MATH 104: MULTIVARIABLE CALCULUS

MIDTERM

NAME:_____

There are four questions. Make sure you justify all your work for complete credit.

Rules

- You have 80 minutes to complete your work..
- Closed books.
- No use of internet, textbooks, computer algebra systems, calculators.
- No collaboration.
- 1 person per bathroom break. When you go to the bathroom, turn in your cellphone and exam until return.

Scores:

(1) _____
(2) _____
(3) _____
(4) _____
Total : _____

Questions

Problem 1 (20 points, 5 points each).   (1) Where does the line

$$x(t) = 2t - 1\,;\, y(t) = 3t + 2\,;\, z(t) = 4t$$

intersect the plane given by $4x + 3y - z = 3$?

(2) What is the surface that the following function a parametrization of? Give a reason.

$$G(\varphi, \theta) = \begin{bmatrix} R\cos\theta\sin\varphi \\ R\sin\theta\sin\varphi \\ R\cos\varphi \end{bmatrix}$$

(3) What is the value of c so that the planes $2cx - y + c^2z = 15$ and $x + 5cy - 3z = 4$ are orthogonal?

Problem 2 (20 points).   (1) Find an equation of the tangent plane
to the surface $z = x \sin(x + y)$ at the point $(-1, 1, 0)$.

(2) Find the linear approximation at $(0, 0)$ of $e^x \cos(xy)$.

Problem 3 (20 points). Given the curve $\gamma(t) = (1, t, t^2)$.

    (1) Compute the velocity, acceleration of γ at time $t = 1$.

    (2) What is the arclength of γ from $t = 1$ to $t = 2$?

Problem 4 (20 points). A manufacturer has modeled its yearly produc-
tion function P as a Cobb-Douglas function

$$P(L, K) = 2L^{0.2}K^{0.8}$$

where L is the number of labor hours (in thousands) and K is the
invested capital (in millions of dollars). Suppose the manufacturer has
20 workers and 2 machines.

  (1) Use differentials to estimate the change in production if the
      company want to employ only 15 workers and increase to 3
      machines.

  (2) Compare the above result to the exact change (not the differen-
      tial approximation).