

## MATH 104: WORKSHEET 2

### 1. Concepts

- (1) Length
- (2) Dot products

**Definition 1.1.** The length of a vector  $\vec{v}$  in  $\mathbb{R}^n$  is

$$|\vec{v}| = \sqrt{v_1^2 + \cdots + v_n^2}.$$

**Definition 1.2.** Scalar multiplication of a number  $c$  and a vector  $\vec{v}$  in  $\mathbb{R}^n$  is

$$c\vec{v} = \langle cv_1, \dots, cv_n \rangle.$$

**Definition 1.3.** Addition between two vectors  $\vec{v}$  and  $\vec{w}$  in  $\mathbb{R}^n$  is

$$\vec{v} + \vec{w} = \langle v_1 + w_1, \dots, v_n + w_n \rangle.$$

**Definition 1.4.** The dot product between two vectors  $\vec{v}$  and  $\vec{w}$  in  $\mathbb{R}^n$  is

$$\vec{v} \cdot \vec{w} = v_1 w_1 + \cdots + v_n w_n.$$

**Theorem 1.5** (Law of cosine).

$$\vec{v} \cdot \vec{w} = |\vec{v}| |\vec{w}| \cos \theta$$

where  $\theta$  is the angle between the two vectors.

**Definition 1.6** (Projections). Given 2 vectors  $\vec{a}$  and  $\vec{b}$  in  $\mathbb{R}^n$ .

Scalar projection of  $\vec{b}$  to  $\vec{a}$  is:

$$\text{comp}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}.$$

Vector projection of  $\vec{b}$  to  $\vec{a}$  is:

$$\text{proj}_{\vec{a}} \vec{b} = \left( \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \right) \frac{\vec{a}}{|\vec{a}|}.$$

## 2. Discussions

*Question 1.* Given two points  $A(x_1, y_1, z_1)$  and  $B(x_2, y_2, z_2)$ . How do you construct a vector  $\vec{AB}$  with the beginning at  $A$  and the end at  $B$ ?

*Question 2.* What are the properties of vectors (Section 12.2)?

*Question 3.* Prove the law of cosine.

*Question 4.* What are the properties of dot product (Section 12.3)?

*Question 5.* Interpret the equation of a plane going through the origin in  $R^3$ :  $ax + by + cz = 0$  .

*Question 6.* What is the value of  $c$  so that the planes  $2cx - y + c^2 = 15$  and  $x + 5cy - 3z = 4$  are orthogonal?

*Question 7.* What is the angle between the planes  $x - 2y + 3z = 6$  and  $2x + 3y - z = 11$ ?