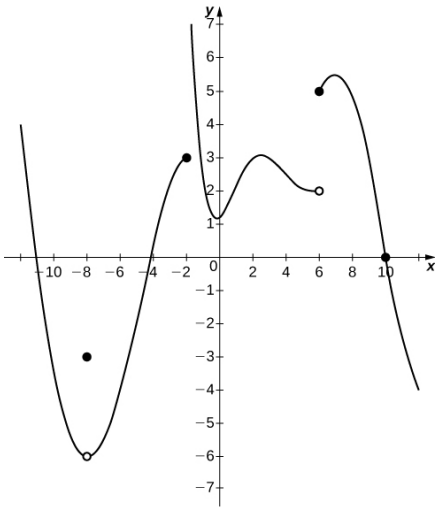
WORKSHEET 2

MATH 101

Fulbright University, Ho Chi Minh City, Vietnam

Question 1. *Consider the following graph of a function $y = f(x)$.*



True or False?

(1) $\lim_{x\to 10} f(x) = 0$
(2) $\lim_{x\to -2^+} f(x) = 3$
(3) $\lim_{x\to -8} f(x) = f(-8)$
(4) $\lim_{x\to 6} f(x) = 5$

Question 2. *What is the limit of the following? If the limit DNE, please specify left and right limits.*

(1) $\lim_{x\to a} \frac{1}{(x-a)^n}$ *where n is odd.*

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(2) $\lim_{x\to a} \frac{1}{(x-a)^n}$ *where* n *is even.*

(3) $\lim_{x\to 2} \frac{x^2-4}{x-2}$

(4) $\lim_{x\to 2} \frac{|x^2-4|}{x-2}$

Theorem 0.1. *The following are VERY important limit laws. The proof of them is out of the scope of this class but I will tell you if you come to office hours.*

Suppose $\lim_{x\to a} f(x) = L$ *and* $\lim_{x\to a} g(x) = M$*. Then,*

(1) $\lim_{x\to a}(f(x) + g(x)) =$
(2) $\lim_{x\to a}(f(x) - g(x)) =$
(3) $\lim_{x\to a}(f(x) \cdot g(x)) =$
(4) $\lim_{x\to a} \frac{f(x)}{g(x)} =$
(5) $\lim_{x\to a}(f(x))^n =$

(6) $\lim_{x\to a}(f(x))^{1/n} =$

Question 3. *Find the limit*

(1) $\lim_{x\to a} x =$
(2) $\lim_{x\to a} c =$
(3) $\lim_{x\to a}(2x-1)\sqrt{x+4} =$

Question 4. *Graph the function*

$$g(x) = \begin{cases} x^3 - 1\,, & x \leqslant 2 \\ 1\,, & x > 2\,. \end{cases}$$

Find

$$\lim_{x\to 2} g(x)$$

Question 5. *What is a polynomial function?*

Theorem 0.2. *Let $p(x)$ and $q(x)$ be polynomial functions. Then,*

$$\lim_{x\to a} p(x) = \ .$$

Question 6. *Find the following limits.*

(1) $\lim_{x\to 4} \dfrac{x^2-16}{x-4}$
(2) $\lim_{h\to 0} \dfrac{(1+h)^2-1}{h}$
(3) $\lim_{x\to 1/2} \dfrac{2x^2+3x-2}{2x-1}$

Theorem 0.3 (Squeeze Theorem)**.** *Let $f(x)$, $g(x)$, and $h(x)$ be functions defined for all $x \neq a$ over an open interval containing a. Suppose:*

$f(x) \leqslant g(x) \leqslant h(x)$ *for all $x \neq a$ in an open interval containing a*

and

$$\lim_{x\to a} f(x) = L = \lim_{x\to a} h(x)$$

where L is a real number. Then,

$$\lim_{x\to a} g(x) = L.$$

Question 7. *We know from our discussion in the last class that*

$$\lim_{x\to 0} \sin\left(\frac{1}{x}\right) = DNE\,.$$

What is it still the same with

$$\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)?\,.$$

Question 8. *Discuss*

(1) Formula (2.18) in Section 2.3

(2) Example 2.25
(3) Checkpoint 2.20