

MATH 104: WORKSHEET 9

1. Concepts

(1) Inverse and Implicit Theorems

2. Discussions

Question 1. (1) What is the inverse function theorem?

(2) What is the implicit function theorem?

Question 2. Consider the function $F : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by

$$F(x, y, z) = \begin{pmatrix} \arctan(x + y) \\ 3x - 2z \\ 1 + e^{y+z} \end{pmatrix}.$$

Is this function locally invertible near $x = y = z = 0$? Is it everywhere locally invertible?

Question 3 (Difficult). The following are three implicitly defined surfaces:

$$\begin{aligned} x^2 + y^2 + z^2 &= C_1 \\ z - \cosh x - \cosh y &= C_2 \\ x - 2yz + y^3 &= C_3. \end{aligned}$$

The surfaces all intersect at one point $(1, 1, 2)$ when $C_1 = \sqrt{6}, C_2 = 2 - e - 1/e, C_3 = -2$. What does this point of mutual intersection do when you wiggle the three constants C_1, C_2, C_3 ? Does it fill in a neighborhood of $(1, 1, 2)$? Or does it trace out some lower-dimensional set?

Question 4. Where can we solve for $y = y(x)$ given that $xe^y - ye^x = 1$?

Question 5. Can you solve for c, d as functions of a, b given that $ab - bc + cd = 8$ and $a + 2b - 2c + 4d = 12$, and you are near $a = 1, b = 2, c = 3, d = 4$?