MATH 170 – 601: MIDTERM 3

GOOD LUCK!

There are three questions. Make sure you justify all your work for complete credit.

Rules

- You have 50 minutes to complete your work and 10 minutes to upload your work.
- Open notes (you can use your notes freely).
- No use of internet, textbooks, computer algebra systems, calculators.
- No collaboration.

Questions

Each sub-problem (a,b) is worth 10 points.

1. Consider the following graph

$$\left\{\{a,b,c,d,e,f,g,h,i,j,k,l\},\{ab,ac,ad,bc,bd,cd,be,ef,eg,fg,gh,hi,hj,hk,hl,ij,ik,il,jk,jl,kl\}\right\}.$$

   (a) Draw the graph.
   (b) Recall for a complete graph (all vertices are connected with each other by an edge) of k vertices, K_k,
   $$P_{K_k}(n) = n(n-1)\dots(n-k+1)\,.$$
   Use the deletion-contraction formula to determine how many ways there are to color the above graph using 10 colors.

2. Consider a sample space of 7 coin tosses.
   (a) What is the probability that the first and the last toss is the same?
   (b) What is the probability that the last toss is head, given that the first toss is head?

3. (a) Compute the following:
   $$\binom{1}{0}+\binom{1}{1},$$
   $$\binom{2}{0}+\binom{2}{1}+\binom{2}{2},$$
   $$\binom{3}{0}+\binom{3}{1}+\cdots+\binom{3}{3}.$$
   (b) What should be the formula for
   $$\binom{n}{0}+\binom{n}{1}+\cdots+\binom{n}{n}.$$

Bonus (5pts): Prove the formula you just guessed.