MATH 104: WORKSHEET 22

1. Concepts

Triple integrals and change of variables

2. Concept

2.1. **Triple integral.** Given a function $f(x, y, z)$ and a 3D region E, the triple integral is denoted by

$$\iiint_E f(x, y, z)dV\,.$$

Depending on the description of the region, one may set up the integral differently. For example, if

$$E = \{(x, y, z) \mid a \leqslant x \leqslant b, g_1(x) \leqslant y \leqslant g_2(x), u_1(x, y) \leqslant z \leqslant u_2(x, y)\}$$

then the integral becomes

$$\int_a^b \int_{g_1(x)}^{g_2(x)} \int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z)\, dzdydz\,.$$

2.2. **Change of variables.** If $x = x(s, t)$, $y = y(s, t)$, then

$$\iint f(x, y)\, dxdy = \iint f(x(s, t), y(s, t)) \left|\frac{\partial(x, y)}{\partial(s, t)}\right| dsdt$$

If $x = x(s, t, u)$, $y = y(s, t, u)$, $z = z(s, t, u)$ then

$$\iiint f(x, y, z)\, dxdydz = \iiint f(x(s, t, u), y(s, t, u), z(s, t, u)) \left|\frac{\partial(x, y, z)}{\partial(s, t, u)}\right| dsdtdu$$

3. Discussion

Question 1. Find the triple integral of the function $f(x, y, z) = x^4 \cos(y+z)$ over the cube $2 \leqslant x \leqslant 5$, $0 \leqslant y \leqslant \pi$, $0 \leqslant z \leqslant \pi$.

Question 2. Find the mass of the tetrahedron in the first octant bounded by the coordinate planes and the plane $x + 2y + 3z = 6$ if the density point (x, y, z) is given by $\delta(x, y, z)$ = x + y + z.

(Hint: mass = integral of density)

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Question 3.
1. Set up and evaluate the triple integral of $f(x, y, z) = x - y + 2z$ over the box $B = [-2, 3] \times [1, 4] \times [0, 2]$
2. Let S be the solid cone bounded by $z = \sqrt{x^2 + y^2}$ and $z = 3$. Draw a picture of this and set up the integral of f over S.