MATH 102: Ideas of Math

Day 11

Oct 2, 2023

Some proof strategies

1. Direct proof
2. Contradiction
3. Contrapositive (new)
4. Proofs with quantifiers

Direct proof

Based on definitions only.

Example

Prove that for $a, b > 0$, $a < b \implies a^2 < b^2$.

Proof by contradiction

To prove that P is true, suppose P is false then derive a contradiction.

Example

Prove that if a is irrational and r is rational, then $a + r$ is irrational.

Proof by contrapositive

In order to prove $P \implies Q$, it is equivalent to prove $\neg Q \implies \neg P$.

Example

Let $m, n \in \mathbb{N}$. Prove that if $mn > 64$, then either $m > 8$ or $n > 8$.

Proof with quantifiers

To prove $\forall x \in X, P(x)$, pick an arbitrary $x \in X$ and try to deduce that $P(x)$ is true. That is, just use the properties of x that are described in the description of X to deduce that $P(x)$ is true.

Example

Prove that the square of every odd integer is odd.

Proof with quantifiers (cont.)

To prove $\exists x \in X, P(x)$, it is enough to find a particular $x_0 \in X$ so that $P(x_0)$ is true.

Example

Prove that there is a natural number that is a perfect square and is one more than a perfect cube.