Multivariable Calculus
Day 9
Partial derivatives

Truong-Son Van

Spring 2023

Fulbright University Vietnam

A word about continuity

A function f is continuous at $\mathbf{a}$ if

$$\lim_{\mathbf{x}\to\mathbf{a}} f(\mathbf{x}) = f(\mathbf{a})\,.$$

$\epsilon - \delta$ definition?

Partial derivative

$$f(x, y) = \frac{x^2 \sin(2y)}{32}$$

Partial derivative and graph

https://www.youtube.com/watch?v=dfvnCHqzK54

Formal definition

Given a function $f(x, y)$. The partial derivative of f with respect to x and (a, b), denoted by $f_x(a, b)$, is defined to be

$$f_x(a, b) = \lim_{h \to 0} \frac{f(a + h, b) - f(a, b)}{h}.$$

Likewise, the partial derivative of f with respect to y and (a, b), denoted by $f_y(a, b)$, is defined to be

$$f_y(a, b) = \lim_{h \to 0} \frac{f(a, b + h) - f(a, b)}{h}.$$

Mixed partial derivatives

One can keep taking partial derivatives derivatives of higher order as each partial derivative again results in a multivariable scalar function.

$$f_{xy} = (f_x)_y = \partial_y(\partial_x f) = \partial_{yx} f \, .$$

Be careful with the notations.

Worksheet

Compute all of the mixed derivatives of second order for the functions

$$f(x, y) = e^x \sin y \,,$$

and

$$u(x, t) = \sin(x - at) \,,$$

where a is a constant.

What do you notice about the mixed derivatives?

Worksheet

You have seen the normal distribution at least once in your FUV life.

1. Look up the formula for the 2 dimensional normal distribution with mean zero and variance t.
2. Compute all the mixed partial derivatives in terms of x, y (or x_1, x_2).
3. Compute the derivative in t of the normal distribution.
4. What do you see?