MATH 102: Ideas of Math

Day 16

Oct 19, 2023

Uniqueness

Uniqueness is typically coupled with existential quantifier.

Definition

The *unique existential quantifier* is the quantifier $\exists!$, defined such that $\exists! x \in X, p(x)$ is short hand for

$$(\exists x \in X, p(x)) \wedge (\forall a \in X, \forall b \in X, [(p(a) \wedge p(b)) \implies a = b]\,.$$

Function

Definition

A function f from a set X to a set Y is a specification of elements $f(x) \in Y$ for $x \in X$ such that

$$\forall x \in X, \exists! y \in Y, y = f(x)\,.$$

Given $x \in X$, the unique element $f(x) \in Y$ is called the value of f at x.
X is called the *domain* of f, and Y is called the *codomain*.
We write $f : X \to Y$ to denote the assertion that f is a function with domain X and codomain Y.