MATH 102: IDEAS OF MATH

WORKSHEET 14

Definition 1. Given a pair of sets A and B, suppose that each element $x \in A$ is associated to a unique element $f(x)$ of B. Then f is said to be a function from A to B. This is denoted by

$$f : A \to B\,.$$

A is called the *domain* of f, B is called the *codomain* of f. The set $\{f(x) : x \in A\}$ is called the *range* of f.

Question 1. What is the vertical line test?

Definition 2. A function $f : A \to B$ is *injective* (of *one-to-one*) if $f(a_1) = f(a_2)$ implies $a_1 = a_2$.

Question 2. Give a few examples of injective function.

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Question 3. Give an equivalent definition of injectivity using contrapositivity.

Definition 3. A function $f : A \to B$ is *surjective* (of *onto*) if for every $b \in B$, there exists some $a \in A$ such that $f(a) = b$.

Question 4. Give a few examples of injective function.

Definition 4. A function is *bijective* if it is both injective and surjective.

Question 5.

(1) Write a strategy to prove injectivity.

(2) Write a strategy to prove surjectivity.

Question 6. Let $\mathbb{R}^+$ denote the nonnegative real numbers. Prove the following.

(a) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = x^2$ is not injective, surjective or bijective.

(b) $g : \mathbb{R}^+ \to \mathbb{R}$ where $g(x) = x^2$ is injective, but not surjective or bijective.

(c) $h : \mathbb{R} \to \mathbb{R}^+$ where $h(x) = x^2$ is surjective, but not injective or bijective.

(d) $k : \mathbb{R}^+ \to \mathbb{R}^+$ where $k(x) = x^2$ is injective, surjective and bijective.

Definition 5. Let A, B be sets. A Cartesian product of A and B, denoted by $A \times B$ is the set of all ordered pair (x, y), where $x \in A$ and $y \in B$.

$$A \times B = \{(x, y) : x \in A, y \in B\}.$$

Question 7. Prove the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ where $f(x, y) = (x + 2y, 2x + 3y)$ is a bijection.