MATH 102: Ideas of Math

Day 9

Sep 26, 2023

More about sets

$$[a, b] = \left\{ x \in \mathbb{R} \mid a \leqslant x \leqslant b \right\},$$
$$[a, b) = \left\{ x \in \mathbb{R} \mid a \leqslant x < b \right\},$$
$$(a, b] = \left\{ x \in \mathbb{R} \mid a < x \leqslant b \right\},$$
$$(a, b) = \left\{ x \in \mathbb{R} \mid a < x < b \right\}.$$

Set comparisons

Definition

Let X be a set. A *subset* of X is a set U such that

$$\forall a, (a \in U \implies a \in X).$$

We write $U \subseteq X$ for the assertion that U is a subset of X.

The notation $U \subsetneq X$ means that U is a *proper subset* of X, that is a subset of X that is not equal to X.

The notation $U \not\subseteq X$ means that U is NOT a subset of X.

In order to prove that U is a subset of X, it is sufficient to take an arbitrary element $a \in U$ and prove that $a \in X$.

Axiom of extentionality

Let X and Y be sets. Then $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$.

Empty sets

Definition
A set is *non-empty* if it contains at least one element. Otherwise, it is *empty*.

Question: How many empty sets are there?