

# Multivariable Calculus

## Day 1

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# Introduction

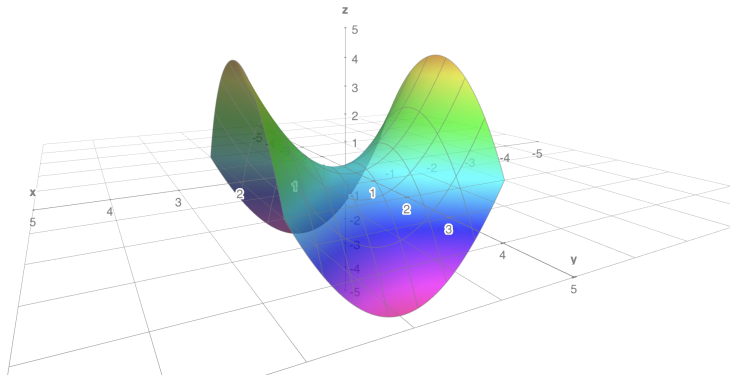
- Instructor: Truong-Son Van
- Time: M & W, 9:45a-11:15a
- Office hours: TBD
- TA: TBD

- Logistics
- Getting started with linear algebra

# Single variable vs. Multivariable

# What's the point?

What do you think is the function that create this graph?



# What's the point?

Life is a function with many variables

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Life is a function with many variables



# What will you learn?

`https://www.tsvan.xyz/MultiCalc/#core-content`



# Assessment

- Homework (40%)
- Quizzes(10%)
- Midterm (20%)
- Final (30%)

DON'T CHEAT!!!

# Final word on style

- Certain things in math have to be delivered while writing.
- So, sometimes I'll deliver my lectures by writing on the iPad / whiteboard.
- Slides/brief notes just contain key points.
- It is your responsibility to update your notes to include the details I speak in class.

# Crash course in linear algebra

First, we need some language from linear algebra.

## Definition

An  $n$ -dimensional Euclidean space  $\mathbb{R}^n$  is the Cartesian product of  $n$  Euclidean spaces  $\mathbb{R}$ .

## Definition

An  $n$ -dimensional vector  $\mathbf{v} \in \mathbb{R}^n$  is a tuple

$$\mathbf{v} = \langle v_1, \dots, v_n \rangle, \quad (1)$$

where  $v_i \in \mathbb{R}$ .

In dimensions less than or equal to 3, we represent a vector geometrically by an arrow, whose length represents the magnitude.

# Examples

- Vector connecting two points  $(1, 2)$  and  $(4, 5)$  in  $\mathbb{R}^2$
- Zero vector
- Unit vector

- Write the general formula for a vector that connects any two points  $A$  and  $B$  in  $\mathbb{R}^n$ .

# Rules to manipulate vectors

Let  $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$  and  $c, d \in \mathbb{R}$ . Then,

$$c(\mathbf{a} + \mathbf{b}) = \langle ca_1 + cb_1, \dots, ca_n + cb_n \rangle = c\mathbf{a} + c\mathbf{b},$$

and

$$(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}.$$



- Elementary vectors have the form

$$\mathbf{e}_i = \langle 0, \dots, 1, \dots, 0 \rangle.$$

Express a vector  $\mathbf{u} \in \mathbb{R}^n$  in terms of these elementary vectors.

- In 3D,

$$\mathbf{e}_1 = \mathbf{i}, \quad \mathbf{e}_2 = \mathbf{j}, \quad \mathbf{e}_3 = \mathbf{k}.$$

Write the vector connecting  $A(1, 2, 3)$  with  $B(-10, -3, 5)$  in terms of elementary vectors.

# Geometric meanings