4. Matrix Decompositions

DATE.

4.1 Determinant and Trace

• Determinant

$$\det(\boldsymbol{A}) = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} \qquad \det(\boldsymbol{T}) = \prod_{i=1}^{n} T_{ii} \,. \qquad \det(\boldsymbol{A}) = \prod_{i=1}^{n} \lambda_i$$

1. *Expansion along column j*

$$\det(\boldsymbol{A}) = \sum_{k=1}^{n} (-1)^{k+j} a_{kj} \det(\boldsymbol{A}_{k,j}) \,. \tag{4.12}$$

2. *Expansion along row j*

$$\det(\boldsymbol{A}) = \sum_{k=1}^{n} (-1)^{k+j} a_{jk} \det(\boldsymbol{A}_{j,k}) \,. \tag{4.13}$$

• Trace

$$\mathrm{tr}(\boldsymbol{A}) := \sum_{i=1}^{n} a_{ii} \,, \quad tr(\boldsymbol{A}) = \sum_{i=1}^{n} \lambda_i$$

4.2 Eigenvalues and Eigenvectors

고유값과 고유벡터 : 선형변형 후에도 값 일정

- λ is an eigenvalue of $\boldsymbol{A} \in \mathbb{R}^{n \times n}$.
- There exists an $\boldsymbol{x} \in \mathbb{R}^n \backslash \{\boldsymbol{0}\}$ with $\boldsymbol{A}\boldsymbol{x} = \lambda \boldsymbol{x}$, or equivalently, $(\boldsymbol{A} - \lambda \boldsymbol{I}_n)\boldsymbol{x} = \boldsymbol{0}$ can be solved non-trivially, i.e., $\boldsymbol{x} \neq \boldsymbol{0}$.
- $\mathrm{rk}(\boldsymbol{A} - \lambda \boldsymbol{I}_n) < n$.
- $\det(\boldsymbol{A} - \lambda \boldsymbol{I}_n) = 0$.

4.3 Cholesky Decomposition

$$\boldsymbol{A} = \boldsymbol{L}\boldsymbol{L}^\top$$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} = \underbrace{\begin{bmatrix} l_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ l_{n1} & \cdots & l_{nn} \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} l_{11} & \cdots & l_{n1} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & l_{nn} \end{bmatrix}}_{L^T}$$

L : Lower-triangular matrix, Cholesky factor of A

4.4 Eigendecomposition and Diagonalization

Diagonal Matrix D : 대각화 가능 행렬

$$\boldsymbol{D} = \begin{bmatrix} c_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & c_n \end{bmatrix}$$

그럼 A : symmetric matrix

대각화

Definition 4.19 (Diagonalizable). A matrix $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ is *diagonalizable* if it is similar to a diagonal matrix, i.e., if there exists an invertible matrix $\boldsymbol{P} \in \mathbb{R}^{n \times n}$ such that $\boldsymbol{D} = \boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P}$.

Theorem 4.20 (Eigendecomposition). *A square matrix* $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ *can be factored into*

고유값분해

$$\boldsymbol{A} = \boldsymbol{P}\boldsymbol{D}\boldsymbol{P}^{-1} \,, \tag{4.55}$$

1. eigenvector, eigenvalue 구하기 (고유벡터, 고유값)
2. 고유값 존재여부 확인
3. 고유벡터로 P 구성하고 $D = P^{-1}AP$ 인 D 구하기