MATH 102: IDEAS OF MATH

WORKSHEET 11

1. Concepts

Definition 1. Let $f : X \to Y$ be a function. A left inverse (or post-inverse) for f is a function $g : Y \to X$ such that $g \circ f = \mathrm{id}_X$.

Definition 2. Let $f : X \to Y$ be a function. A right inverse (or pre-inverse) for f is a function $g : Y \to X$ such that $f \circ g = \mathrm{id}_Y$.

2. Problems

Problem 1. Let $D \subseteq \mathbb{Q}$ be the set of *dyadic rational numbers*, that is

$$D = \left\{ x \in \mathbb{Q} | \exists a \in \mathbb{Z}, \exists n \in \mathbb{N}, x = \frac{a}{2^n} \right\} .$$

Let $k \in \mathbb{N}$ and define $f : D \to D$ by

$$f(x) = \frac{x}{2^k} .$$

Show that f is bijective.

Problem 2.
(1) Are left and right inverses the same?
(2) When can a function have right inverse?
(3) When can a function have left inverse?

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