MATH 102: IDEAS OF MATH

WORKSHEET 6 LIVE SESSION

Problem 2. Prove that
$$\left\{x \in \mathbb{R} | x^2 \leq 1\right\} = [-1, 1] .$$

We want to show that
$$\left\{x \in \mathbb{R} | x^2 \leq 1\right\} \subseteq [-1, 1] ,$$
and
$$\left\{x \in \mathbb{R} | x^2 \leq 1\right\} \supseteq [-1, 1] .$$

($\subseteq$). First, to show $\{x \in \mathbb{R}|x^2 \leq 1\} \subseteq [-1, 1]$, let a be arbitrary in $\{x \in \mathbb{R}|x^2 \leq 1\}$. By definition, $a^2 \leq 1$, which means $a^2 - 1 \leq 0$. Rewriting the left hand side, we have $(a-1)(a+1) \leq 1$. Then it must be the case that $a - 1$ and $a + 1$ have opposite signs. There are two cases for this to happen.

(1) $a - 1 \leq 0$ and $a + 1 \geq 0$. This means, $a \leq 1$ and $a \geq -1$. By definition, $a \in [-1, 1]$.
(2) $a - 1 \geq 0$ and $a + 1 \leq 0$. This means, $a \geq 1$ and $a \leq -1$, which is a contradiction.

Therefore, $a \in [-1, 1]$. Because, a is arbitrary, $\{x \in \mathbb{R}|x^2 \leq 1\} \subseteq [-1, 1]$.

($\supseteq$). Secondly, to show $\{x \in \mathbb{R}|x^2 \leq 1\} \supseteq [-1, 1]$, let a be arbitrary in $[-1, 1]$. By definition, $a \leq 1$ and $a \geq -1$, which means $|a| \leq 1$. This implies, $a^2 \leq 1$. Sine a is arbitrary, $\{x \in \mathbb{R}|x^2 \leq 1\} \supseteq [-1, 1]$. $\square$

Problem 3. Let a be arbitrary in $X \cap Y$. That means, $a \in X \wedge a \in Y$. This means, $a \in X$, which implies that $a \in X \vee a \in Y$. Therefore, $a \in X \cup Y$. Since a is arbitrary, $X \cap Y \subseteq X \cup Y$. $\square$

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