MATH 102: MIDTERM

NAME:_____ ID: _ _ _ _ _ _ _ _ _

GOOD LUCK!

There are five questions. Make sure you justify all your work for complete credit.

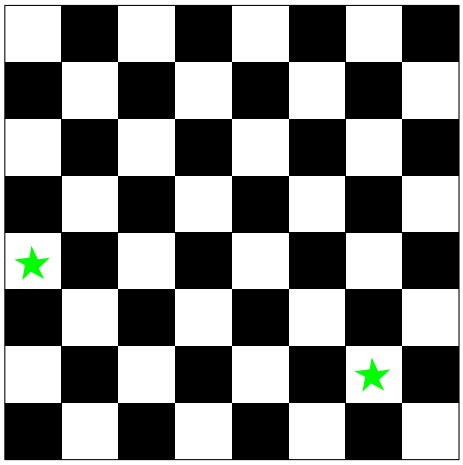
Rules

- You have 80 minutes to complete your work..
- Closed books.
- No use of internet, textbooks, computer algebra systems, calculators.
- No collaboration.
- 1 person per bathroom break. When you go to the bathroom, turn in your cellphone and exam until return.

Date: Oct 12, 2023.

Questions

1. *[20 points.]*
   (a) *[10 points.]* Can you fill the following chessboard with 2x1 domino pieces? Explain your answer.



   (b) *[10 points.]* Show that the square of an odd integer is again odd by direct proof from definition.

2. *[20 points.]* The symmetric difference of two sets A and B is defined as follows

$$A \triangle B = (A \setminus B) \cup (B \setminus A) .$$

   (a) *[10 points.]* Use the Venn diagram to represent $A \triangle B$.
   (b) *[10 points.]* Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{2, 4, 6\}$. What is $A \triangle B$?

3. *[20 points.]* What is Bezout's identity? Give 2 examples.

4. *[20 points.]* Prove that if $a \mid bc$ and $\gcd(a, b) = 1$, then $a \mid c$.

5. *[20 points.]* For any sets A and B, prove that

$$A \triangle B \subseteq A \cup B\,.$$

Hint: Start with "Let x..." and use definitions of the operations.