MATH 104: WORKSHEET 12

1. Concepts

(1) Implicit differentiation
(2) Chain rule. Suppose that u is a differentiable function of the n variables $x_1, x_2, \ldots, x_n$ and each x_j is a differentiable function of the m variables $t_1, t_2, \ldots, t_m$. Then u is a function of $t_1, t_2, \ldots, t_m$ and

$$\frac{\partial u}{\partial t_i} = \frac{\partial u}{\partial x_1}\frac{\partial x_1}{\partial t_i} + \frac{\partial u}{\partial x_2}\frac{\partial x_2}{\partial t_i} + \cdots + \frac{\partial u}{\partial x_n}\frac{\partial x_n}{\partial t_i}$$

for each $i = 1, 2, \ldots, m$.

2. Discussions

Question 1. Suppose $F(x, y, z) = C$. Find formulas for

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

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Question 2. Find $\partial z/\partial x$ and $\partial z/\partial y$ of

$$yz + x \ln y = z^2 \,.$$

Question 3. Let $f(x, y)$, $g(s, t)$, $h(s, t)$ be functions. Compute $\frac{\partial f}{\partial t}$ and $\frac{\partial f}{\partial s}$ if

$$f(x, y) = x \sin y$$
$$g(s, t) = s^5 + \frac{1}{t} \,,$$
$$h(s, t) = e^{s^2} + \ln(st) \,.$$

Question 4. If $z = f(x, y)$, where $x = r \cos\theta$ and $y = r \sin\theta$, then:

(a) Find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$.

(b) Show that:

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2.$$