

MATH 104: WORKSHEET 6

1. Concepts

- (1) Derivatives
- (2) Tangent vector
- (3) Unit tangent vector
- (4) Differentiation rules
- (5) Integrals

2. Discussions

Question 1. Find the unit tangent vectors of the following curves

(1)

$$\vec{r}(t) = \langle \sqrt{t-2}, 3, 1/t^2 \rangle$$

(2)

$$\vec{r}(t) = \langle e^{-t}, t - t^3, \ln t \rangle$$

Question 2. Find the equation of the tangent line to the curve with the given parametric equations at specified point.

(1) $x = t^2 + 1, y = 4\sqrt{t}, z = e^{t^2-t}$ at $(2, 4, 1)$

(2) $x = \ln(t + 1), y = t \cos(2t), z = 2^t$ at $(0, 0, 1)$

Question 3. Find the point of intersection of the tangent lines to the curve $\vec{r}(t) = \langle \sin(\pi t), 2 \sin(\pi t), \cos(\pi t) \rangle$ at the points where $t = 0$ and $t = 1/2$.

Question 4. Evaluate the integral:

(1) $\int_0^2 (t\vec{i} - t^3\vec{j} + 3t^5\vec{k})$

Question 5. Compute

$$\frac{d}{dt}(\vec{u}(t) \cdot \vec{v}(t))$$

where $\vec{u}(t) = \langle \sin t, \cos t, t \rangle$ and $\vec{v}(t) = \langle t, \cos t, \sin t \rangle$.