MATH 104: WORKSHEET 8

1. Concepts

(1) Functions of two variables. A function f of n variables is a rule that assigns to each ordered pair of real numbers $\mathbf{x} \in \mathbb{R}^n$ in a set D a unique real number denoted by $f(\mathbf{x})$. The set D is the *domain* of f and its *range* is the set of values that f takes on, that is,

$$\{f(\mathbf{x}) \mid \mathbf{x} \in D\}.$$

(2) Graph of a function. If f is a function of n variables with domain D, then the *graph* of f is the set of all points $(\mathbf{x}, z)$ in $\mathbb{R}^{n+1}$ such that

$$z = f(\mathbf{x}) \quad \text{and} \quad \mathbf{x} \in D.$$

(3) The *level curves* (contour curves) of a function f of two variables are the curves of (x, y) in the domain with equations

$$f(x, y) = k,$$

where k is a constant (in the range of f).

(4) The *level sets* of a function f of n variables are the sets of $\mathbf{x}$ in the domain with equations

$$f(\mathbf{x}) = k,$$

where k is a constant (in the range of f).

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2. Discussions

Question 1. Find and sketch the domain of

(1)
$$f(x,y) = \frac{\sqrt{x+y+1}}{x-1}$$

(2)
$$f(x,y) = x\ln(y^2 - x)$$

Question 2. The Cobb-Douglas function:

$$P(L, K) = bL^{\alpha}K^{1-\alpha}\,.$$

Sketch the level curves of the function P.