Separation Example:

I illustrate an example with separation with the following variables and domains:

A: {male, female} – protected variable

Y: {yes, no} – ground truth

C: {yes, no} – classification

C	Y	A	P(A,C,Y)
yes	Yes	Male	0.28
yes	Yes	Female	0.24
yes	no	Male	0.03
yes	no	Female	0.04
no	yes	Male	0.07
no	yes	Female	0.06
no	no	Male	0.12
no	no	Female	0.16

C	Y	A	P(C|Y,A)
yes	yes	Male	0.8
yes	yes	Female	0.8
yes	no	Male	0.2
yes	no	Female	0.2
no	Yes	Male	0.2
no	yes	Female	0.2
no	no	Male	0.8
no	no	Female	0.8

C	Y	P(C|Y)
yes	yes	0.8
yes	no	0.2
no	yes	0.2
no	no	0.8

The above probability tables are all based on the joint distribution specified. We can see based on the conditional probabilities for P(C|Y,A) that C is independent of A given Y.

Y	C	A	P(Y|C,A)
yes	yes	Male	0.9032258
yes	yes	Female	0.8571429
yes	no	Male	0.1578947
yes	no	Female	0.1818182
no	Yes	Male	0.2258065
no	yes	Female	0.2142857
no	no	Male	0.6315789
no	no	Female	0.7272727

Y	C	P(Y|C)
yes	yes	0.881356
yes	no	0.317073
no	yes	0.118644
no	no	0.682927

We can also see based on these two tables that sufficiency does not hold.

Sufficiency Example:

I illustrate an example with sufficiency with the following variables and domains:

A: {male, female} – protected variable

Y: {yes, no} – ground truth

C: {yes, no} – classification

C	Y	A	P(A,C,Y)
yes	yes	Male	0.28
yes	yes	Female	0.21
yes	no	Male	0.12
yes	no	Female	0.09
no	yes	Male	0.06
no	yes	Female	0.12
no	no	Male	0.04
no	no	Female	0.08

C	Y	A	P(Y|C,A)
yes	yes	Male	0.7
yes	yes	Female	0.7
yes	no	Male	0.3
yes	no	Female	0.3
no	yes	Male	0.6
no	yes	Female	0.6
no	no	Male	0.4
no	no	Female	0.4

C	Y	P(Y|C)
yes	yes	0.7
yes	no	0.3
no	yes	0.6
no	no	0.4

The above probability tables are based on the joint distribution specified P(A,C,Y). The conditional probability tables show that sufficiency holds.

C	Y	A	P(C|Y,A)
yes	Yes	Male	0.8235294
yes	Yes	Female	0.6363636
yes	No	Male	0.75
yes	NO	Female	0.5625
no	Yes	Male	0.1764706
no	yes	Female	0.3636364
no	no	Male	0.25
no	no	Female	0.4705882

The above table shows that separation does not hold as gender specified by variable A modifies the probability of P(C|Y,A).