MATH 170: MIDTERM 2

GOOD LUCK!

There are three questions. Make sure you justify all your work for complete credit.

Rules

- You have 50 minutes to complete your work and 10 minutes to upload your work.
- Open notes (you can use your notes freely).
- No use of internet, textbooks, computer algebra systems, calculators.
- No collaboration.

Questions

1. Let A, B be sets. Recall that the symmetric difference of two sets A and B is

$$A \triangle B = \{x \mid x \in A \text{ or } x \in B \text{ but not both}\}.$$

Determine whether the following are true or false via the Venn diagram.

$$A = A \cap ((A \cap B) \setminus (A \triangle B)).$$

2. (a) Give an example of a function f from $X = \mathbb{Z}$ to $Y = \mathbb{Z}$ that is not the identity function so that $f \circ f = id_{\mathbb{N}}$.
   (b) What is $f \circ f \circ f \circ f$?

3. Prove by induction that $\forall n \in \mathbb{N}$,

$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$