Mathematics for Machine Learning

Chapter 2. Linear Algebra

한지수=)

Matrix행렬

- m행 n열

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = \begin{bmatrix} | & | & & | \\ a^1 & a^2 & \cdots & a^n \\ | & | & & | \end{bmatrix} = \begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ & \vdots & \\ - & a_m^T & - \end{bmatrix}.$$

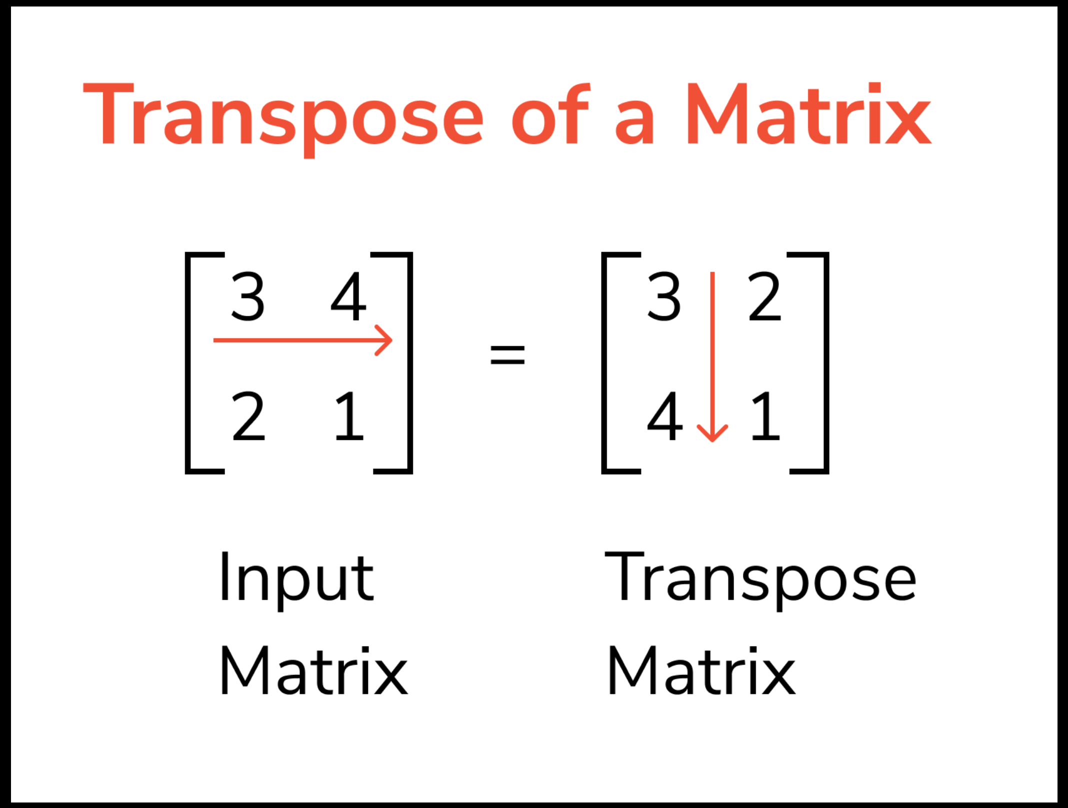
왜 행렬?

$$\left.\begin{aligned} 2x + y - 3z &= -4 \\ 4x - 2y + z &= 9 \\ 3x + 5y - 2z &= 5 \end{aligned}\right\} \rightarrow \begin{array}{c} \left[\begin{array}{ccc:c} 2 & 1 & -3 & -4 \\ 4 & -2 & 1 & 9 \\ 3 & 5 & -2 & 5 \end{array}\right] \end{array} \begin{array}{c} (1) \\ (2) \\ (3) \end{array}$$

Linear Equation에 적용 가능!

Basis

- Transpose: “Flipping” the rows and columns A^T
  - $(A^T)^T = A$
  - $(A^T)_{ij} = A_{ij}$



- Inverse: A^{-1}
  - $A^{-1}A = I = AA^{-1}$

Basis

- Trace: sum of diagonal Elements in the matrix

$$\mathrm{tr}A = \sum_{i=1}^{n} A_{ii}.$$

- Norm: a measure of “length” of the vector

$$\|x\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2}.$$

$$\|x\|_1 = \sum_{i=1}^{n} |x_i|$$

$$\|x\|_\infty = \max_i |x_i|.$$

Basis

- Column Rank: the size of largest subset of columns of A that constitute a linearly independent set

$$\mathrm{tr}A = \sum_{i=1}^{n} A_{ii}.$$

- Row Rank: the largest number of rows of A that constitute a linearly independent set.

$$\|x\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2}.$$

$$\|x\|_1 = \sum_{i=1}^{n} |x_i|$$

$$\|x\|_\infty = \max_i |x_i|.$$

Basis

- Determinant: |A|

$$
\begin{aligned}
|A| &= \sum_{i=1}^{n} (-1)^{i+j} a_{ij} |A_{\setminus i,\setminus j}| \quad \text{(for any } j \in 1, \ldots, n) \\
&= \sum_{j=1}^{n} (-1)^{i+j} a_{ij} |A_{\setminus i,\setminus j}| \quad \text{(for any } i \in 1, \ldots, n)
\end{aligned}
$$

$$
\begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix} = |A| = a \cdot \begin{vmatrix} f & g & h \\ j & k & l \\ n & o & p \end{vmatrix} - b \cdot \begin{vmatrix} e & g & h \\ i & k & l \\ m & o & p \end{vmatrix} + c \cdot \begin{vmatrix} e & f & h \\ i & j & l \\ m & n & p \end{vmatrix} - d \cdot \begin{vmatrix} e & f & g \\ i & j & k \\ m & n & o \end{vmatrix}
$$

Vector Spaces벡터공간

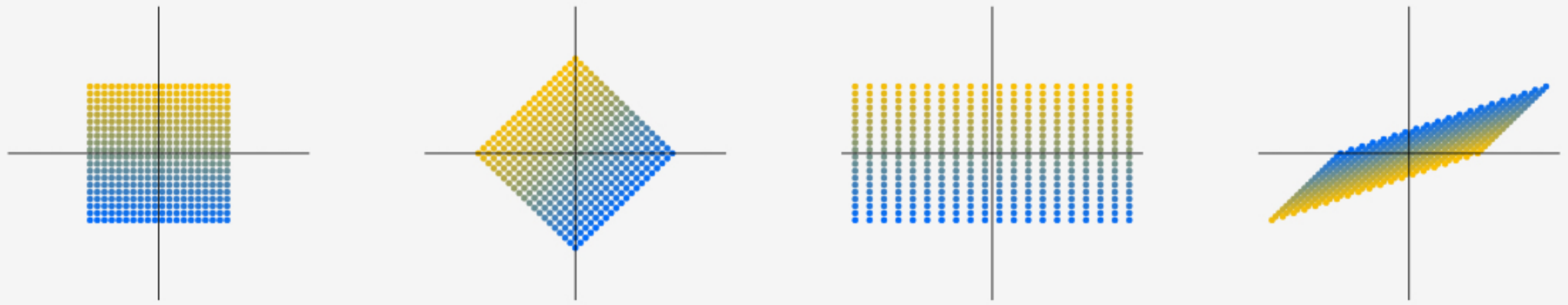
- 벡터 -> 벡터 공간 즉, 벡터는 벡터공간의 원소
- 같은 수의 성분을 가지는 벡터들로 이루어진 공집합이 아닌 집합 V가 있을 때, V에 속하는 임의의 두 벡터 α, β의 일차 결합이 $a\alpha + b\beta$
- Vector SubSpace
- Affine Subspace
  - lines $y = x_0 + \lambda b_1$
  - Planes $y = x_0 + \lambda_1 b_1 + \lambda_2 b_2$
  - Hyperplanes $y = x_0 + \sum_{i=1}^{n-1} \lambda_i b_i$

Linear Mapping선형 변환

- Basis $\{b_1, b_2, b_3, \ldots, b_n\}$ of an n-dimensional vector space V
- For Any $x \in V$ we can obtain a unique representation

$$x = \alpha_1 b_1 + \alpha_2 b_2 + \ldots + \alpha_n b_n$$

Example 2.22 (Linear Transformations of Vectors)



(a) Original data. (b) Rotation by $45°$. (c) Stretch along the horizontal axis. (d) General linear mapping.

We consider three linear transformations of a set of vectors in $\mathbb{R}^2$ with the transformation matrices

$$\boldsymbol{A}_1 = \begin{bmatrix} \cos(\frac{\pi}{4}) & -\sin(\frac{\pi}{4}) \\ \sin(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{A}_3 = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix}. \quad (2.97)$$