

## MATH 104: WORKSHEET 11

### 1. Concepts

- (1) Tangent plane
- (2) Differential: for  $z = f(x, y)$ , the differential (or total differential)  $dz$  around  $(a, b)$  is defined as

$$dz = f_x(a, b) dx + f_y(a, b) dy.$$

- (3)  $\Delta z = f(a + \Delta x, b + \Delta y) - f(a, b)$
- (4) Linear approximation

$$f(x, y, z) \approx f(a, b, c) + f_x(a, b, c)(x - a) + f_y(a, b, c)(y - b) + f_z(a, b, c)(z - c) = L(x, y, z)$$

### 2. Discussions

*Question 1.* (1) What is the vector equation for the tangent plane to the surface  $z = f(x, y)$ ?

- (2) What is the scalar equation for the tangent plane to the surface  $z = f(x, y)$ ?

*Question 2.* Let  $z = f(x, y) = x^2 + 3xy - y^2$ . The input changes from  $(2, 3)$  to  $(2.05, 2.96)$ , compare  $\Delta z$  and  $dz$ .

*Question 3.* Find the tangent plane to the surface at the specified point:

(1)  $z = e^{x-y}$  at  $(2, 2, 1)$

(2)  $z = x \sin(x + y)$  at  $(-1, 1, 0)$

*Question 4.* Find the linearization  $L(x, y)$  of

(1)  $f(x, y) = 1 + x \ln(xy - 5)$  at  $(2, 3)$

(2)  $f(x, y) = x^2 e^y$  at  $(1, 0)$