# MULTIVARIABLE CALCULUS
# WORKSHEET
# INTEGRALS OF VECTOR FUNCTIONS

(1) Compute the length of the curve
$$\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$$
where $t \in [0, 2\pi]$.

(2) Compute the length of the curve made by the graph of the function
$$f(x) = x^3$$
where $x \in [1, 4]$.

(3) Derive a formula to compute the length of a curve made by the graph of a function $f : [a, b] \to \mathbb{R}$.

(Hint: use the previous question as inspiration.)

(4) According to Wikipedia, “In differential geometry of curves, the osculating circle of a sufficiently smooth plane curve at a given point p on the curve has been traditionally defined as the circle passing through p and a pair of additional points on the curve infinitesimally close to p. Its center lies on the inner normal line, and its curvature defines the curvature of the given curve at that point. This circle, which is the one among all tangent circles at the given point that approaches the curve most tightly, was named circulus osculans (Latin for ”kissing circle”) by Leibniz.”

Given a space curve $\mathbf{r}(t)$ that is infinitely differentiable and $\mathbf{r}'(t) \neq 0$ and $\mathbf{r}''(t) \neq 0$, do the following

(a) Without formulas, describe an idea to compute the radius of the osculating circle at any given point on the curve.
(b) Execute your plan.
(c) Relate your result with the derivative of the unit tangent vector.

*Date*: Spring 2023.