WORKSHEET 13

MATH 101

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Optimization

Theorem 1 (First derivative test)**.** *Suppose that f is a continuous function over an interval I containing a critical point c. If f is differentiable over I, except possibly at point c, then $f(c)$ satisfies one of the following descriptions:*

(1) If f' changes sign from positive when $x < c$ to negative when $x > c$, then $f(c)$ is a local maximum of f.
(2) If f' changes sign from negative when $x < c$ to positive when $x > c$, then $f(c)$ is a local minimum of f.
(3) If f' has the same sign for $x < c$ and $x > c$, then $f(c)$ is neither a local maximum nor a local minimum of f.

Theorem 2 (Closed interval method)**.** *To find the absolute extrema of a continuous function f on a closed interval $[a, b]$ we follow the following steps:*

(1) Find the critical points and the values of f at those points
(2) Find the values of f at the end points
(3) Compare all the values from the above steps to find absolute max/min

Definition 1. Let f be a function that is differentiable over an interval I. If f' is increasing over I, we say f is concave up. If f' is decreasing over I, we say f is concave down. The point where f changes concavity is called inflection point.

Theorem 3. *Let f be a function that is twice differentiable over an interval I.*

(1) If $f''(x) > 0$, then f is concave up.
(2) If $f''(x) < 0$, then f is concave down.

Theorem 4 (Second derivative test)**.** *Suppose $f'(c) = 0$ and f'' is continuous over an interval I containing c.*

(1) If $f''(c) > 0$, then f has a local minimum at c.
(2) If $f''(c) < 0$, then f has a local maximum at c.
(3) If $f''(c) = 0$, the test is inconclusive.

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Question 1. *Determine:*

- *Intervals where f is increasing/decreasing*
- *Local minima/maxima of f*
- *Intervals where f is concave up and concave down*
- *Inflection points of f*
- *Sketch curve*

(1) $f(x) = \sin(\pi x) - \cos(\pi x)$ *over* $[-1, 1]$

(2) $f(x) = \sqrt{x} \ln x$ *over* $x > 0$

Question 2. *Consider an inscribed rectangle in the ellipse*

$$\frac{x^2}{4} + y^2 = 1\,.$$

What should the dimensions of the rectangle be to maximize its area? What is the maximum area?

Question 3. *Let's learn some business.*

Let $p(x)$ be the price per unit that the company can charge if it sells x units. Then p is called the ***demand function*** *(or* ***price function****) and we would expect it to be a decreasing function of x. (More units sold corresponds to a lower price.)*

If x units are sold and the price per unit is $p(x)$, then the total revenue is

$$R(x) = x \cdot p(x)$$

where $R(x)$ is called the ***revenue function****.*

A store has been selling 200 flat-screen TVs a week at \$350 each. A market survey indicates that for each \$10 rebate offered to buyers, the number of TVs sold will increase by 20 a week.

Find the demand function and the revenue function. How large a rebate should the store offer to maximize its revenue?