

# Multivariable Calculus

## Day 11

### Differentiability and Directional Derivative

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**Definition**

Let  $\mathbf{u} \in \mathbb{R}^n$ . The directional derivative of  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  at  $a \in \mathbb{R}^n$  in the direction of  $\mathbf{u}$  is the following limit (if exists)

$$D_{\mathbf{u}}f(a) = \lim_{h \rightarrow 0} \frac{f(a + h\mathbf{u}) - f(a)}{h}.$$

## Definition

Let  $f : D \rightarrow \mathbb{R}$  and  $a \in \mathbb{R}^n$ . Let  $z = f(x)$  and  $\Delta z = f(a + \Delta x) - f(a)$ . Then  $f$  is **differentiable at**  $a$  if  $\Delta z$  can be expressed in the form

$$\Delta z = \sum_{i=1}^n \partial_i f(a) \Delta x_i + \epsilon_i \Delta x_i ,$$

where  $\epsilon_i \rightarrow 0$  as  $\Delta x_i \rightarrow (0, 0)$ .

$f$  is said to be **differentiable** if it is differentiable at every point on the domain.

## Theorem

Let  $f(x_1, \dots, x_n), g_i(y_1, \dots, y_m)$  ( $i = 1, \dots, n$ ) be differentiable functions. Then,

$$z(y_1, \dots, y_m) = f(g_1(y_1, \dots, y_m), \dots, g_n(y_1, \dots, y_m))$$

is differentiable and

$$\frac{\partial z}{\partial y_i} = \sum_{j=1}^n \frac{\partial f}{\partial x_j} \frac{\partial g_j}{\partial y_i}.$$

Compute directional derivative in the direction  $\mathbf{u} \neq \mathbf{0}$  of the functions

①  $f(\mathbf{x}) = |\mathbf{x}|^2.$

②

$$f(\mathbf{x}) = \begin{cases} \frac{x_1^2 x_2}{x_1^4 + x_2^2} & \mathbf{x} \neq \mathbf{0} \\ 0 & \mathbf{x} = \mathbf{0}. \end{cases}$$