MATH 104: WORKSHEET 17

1. Concepts

Theorem 1.1 (Lagrange multiplier)**.** *To find the maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$ (assuming that these extreme values exist and $\nabla g \neq 0$ on the surface $g(x, y, z) = k$):*

(1) Find all values of x, y, z, and λ such that

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$$

and

$$g(x, y, z) = k$$

(2) Evaluate f at all the points (x, y, z) that result from step (a). The largest of these values is the maximum value of f; the smallest is the minimum value of f.

2. Discussion

Question 1. A rectangular box without a lid is to be made fro $12m^2$ of cardboard. Find the maximum volume of such a box.

Date: April 14, 2025.

Question 2. Find the extreme values of $f(x, y) = x^2 + 2y^2$ on the disk $x^2 + y^2 \leqslant 1$.

Question 3. Find the extreme values of f subject to two constraints $f(x, y, z) = x + y + z$, $x^2 + z^2 = 2$ and $x + y = 1$.