

# Homework 1

## Multivariable Calculus

Due: Feb 16, 2023, 23:59

Computational problems are graded for completion, each problem is worth 1 points.

Conceptual problems are graded for correctness, each problem is worth 5 points.

Show all your work to get full credits for each problem.

## 1 Computational

Do the following problems in Stewart's calculus textbook, 8th edition.

Section 12.2: 7, 20, 22, 24

Section 12.3: 2-10

Section 12.4: 1-7

## 2 Conceptual

**Problem 1.** A street vendor sells  $a$  banh mi,  $b$  banh bao,  $c$  coconuts on a given day. He charges \$ 25,000 VND for a banh mi, \$20,000 VND for a banh bao, \$ 15,000 VND for a cononut. If  $\mathbf{A} = \langle a, b, c \rangle$  and  $\mathbf{P} = \langle 25, 20, 15 \rangle$ , what is the meaning of the dot product  $\mathbf{A} \cdot \mathbf{P}$ ?

**Problem 2.** Let  $\mathbf{r} = \langle x, y \rangle$  and  $\mathbf{r}_1 = \langle x_1, y_1 \rangle$ ,  $\mathbf{r}_2 = \langle x_2, y_2 \rangle$ . Describe the set of all the points  $(x, y)$  such that  $|\mathbf{r} - \mathbf{r}_1| + |\mathbf{r} - \mathbf{r}_2| = k$  where  $k > |\mathbf{r}_1 - \mathbf{r}_2|$ .

**Problem 3.** Prove the law of cosine in  $\mathbb{R}^2$ . In other words, prove that, If  $\theta$  is the angle between the vectors  $\mathbf{u}$  and  $\mathbf{v}$  in  $\mathbb{R}^2$ , then

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos \theta .$$

Hint:

Step 1: prove the high school trigonometry identity  $c^2 = a^2 + b^2 - 2ab \cos \theta$ .  
(Look it up (and cite) if you don't know it)

Step 2: use the identity in step 1.

**Problem 4.** Suppose  $\mathbf{a}, \mathbf{b}, \mathbf{c}$  are nonzero vectors in  $\mathbb{R}^3$ . A formula to find the volume of the parallelepiped created by these vectors is

$$V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|.$$

1. Prove this formula.
2. Test this formula with an example of a rectangular box constructed by yourself.