Multivariable Calculus
Integration

Spring 2024

Double integral over rectangle R

Definition
The double integral of f over the rectangle R is

$$\iint_R f(x,y)\,dA = \lim_{\substack{m\to\infty \\ n\to\infty}} \sum_{i=1}^{n}\sum_{j=1}^{m} f(x_i^*, y_j^*)\cdot \mathrm{Area}(R_{ij})$$

if this limit exists. Here, (x_i^*, y_j^*) is a point inside $R_{ij} = [x_{i-1}, x_i] \times [y_{j-1}, y_j]$.

Let $f(x, y) = x^2 y$. Set up a Riemann sum for this function with 3×4 sub-regions inside the domain $[2, 6] \times [1, 4]$, (x_i^*, y_j^*) is the upper right corner of each box.

Fubini Theorem

Let $R = [a, b] \times [c, d]$. If $f : \mathbb{R}^2 \to \mathbb{R}$ is continuous on R, then

$$\iint_R f(x, y) dA = \int_a^b \left[\int_c^d f(x, y)\, dy \right] dx = \int_c^d \left[\int_a^b f(x, y)\, dx \right] dy.$$