Multivariable Calculus
Day 11
Differentiability and Directional Derivative

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Directional derivative

Definition
Let $\mathbf{u} \in \mathbb{R}^n$. The directional derivative of $f : \mathbb{R}^n \to \mathbb{R}$ at $a \in \mathbb{R}^n$ in the direction of $\mathbf{u}$ is the following limit (if exists)

$$D_{\mathbf{u}} f(a) = \lim_{h \to 0} \frac{f(a + h\mathbf{u}) - f(a)}{h}.$$

Differentiability

Definition
Let $f : D \to \mathbb{R}$ and $a \in \mathbb{R}^n$. Let $z = f(x)$ and $\Delta z = f(a + \Delta x) - f(a)$. Then f is **differentiable at** a if Δz can be expressed in the form

$$\Delta z = \sum_{i=1}^{n} \partial_i f(a) \Delta x_i + \epsilon_i \Delta x_i \,,$$

where $\epsilon_i \to 0$ as $\Delta x_i \to (0, 0)$.

f is said to be **differentiable** if it is differentiable at every point on the domain.

Chain rule

Theorem
Let $f(x_1, \ldots, x_n), g_i(y_1, \ldots, y_m)$ *(*$i = 1, \ldots, n$*) be differentiable functions. Then,*

$$z(y_1, \ldots, y_m) = f(g_1(y_1, \ldots, y_m), \ldots, g_n(y_1, \ldots, y_m))$$

is differentiable and

$$\frac{\partial z}{\partial y_i} = \sum_{j=1}^{n} \frac{\partial f}{\partial x_j} \frac{\partial g_j}{\partial y_i}.$$