Chap 6 Probability and Distribution

기본용어

Sample space Ω

event space $\mathcal{A}$ probability P

Probability $\begin{cases} \text{Discrete} & \text{joint시 } P(X=x_i, Y=y_j) = \frac{n_{ij}}{N} \\ \text{Continuous} & P(a \le X \le b) = \int_a^b f(x)dx \end{cases}$

Probability of F

Cumulative distribution Function

$$F_X(x) = P(X_1 \le x_1, \cdots X_D \le x_D)$$

Rule

① Sum rule

$$p(x) = \begin{cases} \sum_{y \in \mathcal{Y}} p(x,y) & \text{if } y \text{ is discrete} \\ \int_{\mathcal{Y}} p(x,y)dy & \text{if } y \text{ is continuous} \end{cases}$$

② Product rule

$$p(x,y) = p(y|x)p(x)$$

③ Bayes' theorem

$$p(x|y) = \frac{p(y|x)p(x)}{p(y)}$$

Properties of PDF

① Mean

$$E_X[g(x)] = \int_{\mathcal{X}} g(x)p(x)dx$$

$$E_X[f(x)] = aE_X(g(x)) + bE_X[h(x)]$$

$$(f(x) = ag(x) + bh(x))$$

② Covariance

$$Cov_{X,Y}[x,y] = E_{X,Y}[(x - E_X[x])(y - E_Y[y])]$$

$$= E[xy] - E[x]E[y]$$

③ Variance

$$V_X[x] = Cov_X[x, x] = E_X[xx^T] - E_X[x]E_X[x]^T$$

$$= E_X[x^2] - (E_X[x])^2$$

④ Correlation

$$corr[x,y] = \frac{Cov[x,y]}{\sqrt{V[x]V[y]}} \in [-1, 1]$$

Statistical Independence

1) Statistical Independence

$p(x,y) = p(x)p(y) \Leftrightarrow X, Y$ statistical Independent

2) Conditional Independence

X, Y conditional independent $\Leftrightarrow p(x, y|z) = p(x|z)\,p(y|z)$

↳ If uncorrelated X, Y, $Var[X+Y] = Var[X] + Var[Y]$

$cov[x, y] = 0$

Distribution

1) Gausian Distribution $X \sim N(\mu, \Sigma)$ → Exponential Family.

\+ Standard Normal distribution $\mu = 0, \Sigma = I$

2) Bernoulli Distribution $Ber(\mu)$

$p(x|\mu) = \mu^x (1-\mu)^{1-x} \quad x \in \{0, 1\}$,

$E[x] = \mu$,

$V[x] = \mu(1-\mu)$

3) Binomial Distribution

$p(m|N, \mu) = \binom{N}{m} \mu^m (1-\mu)^{N-m}$

$E[m] = \mu N \quad V[m] = N\mu(1-\mu)$

4) Beta Distribution + Conjugacy