MATH 102: IDEAS OF MATH

WORKSHEET 12

1. Concepts

Theorem 1.1 (The Induction Principle)**.** *Suppose that we have a sequence of statements $P(n)$ labeled by the natural numbers $1, 2, \ldots$ such that we know that*

(1) $P(1)$ is true, and
(2) $\big(P(1) \wedge P(2) \wedge \cdots \wedge P(n)\big) \Rightarrow P(n+1)$.

Then all the statements $P(1), P(2), \ldots$ are true.

We will not discuss the proof of this theorem. However, curious minds can find the proof in Section 4.2 of Newstead.

Structure of induction proof. Induction proofs typically have the following structure:

(1) Identify the statements $P(n)$.
(2) Step 2 is also called *base of induction*: prove $P(0)$ or $P(1)$ (sometimes it doesn't make sense to talk about $P(0)$).
(3) Assume that all $P(k)$ with $k \leqslant n$ are true – this is called the *induction hypothesis*. Now perform the *step of induction*: prove that $P(n+1)$ is true.
(4) Finally, conclude by the Principle of Induction that all $P(n)$ are true.

□

2. Problems

Problem 1. Prove that

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}\,.$$

Problem 2. Prove that for all natural number n, $n^3 - n$ is divisible by 3.

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Problem 3. Prove by induction that for all $n \in \mathbb{N}$,

$$2^0 + 2^1 + 2^3 + \cdots + 2^n = 2^{n+1} - 1\,.$$

Problem 4. For all $n \geqslant 4$, we have $3n < 2^n$.