Multivariable Calculus
Day 3
Some toy examples in $\mathbb{R}^3$

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Equations for lines and planes

Equation for a line

A line is a collection of points that is parallel to a vector and goes through a

$$L = \{\mathbf{r}(t) \,|\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}, t \in \mathbb{R}\},$$

where r_0 is the initial position and $\mathbf{v}$ is the direction. The equation for $\mathbf{r}(t)$ is called a **vector equation for a line** L.

Equation for a line

Let $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ and $\mathbf{r}_0 = (x_0, y_0, z_0)$. The **parametric equations** of L is the following system of equations

$$
\begin{aligned}
x &= x_0 + v_1 t\,, \\
y &= y_0 + v_2 t\,, \\
z &= z_0 + v_3 t\,.
\end{aligned}
$$

This leads to the **symmetric equations** of L

$$
\frac{x - x_0}{v_1} = \frac{y - y_0}{v_2} = \frac{z - z_0}{v_3}\,.
$$

Worksheet

1.
   - Find parametric equations and symmetric equations of the line that passes through the points $A(2, 4, -4)$ and $B(3, -1, 1)$.
   - At what point does this line intersect the xy-plane?
2. What would be an equation that describe the line segment connecting two points A and B in $\mathbb{R}^3$?

Equation for plane

A plane is a collection of points that is perpendicular to one specific direction

$$P = \{\mathbf{r} \,|\, \mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0\}\,.$$

$\mathbf{n}$ is the perpendicular vector to the plane called the normal vector.

In higher dimension, planes are called hyperplanes. This is an important concept in data classification.

Worksheet

1. Find an equation of the plane through the point $(2, 4, -1)$ with normal vector $\mathbf{n} = \langle 2, 3, 4 \rangle$. Sketch the plane on the coordinate system.