Question 3: Is your Bayes Network Fair?

Example 1: Separation holds, but not sufficiency -- *(P(A | C, Y) = P(A | C)) and (P(A | Y, C) != P(A | Y))*

C	Y	A	Probability
YES	YES	YES	0.2
YES	YES	NO	0.15
YES	NO	YES	0.2
YES	NO	NO	0.15
NO	YES	YES	0.05
NO	YES	NO	0.1
NO	NO	YES	0.05
NO	NO	NO	0.1

Separation:
P(A=yes | C=yes, Y=yes) = P(A=yes, C=yes, Y=yes)/P(C=yes, Y=yes) = 0.2/0.35 = 0.57
P(A=yes | C=yes) = P(A=yes, C=yes)/P(C=yes) = 0.4/0.7 = 0.57
Therefore, P(A=yes | C=yes, Y=yes) = P(A=yes | C=yes)

P(A=yes | C=yes, Y=no) = P(A=yes, C=yes, Y=no)/P(C=yes, Y=no) = 0.2/0.35 = 0.57
P(A=yes | C=yes) = P(A=yes, C=yes)/P(C=yes) = 0.4/0.7 = 0.57
Therefore, P(A=yes | C=yes, Y=no) = P(A=yes | C=yes)

P(A=yes | C=no, Y=yes) = P(A=yes, C=no, Y=yes)/P(C=no, Y=yes) = 0.05/0.15 = 0.33
P(A=yes | C=no) = P(A=yes, C=no)/P(C=no) = 0.1/0.3 = 0.33
Therefore, P(A=yes | C=no, Y=yes) = P(A=yes | C=no)

P(A=yes | C=no, Y=no) = P(A=yes, C=no, Y=no)/P(C=no, Y=no) = 0.05/0.15 = 0.33
P(A=yes | C=no) = P(A=yes, C=no)/P(C=no) = 0.1/0.3 = 0.33
Therefore, P(A=yes | C=no, Y=no) = P(A=yes | C=no)

Thus, *P(A | C, Y) = P(A | C)* and the joint distribution is separated

Sufficiency:
P(A=yes | C=yes, Y=yes) = P(A=yes, C=yes, Y=yes)/P(C=yes, Y=yes) = 0.2/0.35 = 0.57
P(A=yes | Y=yes) = P(A=yes, Y=yes)/P(Y=yes) = 0.25/0.5 = 0.5
Therefore, P(A | Y, C) != P(A | Y)

Thus, the joint distribution is not sufficient

Example 2: Sufficiency holds, but not separation -- *(P(A | Y, C) = P(A | Y)) and (P(A | C, Y) != P(A | C))*

C	Y	A	Probability
YES	YES	YES	0.2
YES	YES	NO	0.05
YES	NO	YES	0.1
YES	NO	NO	0.15
NO	YES	YES	0.2
NO	YES	NO	0.05
NO	NO	YES	0.1
NO	NO	NO	0.15

Sufficiency:
P(A=yes | C=yes, Y=yes) = P(A=yes, C=yes, Y=yes)/P(C=yes, Y=yes) = 0.2/0.25 = 0.8
P(A=yes | Y=yes) = P(A=yes, Y=yes)/P(Y=yes) = 0.4/0.5 = 0.8
Therefore, P(A=yes | C=yes, Y=yes) = P(A=yes | Y=yes)

P(A=yes | C=yes, Y=no) = P(A=yes, C=yes, Y=no)/P(C=yes, Y=no) = 0.1/0.25 = 0.4
P(A=yes | Y=no) = P(A=yes, Y=no)/P(Y=no) = 0.2/0.5 = 0.4
Therefore, P(A=yes | C=yes, Y=no) = P(A=yes | Y=no)

P(A=yes | C=no, Y=yes) = P(A=yes, C=no, Y=yes)/P(C=no, Y=yes) = 0.2/0.25 = 0.8
P(A=yes | Y=yes) = P(A=yes, Y=yes)/P(Y=yes) = 0.4/0.5 = 0.8
Therefore, P(A=yes | C=no, Y=yes) = P(A=yes | Y=yes)

P(A=yes | C=no, Y=no) = P(A=yes, C=no, Y=no)/P(C=no, Y=no) = 0.1/0.25 = 0.4
P(A=yes | Y=no) = P(A=yes, Y=no)/P(Y=no) = 0.2/0.5 = 0.4
Therefore, P(A=yes | C=no, Y=no) = P(A=yes | Y=no)

Thus, $P(A \mid Y, C) = P(A \mid Y)$ and the joint distribution is sufficient

Separation:
P(A=yes | C=yes, Y=yes) = P(A=yes, C=yes, Y=yes)/P(C=yes, Y=yes) = 0.2/0.25 = 0.8
P(A=yes | C=yes) = P(A=yes, C=yes)/P(C=yes) = 0.3/0.5 = 0.6
Therefore, P(A=yes | C=yes, Y=yes) != P(A=yes | C=yes)

Thus, the joint distribution is not separated