



We consider Hyperlipidemia as the ground truth.

1) Proof when separation holds, sufficiency fails.

By calculation we can get:

$$P(vg = '<400g/d' \mid hl = 'YES', ht = 'YES') = P(vg = '<400g/d' \mid hl = 'YES') = 0.579$$

Where we can see separation holds.

Now we check if sufficiency holds:

$$P(hl = 'YES' \mid vg = '<400g/d', ht = 'YES') = 0.6390947454432463$$

$$P(hl = 'YES' \mid vg = '<400g/d') = 0.5821287241184987$$

The data above shows that sufficiency does not hold.

2) Proof when sufficiency holds, separation fails.

By calculation we can get:

$$P(hl = 'YES' | co = 'YES', bmi = '\sim 18.5', ht = 'NO')$$

$$= P(hl = 'YES' | co = 'YES', bmi = '\sim 18.5')$$

$$= 0.21850713898427612$$

Where we can see sufficiency holds.

Then we check if separation holds:

$$P(co = 'YES', bmi = '\sim 18.5' | hl = 'YES', ht = 'NO') = 0.1325567200235734$$

$$P(co = 'YES', bmi = '\sim 18.5' | hl = 'YES') = 0.09385561378061805$$

The data above shows that separation does not hold.

We can conclude that: when sufficiency holds, separation does not, and when separation holds, sufficiency does not.