WORKSHEET 4

MATH 101

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Please make sure you have a graphical example for each of the definitions below.

Theorem 1 (Composite Function Theorem)**.** *If $f(x)$ is continuous at L and $\lim_{x\to a} g(x) = L$, then*

$$\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x)) = f(L)\,.$$

Question 1. *Find the limits:*

(1) $\lim_{x\to\pi} \sin(x + \sin x)$

(2) $\lim\limits_{x\to 1} \ln\left(\dfrac{5-x^2}{1+x}\right)$

Theorem 2 (Intermediate Value Theorem)**.** *Let f be continuous over a closed, bounded interval $[a, b]$. If z is any real number between $f(a)$ and $f(b)$, then there exists a number c in $[a, b]$ so that $f(c) = z$.*

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Question 2. *(1) Show that there is a solution to the equation*

$$x^3 + 5x^2 + 3x - 9 = 0$$

(2) Show that $f(x) = x^3 - x^2 - 3x + 1$ *has at least one zero over* $[0, 1]$.

(3) Show that $f(x) = x - \cos x$ *has at least one zero.*

Question 3. *Do the graphs of* $f(x) = 2^x$ *and* $g(x) = x^3$ *cross at some point?*

Question 4. *A ball is thrown into the air and the vertical position is given by*

$$x(t) = -4.9t^2 + 25t + 5.$$

Use the Intermediate Value Theorem to show that the ball must land on the ground sometime between 5 sec and 6 sec after the throw.

Question 5. *A Tibetan monk leaves the monastery at 7:00 am and takes his usual path to the top of the mountain, arriving at 7:00 pm. The following morning, he starts at 7:00 am at the top and takes the same path back, arriving at the monastery at 7:00 pm. Use the Intermediate Value Theorem to show that there is a point on the path that the monk will cross at exactly the same time of day on both days.*