MATH 104: WORKSHEET 7

1. Concepts

(1) Differentiation Rules

2. Discussions

Question 1. Consider the following three derivatives of functions

$$[Df]_0 = \begin{bmatrix} 1 & -3 & 0 \\ 2 & 7 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & -1 \end{bmatrix},$$

$$[Dg]_0 = \begin{bmatrix} 4 & 1 \\ -2 & 0 \\ -1 & 1 \end{bmatrix},$$

$$[Dh]_0 = \begin{bmatrix} 3 & -1 & 2 & 0 \\ -2 & -1 & 1 & 4 \\ 5 & 0 & 4 & 2 \end{bmatrix}.$$

Which derivatives can you compute at the origin among:

$$f \circ g, \quad g \circ f, \quad g \circ h, \quad h \circ g, \quad f \circ h, \quad h \circ f?$$

Question 2. Consider the functions

$$f(x, y, z) = \begin{pmatrix} x^2 + yz^2 \\ 2x + y^3 - z) \end{pmatrix}$$

and

$$g(u, v) = \begin{pmatrix} u^2 - v^2 \\ uv \\ 3u - 2v \end{pmatrix}$$

(1) Compute $[Df]$ and $[Dg]$
(2) Compute $[D(f \circ g)]_a$ where $a = (1, 1, 1)$.
(3) If all inputs of $f \circ g$ equal 1 what is the most sensitive output of $f \circ g$?

Question 3. Consider the function $f(x) = u(x)^{v(x)}$, where $f, u, v : \mathbb{R} \to \mathbb{R}$ are all single variable functions.

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(1) Compute $f'(x)$ using you technique from Calculus.
(2) Find $g : \mathbb{R}^2 \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}^2$ so that you can rephrase $f = g \circ h$. Compute $[Df]_x$ using the multivariable chain rule and compare with the previous result.

Question 4. What is the derivative of the function $f(x) = (Q(x))^2$ where $Q(x) = x \cdot Ax$?

Here, A is a square $n \times n$ matrix and $x \in \mathbb{R}^n$.

Question 5. Let $f, h : \mathbb{R} \to \mathbb{R}$ and define

$$F(x) = \int_0^{h(x)} f(t)\, dt.$$

Determine $[DF]$ using what we learn in this class. Check your answer with what you learned in Calculus.