MATH 102: FINAL

NAME:_____ ID: _ _ _ _ _ _ _ _ _

There are 3 problems.
Make sure you justify all your work for complete credit.

SUBMITTED PROBLEMS: _____

Rules

- You have 80 minutes to complete your work..
- Closed books.
- No use of internet, textbooks, computer algebra systems, calculators.
- No collaboration.
- 1 person per bathroom break. When you go to the bathroom, turn in your cellphone and exam until return.

Good luck!

Date: Dec 3, 2023.

Questions

1. *[60 points.]* Translate the following into symbols, then negate the symbolic sentence, and translate the negation back to English.
   (a) Every natural number, when squared, remains a natural number.

   (b) Every real number has a cube root in the reals.

   (c) Not every integer has a square root in the reals.

   (d) There exists a smallest natural number.

(e) There exists a largest negative integer.

(f) Every real number, when multiplied by zero, equals zero.

2. *[40 points.]*
   (a) *[10 points.]* Give a definition of function.

   (b) *[10 points.]* Give a definition of bijective function.

   (c) *[10 points.]* Give an example of a bijective function with both domain and co-domain to be $\mathbb{R}$.

(d) *[10 points.]* Prove that the function you gave in the previous part is a bijection.

3. *[20 points.]* Prove by contradiction that if $A \cap C \subseteq B$ and $a \in C$, then $a \notin A \backslash B$.