MATH 104: WORKSHEET 6

1. **Concepts**

(1) Multivariable functions
(2) Partial derivatives
(3) Derivative as matrix of partial derivatives

2. **Discussions**

Question 1. Let
$$F(x, y, z) = \frac{x^2 \sqrt{y^3}}{z^4}.$$
Compute
$$\frac{\partial F}{\partial x}, \quad \frac{\partial F}{\partial y}, \quad \frac{\partial F}{\partial z}$$

Question 2. Recall that the equation for a paraboloid is
$$z = x^2 + y^2.$$
This can be parametrized by the following equation $G : \mathbb{R}^2 \to \mathbb{R}^3$,
$$G(s, t) = \begin{pmatrix} s \\ t \\ s^2 + t^2 \end{pmatrix}.$$
Compute the derivative of this parametrization.

Question 3. The Cobb-Douglas model is a classic model in economics. It says production, P, is a function of materials, M, and labor L via the following relationship
$$P = \kappa M^\alpha L^\beta$$
where $\kappa > 0$, $0 < \alpha, \beta < 1$ and $\alpha + \beta = 1$. If the investment in labor is increased and the investment in materials is decreased at an equal rate, what is the impact on production?

Question 4. Consider a function f such that, at a particular point a,
$$[Df]_a \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}.$$

Date: 01/28/2024.

(1) How many inputs does f have?
(2) What happens if inputs change at rates $\vec{h} = \langle -2, 2 \rangle$?
(3) What if $\vec{h} = \langle 3, 3 \rangle$? Can you do this?
(4) Can you do the previous problem if you know

$$[Df]_a \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}?$$