

Multivariable Calculus

Day 17

Integration

Spring 2023

Some properties of integrals

- ① Let U, V be disjoint domains, then

$$\iint_{U \cup V} f \, dA = \iint_U f \, dA + \iint_V f \, dA.$$

②

$$\iint_U (f + g) \, dA = \iint_U f \, dA + \iint_U g \, dA.$$

- ① Set up a problem with double integral to find the area of triangle with vertices $(0, 0)$, $(2, 0)$, $(2, 3)$.
- ② Let $f(x, y) = x^2y$. Find

$$\iint_D f(x, y) dA$$

where D is the triangle with vertices $(0, 0)$, $(2, 0)$, $(2, 3)$.

- ③ Compute

$$\iint_D e^{-y^2} dA$$

- ❶ Let B be a 2×2 matrix that is invertible (the determinant is non-zero). We can think of B as a function $B : \mathbb{R}^2 \rightarrow \mathbb{R}^2$.

Let now $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a function such that

$$f(x, y) = xy.$$

- ❶ What is

$$\iint_D f \, dA,$$

where D is the rectangle with vertices $(1, 1), (1, 6), (5, 1), (5, 6)$.

- ❷ Let $y = Bx$. Which of the following expression makes sense (could be more than 1)?

- ❶ $\iint_{B(D)} f(Bx) \, dA$
- ❷ $\iint_{B(D)} f(y) \, dA$
- ❸ $\iint_D f(Bx) \, dA$
- ❹ $\iint_{B(D)} f(x) \, dA$

Find the relationship between

$$\iint_{B(D)} f(y) \, dA$$

and

$$\iint_D f(x) \, dA.$$

Change of coordinates

Let f be a function of (x, y) defined on the domain D . Let

$$\begin{pmatrix} x \\ y \end{pmatrix} = \varphi(u, v)$$

for some coordinate change function $\varphi : D \rightarrow S$.

Theorem

If f is continuous, then

$$\int_S f \, dA = \int_D (f \circ \varphi) |\det D\varphi| \, dA.$$

Example

Compute the following integral

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \exp\left(-\frac{x^2}{2}\right) dx .$$