MATH 104: HOMEWORK 1

DUE DATE: IN CLASS – MONDAY, JAN 22, 2024

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The problems are taken from the Calculus Blue Guide by Rob Ghrist.

Problem 1. Consider the following planes in $\mathbb{R}^3$, where C is a constant:

$$2Cx - 3y + (C+4)z = 5$$

and

$$(C+1)x + Cy - z = 1\,.$$

(1) Assuming that $C = 1$ and $z = 0$, find the point of intersection of these two planes.
(2) Assuming that $C = 0$, find a vector that points along the line of intersection between two planes.

Problem 2. Consider the vectors

$$\vec{a} = \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix}, \qquad \vec{b} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}.$$

(1) Write down an implicit equation for a plane passing through the point $(1, 2, 3)$ and orthogonal to $\vec{a}$.
(2) Write down a parametrized equation of a line passing through the point $(1, 2, 3)$ and tangent to $\vec{b}$.

Problem 3. Consider the following parametrized plane given by

$$F(u, v) = \begin{pmatrix} 2u - v \\ u + v - 3 \\ 3u + v + 1 \end{pmatrix}.$$

(1) Give two nonzero vectors tangent to the plane.
(2) Find the point on this plane which intersects the z-axis.

Problem 4. Consider the following vector in $\mathbb{R}^6$

$$\vec{v} = 2\vec{e}_1 + \vec{e}_3 - \vec{e}_4 + \vec{e}_5 - 3\vec{e}_6\,.$$

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(1) Compute the length of v.
(2) Give an example of a vector with the same length as $\vec{v}$ but which is not parallel to $\vec{v}$.
(3) Is there a vector with all entries positive that is parallel to v? If so, give an example. If not, explain why not.

Problem 5. Which of the following five vectors in $\mathbb{R}^5$ is longest/shortest?

$$\vec{u} = \begin{pmatrix} 1 \\ 1 \\ -5 \\ 0 \\ 3 \end{pmatrix}, \vec{v} = \begin{pmatrix} -5 \\ 2 \\ 2 \\ 2 \\ 0 \end{pmatrix}, \vec{w} = \begin{pmatrix} 0 \\ 3 \\ 5 \\ 0 \\ -2 \end{pmatrix}, \vec{x} = \begin{pmatrix} 3 \\ 0 \\ -2 \\ 0 \\ -5 \end{pmatrix}, \vec{y} = \begin{pmatrix} -5 \\ 0 \\ 3 \\ 2 \\ -1 \end{pmatrix}.$$

Problem 6. (1) Write down an implicit equation of a plane which intersects the x-axis at 3; the y-axis at -2; the z-axis at -5.
(2) Give an example of a vector that is orthogonal to this plane.

Problem 7. (1) Write down a parametrization of a line that passes through the point $(1, 3, -5)$ and $(2, 4, 0)$ using a parameter s.
(2) Write down a parametrization of a line passes through the points $(2, 4, 0)$ and $(3, 0, -2)$ using a parameter t.
(3) Write down a parametrization of a plane that passes through the points $(1, 3, -5), (2, 4, 0), (3, 0, -2)$ using parameters s and t.

Problem 8. For what values of constant C are the planes given by

$$3Cx + 16y + Cz = 5\,, \qquad 12x + Cy + 4z = 17$$

parallel to each other?

Problem 9. Consider the points $P = (2, -3, 5)$ and $Q = (4, 1, 7)$.
(1) What is the distance between P and Q?
(2) Give a parametrization of a line passing through both P and Q.

Problem 10. Consider the line parametrized via

$$\gamma(s) = \begin{pmatrix} 2s - 1 \\ 3s + 2 \\ 4s \end{pmatrix}.$$

(1) Find a point where this line intersects the plane given implicitly by
$$2x - 3y + z = 10\,.$$
(2) Does this line intersect this plane orthogonally? Explain.

Problem 11. Consider the plane given by $4x + 12y - 5z = 6$.

(1) Give an example of a vector tangent to the plane; and a vector that is orthogonal to the plane, noting which is which.
(2) Parametrize a line that is orthogonal this plane at the point $(1, 1, 2)$.

Problem 12. Write down parametrizations of a line between the following points (using a parameter t from 0 to 1):

(1) From $(0, 1)$ to $(3, 0)$ in the plane.
(2) From $(0, 1, 7, 2, -3)$ to $(3, 0, 5, -4, 8)$ in $\mathbb{R}^5$.

Problem 13. Consider the following parametrized lines

$$\gamma_1(t_1) = \begin{pmatrix} 3 + 2t_1 \\ 5 + 3t_1 \\ 7 + 4t_1 \end{pmatrix}, \qquad \gamma_2(t_2) = \begin{pmatrix} -1 + t_2 \\ -2 + 2t_2 \\ 11 - 4t_2 \end{pmatrix}.$$

(1) Find the point at which these two lines intersect.
(2) Write down a parametrized plane $S(t_1, t_2)$ which contains both lines.