# MATH 104: Multivariable Calculus Oral Exam

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## Rules

- 4 questions, 90 minutes
- Closed books

## Scores

Problem 1. ____/24

Problem 2. ____/16

Problem 3. ____/20

Problem 4. ____/20

Total _____/100

# Questions

*Problem* 1. (4 points each subproblem) Let $f : \mathbb{R}^2 \to \mathbb{R}$.

(a) What does it mean for $f$ to be differentiable at $(a, b)$?

(b) Write directional derivative of function $f$ in the direction $\mathbf{u}$ in terms of partial derivative/gradient of $f$.

(c) What is the meaning of $\nabla f$, the gradient of a function $f : \mathbb{R}^n \to \mathbb{R}$?

(d) What is a parametric curve?

(e) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function and $C$ be a smooth curve in $\mathbb{R}^2$ parametrized by $\mathbf{r} : [a, b] \to \mathbb{R}^2$. Write down the formula to compute the line integral of $f$ along $C$.

(f) Let $\mathbf{F} : \mathbb{R}^2 \to \mathbb{R}^2$ be a vector field and $C$ be a smooth curve in $\mathbb{R}^2$ parametrized by $\mathbf{r} : [a, b] \to \mathbb{R}^2$. Write down a formula to compute the line integral of $\mathbf{F}$ along $C$.

(g) What is Green's theorem?

(h) What is the second derivative test for optimization?

*Problem* 2 (16 points). Compute the following

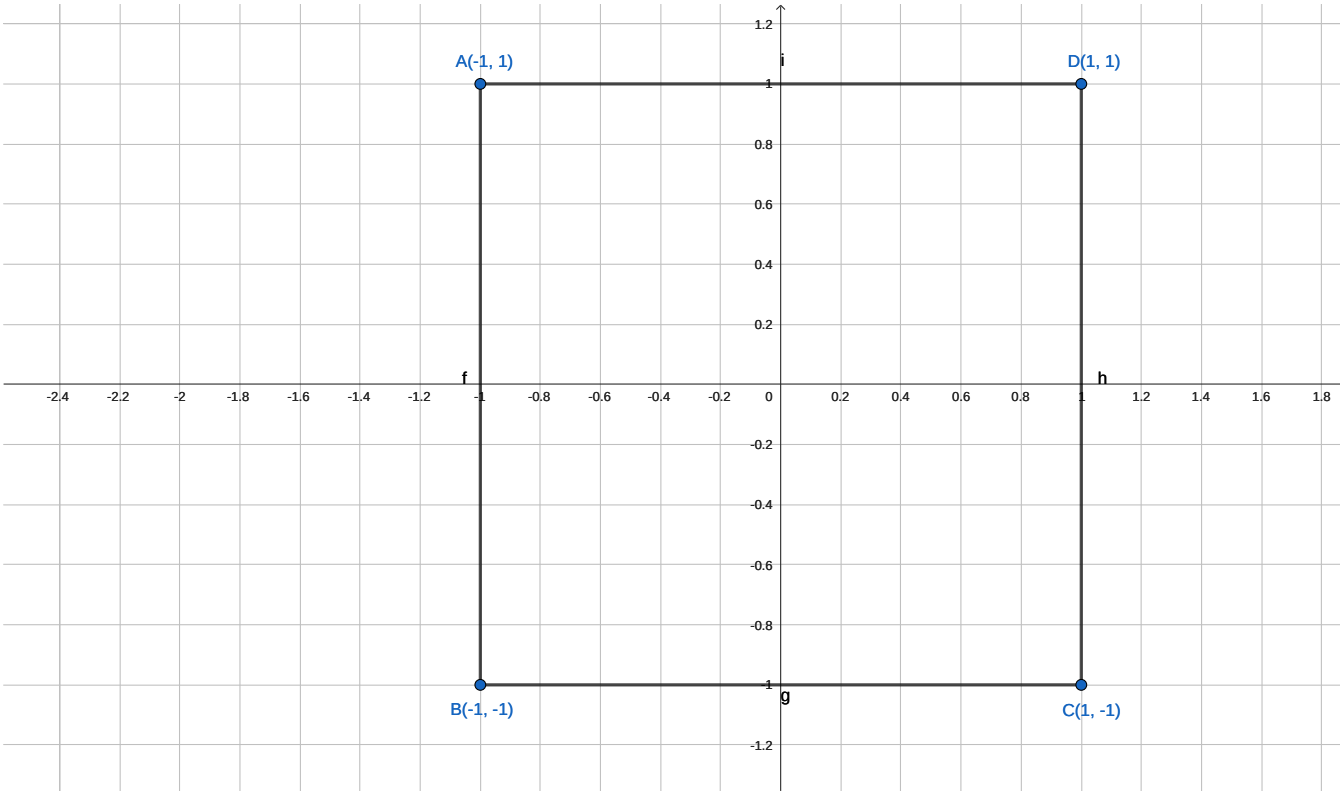
$$\int_0^1 \int_{3y}^3 e^{x^2}\, dxdy\,.$$

*Problem* 3. (a) (5 points) Consider the vector field

$$\mathbf{F}(x, y) = x^2\mathbf{j}\,.$$

Is the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ independent of path?

(b) (10 points) Compare the path integrals of $\mathbf{F}$ on two paths $A \to B \to C$ and $A \to D \to C$, where the paths are from the figure below.




(c) (5 points) Are parts (a) and (b) consistent with each other? Why or why not?

*Problem* 4. (a) (10 points) State the change of variable theorem. That is, for a change of coordinate $\varphi : D \to S$ such that

$$\begin{pmatrix} x \\ y \end{pmatrix} = \varphi(u, v)\,,$$

what is the formula for $\iint_S f\, dA$?

(b) (10 points) Evaluate

$$\iint_S xy\, dA$$

where $S$ is the hollowed disk with radius between 1 and 2 and has center at $(0, 0)$.