Q4. First-order Logic and Resolution **(worth 40/100 marks)**

Consider a first-order language L that consists of constant symbols (or 0-ary functions) $o1, o2, o3$, a binary relation symbol $scarier\_than$, and predicate symbol $mammal$. Now say we have a Knowledge Base that consists of the following sentences:

- $mammal(o1)$
- $mammal(o2)$
- $scarier\_than(o2, o1)$
- $scarier\_than(o3, o2)$

Consider a model (or interpretation) $M =< D, \phi, \psi, V >$ such that:

- $D = \{Rabbit, Koala, Alligator\}$
- $\psi(mammal) = \{Rabbit, Koala\}$
- $\psi(scarier\_than) = \{\langle Koala, Rabbit\rangle, \langle Rabbit, Alligator\rangle, \langle Alligator, Koala\rangle\}$
- $\phi(o_1) = Rabbit$
- $\phi(o_2) = Koala$
- $\phi(o_3) = Alligator$

1. **(worth 3 marks)** Does the model M satisfy the Knowledge Base? **Explain** why or why not.
   Solutions: Yes.

   The first and second sentences are satisfied because $\phi(o_1) = Rabbit, \phi(o_2) = Koala$ and $\psi(mammal) = \{Rabbit, Koala\}$.

   The third sentence is satisfied because $\phi(o_1) = Rabbit, \phi(o_2) = Koala$ and $\langle Koala, Rabbit\rangle \in \psi(scarier\_than)$.

   The last sentence is satisfied because $\phi(o_3) = Alligator, \phi(o_2) = Koala$ and $\langle Alligator, Koala\rangle \in \psi(scarier\_than)$.

2. **(worth 4 marks)** Suppose we add the following sentence to the Knowledge Base:

$$\forall x \forall y \forall z((scarier\_than(x, y) \land scarier\_than(y, z)) \rightarrow scarier\_than(x, z))$$

Does the model M satisfy the Knowledge Base? **Explain** why or why not.

Solutions: No. Consider for example $\mathcal{M}_1 \in \mathfrak{B}$ where
$\langle Koala, Rabbit\rangle \in \psi(scarier\_than)$
$\langle Alligator, Koala\rangle \in \psi(scarier\_than)$
but $\langle Alligator, Rabbit\rangle \notin \psi(scarier\_than)$

3. **(Worth 9 marks; 3 marks each)** For each of the pairs below, give the most general unifier (MGU) or state why no unifier exists. If a unifier exists, provide the expression that results from the unification. In all of the expressions that follow, a, b, and c are constants and X, Y, and Z are variables.

   (a) $P(g(X), f(a), Y)$ and $P(Y, f(X), g(Z))$

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   (b) $P(g(X), Y, f(X, Y))$ and $P(Z, h(X), f(c, h(X)))$

   (c) $f(g(X, X), h(b, X))$ and $f(Y, h(b, q(Y, Y)))$

Notes

a g(X) P(g(X), f(X), g(Z))

3a. $P(g(X), f(a), X)$ $P(Y, f(X), g(Z))$

$\sigma = \{Y = g(X)\}$ $\theta = \{X = a\}$

$\sigma\theta = \{Y = g(a), X = a\}$ $\psi = \{Z = a\}$

$\sigma\theta\psi = \{Y = g(a), X = a, Z = a\}$

3a. final solution: {Y = g(a), X=a, Z=a}

c h(X) c h(X) g(X) c c

3b. $P(g(X), Y, f(X, Y))$ $P(Z, h(X), f(c, h(X)))$

$\sigma = \{Z = g(X)\}$ $\theta = \{Y = h(X)\}$

$\sigma\theta = \{Z = g(X), Y = h(X)\}$ $\psi = \{X = c\}$

$\sigma\theta\psi = \{Z = g(c), Y = h(c), X = c\}$

3b. final solution: {Z = g(c), Y=h(c), X=c}

3c. $f(g(X,X), h(b,X))$ $f(Y, h(b, g(Y,Y)))$

$\sigma = \{Y = g(X,X)\}$

$\theta = ?$ cannot unify X & g(g(X,X), g(X,X))

NOT UNIFIABLE

3c. final solution: Not unifiable! Cannot unify X and q(g(X,X), g(X,X))

4. **(worth 10 marks; 2 marks each)** Given predicates and relations P, Q, R and constants a, b, c. Convert the following sentences to clausal form.

   1 $\forall x[R(x) \rightarrow P(a, x)]$

   2 $Q(a, c) \vee Q(b, c)$

   3 $\forall x[\exists z(R(z) \wedge Q(x, z)) \rightarrow \forall w.\neg P(w, x)]$

   4 $R(c)$

   5 $\forall x[\forall y(R(y) \rightarrow P(x, y)) \rightarrow \exists z.P(z, x)]$

   Solution:

   1. $[\neg R(x), P(a, x)]$
   2. $[Q(a, c), Q(b, c)]$
   3. $[\neg R(z), \neg Q(x, z), \neg P(w, x)]$
   4. $[R(c)]$
   5.1. $[R(f(x)), P(g(x), x)]$
   5.2. $[\neg P(x, f(x)), P(g(x), x)]$

5. **(worth 14 marks)** Use the clauses you generated in part (4) to show using Resolution (by refutation or forward chaining) that $\{1, 2, 3, 4, 5\} \models Q(b, c)$. Annotate each step using the notation from the slides (i.e. *R.[X,Y]*$\{C$*=*$d\}$ where X and Y denote clauses and the positions of literals and any substitutions (e.g. d for C) are listed within curly brackets ($\{\}$).

Solution:

Note that in the notation below $w/g(a)$ is the same as $w = g(a)$, i.e. is a substitution.
1. $[\neg R(x), P(a, x)]$
2. $[Q(a, c), Q(b, c)]$
3. $[\neg R(z), \neg Q(x, z), \neg P(w, x)]$
4. $[R(c)]$
5. $[R(f(x)), P(g(x), x)]$
6. $[\neg P(x, f(x)), P(g(x), x)]$
7. $[\neg Q(b, c)]$
8. $[7, 2]$ $Q(a, c)$
9. $[8, 3]\{x/a, z/c\}$ $[\neg R(c), \neg P(w, a)]$
10. $[9, 4]$ $[\neg P(w, a)]$
11. $[10, 6]\{x/a, w/g(a)\}$ $[\neg P(a, f(a))]$
12. $[11, 1]\{x/f(a)\}[\neg R(f(a))]$
13. $[12, 5]\{x/a\}$ $[P(g(a), a)]$
14. $[13, 10]\{w/g(a)\}$ []