MATH 104: WORKSHEET 20

1. Concepts

Double integral over general regions

Definition 1.1. A plane region D is said to be of *type I* if it lies between the graphs of two continuous functions of x

$$D = \{(x, y) \mid a \leqslant x \leqslant b\,, g_1(x) \leqslant y \leqslant g_2(x)\}$$

The integral over type I would be

$$\iint_D f(x, y)\, dA \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dydx\,.$$

Definition 1.2. A plane region D is said to be of *type II* if it lies between the graphs of two continuous functions of x

$$D = \{(x, y) \mid c \leqslant y \leqslant d\,, h_1(y) \leqslant x \leqslant h_2(y)\}$$

The integral over type II would be

$$\iint_D f(x, y)\, dA \int_a^b \int_{h_1(y)}^{h_2(y)} f(x, y)\, dxdy\,.$$

If $D = D_1 \cup D_2$ where D_1 and D_2 don't overlap, then

$$\iint_D f(x, y)\, dA = \iint_{D_1} f(x, y)\, dA + \iint_{D_2} f(x, y) dA\,.$$

The area of a region

$$A(D) = \iint_D 1\, dA\,.$$

If $m \leqslant f(x, y) \leqslant M$, then

$$mA(D) \leqslant \iint_D f(x, y)\, dA \leqslant MA(D)\,.$$

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2. Discussions

Evaluate

(1) $\iint_D x \cos y \, dA$ where D is the region bounded by $y = 0, y = x^2, x = 1$

(2) $\iint_D (x^2 + 2y) \, dA$ where D is bounded by $y = x, y = x^3, x \geqslant 0$.

Question 1. Find the volume of the given solid under the plane $3x + 2y - z = 0$ and above the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

Question 2. Evaluate

(1)
$$\int_0^1 \int_x^1 \sin(y^2)\, dy\, dx$$

(2)
$$\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3+1}\, dydx$$

Question 3. Estimate upper and lower bound of the integral
$$\iint_S \sqrt{4-x^2y^2}\, dA$$
where $S = \{(x,y) \mid x^2+y^2 \leqslant 1\,, x \geqslant 0\}$.