Different Quotient → Derivative → Partial Derivative → Chainrule
Gradient
→ Jacobian / Hessian Matrix
→ Multivariate Taylor Series

Chapter 5 Vector Calulus

Difference Quotient 차분몫

$$\frac{\delta y}{\delta x} := \frac{f(x+\delta x) - f(x)}{\delta x}$$

Derivative differentiation 미분

$$\frac{df}{dx} := \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Partial Derivative 편미분

$$\frac{\partial f}{\partial x_1} = \lim_{h \to 0} \frac{f(x_1+h, x_2, \quad x_n) - f(x)}{h}$$

$$\nabla_x f = \operatorname{grad} f = \frac{df}{dx}$$

$$= \begin{bmatrix} \frac{\partial f(x)}{\partial x_1} & \frac{\partial f(x)}{\partial x_2} & \cdots & \frac{\partial f(x)}{\partial x_n} \end{bmatrix} \in R^{1 \times n}$$

column vector 형태

Chain Rule

$$\frac{df}{dt} = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} \end{bmatrix} \begin{bmatrix} \frac{\partial x_1(t)}{\partial t} \\ \frac{\partial x_2(t)}{\partial t} \end{bmatrix}$$

$$= \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t}$$

Jacobian Matrix Jacobian 행렬

$$J = \nabla_x f = \frac{df(x)}{dx}$$

$$= \begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \cdots & \frac{\partial f_1(x)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m(x)}{\partial x_1} & \cdots & \frac{\partial f_m(x)}{\partial x_n} \end{bmatrix}$$

$$J(i,j) = \frac{\partial f_i}{\partial x_j}$$

Gradient 기울기 of a Vector-Valued Function

$$\frac{df}{dx} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_M}{\partial x_1} & \cdots & \frac{\partial f_M}{\partial x_N} \end{bmatrix} = \begin{bmatrix} A_{11} & \cdots & A_{1N} \\ \vdots & \ddots & \vdots \\ A_{M1} & \cdots & A_{MN} \end{bmatrix}$$

$$= A \in R^{M \times N}$$

Higer-Order Derivatives

- Taylor Polynomial

$$T_n(x) := \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k$$

- Taylor Series

$$T_\infty(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k$$

- Multivariate Taylor Series

$$f(x) = \sum_{k=0}^{\infty} \frac{D_x^k f(x_0)}{k!}(x-x_0)^k$$

- Taylor Polynomial

$$T_n(x) = \sum_{k=0}^{n} \frac{D_x^k f(x_0)}{k!}(x-x_0)^k$$

Hessian Matrix Hessian 행렬

$$H_f = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

$$(H_f)_{i,j} = \frac{\partial^2 f}{\partial x_i \partial x_j}$$

$$= \frac{\partial}{\partial x_i}\left(\frac{\partial f}{\partial x_j}\right)$$