Multivariable Calculus
Day 1

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Introduction

- Instructor: Truong-Son Van
- Time: T & Th, 9:45a-11:15a
- Office hours: M & W, 10a-11a (or by appointment)
- TA: TBD
- Discussion board: Piazza

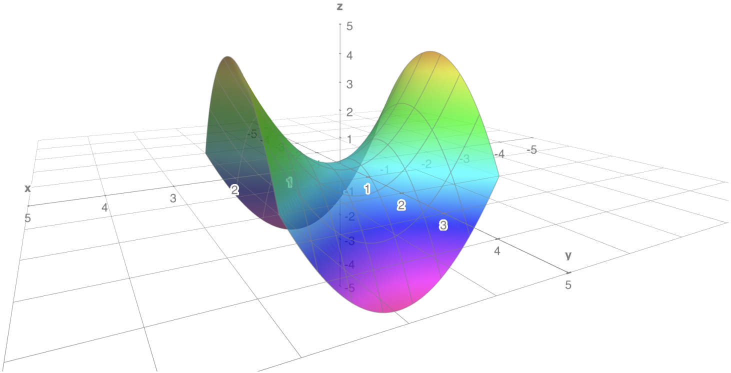
Overview

- Logistics
- Getting started with linear algebra

Single variable vs. Multivariable

What's the point?

What do you think is the function that create this graph?



What's the point?

Life is a function with many variables

What's the point?

Life is a function with many variables

What will you learn?

https://www.tsvan.xyz/MultiCalc/#core-content

Assessment

- Homework (25%)
- Worksheets (15%)
- Midterm (15%)
- Final (30%)
- Project (15%)

DON'T CHEAT!!!

Final word on style

- Certain things in math have to be delivered while writing.
- So, sometimes I'll deliver my lectures by writing on the iPad / whiteboard.
- Slides/brief notes just contain key points.
- It is your responsibility to update your notes to include the details I speak in class.

Crash course in linear algebra

First, we need some language from linear algebra.

Vectors

Definition

An n-dimensional Euclidean space $\mathbb{R}^n$ is the Cartesian product of n Euclidean spaces $\mathbb{R}$.

Definition

An n-dimensional vector $\mathbf{v} \in \mathbb{R}^n$ is a tuple

$$\mathbf{v} = \langle v_1, \ldots, v_n \rangle \,, \tag{1}$$

where $v_i \in \mathbb{R}$.

In dimensions less than or equal to 3, we represent a vector geometrically by an arrow, whose length represents the magnitude.

Examples

- Vector connecting two points $(1, 2)$ and $(4, 5)$ in $\mathbb{R}^2$
- Zero vector
- Unit vector

Worksheet

- Write the general formula for a vector that connects any two points A and B in $\mathbb{R}^n$.

Rules to manipulate vectors

Let $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$ and $c, d \in \mathbb{R}$. Then,

$$c(\mathbf{a} + \mathbf{b}) = \langle ca_1 + cb_1, \dots, ca_n + cb_n \rangle = c\mathbf{a} + c\mathbf{b}\,,$$

and

$$(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}\,.$$

Worksheet

- Elementary vectors have the form

$$\mathbf{e_i} = \langle 0, \dots, 1, \dots, 0 \rangle \, .$$

  Express a vector $\mathbf{u} \in \mathbb{R}^n$ in terms of these elementary vectors.
- In 3D,

$$\mathbf{e_1} = \mathbf{i} \, , \qquad \mathbf{e_2} = \mathbf{j} \, , \qquad \mathbf{e_3} = \mathbf{k} \, .$$

  Write the vector connecting $A(1, 2, 3)$ with $B(-10, -3, 5)$ in terms of elementary vectors.

Geometric meanings

Dot product

Definition

The dot product of vectors $\mathbf{u} = \langle u_1, \ldots, u_n \rangle$ and $\mathbf{v} = \langle v_1, \ldots, v_n \rangle$ in $\mathbb{R}^n$ is the scalar

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + \cdots + u_n v_n \,.$$

Geometric meaning

Theorem

If θ is the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$, then

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos \theta \, .$$

Geometric meaning

Theorem

If θ is the angle between the vectors $\mathbf{u}$ *and* $\mathbf{v}$*, then*

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos \theta \,.$$

Proof in $\mathbb{R}^2$.

Homework $\square$

Geometric meaning

Theorem

If θ is the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$, then

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos \theta .$$

Proof in $\mathbb{R}^2$.

Homework □

Corollary

Two vectors $\mathbf{u}$ and $\mathbf{v}$ are orthogonal to each other if $\mathbf{u} \cdot \mathbf{v} = 0$.

Projection

Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$. The component of $\mathbf{u}$ in the direction of $\mathbf{v}$ is the scalar

$$\mathrm{comp}_{\mathbf{v}}\mathbf{u} =$$

and the projection of $\mathbf{u}$ onto $\mathbf{v}$ is the vector

$$\mathrm{proj}_{\mathbf{v}}\mathbf{u} =$$