MATH 102: Ideas of Math

Day 17

Nov 1, 2023

Agenda

1. Final, project discussion
2. This week: function and relation
   2.1 Velleman chapters 4, 5
   2.2 Newstead chapter 3
3. Next week: induction

Function

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Definition (Newstead, Chapter 4)

A function f from a set X to a set Y is a specification of elements $f(x) \in Y$ for $x \in X$ such that

$$\forall x \in X, \exists! y \in Y, y = f(x).$$

Given $x \in X$, the unique element $f(x) \in Y$ is called the value of f at x.
X is called the *domain* of f, and Y is called the *codomain*.
We denote the *range* of f is

$$f(X) = \{f(x) \mid x \in X\}.$$

We write $f : X \to Y$ to denote the assertion that f is a function with domain X and codomain Y.
We sometimes write $\mathrm{Dom}(f)$ to mean domain of f and $\mathrm{Ran}(f)$ to mean the range of f.

Cartesian Product

Definition

Let X, Y be sets. The *cartesian product* of X and Y is the set $X \times Y$, defined by

$$X \times Y = \{(a, b) | a \in X \land b \in Y\} \, .$$

The elements $(a, b) \in X \times Y$ are called *ordered pairs*, whose defining property is that

$$\forall x \in X, \forall y \in Y, (a, b) = (x, y) \iff a = x \land b = y \, .$$

Graph of a function

Definition

Let $f : X \to Y$ be a function. The *graph* of f is the subset $\mathrm{Gr}(f) \subseteq X \times Y$ defined by

$$\mathrm{Gr}(f) = \{(x, f(x)) | x \in X\} = \{(x, y) \in X \times Y | y = f(x)\} .$$

Relation

Definition

Let A, B be sets. Then the set $R \subseteq A \times B$ is called a relation from A to B. We also define the domain and range of a relation R.

$$\begin{aligned}\mathrm{Dom}(R) &= \{a \in A \mid \exists b \in B, (a, b) \in R\}\\ \mathrm{Ran}(R) &= \{b \in B \mid \exists a \in A, (a, b) \in R\}.\end{aligned}$$

If $(x, y) \in R$, then we say that x is related to y by R and write xRy.

Examples

1. $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} | x > y\}$ is the relation from $\mathbb{R}$ to $\mathbb{R}$. xRy here means $x > y$.

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2. Let P be the set of all people at FUV, and C be the set of all courses at FUV. Let $E = \{(p, c) \in P \times C \mid p \text{ is enrolled in course } c\}$. Then E is a relation from P to C. xEy means x is enrolled in y.