CALCULUS

FINAL A

NAME: ______________________________________

ID: ______________

SCORE: _________/ 100

RULES:

- You have 80 minutes to complete the exam.
- There are 5 questions and 100 points in total.
- You can use a non-graphing calculator.
- If you need to go to the restroom, please turn in your cellphone before.
- If you need hints, 1 hint is worth 3 points.

Date: December 9, 2024.

Problem 1 (20 points). State definitions or theorems.

(1) Give definition for a function f being differentiable at a point $x = a$.

(2) What is the intermediate value theorem?

(3) What is the antiderivative of a function $f(x)$?

(4) What is the mean value theorem?

Problem 2 (20 points). Compute the following:

(1)

$$\lim_{x\to 1} \frac{e^{x-1}-1}{x^3-1}$$

(2)

$$\frac{d}{dx}\left(e^{e^x}\sin(x)\right)$$

(3)

$$\int (e^2 - 3x^5)\, dx$$

(4)

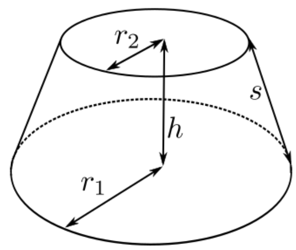
$$\int \sin(x^2) x\, dx$$

Problem 3 (20 points).

(1) What is the quotient rule?

(2) Derrive the quotient rule from definition.

Problem 4 (20 points). Find the volume of the conical frustrum of height 5, the radius of the base is 10, and the radius on the top is 5.



Problem 5 (20 points). You own a TV company. Let $p(x)$ be the price per unit that the company can charge if it sells x units. Then p is called the **demand function** (or **price function**) and we would expect it to be a decreasing function of x. (More units sold corresponds to a lower price.)

If x units are sold and the price per unit is $p(x)$, then the total revenue is

$$R(x) = x \cdot p(x)$$

where $R(x)$ is called the **revenue function**.

A store has been selling 200 flat-screen TVs a week at \$350 each. A market survey indicates that for each \$10 rebate offered to buyers, the number of TVs sold will increase by 20 a week.

Find the demand function and the revenue function. How large a rebate should the store offer to maximize its revenue?