CSC384 A4 Q3

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Question 3: Fairness

1. Separation but not sufficiency:

C	Y	A	Probability
c	y	a	0.35
c	y	$\neg a$	0
c	$\neg y$	a	0.21
c	$\neg y$	$\neg a$	0.18
$\neg c$	y	a	0
$\neg c$	y	$\neg a$	0
$\neg c$	$\neg y$	a	0.14
$\neg c$	$\neg y$	$\neg a$	0.12

Proof:

$$\begin{aligned} P(c \mid y) &= \frac{P(c \wedge y)}{P(y)} \\ &= \frac{0.35}{0.35} \\ &= 1 \end{aligned}$$

$$\begin{aligned} P(c \mid y, a) &= \frac{P(c \wedge y \wedge a)}{P(y \mid a)P(a)} \\ &= \frac{P(c \wedge y \wedge a)}{\frac{P(y \wedge a)}{P(a)}P(a)} \\ &= \frac{P(c \wedge y \wedge a)}{P(y \wedge a)} \\ &= \frac{0.35}{0.35} \\ &= 1 \end{aligned}$$

So, $P(c \mid y, a) = P(c \mid y)$ and separation holds.

$$\begin{aligned} P(y \mid c) &= \frac{P(y \wedge c)}{P(c)} \\ &= \frac{0.35}{0.74} \\ &= 0.473 \\ P(y \mid c, a) &= \frac{P(y \wedge c \wedge a)}{P(c \wedge a)} \\ &= \frac{0.35}{0.56} \\ &= 0.625 \end{aligned}$$

So, $P(y \mid c, a) \neq P(y \mid c)$ and sufficiency does not hold.

2. Sufficiency but not separation:

C	Y	A	Probability
c	y	a	0.35
c	y	$\neg a$	0
c	$\neg y$	a	0
c	$\neg y$	$\neg a$	0
$\neg c$	y	a	0.21
$\neg c$	y	$\neg a$	0.18
$\neg c$	$\neg y$	a	0.14
$\neg c$	$\neg y$	$\neg a$	0.12

Proof:

$$\begin{aligned} P(y \mid c) &= \frac{P(y \wedge c)}{P(c)} \\ &= \frac{0.35}{0.35} \\ &= 1 \\ P(y \mid c, a) &= \frac{P(y \wedge c \wedge a)}{P(c \wedge a)} \\ &= \frac{0.35}{0.35} \\ &= 1 \end{aligned}$$

So, $P(y \mid c, a) = P(y \mid c)$ and sufficiency holds.

$$
\begin{aligned}
P(c \mid y) &= \frac{P(c \wedge y)}{P(y)} \\
&= \frac{0.35}{0.74} \\
&= 0.473 \\
P(c \mid y, a) &= \frac{P(c \wedge y \wedge a)}{P(y \wedge a)} \\
&= \frac{0.35}{0.56} \\
&= 0.625
\end{aligned}
$$

So, $P(c \mid y, a) \neq P(c \mid y)$ and separation does not hold.