MATH 102: IDEAS OF MATH

WORKSHEET 10

1. Concepts

Definition 1 (Injective functions)**.** Let $f : A \to B$ be a function. f is said to be *injective* (or *one-to-one*) if

$$\forall a_1, a_2 \in A, f(a_1) = f(a_2) \implies a_1 = a_2 \,.$$

An injective function is called an *injection*.

Definition 2 (Surjective functions)**.** Let $f : A \to B$ be a function. f is said to be *surjective* (or *onto*) if

$$\forall b \in B \exists a \in A, f(a) = b \,.$$

A surjective function is called a *surjection*.

Definition 3 (Bijective function)**.** A function f is *bijective* if it is both injective and surjective.

A bijective function is called a *bijection*.

Definition 4. Let A be a set. A function $f : A \to A$ is called an identity function if $f(a) = a$.

An identity function on a set A is denoted by id_A.

Definition 5. Let $f : A \to B$ and $g : B \to C$ be functions. The composition of g and f, denoted by $g \circ f : A \to C$, is defined to be

$$(g \circ f)(a) = g(f(a)) \,,$$

for all $a \in A$.

2. Problems

Problem 1. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to [0, \infty)$ be such that $f(x) = 2+5x$, $g(x) = x^2 + 3$. Find

(1) Domain and codomain of $g \circ f$,
(2) A formula for $g \circ f$.

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Problem 2. Let $A = \mathbb{R} \setminus \{1\}$, and let $f : A \to A$ be defined as follows: $f(x) = \frac{x+1}{x-1}$.
a. Show that f is one-to-one and onto.
b. Show that $f \circ f = \mathrm{id}_A$.

Problem 3. Let $A = \mathcal{P}(\mathbb{R})$. Define $f : \mathbb{R} \to A$ by the formula $f(x) = \{y \in \mathbb{R} \mid y^2 < x\}$.
a. Find $f(2)$.
b. Is f one-to-one? Is it onto?

Problem 4. Suppose $f : A \to B$ and $g : B \to C$.

(1) If f and g are both one-to-one, then so is $g \circ f$.
(2) If f and g are both onto, then so is $g \circ f$.