

WORKSHEET 18

MATH 101

Fulbright University, Ho Chi Minh City, Vietnam

Integration techniques

Integration by Parts.

- (1) Indefinite integration by parts is given by the the following formula:

$$\int u(x)v'(x) dx = u(x)v(x) - \int u'(x)v(x) dx .$$

- (2) Definite integration by parts is given by the following formula:

$$\int_a^b u(x)v'(x) dx = u(x)v(x)\Big|_a^b - \int_a^b u'(x)v(x) dx .$$

Here,

$$f(x)\Big|_a^b = f(b) - f(a) .$$

Example 1. Evaluate:

(1)

$$\int x \cos x dx$$

(2)

$$\int t^2 e^t dt$$

Problem 1. Evaluate:

(1)

$$\int t^2 \sin(\beta t) dt$$

(2)

$$\int \ln \sqrt{x} dx$$

(3)

$$\int \tan^{-1}(2y) dy$$

(4)

$$\int (\ln x)^2 dx$$

(5)

$$\int e^{2\theta} \sin(3\theta) d\theta$$

(6)

$$\int_0^{1/2} x \cos(\pi x) dx$$

(7)

$$\int_1^5 \frac{\ln R}{R^2} dR$$

Integration by Substitution. Let $u = g(x)$. Then,

$$\int f(g(x))g'(x) dx = \int f(u) du ,$$

and

$$\int_a^b f(g(x))g'(x) dx = \int_{g(a)}^{g(b)} f(u) du .$$

Example 2. Evaluate:

(1)

$$\int \cos x \sin x dx$$

(2)

$$\int_0^1 x e^{4x^2+3} dx$$

Problem 2. Evaluate:

$$(1) \int \frac{e^u}{(1-e^u)^2} du$$

$$(2) \int \frac{a + bx^2}{\sqrt{3ax + bx^3}} dx$$

$$(3) \int \frac{(\ln x)^2}{x} dx$$

$$(4) \int \sec^2 \theta \tan^3 \theta d\theta$$

$$(5) \int e^x \sqrt{1 + e^x} dx$$

$$(6) \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$$

$$(7) \int \frac{z^2}{z^3 + 1} dz$$

$$(8) \int \sin x \sin(\cos x) dx$$

$$(9) \int x\sqrt{x+2} dx$$