MATH 102: IDEAS OF MATH

WORKSHEET 11
ADAPTED FROM HTTPS://WWW.MATH.TORONTO.EDU/LSHORSER/SUMMER2017/PUMPII-QUANTIFIERS-COMPLETE.PDF

1. Review

Problem 1. Consider the example in the book:

(1) Good food is not cheap.
(2) Cheap food is not good.

Do the two sentences mean the same?

Problem 2. Watch https://www.youtube.com/watch?v=fQ3Md3CZxks

What's going on? Sketch the truth table to describe the situation in the video.

Problem 3 (name of the problem). Watch https://www.youtube.com/watch?v=zqSlij2Idgg

What's going on? Sketch the truth table to describe the situation in the video.

Problem 4 (name of the problem). Watch https://www.youtube.com/watch?v=juFsA25b9EY

What's going on? Sketch the truth table to describe the situation in the video.

Problem 5 (name of the problem). Watch https://www.youtube.com/watch?v=BUqMNVnELzE&list=PLMpofmkxKHBJfta_JzekLbWGHUSLUJoLt

What's going on? Sketch the truth table to describe the situation in the video.

2. Quantifiers

Open sentences are sometimes technically called predicates. Because open sentences doesn't have a truth value as they may have unknowns, we denote them as something that look like a function $S(x)$.

For example, we may use $S(n)$ to denote the sentence "n is even."

Then, $S(1)$ is false and $S(10)$ is true.

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Definition 1. In logic, quantifiers are very important. The most important among them are:

(1) Universal quantifier: $\forall$, means "for all", "for every", "for each"
(2) Existential quantifier: $\exists$, means "there exists", "there is at least one"

Problem 6. Translate the logical expression into English and determine their truth values.

(1) $\forall x \in \mathbb{Z}, 2 \mid x \implies x \mid 4$

(2) $\forall x \in \mathbb{Z}, 4 \mid x \implies 2 \mid x$

(3) $\exists x \in \mathbb{N} : x < -2$

(4) $\forall x \in \mathbb{Z}, \forall y \mathbb{Z}, x - y = y - x$

(5) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, xy = yx$

(6) $\forall x \in \mathbb{R}, \forall y \in R, x + y = 1$

(7) $\exists x \in \mathbb{R}, \forall y \in R, x + y = 1$

(8) $\forall x \in \mathbb{R}, \exists y \in R, x + y = 1$

Definition 2 (Negation with quantifiers)**.** What do you think the following would be equivalent with?

(1) $\sim (\forall x \in A, S(x))$

(2) $\sim (\exists x \in A, S(x))$

Problem 7. Negate all the statements in the previous problem.

Problem 8. True or False?

If everyone loves my baby and my baby loves only me, then I am my own baby.