MATH 312: FINAL

NAME: _____

Instructions

(1) Pick 4 out of 5 questions to do. Specify your choices. If you don't I'll take the minimum of all possible points.
(2) Show your work, be as thorough as possible.
(3) You have 80 minutes

Date: May 12, 2025.

Question 1. *State and prove the monotone class theorem.*

Question 2. *State and prove Fatou's lemma.*

Question 3 (True or False? Justify. 20 points.)**.** *Show that every compact Hausdorff space is normal.*

Question 4 (True or False? Justify. 20 points.)**.** *Let $f : (X, \tau_X) \to (Y, \tau_Y)$ be continuous. If K is compact, then $f(K)$ is compact.*

Question 5 (True or False? Justify. 20 points.)**.** *If $f_n \to f$ in measure then there is a subsequence $f_{n_j} \to f$ almost everywhere.*