

WORKSHEET 6

MATH 101

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Useful definitions:

$$\begin{aligned}(1) \quad \tan x &= \frac{\sin x}{\cos x} \\(2) \quad \cot x &= \frac{\cos x}{\sin x} \\(3) \quad \csc x &= \frac{1}{\sin x} \\(4) \quad \sec x &= \frac{1}{\cos x}\end{aligned}$$

Question 1. (1) *Show the chain rule:*

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x).$$

(2) *Apply the chain rule to show the quotient rule:*

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

(3) *Apply the above rules when*

$$f(x) = x^3 - \pi x$$

and

$$g(x) = \frac{1}{x^2 + 3}$$

Question 2. For this part, you should review <https://openstax.org/books/calculus-volume-1/pages/2-3-the-limit-laws> for the limits $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ and $\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} = 0$.

Show that

$$(1) \quad (\sin x)' = \cos x$$

(Hint: $\sin(x + y) = \sin x \cos y + \sin y \cos x$)

$$(2) \quad (\cos x)' = -\sin x$$

(Hint: $\cos(x + y) = \cos x \cos y - \sin x \sin y$)

Question 3. Use all the rules above to find

$$(1) \quad (\cot x)'$$

$$(2) \quad (\tan x)'$$

$$(3) \ (\csc x)'$$

$$(4) \ (\sec x)'$$