MATH 104: WORKSHEET 10

1. Concepts

(1) Gradient
(2) Tangent plane
(3) Linear approximation
(4) Steepest Ascent

2. Discussions

This part really is starting to get to the applications of the theory we learned before.

Question 1. Let $f : \mathbb{R}^n \to \mathbb{R}$. What are the differences of the derivative of f and the gradient of f ?

Question 2. Compute the gradient of the function

$$f(x, y) = ax^2 + by^2 - 2xy.$$

Question 3. At what points in the plane are level sets of $g = x^2 + y^2 - 2xy$ and $f = 2y - 3x$ are orthogonal?

Question 4. Why is the gradient orthogonal to the level set?

Question 5. Compute the tangent spaces to the following:

(1) Implicit tangent plane to $xyz - 2xy^3 + 3z^2 = 0$ at the point $(3, 1, 1)$.

(2) Parametrized tangent line to

$$\gamma(t) = \begin{pmatrix} t^2 \\ -3t \\ t^3 \end{pmatrix}$$

(3) Parametrized tangent plane to

$$S(t_1, t_2) = \begin{pmatrix} t_1 + 3t_2 \\ t_1 t_2 \\ 2t_1^2 - t_2^3 \end{pmatrix}$$

Question 6. What are the linear approximations of the following

(1) 1D: $f(x) = e^{-x^2}$ around the point $a = 1$

(2) 2D: $f(\mathbf{x}) = e^{-|\mathbf{x}|^2}$ around the point $a = (1, 1)$

Date: 03/04/2024.