Chapter3. Analytic Geometry

DATE.

3.1 Norms

Def. vector space V에서의 norm

$\|\cdot\| : V \to \mathbb{R}$, $x \mapsto \|x\|$

추가 특성) $\|\lambda x\| = |\lambda|\|x\|$

$\|x+y\| \le \|x\| + \|y\|$

$\|x\| \ge 0$ & $\|x\| = 0 \Leftrightarrow x = 0$

Manhattan Norm $\|x\|_1$, l_1 norm

$\|x\|_1 := \sum_{i=1}^{n} |x_i|$ 모든 요소 절댓값의 합

Euclidean Norm $\|x\|_2$, l_2 norm

$\|x\|_2 := \sqrt{\sum_{i=1}^{n} x_i^2} = \sqrt{x^T x}$

3.2 Inner Products 내적공간

: inner product / dot product / Scalar product

$x^T y = \sum_{i=1}^{n} x_i y_i$

$\langle x, y \rangle := x_1 y_1 - (x_1 y_2 + x_2 y_1) + 2 x_2 y_2$

$:= \hat{x}^T A \hat{y}$

1. Symmetric, Positive

$x^T A x > 0 \Leftrightarrow \langle x, y \rangle = \hat{x}^T A \hat{y}$

3.3 Lengths and Distances

norm $\|x\| := \sqrt{\langle x, x \rangle}$

$|\langle x, y \rangle| \le \|x\|\|y\|$

(Euclidean)

Distance $d(x,y) := \|x-y\| = \sqrt{\langle x-y, x-y \rangle}$

3.4 Angles and Orthogonality

직교

$-1 \le \dfrac{\langle x, y \rangle}{\|x\|\|y\|} \le 1$

cos ω (angle btw x & y)

$\cos\omega = \dfrac{\langle x, y \rangle}{\sqrt{\langle x, x \rangle \langle y, y \rangle}} = \dfrac{x^T y}{\sqrt{x^T x y^T y}}$

Orthogonality.

orthogonal $x \perp y \Leftrightarrow \langle x, y \rangle = 0$

+ if unit vector x, y : orthonormal

• Orthogonal matrix

$AA^T = I = A^T A \Leftrightarrow A^T = A^{-1}$

3.5 Orthonormal Basis (ONB)

정규직교기저

: vector space V and basis $\{b_1, \cdots b_n\}$ of V

$\langle b_i, b_j \rangle = 0$ for $i \ne j$

$\langle b_i, b_i \rangle = 1$

3.6 Orthonormal Complement

직교여공간

M-dimensional vector space U

its orthogonal complement $U^\perp$

$\Leftrightarrow U \cap U^\perp = \{0\}$