MATH 104: WORKSHEET 21

1. Concepts

Double integral in polar coordinates

2. Concept

Definition 2.1. We can represent the points on th Cartesian plane using polar coordinates (r, θ) and (x, y) by writing

$$r^2 = x^2 + y^2 \qquad x = r\cos\theta \qquad y = r\sin\theta$$

If f is continuous on a polar region of the form

$$D = \{(r, \theta) \mid \alpha \leqslant \theta \leqslant \beta\,, h_1(\theta) \leqslant r \leqslant h_2(\theta)\}$$

then

$$\iint_D f(x, y)\, dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta) r\, dr\, d\theta\,.$$

3. Discussion

Question 1. Compute the integral

$$\int_{\mathbb{R}} e^{-x^2}\, dx\,.$$

Question 2. Find the volume of the solid bounded by the plane $z = 0$ and the paraboloid $z = 1 - x^2 - y^2$.

Question 3. Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy-plane, and inside the cylinder $x^2 + y^2 = 2x$.

Date: April 28, 2025.