MATH 104: Multivariable Calculus Oral Exam

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Rules

- 4 questions, 90 minutes

- Closed books

Scores

Problem 1. _____/24

Problem 2. _____/16

Problem 3. _____/20

Problem 4. _____/20

Total _____/100

Questions

Problem 1. (4 points each subproblem) Let $f : \mathbb{R}^2 \to \mathbb{R}$.

(a) What does it mean for f to be differentiable at (a, b)?

(b) Write directional derivative of function f in the direction $\mathbf{u}$ in terms of partial derivative/gradient of f.

(c) What is the meaning of ∇f, the gradient of a function $f : \mathbb{R}^n \to \mathbb{R}$?

(d) What is a parametric curve?

(e) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function and C be a smooth curve in $\mathbb{R}^2$ parametrized by $\mathbf{r} : [a, b] \to \mathbb{R}^2$. Write down the formula to compute the line integral of f along C.

(f) Let $\mathbf{F} : \mathbb{R}^2 \to \mathbb{R}^2$ be a vector field and C be a smooth curve in $\mathbb{R}^2$ parametrized by $\mathbf{r} : [a, b] \to \mathbb{R}^2$. Write down a formula to compute the line integral of $\mathbf{F}$ along C.

(g) What is Green's theorem?

(h) What is the second derivative test for optimization?
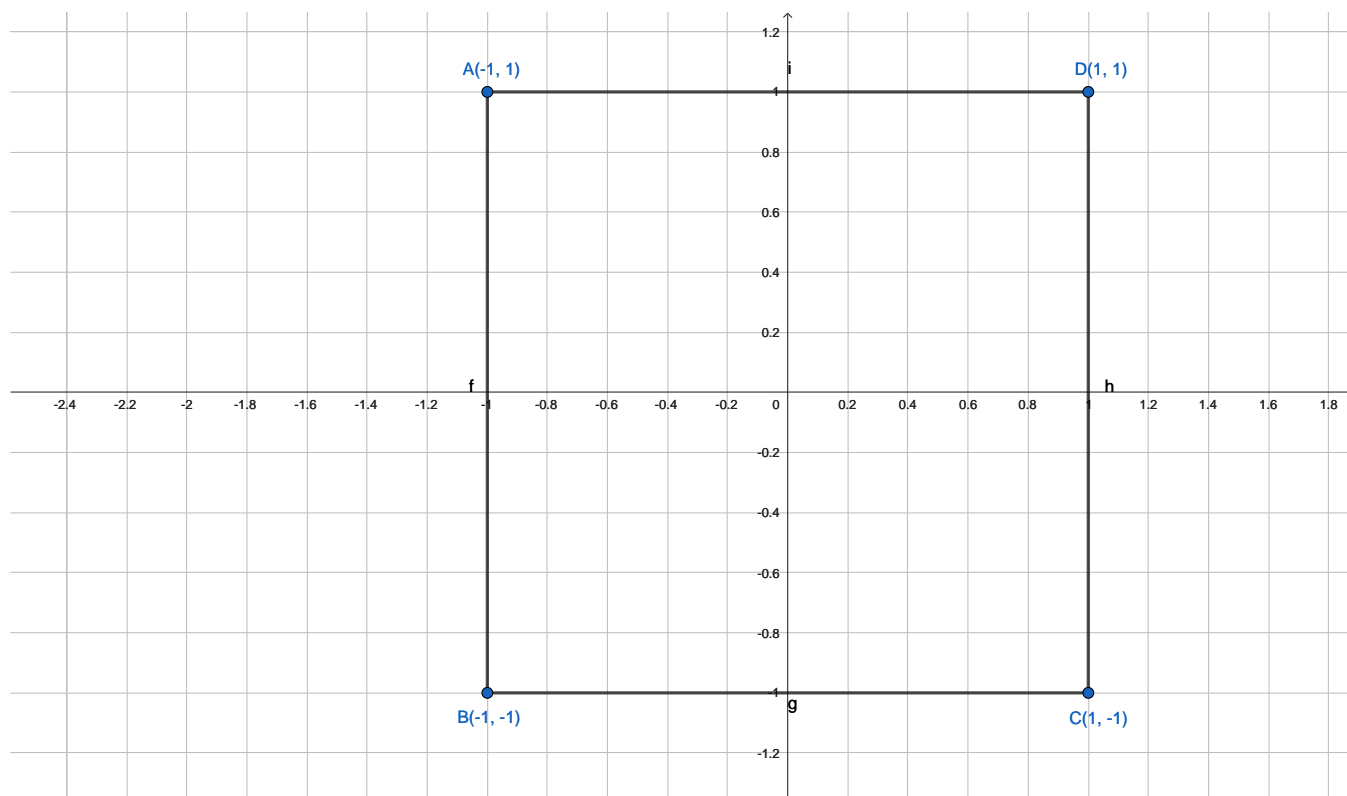
Problem 2 (16 points). Compute the following

$$\int_0^1 \int_{3y}^3 e^{x^2} \, dx dy \, .$$

Problem 3. (a) (5 points) Consider the vector field

$$\mathbf{F}(x, y) = x^2 \mathbf{j}.$$

Is the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ independent of path?

(b) (10 points) Compare the path integrals of $\mathbf{F}$ on two paths $A \to B \to C$ and $A \to D \to C$, where the paths are from the figure below.

(c) (5 points) Are parts (a) and (b) consistent with each other? Why or why not?

Problem 4. (a) (10 points) State the change of variable theorem. That is, for a change of coordinate $\varphi : D \to S$ such that

$$\begin{pmatrix} x \\ y \end{pmatrix} = \varphi(u, v),$$

what is the formula for $\iint_S f\, dA$?

(b) (10 points) Evaluate

$$\iint_S xy\, dA$$

where S is the hollowed disk with radius between 1 and 2 and has center at $(0,0)$.