MATH 102: IDEAS OF MATH

WORKSHEET 12

Things to remember from previous worksheets:

(1) $P \implies Q$ is equivalent to $\sim P \vee Q$.

Definition 1 (Negation with quantifiers)**.** What do you think the following would be equivalent with?

(1) $\sim (\forall x \in A, S(x))$

(2) $\sim (\exists x \in A, S(x))$

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Problem 1. Negate all the statements.

(1) $\forall x \in \mathbb{Z}, 2 \mid x \implies x \mid 4$

(2) $\forall x \in \mathbb{Z}, 4 \mid x \implies 2 \mid x$

(3) $\exists x \in \mathbb{N} : x < -2$

(4) $\forall x \in \mathbb{Z}, \forall y \mathbb{Z}, x - y = y - x$

(5) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, xy = yx$

(6) $\forall x \in \mathbb{R}, \forall y \in R, x + y = 1$

(7) $\exists x \in \mathbb{R}, \forall y \in R, x + y = 1$

(8) $\forall x \in \mathbb{R}, \exists y \in R, x + y = 1$

Problem 2. Translate each of the following into symbolic sentences, negate, and translate the nagation to English.

(1) Every natural number, when squared, remains a natural number.

(2) Every real number has a cube root in the reals.

(3) Not every integer has a square root in the reals.

(4) There exists a smallest natural number.

(5) There exists a largest negative number.

(6) Every real number, when multiplied by zero, equals 0.

Problem 3. Try the following puzzle in the book:

> You are shown a set of four cards placed on a table (pictured below), each of which has a number on one side and a letter on the other side. Which card, or cards, must you turn over in order to determine whether the following is true or false: If a card shows an even number on one face, then its opposite face is an H?