Multivariable Calculus
Day 17
Integration

Spring 2023

Change of coordinates

Let f be a function of (x, y) defined on the domain D. Let

$$\begin{pmatrix} x \\ y \end{pmatrix} = \varphi(u, v)$$

for some coordinate change function $\varphi : D \to S$.

Theorem
If f is continuous, then

$$\int_S f \, dA = \int_D f \circ \varphi |\det D\varphi| \, dA$$

Worksheet

1. Set up a problem with double integral to find the area of triangle with vertices $(0,0), (2,0), (2,3)$.
2. Let $f(x,y) = x^2 y$. Find
$$\iint_D f(x,y)\, dA$$
where D is the triangle with vertices $(0,0), (2,0), (2,3)$.
3. Compute
$$\iint_D e^{-y^2}\, dA$$