MATH 102: IDEAS OF MATH

WORKSHEET 5

Definition 1. A *set* is a collection of objects.

Set of objects x satisfying some property $P(x)$ is denoted by

$$\left\{x \,\middle|\, P(x)\right\}. \tag{0.1}$$

Denote

(1) The set of all integers to be $\mathbb{Z}$
(2) The set of all natural numbers to be $\mathbb{N}$
(3) the set of all rational numbers to be $\mathbb{Q}$

Problem 1. From high school knowledge, try to describe $\mathbb{Q}$ in terms of $\mathbb{Z}$ using set notation (0.1).

Problem 2. What's the difference between a logical formula and a propositional formula?

Problem 3. We short hand the phrase "x belongs to set X" by $x \in X$.

Problem 4. Represent the following sentences in logical formula form. Some sentences need to be rephrased so that things are clear to identify variables and predicates.

(1) $x - y$ is rational.
(2) Every even natural number $n \geqslant 2$ is divisible by k.
(3) There is an integer that is divisible by every integer.
(4) There is no greatest odd integer.
(5) Between any two distinct rational numbers is a third distinct rational number.
(6) If any integer has a rational square root, then that root is an integer.

Problem 5. Translate the following into English.

$$\forall a \in \mathbb{R}, (a \geqslant 0 \implies \exists b \in \mathbb{R}, a = b^2).$$

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