\{R_r\}_1$
2	ω_2	S_2	$\cancel{\{R\}_2}$	α_2	$\{R_p\}_2$	$\{R_r\}_2$
3	ω_3	S_3	$\cancel{\{R\}_3}$	α_3	$\{R_p\}_3$	$\{R_r\}_3$
⋮	⋮	⋮	⋮	⋮	⋮	⋮

… Rigid Response

… Rigid Response Calculations

- The combined periodic and combined rigid modes are finally combined to give the total response.

$$\{R_t\} = \sqrt{\{R_r\}^2 + \{R_p\}^2}$$

mode	frequency	spectrum value	response	rigid response coefficient	periodic component	rigid component
1	ω_1	S_1	$\{R\}_1$	α_1	$\{R_p\}_1$	$\{R_r\}_1$
2	ω_2	S_2	$\{R\}_2$	α_2	$\{R_p\}_2$	$\{R_r\}_2$
3	ω_3	S_3	$\{R\}_3$	α_3	$\{R_p\}_3$	$\{R_r\}_3$
⋮	⋮	⋮	⋮	⋮	⋮	⋮

G. Missing Mass Response

- Recall: The Ratio of Effective Mass to Total Mass is provided in the Participation Factor Summary following a Modal analysis.

Details of "Solution Information"	
Solution Information	
Solution Output	Participation Factor Summary
Summary Type	All

- Generally, we cannot practically extract all possible modes to account for 100% of the mass of the structure.
  - Many structures may have important natural vibration modes at frequencies higher than the ZPA frequency, f_{ZPA}.

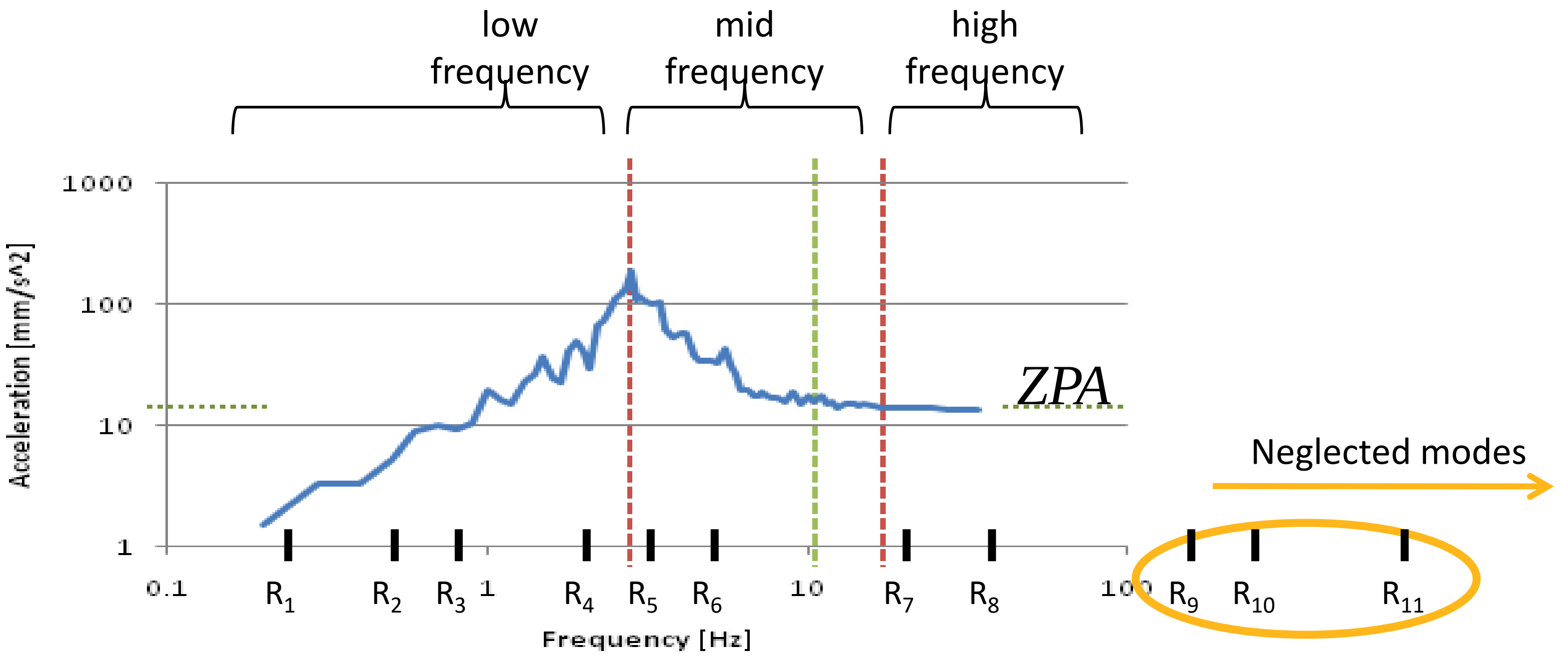
$$\gamma_i = \{\varphi\}_i^T [M]\{D\} \qquad M_{eff,i} = \gamma_i^2$$

Ratio of Effective Mass to Total Mass

Mode	Frequency [Hz]	X Direction	Y Direction	Z Direction
1	180.09	0.28052	2.2659e-002	5.6037e-005
2	208.39	3.5778e-003	0.49808	5.1925e-002
3	295.82	2.0599e-003	0.20983	0.16269
4	390.9	8.8629e-002	2.9912e-002	1.481e-003
5	441.91	0.41811	4.9148e-003	3.282e-004
6	533.38	5.6619e-003	2.6269e-002	2.3887e-003
7	764.9	1.2904e-005	3.3971e-004	2.6758e-002
8	816.39	4.2024e-006	7.1281e-005	9.1368e-007
9	919.42	5.4028e-005	1.0979e-003	1.5054e-003
10	1009.4	5.0605e-004	4.5757e-003	5.9962e-002
Sum		0.79914	0.79775	0.30709

… Missing Mass Response

- So far, we have addressed the low-, mid-, and high-frequency ranges of the spectrum.
  - We could have many modes extend far beyond f_{ZPA}.



Ansys

… Missing Mass Response

- Idea: if we can figure out how much mass is missing from the modal analysis, then we don't have to extract all modes above f_{ZPA}.
- Its effect can be lumped into an additional response

Ratio of Effective Mass to Total Mass

Mode	Frequency [Hz]	X Direction	Y Direction	Z Direction
1	180.09	0.28052	2.2659e-002	5.6037e-005
2	208.39	3.5778e-003	0.49808	5.1925e-002
3	295.82	2.0599e-003	0.20983	0.16269
4	390.9	8.8629e-002	2.9912e-002	1.481e-003
5	441.91	0.41811	4.9148e-003	3.282e-004
6	533.38	5.6619e-003	2.6269e-002	2.3887e-003
7	764.9	1.2904e-005	3.3971e-004	2.6758e-002
8	816.39	4.2024e-006	7.1281e-005	9.1368e-007
9	919.42	5.4028e-005	1.0979e-003	1.5054e-003
10	1009.4	5.0605e-004	4.5757e-003	5.9962e-002
Sum		0.79914	0.79775	0.30709

… Missing Mass Response

- Recall, above f_{ZPA}, the *acceleration response* is rigid (in-phase), so it can be fairly accurately represented by a static acceleration analysis.
    1. We can calculate what the total inertia force should be above f_{ZPA}.

$$\{F_T\} = -[M]\{D\}S_{ZPA}$$

    2. We can also calculate the inertia force contributed by each mode.

$$\{F_j\} = -[M]\{\varphi\}_j\gamma_j S_{ZPA}$$

    3. The sum of inertia force contributed by all modes is simply

$$\sum_{j=1}^{N}\{F_j\} = -\sum_{j=1}^{N}[M]\{\varphi\}_j\gamma_j S_{ZPA}$$

… Missing Mass Response

- The "missing" inertia force must simply be the difference between the total inertia force and the sum of modal inertia forces.

$$\{F_M\} = \{F_T\} - \sum_{j=1}^{N}\{F_j\} = [M]\left(\sum_{j=1}^{N}\{\varphi\}_j\gamma_j - \{D\}\right)S_{ZPA}$$

- The "missing mass response" is the static shape due to the missing inertia forces.

$$\{R_M\} = [K]^{-1}\{F_M\}$$

- The missing mass response is a *pseudo-static response* to an *acceleration base excitation*.

… Missing Mass Response

- If rigid response is included, then the missing mass is added to the rigid response.

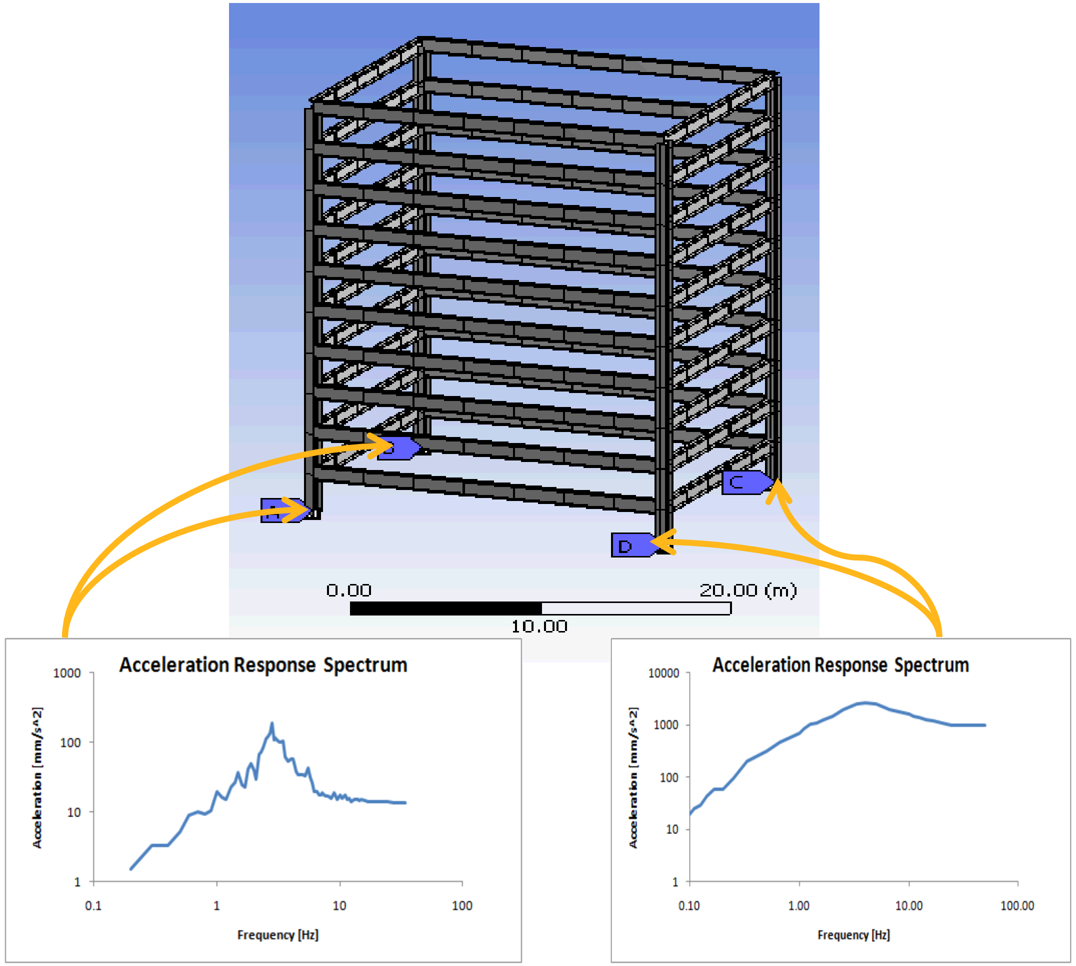
$$\{R_r\} = \sum_{i=1}^{N} \{R_r\}_i + \{R_M\}$$

- The total response is then calculated by combining the periodic and rigid components.

$$\{R\} = \sqrt{\{R_p\}^2 + \{R_r\}^2}$$

H. Multi-Point Response Spectrum (MPRS) Analysis

- In multi-point response spectrum analysis, different constrained points can be subjected to different spectra.
- Up to 100 different excitations are allowed.
- The structure is linear (i.e. constant stiffness and mass).



… Multi-Point Response Spectrum (MPRS) Analysis

- Each spectrum is treated individually first, so all of the previous information on single-point response spectrum applies
  - closely spaced modes (SRSS, CQC, ROSE)
  - rigid response (Lindley-Yow, Gupta)
  - missing mass
- Finally, the response of the structure is obtained by combining the responses to each spectrum using the SRSS method.

$$\{R_{MPRS}\} = \sqrt{\{R_{SPRS}\}_1^2 + \{R_{SPRS}\}_2^2 + \cdots}$$

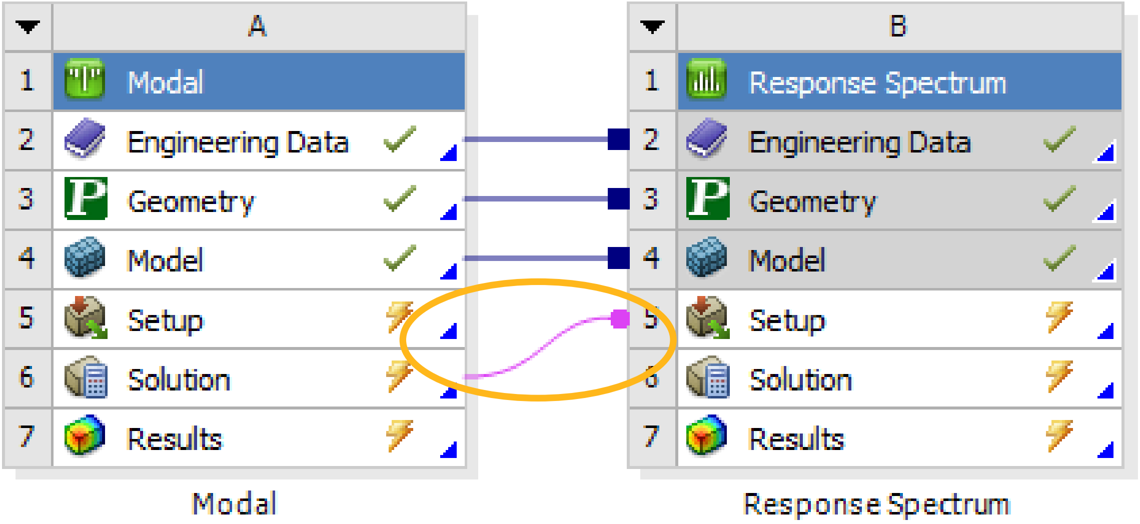
- where,

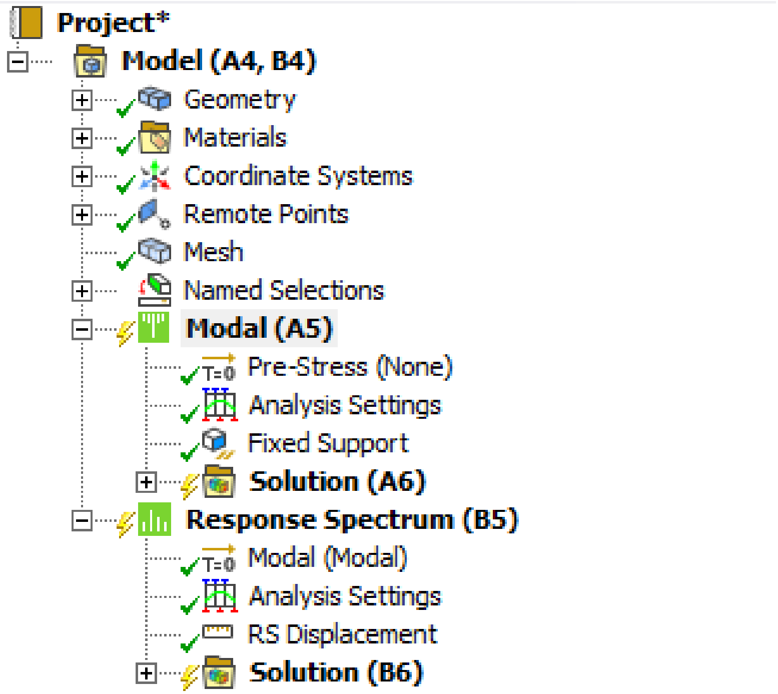
$\{R_{MPRS}\}$ is the total response of the MPRS analysis
$\{R_{SPRS}\}_1$ is the total response of the SPRS analysis for spectrum 1
$\{R_{SPRS}\}_2$ is the total response of the SPRS analysis for spectrum 2
Etc.

I. Recommendations

1. Setup

- Set up a response spectrum analysis in the schematic by linking a modal system to a response spectrum system *at the solution level*.

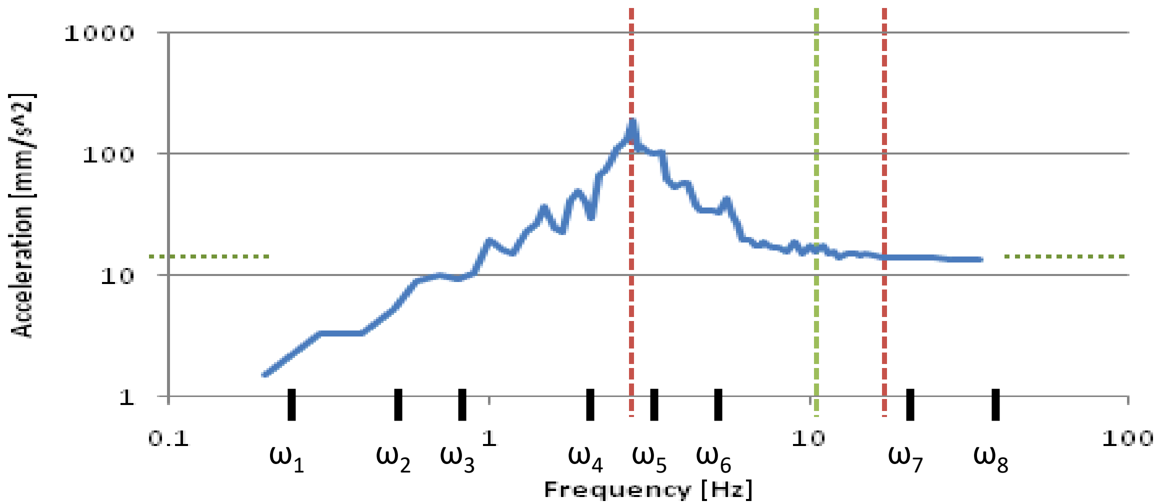




… Recommendations

2. Solution Procedure

- The recommended solution method is generally specified by your design code.
  - combination method
  - rigid response method
  - missing mass effects
- Alternatively, the best solution method can be determined by
  - extracting the modes to be used for combination and
  - comparing them to the response spectrum



Ansys

… Recommendations

… Solution Procedure

- Modes only in low-frequency region
  - SRSS (or CQC/ROSE for closely spaced modes).
  - No rigid response effects. No missing mass effects.
- Modes only in mid- to high-frequency region
  - SRSS (or CQC/ROSE for closely spaced modes).
  - Rigid response by Lindley or Gupta method. Missing mass on.
- Modes in all frequency regions
  - SRSS (or CQC/ROSE for closely spaced modes).
  - Rigid response by Gupta method. Missing mass on.

J. Analysis Settings

Points to Remember

- The excitation is applied in the form of a response spectrum:
  - displacement,
  - velocity, or
  - acceleration units.
- The excitation must be applied at *fixed degrees of freedom*.
- The response spectrum is calculated based on modal responses. A modal analysis is therefore a prerequisite.

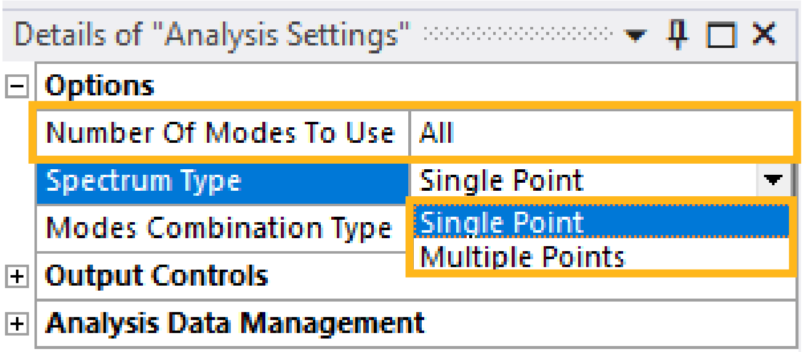
J. Analysis Settings

Points to Remember

- If response strains or stresses are of interest, then the calculation of modal strains and stresses must be performed in the modal analysis.
  - This will happen automatically when the Modal and Response Spectrum analysis systems are linked in the Project Schematic as shown previously; however, depending on the sequence of events leading up to combined solution of the linked systems, you may end up having to solve the Modal analysis twice. This situation can be prevented by setting “Future Analysis” to “MSUP Analyses” in the Analysis Settings Details of the Modal system before its first solution.
- The results are in terms of the *maximum response*.

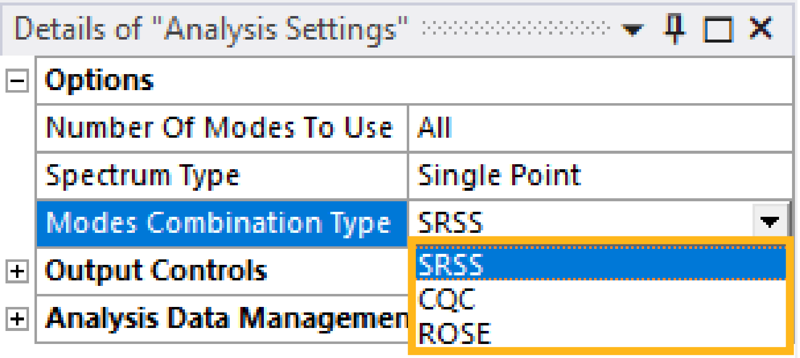
… Analysis Settings

- Analysis Settings > Options
  - Number of Modes To Use:
    - It is recommended to include all the modes whose frequencies span 1.5 times the maximum frequency defined in the input response spectrum.



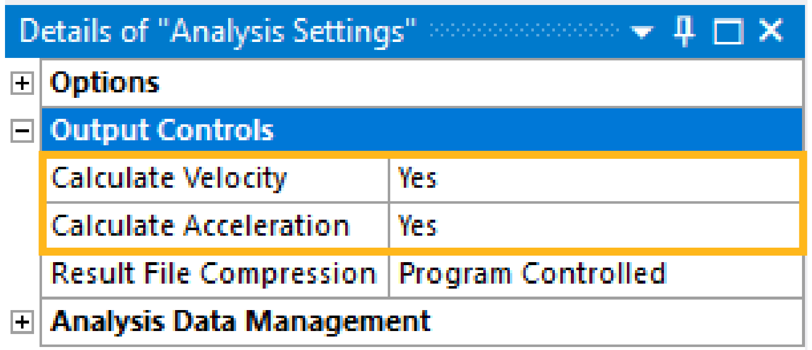
  - Spectrum Type:
    - Single Point
    - Multiple Points

  - Modes Combination Type:
    - SRSS method (more conservative),
    - CQC method
    - ROSE method



… Analysis Settings

- Analysis Settings > Output Controls
  - By default, only displacement responses are calculated.
  - To include velocity and/or acceleration responses, set their respective Output Controls to Yes.



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… Analysis Settings

- Analysis Settings > Damping
  - Like other Mode Superposition (MSUP) analyses, the damping matrix [C] for Response Spectrum is not calculated explicitly, but instead damping is defined directly in terms of a damping ratio ξ^d for mode i :

$$\xi_i^d = \xi + \xi_i^m + \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2}$$

ξ : constant modal damping ratio (DMPRAT)

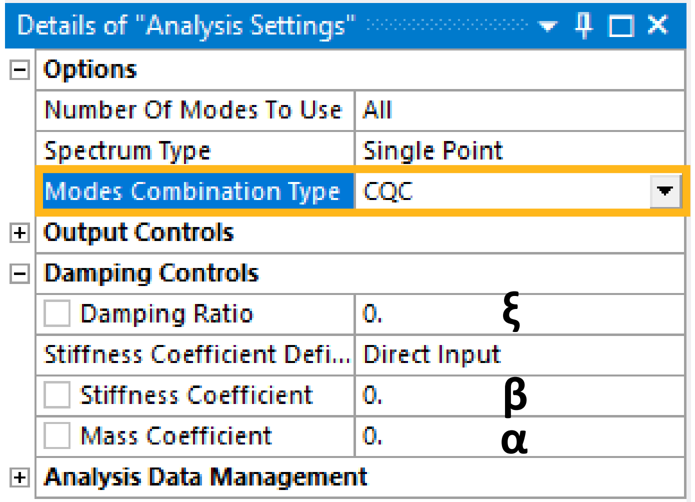
ξ_i^m: modal damping ratio for mode shape i (MP,DMPR during modal analysis)

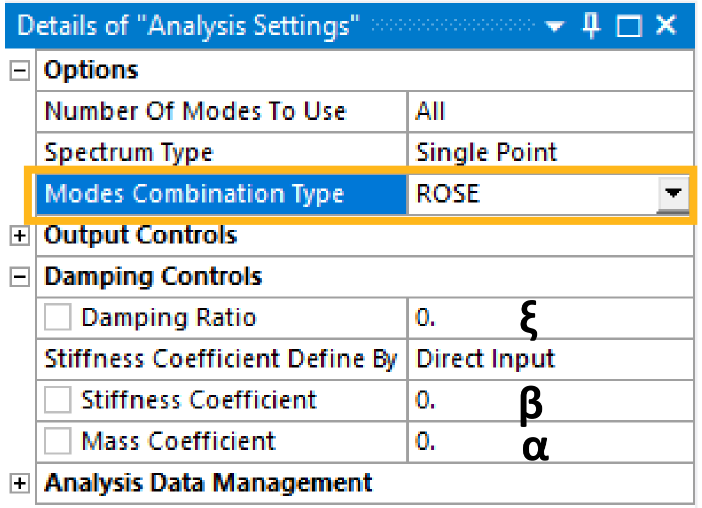
α : Global Mass matrix multiplier (alpha damping, ALPHAD)

β : Global k-Matrix Multiplier (beta damping, BETAD)

… Analysis Settings

- Analysis Settings > Damping
  - Recall from Section B that damping was considered during generation of the spectral input.
  - For SRSS, assigned damping values are used to interpolate between multiple spectral input curves, each defined at a different damping value.
    - *A current limitation exists in Mechanical whereby it's not possible to define multiple spectrum input curves; therefore there is no mechanism to define damping when the SRSS mode combination method is chosen.*
  - For CQC and ROSE combination methods, Constant Damping Ratio and Rayleigh damping constants α and β are available and are used to couple closely spaced modes (Section E).
    - Values for ξ, α, and β can be entered on a Global basis via the Damping Controls section of Analysis Settings:





… Analysis Settings

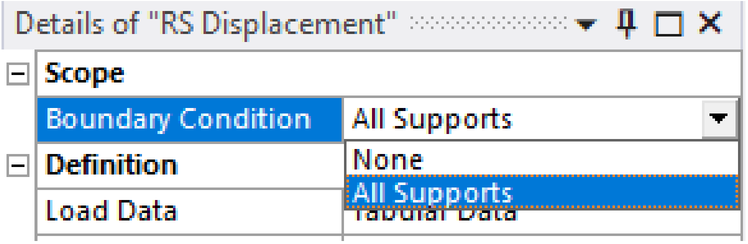
- The value for ξ^m is entered on a material basis within Engineering Data as part of the undamped Modal Analysis:

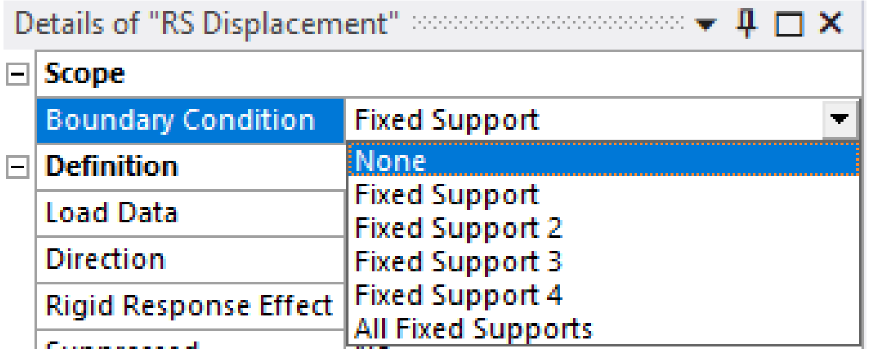
Properties of Outline Row 3: Structural Steel

	A	B
1	Property	Value
2	Material Field Variables	Table
3	Density	7850
4	Material Dependent Damping	
5	Damping Ratio	0.01 ξ^m
6	Constant Structural Damping Coefficient	= 0.02
7	Isotropic Elasticity	

K. Loads and Supports

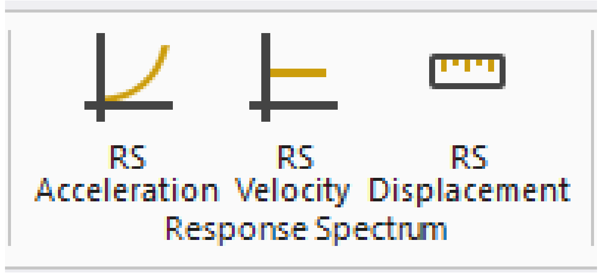
- Supported boundary condition types include:
  - fixed support,
  - displacement,
  - remote displacement, and
  - body-to-ground springs.
- For a Single Point spectrum type, input excitation spectrums are applied to all boundary condition types defined in the model.
- For Multiple Points however, each input excitation spectrum is associated to only one boundary condition type.





… Loads and Supports

- Three types of input excitation spectrum are supported:
  - displacement input excitation (RS Displacement),
  - velocity input excitation (RS Velocity), and
  - acceleration input excitation (RS Acceleration).

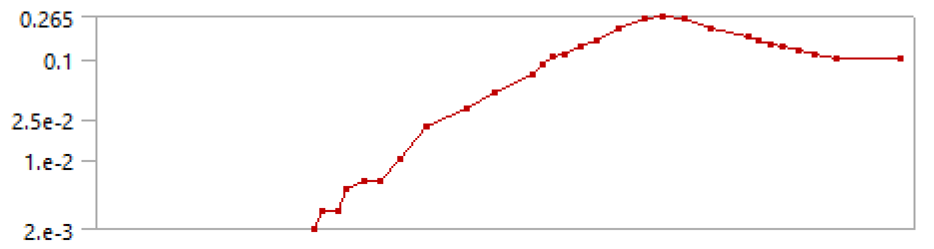


- For each spectrum value, there is one corresponding frequency.
  - Scale Factor may be applied to convert data; here, acceleration values in tabular data are supplied in g's then scaled into m/s^2 by the supplied scale factor.

Details of "RS Acceleration"

Scope	
Boundary Condition	All Supports
Definition	
Load Data	Tabular Data
Scale Factor	9.806
Direction	Y Axis
Missing Mass Effect	No
Rigid Response Effect	No
Suppressed	No

Graph



Messages Graph

Tabular Data

	Frequency [Hz]	Acceleration [(m/s^2)]
1	0.1	2.e-003
2	0.11	3.e-003
3	0.13	3.e-003
4	0.14	5.e-003
5	0.17	6.e-003
6	0.2	6.e-003
7	0.25	1.e-002
8	0.33	2.1e-002
9	0.5	3.2e-002
10	0.67	4.7e-002

Ansys

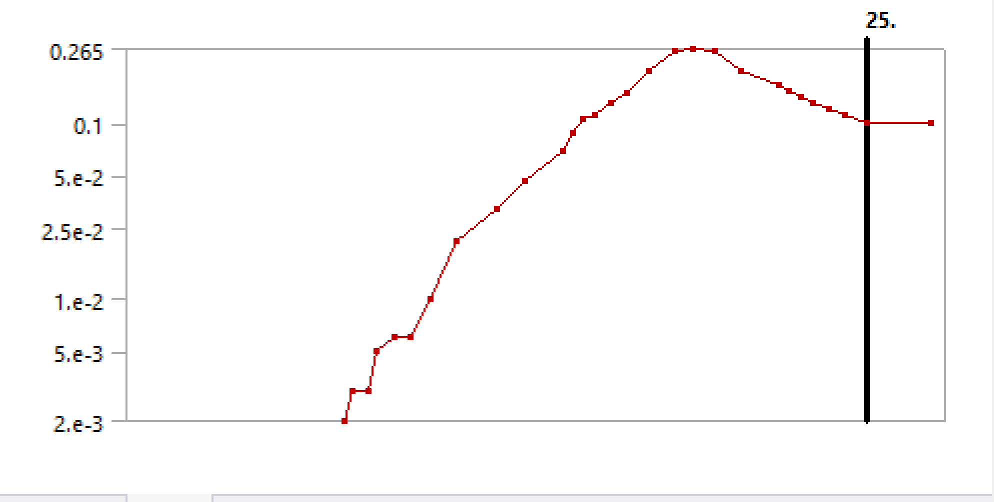
… Loads and Supports

- Missing Mass effect:
  - If set to YES, includes contribution of high frequency modes in the total response calculation.
  - Only applicable to RS Acceleration excitation.
  - ZPA (Zero Period Acceleration) needs to be defined.
    - Note that if a Scale Factor is used, it is also applied to the ZPA value

Details of "RS Acceleration"

Scope	
Boundary Condition	All Supports
Definition	
Load Data	Tabular Data
Scale Factor	9.806
Direction	Y Axis
Missing Mass Effect	Yes
Missing Mass Effect ZPA	0.1 m/s^2
Rigid Response Effect	No
Suppressed	No

Graph



Tabular Data

	Frequency [Hz]	Acceleration [(m/s^2)]
16	2.	0.15
17	2.5	0.2
18	3.33	0.255
19	4.	0.265
20	5.	0.255
21	6.67	0.2
22	10.	0.165
23	11.11	0.153
24	12.5	0.14
25	14.29	0.131
26	16.67	0.121
27	20.	0.111
28	25.	0.1
29	50.	0.1

… Loads and Supports

- Rigid Response Effect:
  - Specify option to include or not include rigid responses to the total response calculation by setting Rigid Response Effect to Yes or No.
  - The rigid responses normally occur in the frequency range that is lower than that of missing mass responses but is higher than that of periodic responses.
  - Applicable to RS Acceleration, RS Velocity, and RS Displacement
    - Gupta method requires f_1 and f_2
    - Lindley method requires ZPA (Scale factor applies to ZPA as well)

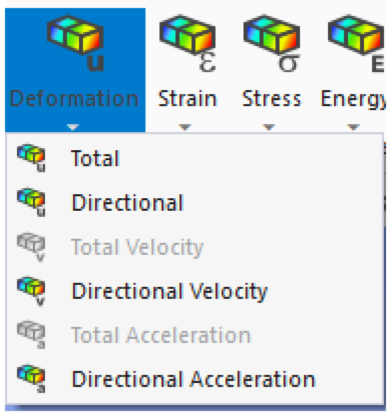
Details of "RS Acceleration"

Scope	
Boundary Condition	All Supports
Definition	
Load Data	Tabular Data
Scale Factor	9.806
Direction	Y Axis
Missing Mass Effect	Yes
Missing Mass Effect ZPA	0.1 m/s^2
Rigid Response Effect	Yes
Rigid Response Effect Type	Rigid Response Effect Using Gupta
Rigid Response Effect Freq Begin	4. Hz
Rigid Response Effect Freq End	18. Hz
Suppressed	No

Details of "RS Acceleration"

Scope	
Boundary Condition	All Supports
Definition	
Load Data	Tabular Data
Scale Factor	9.806
Direction	Y Axis
Missing Mass Effect	Yes
Missing Mass Effect ZPA	0.1 m/s^2
Rigid Response Effect	Yes
Rigid Response Effect Type	Rigid Response Effect Using Lindley
Rigid Response Effect ZPA	0.1 m/s^2
Suppressed	No

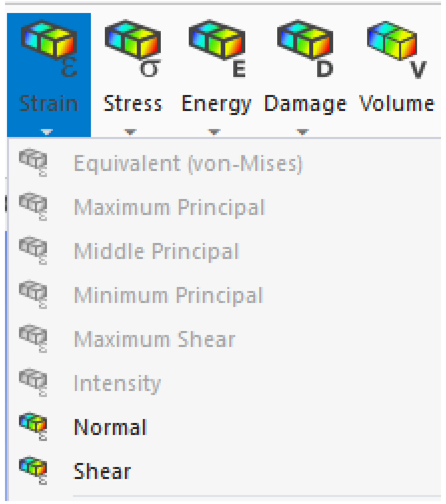
L. Results

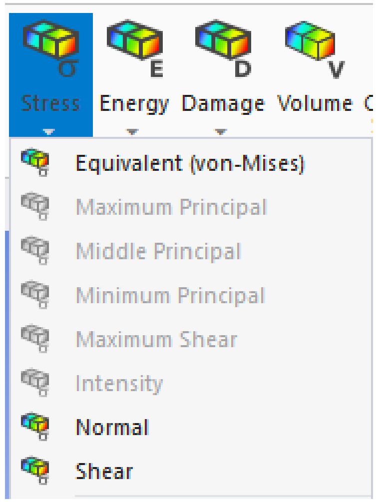


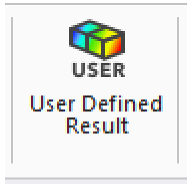
- Applicable Deformation results are:
  - Total, Directional (X,Y,Z)
  - Directional Velocity (provided it has been requested in Output Controls)
  - Directional Acceleration (provided it has been requested in Output Controls)

- If strain/stress are requested (during modal analysis), applicable results are normal strain and stress, shear strain and stress, and equivalent stress. Membrane and bending stress are also available for surface (shell) models.

- User-Defined Results are also available.



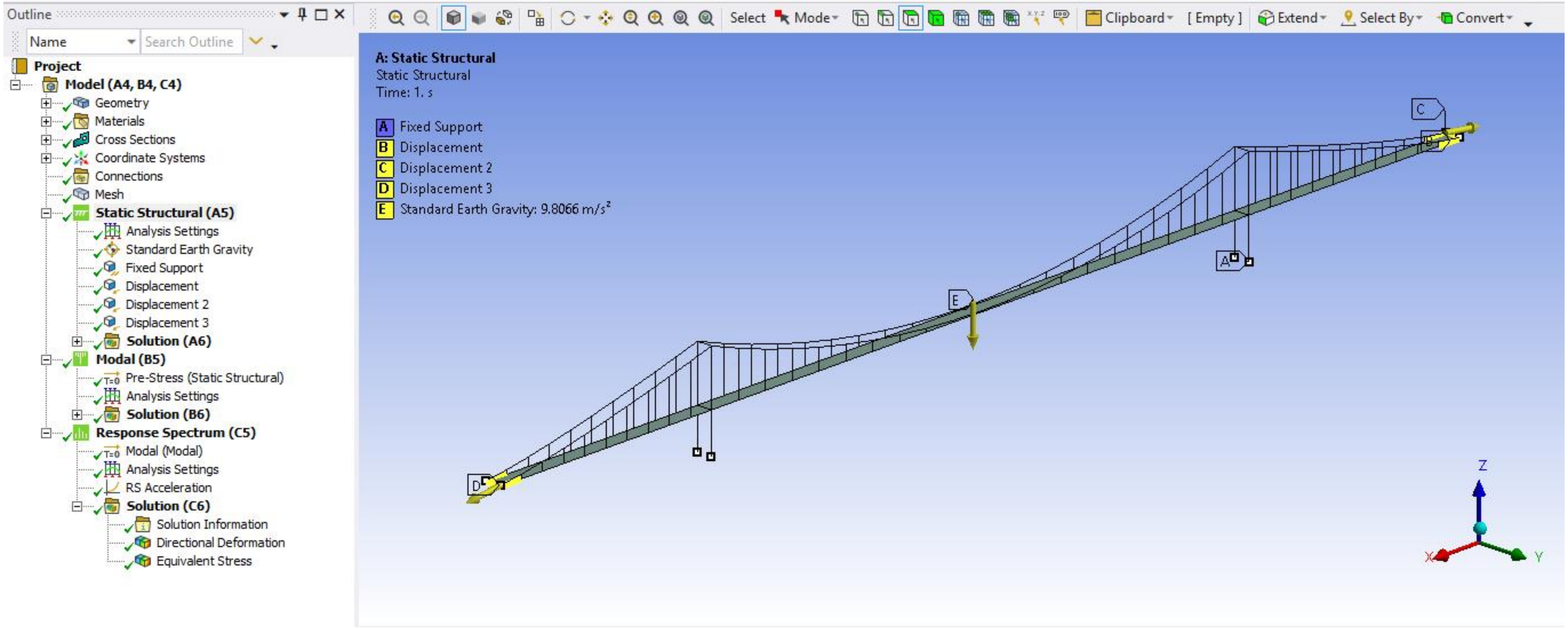




… Results

- To review reaction results, Force Reaction / Moment Reaction probes can be scoped to any boundary condition that was used to apply the base excitation. This includes Body-Ground Spring supports.
- Should be used carefully since reactions are summed by mode combinations and thus phase information will be partially lost.

Workshop 07.1: Suspension Bridge



Ansys

End of presentation