# Question 3: Is your Bayes Network Fair?

## Part A

Example of Sufficiency holds but not Separation

Take $P(A, C, Y)$ for:

$A$ = Gender (gd) [Protected Attribute]
$C$ = Hyperlipidemia (hl) [Classification]
$Y$ = Vegetables (vg) [Ground Truth]

We want to show that:

$P(\texttt{vg}|\texttt{hl}) = P(\texttt{vg}|\texttt{hl}, \texttt{gd})$ for Sufficiency
$P(\texttt{hl}|\texttt{vg}) \neq P(\texttt{hl}|\texttt{vg}, \texttt{gd})$ for Separation

By Table 1 and Table 2: $P(\texttt{vg}|\texttt{hl}) = P(\texttt{vg}|\texttt{hl}, \texttt{gd})$ is true so Sufficiency holds

By Table 3 and Table 4: $P(\texttt{hl}|\texttt{vg}) \neq P(\texttt{hl}|\texttt{vg}, \texttt{gd})$ is true so Separation does not hold

□

## Part B

Example of Separation holds but not Sufficiency

Take $P(A, C, Y)$ for:

$A$ = Gender (gd) [Protected Attribute]
$C$ = Diabetes (db) [Classification]
$Y$ = Hyperlipidemia (hl) [Ground Truth]

We want to show that:

$P(\texttt{db}|\texttt{hl}) = P(\texttt{db}|\texttt{hl}, \texttt{gd})$ for Separation
$P(\texttt{hl}|\texttt{db}) \neq P(\texttt{hl}|\texttt{db}, \texttt{gd})$ for Sufficiency

By Table 5 and Table 6: $P(\texttt{db}|\texttt{hl}) = P(\texttt{db}|\texttt{hl}, \texttt{gd})$ is true so Separation holds

By Table 7 and Table 8: $P(\texttt{hl}|\texttt{db}) \neq P(\texttt{hl}|\texttt{db}, \texttt{gd})$ is true so Sufficiency does not hold

□

## Tables

| | | hl | |
|---|---|---|---|
| | | Yes | No |
| $P(\texttt{vg}|\texttt{hl})$ | <400g/d | 0.579 | 0.283 |
| | 400-500g/d | 0.579 | 0.283 |
| | >500g/d | 0.579 | 0.283 |

Table 1: $P(\texttt{vg}|\texttt{hl})$ : Sufficiency 1A

| $P(\texttt{vg}\|\texttt{hl},\texttt{gd})$ | | hl, gd | | | |
|---|---|---|---|---|---|
| | | YES, Male | NO, Male | YES, Female | NO, Female |
| | <400g/d | 0.579 | 0.283 | 0.579 | 0.283 |
| | 400-500g/d | 0.579 | 0.283 | 0.579 | 0.283 |
| | >500g/d | 0.579 | 0.283 | 0.579 | 0.283 |

Table 2: $P(\texttt{vg}|\texttt{hl},\texttt{gd})$ : Sufficiency 2A

| $P(\texttt{hl}\|\texttt{vg})$ | | vg | | |
|---|---|---|---|---|
| | | <400g/d | 400-500g/d | >500g/d |
| | YES | 0.58213 | 0.37376 | 0.19183 |
| | NO | 0.58213 | 0.37376 | 0.19183 |

Table 3: $P(\texttt{hl}|\texttt{vg})$ : Separation 1A

| $P(\texttt{hl}\|\texttt{vg},\texttt{gd})$ | | vg, gd | | | | | |
|---|---|---|---|---|---|---|---|
| | | <400g/d, Male | 400-500g/d, Male | >500g/d, Male | <400g/d, Female | 400-500g/d, Female | >500g/d, Female |
| | YES | 0.61689 | 0.40824 | 0.21529 | 0.54151 | 0.33600 | 0.16753 |
| | NO | 0.61689 | 0.40824 | 0.21529 | 0.54151 | 0.33600 | 0.16753 |

Table 4: $P(\texttt{hl}|\texttt{vg},\texttt{gd})$ : Separation 2A

| $P(\texttt{db}\|\texttt{hl})$ | | hl | |
|---|---|---|---|
| | | YES | NO |
| | YES | 0.64594 | 0.38504 |
| | NO | 0.64594 | 0.38504 |

Table 5: $P(\texttt{db}|\texttt{hl})$: Separation 1B

| $P(\texttt{db}\|\texttt{hl},\texttt{gd})$ | | hl, gd | | | |
|---|---|---|---|---|---|
| | | YES, Male | NO, Male | YES, Female | NO, Female |
| | YES | 0.64594 | 0.38504 | 0.64594 | 0.38504 |
| | NO | 0.64594 | 0.38504 | 0.64594 | 0.38504 |

Table 6: $P(\texttt{db}|\texttt{hl},\texttt{gd})$: Separation 2B

| $P(\texttt{hl}\|\texttt{db})$ | | db | |
|---|---|---|---|
| | | YES | NO |
| | YES | 0.53321 | 0.28162 |
| | NO | 0.53321 | 0.28162 |

Table 7: $P(\texttt{hl}|\texttt{db})$ : Sufficiency 1B

| $P(\texttt{hl}\|\texttt{db},\texttt{gd})$ | | db, gd | | | |
|---|---|---|---|---|---|
| | | YES, Male | NO, Male | YES, Female | NO, Female |
| | YES | 0.56902 | 0.31183 | 0.49199 | 0.24946 |
| | NO | 0.56902 | 0.31183 | 0.49199 | 0.24946 |

Table 8: $P(\texttt{hl}|\texttt{db},\texttt{gd})$: Sufficiency 2B