# MATH 102: IDEAS OF MATH

WORKSHEET 14

**Definition 1.** Given a pair of sets $A$ and $B$, suppose that each element $x \in A$ is associated to a unique element $f(x)$ of $B$. Then $f$ is said to be a function from $A$ to $B$. This is denoted by

$$f : A \to B\,.$$

$A$ is called the *domain* of $f$, $B$ is called the *codomain* of $f$. The set $\{f(x) : x \in A\}$ is called the *range* of $f$.

*Question* 1. What is the vertical line test?

**Definition 2.** A function $f : A \to B$ is *injective* (of *one-to-one*) if $f(a_1) = f(a_2)$ implies $a_1 = a_2$.

*Question* 2. Give a few examples of injective function.

*Date*: Nov 14, 2024.

*Question* 3. Give an equivalent definition of injectivity using contrapositivity.

**Definition 3.** A function $f : A \to B$ is *surjective* (of *onto*) if for every $b \in B$, there exists some $a \in A$ such that $f(a) = b$.

*Question* 4. Give a few examples of injective function.

**Definition 4.** A function is *bijective* if it is both injective and surjective.

*Question* 5. (1) Write a strategy to prove injectivity.

(2) Write a strategy to prove surjectivity.

*Question* 6. Let $\mathbb{R}^+$ denote the nonnegative real numbers. Prove the following.

(a) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = x^2$ is not injective, surjective or bijective.

(b) $g : \mathbb{R}^+ \to \mathbb{R}$ where $g(x) = x^2$ is injective, but not surjective or bijective.

(c) $h : \mathbb{R} \to \mathbb{R}^+$ where $h(x) = x^2$ is surjective, but not injective or bijective.

(d) $k : \mathbb{R}^+ \to \mathbb{R}^+$ where $k(x) = x^2$ is injective, surjective and bijective.

**Definition 5.** Let $A, B$ be sets. A Cartesian product of $A$ and $B$, denoted by $A \times B$ is the set of all ordered pair $(x, y)$, where $x \in A$ and $y \in B$.

$$A \times B = \{(x, y) : x \in A, y \in B\}.$$

*Question* 7. Prove the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ where $f(x, y) = (x + 2y, 2x + 3y)$ is a bijection.