Question 3

Show an example where sufficiency holds but not separation, and one where separation holds but not sufficiency.

A = gender (protected attribute)

C = log-probability of hyperlipidmia

Y = has hyperlipidmia or not (ground truth)

P (A, C, Y) separation holds: A independent of C

Showing P(A indep C | Y)

A	C	Y	P(A, C, Y)
Female	**0.1**	**True**	**0.4**
Female	**0.9**	**True**	**0.7**
Female	**0.9**	**False**	**0.6**
Female	**0.1**	**False**	**0.3**
Male	**0.1**	**True**	**0.4**
Male	**0.9**	**True**	**0.7**
Male	**0.9**	**False**	**0.6**
Male	**0.1**	**False**	**0.4**

In this example separation holds because A is independent of C, for example when (A = Female, C = 0.1, Y = True), P(A,C,Y) is 0.4 and when (A = Male, C = 0.1, Y = True) P(A, C, Y) is still 0.4.

Sufficiency doesn't hold because A is not independent of Y, for example P(A = Female, C = 0.9, Y = True) = 0.7 which does not equal P(A = Female, C = 0.9, Y = False) which is 0.6.

P(A, C, Y) sufficiency holds: A independent of Y

Showing P(A indep Y | C)

A	C	Y	P(A, C, Y)
Female	**0.2**	**True**	**0.3**
Female	**0.8**	**True**	**0.3**
Female	**0.8**	**False**	**0.7**
Female	**0.2**	**False**	**0.7**
Male	**0.2**	**True**	**0.3**
Male	**0.8**	**True**	**0.3**
Male	**0.8**	**False**	**0.7**
Male	**0.2**	**False**	**0.7**

In this example sufficiency holds because A is independent of Y, for example when (A = Female, C = 0.2, Y = True), P(A,C,Y) is 0.3 and when (A = Male, C = 0.2, Y = True) P(A, C, Y) it is still 0.3.