Multivariable Calculus
Day 18
Integration

Spring 2023

Worksheet

Let B be a 2×2 matrix that is invertible (the determinant is non-zero). We can think of B as a function $B : \mathbb{R}^2 \to \mathbb{R}^2$.

Let now $f : \mathbb{R}^2 \to \mathbb{R}$ be a function such that

$$f(x, y) = xy\,.$$

Let D is the rectangle with vertices $(1, 1), (1, 6), (5, 1), (5, 6)$. Find the relationship between

$$\iint_{B(D)} f(y)\, dA$$

and

$$\iint_{D} f(x)\, dA\,.$$

Change of coordinates

Let f be a function of (x, y) defined on the domain D. Let

$$\begin{pmatrix} x \\ y \end{pmatrix} = \varphi(u, v)$$

for some coordinate change function $\varphi : D \to S$.

Theorem
If f is continuous, then

$$\int_S f \, dA = \int_D (f \circ \varphi) \, |\det D\varphi| \, dA \, .$$

Example

Compute the following integral

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \exp\left(-\frac{x^2}{2}\right) dx\,.$$

Worksheet