

MATH 102: IDEAS OF MATH

WORKSHEET 3

1. Statement

Problem 1.1. Classify the following as statements or non-statemetns:

- (1) I had a hearty laugh at the joke!
- (2) He had a fever of 102° but still completed his task.
- (3) Why would anyone want to do this?
- (4) If you go to the city, you will find what you are seeking.
- (5) I am so happy today!
- (6) $x > x - 1$.
- (7) For all integers x , $x + 2$ is even.

Problem 1.2. Classify whether the following statements are propositions.

- (1) The Sun is a star.
- (2) Every prime number is odd.
- (3) The Moon is a star.
- (4) Every even number
- (5) If 4 is a prime, then FUV is the best university in the world.
- (6) “ $P \implies Q$ ” has the same meaning as “ $\neg P \vee Q$ ”.

2. Logical equivalence

Definition 2.1. Let P and Q be logical formulae. We say that P and Q are logically equivalent and write $P \equiv Q$ if Q can be derived from P and P can be derived from Q .

Problem 2.1. Show that

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R),$$

where P, Q, R are propositional variables.

It turns out that the following is true.

Theorem 2.2. *Two propositional formulae are logically equivalent if and only if their truth values are the same under any assignment of truth values to their constituent propositional variables.*

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Even though the proof of this theorem is beyond the scope of this class, we can use this to our advantage to check whether P and Q are logically equivalent by just looking at their truth tables.

Theorem 2.3. *Two propositional formulae are logically equivalent if and only if they have the same truth table.*

Problem 2.2. Check the theorem above by redo Problem 1.2.6 and Problem 2.2