HOMEWORK 1

Graded for accuracy: 1, 2.
Graded for completion: 3, 4, 5.
Instructions: Problems that are graded for accuracy must be correct to get points. Problems that are graded for completion must show some trying effort.

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1. 15 children participate in an Easter Egg hunt. If they bring back a total of 100 eggs, must two of them bring back the same number of eggs?
   Warning: some kids might not find any Easter eggs.

2. Suppose we pick $n + 1$ numbers from the set $\{1, 2, \ldots, 2n\}$ where $n \geqslant 1$. Let $j \geqslant 1$ be a divisor of n. Is it true or false that there are always two numbers have a difference of j apart from each other? Why?

3. Show that if you select 6 numbers from the set $\{1, 2, \ldots, 10\}$, one must be a multiple of another.
   Hint: label pigeonholes by pairs of numbers.

4. The Earth has more than 7 billion people and almost no one lives 100 years. Suppose this longevity fact remains true. How do you know that some year soon, more than 50 million people will die?
   Hint: "soon" means within 100 years.

5. You have 10 pairs of socks, five black and five rainbow, but they are all mixed up in a drawer. It's 5 a.m. in the morning, and you don't want to turn on the lights in your dark room. How many socks must you pull out to guarantee that you have a pair of one color? How many must you pull out to have two good pairs (one black pair and one rainbow pair is okay)?