

## MATH 102: MIDTERM

NAME: \_\_\_\_\_ ID: \_ \_ \_ \_ \_

There are five questions PICK ONLY 3 to submit. I'll record the worst problems if you submit them all.

Make sure you justify all your work for complete credit.

SUBMITTED PROBLEMS: \_\_\_\_\_

### Rules

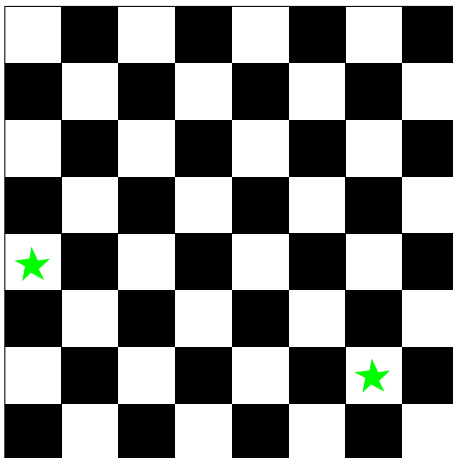
- You have 80 minutes to complete your work..
- Closed books.
- No use of internet, textbooks, computer algebra systems, calculators.
- No collaboration.
- 1 person per bathroom break. When you go to the bathroom, turn in your cellphone and exam until return.

Good luck!

## Questions

1. [20 points.]

- (a) [10 points.] Can you fill the following chessboard with  $2 \times 1$  domino pieces? Explain your answer.



- (b) [10 points.] Show that the square of an odd integer is again odd by direct proof from definition.

2. [20 points.] The symmetric difference of two sets  $A$  and  $B$  is defined as follows

$$A \triangle B = (A \setminus B) \cup (B \setminus A).$$

- (a) [10 points.] Use the Venn diagram to represent  $A \triangle B$ .  
(b) [10 points.] Let  $A = \{1, 2, 3, 4, 5, 6, 7\}$  and  $B = \{2, 4, 6\}$ . What is  $A \triangle B$ ?

3. [20 points.]

(a) [5 points.] What is Euclid's algorithm? Make sure you state all the conditions.

(b) [5 points.] Give an example to show how Euclid's algorithm works.

(c) [5 points.] What is Bezout's identity? Make sure you state all the conditions.

(d) [5 points.] Give an example to show how Bezout's identity works.

4. [20 points.] Prove that if  $a \mid bc$  and  $\gcd(a, b) = 1$ , then  $a \mid c$ .

5. *[20 points.]* For any sets  $A$  and  $B$ , prove that

$$A \Delta B \subseteq A \cup B.$$

Hint: Start with “Let  $x \dots$ ” and use definitions of the operations.