

	A = 0	
	Y=0	Y=1
C=0	a	b
C=1	e	f

	A = 1	
	Y=0	Y=1
C=0	c	d
C=1	g	h

Separation:

$$- P(C=0 | Y=0, A=0) = P(C=0 | Y=0, A=1)$$

$$\Rightarrow \frac{P(C=0, Y=0, A=0)}{P(Y=0, A=0)} = \frac{P(C=0, Y=0, A=1)}{P(Y=0, A=1)}$$

$$\Rightarrow \frac{a}{a+e} = \frac{c}{c+g} \Rightarrow ac + ag = ac + ae$$

$$\Rightarrow e = g$$

$$- P(C=0 | Y=1, A=0) = P(C=0 | Y=1, A=1)$$

$$\Rightarrow \frac{b}{b+f} = \frac{d}{d+h} \Rightarrow b = f$$

$$- P(C=1 | Y=0, A=0) = P(R=1 | Y=0, A=1)$$

$$\Rightarrow \frac{e}{a+e} = \frac{g}{c+g} \Rightarrow a=c$$

$$- P(C=1 | Y=1, A=0) = P(C=1 | Y=1, A=1)$$

$$\Rightarrow \frac{f}{b+f} = \frac{h}{d+h} \Rightarrow d=h$$

- With the following restrictions:

$$\begin{array}{ll} - e=g & - a=c \\ - b=f & - d=h. \end{array}$$

Our joint table becomes:

		A = 0		A = 1	
		Y=0	Y=1	Y=0	Y=1
C=0		a	b	a	d
C=1		e	b	e	d

Let's use:

$$a = 0.1$$

$$b = 0.15$$

$$c = 0.2$$

$$d = 0.05$$

Our table becomes:

		A=0				A=1	
		Y=0	Y=1			Y=0	Y=1
C=0	0.1	0.15	C=0	0.1	0.05		
C=1	0.2	0.15	C=1	0.2	0.05		

- Show it is separate not sufficient:

Sufficiency implies:

$$P(Y=0|C=0, A=0) = P(Y=0|C=0, A=1)$$

$$\Rightarrow \frac{a}{a+b} = \frac{c}{c+d} \Rightarrow \frac{0.1}{0.1+0.2} = \frac{0.1}{0.1+0.05}$$

$$\Rightarrow 0.33 \neq 0.667$$

To create a sufficient distribution:

$$- P(Y=0|C=0, A=0) = P(Y=0|C=0, A=1)$$

$$\Rightarrow \frac{a}{a+b} = \frac{c}{c+d} \Rightarrow ad = bc$$

$$- P(Y=1|C=1, A=0) = P(Y=1|C=1, A=1)$$

$$\Rightarrow \frac{f}{e+f} = \frac{h}{g+h} \Rightarrow eh = fg$$

- Our restrictions are:

$$\begin{aligned} ad &= bc \\ eh &= fg \end{aligned}$$

$$\begin{aligned} e &\neq g & a &\neq c \\ b &\neq f & d &\neq h \end{aligned}$$

lets set:

		A=0				A=1	
		Y=0	Y=1			Y=0	Y=1
C=0		0.05	0.15	C=0		0.1	0.3
C=1		0.05	0.05	C=1		0.15	0.15

Let's show it is ~~not~~ separate:

$$P(C=0 | Y=0, A=0) = P(C=0 | Y=0, A=1)$$

$$\frac{a}{a+e} = \frac{c}{c+g}$$

$$\frac{0.05}{0.05+0.05} = \frac{0.1}{0.1+0.15}$$

$$0.5 \neq 0.4.$$

Not separate.

As long as ~~as~~ $C \not\perp Y$ ~~no~~, they
cannot be separate AND ~~is~~ sufficient.