MATH 102: Ideas of Math

Day 10

Sep 28, 2023

More about sets

$$
\begin{aligned}
[a, b] &= \{x \in \mathbb{R} \mid a \leqslant x \leqslant b\}\,, \\
[a, b) &= \{x \in \mathbb{R} \mid a \leqslant x < b\}\,, \\
(a, b] &= \{x \in \mathbb{R} \mid a < x \leqslant b\}\,, \\
(a, b) &= \{x \in \mathbb{R} \mid a < x < b\}\,.
\end{aligned}
$$

Set comparisons

Definition

Let X be a set. A *subset* of X is a set U such that

$$\forall a, (a \in U \implies a \in X) .$$

We write $U \subseteq X$ for the assertion that U is a subset of X.
The notation $U \subsetneq X$ means that U is a *proper subset* of X, that is a subset of X that is not equal to X.
The notation $U \not\subseteq X$ means that U is NOT a subset of X.

In order to prove that U is a subset of X, it is sufficient to take an arbitrary element $a \in U$ and prove that $a \in X$.

Axiom of extentionality

Let X and Y be sets. Then $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$.

Empty sets

Definition

A set is *non-empty* if it contains at least one element. Otherwise, it is *empty*.

Question: How many empty sets are there?

Set operations

Definition (Pairwise intersection)

Let X and Y be sets. The *pairwise intersection* of X and Y, denoted $X \cap Y$ is defined by

$$X \cap Y \stackrel{\text{def}}{=} \{a \mid a \in X \wedge a \in Y\}.$$

Set operations

Definition (Pairwise union)

Let X and Y be sets. The *pairwise union* of X and Y, denoted $X \cup Y$ is defined by

$$X \cup Y \stackrel{\text{def}}{=} \{a \mid a \in X \vee a \in Y\}.$$

Set operations

Definition (Relative complement)

Let X and Y be sets. The *relative complement* of Y and X, denoted $Y \setminus X$ is defined by

$$Y \setminus X \stackrel{\text{def}}{=} \{a \mid a \in Y \wedge a \notin X\}\,.$$

Indexed families of sets

Definition

An *indexed family of sets* is a specification of a set X_i for each element i of some *indexing set* I. We write $\{X_i \mid i \in I\}$ for the indexed family of sets.

Example

$$\left\{ \left[0, 1 + \frac{1}{n}\right) \,\middle|\, n \in \mathbb{N} \right\}$$