

Multivariable Calculus

Day 2

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Dot product

Definition

The dot product of vectors $\mathbf{u} = \langle u_1, \dots, u_n \rangle$ and $\mathbf{v} = \langle v_1, \dots, v_n \rangle$ in \mathbb{R}^n is the scalar

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + \cdots + u_n v_n.$$

Theorem

If θ is the angle between the vectors \mathbf{u} and \mathbf{v} , then

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| |\mathbf{v}| \cos \theta .$$

Geometric meaning

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$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta .$$

Proof in \mathbb{R}^2 .

Homework



Geometric meaning

Theorem

If θ is the angle between the vectors \mathbf{u} and \mathbf{v} , then

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| |\mathbf{v}| \cos \theta .$$

Proof in \mathbb{R}^2 .

Homework □

Corollary

Two vectors \mathbf{u} and \mathbf{v} are orthogonal to each other if $\mathbf{u} \cdot \mathbf{v} = 0$.

Projection

Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$. The component of \mathbf{u} in the direction of \mathbf{v} is the scalar

$$\text{comp}_{\mathbf{v}} \mathbf{u} =$$

and the projection of \mathbf{u} onto \mathbf{v} is the vector

$$\text{proj}_{\mathbf{v}} \mathbf{u} =$$