Prob	Classification C	Ground truth Y	protected attribute A
x_1	T	T	T
x_2	T	F	T
x_3	T	T	F
x_4	T	F	F
x_5	F	F	F
x_6	F	T	F
x_7	F	F	T
x_8	F	T	T

① $P(C=c \mid Y=y, A=a)$

$$= \frac{P(C=c \cap Y=y \cap A=a)}{P(Y=y \cap A=a)}$$

② $P(C=c \mid Y=y)$

$$= \frac{P(C=c \cap Y=y)}{P(Y=y)}$$

③ $P(Y=y \mid C=c \cap A=a)$

$$= \frac{P(Y=y \cap C=c \cap A=a)}{P(C=c \cap A=a)}$$

④ $P(Y=y \mid C=c)$

$$= \frac{P(Y=y \cap C=c)}{P(C=c)}$$

- ~~WTS~~ ∃ c, y, a s.t.

  ~~① = ② AND ③ ≠ ④~~

- WTS ∃ c, y, a s.t.

  ~~① ≠ ② AND ③ = ④~~

Want $x_1, \ldots, x_8$ such that

$\boxed{I}$ ① = ② and ③ ≠ ④   Separated but not sufficient

and

$\boxed{II}$ ① ≠ ② and ③ = ④   Sufficient but not separated.

For:	C	Y	A	Prob
	T	T	T	$x_1 = 0.10$
	T	F	T	$x_2 = 0.10$
	T	T	F	$x_3 = 0.10$
	T	F	F	$x_4 = 0.10$
	F	F	F	$x_5 = 0.10$
	F	T	F	$x_6 = 0.10$
	F	F	T	$x_7 = 0.10$
	F	T	T	$x_8 = 0.30$

$\boxed{II}$ is True.

① $= P(C=T \mid Y=T, A=T)$

$\qquad = \dfrac{x_1}{x_1 + x_8} = \dfrac{0.10}{0.40} = 0.25$

② $= P(C=T \mid Y=T)$

$\qquad = \dfrac{x_1 + x_3}{x_1 + x_3 + x_6 + x_8}$

$\qquad = \dfrac{0.20}{0.60} = 0.3333\ldots$

$\Rightarrow$ ① ≠ ②

But ③ $= P(Y=T \mid C=T \cap A=T)$

$\qquad = \dfrac{x_1}{x_1 + x_2} = \dfrac{0.10}{0.20} = 0.5$

And ④ $= P(V=T \mid C=T)$

$\qquad = \dfrac{x_1 + x_3}{x_1 + x_2 + x_3 + x_4} = \dfrac{0.20}{0.40} = 0.5$

$\Rightarrow$ ③ = ④

For: C Y A   Prob $\boxed{I}$ in True

C	Y	A	Prob
T	T	T	$x_1 = 0.10$
T	F	T	$x_2 = 0.30$
T	T	F	$x_3 = 0.10$
T	F	F	$x_4 = 0.10$
F	F	F	$x_5 = 0.10$
F	T	F	$x_6 = 0.10$
F	F	T	$x_7 = 0.10$
F	T	T	$x_8 = 0.10$

① $= P(C=T|Y=T \wedge A=T) = \dfrac{x_1}{x_1 + x_8} = \dfrac{0.10}{0.20} = 0.5$

② $= P(C=T|Y=T) = \dfrac{x_1 + x_3}{x_1 + x_3 + x_6 + x_8} = \dfrac{0.20}{0.40} = 0.5$

$\Rightarrow$ $\boxed{① = ②}$

But ③ $= P(Y=T|C=T \wedge A=T) = \dfrac{x_1}{x_1 + x_2} = \dfrac{0.10}{0.40} = 0.25$

And ④ $= P(Y=T|C=T) = \dfrac{x_1 + x_3}{x_1 + x_2 + x_3 + x_4} = \dfrac{0.20}{0.60} = 0.333..$

$\Rightarrow$ $\boxed{③ \neq ④}$