Homework 6

Multivariable Calculus

Due: April 10, 2023, 23:59

Computational problems are graded for completion, each problem is worth 1 points.

Conceptual problems are graded for correctness, each problem is worth 5 points.

Show all your work to get full credits for each problem.

1 Computational

Do the following problems in Stewart's calculus textbook, 8th edition.

Section 14.7: 16-20, 21, 22, 31-38

Section 14.8: 6-10

2 Conceptual

Problem 1. Do the following:

1. Formulate an optimization problem to find the shortest distance between a point $P_0(x_0, y_0, z_0)$ and a plane $Ax + By + Cz = D$.

2. Solve the above problem.

Problem 2. The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the point on this ellipse tat are nearest and farthest from the origin.

Problem 3. Consider the problem of minimizing the function $f(x, y) = x$ on the curve $y^2 + x^4 - x^3 = 0$.

1. Try using the Lagrange multipliers to solve the problem.

2. Show that the minimum value is $f(0,0) = 0$ but the Lagrange condition $\nabla f(0,0) = \lambda \nabla g(0,0)$ is not satisfied for any λ.

3. Explain why the Lagrange multipliers fail to find the minimum value in this case.