MATH 102: IDEAS OF MATH

WORKSHEET 4

1. Logical equivalence

Definition 1.1. Let P and Q be logical formulae. We say that P and Q are logically equivalent and write $P \equiv Q$ if Q can be derived from P and P can be derived from Q.

Problem 1.1. Show that

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)\,,$$

where P, Q, R are propositional variables.

It turns out that the following is true.

Theorem 1.2. *Two propositional formulae are logically equivalent if and only if their truth values are the same under any assignment of truth values to their constituent propositional variables.*

2. Truth table

Theorem 2.1 (De Morgan's laws for logical operators)**.** *Let P, Q be propositional variables. Then,*

(1) $\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$,
(2) $\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$.

Theorem 2.2 (Distributive laws)**.** *Let P, Q, R be propositional variables. Then,*

(1) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$,
(2) $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$.

Theorem 2.3 (Absorption laws)**.** *Let P, Q be propositional variables. Then,*

(1) $P \vee (P \wedge Q) \equiv P$,
(2) $P \wedge (P \vee Q) \equiv P$.

Problem 2.1. Prove the theorems above by truth tables.

Date: September 13, 2023.

There are a few other laws that are useful to know.

Theorem 2.4 (Tautology laws). *(1) $P\wedge$ a tautology $\equiv P$*
(2) $P\vee$ a tautology $\equiv$ tautology
(3) $\neg$ (a tautology) is a contradiction.

Theorem 2.5 (Contradiction laws). *(1) $P\wedge$ contradiction $\equiv$ contradiction.*
(2) $P\vee$ contradiction $\equiv P$
(3) $\neg$ (a contradiction) is a tautology.

Problem 2.2 (Problem 9 in section 1.2). Use truth table to determine which of these statements are tautologies, which are contradictions, and which are neither:

(1) $(P \vee Q) \wedge (\neg P \vee \neg Q)$
(2) $(P \vee Q) \wedge (\neg P \wedge \neg Q)$
(3) $(P \vee Q) \vee (\neg P \vee \neg Q)$
(4) $[P \wedge (Q \vee \neg R)] \vee (\neg P \vee R)$.