Quantum Computation Introduction

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Logistics

Info (dates, speakers, references) is hosted on:
https://www.tsvan.xyz/reading.html
Follow John Preskill's notes
http:
//theory.caltech.edu/~preskill/ph219/ph219_2021-22.html
Helpful videos:

1. Preskill's class: https://www.youtube.com/playlist?list=PL0ojjrEqIyPy-1RRD8cTD_lF1hflo89Iu
2. UC Berkeley Vazirani's class: https://www.youtube.com/playlist?list=PLXEJgM3ycgQW5ysL69uaEdPoof4it6seB

Basics: Complex Vector Space

A *complex vector space* is a non-empty set $\mathbb{V}$, whose elements we call vectors, with three operations

1. Addition $+ : \mathbb{V} \times \mathbb{V} \to \mathbb{V}$
2. Negation $- : \mathbb{V} \to \mathbb{V}$
3. Scalar multiplication: $\cdot : \mathbb{C} \times \mathbb{V} \to \mathbb{V}$

and a distinguished element called *zero vector* $0 \in \mathbb{V}$. The operations above obey usual rules with the scalar multiplication obeys rules for complex numbers.
In QM, vectors will be donoted by $|\varphi\rangle$.

Example 1

$\mathbb{C}^n$, space of complex polynomials $\mathbb{C}[x]$, space of complex-valued square integrable functions $L^2(\mathbb{R};\mathbb{C})$.

(Un)fortunately, we do have to work with complex numbers...

Basics: Inner Product

Definition 2

Given a vector space $\mathbb{V}$, an *inner product* (dot product or scalar product) is a function

$$\langle \cdot, \cdot \rangle : \mathbb{V} \times \mathbb{V} \to \mathbb{C}$$

that satisfies

1. $\langle V, V \rangle \geq 0$ and $\langle V, V \rangle = 0$ iff $V = 0$.
2. $\langle V_1 + V_2, V_3 + V_4 \rangle = \langle V_1, V_3 \rangle + \langle V_2, V_4 \rangle$.
3. $\langle cV_1, V_2 \rangle = c\langle V_1, V_2 \rangle$ and $\langle V_1, cV_2 \rangle = \overline{c}\langle V_1, V_2 \rangle$.
4. $\langle V_1, V_2 \rangle = \overline{\langle V_2, V_1 \rangle}$.

Basics: Hilbert Space

Definition 3

A Hilbert space is a complete vector space with an inner product.

Example 4

1. $\mathbb{C}^n$ wiht the usual inner product
2. $L^2(\mathbb{R}; \mathbb{C})$ where

$$\langle f, g \rangle = \int \overline{f} g$$

Fact: finite dimensional Hilbert spaces are isomorphic to $\mathbb{R}^n$. This concept is more useful in infinite dimensional spaces.

Basics: Hilbert Space

Definition 5

Given a Hilbert space $\mathcal{H}$, a hermitian operator is a mapping $A : \mathcal{H} \to \mathcal{H}$ such that $A = A^\dagger$, where $A^\dagger : \mathrm{Dom(A^\dagger)} \subset \mathcal{H} \to \mathcal{H}$ is defined by the following relation

$$\langle Ax, y \rangle = \langle x, A^\dagger y \rangle .$$

- For a bounded operator or in finite dimensions, hermitian is the same with self-adjoint. Issues come up when we deal with unbounded operators in infinite dimensions.
- We mostly deal with finite dimensions, however. So this is not an issue. Just get used to the terminology of physicists.

Basics: Quantum Mechanics

Basics: Quantum Mechanics (đểu)

We approach this from the axiomatic point of view.

Axiom 1 (States)

A state is a complete description of a physical system. In quantum mechanics, a state is a ray in a Hilbert space.