

MATH 312: MIDTERM

NAME: _____

Instructions

- (1) Pick 5 out of 6 questions to do. Specify your choices. If you don't I'll take the minimum of all possible points.
- (2) Show your work, be as thorough as possible.
- (3) You have 80 minutes

Question 1 (Quick questions, 5 points each). *Complete the following:*

(1) *Give the definition of a topological space.*

(2) *Give the definition of a measure space (include definitions of sigma algebra and measure).*

(3) *Give an example of a topological space where every set is a clopen set.*

(4) *What does it mean for a set to be compact in a topology?*

(5) *What is a continuous function from (X, τ_X) to (Y, τ_Y) ?*

Question 2 (True or False? Justify. 20 points.). *A set K in a topological space (X, τ) is closed if it is compact.*

Question 3 (True or False? Justify. 20 points.). *A closed subset of a compact set is compact.*

Question 4 (True or False? Justify. 20 points.). *Let $f : (X, \tau_X) \rightarrow (Y, \tau_Y)$ be continuous. If K is compact, then $f(K)$ is compact.*

Question 5 (20 points). *Let (X, τ_X) and (Y, τ_Y) be topological spaces where Y is compact. If N is an open set of $X \times Y$ containing the slice $\{x_0\} \times Y$ of $X \times Y$. Prove that there exists a neighborhood W of x_0 such that $W \times Y \subseteq N$.*

Question 6 (20 points). *Complete the following:*

- (1) *Let c_{nj} be non-negative real numbers that increase in n to c_j for each j . Suppose $\sum_{j=1}^{\infty} c_j < \infty$, show*

$$\lim_{n \rightarrow \infty} \sum_{j=1}^{\infty} c_{nj} = \sum_{j=1}^{\infty} c_j.$$

- (2) *(True or False? Justify) Let μ_1, μ_2, \dots be probability measures on a measurable space (X, \mathcal{A}) , that is $\mu_n(X) = 1$ for all $n \in \mathbb{N}$. Suppose that $\mu_n(A)$ is increasing for each $A \in \mathcal{A}$. Define*

$$\mu(A) = \lim_{n \rightarrow \infty} \mu_n(A).$$

Then, μ is a measure on (X, \mathcal{A}) .