Multivariable Calculus
Day 17
Integration

Spring 2023

Some properties of integrals

1. Let U, V be disjoint domains, then

$$\iint_{U \cup V} f\, dA = \iint_U f\, dA + \iint_V f\, dA.$$

2.

$$\iint_U (f + g)\, dA = \iint_U f\, dA + \iint_V f\, dA.$$

1. Set up a problem with double integral to find the area of triangle with vertices $(0,0), (2,0), (2,3)$.

2. Let $f(x,y) = x^2 y$. Find

$$\iint_D f(x,y)\, dA$$

   where D is the triangle with vertices $(0,0), (2,0), (2,3)$.

3. Compute

$$\iint_D e^{-y^2}\, dA$$

Worksheet

1. Let B be a 2×2 matrix that is invertible (the determinant is non-zero). We can think of B as a function $B : \mathbb{R}^2 \to \mathbb{R}^2$.

   Let now $f : \mathbb{R}^2 \to \mathbb{R}$ be a function such that

   $$f(x, y) = xy \,.$$

   1. What is

      $$\iint_D f \, dA \,,$$

      where D is the rectangle with vertices $(1, 1), (1, 6), (5, 1), (5, 6)$.
   2. Let $y = Bx$. Which of the following expression makes sense (could be more than 1)?
      1. $\iint_{B(D)} f(Ax) \, dx$
      2. $\iint_{B(D)} f(y) \, dy$
      3. $\iint_D f(Bx) \, dx$
      4. $\iint_{B(D)} f(x) \, dx$

Find the relationship between

$$\iint_{B(D)} f(y)\, dy$$

and

$$\iint_D f(x)\, dx\,.$$

Change of coordinates

Let f be a function of (x, y) defined on the domain D. Let

$$\begin{pmatrix} x \\ y \end{pmatrix} = \varphi(u, v)$$

for some coordinate change function $\varphi : D \to S$.

Theorem
If f is continuous, then

$$\int_S f \, dA = \int_D (f \circ \varphi) \, |\det D\varphi| \, dA \, .$$

Example

Compute the following integral

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \exp\left(-\frac{x^2}{2}\right) \, dx \,.$$