

# MATH 101 Calculus Final Exam

Fulbright University Vietnam  
December 12th, 2023

Student Full Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

Question	Maximal Points	Your Points
1	25	
2	25	
3	25	
4	25	
Total	100	

## Instructions:

1. You have **eighty (80) minutes** to complete the exam.
2. This exam contains 4 problems. Some pages are intentionally left blank for you to write your solutions. If you run out of space you can use the backs of the pages, but please indicate clearly on the front pages that you have done so.
3. Phones are not allowed. Please turn off your phones during the exam. If you need to use the restroom, please turn in your phones before leaving the room.
4. **Show and justify your work unless indicated otherwise.**
5. Please sign the Honor Code statement below.

In recognition of and in the spirit of the Fulbright University Vietnam Honor Code, I certify that I will neither give nor receive unpermitted aid on this examination.

Signature: \_\_\_\_\_

**Problem 1.** Suppose that  $A = \{1, 2, 3\}$ ,  $B = \{4, 5\}$ ,  $C = \{6, 7, 8\}$ ,  $R = \{(1, 7), (3, 6), (3, 7)\}$ , and  $S = \{(4, 7), (4, 8), (5, 6)\}$ . Note that  $R$  is a relation from  $A$  to  $C$ , and  $S$  is a relation from  $B$  to  $C$ .

1. Represent the relations via Venn diagrams and arrows.
2. Write down in the set notation the following relation:  $S^{-1} \circ R$ .

**Problem 2.** *True or False? Give some reason for your answers.*

1.  $\emptyset \in \{\{\emptyset\}\}$

2.  $\{\emptyset\} \in \{\{\emptyset\}\}$

3.  $\mathcal{P}(\emptyset) \in \mathcal{P}(\mathcal{P}(\emptyset))$

4.  $\mathcal{P}(\mathcal{P}(\emptyset)) \in \{\emptyset, \{\emptyset, \{\emptyset\}\}\}$

**Problem 3.** *Prove by induction that  $\forall n \in \mathbb{N}$ , we have*

$$1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1).$$

**Problem 4.** 1. Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be functions. Prove that if  $f$  and  $g$  are both injective, then so is  $g \circ f$ .

2. let  $f : (0, \infty) \rightarrow (0, \infty)$  and  $g : (0, \infty) \rightarrow (0, \infty)$  be defined as

$$f(x) = x^3,$$

and

$$g(x) = \frac{1}{x}.$$

(a) Find a formula for  $g \circ f$ .

(b) Is  $g \circ f$  injective and why?