MATH 104: WORKSHEET 10

1. **Concepts**

(1) Partial derivative

$$\partial_x f(x, y, z) = f_x(x, y, z) = \lim_{h \to 0} \frac{f(x + h, y, z) - f(x, y, z)}{h}$$

$$\partial_y f(x, y, z) = f_y(x, y, z) = \lim_{h \to 0} \frac{f(x, y + h, z) - f(x, y, z)}{h}$$

$$\partial_z f(x, y, z) = f_z(x, y, z) = \lim_{h \to 0} \frac{f(x, y, z + h) - f(x, y, z)}{h}$$

(2) Gradient

$$\nabla f(x, y, z) = \langle f_x(x, y, z), f_y(x, y, z), f_z(x, y, z) \rangle .$$

2. **Discussions**

Question 1. Find all partial derivatives of

(1)
$$f(x, y) = 3x^3 - 2x^2 y^5$$

(2)

$$f(x, y) = \frac{xy^2}{x+1}$$

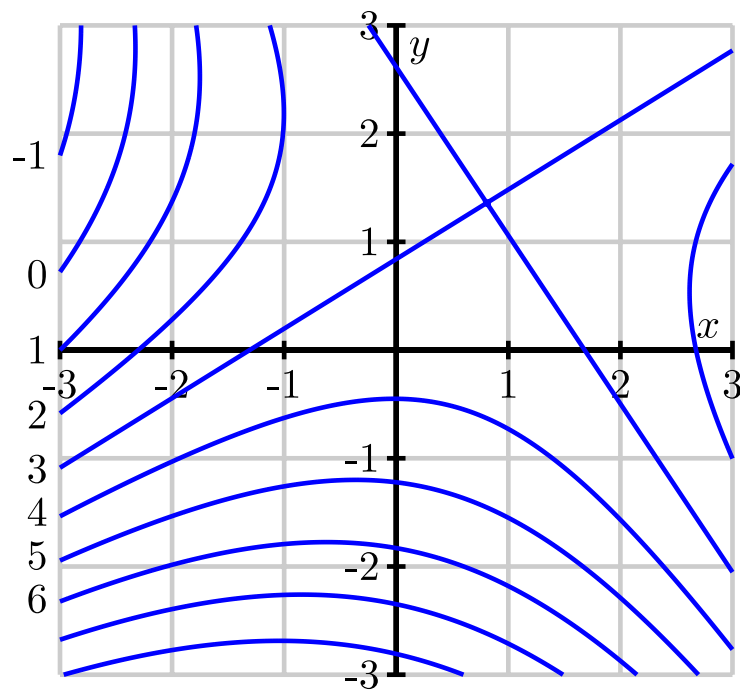(3)

$$g(r, s) = rs \cos(r)$$

(4)

$$f(w, x, y) = (6w + 1) \cos(3x^2 + 4xy^3 + y)$$

(5)

$$q(x, t, z) = \frac{x2^t z^3}{1 + x^2}.$$

Question 2. Consider the following picture

(1) Estimate the partial derivative $f_x(-2, -1)$. (Hint: How can values of f that are of the form $f(-2 + h)$ and $f(-2 - h)$ be used to compute a symmetric difference quotient?)

(2) Estimate the partial derivative $f_y(-2, -1)$.

(3) Estimate the partial derivatives $f_x(-1, 2)$ and $f_y(-1, 2)$.

(4) Locate, if possible, one point (x, y) where $f_x(x, y) = 0$.

(5) Locate, if possible, one point (x, y) where $f_x(x, y) < 0$.

(6) Locate, if possible, one point (x, y) where $f_y(x, y) > 0$.