
A4Q3

Assume $X = \text{Prediction}$, $Y = \text{Hyperlipidemia}$, $Z = \text{Gender}$, then

- **Separated**

- $P(\text{Prediction} = \text{YES} | \text{Hyperlipidemia} = \text{YES}, \text{Gender} = \text{Female}) =$

- $P(\text{Prediction} = \text{YES} | \text{Hyperlipidemia} = \text{YES})$

- Becomes to $P(X = \text{YES} | Y = \text{YES}, Z = \text{Female}) = P(X = \text{YES} | Y = \text{YES})$

- **Sufficient**

- $P(\text{Hyperlipidemia} = \text{YES} | \text{Prediction} = \text{YES}, \text{Gender} = \text{Female}) =$

- $P(\text{Hyperlipidemia} = \text{YES} | \text{Prediction} = \text{YES})$

- Becomes to $P(Y = \text{YES} | X = \text{YES}, Z = \text{Female}) = P(Y = \text{YES} | X = \text{YES})$

- **Separated and not Sufficient example**

X	YES		NO		
	Z	Male	Female	Male	Female
Y					
YES		0.1	0.075	0.1	0.075
NO		0.15	0.175	0.15	0.175

● Sufficient and not Separated example

X \ Z	YES		NO	
	Male	Female	Male	Female
Y				
YES	0.1	0.075	0.15	0.175
NO	0.1	0.075	0.15	0.175

$$P(x, y) = P(x, y, z) + P(x, y, \sim z) = 0.7$$

$$P(x, z) = P(x, y, z) + P(x, \sim y, z) = 0.6$$

$$P(y, z) = P(x, y, z) + P(\sim x, y, z) = 1$$

x \ Z	Prediction = YES		Prediction = NO	
	Gender = Male	Gender = Female	Gender = Male	Gender = Female
Y				
Hyperlipidemia = YES	0.4	0.3	0.6	0.7
Hyperlipidemia = NO	0.6	0.7	0.4	0.3

$$P(x, y) = P(x, y, z) + P(x, y, \sim z) = 0.7$$

$$P(x, z) = P(x, y, z) + P(x, \sim y, z) = 1$$

$$P(y, z) = P(x, y, z) + P(\sim x, y, z) = 1$$

$$P(X, Y, Z) = \frac{P(X, Y) \times P(X, Z)}{P(X)} \neq \frac{P(X, Y) \times P(Y, Z)}{P(Y)}$$

Assume Prediction = YES is X, Hyperlipidemia = YES is Y, Gender = Female is Z

If separated, then

$$P(X|Y, Z) = P(X|Y)$$

$$\frac{P(X, Y, Z)}{P(Y, Z)} = \frac{P(X, Y)}{P(Y)}$$

$$P(X, Y, Z) = \frac{P(X, Y) \times P(Y, Z)}{P(Y)}$$

If sufficient, then

$$P(Y|X, Z) = P(Y|X)$$

$$P(X, Y, Z) = \frac{P(X, Y) \times P(X, Z)}{P(X)}$$

So

$$P(X, Y, Z) = \frac{P(X, Y) \times P(Y, Z)}{P(Y)} = \frac{P(X, Y) \times P(X, Z)}{P(X)}$$

$$\frac{P(Y, Z)}{P(Y)} = \frac{P(X, Z)}{P(X)}$$

$$P(Z|Y) = P(Z|X)$$

Or try to use apply Bayes rule for the separation and you get terms similar to
Sufficient

Add new node of BN, and create own prob table