Multivariable Calculus
Day 19
Applications of change of variables

Spring 2023

Worksheet

Evaluate the following integral

1. $$\iint_R \frac{x-2y}{3x-y}\, dA\,,$$
   where R is the parallelogram enclosed by the lines
   $x-2y=0, x-2y=4, 3x-y=1, 3x-y=8.$

2. $$\iint_R \sin(9x^2+4y^2)\, dA$$
   where R is the region in the first quadrant bounded by the ellipse $9x^2+4y^2=1$.

Polar Coordinate

In $\mathbb{R}^2$, when the region of integration is a section of a disk centered at 0. Let

$$\begin{pmatrix} x \\ y \end{pmatrix} = \varphi(r, \theta) = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix},$$

where $a \le r \le b$ and $\alpha \le \theta \le \beta$.

Example

Compute the following integral

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \exp\left(-\frac{x^2}{2}\right)\, dx \, .$$

Cylindrical Coordinate

In $\mathbb{R}^3$, when the region of integration is part of a cylinder. Let

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \varphi(r, \phi, \theta) = \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ z \end{pmatrix},$$

where $a \leq r \leq b$, $\alpha \leq \theta \leq \beta$.

Spherical Coordinate

In $\mathbb{R}^3$, when the region of integration is a section of a ball centered at 0. Let

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \varphi(\rho, \phi, \theta) = \begin{pmatrix} \rho \sin\phi \cos\theta \\ \rho \sin\phi \sin\theta \\ \rho \cos\phi \end{pmatrix},$$

where $a \leq \rho \leq b$, $\alpha \leq \theta \leq \beta$, and $c \leq \phi \leq d$.

Worksheet

1. Compute the volume of a solid sphere with radius R.