CALCULUS

MINI EXAM 1
FIRST SECTION

NAME: _____

ID: _____

SCORE: _____/ 80

RULES:

- You have 30 minutes to complete the exam.
- There are 3 questions and 80 points in total.
- You can use a non-graphing calculator.
- If you need to go to the restroom, please turn in your cellphone before.

Date: September 9, 2024.

Problem 1 (30 points). (1) (10 points) State the mean value theorem (you can use your own words)

(2) (20 points) Show that the equation $e^x - \sin x = 0$ has a solution between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

Problem 2 (30 points). Do the limits below exist? Give reason.

(1) $\lim_{x\to 0} x \sin\left(\frac{1}{x}\right)$

(2) $\lim_{x\to 1} \frac{x^2-4}{x-1}$

(3) $\lim_{x\to 2} \frac{x^2-4}{x-2}$

Problem 3 (20 points). Consider

$$f(x) = \begin{cases} x + k\,, & x < 0 \\ \cos x\,, & x \geqslant 0\,. \end{cases}$$

Find k to make $f(x)$ continuous at $x = 0$.