3.7 Inner Product of functions

$u: \mathbb{R} \to \mathbb{R},\ v: \mathbb{R} \to \mathbb{R}$

$$\langle u, v \rangle := \int_a^b u(x)\, v(x)\, dx$$

$\langle u, v \rangle = 0$일시 두 함수 u, v는 orthogonal

3.8 Orthogonal Projections

vector space V, $U \subseteq V$

$\pi : V \to U$ is called projection
if $\pi^2 = \pi \circ \pi = \pi$

projection matrix P_π, $P_\pi^2 = P_\pi$

* on Lines

① $\langle x - \underbrace{\pi_U(x)}_{\lambda b}, b \rangle = 0$

$\rightarrow \langle x, b \rangle - \lambda \langle b, b \rangle = 0$

$\rightarrow \lambda = \dfrac{\langle b, x \rangle}{\|b\|^2} = \dfrac{b^T x}{\|b\|^2}$

② $\pi_U(x) = b\lambda = \dfrac{bb^T}{\|b\|^2} x$

$\Rightarrow P_\pi = \dfrac{bb^T}{\|b\|^2}$

* on General Subspace

$\pi_U(x) = B\lambda$

$$\begin{bmatrix} b_1^T \\ \vdots \\ b_m^T \end{bmatrix} [x - B\lambda] = 0 \iff B^T(x - B\lambda) = 0$$

$$\iff B^T B \lambda = B^T x$$

$\lambda = (B^T B)^{-1} B^T x$

$\pi_U(x) = B(B^T B)^{-1} B^T x$

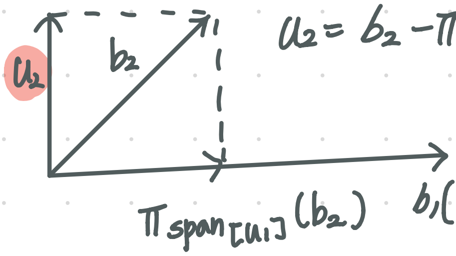
$\therefore P_\pi = B(B^T B)^{-1} B^T$

* Gram-Schmidt Orthogonalization

on basis $(b_1, \cdots b_n)$ of V,
orthogonal basis $(u_1, u_2, \cdots u_n)$

$u_1 := b_1$

$u_k := b_k - \pi_{\mathrm{span}[u_1, \cdots u_{k-1}]}(b_k)$
$(k = 2 \ldots n)$



$u_2 = b_2 - \pi_{\mathrm{span}[u_1]} b_2$

* on Affine Subspace

$\pi_L(x) = x_0 + \pi_U(x - x_0)$

3.9 Rotations

preserve angles, distances
(+α) dimension × commutative

: linear mapping that rotates a plane by θ angle

$R(\theta) = [\Phi(e_1), \Phi(e_2)]$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

+ on n dimensions

$$R_{ij}(\theta) := \begin{bmatrix} I_{i-1} & 0 & \cdots & \cdots & 0 \\ 0 & \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 0 & I_{j-i-1} & 0 & 0 \\ 0 & \sin\theta & 0 & \cos\theta & 0 \\ 0 & \cdots & \cdots & 0 & I_{n-j} \end{bmatrix} \in \mathbb{R}^{n \times n}$$