**MATH 104: WORKSHEET 19**

# 1. Concepts

$R = [a, b] \times [c, d]$.

**Definition 1.1.** The double integral of $f$ over the rectangle $R$ is

$$\iint_R f(x, y)\, dA = \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$$

if the limit exists.

**Theorem 1.2** (Fubini)**.** *If $f$ is continuous, then*

$$\int_R f(x, y)\, dA = \int_a^b \left[\int_c^d f(x, y)dy\right] dx = \int_c^d \left[\int_a^b f(x, y)dx\right] dy\,.$$

**Definition 1.3** (Average)**.**

$$f_{\text{ave}} = \frac{1}{A(R)} \iint_R f(x, y)\, dA\,.$$

# 2. Discussions

*Question* 1. Evaluate the integrals

(1) $\iint_R x^2 y\, dA$ where $R = [0, 1] \times [1, 2]$.
(2) $\iint_R (x - 3y^2)\, dA$ where $R = \{(x, y) \mid 0 \leqslant x \leqslant 2, 1 \leqslant y \leqslant 2\}$
(3) $\iint_R y \sin(xy)\, dA$ where $R = [1, 2] \times [0, \pi]$.

*Question* 2. Consider the solid that lies above the square (in the $xy$-plane)

$$R = [0, 1] \times [0, 1],$$

and below the elliptic paraboloid

$$z = 25 - x^2 + xy - 4y^2.$$

Estimate the volume by dividing $R$ into 9 equal squares and choosing the sample points to lie in the midpoints of each square.

*Question* 3. Find the average value of $f$ over the given rectangle:

$$f(x, y) = e^y \sqrt{x + e^y}, \quad R = [0, 4] \times [0, 1].$$

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