

ANALYSIS OF NEURONAL INTERACTIONS USING INFORMATION- THEORETICAL MODELS

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EXCITABLE CELLS AND NEURONS

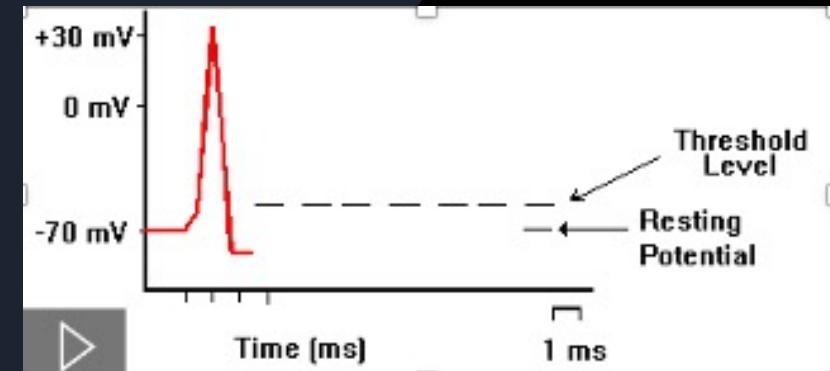
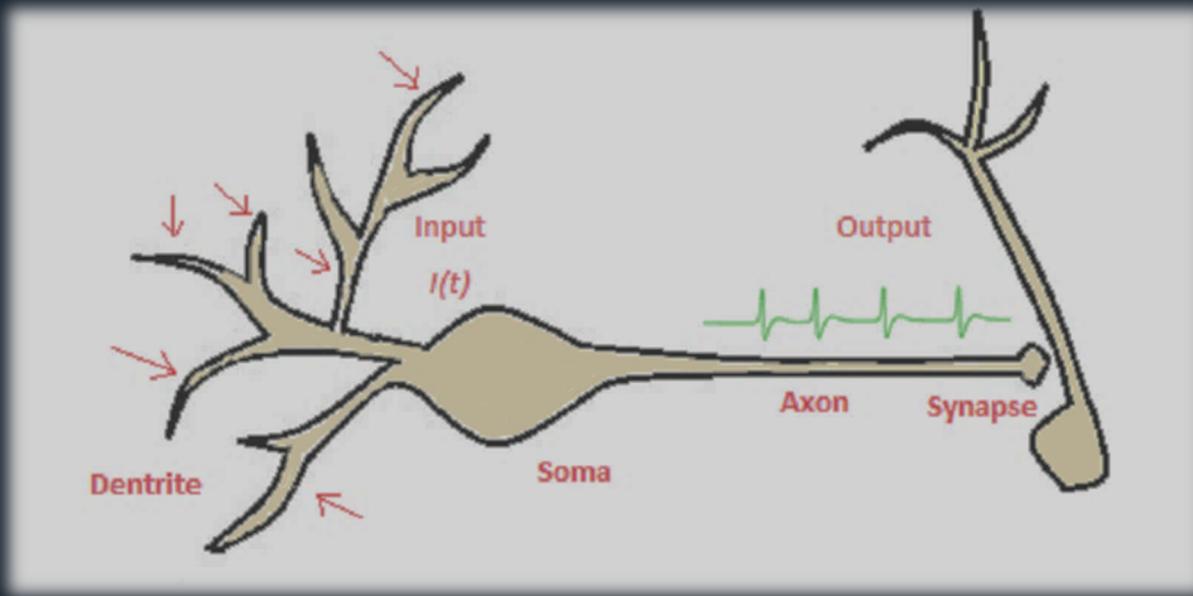


Example: Nerve cells, muscle cells, heart cells and secretary cells such as pituitary cell, pancreatic beta cells.



The signals circulating neurons forms the basis of memories, thoughts and emotions.

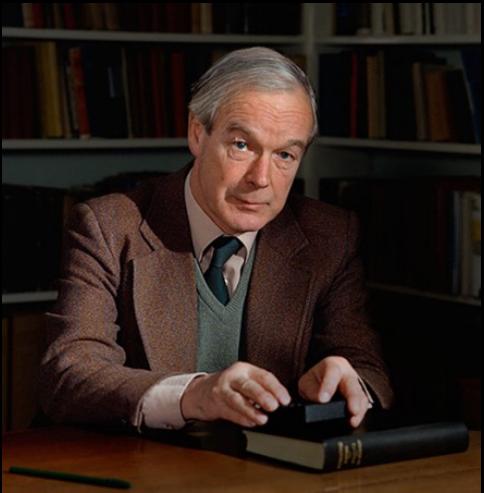
NEURON ACTION POTENTIAL



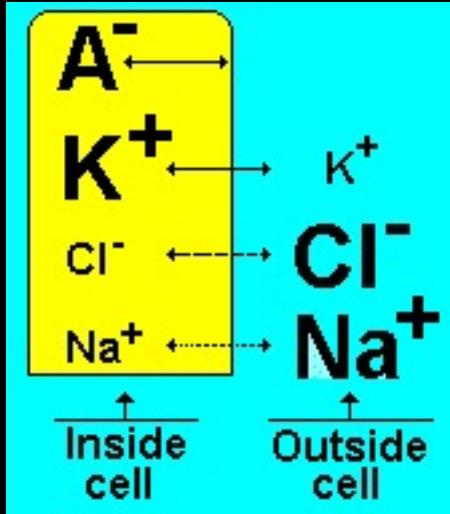
Neurons send messages electrochemically. It means that chemicals cause an electrical signal.

MODELLING ACTION POTENTIAL IN NEURONS

HODGKIN HUXLEY MODEL



$$C \frac{dV}{dt} = \bar{g}_{Na} m^3 h (V_{Na} - V) + \bar{g}_K n^4 (V_K - V) + g_L (V_L - V) + I_{ext}$$



Alan Hodgkin and Andrew Huxley conducted experiment to understand the movement of ions in a nerve cell during an action potential

MORRIS LECAR MODEL

The Morris–Lecar neuron model is a 2-D nonlinear differential system.

In comparison to the four-dimensional Hodgkin–Huxley model, it is considered a simpler model.

$$C \frac{dV}{dt} = I - g_L(V - V_L) - g_{Ca}M_{ss}(V - V_{Ca}) - g_KN(V - V_K)$$

$$\frac{dN}{dt} = \frac{N_{ss} - N}{\tau_N}$$

where

$$M_{ss} = \frac{1}{2} \cdot \left(1 + \tanh \left[\frac{V - V_1}{V_2} \right] \right)$$

$$N_{ss} = \frac{1}{2} \cdot \left(1 + \tanh \left[\frac{V - V_3}{V_4} \right] \right)$$

$$\tau_N = 1 / \left(\varphi \cosh \left[\frac{V - V_3}{2V_4} \right] \right)$$

I : applied current

C : membrane capacitance

g_L , g_{Ca} , g_K : leak, Ca^{++} , and K^+ conductances through membranes channel

V_L , V_{Ca} , V_K : equilibrium potential of relevant ion channels

V_1 , V_2 , V_3 , V_4 : tuning parameters for steady state and time constant

φ : reference frequency

HINDMARSH ROSE MODEL

This is non-conductance model

In experiments, Hindmarsh and Rose observed Spiking behaviour of the membrane potential

Membrane potential: Electric potential between the interior and the exterior of a biological cell.

$x(t)$ =Membrane potential
 $y(t)$ =Spiking variable
 $z(t)$ =Adaption current

$$\dot{x} = y - ax^3 + bx^2 + I - z$$

$$\dot{y} = c - dx^2 - y$$

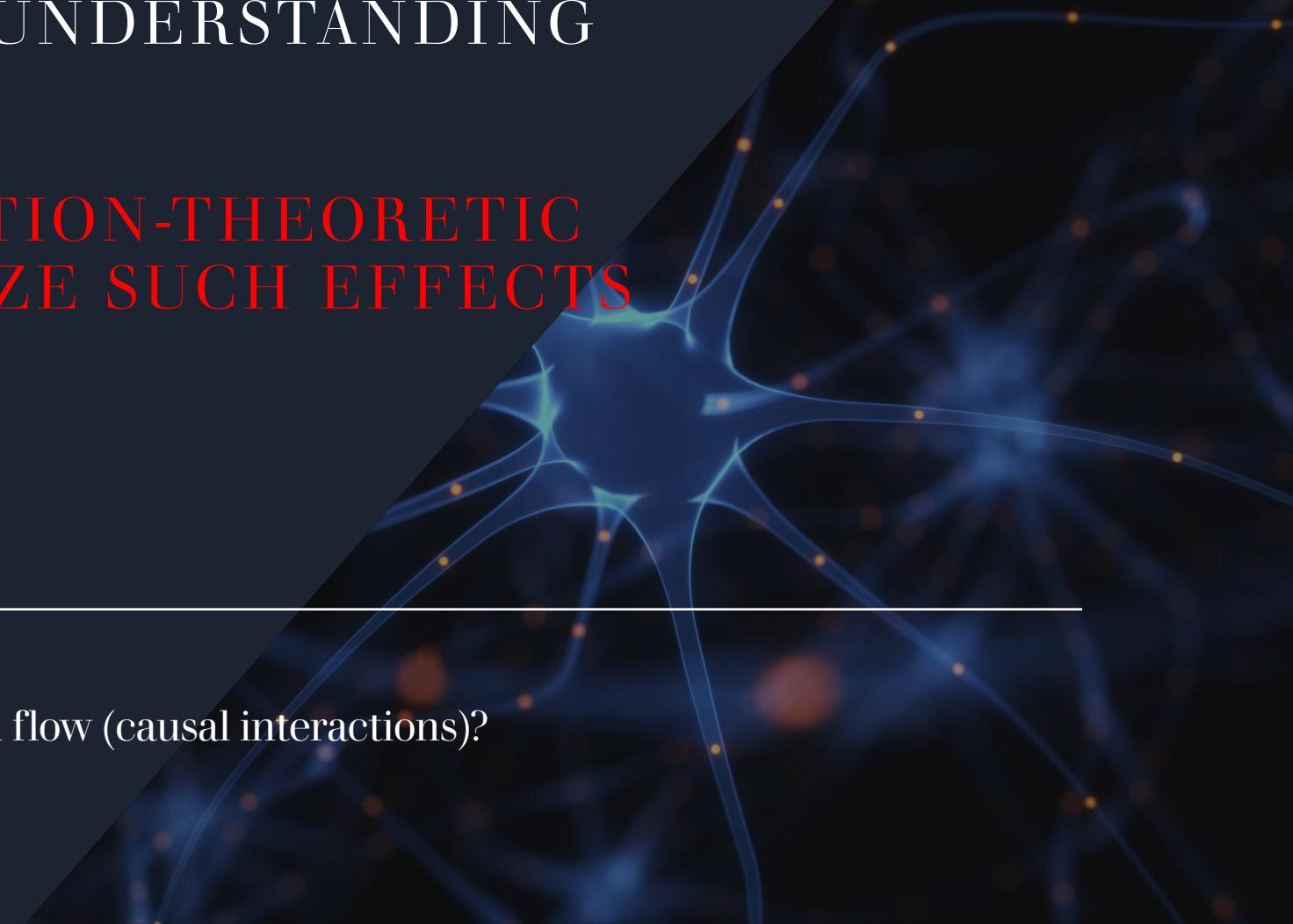
$$\dot{z} = r(s(x - x_1) - z),$$

UNDERSTAND AND ANALYZE
INTERACTION BETWEEN NEURONS
WILL GIVE BETTER UNDERSTANDING
HOW BRAIN WORKS

WE APPLY INFORMATION-THEORETIC
METHODS TO ANALYZE SUCH EFFECTS

Which neurons are related?

What is the direction of the information flow (causal interactions)?



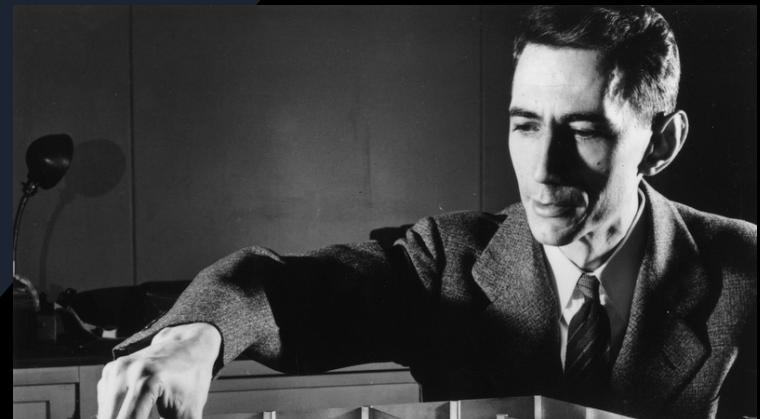
INFORMATION THEORY

Entropy: Fundamental information theoretical quantity to describe the average uncertainty of a system.

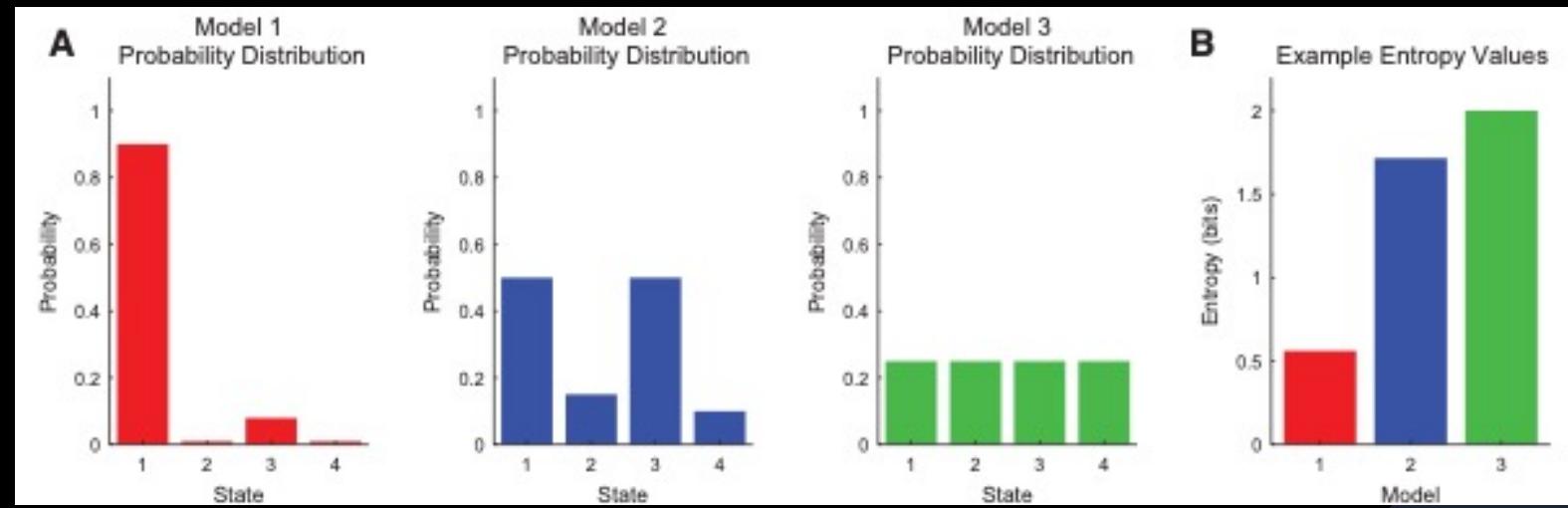
$$H(X) = - \sum_{x \in X} p(x) \log(p(x)))$$

where H denotes the entropy of a random variable “X” and p(x) represents its probability density function (pdf).

Claude Shannon



INFORMATION is reduction in uncertainty



Systems with varied distribution , the entropy is lower
 Systems with uniform distributions, the entropy is higher

MUTUAL INFORMATION (MI)

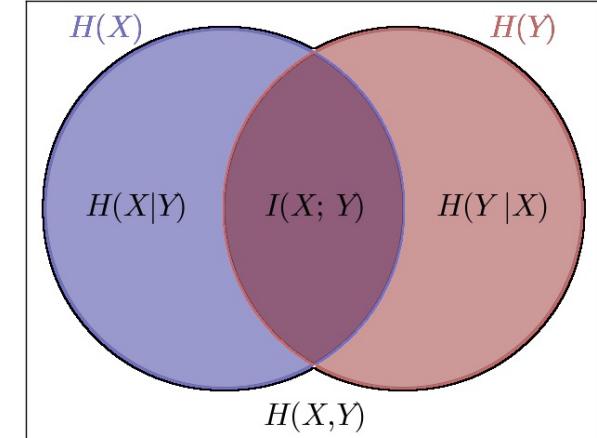
- Conditional Entropy which quantifies the uncertainty in a variable given the state of another variable.

- MI ($I(X;Y)$):

$$I(X;Y) = H(X) - H(X|Y) = \sum_{x \in X, y \in Y} p(x,y) \log_2 \left(\frac{p(x,y)}{p(x)p(y)} \right)$$

We can quantity reduction in uncertainty so we can quantity information

$$H(Y|X) = - \sum_{x \in \mathcal{X}, y \in \mathcal{Y}} p(x,y) \log \frac{p(x,y)}{p(x)}$$



TRANSFER ENTROPY(TE)

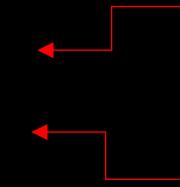
To identify the direction of information flow from one variable to another, we use Transfer Entropy which is capable of detecting the direction.

Information Flow: Information we learn from the past state of one variable about the current state of the other

The TE in two directions are calculated from data by using the following equations

$$\textcolor{red}{TE}_{\mathbf{v}_1 \mathbf{v}_2} = T(V_{2(i+1)} | \mathbf{V}_{2(i)}^{(k)}, \mathbf{V}_{1(i)}^{(l)}) = \sum_{v_{2(i+1)}, \mathbf{v}_{2(i)}^{(k)}, \mathbf{v}_{1(i)}^{(l)}} p(v_{2(i+1)}, \mathbf{v}_{2(i)}^{(k)}, \mathbf{v}_{1(i)}^{(l)}) \log_2 \frac{p(v_{2(i+1)} | \mathbf{v}_{2(i)}^{(k)}, \mathbf{v}_{1(i)}^{(l)})}{p(v_{2(i+1)} | \mathbf{v}_{2(i)}^{(k)})},$$

$$\textcolor{red}{TE}_{\mathbf{v}_2 \mathbf{v}_1} = T(V_{1(i+1)} | \mathbf{V}_{1(i)}^{(k)}, \mathbf{V}_{2(i)}^{(l)}) = \sum_{v_{1(i+1)}, \mathbf{v}_{1(i)}^{(k)}, \mathbf{v}_{2(i)}^{(l)}} p(v_{1(i+1)}, \mathbf{v}_{1(i)}^{(k)}, \mathbf{v}_{2(i)}^{(l)}) \log_2 \frac{p(v_{1(i+1)} | \mathbf{v}_{1(i)}^{(k)}, \mathbf{v}_{2(i)}^{(l)})}{p(v_{1(i+1)} | \mathbf{v}_{1(i)}^{(k)})},$$

$$TE_{V_1 V_2} = T \left(V_{2(i+1)} \middle| V_{2(i)}^{(k)}, V_{1(i)}^{(l)} \right) = \sum_{v_{2(i+1)}, v_{2(i)}^{(k)}, v_{1(i)}^{(l)}} p(v_{2(i+1)}, v_{2(i)}^{(k)}, v_{1(i)}^{(l)}) \log_2 \frac{p(v_{2(i+1)} | v_{2(i)}^{(k)}, v_{1(i)}^{(l)})}{p(v_{2(i+1)} | v_{2(i)}^{(k)})},$$


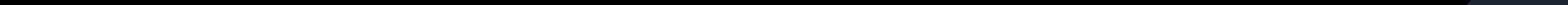
$$TE_{V_2 V_1} = T \left(V_{1(i+1)} \middle| V_{1(i)}^{(k)}, V_{2(i)}^{(l)} \right) = \sum_{v_{1(i+1)}, v_{1(i)}^{(k)}, v_{2(i)}^{(l)}} p(v_{1(i+1)}, v_{1(i)}^{(k)}, v_{2(i)}^{(l)}) \log_2 \frac{p(v_{1(i+1)} | v_{1(i)}^{(k)}, v_{2(i)}^{(l)})}{p(v_{1(i+1)} | v_{1(i)}^{(k)})},$$

$$\begin{aligned} \boldsymbol{v}_{1(i+1)}^{(k)} &= \{v_{1(i)}, v_{1(i-1)}, \dots, v_{1(i-k)}\} \\ \boldsymbol{v}_{2(i+1)}^{(l)} &= \{v_{2(i)}, v_{2(i-1)}, \dots, v_{2(i-l)}\} \end{aligned}$$

NetTE

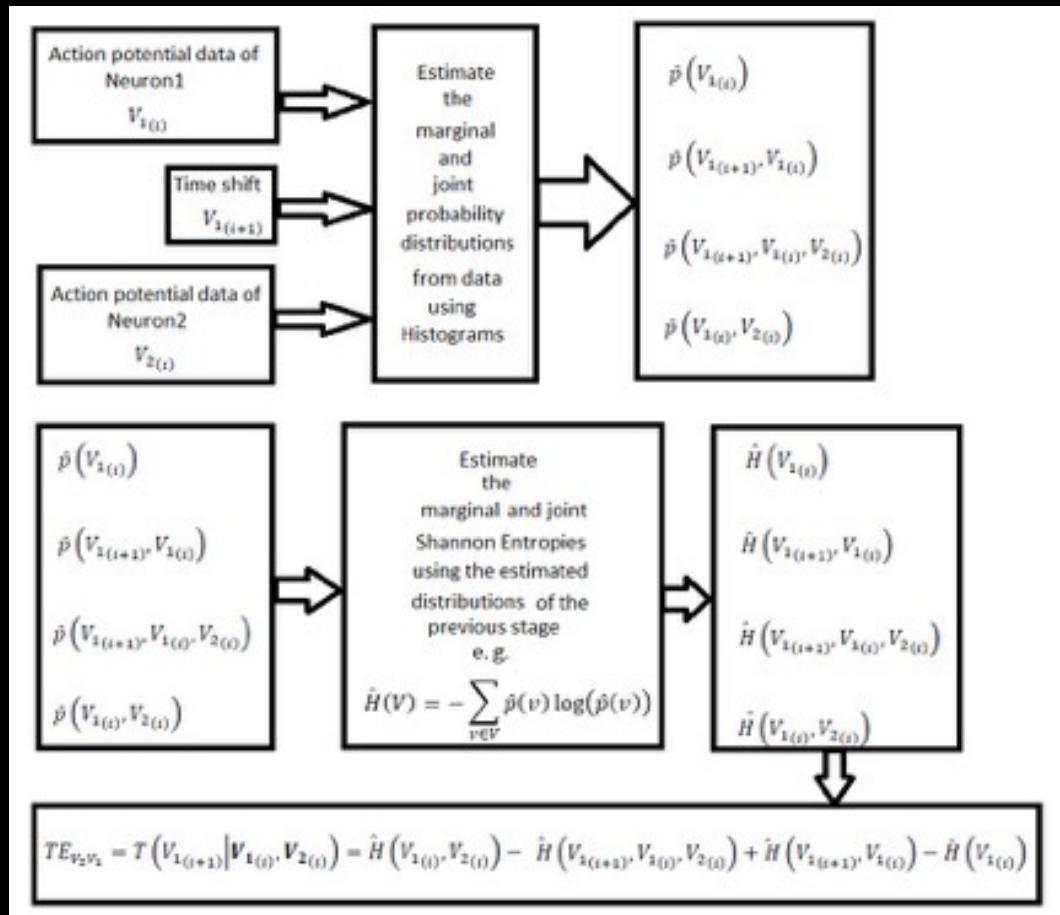
$$NetTE_{XY} = \max(TE_{YX}, TE_{XY}) - \min(TE_{YX}, TE_{XY})$$

- The NetTE allows us to compare the relative values of information flow in both directions giving a sense of main interaction direction between the two variables X and Y.



Estimation Of TE From Data

$$H(V_{1(i)}, V_{2(i)}), H(V_{1(i+1)}, V_{1(i)}, V_{2(i)}), H(V_{1(i+1)}, V_{1(i)}), H(V_{1(i)}).$$

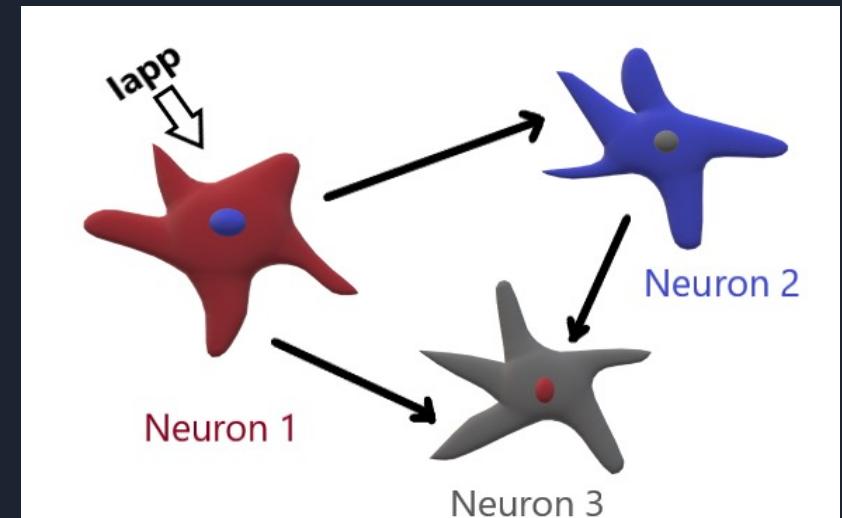


Finally, TE from V_2 to V_1 is computed

SIMULATIONS



Coupling For All 3 Neuron Model

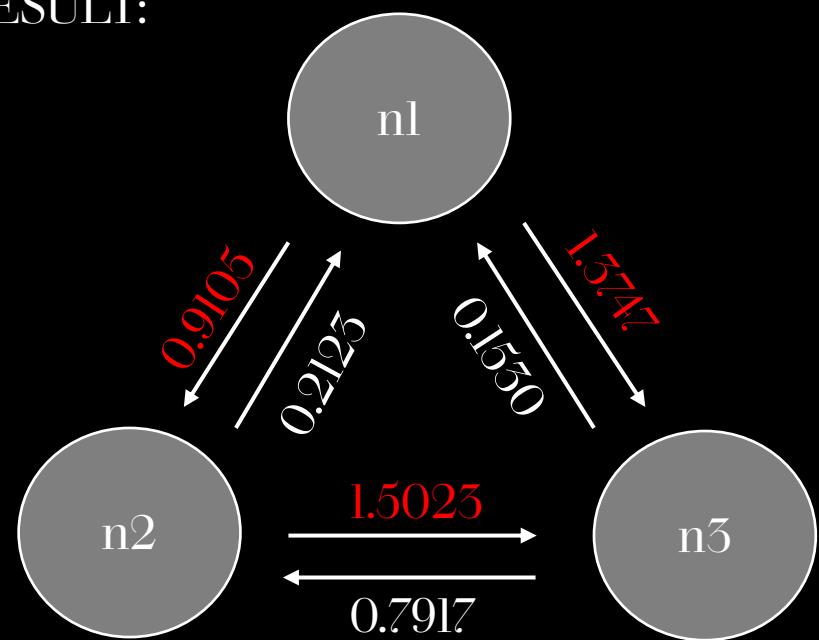


HH Model Simulation

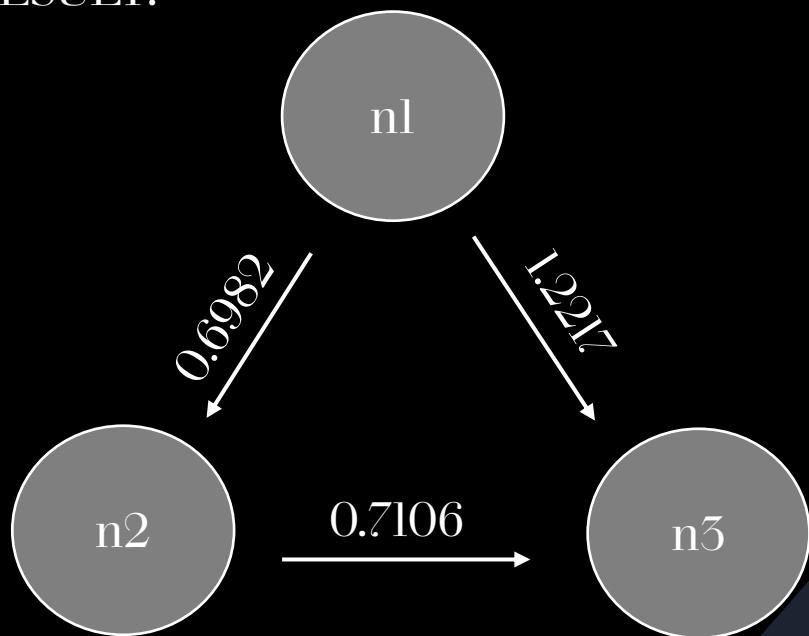
$I_{Syn_{i-j}} = k_x(V_i - V_j)$ is the synaptic current defined from pre-synaptic neuron i to post-synaptic neuron j.

$$C_m \frac{dV_1}{dt} = I_{app} - I_{Na,1} - I_{K,1} - I_{L,1}$$
$$C_m \frac{dV_2}{dt} = -I_{Na,2} - I_{K,2} - I_{L,2} - I_{Syn_{1-2}}$$
$$C_m \frac{dV_3}{dt} = -I_{Na,3} - I_{K,3} - I_{L,3} - I_{Syn_{1-3}} - I_{Syn_{2-3}}$$

TE RESULT:



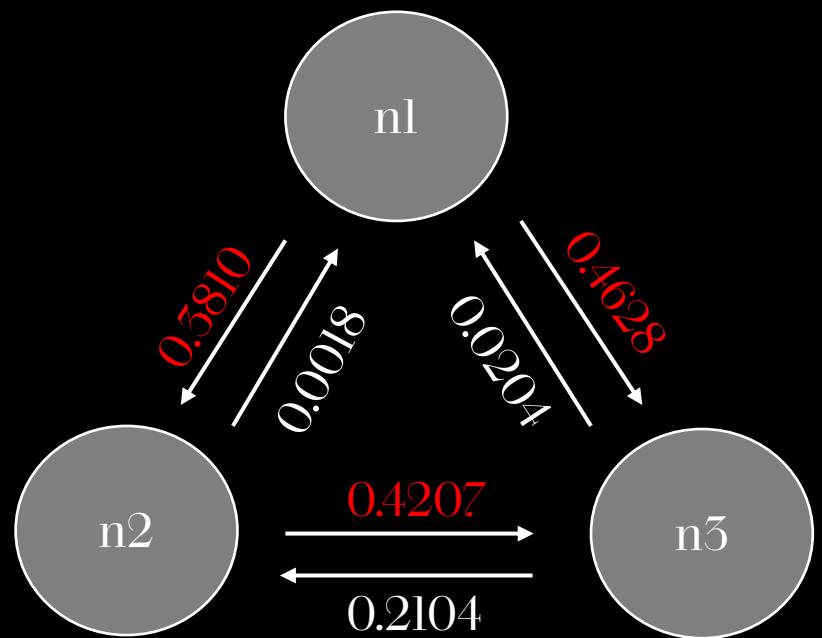
NETTE RESULT:



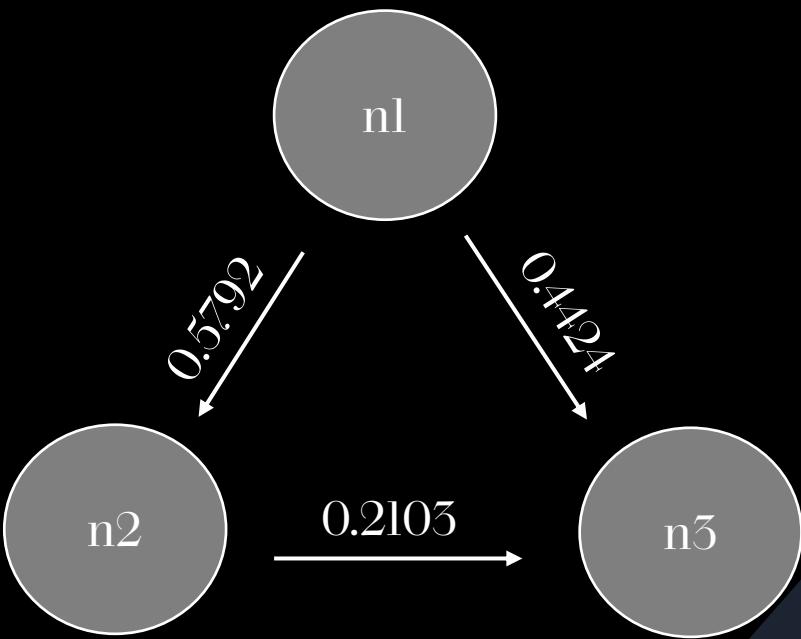
nl:Neuron1 n2:Neuron2 n3:Neuron3

ML 3 Neuron Model Simulation

TE RESULT:



NETTE RESULT:



nl:Neuron1 n2:Neuron2 n3:Neuron3

ML 2 Neuron Model Simulation

Model Simulation Coupling:



TE RESULT:

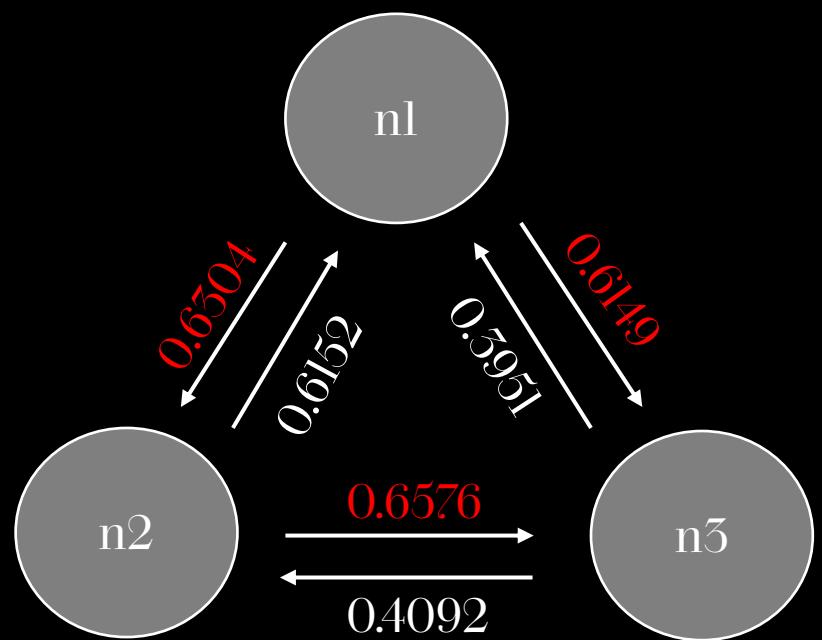


NETTE RESULT:

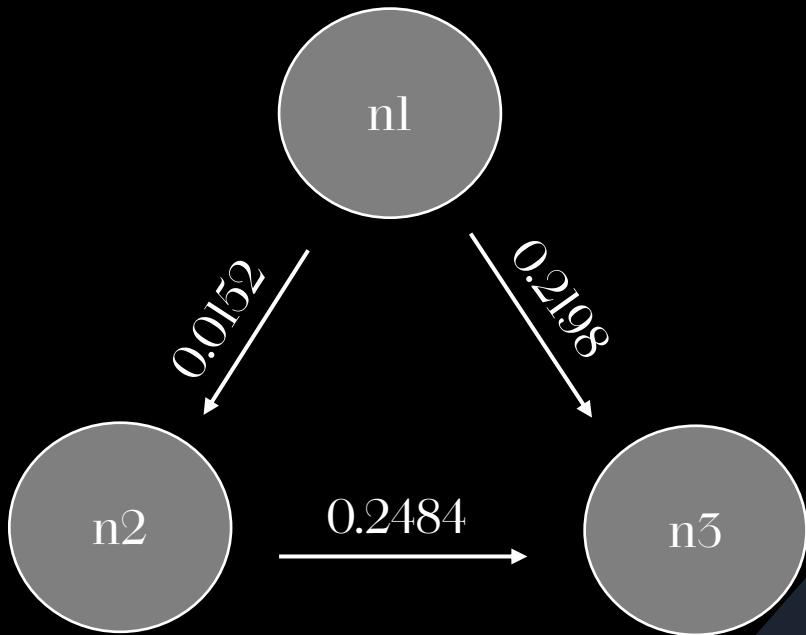


HR 3 Neuron Model Simulation

TE RESULT:



NETTE RESULT:



nl:Neuron1 n2:Neuron2 n3:Neuron3

HR 2 Neuron Model Simulation

Model Simulation Coupling:



TE RESULT:



NETTE RESULT:



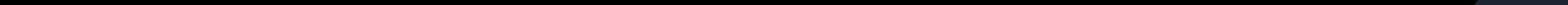
Summary

- Neuronal interactions were analyzed by information-theoretic methods
 - Different neural interaction models have been considered
 - Cause and effect between neurons have been analyzed using these models
 - Successful interaction modelling for each case
-

THANK YOU

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