MATH 102: IDEAS OF MATH

WORKSHEET 6

Notations:

(1) $\mathbb{R}$ is the set of all real numbers
(2) $\mathbb{N}$ is the set of all natural numbers
(3) $\mathbb{Z}$ is the set of integers
(4) $\emptyset = \{\}$ is the empty set
(5) $(a, b) = \{x \in \mathbb{R} : a < x < b\}$
(6) $[a, b) = \{x \in \mathbb{R} : a \leqslant x < b\}$
(7) $(a, b] = \{x \in \mathbb{R} : a < x \leqslant b\}$
(8) $[a, b] = \{x \in \mathbb{R} : a \leqslant x \leqslant b\}$

Problem 1. What are the following?

(1) Union:

(2) Intersection:

(3) Subtraction:

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(4) Complement:

Problem 2. (1) Prove that $\mathbb{Q} \subseteq \mathbb{R}$.

(2) Prove that $\mathbb{R} \not\subseteq \mathbb{Q}$.

Problem 3. Prove that,

$$\left\{ x \in \mathbb{R} \;\middle|\; x^2 \leqslant 1 \right\} = [-1, 1]\,.$$

Problem 4. Prove that for any two sets X and Y,

$$X \cap Y \subseteq X \cup Y\,.$$

Problem 5. Prove that

$$\bigcap_{n \geqslant 1} \left[0, 1 + \frac{1}{n}\right) = [0, 1]\,.$$

Problem 6. Prove that $A \setminus (B \setminus C) \subseteq (A \setminus B) \cup C$.

Problem 7. Prove that for every integer, the remainder when x^2 is divided by 4 is either 0 or 1.

Definition 1. Let X be a set. The *power set* of X, written $\mathcal{P}(X)$, is the set of all subsets of X.

$$\mathcal{P}(X) = \{A : A \subseteq X\}.$$

For example,

$$\mathcal{P}(\{1,2\}) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}.$$

Problem 8. Write out elements of

(1) $\mathcal{P}(\{1,2,3\})$,
(2) $\mathcal{P}(\emptyset)$,
(3) $\mathcal{P}(\mathcal{P}(\emptyset))$,
(4) $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))$.

The following problem is to help you clear some of the common confusion between the use of $\in$ and $\subseteq$ in power set.

Problem 9. True or false and prove your claim.

(1) $\mathcal{P}(\emptyset) \in \mathcal{P}(\mathcal{P}(\emptyset))$

(2) $\emptyset \in \{\{\emptyset\}\}$

(3) $\{\emptyset\} \in \{\{\emptyset\}\}$

(4) $\mathcal{P}(\mathcal{P}(\emptyset)) \in \{\emptyset, \{\emptyset, \{\emptyset\}\}\}$

Problem 10. True or False and prove your claim.

(1) $\mathcal{P}(X \cup Y) = \mathcal{P}(X) \cup \mathcal{P}(Y)$,

(2) $\mathcal{P}(X \cap Y) = \mathcal{P}(X) \cap \mathcal{P}(Y)$.