

MATH 104: WORKSHEET 3

1. Concepts

Theorem 1.1 (Law of cosine).

$$\vec{v} \cdot \vec{w} = |\vec{v}| |\vec{w}| \cos \theta$$

where θ is the angle between the two vectors.

Definition 1.2 (Projections). Given 2 vectors \vec{a} and \vec{b} in \mathbb{R}^n .

Scalar projection of \vec{b} to \vec{a} is:

$$\text{comp}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}.$$

Vector projection of \vec{b} to \vec{a} is:

$$\text{proj}_{\vec{a}} \vec{b} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \right) \frac{\vec{a}}{|\vec{a}|}.$$

Definition 1.3 (Cross product). Let $\vec{v} = \langle v_1, v_2, v_3 \rangle$ and $\vec{w} = \langle w_1, w_2, w_3 \rangle$ be vectors in \mathbb{R}^3 . The cross product of \vec{v} and \vec{w} is defined by

$$\vec{v} \times \vec{w} = \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix}.$$

Theorem 1.4.

$$|\vec{v} \times \vec{w}| = |\vec{v}| |\vec{w}| |\sin \theta|.$$

2. Problems

Question 1. What is the angle between the planes $x - 2y + 3z = 6$ and $2x + 3y - z = 11$?

Question 2. What is the definition for general plane that doesn't go through the origin?

Question 3. What is the projected length (component) of vector \vec{w} onto the vector \vec{v} :

$$\vec{w} = \begin{pmatrix} 5 \\ -6 \\ 2 \\ -7 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 0 \\ 3 \\ 4 \\ 0 \end{pmatrix}.$$

Question 4. Show

$$\vec{v} \times \vec{w} = -\vec{w} \times \vec{v}.$$

Question 5. What is the geometric interpretation of $|\vec{v} \times \vec{w}|$?

Question 6. What is the geometric interpretation of $|\vec{a} \cdot (\vec{v} \times \vec{w})|$?