WORKSHEET 10

MATH 101

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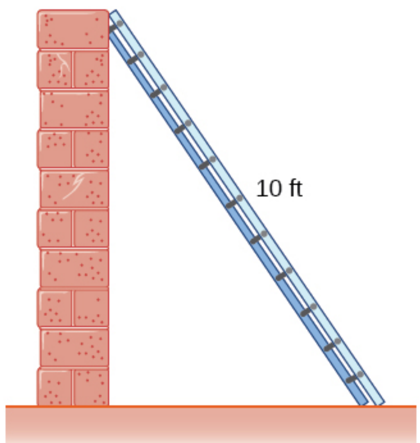
Related rates

Question 1. *A 10-ft ladder is leaning against a wall. If the top of the ladder slides down the wall at a rate of 2 ft/sec, how fast is the bottom moving along the ground when the bottom of the ladder is 5 ft from the wall?*



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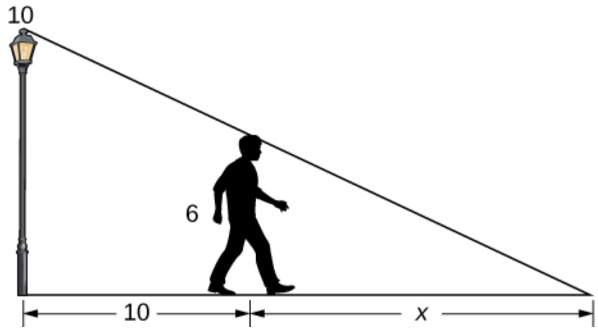
Question 2. *A 6-ft-tall person walks away from a 10-ft lamppost at a constant rate of 3ft/sec. What is the rate that the tip of the shadow moves away from the pole when the person is 10ft away from the pole?*



Approximations

The goal of linear approximation is to find a linear function that can best represent a function $f(x)$ near a particular point $x = a$. It turns out, the equation for the tangent line provides the best linear approximation for $f(x)$!

If $y = f(x)$, then the linear approximation of $f(x)$ when $x = a$ is

$$L(x) = f(a) + f'(a)(x - a)\,.$$

Question 3. *(1) Let $f(x) = e^{x^2}$. Find $L(x)$ at $x = 1$.*

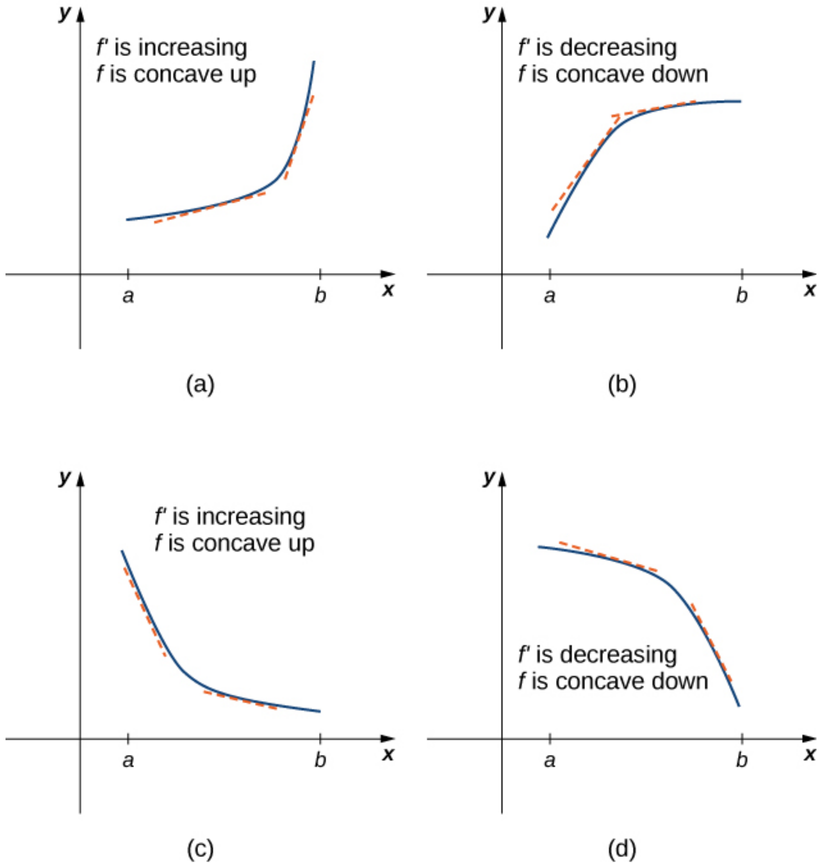
(2) Invent a way to compare the error between $f(x)$ and $L(x)$ with that between $f(x)$ and $g(x) = e + x - 1$.

The linear approximation just captures the slope of the function at a particular point. Let's reverse engineer the problem. If you knew $L(x)$ must look something like $b(x-a)+c$, then c must be $f(a)$ and and b must be $f'(a)$.

If you were to use a quadratic function to approximate $f(x)$, how can you find that magical quadratic function?

Optimization

The meaning of second derivative is that it tells us about the concavity of the graph of a function.



Definition 1. Let f be a function defined over an interval I and let $c \in I$. We say f has an absolute maximum on I at c if $f(c) \geqslant f(x)$ for all $x \in I$. We say f has an absolute minimum on I at c if $f(c) \leqslant f(x)$ for all $x \in I$. If f has an absolute maximum on I at c or an absolute minimum on I at c, we say f has an absolute extremum on I at c.

Theorem 1. *If f is a continuous function over the closed, bounded interval $[a, b]$, then there is a point in $[a, b]$ at which f has an absolute maximum over $[a, b]$, and there is a point in $[a, b]$ at which f has an absolute minimum over $[a, b]$.*