# Multivariable Calculus
# Day 18
# Integration

Spring 2023

# Worksheet

Let $B$ be a $2 \times 2$ matrix that is invertible (the determinant is non-zero). We can think of $B$ as a function $B : \mathbb{R}^2 \to \mathbb{R}^2$.

Let now $f : \mathbb{R}^2 \to \mathbb{R}$ be a function such that

$$f(x, y) = xy \, .$$

Let $D$ is the rectangle with vertices $(1,1), (1,6), (5,1), (5,6)$. Find the relationship between

$$\iint_{B(D)} f(y) \, dA$$

and

$$\iint_{D} f(x) \, dA \, .$$

# Change of coordinate

A coordinate transformation is a function $\varphi$, which is bijective and differentiable for which $D\varphi$ is invertible at all points in the domain. Here,

$$D\varphi = \begin{pmatrix} \partial_1\varphi_1 & \partial_2\varphi_1 \\ \partial_1\varphi_2 & \partial_2\varphi_2 \end{pmatrix} .$$

# Worksheet

Find the image of the following transformations. Determine whether they are coordinate transformation or not?

1. (Ex. 15.9.1)

$$x = u^2 - v^2 , \qquad y = 2uv .$$

$$S = \{(u, v) | 0 \leq u, v \leq 1\}$$

2. $$x = u + v , \qquad y = u - v .$$

$$S = \{(u, v) | 0 \leq u, v \leq 1\}$$

# Change of coordinates

Let $f$ be a function of $(x, y)$ defined on the domain $D$. Let

$$\begin{pmatrix} x \\ y \end{pmatrix} = \varphi(u, v)$$

for some coordinate change function $\varphi : D \to S$.

**Theorem**
*If $f$ is continuous, then*

$$\int_S f\, dA = \int_D (f \circ \varphi) \left| \det D\varphi \right| dA\,.$$

# Example

Compute the following integral

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \exp\left(-\frac{x^2}{2}\right)\, dx\,.$$

# Worksheet

Evaluate the following integral

1. $$\iint_R \frac{x-2y}{3x-y}\, dA\,,$$
   where $R$ is the parallelogram enclosed by the lines
   $x-2y=0, x-2y=4, 3x-y=1, 3x-y=8.$

2. $$\iint_R \sin(9x^2+4y^2)\, dA$$
   where $R$ is the region in the first quadrant bounded by the ellipse $9x^2+4y^2=1$.