WORKSHEET 14

MATH 101

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Mean Value Theorem

Theorem 1. *If $f(x)$ is continuous over interval $[a, b]$, then there is at least one point $c \in [a, b]$ such that*

$$f(c) = \frac{1}{b-a} \int_a^b f(x)\, dx\,.$$

Problem 1. True or False. If f is continuous over $[a, b]$ and is not equal to a constant, there is at least one point $M \in [a, b]$ such that $f(M) > \frac{1}{b-a} \int_a^b f(t)\, dt$ and at least one point $m \in [a, b]$ such that $f(m) < \frac{1}{b-a} \int_a^b f(t)\, dt$.

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Problem 2. Fidn the average value of the function $f(x) = 8 - 2x$ over the interval $[0, 4]$ and find c such that $f(c)$ equals to the average of the function over $[0, 4]$.

Problem 3. Given $\int_0^3 (2x^2 - 1)\, dx = 15$, find c such that $f(c)$ equals to the average value of $f(x) = 2x^2 - 1$ over $[0, 3]$.

Theorem 2 (Fundamental Theorem of Calculus, Part 1)**.** *If $f(x)$ is continuous over an interval $[a, b]$, and the function $F(x)$ is defined by*

$$F(x) = \int_a^x f(t)\, dt, ,$$

then $F'(x) = f(x)$ over $[a, b]$.

Problem 4. Evaluate the following:

(1)

$$\frac{d}{dx} \int_1^x e^{-t^2}\, dt$$

(2)

$$\frac{d}{dx}\int_1^x e^{\cos t}\,dt$$

(3)

$$\frac{d}{dx}\int_3^x \sqrt{9-y^2}\,dy$$

(4)

$$\frac{d}{dx}\int_3^x \frac{ds}{\sqrt{16-s^2}}$$

(5)

$$\frac{d}{dx}\int_x^{2x} t\,dt$$

Theorem 3 (Fundamental Theorem of Calculus, Part 2)**.** *If f is continuous over the interval $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$, then*

$$\int_a^b f(x)\,dx = F(b) - F(a)\,.$$

Problem 5 (name of the problem)*.* In the following exercises, identify the roots of the integrand to remove absolute values, then evaluate using the Fundamental Theorem of Calculus, Part 2.

(1)

$$\int_{-2}^{3} |x|\,dx$$

(2)

$$\int_{-2}^{4} |t^2 - 2t - 3|\,dt$$

(3)

$$\int_{0}^{\pi} |\cos t|\,dt$$

(4)

$$\int_{-2}^{4} |t^2 - 2t - 3| \, dt$$

(5)

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin t| \, dt$$