MATH 104: MULTIVARIABLE CALCULUS

MIDTERM

NAME:______________________________

There are five questions. Make sure you justify all your work for complete credit.

Rules

- You have 80 minutes to complete your work..
- Closed books.
- No use of internet, textbooks, computer algebra systems, calculators.
- No collaboration.
- 1 person per bathroom break. When you go to the bathroom, turn in your cellphone and exam until return.

Scores:

(1) ______
(2) ______
(3) ______
(4) ______
(5) ______
Total : ______

Date: March 11, 2024.

Questions

Problem 1 (20 points, 5 points each). (1) Where does the line

$$x(t) = 2t - 1\,;\, y(t) = 3t + 2\,;\, z(t) = 4t$$

intersect the plane given by $4x + 3y - z = 3$?

(2) What is the surface that the following function a parametrization of? Give a reason.

$$G(\varphi, \theta) = \begin{bmatrix} R\cos\theta\sin\varphi \\ R\sin\theta\sin\varphi \\ R\cos\varphi \end{bmatrix}$$

(3) In $\mathbb{R}^4$, what is the intersection of the (x_1, x_2) and (x_3, x_4) plane?

(4) What is the value of c so that the planes $2cx - y + c^2 = 15$ and $x + 5cy - 3z = 4$ are orthogonal?

Problem 2 (20 points, 5 points each). Consider the following functions

$$f(u,v,w) = \begin{pmatrix} u^2v^{-3}w \\ 2u-5w \\ uv-vw \end{pmatrix},$$

and

$$g(a,b,c) = \begin{pmatrix} \ln(ab) \\ abc \\ b^2+5c \end{pmatrix},$$

(1) Compute the derivative of f at the point $(1,-1,2)$.
(2) What is the most sensitive output of f at the point $(1,-1,2)$?
(3) Compute $[Dg]_{(a,b,c)}$.
(4) Compute the derivative of $g \circ f$ at the point $(1,-1,2)$.

Problem 3 (20 points). (1) Find an equation of the tangent plane to the surface $z = x\sin(x+y)$ at the point $(-1, 1, 0)$.

(2) Find the linear approximation at $(0, 0)$ of $e^x\cos(xy)$.

Problem 4 (20 points). Given the curve $\gamma(t) = (\sin t, \cos 3t, t^2)$.

(1) Compute the velocity, acceleration of γ at time $t = 1$.
(2) What is the arclength of γ from $t = 1$ to $t = 2$?

Problem 5 (20 points). A manufacturer has modeled its yearly production function P (the value of its entire production, in millions of dollars) as a Cobb-Douglas function

$$P(L, K) = 1.47L^{0.65}K^{0.35}$$

where L is the number of labor hours (in thousands) and K is the invested capital (in millions of dollars). Suppose that when $L = 30$ and $K = 8$, the labor force is decreasing at a rate of 2000 labor hours per year and capital is increasing at a rate of $\$500,000$ per year. Find the rate of change of production.