

Multivariable Calculus

Day 5

Integrals of vector functions

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Let

$$\mathbf{r}(t) = \langle \cos(t), t, \sin(t) \rangle$$

- Compute $\mathbf{s}(t) = \mathbf{r}'(t)/|\mathbf{r}'(t)|$
- Compute $\mathbf{s}'(t) \cdot \mathbf{r}'(t)$

Integrals

Coordinate-wise integrals

Indefinite integral

$$\int \mathbf{r}(t) dt = \left\langle \int r_1(t) dt, \int r_2(t) dt, \int r_3(t) dt \right\rangle = \mathbf{R}(t) + \mathbf{C}.$$

Definite integral

$$\int_a^b \mathbf{r}(t) dt = \left\langle \int_a^b r_1(t) dt, \int_a^b r_2(t) dt, \int_a^b r_3(t) dt \right\rangle.$$

Fundamental Theorem of Calculus

$$\int_a^b \mathbf{r}(t) dt = ?$$

Compute

$$\mathbf{R}(t) = \int (\cos(t)\mathbf{i} + \mathbf{j} - 2t\mathbf{k}) dt .$$

Compute

$$\int_0^{\pi} \langle \cos(t), 1, -2t \rangle dt .$$

Arc length

Compute the length of the line segment

$$\mathbf{r}(t) = \langle 1, 1, 1 \rangle + t\langle 2, 2, 2 \rangle, t \in [0, 4].$$

A vector function that is continuous.

Example

$$\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$$

Compute the length of the following curve

$$\mathbf{r}(t) = \begin{cases} \langle 1, 1, 1 \rangle + t\mathbf{i}, & t \in [0, 1] \\ \mathbf{r}(1) + t\mathbf{j}, & t \in [1, 2]. \end{cases}$$

Question

How can you compute the length of any curve?

- Compute the length of the curve

$$\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$$

where $t \in [0, 2\pi]$.

- Compute the length of the curve made by the graph of the function

$$f(x) = x^3$$

where $x \in [1, 4]$.