MATH 104: WORKSHEET 1

1. Concepts

(1) Lines & planes in 2D & 3D
(2) Curves & surfaces in 2D & 3D
(3) Implicit and parametric representations

2. Discussions

Question 1. What happens if you take the equation of a line in 2D, say $2x - 3y = 7$ and interpret it in 3D?

Question 2. (1) What does each of

$$3x + y - z = 4$$

and

$$x - 2y + z = 1$$

represent?
(2) If taking both of the above equation together, what do they represent? Is there another way to represent this object?

Question 3. Where does the line

$$x(t) = 2t - 1\,; y(t) = 3t + 2\,; z(t) = 4t$$

intersect the plane given by $4x + 3y - z = 3$?

What happens if it's not a plane but a more general surface?

Question 4. (1) Given two lines in 2D, what is there intersection?
(2) What could it be?
(3) What about intersection two lines in 3D?
(4) What about intersection of a line and a plane in 3D?
(5) What about intersection of two planes in 3D?

Question 5. In $\mathbb{R}^4$, what is the intersection of the (x_1, x_2) and (x_3, x_4) plane?

Question 6. Compute a parametrization of the line in $\mathbb{R}^4$ that passes through $(1, 2, 3, 4)$ and the origin.

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Question 7. A parametrization of a sphere of radius R with center at the origin is

$$G(\varphi, \theta) = \begin{bmatrix} R\cos\theta\sin\varphi \\ R\sin\theta\sin\varphi \\ R\cos\varphi \end{bmatrix}$$

How do you know that this describe a sphere?

Question 8 (Exploration, not important for this course but VERY interesting and deep!). Answer the following

(1) What is the volume of a unit cube in $\mathbb{R}^n$?
(2) Look up Wikipedia to answer the question, what is the volume of the ball of radius 1 in $\mathbb{R}^n$? What happens when $n \to \infty$?