

Question 3

Denote Gender as G, Prediction as P and Hyperlipidemia as H.

I used the formula below to calculate the probabilities in Excel:

$$P(P = \text{YES} \mid H = \text{YES}, G = \text{Female}) = \frac{P(P = \text{YES}, H = \text{YES}, G = \text{Female})}{P(P = \text{YES}, H = \text{YES}, G = \text{Female}) + P(P = \text{NO}, H = \text{YES}, G = \text{Female})}$$

$$P(P = \text{YES} \mid H = \text{YES}) = \frac{\sum_G P(P = \text{YES}, H = \text{YES})}{\sum_{P,G} P(H = \text{YES})}$$

$$P(H = \text{YES} \mid P = \text{YES}, G = \text{Female}) = \frac{P(P = \text{YES}, H = \text{YES}, G = \text{Female})}{P(P = \text{YES}, H = \text{YES}, G = \text{Female}) + P(P = \text{YES}, H = \text{NO}, G = \text{Female})}$$

$$P(H = \text{YES} \mid P = \text{YES}) = \frac{\sum_P P(P = \text{YES}, H = \text{YES})}{\sum_{H,G} P(P = \text{YES})}$$

Separated but not Sufficient

| Table1 | | |
|--|------------------|-------------|
| G (Gender) | Female | Male |
| H (Hyperlipidemia) | YYYYYY NNN | YY NN |
| P (Prediction) | YYNNNN NNY | NY NN |
| Note: Y = YES, N = NO | | |
| | | |
| Values for P, H, G | Num of Instances | Probability |
| ('YES', 'YES', 'Female') | 3.00 | 0.23076923 |
| ('YES', 'YES', 'Male') | 1.00 | 0.07692308 |
| ('YES', 'NO', 'Female') | 1.00 | 0.07692308 |
| ('YES', 'NO', 'Male') | 0.00 | 0 |
| ('NO', 'YES', 'Female') | 3.00 | 0.23076923 |
| ('NO', 'YES', 'Male') | 1.00 | 0.07692308 |
| ('NO', 'NO', 'Female') | 2.00 | 0.15384615 |
| ('NO', 'NO', 'Male') | 2.00 | 0.15384615 |
| Sum | 13.00 | 1 |
| Separation | | |
| $P(P = \text{YES} \mid H = \text{YES}, G = \text{Female})$ | | 0.50 |
| $P(P = \text{YES} \mid H = \text{YES})$ | | 0.50 |
| Sufficiency | | |
| $P(H = \text{YES} \mid P = \text{YES}, G = \text{Female})$ | | 0.75 |
| $P(H = \text{YES} \mid P = \text{YES})$ | | 0.80 |

Sufficient but not Separated

| Table2 | | |
|---------------------------------|------------------|-------------|
| G (Gender) | Female | Male |
| H (Hyperlipidemia) | YYNNN NNY | NY NN |
| P (Prediction) | YYYYY NNN | YY NN |
| | | |
| Values for P, H, G | Num of Instances | Probability |
| ('YES', 'YES', 'Female') | 3.00 | 0.2307692 |
| ('YES', 'YES', 'Male') | 1.00 | 0.0769231 |
| ('YES', 'NO', 'Female') | 3.00 | 0.2307692 |
| ('YES', 'NO', 'Male') | 1.00 | 0.0769231 |
| ('NO', 'YES', 'Female') | 1.00 | 0.0769231 |
| ('NO', 'YES', 'Male') | 0.00 | 0 |
| ('NO', 'NO', 'Female') | 2.00 | 0.1538462 |
| ('NO', 'NO', 'Male') | 2.00 | 0.1538462 |
| Sum | 13.00 | 1 |
| Separation | | |
| P (P =YES H =YES, G = Female) | | 0.75 |
| P (P =YES H =YES) | | 0.80 |
| Sufficiency | | |
| P (H =YES P =YES, G = Female) | | 0.50 |
| P (H =YES P =YES) | | 0.50 |

Conclusion

Separation and Sufficiency cannot hold together. For example, if we want to modify the values in Table 1 in order to hold sufficiency, the separation property would no longer hold any more. We cannot ensure both properties. Therefore, we can't enforce Sufficiency and Separation at the same time.

Note:

- Example details are shown in the two tables attached.
- Probability is calculated by $\frac{\text{num of instances}}{\text{total instances}}$ for each entry in the joint probability table.