

Q3) a- Give one example where separation holds but not sufficiency

Separation equation: $P(C|Y, \text{gender} = \text{'Male/Female'}) = P(C|Y)$

Sufficiency equation: $P(Y|C, \text{gender} = \text{'Male/Female'}) = P(Y|C)$

C= BMI and Y=Hyperlipidemia and A=Gender

Separation Check

	Hyperlipidemia= YES	Hyperlipidemia= NO
BMI = ~18.5	$P(C = \sim 18.5 Y = \text{YES}, \text{gender} = \text{'Male'}) = \mathbf{0.1985297}$ $P(C = \sim 18.5 Y = \text{YES}) = \mathbf{0.1985297}$	$P(C = \sim 18.5 Y = \text{NO}, \text{gender} = \text{'Male'}) = \mathbf{0.49179708}$ $P(C = \sim 18.5 Y = \text{NO}) = \mathbf{0.49179708}$
BMI = ~24.0	$P(C = \sim 24.0 Y = \text{YES}, \text{gender} = \text{'Male'}) = \mathbf{0.4696328}$ $P(C = \sim 24.0 Y = \text{YES}) = \mathbf{0.4696328}$	$P(C = \sim 18.5 Y = \text{NO}, \text{gender} = \text{'Male'}) = \mathbf{0.36267231}$ $P(C = \sim 18.5 Y = \text{NO}) = \mathbf{0.36267231}$
BMI = ~28.0	$P(C = \sim 28.0 Y = \text{YES}, \text{gender} = \text{'Male'}) = \mathbf{0.32815359}$ $P(C = \sim 28.0 Y = \text{YES}) = \mathbf{0.32815359}$	$P(C = \sim 18.5 Y = \text{NO}, \text{gender} = \text{'Male'}) = \mathbf{0.119463616}$ $P(C = \sim 18.5 Y = \text{NO}) = \mathbf{0.119463616}$
BMI = <18.5	$P(C = < 18.5 Y = \text{YES}, \text{gender} = \text{'Male'}) = \mathbf{0.003683854}$ $P(C = < 18.5 Y = \text{YES}) = \mathbf{0.0036838540}$	$P(C = \sim 18.5 Y = \text{NO}, \text{gender} = \text{'Male'}) = \mathbf{0.0260669856}$ $P(C = \sim 18.5 Y = \text{NO}) = \mathbf{0.0260669856}$

The right-hand side and left-hand side of the separation equation are equal for all values; **hence separation holds**. To ensure the conditional independence of BMI and gender, the value of gender was set to 'Female' and the results were exactly like the table above.

Sufficiency check

A=Gender='Male'

	BMI = ~18.5	BMI = ~24.0	BMI = ~28.0	BMI = <18.5
Hyperlipidemia= YES	$P(Y='YES' C \sim 18.5, \text{gender}='Male') =$ 0.24110864246533822 $P(Y='YES' C \sim 18.5) =$ 0.21560500000000002	$P(Y='YES' C \sim 24.0, \text{gender}='Male') =$ 0.5047417852429753 $P(Y='YES' C \sim 24.0) =$ 0.46857000000000004	$P(Y='YES' C \sim 28.0, \text{gender}='Male') =$ 0.6837341865229888 $P(Y='YES' C \sim 28.0) =$ 0.6516120000000001	$P(Y='YES' C < 18.5, \text{gender}='Male') =$ 0.10009280710767367 $P(Y='YES' C < 18.5) =$ 0.08777999999999998
Hyperlipidemia= NO	$P(Y='NO' C \sim 18.5, \text{gender}='Male') =$ 0.7588913575346617 $P(Y='NO' C \sim 18.5) =$ 0.784395	$P(Y='NO' C \sim 24.0, \text{gender}='Male') =$ 0.49525821475702475 $P(Y='NO' C \sim 24.0) =$ 0.53143000000000001	$P(Y='NO' C \sim 28.0, \text{gender}='Male') =$ 0.31626581347701127 $P(Y='NO' C \sim 28.0) =$ 0.348388	$P(Y='NO' C < 18.5, \text{gender}='Male') =$ 0.8999071928923262 $P(Y='NO' C < 18.5) =$ 0.91222

A=Gender='Female'

	BMI = ~18.5	BMI = ~24.0	BMI = ~28.0	BMI = <18.5
Hyperlipidemia= YES	$P(Y='YES' C \sim 18.5, \text{gender}='Female') =$ 0.18899641307583723 $P(Y='YES' C \sim 18.5) =$ 0.21560500000000002	$P(Y='YES' C \sim 24.0, \text{gender}='Female') =$ 0.4277674560469438 $P(Y='YES' C \sim 24.0) =$ 0.46857000000000004	$P(Y='YES' C \sim 28.0, \text{gender}='Female') =$ 0.6132638858790895 $P(Y='YES' C \sim 28.0) =$ 0.6516120000000001	$P(Y='YES' C < 18.5, \text{gender}='Female') =$ 0.07542975141421693 $P(Y='YES' C < 18.5) =$ 0.08777999999999998
Hyperlipidemia= NO	$P(Y='NO' C \sim 18.5, \text{gender}='Female') =$ 0.8110035869241627 $P(Y='NO' C \sim 18.5) =$ 0.784395	$P(Y='NO' C \sim 24.0, \text{gender}='Female') =$ 0.5722325439530562 $P(Y='NO' C \sim 24.0) =$ 0.53143000000000001	$P(Y='NO' C \sim 28.0, \text{gender}='Female') =$ 0.3867361141209104 $P(Y='NO' C \sim 28.0) =$ 0.348388	$P(Y='NO' C < 18.5, \text{gender}='Female') =$ 0.924570248585783 $P(Y='NO' C < 18.5) =$ 0.91222

Sufficiency does not hold since the left hand side and right hand side of the equation are not equal.

Q3) b- Give one example where sufficiency holds but not separation

Separation equation: $P(C|Y, \text{gender}=\text{'Male/Female'}) = P(C|Y)$

Sufficiency equation: $P(Y|C, \text{gender}=\text{'Male/Female'}) = P(Y|C)$

C= Hyperlipidemia and Y=Region and A=Gender

Sufficiency Check

	Hyperlipidemia=YES	Hyperlipidemia=NO
Region= Countryside	$P(Y=\text{'Countryside'} C=\text{'YES'}, \text{gender}=\text{'Male'})=$ 0.4967974089058388 $P(Y=\text{'Countryside'} C=\text{'YES'})=$ 0.4967974089058388	$P(Y=\text{'Countryside'} C=\text{'NO'}, \text{gender}=\text{'Male'})=$ 0.467759242485746 $P(Y=\text{'Countryside'} C=\text{'NO'})=$ 0.467759242485746
Region= City	$P(Y=\text{'City'} C=\text{'YES'}, \text{gender}=\text{'Male'})=$ 0.503202591094161 $P(Y=\text{'City'} C=\text{'YES'})=$ 0.503202591094161	$P(Y=\text{'City'} C=\text{'NO'}, \text{gender}=\text{'Male'})=$ 0.5322407575142535 $P(Y=\text{'City'} C=\text{'NO'})=$ 0.5322407575142535

The right side of the sufficiency equation and the left side are equal for all values **hence sufficiency holds**. To ensure conditional independency between Region and gender, gender was set to female and the results were exactly like the table above.

Separation Check with gender='Male'

	Region= Countryside	Region= City
Hyperlipidemia=YES	$P(C=\text{'YES'} Y=\text{'Countryside'}, \text{gender}=\text{'Male'})=$ 0.45530583084984777 $P(C=\text{'YES'} Y=\text{'Countryside'})=$ 0.41967472381515214	$P(C=\text{'YES'} Y=\text{'City'}, \text{gender}=\text{'Male'})=$ 0.4266366780681491 $P(C=\text{'YES'} Y=\text{'City'})=$ 0.391636012419724
Hyperlipidemia=NO	$P(C=\text{'NO'} Y=\text{'Countryside'}, \text{gender}=\text{'Male'})=$ 0.5446941691501523 $P(C=\text{'NO'} Y=\text{'Countryside'})=$ 0.580325276184848	$P(C=\text{'NO'} Y=\text{'City'}, \text{gender}=\text{'Male'})=$ 0.573363321931851 $P(C=\text{'NO'} Y=\text{'City'})=$ 0.608363987580276

Separation Check with gender=`Female`

	Region= Countryside	Region= City
Hyperlipidemia=YES	$P(C=\text{`YES`} Y=\text{`Countryside`}, \text{gender}=\text{`Female`})=$ 0.38008473760328293 $P(C=\text{`YES`} Y=\text{`Countryside`})=$ 0.41967472381515214	$P(C=\text{`YES`} Y=\text{`City`}, \text{gender}=\text{`Female`})=$ 0.3530817658296224 $P(C=\text{`YES`} Y=\text{`City`})=$ 0.391636012419724
Hyperlipidemia=NO	$P(C=\text{`NO`} Y=\text{`Countryside`}, \text{gender}=\text{`Female`})=$ 0.619915262396717 $P(C=\text{`NO`} Y=\text{`Countryside`})=$ 0.580325276184848	$P(C=\text{`NO`} Y=\text{`City`}, \text{gender}=\text{`Female`})=$ 0.6469182341703775 $P(C=\text{`NO`} Y=\text{`City`})=$ 0.608363987580276

The right hand side and left hand side of the separation equation are not equal **hence Separation does not hold.**