Multivariable Calculus
Day 15
Optimization (cont.)

Spring 2023

Constrained optimization

Constrained optimization takes various forms, depending on the assumptions. We will deal with the most straight forward form. The problem we will study is the following:

Maximize/minimize a function $f : D \to \mathbb{R}$, subject to a constraint (side condition) of the form $g(\mathbf{x}) = k$, for some constant $k \in \mathbb{R}$.

Typically, people will write as follows

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & g(x) = k\,. \end{aligned}$$

Constrained optimization

Theorem (Method of Lagrange Multiplier)
Suppose the maximum/minimum values of f exist and $\nabla g(\mathbf{x}) \neq 0$ where $g(\mathbf{x}) = k$. To find the maximum and minimum values of f subject to constraint $g(\mathbf{x}) = k$, we do the following:

1. *Find all values of $\mathbf{x}$ and $\lambda \in \mathbb{R}$ such that*

$$\nabla f(\mathbf{x}) = \lambda \nabla g(\mathbf{x}),$$

   and

$$g(\mathbf{x}) = k.$$

2. *Evaluate f at all the points $\mathbf{x}$ that result from step 1. The largest of these values is the maximum of f; the smallest is the minimum value of f.*

Example

https://youtu.be/hQ4UNu1P2kw

Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(1, 1, 1)$.

Two constraints

Problem 15 from

`https://activecalculus.org/multi/S-10-8-Lagrange-Multipliers.html`