MATH 104: MULTIVARIABLE CALCULUS

FINAL

NAME:______________________________

There are four questions. Make sure you justify all your work for complete credit.

Rules

- You have 80 minutes to complete your work..
- Closed books.
- No use of internet, textbooks, computer algebra systems, calculators.
- No collaboration.
- 1 person per bathroom break. When you go to the bathroom, turn in your cellphone and exam until return.

Scores:

(1) ______
(2) ______
(3) ______
Total : ______

Date: May 29, 2025.

Questions

Problem 1 (20 points, 10 points each). Evaluate

(1)
$$\iiint_E y\,dV\,,$$
where $E = \{(x, y, z) | 0 \leqslant x \leqslant 3\,, 0 \leqslant y \leqslant x\,, x - y \leqslant z \leqslant x + y\}$.

(2) The volume of solid enclosed by the paraboloid $z = (x-2)^2 + (y-2)^2$ and the plane $z = 4$.

Problem 2 (30 points, 10 points each). Evaluate the integrals

(1)
$$\int_{-\infty}^{\infty} e^{-x^2}\, dx\,.$$

(2)
$$\iint_D \sin\sqrt{x^2+y^2}\, dA$$
where D is the upper half of the disk center at the origin and radius 4.

(3)
$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dydx\,.$$

Problem 3 (50 points, 25 each). (1) The total production P of a certain product depends on the amount L of labor used and the amount K of capital investment. The Cobb-Douglas model $P = bL^{\alpha}K^{1-\alpha}$ follows from certain economic assumptions, where b and α are positive constants and $\alpha < 1$. If the cost of a unit of labor is m and the cost of a unit of capital is n, and the company can spend only p dollars as its total budget, then maximizing the production P is subject to the constraint $mL + nK = p$. Show that the maximum production occurs when

$$L = \frac{\alpha p}{m} \quad \text{and} \quad K = \frac{(1-\alpha)p}{n}$$

(2) Suppose that the production is fixed at $bL^\alpha K^{1-\alpha} = Q$, where Q is a constant. What values of L and K minimize the cost function $C(L, K) = (mL)^2 + (nK)^3$?