

Example 1: Separation holds but not sufficiency

Set A = Gender, C = Diabetes, Y = Hyperlipidemia (using same definitions as the handout)

Joint probability distribution for Hyperlipidemia, Gender, and Diabetes:

	Gender = Male		Gender = Female	
	Diabetes = YES	Diabetes = NO	Diabetes = YES	Diabetes = NO
Hyperlipidemia = YES	0.1494	0.0819	0.1123	0.0615
Hyperlipidemia = NO	0.1132	0.1807	0.1159	0.1851

Gender is conditionally independent of Diabetes given Hyperlipidemia.

→ $P(\text{Gender} \mid \text{Hyperlipidemia}) = P(\text{Gender} \mid \text{Hyperlipidemia}, \text{Diabetes})$

$P(\text{Gender} = \text{Male} \mid \text{Hyperlipidemia} = \text{YES})$	0.571
$P(\text{Gender} = \text{Male} \mid \text{Hyperlipidemia} = \text{YES}, \text{Diabetes} = \text{YES})$	0.571
$P(\text{Gender} = \text{Male} \mid \text{Hyperlipidemia} = \text{YES}, \text{Diabetes} = \text{NO})$	0.5709999999999998
$P(\text{Gender} = \text{Male} \mid \text{Hyperlipidemia} = \text{NO})$	0.49399999999999994
$P(\text{Gender} = \text{Male} \mid \text{Hyperlipidemia} = \text{NO}, \text{Diabetes} = \text{YES})$	0.49399999999999994
$P(\text{Gender} = \text{Male} \mid \text{Hyperlipidemia} = \text{NO}, \text{Diabetes} = \text{NO})$	0.494

Diabetes is conditionally independent of Gender given Hyperlipidemia.

→ $P(\text{Diabetes} \mid \text{Hyperlipidemia}) = P(\text{Diabetes} \mid \text{Hyperlipidemia}, \text{Gender})$

$P(\text{Diabetes} = \text{YES} \mid \text{Hyperlipidemia} = \text{YES})$	0.6459421684394565
$P(\text{Diabetes} = \text{YES} \mid \text{Hyperlipidemia} = \text{YES}, \text{Gender} = \text{Male})$	0.6459421684394565
$P(\text{Diabetes} = \text{YES} \mid \text{Hyperlipidemia} = \text{YES}, \text{Gender} = \text{Female})$	0.6459421684394565
$P(\text{Diabetes} = \text{YES} \mid \text{Hyperlipidemia} = \text{NO})$	0.3850423492316574
$P(\text{Diabetes} = \text{YES} \mid \text{Hyperlipidemia} = \text{NO}, \text{Gender} = \text{Male})$	0.38504234923165737
$P(\text{Diabetes} = \text{YES} \mid \text{Hyperlipidemia} = \text{NO}, \text{Gender} = \text{Female})$	0.3850423492316574

Therefore, separation holds (A is independent of C given Y).

Gender is **not** conditionally independent of Hyperlipidemia given Diabetes.

→ $P(\text{Gender} \mid \text{Diabetes}, \text{Hyperlipidemia} = \text{YES}) \neq P(\text{Gender} \mid \text{Diabetes}, \text{Hyperlipidemia} = \text{NO})$

$P(\text{Gender} = \text{Male} \mid \text{Diabetes} = \text{YES}, \text{Hyperlipidemia} = \text{YES})$	0.571
$P(\text{Gender} = \text{Male} \mid \text{Diabetes} = \text{YES}, \text{Hyperlipidemia} = \text{NO})$	0.49399999999999994

Therefore, sufficiency does not hold (A is not independent of Y given C).

Example 2: Sufficiency holds but not separation

By setting $C = \text{Hyperlipidemia}$ (instead of Diabetes) and $Y = \text{Diabetes}$ (instead of Hyperlipidemia), Example 1 can be used to show that sufficiency holds but not separation.

Hence, sufficiency and separation can't be enforced at the same time.