Separation holds, but not sufficiency

Let A = gender (gd), C = region (rg), Y = hyperlipidemia (hl)

P(rg|hl, gd) = P(rg|hl)

P(rg=Countryside|hl=YES, gd=Male)	0.4967974089058388	P(rg=Countryside|hl=YES, gd=Female)	0.4967974089058388
P(rg=City|hl=YES, gd=Male)	0.5032025910941611	P(rg=City|hl=YES, gd=Female)	0.5032025910941611
P(rg=Countryside|hl=NO, gd=Male)	0.4677592424857465	P(rg=Countryside|hl=NO, gd=Female)	0.4677592424857465
P(rg=City|hl=NO, gd=Male)	0.5322407575142535	P(rg=City|hl=NO, gd=Female)	0.5322407575142535

This proves **Separation** holds because the protected attribute (gender) is conditionally independent from the classification (region) when given the ground truth (hyperlipidemia).

P(hl|rg, gd) != P(hl|rg)

P(hl=YES|rg=Countryside, gd=Male)	0.45530583084984777	P(hl=YES|rg=Countryside, gd=Female)	0.38008473760328293
P(hl=YES|rg=City, gd=Male)	0.4266366780681491	P(hl=YES|rg=City, gd=Female)	0.3530817658296224
P(hl=NO|rg=Countryside, gd=Male)	0.5446941691501523	P(hl=NO|rg=Countryside, gd=Female)	0.619915262396717
P(hl=NO|rg=City, gd=Male)	0.573363321931851	P(hl=NO|rg=City, gd=Female)	0.6469182341703775

This proves **Sufficiency** does not hold because the protected attribute (gender) effects the probabilities.

Sufficiency holds, but not separation

Let A = gender (gd), C = hyperlipidemia (hl), Y = Central Obesity (co)

P(co|hl, gd) = P(co|hl)

P(co=YES|hl=YES, gd=Male)	0.7876943228020956	P(co=YES|hl=YES, gd=Female)	0.7876943228020956
P(co=NO|hl=YES, gd=Male)	0.21230567719790436	P(co=NO|hl=YES, gd=Female)	0.21230567719790436
P(co=YES|hl=NO, gd=Male)	0.583203690852557	P(co=YES|hl=NO, gd=Female)	0.583203690852557
P(co=NO|hl=NO, gd=Male)	0.416796309147443	P(co=NO|hl=NO, gd=Female)	0.416796309147443

This proves **Sufficiency** holds because the protected attribute (gender) is conditionally independent from the ground truth (central obesity) when given the classification (hyperlipidemia).

P(hl|co, gd) != P(hl|co)

P(hl=YES|co=YES, gd=Male)	0.5152677192429572	P(hl=YES|co=YES, gd=Female)	0.4381081553691644
P(hl=YES|co=NO, gd=Male)	0.2861709298888966	P(hl=YES|co=NO, gd=Female)	0.22723545270995207
P(hl=NO|co=YES, gd=Male)	0.4847322807570428	P(hl=NO|co=YES, gd=Female)	0.5618918446308354
P(hl=NO|co=NO, gd=Male)	0.7138290701111034	P(hl=NO|co=NO, gd=Female)	0.7727645472900478

This proves **Separation** does not hold because the protected attribute (gender) effects the probabilities.