

# Multivariable Calculus

## Day 13

### Optimization

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Spring 2023

# Motivations

*'Nothing takes place in the world whose meaning is not that of some maximum or minimum.'*

Leonhard Euler

# Motivations

- Economics
- Data analysis
- Machine learning
- Physics
- Even the stuff that seems the most random like flipping a coin (and Brownian motion) comes from an optimization problem!

## Some toy problems: Economics

A store tries to optimize its sale of Apple Macbook Air and Lenovo Thinkpad. A Macbook Air has the retail price \$1000 and a Thinkpad \$700. The importation costs for the two laptops would be \$900 for the Macbook Air and \$500 for the Thinkpad. The initial capital of the store is \$100,000.

How many Macbook Airs and Thinkpads should the store import if it wants to optimize its profit?

## Some toy problems: Data analysis

Suppose a scientist has a collection of paired data  $\{(x_i, y_i)\}_{i=1}^N$ . What is the formula for line that “best fits” with the data?

## Some toy problems: Machine learning



What is the best label for the following?

## Some toy problems: Machine learning



What is the best label for the following?

(Cat)

## Some toy problems: physics

Fermat's principle: light takes path that minimizes the time it travels.

What is the path?

## Some toy problems: information theory

A long string of characters is picked from the 26 alphabets.

How should we distribute the alphabets so that the string looks as chaotic as possible?

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Claude Shannon's proposal: use the entropy function

$$-\sum_{i=1}^{26} p(x_i) \log(p(x_i)),$$

where  $x_i$  is the  $i$ th alphabet and  $p(x_i)$  is the probability that  $x_i$  will appear at random.  
This measures how chaotic a certain probabilistic behavior could be.

<https://people.math.harvard.edu/~ctm/home/text/others/shannon/entropy/entropy.pdf>

<https://www.youtube.com/watch?v=v68zYyaEmEA>

**Let's start with the basics...**