MATH 170: HOMEWORK 3

DUE: SEPTEMBER 27, 2021

Graded for accuracy: 2, 3.
Graded for completion: 1.
Instructions: Problems that are graded for accuracy must be correct to get points. Problems that are graded for completion must show some trying effort.

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1. A practice in inductive logic.
   a. Give an argument that supports the statement "Having a pet is good".
   b. Give an argument that supports the statement "Having a pet is bad".
   c. Which of your arguments do you find stronger than another?
   d. Is the statement from part (a) a *mathematical statement*? Why / why not?

One possible answer. (a) My mother has a pet (me) and she is happy. My grandfather has a pet (my father) and he is happy. My sister has a cat and she is happy. Therefore, having a pet is good.
(b) My father is always stressed about his pet (me). My niece was bitten by a dog. My friend Jake has to clean up for his cat everyweek and it seems torturing. Therefore, having a pet is bad.
(c) (b) seems stronger to me.
(d) The statement in part (a) is not a mathematical statement because it depends on the background theory of each person. There is no abstract principle that one can go to deduce the truth value of this statement.

□

2. For each of the following plain English statements, translate it into a symbolic logic propositional formula. Propositional variables in your formulae should represent the simplest propositions that they can.

(Note that each statement doesn't need to be true. Your job is just to translate to symbolic logic.)

a. Guinea pigs are quiet, but they are loud when they are hungry.

b. $\sqrt{2}$ can't be an integer because it is a rational number.

Answer. (a) Let P = "Guinea pigs are quiet", and R = "guinea pigs are hungry". Then the propositional formula is

$$P \vee (R \implies \neg P)\,.$$

(b) Let P = "$\sqrt{2}$ is an integer", Q = "$\sqrt{2}$ is a rational number". Then the propositional formular is

$$Q \implies \neg P\,.$$

□

3. For fixed $n \in \mathbb{N}$, let P represent the proposition 'n is even', Q represent the proposition 'n is prime' and R represent the proposition '$n = 2$'. For each of the following propositional formulae, translate it into plain English and determine whether it is true for all $n \in \mathbb{N}$, true for some values of n and false for some values of n, or false for all $n \in \mathbb{N}$.

a. $(P \wedge Q) \implies R$

b. $Q \wedge (\neg R) \implies (\neg P)$

c. $((\neg P) \vee (\neg Q)) \vee (\neg R)$

d. $(Q \wedge P) \vee (\neg R)$

Answer. (a) This is true for all n. To see this, we need to plug in all n into the following statement to see if it is true.
"If n is even and n is prime then $n = 2$."
There are two cases to consider:
Case 1: $n = 2$. In this case, P is true and Q is true. Therefore $P \wedge Q$ is true. Of course, $2 = 2$, so R is true. As the premise and conclusion are both true, this statement is true.
Case 2: $n \neq 2$. There are two sub-cases here. If n is even, then n is not a prime, so Q is false, which makes $P \wedge Q$ false. Thus, the premise is false, making the whole statement true.
If n is odd, then P is false, which makes $P \wedge Q$ false. Thus, the premise is false, making the whole statement true.

(b) English translation:
"If n is prime and n is not 2 then n is odd."
This statement is true for all n. Again, there are two cases.

Case 1: $n = 2$. This makes $\neg R$ false and therefore $Q \wedge (\neg R)$ is false, making the premise false. Therefore, the whole statement is true.
Case 2: $n \neq 2$. There are two sub-cases as well. If n is prime (thus Q is true), then n is odd (thus $\neg P$ is true). Both premise and conclusion are true, which means the statement is true.
If n is not prime then Q is false, making the premise false. Therefore, the whole statement is true.

(c) English translation:
"n is odd or n is not a prime or n is not 2."
This statement is false for $n = 2$ but true for $n = 25$. Therefore, it is true for some n and true for some n.

(d) English translation:
"n is an even prime or n is not 2."
This statement is true for all n.
Again, there are two cases.
Case 1: $n = 2$. This makes P and Q both true; therefore, $P \wedge Q$ is true by the rule of conjunction. Therefore, by the rule of disjunction, the statement is true.
Case 2: $n \neq 2$. This makes $\neg R$ true. By the rule of disjunction, the whole statement is true.

$\square$