WORKSHEET 14

MATH 101

Fulbright University, Ho Chi Minh City, Vietnam

Antiderivative

Definition 1. A function F is an *antiderivative* of the function f if

$$F'(x) = f(x)$$

for all x in the domain of f.

Theorem 1. *Let F be an antiderivative of f over an interval I. Then, $F(x) + C$ is also an antiderivative of f.*

Problem 1. Find the antiderivatives of the following:

(1) $f(x) = 3x^2$

(2) $f(x) = \frac{1}{x}$

(3) $f(x) = \cos x$

Date: October 30, 2024.

(4) $f(x) = e^x$

Definition 2. Given a function f, the *indefinite integral* of f, denoted by

$$\int f(x)\,dx\,,$$

is the most general antiderivative of f. The expression $f(x)$ is called the *integrand* and x is called the variable of integration.

Problem 2. Verify that $F(x)$ is an antiderivative of $f(x)$.

(1) $F(x) = 5x^3 + 2x^2 + 3x + 1,\ f(x) = 15x^2 + 4x + 3$

(2) $F(x) = x^2 + 4x + 1,\ f(x) = 2x + 4$

Problem 3. Find an antiderivative of the following functions

(1) $f(x) = \frac{1}{x} + x$

(2) $f(x) = e^x - 3x^2 + \sin x$

(3) $f(x) = 2\sin(x) + \sin(2x)$

(4) $f(x) = \frac{1}{2}e^{-4x} + \sin x$

(5) $f(x) = \tan x \cos x$

Problem 4. We now have the language to discuss about differential equations. The first problem that everyone learns in differential equations is the so-called *Initial Value Problem*.

https://youtu.be/p_di4Zn4wz4?si=l-eqAJCQSLXFJGb5