

MATH 104: WORKSHEET 7

1. Concepts

- (1) Multivariable functions
- (2) Partial derivatives
- (3) Derivative as matrix of partial derivatives
- (4) Derivative as transformation

2. Discussions

Question 1. Consider a function f such that, at a particular point a ,

$$[Df]_a \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}.$$

- (1) How many inputs does f have?
- (2) What happens if inputs change at rates $\vec{h} = \langle -2, 2 \rangle$?
- (3) What if $\vec{h} = \langle 3, 3 \rangle$? Can you do this?
- (4) Can you do the previous problem if you know

$$[Df]_a \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}?$$

Question 2. Explain the velocity vector of a parametrized curve $\gamma(t)$ in terms of the definition of a derivative.

Question 3. Consider the following

$$f \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} u^2 v^{-3} w \\ 2u - 5w \\ uv - vw \end{pmatrix}.$$

- (1) Compute the derivative $[Df]$.
- (2) Evaluate this derivative at the point where $u = 1, v = -1, w = 2$.
- (3) If, at this point, all the inputs are decreasing at the same rate, which output is increasing the most?

Question 4. From Calculus, we know that if $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$, then

$$(g \circ f)'(x) = g'(f(a))f'(a).$$

Now, if we have $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ and $g : \mathbb{R}^m \rightarrow \mathbb{R}^l$, what is your guess of $[D(g \circ f)]_a$?