MATHEMATICAL INVESTIGATIONS OF SOME PHYSICAL MODELS

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For God and my parents

Contents

Preface and Acknowledgements

The main message of this thesis is:

> “ Mechanics is the paradise of the mathematical sciences, because by means of it one comes to the fruits of mathematics.’ ’ — Leonardo da Vinci[1]

[1]Source: `https://mathshistory.st-andrews.ac.uk/Biographies/Leonardo/quotations/`

Part I

Continuum-mechanics-type equations

1 Optimal heat transfer

1.1 Motivations

Heat transfer is important in both daily life and the industry. Almost everyone in the United States has either an air-conditioner or a heater; nuclear plants and data centers need sophisticated heat transfer systems to stay cool.

1.2 Mathematical background

The main tools for our analysis come from probability. In this section, I will re-call some standard facts that are essential to our understanding of the problem.

2
Peridynamics

Part II

Coagulation-Fragmentation equations

3

Introduction to Coagulation-Fragmentation equations

4

Well-posedness of a critical Coagulation-Fragmentation equation

5

Metastability of a Coagulation-Fragmentation equation