MATH 104: HOMEWORK 3

DUE DATE: IN CLASS – MONDAY, MARCH 4, 2024

Fulbright University, Ho Chi Minh City, Vietnam

Problem 1. Consider the following function

$$f(u,v,w) = \begin{pmatrix} u^2v^{-3}w \\ 2u-5w \\ uv-vw \end{pmatrix}.$$

(1) Compute the derivative $[Df]$
(2) Evaluate this derivative at the point $(1,-1,2)$.
(3) What is the most sensitive output?

Problem 2. Consider the following square matrix, depending on variables x and y

$$A = \begin{pmatrix} x & 1 & 7x \\ 0 & 2 & y \\ 0 & x & 3y \end{pmatrix}.$$

(1) Compute and simplify the determinant $\det(A)$.
(2) Define $\mathrm{sum}(A)$ to be the sum of all nine entries of the matrix A. Compute and simplify $\mathrm{sum}(A)$.
(3) Define

$$f(x,y) = \begin{pmatrix} \det(A) \\ \mathrm{sum}(A) \end{pmatrix}.$$

Compute $[Df]$.

Problem 3. Consider the parametrized surface in 3D given by

$$F(s,t) = \begin{pmatrix} s^3-2t^2+7 \\ (s-1)(t-2) \\ s^2-t^2 \end{pmatrix}$$

(1) Compute the derivative of F.
(2) Evaluate the derivative of F at $s=1$ and $t=2$.

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(3) Assume that you start at the point on the surface in which $s = 1$ and $t = 2$. Is it possible to increase both s and t at non-zero rates so that both y- and z- components on the surface change at rate zero? If so, at what rates should these inputs be increased?

Problem 4. Consider the following functions

$$f(a, b, c) = \begin{pmatrix} \ln(ab) \\ abc \\ b^2 + 5c \end{pmatrix},$$

$$g(x, y, z) = \begin{pmatrix} x^2 + y^2 z \\ 2x - y + 3z \\ 3x - 4y \end{pmatrix}$$

(1) Compute the derivative $[Dg]$.
(2) Compute the derivative $[Df]$.
(3) Use the chain rule to evaluate $[D(f \circ g)]_a$, where $a = (1, 0, 0)$.

Problem 5. Let $f : \mathbb{R}^n \to \mathbb{R}, g : \mathbb{R}^m \to \mathbb{R}^n$ be differentiable functions. Show, using the Chain Rule for multivariable functions, that if

$$z(y_1, \ldots, y_m) = (f \circ g)(y_1, \ldots, y_m)$$

then

$$\frac{\partial z}{\partial y_i} = \sum_{j=1}^{n} \frac{\partial f}{\partial x_j} \frac{\partial g_j}{\partial y_i}.$$