

# Multivariable Calculus

## Day 19

### Applications of change of variables

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Spring 2023

Evaluate the following integral

1

$$\iint_R \frac{x - 2y}{3x - y} dA,$$

where  $R$  is the parallelogram enclosed by the lines  
 $x - 2y = 0, x - 2y = 4, 3x - y = 1, 3x - y = 8$ .

In  $\mathbb{R}^2$ , when the region of integration is a section of a disk centered at 0. Let

$$\begin{pmatrix} x \\ y \end{pmatrix} = \varphi(r, \theta) = \begin{pmatrix} r \cos \theta \\ r \sin \theta \end{pmatrix},$$

where  $a \leq b$  and  $\alpha \leq \theta \leq \beta$ .

## Example

Compute the following integral

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \exp\left(-\frac{x^2}{2}\right) dx .$$

# Cylindrical Coordinate

In  $\mathbb{R}^3$ , when the region of integration is part of a cylinder. Let

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \varphi(r, \phi, \theta) = \begin{pmatrix} r \cos \theta \\ r \sin \theta \\ z \end{pmatrix},$$

where  $a \leq r \leq b$ ,  $\alpha \leq \theta \leq \beta$ .

# Spherical Coordinate

In  $\mathbb{R}^3$ , when the region of integration is a section of a ball centered at 0. Let

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \varphi(r, \phi, \theta) = \begin{pmatrix} r \sin \phi \cos \theta \\ r \sin \phi \sin \theta \\ r \cos \phi \end{pmatrix},$$

where  $a \leq r \leq b$ ,  $\alpha \leq \theta \leq \beta$ , and  $c \leq \phi \leq d$ .

- 1 Compute the volume of a solid sphere with radius  $R$ .
- 2 For which  $p$  is the following finite?

$$\int_{x^2+y^2+z^2>1} \frac{1}{(x^2 + y^2 + z^2)^{p/2}} \cdot$$

3

$$\iint_R \sin(9x^2 + 4y^2) dA$$

where  $R$  is the region in the first quadrant bounded by the ellipse  $9x^2 + 4y^2 = 1$ .