Example 1: separation holds but not sufficiency

Let A, C, Y be three binary variables each takes on the value of 0 or 1.
Construct the bay's net with the following CPT:

Y	P(Y)
0	0.1
1	0.9

C	Y	P(C|Y)
0	0	0.1
0	1	0.5
1	0	0.9
1	1	0.5

A	Y	P(A|Y)
0	0	0.5
0	1	0.1
1	0	0.5
1	1	0.9

We obtain the table for P(Y, C, A) as the following:

Y	C	A	P(Y, C, A)
0	0	0	0.005
0	0	1	0.005
0	1	0	0.045
0	1	1	0.045
1	0	0	0.045
1	0	1	0.405
1	1	0	0.045
1	1	1	0.405

We can check for separation using P(A|C, Y) = P(A|Y). We obtain the table for P(A|C, Y) as the following. Comparing with the CPT for P(A|Y), we find out that they equal no matter the value for C.

A	Y	C	P(A|Y, C)
0	0	0	0.5
0	1	0	0.1
1	0	0	0.5
1	1	0	0.9
0	0	1	0.5
0	1	1	0.1
1	0	1	0.5
1	1	1	0.9

We find that sufficiency does not hold since P(A|Y, C) != P(A|C). We obtain that P(A=0|Y=0, C=0) = 0.5 but P(A=0|C=0) = 0.16666666666666669

Example 2: sufficiency holds but not separation

Let A, C, Y be three binary variables each takes on the value of 0 or 1.
Construct the bay's net with the following CPT:

Y	P(Y)
0	0.1
1	0.9

C	Y	P(C|Y)
0	0	0.1
0	1	0.5
1	0	0.9
1	1	0.5

A	C	P(A|C)
0	0	0.5
0	1	0.1
1	0	0.5
1	1	0.9

We obtain the table for P(Y, C, A) as the following:

Y	C	A	P(Y, C, A)
0	0	0	0.005
0	0	1	0.005
0	1	0	0.009
0	1	1	0.081
1	0	0	0.225
1	0	1	0.225
1	1	0	0.045
1	1	1	0.405

We can check for sufficiency using P(A|C, Y) = P(A|C). We obtain the table for P(A|C, Y) as the following. Comparing with the CPT for P(A|C), we find out that they equal no matter the value for Y.

A	C	Y	P(A|C, Y)
0	0	0	0.5
0	1	0	0.1
1	0	0	0.5
1	1	0	0.9
0	0	1	0.5
0	1	1	0.1
1	0	1	0.5
1	1	1	0.9

We find that separation does not hold since P(A|Y, C) != P(A|Y). We obtain that P(A=0|Y=0, C=0) = 0.5 but P(A=0|Y=0) = 0.3