

Example 1: separation holds, but not sufficiency

Let C = Hypertension, Y = Hyperlipidemia, and A = Gender

Want to prove:

$$P(\text{Hypertension} \mid \text{Hyperlipidemia}, \text{Gender}) = P(\text{Hypertension} \mid \text{Hyperlipidemia})$$

$$P(\text{Hypertension} \mid \text{Hyperlipidemia}, \text{Gender}) = [0.5560581928250615, 0.4439418071749385]$$

$$P(\text{Hypertension} \mid \text{Hyperlipidemia}) = [0.5560581928250615, 0.4439418071749385]$$

Since LHS == RHS, separation holds

Want to prove:

$$P(\text{Hyperlipidemia} \mid \text{Hypertension}, \text{Gender}) \neq P(\text{Hyperlipidemia} \mid \text{Hypertension})$$

$$P(\text{Hyperlipidemia} \mid \text{Hypertension}, \text{Gender}) = [0.5001088390530728, 0.4998911609469271]$$

$$P(\text{Hyperlipidemia} \mid \text{Hypertension}) = [0.4639580370746129, 0.5360419629253871]$$

Since LHS != RHS, sufficiency doesn't hold

Thus, separation holds, but not sufficiency

Example 2: sufficiency holds, but not separation

Let C = Hyperlipidemia, Y = Hypertension, A = Gender

Want to prove:

$$P(\text{Hyperlipidemia} \mid \text{Hypertension}, \text{Gender}) \neq P(\text{Hyperlipidemia} \mid \text{Hypertension})$$

$$P(\text{Hyperlipidemia} \mid \text{Hypertension}, \text{Gender}) = [0.5001088390530728, 0.4998911609469271]$$

$$P(\text{Hyperlipidemia} \mid \text{Hypertension}) = [0.4639580370746129, 0.5360419629253871]$$

Since LHS \neq RHS, separation doesn't hold

Want to prove:

$$P(\text{Hypertension} \mid \text{Hyperlipidemia}, \text{Gender}) = P(\text{Hypertension} \mid \text{Hyperlipidemia})$$

$$P(\text{Hypertension} \mid \text{Hyperlipidemia}, \text{Gender}) = [0.5560581928250615, 0.4439418071749385]$$

$$P(\text{Hypertension} \mid \text{Hyperlipidemia}) = [0.5560581928250615, 0.4439418071749385]$$

Since LHS == RHS, sufficiency holds.

Thus, sufficiency holds, but not separation.