Multivariable Calculus
Day 22
Vector Calculus: Line integrals (cont.)

Spring 2023

Recap

Fundamental Theorem for line integrals

Theorem (Fundamental Theorem for line integrals)
Let C be a smooth curve given by the parametrization $\mathbf{r}(t)$, $a \leq t \leq b$. Let f be a differentiable function of two or three variables whose gradient vector ∇f is continuous on C. Then,

$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a)).$$

Worksheet

Definition
A **closed curve** is a curve that starts and ends at the same point.

A **simple closed curve** is a closed curve that never crosses itself.

Corollary
If C is a closed curve and $f : \mathbb{R}^n \to \mathbb{R}$ is a smooth function, then

$$\oint_C \nabla f \cdot d\mathbf{r} = 0\,.$$

Definition
A vector field $\mathbf{F}$ is called a **conservative vector field** if it is the gradient of some scalar function, that is there exists a function f such that

$$\nabla f = \mathbf{F}\,.$$

Independence of path

Worksheet

1. True or False? If $\mathbf{F}$ is a conservative vector field, then

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = 0\,.$$

Definition
Let $\mathbf{F}$ be a continuous vector field with domain D, we say that the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

is **independent of path** if

$$\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$$

Independence of path and conservative vector fields

Theorem
$\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path in D if and only if $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for every closed path C in D.