KR Questions!

Thanks to Sheila McIlraith,
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Convert these sentences to First Order Logic

1. Nobody likes taxes.
2. Some people like anchovies.
3. Emma is a Doberman pincher and a good dog.

Convert these sentences to First Order Logic

1. Marcus was a man.
2. Marcus was a Roman.
3. All men are people.
4. Caesar was a ruler.
5. All Romans were either loyal to Caesar or hated him (or both).
6. Everyone is loyal to someone.
7. People only try to assassinate rulers they are not loyal to.
8. Marcus tried to assassinate Caesar.

Note that next tutorial, we will use this KB to answer the query:

Who hated Ceaser?

Models

Let $\mathcal{M}$ be a model for $\mathcal{L}$, with domain $D = \{a, b, c, d\}$, and interpretation function σ:

1. $A^\sigma = a$, $B^\sigma = b$, $C^\sigma = c$, $D^\sigma = d$.
2. $R^\sigma = \{(b, a), (c, d)\}$.
3. $P^\sigma = \{b, c\}$.
4. $Q^\sigma = \{a, d\}$.

Which of the following formulas are satified by $\mathcal{M}$?

$$R(C, B) \vee R(B, A)$$

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Which of the following formulas are satified by $\mathcal{M}$?

$$\forall x.P(x) \wedge \neg Q(x).$$

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Which of the following formulas are satified by $\mathcal{M}$?

$$\forall x.P(x) \to \neg\exists y.R(y, x).$$

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4. $Q^\sigma = \{a, d\}$.

Which of the following formulas are satified by $\mathcal{M}$?

1. $\forall x. Q(x) \to \neg \exists y. R(y, x)$.

Models

Let $\mathcal{M}$ be a model for $\mathcal{L}$, with domain $D = \{a, b, c, d\}$, and interpretation function σ:

1. $A^\sigma = a$, $B^\sigma = b$, $C^\sigma = c$, $D^\sigma = d$.
2. $R^\sigma = \{(b, a), (c, d)\}$.
3. $P^\sigma = \{b, c\}$.
4. $Q^\sigma = \{a, d\}$.

Which of the following formulas are satified by $\mathcal{M}$?

1. $\forall x. Q(x) \rightarrow \exists y. R(y, x)$.

Models

Let $\mathcal{M}$ be a model for $\mathcal{L}$, with domain $D = \{a, b, c, d\}$, and interpretation function σ:

1. $A^\sigma = a$, $B^\sigma = b$, $C^\sigma = c$, $D^\sigma = d$.
2. $R^\sigma = \{(b, a), (c, d)\}$.
3. $P^\sigma = \{b, c\}$.
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Which of the following formulas are satified by $\mathcal{M}$?

$$\exists x, y.(P(x) \wedge Q(y)) \rightarrow R(y, x).$$

To find the MGU of two formulas f and g.

1. $k = 0$; $\sigma_0 = \{\}$; $S_0 = \{f,g\}$
2. If S_k contains an identical pair of formulas stop, and return σ_k as the MGU of f and g.
3. Else find the disagreement set $D_k=\{e_1,e_2\}$ of S_k
4. If $e_1 = V$ a variable, and $e_2 = t$ a term not containing V (or vice-versa) then let
   $\sigma_{k+1} = \sigma_k \{V=t\}$ (Compose the additional substitution)

   $S_{k+1} = S_k\{V=t\}$ (Apply the additional substitution)

   $k = k+1$
   GOTO 2
5. Else stop, f and g cannot be unified.

Unify!

$$P(\ g(h(x))\ ,\ f(g(h(b)))\ ,\ f(x)\ )$$
$$P(\ y\ ,\ f(y)\ ,\ z\ )$$

$$P(\ g(h(x))\ ,\ f(h(y))\ ,\ y\ )$$
$$P(\ g(z)\ ,\ f(z)\ ,\ h(a)\ )$$

$$P(\ x\ ,\ h(b)\ ,\ h(x)\ )$$
$$P(\ f(g(y))\ ,\ y\ ,\ h(f(g(h(a))))\ )$$

$$P(\ x\ ,\ g(x)\ ,\ z\ )$$
$$P(\ f(y)\ ,\ g(f(b))\ ,\ h(y)\ )$$

$$P(\ f(g(x))\ ,\ g(b)\ ,\ h(x)\ )$$
$$P(\ f(y)\ ,\ y\ ,\ h(c)\ )$$

$$P(\ x\ ,\ h(x)\ ,\ h(y)\ )$$
$$P(\ f(g(z))\ ,\ h(f(g(b)))\ ,\ h(z)\ )$$

How to convert First Order Logic to Clausal Form

1. **Eliminate Implications.**
2. **Move Negations inwards (and simplify ¬¬).**
3. **Standardize Variables.**
4. **Skolemize.**
5. **Convert to Prenix Form.**
6. **Distribute conjunctions over disjunctions.**
7. **Flatten nested conjunctions and disjunctions.**
8. **Convert to Clauses.**

Resolution Question

Consider the following sentences:

1. Marcus was a man.
2. Marcus was a Roman.
3. All men are people.
4. Caesar was a ruler.
5. All Romans were either loyal to Caesar or hated him (or both).
6. Everyone is loyal to someone.
7. People only try to assassinate rulers they are not loyal to.
8. Marcus tried to assassinate Caesar.

Answer the query: Who hated Caeser?

Convert to FoL, then Clausal Form!

1. All hounds howl at night.
2. Anyone who has any cats will not have any mice.
3. Light sleepers do not have anything which howls at night.
4. John has either a cat or a hound.

5. (Conclusion) If John is a light sleeper, then John does not have any mice.

Now, prove the conclusion!

Every child loves Santa.

Everyone who loves Santa loves any reindeer.

Rudolph is a reindeer, and Rudolph has a red nose.

Anything which has a red nose is weird or is a clown.

No reindeer is a clown.

Scrooge does not love anything which is weird.

(Conclusion) Scrooge is not a child.

Convert the sentences above to clausal form, and prove the conclusion!