

# Quiz 4: Bayes Nets

⚠ This is a preview of the published version of the quiz

Started: Aug 10 at 10:52am

## Quiz Instructions

For the first 7 questions below, refer to the following Bayesian Net. All of the variables are boolean.

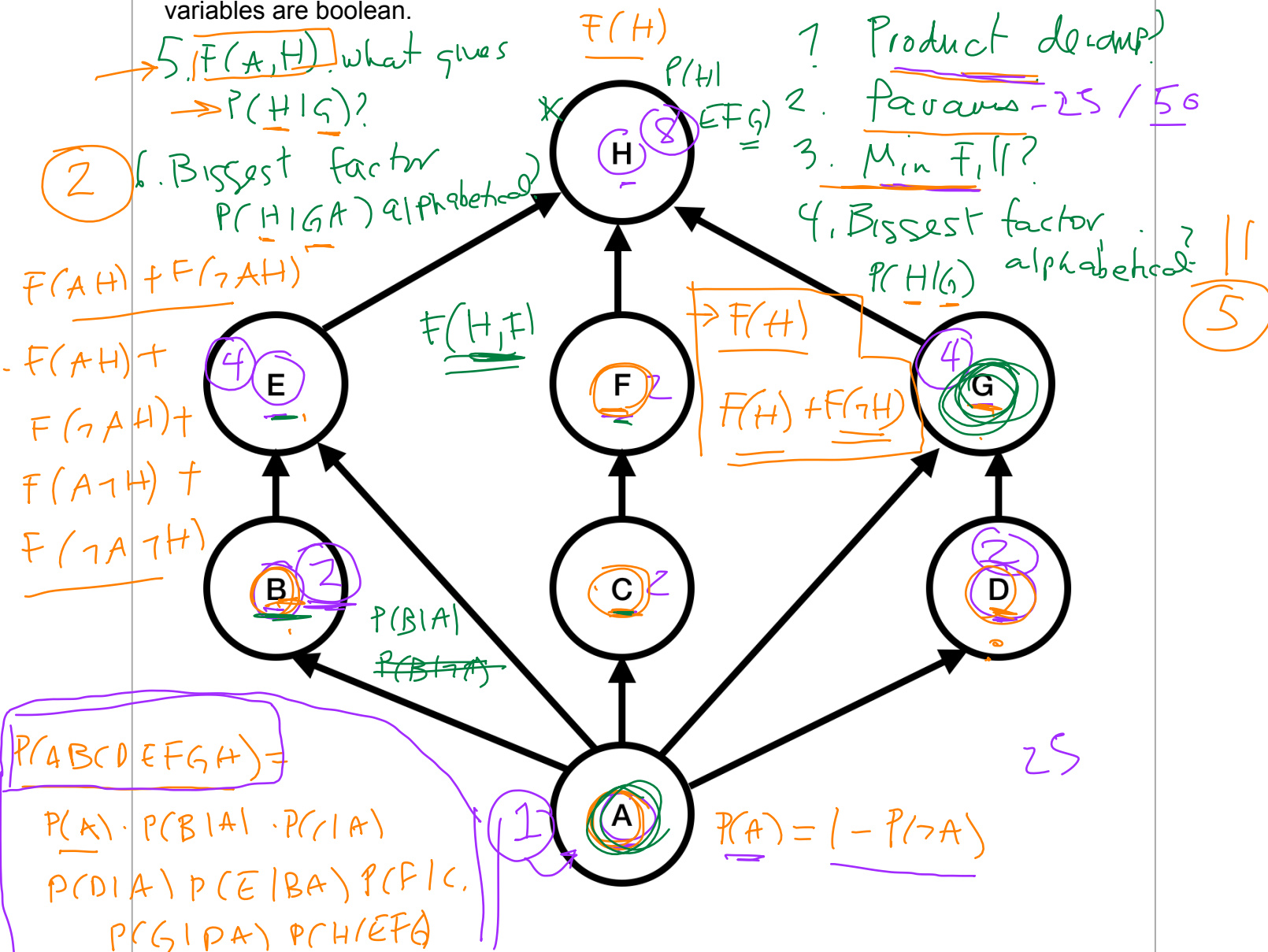


Figure 1

## Question 1

1 pts

What's the **product decomposition** for  $P(A,B,C,D,E,F,G,H)$ , as specified by this network?

- ☐  $P(A)*P(B|A)*P(C)*P(D|A)*P(E|A,B,F)*P(F|C)*P(G|D,A)*P(H|E,F)$
- ☐  $P(A)*P(B)*P(C)*P(D)*P(E|A,B,F)*P(F|C)*P(G|D,A)*P(H|E,F)$
- ☐  $P(A)*P(B|A)*P(C|A)*P(D|A,B)*P(E|A,B,C)*P(F|C)*P(G|D,A)*P(H|E,F,G)$
- ☐  $P(A)*P(B|A)*P(C|A)*P(D|A)*P(E|A,B)*P(F|C)*P(G|D,A)*P(H|E,F,G)$

## Question 2

2 pts

How many **parameters** will you need to specify the CPTs of the network, assuming all variables are boolean?

## Question 3

3 pts

Which variables might you pick first in the network above, if you were using the **min-fill heuristic** to eliminate variables in the Variable Elimination algorithm?

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E

☐ F☐ G☐ H**Question 4****3 pts**

Assume you have been asked to use Variable Elimination to calculate  $P(H=True|G=True)$ , and that you perform elimination of variables in **ALPHABETICAL ORDER**. What is the size of the **largest factor** you will create during the elimination process (i.e. how many variables with this largest factor have in scope)?

**Question 5****1 pts**

Let's again say you are asked to use Variable Elimination to calculate  $P(H=True|G=True)$ , but you leave A as the last variable that you eliminate. As you near the end of the elimination process, you will be left with a factor that is a function of A and H alone (i.e.  $F(A,H)$ ). **Which of the expressions below will yield  $P(H=True|G=True)$ ?**

*Note  $P(A)$  represents the input CPT attached to the variable A in the equations below.*

☐  $F(A=True, H=True) * P(A=True) * F(A=False, H=True) * P(A=False)$ 
☐  $F(A=True, H=True) * P(A=True) + F(A=False, H=True) * P(A=False)$ 
☐  $F(A=True, H=True) + F(A=False, H=True)$ 
☐  $F(A=True, H=True) / (F(A=False, H=True) + F(A=False, H=False))$ 
☐  $(F(A=True, H=True) + F(A=False, H=True)) / (F(A=True, H=True) + F(A=False, H=True) + F(A=True, H=False) + F(A=False, H=False))$ 
☐  $F(A=True, H=True) * P(A=True) / (F(A=False, H=True) * P(A=False) + F(A=False, H=True) * P(A=True))$

## Question 6

3 pts

Assume instead that you have been asked to use Variable Elimination to calculate  $P(H=True|G=True, A=True)$ , and that you are performing elimination of variables in **ALPHABETICAL ORDER**. What is the size of the **largest factor** you will create during the elimination process (i.e. how many variables with this largest factor have in scope)?

## Question 7

8 pts

Once again assume you have been asked to use Variable Elimination to calculate  $P(H=True|G=True, A=True)$ , and that you again perform elimination of variables in **ALPHABETICAL ORDER**. Near the end of your process, you are left with a factor that is a function of H alone. **Before the normalization step in the VE algorithm**, what distribution does this factor represent?

☐  $P(H|G=True, A=True)$ 
☒  $P(H, A=True, G=True)$ 
☐  $P(H|G=True)$ 
☐  $P(H|A=True)$ 
☐  $P(H, A=True|G=True)$ 
☐  $P(H, G=True|A=True)$ 

Handwritten notes:

Numerator

$P(H, A, G)$

$\rightarrow P(H, A, G) + P(\neg H, A, G)$

$* P(A, G)$

Denominator

When driving to or from your grandma's house, your family likes to play a game that involves guessing the make, colour and destination of the vehicles that pass you on

the highway. If you guess correctly, you win! To help you play, you've decide to build a Bayesian Network.

You note that:

$$P(G, colour, Type, Exit) =$$

1. There are only ever CARS or TRUCKS on the highway. You also note that, for every single trip TO grandma's house, you make one trip FROM it.

$$P(G)$$

2. There are only three colours of vehicle that pass you: **red**, **blue** and **orange**. **Red** cars pass more frequently than blue ones, and **blue** ones pass more frequently than **orange** ones (the ratio is 5:3:2). Note that two cars NEVER pass at the same time.

$$P(colour)$$

3. When you are driving TO grandma's, 9 of every 10 **red** vehicles that pass are CARS; 1 in every 10 red vehicles is a TRUCK. When you are driving FROM grandma's, the **red** vehicles that pass are always CARS.

$$P(Type | colour, G)$$

4. The **blue** vehicles that pass are always TRUCKS when you are going TO grandma's, but when you are coming FROM grandma's, 1 in 10 that pass is a CAR.

5. The **orange** vehicles that pass are CARS 8 of every 10 times, whether you are going TO or FROM grandma's house.

$$P(Exit | colour)$$

6. Every tenth **red** and **blue** vehicle EXITS the highway immediately after it passes you.

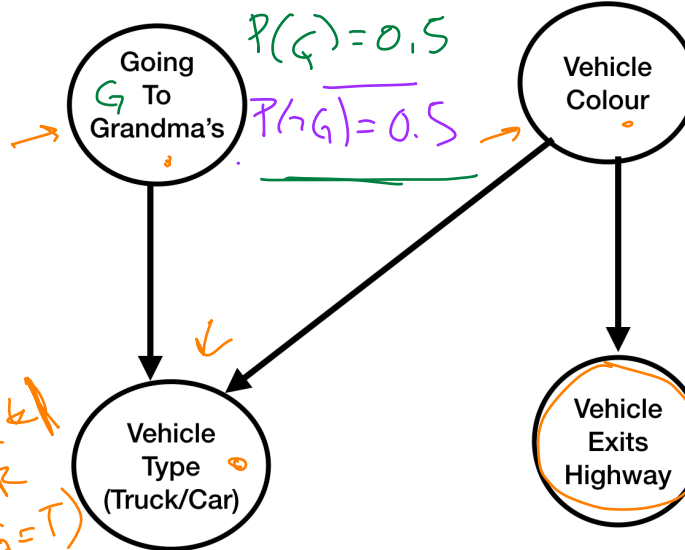
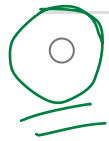
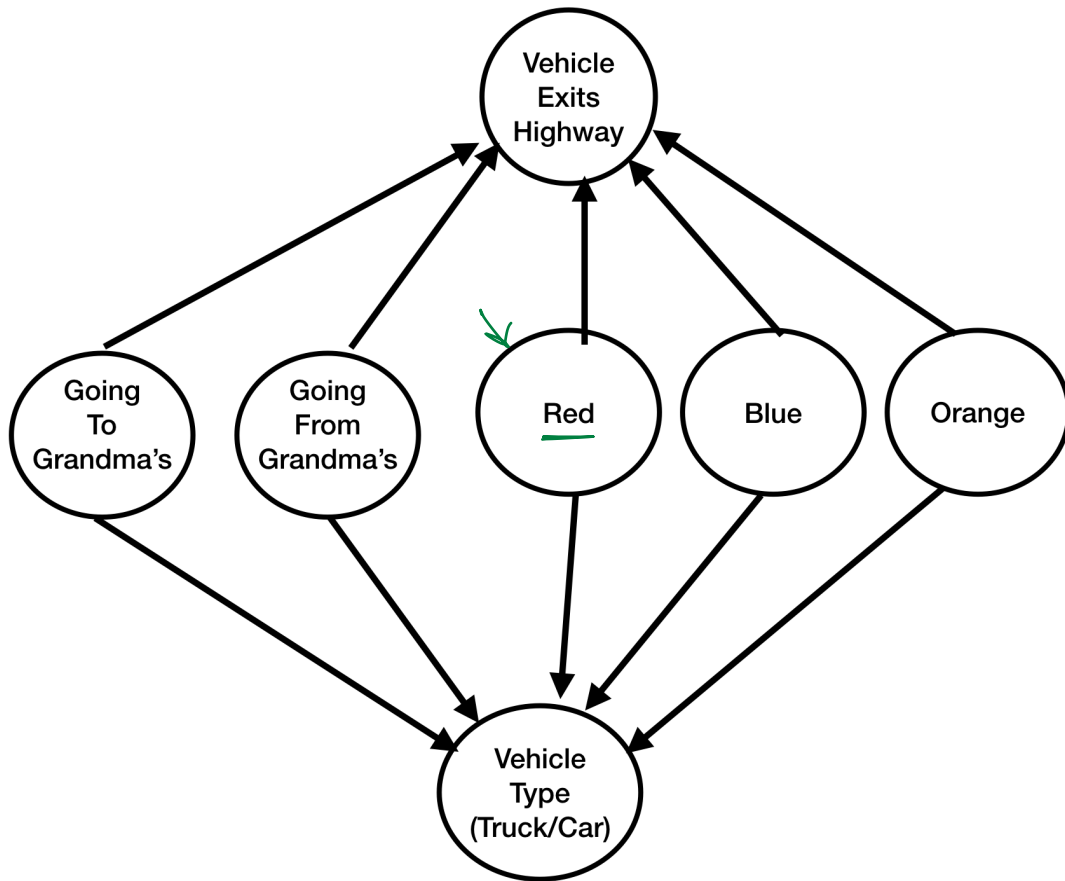
7. Every fourth **orange** vehicle EXITS the highway immediately after it passes you.

## Question 8

3 pts

Which network is **best suited** to express the details of this problem?

☐



$$P(G) = 0.5$$

$$P(\neg G) = 0.5$$

$$\begin{aligned} P(R) &= 0.5 \\ P(B) &= 0.3 \\ P(O) &= 0.2 \end{aligned}$$

$$P(\text{Type} = \text{Truck} | \text{Colour} = R, G = T)$$

$$P(\text{Truck} | R, G) = 0.1$$

$$P(\text{Truck} | R, \neg G) = 0$$

$$P(\text{Truck} | B, G) = 1$$

$$P(\text{Truck} | B, \neg G) = 0.9$$

$$P(\text{Truck} | O, G) = 0.2$$

$$P(\text{Truck} | O, \neg G) = 0.2$$

$$P(\text{Exit} | R) = 0.1$$

$$P(\text{Exit} | B) = 0.1$$

$$P(\text{Exit} | O) = 0.9$$

$$P(\text{Truck})? \quad P(\text{Type} = \text{Truck})$$

$$P(\text{Exit} | G, \text{Car})?$$

$$\begin{aligned}
 P(\text{Type} = \text{Truck}) &= \sum_{G \in \mathcal{G}} \sum_{C \in \mathcal{C}} P(G, \text{Type} = \text{Truck}, \text{Color}) = \\
 &= \sum_{G \in \mathcal{G}} \sum_{C \in \mathcal{C}} P(\text{Truck} | G, \text{Color}) \cdot P(G) \cdot P(\text{Color})
 \end{aligned}$$

2 · 3 = 6

0.35

$$= * P(\text{Truck} | G, R) \cdot P(G) \cdot P(R) +$$

→ 0.1 · 0.5 · 0.5

$$+ P(\text{Truck} | \neg G, R) \cdot P(\neg G) \cdot P(R) +$$

→ 0.1 · 0.5 · 0.5

$$+ P(\text{Truck} | G, B) \cdot P(G) \cdot P(B) +$$

→ 1 · 0.5 · 0.3

$$+ P(\text{Truck} | \neg G, B) \cdot P(\neg G) \cdot P(B) +$$

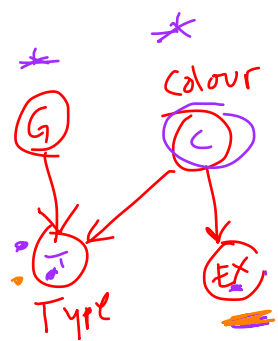
→ 0.9 · 0.5 · 0.3

$$+ P(\text{Truck} | G, O) \cdot P(G) \cdot P(O) +$$

→ 0.2 · 0.5 · 0.2

$$+ P(\text{Truck} | \neg G, O) \cdot P(\neg G) \cdot P(O)$$

→ 0.2 · 0.5 · 0.2



$$P(\text{Exit} | \text{Type} = \text{Car}, G) = P(\text{Exit}, \text{Type} = \text{Car}, G) *$$

$$P(\text{Type} = \text{Car}, G) = P(\text{Exit}, \text{Type} = \text{Car}, G) +$$

$$P(\neg \text{Exit}, \text{Type} = \text{Car}, G)$$

Colour

← P(Car, R, G, Exit)

$$* P(\text{Car} | R, G) \cdot P(R) \cdot P(G) \cdot P(\text{Exit} | R) +$$

$$* P(\text{Car} | B, G) \cdot P(B) \cdot P(G) \cdot P(\text{Exit} | B) +$$

$$* P(\text{Car} | O, G) \cdot P(O) \cdot P(G) \cdot P(\text{Exit} | O)$$

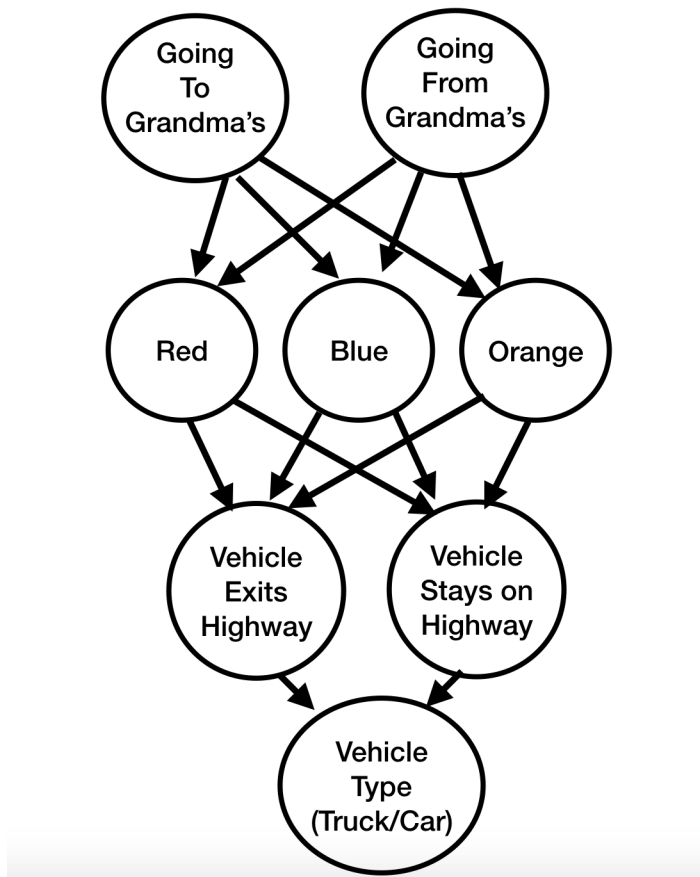
$$= \left\{ \begin{array}{l} 0.9 \cdot 0.5 \cdot 0.5 \cdot 1 \\ 0 \cdot 0.3 \cdot 0.5 \cdot 1 \\ 0.8 \cdot 0.2 \cdot 0.5 \cdot 1 \end{array} \right\} *$$

$$\begin{aligned}
 P(\neg \text{Exit}, \text{Car}, G) &= P(\text{Car} | R, G) \cdot P(R) \cdot P(G) \cdot P(\neg \text{Exit} | R) + \\
 &P(\text{Car} | B, G) \cdot P(B) \cdot P(G) \cdot P(\neg \text{Exit} | B) + \\
 &P(\text{Car} | O, G) \cdot P(O) \cdot P(G) \cdot P(\neg \text{Exit} | O)
 \end{aligned}$$

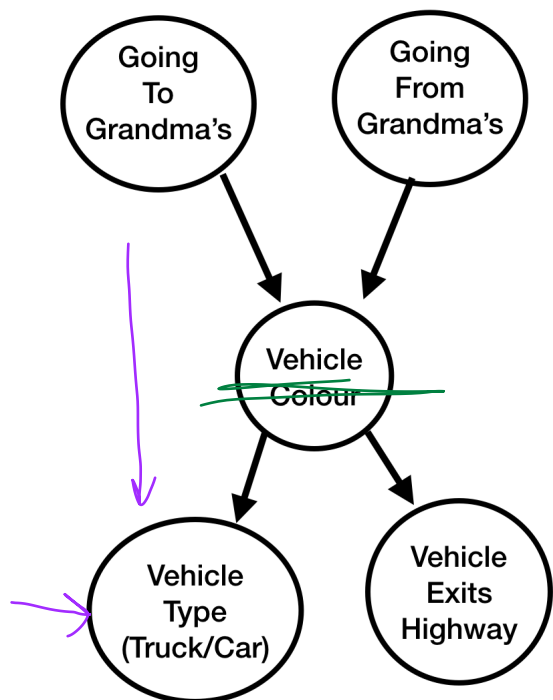
A  
A+B

$$= 0.9 \cdot 0.5 \cdot 0.5 \cdot 0.9 + 0 + 0.8 \cdot 0.2 \cdot 0.5 \cdot 0.6 \quad * = B$$

≈ 1.78

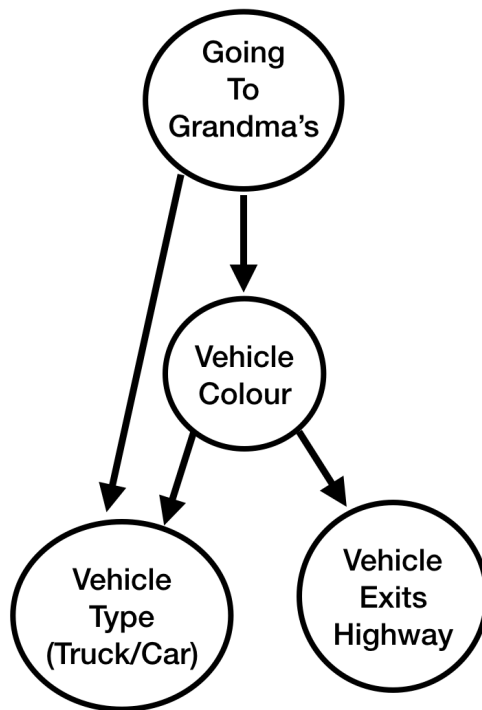


○



○



**Question 9****5 pts**

What is the probability that a vehicle that passes you is a **TRUCK**?

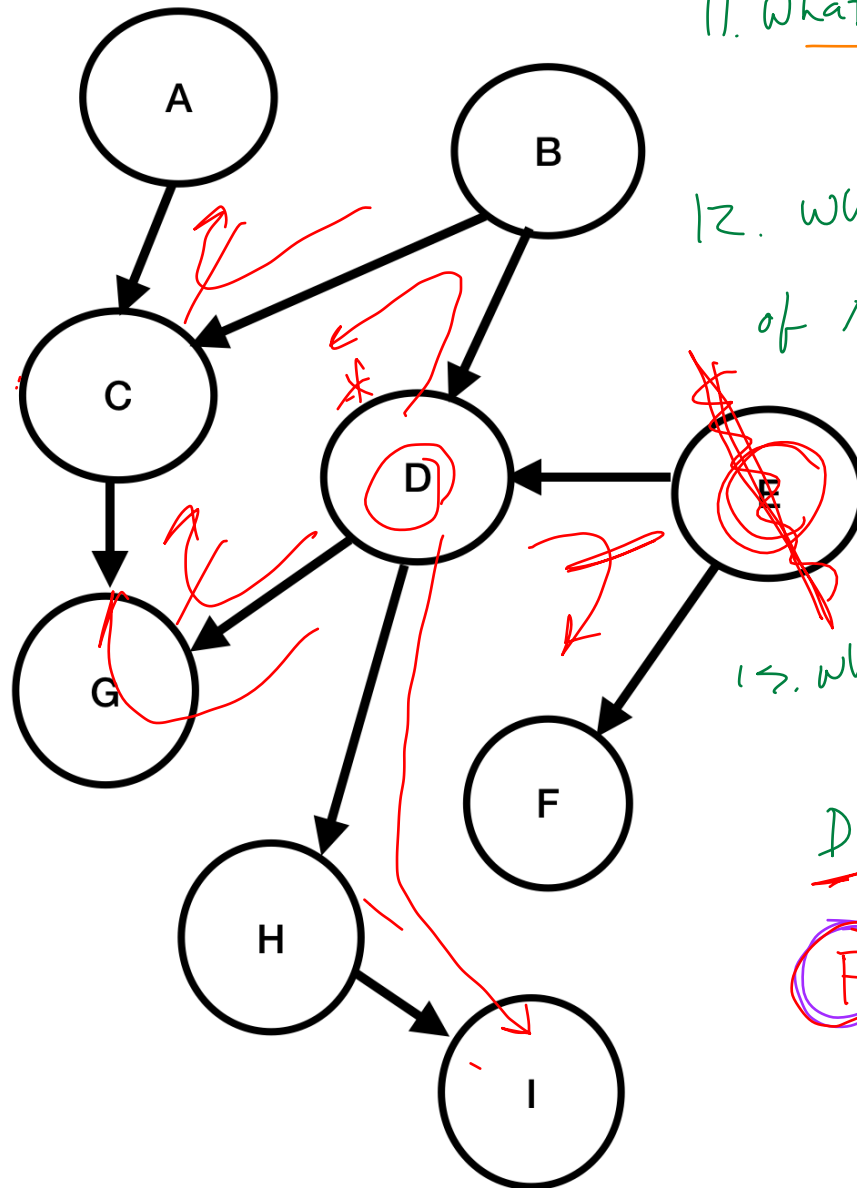
Express your answer to two decimal places. [Need a calculator? Click here.](https://www.google.com/search?q=calculator)  
(<https://www.google.com/search?q=calculator>)

 $P(\text{Truck})$ **Question 10****5 pts**

Assume you are driving **TO** grandma's. A **CAR** passes you. What is the probability it will **EXIT** the highway?

Express your answer to two decimal places. [Need a calculator? Click here.](https://www.google.com/search?q=calculator)  
(<https://www.google.com/search?q=calculator>)

For the questions that follow, refer to the network below.



11. What's ind of I? A

12. What's c.i. of A given G?

13. What's c.i. of D given E?

F A

Figure 2

**Question 11****2 pts**

Which variables are independent of I?

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ F
- ☐ G
- ☐ H
- ☐ None of the Above

**Question 12****2 pts**

Which variables are conditionally independent of A given G?

- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ F
- ☐ H
- ☐ I
- ☐ None of the Above

**Question 13****2 pts**

Which variables are conditionally independent (or just independent) of D given E?

☐ A☐ B☐ C☐ F☐ G☐ H☐ I☐ None of the Above

Quiz saved at 10:53am

**Submit Quiz**