

- Separation: A is independent of C given Y , i.e.

$$P(A|CY) = P(A|Y) \text{ for all values in } Y\text{'s domain}$$

- Sufficiency A is independent of Y given C , i.e.

$$P(A|YC) = P(A|C) \text{ for all values in } C\text{'s domain}$$

First I'll show that if we have a joint distribution that satisfies sufficiency, it can't satisfy separation. Consider the variables

$$A, \quad \text{domain}(A) = \{a1, a2\}$$

$$C, \quad \text{domain}(C) = \{c1, c2\}$$

$$Y, \quad \text{domain}(Y) = \{y1, y2\}$$

Where all of there values have equal (0.5) probability, so in the joint, $\forall a \in \text{domain}(A), c \in \text{domain}(C), y \in \text{domain}(Y), P(A = a, C = c, Y = y) = 0.125$ (i.e. each event in the joint has equal probability)