Multivariable Calculus
Day 18
Integration

Spring 2023

Worksheet

Let B be a 2×2 matrix that is invertible (the determinant is non-zero). We can think of B as a function $B : \mathbb{R}^2 \to \mathbb{R}^2$.

Let now $f : \mathbb{R}^2 \to \mathbb{R}$ be a function such that

$$f(x, y) = xy \,.$$

Let D is the rectangle with vertices $(1, 1), (1, 6), (5, 1), (5, 6)$. Find the relationship between

$$\iint_{B(D)} f(y) \, dA$$

and

$$\iint_{D} f(x) \, dA \,.$$

Change of coordinate

A coordinate transformation is a function φ, which is bijective and differentiable for which $D\varphi$ is invertible at all points in the domain. Here,

$$D\varphi = \begin{pmatrix} \partial_1\varphi_1 & \partial_2\varphi_1 \\ \partial_1\varphi_2 & \partial_2\varphi_2 \end{pmatrix} .$$

Worksheet

Find the image of the following transformations. Determine whether they are coordinate transformation or not?

1. (Ex. 15.9.1)

$$x = u^2 - v^2\,, \qquad y = 2uv\,.$$

$$S = \{(u, v) | 0 \leq u, v \leq 1\}$$

2. $$x = u + v\,, \qquad y = u - v\,.$$

$$S = \{(u, v) | 0 \leq u, v \leq 1\}$$

Change of coordinates

Let f be a function of (x, y) defined on the domain D. Let

$$\begin{pmatrix} x \\ y \end{pmatrix} = \varphi(u, v)$$

for some coordinate change function $\varphi : D \to S$.

Theorem
If f is continuous, then

$$\int_S f \, dA = \int_D (f \circ \varphi) \, |\det D\varphi| \, dA \, .$$

Example

Compute the following integral

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \exp\left(-\frac{x^2}{2}\right) \, dx \, .$$

Worksheet

Evaluate the following integral

1. $$\iint_R \frac{x-2y}{3x-y}\, dA\,,$$
   where R is the parallelogram enclosed by the lines $x-2y=0, x-2y=4, 3x-y=1, 3x-y=8.$

2. $$\iint_R \sin(9x^2+4y^2)\, dA$$
   where R is the region in the first quadrant bounded by the ellipse $9x^2+4y^2=1.$