Multivariable Calculus

Day 19

Applications of change of variables

Worksheet

Evaluate the following integral

0

$$\iint_{R} \frac{x - 2y}{3x - y} \, dA \,,$$

where R is the parallelogram enclosed by the lines

$$x - 2y = 0, x - 2y = 4, 3x - y = 1, 3x - y = 8.$$



$$\iint_{R} \sin(9x^2 + 4y^2) \, dA$$

where R is the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$.

Polar Coordinate

In \mathbb{R}^2 , when the region of integration is a section of a disk centered at 0. Let

$$\begin{pmatrix} x \\ y \end{pmatrix} = \varphi(r,\theta) = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix}$$
,

where $a \le r \le b$ and $\alpha \le \theta \le \beta$.

Example

Compute the following integral

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \exp\left(-\frac{x^2}{2}\right) \, dx \, .$$

Cylindrical Coordinate

In \mathbb{R}^3 , when the region of integration is part of a cylinder. Let

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \varphi(r, \phi, \theta) = \begin{pmatrix} r \cos \theta \\ r \cos \theta \\ z \end{pmatrix},$$

where $a \le r \le b$, $\alpha \le \theta \le \beta$.

Spherical Coordinate

In \mathbb{R}^3 , when the region of integration is a section of a ball centered at 0. Let

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \varphi(\rho, \phi, \theta) = \begin{pmatrix} \rho \sin \phi \cos \theta \\ \rho \sin \phi \sin \theta \\ \rho \cos \phi \end{pmatrix},$$

where $a \le \rho \le b$, $\alpha \le \theta \le \beta$, and $c \le \phi \le d$.

Worksheet

lacktriangle Compute the volume of a solid sphere with radius R.