MATH 170: HOMEWORK 6

DUE: OCTOBER 29, 2021

Graded for accuracy: 1,2, 4.
Graded for completion: 3.
Instructions: Problems that are graded for accuracy must be correct to get points. Problems that are graded for completion must show some trying effort.

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1. Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$, $g : [0, \infty) \to \mathbb{R}$, $g(x) = \sqrt{x}$ Determine the formulas for the following compositions of functions.
   (a) $f \circ g$
   (b) $g \circ f$
   (c) $g \circ g$
   (d) $f \circ g \circ g$

2. Let $f : X \to Y$ be a function between two sets.
   (a) Assume that f is a bijection. Show that then you can define a function $g : Y \to X$ such that $f \circ g$ is the identity function of Y and $g \circ f$ is the identity function of X.
   (b) Assume that $g : Y \to X$ is a function as in the previous part. Prove that then f is necessarily a bijection.

3. Watch Vi Hart's videos about Fibonacci spirals in nature:
   Part 1: https://www.youtube.com/watch?v=ahXIMUkSXX0
   Part 2: https://www.youtube.com/watch?v=lOIP_Z_-0Hs
   Part 3: https://www.youtube.com/watch?v=14-NdQwKz9w
   (a) Create an angle 137.5° and draw approximately 30 petals, each 137.5° from the previous one.
   (b) When you are done with part (a), mark the spirals and count the number of them.
   (c) After watching the third part, in your own words explain why we almost always get a Fibonacci number of spirals in plants.

4. Prove by induction that for every $n \in \mathbb{N}$
$$1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1 \,.$$