MATH 104: WORKSHEET 10

1. Concepts

(1) Partial derivative

$$\partial_x f(x,y,z) = f_x(x,y,z) = \lim_{h\to 0} \frac{f(x+h,y,z) - f(x,y,z)}{h}$$

$$\partial_y f(x,y,z) = f_y(x,y,z) = \lim_{h\to 0} \frac{f(x,y+h,z) - f(x,y,z)}{h}$$

$$\partial_z f(x,y,z) = f_z(x,y,z) = \lim_{h\to 0} \frac{f(x,y,z+h) - f(x,y,z)}{h}$$

(2) Gradient

$$\nabla f(x,y,z) = \langle f_x(x,y,z), f_y(x,y,z), f_z(x,y,z)\rangle\,.$$

2. Discussions

Question 1. Find all partial derivatives of

(1)

$$f(x,y) = 3x^3 - 2x^2y^5$$

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(2)

$$f(x, y) = \frac{xy^2}{x + 1}$$

(3)

$$g(r, s) = rs \cos(r)$$

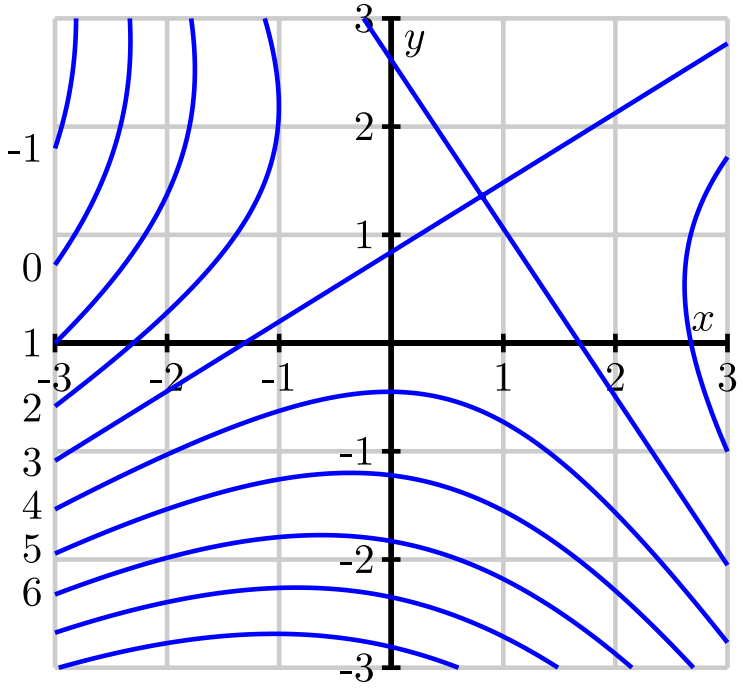
(4)

$$f(w, x, y) = (6w + 1) \cos(3x^2 + 4xy^3 + y)$$

(5)

$$q(x,t,z) = \frac{x2^t z^3}{1+x^2}.$$

Question 2. Consider the following picture



(1) Estimate the partial derivative $f_x(-2,-1)$. (Hint: How can values of f that are of the form $f(-2+h)$ and $f(-2-h)$ be used to compute a symmetric difference quotient?)

(2) Estimate the partial derivative $f_y(-2,-1)$.

(3) Estimate the partial derivatives $f_x(-1,2)$ and $f_y(-1,2)$.

(4) Locate, if possible, one point (x,y) where $f_x(x,y)=0$.

(5) Locate, if possible, one point (x,y) where $f_x(x,y)<0$.

(6) Locate, if possible, one point (x, y) where $f_y(x, y) > 0$.