MATH 104: WORKSHEET 4

1. Concepts

(1) Matrices
(2) Row-reduction

2. Discussions

Question 1. What are the sizes of the following matrices? Which pairwise products are well-defined?

$$A = \begin{pmatrix} 5 & 1 \\ 1 & 3 \\ -2 & 0 \end{pmatrix}, B = \begin{pmatrix} -3 & 0 & -5 \\ -1 & 7 & 1 \\ 2 & 4 & 1 \end{pmatrix}, C = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}.$$

Question 2. Find the fastest way to compute this

$$\begin{pmatrix} 5 & -7 & 1 & -9 \\ 1 & 3 & 1 & 1 \\ 2 & -11 & 3 & 4 \\ 2 & 64 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 4 \\ 0 \end{pmatrix}$$

Question 3. Let $\vec{u}, \vec{v} \in \mathbb{R}^n$.

(1) What does $\vec{u}^T \vec{v}$ mean (dimension)? Is it the same with $\vec{v}^T \vec{u}$?
(2) What does $\vec{u}\vec{v}^T$ mean (dimension)? Is it the same with $\vec{v}\vec{u}^T$?

Question 4. Solve the following equation

$$x - y + 2z = 0$$
$$2x + y - 3z = 1$$
$$-3x + 2y + z = 2.$$

Question 5. For what value of c does the following system have solution?

$$x + y + cz = 1$$
$$x + cy + z = 1$$
$$cx + y + z = c$$

Question 6. Parametrize the intersection of the planes

$$3x + y - z = 4$$

and

$$x - 2y + z = 1.$$

Set this up as a row-reduction problem to obtain the answer.