

# CALCULUS

## MINI EXAM 3

NAME: \_\_\_\_\_

ID: \_\_\_\_\_

SCORE: \_\_\_\_\_ / 80

### RULES:

- You have 30 minutes to complete the exam.
- There are 3 questions and 80 points in total.
- You can use a non-graphing calculator.
- If you need to go to the restroom, please turn in your cellphone before.
- If you need hints, 1 hint is worth 3 points.

*Problem 1* (20 points). A 170-cm-tall person walks toward a wall at a rate of 0.5 m/sec. A spotlight is located on the ground 10m from the wall. How fast does the height of the person's shadow on the wall change when the person is 5 m from the wall?

*Problem 2* (20 points). Consider the following function

$$f(x) = \cos x + e^x$$

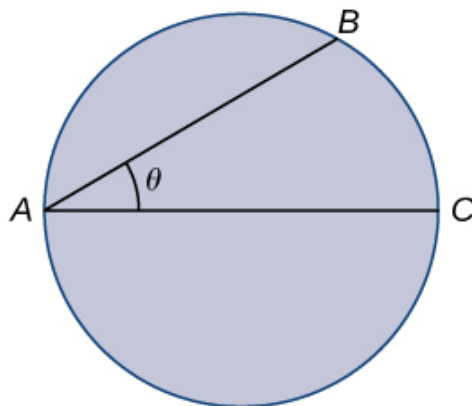
- (1) Find the linear approximation  $L(x)$  of  $f(x)$  around  $a = 1$ .

- (2) Find the Taylor series (to the 3rd power) of the function  $f(x)$  around  $a = 0$ .

*Problem 3* (20 points). (1) Sketch the graph of a function  $f$  that is continuous on  $[1, 5]$  that has absolute maximum at 5, absolute minimum at 2, local maximum at 3, local minima at 2 and 4.

(2) Let  $m$  be the number of local minima and  $M$  be the number of local maxima. Can you create a function where  $M > m + 3$ ? Draw a graph to support your claim.

*Problem 4* (20 points). Consider a lifeguard at position  $A$  at a circular pool with diameter 30m. He must reach someone who is drowning on the exact opposite side of the pool, at position  $C$ . He might swim to position  $B$  first and then run to  $C$ . The lifeguard swims with a speed  $1m/s$  and runs around the pool at speed  $2m/s$ .



- (1) Find a function that measures the total amount of time it takes to reach the drowning person as a function of the swim angle,  $\theta$ .
  
  
  
  
  
  
  
  
  
  
- (2) Find at what angle  $\theta$  the lifeguard should swim to reach the drowning person in the least amount of time.