Multivariable Calculus
Day 5
Integrals of vector functions

Truong-Son Van

Spring 2023

Fulbright University Vietnam

Refresher

Let

$$\mathbf{r}(t) = \langle \cos(t), t^2, \sin(t) \rangle$$

- Compute $\mathbf{s}(t) = \mathbf{r}'(t)/|\mathbf{r}'(t)|$
- Compute $\mathbf{s}'(t) \cdot \mathbf{r}'(t)$

Integrals

Coordinate-wise integrals

Indefinite integral

$$\int \mathbf{r}(t)\,dt = \left\langle \int r_1(t)\,dt, \int r_2(t)\,dt, \int r_3(t)\,dt \right\rangle = \mathbf{R}(t) + \mathbf{C}\,.$$

Definite integral

$$\int_a^b \mathbf{r}(t)\,dt = \left\langle \int_a^b r_1(t)\,dt, \int_a^b r_2(t)\,dt, \int_a^b r_3(t)\,dt \right\rangle\,.$$

Fundamental Theorem of Calculus

$$\int_a^b \mathbf{r}(t)\,dt = ?$$

Worksheet

Compute

$$\mathbf{R}(t) = \int (\cos(t)\mathbf{i} + \mathbf{j} - 2t\mathbf{k})\, dt\,.$$

Compute

$$\int_0^{\pi} \langle \cos(t), 1, -2t \rangle\, dt\,.$$

Arc length

Worksheet

Compute the length of the line segment

$$\mathbf{r}(t) = \langle 1, 1, 1 \rangle + t\langle 2, 2, 2 \rangle \, , \, t \in [0, 4] \, .$$

Space curve

A vector function that is continuous.

Example

$$\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$$

Worksheet

Compute the length of the following curve

$$\mathbf{r}(t) = \begin{cases} \langle 1,1,1\rangle + t\mathbf{i}\,, t \in [0,1] \\ \mathbf{r}(1) + (t-1)\mathbf{j}\,, t \in [1,2]\,. \end{cases}$$

Question

How can you compute the length of any curve?

Worksheet

- Compute the length of the curve

$$\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$$

  where $t \in [0, 2\pi]$.

- Compute the length of the curve made by the graph of the function

$$f(x) = x^3$$

  where $x \in [1, 4]$.