WORKSHEET 14

MATH 101

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Definite Integral

Definition 1. If $f(x)$ is a function defined on an interval $[a, b]$, the **definite integral** of f from a to b is given by

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x,$$

provided the limit exists. If this limit exists, the function $f(x)$ is said to be integrable on $[a, b]$, or is an **integrable function**.

Problem 1. Evaluate the integral using definition of definite integral

(1)

$$\int_0^3 x^2\,dx\,.$$

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(2)

$$\int_{-3}^{3} x\,dx\,.$$

Problem 2. Using Geometric Formula to calculate the following integrals:

(1)

$$\int_{3}^{6} \sqrt{9-(x-3)^2}\,dx$$

(2)

$$\int_{0}^{2} \sqrt{2x-x^2}\,dx + \int_{2}^{6} \sqrt{-12+8x-x^2}\,dx + \int_{6}^{12} \sqrt{-72+18x-x^2}\,dx$$

(3)
$$\int_0^2 (1 - |x - 1|)\, dx$$

Proposition 1. *Properties of Definite Integral.*

(1)
$$\int_a^a f(x)\, dx = 0$$
If the limits of integration are the same, the integral is just a line and contains no area.

(2)
$$\int_b^a f(x)\, dx = -\int_a^b f(x)\, dx$$
If the limits are reversed, then place a negative sign in front of the integral.

(3)
$$\int_a^b [f(x) + g(x)]\, dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$$
The integral of a sum is the sum of the integrals.

(4)
$$\int_a^b [f(x) - g(x)]\, dx = \int_a^b f(x)\, dx - \int_a^b g(x)\, dx$$
The integral of a difference is the difference of the integrals.

(5)
$$\int_a^b cf(x)\, dx = c\int_a^b f(x)\, dx$$
for constant c. The integral of the product of a constant and a function is equal to the constant multiplied by the integral of the function.

(6)
$$\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$$
Although this formula normally applies when c is between a and b, the formula holds for all values of a, b, and c, provided $f(x)$ is integrable on the largest interval.