MATH 104: Multivariable Calculus (brief notes)

Truong-Son Van

Contents

Spring 2023

Syllabus

Key information

- Instructor: Truong-Son Van
- Email: son.van+104@fulbright.edu.vn
- Class time: T & Th: 9:45a - 11:15a
- Class Location: CR 502
- Office hours: M & W, 10a-11a (or by appointment)
- Prerequisites: Calculus (MATH 101)
- TA: Tran Lan Phuc, phuc.tran.200077@student.fulbright.edu.vn
- TA office hours: Friday, 8:30-10:30 AM by prior appointment

Textbooks and references

It is highly recommended that students read the textbooks.

1. In-class worksheets.
2. Active Calculus: Multivariable by Schlicker et al. 2018 edition. (https://activecalculus.org/multi/preface-2.html)
3. Thomas' Calculus: Early Transcendentals by Hass, Heil, et al. 14^{th} edition.
4. Calculus Early Transcendental by Stewart. 8^{th} edition.
5. Anything you can find on Google would work. Calculus is a subject that people have written about so much. So, there's no excuse for not having access to the knowledge.
6. 3-D grapher: https://www.math3d.org/

7. Very good graphers: https://www.desmos.com/, https://www.geogebra.org/

Course description

How do we describe the trajectory of a space shuttle? How is the human body affected by scuba diving to different depths for different lengths of time? The mathematics required to describe most real life systems involves functions of more than one variable. The concepts of the derivative and integral from a first course in calculus must therefore be extended to higher dimensional settings. In this course students will be guided through the essential ideas of multivariable calculus, including partial derivatives, multiple integrals and vector calculus, and their applications. These mathematical tools are used extensively in the physical sciences and engineering, and in other areas including economics and computer graphics.

Learning objectives

After the course, students are expected to:

- Be confident in handling functions of two or more variables and familiar with how they can be represented graphically
- Understand the key concepts of multivariable calculus, including partial derivatives, the gradient vector, multiple integrals, line and surface integrals, the divergence and curl of a vector function
- Know how such derivatives and integrals are calculated and some of their uses
- Be able to apply these ideas to real world problems
- Have improved analytic, computational and problem solving skills

Assessment

During the course, students are expected to compute their own percentage points based on the following scheme. The instructor is not responsible for providing the running percentage.

Form of assessment	Weight
Weekly homeworks	25%
Class worksheets	15%
Mini-project	15%
Midterm	15%
Final	30%

The following is the non-negotiable letter grade breakdown. It is based on common practice in the United States for standard courses such as Calculus.

Letter Grade	Percentage
A	[93,100]
A-	[90,93)
B+	[87,90)
B	[83,87)
B-	[80, 83)
C+	[77,80)
C	[73,77)
C-	[70,73)
D+	[67,70)
D	[60, 66)
F	[0,60)

Core content

1. Introduction

- Functions of two variables
- Graphs in three dimensions, surfaces and level curves
- Functions of three or more variables
- Limits and continuity
- Vectors (review)

2. Partial Derivatives

- Partial derivatives
- Tangent planes, linear approximations and differentials
- Chain rule
- Directional derivatives and gradient vectors
- Extrema and optimization
- Lagrange Multipliers

3. Multiple Integrals

- Double integrals
- Double integrals in polar coordinates
- Triple integrals
- Triple integrals in cylindrical and spherical coordinates
- Applications of multiple integrals

4. Vector Calculus

- Vector functions and their derivatives
- Vector fields
- Line integrals
- The fundamental theorem of line integrals
- Green's Theorem
- Parametric surfaces and surface integrals
- Curl and divergence
- Divergence Theorem
- Stokes Theorem

Project description

The goal of the project is for you to find great applications of calculus in different fields (economics, physics, chemistry, art, computer science, etc). There's no constraint on what you can do.

Project Guidelines

- You may work alone or in team of two.
- You need to discuss with me a proposal for your project. The proposal should include your group members names & IDs, a brief description of the content of your project (as clear and explicit as possible), how you plan to present it, and a suggested grading rubric.
- You will need to deliver a 15 minutes presentation as well as a 5-page-minimum write up in 1.5 spacing).

Project Timeline

- Project proposal: March 24
- Presentation: May 16 &18
- Write-up: May 25

Possible topics

1. Create an interactive demonstration (GeoGebra/Python/...) of something studied in the course.
2. Implement gradient descent on a real world problem.
3. Discuss the theory behind gradient descent (convergence, speed of convergence, etc.)
4. Numerical scheme to solve partial differential equations
5. Neural network
6. Whatever you can think of

Project grading rubric

TBD as I'm asking you to give me the grading rubric.

Late assignments

- 15% of the possible total mark will be deducted for every 24 hrs (or part of 24 hrs) after the deadline. Work more than 2 days late will not be accepted.
- Except for exceptional circumstances (see definition), I will not extend the deadlines.

Time expectations

Some materials require time to be accustomed to. Some students are quicker than others. However, on average, you should expect 10-15 hours per week (including class time) on the materials in order to know the subject relatively well.

Collaboration & Plagiarism

Plagiarism is the act of submitting the intellectual property of another person as your own. It is one of the most serious of academic offenses. Acts of plagiarism include, but are not limited to:

- Copying, or allowing someone to copy, all or a part of another person's work and presenting it as your own, or not giving proper credit.
- Purchasing a paper from someone (or a website) and presenting it as your own work.
- Re-submitting your work from another course to fulfill a requirement in another course.

Further details can be found in the Code of Academic Integrity [link].

Learning Support

In addition to your course instructors, there are other resources available to support your academic work at Fulbright, including one-on-one consultations with learning support staff, supplementary workshops, and both individual and group tutoring and mentoring in course content, language learning, and academic skills. If you would like to request learning support, please contact Fulbright Learning Support (https://learning-support.notion.site).

Wellbeing

Mental health and wellbeing are essential for the success of your academic journey. The Fulbright Wellness Center provides various services including counseling, safer community, and accessibility services. If you are experiencing undue personal or academic stress, are feeling unsafe, or would like to know more about issues related to wellbeing, please contact the Wellness Center via wellness@fulbright.edu.vn or visit the Wellness Center office on Level 5 of the Crescent campus.

For more information, pleaes check https://onestop.fulbright.edu.vn/s/article/Health-and-Wellness-Introduction

Tentative Course Schedule

The following schedule will be updated as we go so that students will know what to read before/after class.

Date	Session	Content	Deadlines
Feb 7 & 9	1&2	Linear algebra crash course & Vector-valued functions	
Feb 14 & 16	3 & 4	Integration: arc length	
Feb 21 & 23	5 & 6	Introduction and functions of several variables	
Feb 28 & Mar 2	7 & 8	Limits & continuity	
Mar 7 & 9	9 & 10	Partial derivatives & Tangent planes, chain rules & higher derivatives	
Mar 14 & 16	11 & 12	Directional derivatives, gradient vectors & Extrema and Optimization	
Mar 21 & 23	13 & 14	Extrema and Optimization (cont.)	
Mar 28	15	Midterm exam	
Mar 30		**Break**	
Apr 4 & 6	16 & 17	Double integrals	
Apr 11	18	Applications of Double Integrals	
Apr 13	19	Surface Area	
Apr 18 & 20	20 & 21	Triple Integrals	
Apr 25 & 27	22 & 23	Change of variables	
May 2		**Break**	
May 4	24		
May 9	25		
May 11	26		
May 16 & 18	27 & 28	Presentation	
May 25		Project Write-up	