CALCULUS

MINI EXAM 3
FIRST SECTION

NAME: _____

ID: _____

SCORE: _____/ 80

RULES:
- You have 30 minutes to complete the exam.
- There are 3 questions and 80 points in total.
- You can use a non-graphing calculator.
- If you need to go to the restroom, please turn in your cellphone before.
- If you need hints, 1 hint is worth 3 points.

Problem 1 (20 points). A 170-cm-tall person walks toward a wall at a rate of 0.5 m/sec. A spotlight is located on the ground 10m from the wall. How fast does the height of the person's shadow on the wall change when the person is 5 m from the wall?

*Problem* 2 (20 points). Consider the following function
$$f(x) = \sin x + x^2$$
(1) Find the linear approximation $L(x)$ of $f(x)$ around $a = 1$.

(2) Find the Taylor series (to the 3rd power) of the function $f(x)$ around $a = 1$.

Problem 3 (20 points). (1) Sketch the graph of a function f that is continuous on $[1, 5]$ that has absolute maximum at 5, absolute minimum at 2, local maximum at 3, local minima at 2 and 4.

(2) Let m be the number of local minima and M be the number of local maxima. Can you create a function where $M > m + 2$? Draw a graph to support your claim.

Problem 4 (20 points). Consider a lifeguard at position A at a circular pool with diameter 50m. He must reach someone who is drowning on the exact opposite side of the pool, at position C. The lifeguard swims with a speed v and runs around the pool at speed $w = 2v$.
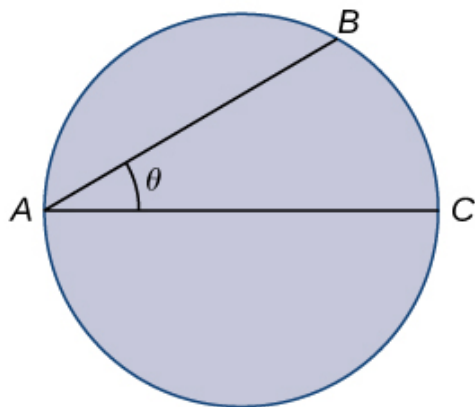
(1) Find a function that measures the total amount of time it takes to reach the drowning person as a function of the swim angle, θ.

(2) Find at what angle θ the lifeguard should swim to reach the drowning person in the least amount of time.