

## MATH 104: WORKSHEET 10

### 1. Concepts

- (1) Optimization
- (2) Necessary condition
- (3) Second derivative test for 2D

### 2. Discussions

*Problem 2.1.* (1) Find critical points of the following functions:

(a)  $f(x, y) = |x| + |y|$ .

(b)  $f(x, y) = x^2 + y^2 - qxy$  where  $q \in \mathbb{R}$  is a given constant.

(c)  $xy + \frac{2}{x} + \frac{4}{y}$

- (2) Are the critical points you found above minima, maxima or neither.

*Problem 2.2.* A rectangular box without a lid is to be made from  $12 \text{ m}^2$  of cardboard. Find the maximum volume of such a box.

*Problem 2.3.* A model for the yield  $Y$  of an agricultural crop as a function of the nitrogen level  $N$  and phosphorus level  $P$  in the soil (measured in appropriate units) is

$$Y(N, P) = kNP e^{-N-P}$$

where  $k$  is a positive constant. What levels of nitrogen and phosphorus result in the best yield?