

WORKSHEET 8

MATH 101

Fulbright University, Ho Chi Minh City, Vietnam

Question 1. *Finding the tangent curve to the graph at a given point*

(1) $x^2 + y^2 = 4$ at $(2, 0)$

(2) $x^4y - xy^3 = -2$ at $(-1, -1)$

(3) $\tan(xy) = y$ at $(\pi/4, 1)$

Question 2. (1) *The binomial theorem says that*

$$\begin{aligned}(1+a)^n &= 1 + \binom{n}{1}a + \binom{n}{2}a^2 + \cdots + \binom{n}{n-1}a^{n-1} + \binom{n}{n}a^n \\ &= 1 + na + \binom{n}{2}a^2 + \cdots + na^{n-1} + a^n.\end{aligned}$$

One definition of the number e is that

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

Show that

$$(e^x)' = e^x.$$

Question 3. (1) Recall that the natural log function is the inverse function of e^x . Use this fact and the chain rule to show that

$$(\ln x)' = \frac{1}{x}.$$

(2) Find a formula for $(a^x)'$ where a is a real number.

Question 4. Find the equation of the tangent line to the graph

$$f(x) = x5^x$$

at point $x = 1$.