MATH 104: WORKSHEET 6

1. Concepts

(1) Multivariable functions
(2) Partial derivatives
(3) Derivative as matrix of partial derivatives

2. Discussions

Question 1. Let

$$F(x, y, z) = \frac{x^2\sqrt{y^3}}{z^4}.$$

Compute

$$\frac{\partial F}{\partial x}, \quad \frac{\partial F}{\partial y}, \quad \frac{\partial F}{\partial z}$$

Question 2. Recall that the equation for a paraboloid is

$$z = x^2 + y^2.$$

This can be parametrized by the following equation $G : \mathbb{R}^2 \to \mathbb{R}^3$,

$$G(s, t) = \begin{pmatrix} s \\ t \\ s^2 + t^2 \end{pmatrix}.$$

Compute the derivative, as a matrix, of this parametrization.

Does it matter how we write the matrix?

Question 3. The Cobb-Douglas model is a classic model in economics. It says production, P, is a function of materials, M, and labor L via the following relationship

$$P = \kappa M^\alpha L^\beta$$

where $\kappa > 0$, $0 < \alpha, \beta < 1$ and $\alpha + \beta = 1$. If the investment in labor is increased and the investment in materials is decreased at an equal rate, what is the impact on production?

Question 4. Consider a function f such that, at a particular point a,

$$[Df]_a \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}.$$

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(1) How many inputs does f have?
(2) What happens if inputs change at rates $\vec{h} = \langle -2, 2\rangle$?
(3) What if $\vec{h} = \langle 3, 3\rangle$? Can you do this?
(4) Can you do the previous problem if you know
$$[Df]_a \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}?$$

Question 5. Explain the velocity vector of a parametrized curve $\gamma(t)$ in terms of the definition of a derivative.