MATH 102: HOMEWORK 5

DUE DATE: TUES, NOV 14

Problem 1. Describe $\mathbb{R} \times \mathbb{N}$ using set notation. What is a graphical representation of this set? (For this, make sure you know how to insert figures in LaTeX).

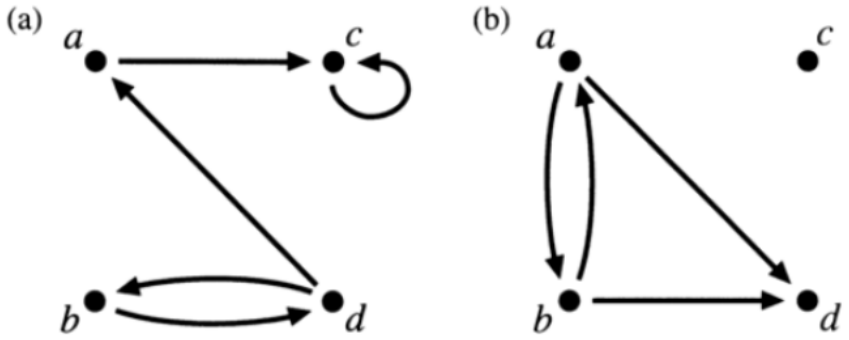
Problem 2.
(1) Is the set $([0,1] \cup [1,2]) \times ([2,3] \cup [3,4])$ the same with $([0,1] \times [2,3]) \cup ([1,2] \times [3,4])$?
(2) Let A, B, C, D be sets. What must be true for the relationship between $(A \cup B) \times (C \cup D)$ and $(A \times C) \cup (B \times D)$?
(3) Prove your claim in (2).

Problem 3. Prove that
(1) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
(2) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

Problem 4. Let R be a relation from A to B and S be a relation from B to C. We define the composition of relations $S \circ R$ as follows

$$S \circ R = \{(a, c) \in A \times C \mid \exists b \in B, (a, b) \in R \wedge (b, c) \in S\}.$$

Consider the following graphs.



Let R be the relation in (a) and S be the relation in (b).
(1) Write, in set notation, $S \circ R$.
(2) Draw $S \circ R$.

Date: November 8, 2023.