MATH 104: WORKSHEET 13

1. Concepts

(1) Directional derivative in the direction of unit vector $\approxeq$.

$$D_{\mathbf{u}}f(\mathbf{x_0}) = \lim_{h\to 0} \frac{f(\mathbf{x_0} + h\mathbf{u}) - f(\mathbf{x_0})}{h}$$

(2)

Theorem 1.1.

$$D_{\mathbf{u}}f(x, y) = \nabla f(x, y) \cdot \mathbf{u}\,.$$

(3)

Theorem 1.2. *Suppose f is a differentiable function of two or three variables. The maximum valule of the directional derivative $D_{\mathbf{u}}f(\mathbf{x})$ is $|\nabla f(\mathbf{x})|$ and it occurs when $\mathbf{u}$ has the same direction as the gradient vector $\nabla f(\mathbf{x})$.*

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2. Discussions

Question 1. Find the direction in which the directional derivative of $f(x, y) = x^2 + xy^3$ at the point $(2, 1)$ has the value 2.

Question 2. Suppose that the temperature at a point (x, y, z) in space is given by

$$T(x, y, z) = \frac{80}{1 + x^2 + 2y^2 + 3z^2},$$

where T is measured in degrees Celsius and x, y, z in meters. In which direction does the temperature increase fastest at the point $(1, 1, -2)$? What is the maximum rate of increase?