MATH 104: WORKSHEET 1

1. Concepts

(1) Surfaces
(2) Vectors

2. Discussions

Definition 2.1 (Line in 2D)**.** And equation with two variables $f(x, y) = 0$ represents a line in $\mathbb{R}^2$.

Definition 2.2 (Surface in 3D)**.** And equation with three variables $f(x, y, z) = 0$ represents a surface in $\mathbb{R}^3$.

Definition 2.3 (Hypersurface)**.** And equation with n variables $f(x_1, \ldots, x_n) = 0$ represents a hypersurface in $\mathbb{R}^n$.

Question 1. What are the surfaces that the following equations represent in $\mathbb{R}^3$?

(1) $(x-1)^2 + (y-1)^2 + (z-2)^2 = 9$

(2) $(x-1)^2 + \frac{1}{4}(y-3)^2 = 4$, $1 \leqslant z \leqslant 2$

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(3) $3x + 3y - 5z = 0$

Definition 2.4. A point $x \in \mathbb{R}^3$ is a tuple

$$x = (x_1, x_2, x_3)\,.$$

A vector $\vec{x} \in \mathbb{R}^3$ is a tuple

$$\vec{v} = \langle v_1, v_2, v_3 \rangle\,.$$

A vector represents a quantity that has a length and a direction (starting and ending points are not important).

Question 2. What is the length of

(1) $\langle 1, 2, 3 \rangle$?

(2) $\vec{v} + \vec{w}$ where $\vec{v} = \langle 1, 2, 3 \rangle$ and $\vec{w} = \langle 2, 3, 1 \rangle$?

(3) $\vec{w} = 10\vec{v}$ where $\vec{v} = \langle 1, 2, 3 \rangle$?