

CALCULUS

FINAL A

NAME: _____

ID: _____

SCORE: _____/ 100

RULES:

- You have 80 minutes to complete the exam.
- There are 5 questions and 100 points in total.
- You can use a non-graphing calculator.
- If you need to go to the restroom, please turn in your cellphone before.
- If you need hints, 1 hint is worth 3 points.

Problem 1 (20 points). State definitions or theorems.

- (1) Give definition for a function f being differentiable at a point $x = a$.

- (2) What is the intermediate value theorem?

(3) What is the antiderivative of a function $f(x)$?

(4) What is the mean value theorem?

Problem 2 (20 points). Compute the following:

(1)

$$\lim_{x \rightarrow 1} \frac{e^{x-1} - 1}{x^3 - 1}$$

(2)

$$\frac{d}{dx} \left(e^{e^x} \sin(x) \right)$$

(3)

$$\int (e^2 - 3x^5) dx$$

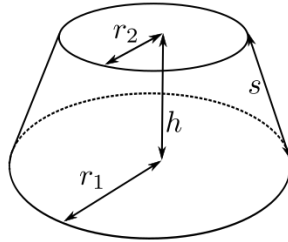
(4)

$$\int \sin(x^2)x dx$$

Problem 3 (20 points). (1) What is the quotient rule?

(2) Derrive the quotient rule from definition.

Problem 4 (20 points). Find the volume of the conical frustrum of height 5, the radius of the base is 10, and the radius on the top is 5. You must use calculus to do this problem.



Problem 5 (20 points). You own a TV company. Let $p(x)$ be the price per unit that the company can charge if it sells x units. Then p is called the **demand function** (or **price function**) and we would expect it to be a decreasing function of x . (More units sold corresponds to a lower price.)

If x units are sold and the price per unit is $p(x)$, then the total revenue is

$$R(x) = x \cdot p(x)$$

where $R(x)$ is called the **revenue function**.

A store has been selling 200 flat-screen TVs a week at \$350 each. A market survey indicates that for each \$10 rebate offered to buyers, the number of TVs sold will increase by 20 a week.

Find the demand function and the revenue function. How large a rebate should the store offer to maximize its revenue?