WORKSHEET 17

MATH 101

Fulbright University, Ho Chi Minh City, Vietnam

Integration techniques

Integration by Parts.

(1) Indefinite integration by parts is given by the the following formula:

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx\,.$$

(2) Definite integration by parts is given by the following formula:

$$\int_a^b u(x)v'(x)\,dx = u(x)v(x)\Big|_a^b - \int_a^b u'(x)v(x)\,dx\,.$$

Here,

$$f(x)\Big|_a^b = f(b) - f(a)\,.$$

Example 1. Evaluate:

(1)

$$\int x\cos x\,dx$$

(2)

$$\int t^2 e^t\,dt$$

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Problem 1. Evaluate:

(1)

$$\int t^2 \sin(\beta t)\, dt$$

(2)

$$\int \ln \sqrt{x}\, dx$$

(3)

$$\int \tan^{-1}(2y)\, dy$$

(4)

$$\int (\ln x)^2\, dx$$

(5)

$$\int e^{2\theta} \sin(3\theta)\, d\theta$$

(6)

$$\int_0^{1/2} x \cos(\pi x)\, dx$$

(7)

$$\int_1^5 \frac{\ln R}{R^2}\, dR$$

Integration by Substitution. Let $u = g(x)$. Then,

$$\int f(g(x))g'(x)\, dx = \int f(u)\, du\,,$$

and

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du\,.$$

Example 2. Evaluate:

(1)

$$\int \cos x \sin x\,dx$$

(2)

$$\int_0^1 xe^{4x^2+3}\,dx$$

Problem 2. Evaluate:

(1) $\displaystyle\int \frac{e^u}{(1-e^u)^2}\,du$

(2) $\displaystyle\int \frac{a+bx^2}{\sqrt{3ax+bx^3}}\,dx$

(3) $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$

(4) $\displaystyle\int \sec^2\theta\tan^3\theta\,d\theta$

(5) $\displaystyle\int e^x\sqrt{1+e^x}\,dx$

(6) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

(7) $\int \frac{z^2}{z^3+1}\,dz$

(8) $\int \sin x \sin(\cos x)\,dx$

(9) $\int x\sqrt{x+2}\,dx$