Let

- C = {YES, NO} be a classification
- Y = {YES, NO} be a label representing 'ground truth'
- A = {male, female} represents gender

Sufficiency means A is independent of Y given C

Separation means A is independent of C given Y

Example 1: sufficiency holds but not separation

A	C	Y	P(A,C,Y)
male	YES	YES	0.144
male	YES	NO	0.036
male	NO	YES	0.042
male	NO	NO	0.378
female	YES	YES	0.064
female	YES	NO	0.016
female	NO	YES	0.032
female	NO	NO	0.288

We need to show that

- $P(Y|A,C) = P(Y|C)$ for all possible values of A, C, Y
- $P(C|A,Y) \neq P(C|Y)$ for some values of A,C,Y

From the table above we get the following three aggregation tables:

<table>
<tr><td>C</td><td>Y</td><td>P(C,Y)</td><td>P(C)</td></tr>
<tr><td>YES</td><td>YES</td><td>P(Y= YES, C = YES) = 0.144 + 0.064 = 0.208</td><td rowspan="2">P(C = YES) = 0.144 + 0.036 + 0.064 + 0.016 = 0.26</td></tr>
<tr><td>YES</td><td>NO</td><td>P(Y= NO, C = YES) = 0.036 + 0.016 = 0.052</td></tr>
<tr><td>NO</td><td>YES</td><td>P(Y= YES, C = NO) = 0.042 + 0.032 = 0.074</td><td rowspan="2">P(C = NO) = 0.042 + 0.378 + 0.032 + 0.288 = 0.74</td></tr>
<tr><td>NO</td><td>NO</td><td>P(Y= NO, C = NO) = 0.378 + 0.288 = 0.666</td></tr>
</table>

P(Y = YES) = 0.144 + 0.042 + 0.064 + 0.032 = 0.282

P(Y = NO) = 0.036 + 0.378 + 0.016 + 0.288 = 0.718

A	C	P(A,C)
male	YES	P(A = male, C = YES) = 0.144 + 0.036 = 0.18
male	NO	P(A = male, C = NO) = 0.042 + 0.378 = 0.42
female	YES	P(A = female, C = YES) = 0.064 + 0.016 = 0.08
female	NO	P(A = female, C = NO) = 0.032 + 0.288 = 0.32

A	Y	P(A,Y)
male	YES	P(A = male, Y = YES) = 0.144 + 0.042 = 0.186
male	NO	P(A = male, Y = NO) =0.036 + 0.378 = 0.414
female	YES	P(A = female, Y = YES) = 0.064 + 0.032 = 0.096
female	NO	P(A = female, Y = NO) = 0.016 + 0.288 = 0.304

Using data above, we get

A	C	Y	P(Y|A,C)	P(Y|C)	P(C|A,Y)	P(C|Y)
male	YES	YES	$\frac{P(Y=YES,A=male,C=YES))}{P(A=male,C=YES)} = \frac{0.144}{0.18} = 0.8$	$\frac{P(Y=YES,C=YES))}{P(C=YES)} = \frac{0.208}{0.26} = 0.8$	$\frac{P(Y=YES,A=male,C=YES))}{P(A=male,Y=YES)} = \frac{0.144}{0.186} = 0.774$	$\frac{P(C=YES,Y=YES))}{P(Y=YES)} = \frac{0.208}{0.282} = 0.738$
male	YES	NO	$\frac{P(Y=NO,A=male,C=YES))}{P(A=male,C=YES)} = \frac{0.036}{0.18} = 0.2$	$\frac{P(Y=NO,C=YES))}{P(C=YES)} = \frac{0.052}{0.26} = 0.2$	$\frac{P(Y=NO,A=male,C=YES))}{P(A=male,Y=NO)} = \frac{0.036}{0.414} = 0.087$	$\frac{P(C=YES,Y=NO))}{P(Y=NO)} = \frac{0.052}{0.718} = 0.072$
male	NO	YES	$\frac{P(Y=YES,A=male,C=NO))}{P(A=male,C=NO)} = \frac{0.042}{0.42} = 0.1$	$\frac{P(Y=YES,C=NO))}{P(C=NO)} = \frac{0.074}{0.74} = 0.1$	$\frac{P(Y=YES,A=male,C=NO))}{P(A=male,Y=YES)} = \frac{0.042}{0.186} = 0.226$	$\frac{P(C=NO,Y=YES))}{P(Y=YES)} = \frac{0.074}{0.282} = 0.262$
male	NO	NO	$\frac{P(Y=NO,A=male,C=NO))}{P(A=male,C=NO)} = \frac{0.378}{0.42} = 0.9$	$\frac{P(Y=NO,C=NO))}{P(C=NO)} = \frac{0.666}{0.74} = 0.9$	$\frac{P(Y=NO,A=male,C=NO))}{P(A=male,Y=NO)} = \frac{0.378}{0.414} = 0.913$	$\frac{P(C=NO,Y=NO))}{P(Y=NO)} = \frac{0.666}{0.718} = 0.928$
female	YES	YES	$\frac{P(Y=YES,A=female,C=YES))}{P(A=female,C=YES)} = \frac{0.064}{0.08} = 0.8$	$\frac{P(Y=YES,C=YES))}{P(C=YES)} = \frac{0.208}{0.26} = 0.8$	$\frac{P(Y=YES,A=female,C=YES))}{P(A=female,Y=YES)} = \frac{0.064}{0.096} = 0.667$	$\frac{P(C=YES,Y=YES))}{P(Y=YES)} = \frac{0.208}{0.282} = 0.738$
female	YES	NO	$\frac{P(Y=NO,A=female,C=YES))}{P(A=female,C=YES)} = \frac{0.016}{0.08} = 0.2$	$\frac{P(Y=NO,C=YES))}{P(C=YES)} = \frac{0.052}{0.26} = 0.2$	$\frac{P(Y=NO,A=female,C=YES))}{P(A=female,Y=NO)} = \frac{0.016}{0.304} = 0.053$	$\frac{P(C=YES,Y=NO))}{P(Y=NO)} = \frac{0.052}{0.718} = 0.072$
female	NO	YES	$\frac{P(Y=YES,A=female,C=NO))}{P(A=female,C=NO)} = \frac{0.032}{0.32} = 0.1$	$\frac{P(Y=YES,C=NO))}{P(C=NO)} = \frac{0.074}{0.74} = 0.1$	$\frac{P(Y=YES,A=female,C=NO))}{P(A=female,Y=YES)} = \frac{0.032}{0.096} = 0.333$	$\frac{P(C=NO,Y=YES))}{P(Y=YES)} = \frac{0.074}{0.282} = 0.262$
female	NO	NO	$\frac{P(Y=NO,A=female,C=NO))}{P(A=female,C=NO)} = \frac{0.288}{0.32} = 0.9$	$\frac{P(Y=NO,C=NO))}{P(C=NO)} = \frac{0.666}{0.74} = 0.9$	$\frac{P(Y=NO,A=female,C=NO))}{P(A=female,Y=NO)} = \frac{0.288}{0.304} = 0.947$	$\frac{P(C=NO,Y=NO))}{P(Y=NO)} = \frac{0.666}{0.718} = 0.928$

This shows that sufficiency holds but not separation.

Example 2: separation holds but not sufficiency

A	C	Y	P(A,C,Y)
male	YES	YES	0.144
male	YES	NO	0.042
male	NO	YES	0.036
male	NO	NO	0.378
female	YES	YES	0.064
female	YES	NO	0.032
female	NO	YES	0.016
female	NO	NO	0.288

We need to show that

- $P(C|A,Y) = P(C|Y)$ for all possible values of A, C, Y
- $P(Y|A,C) \neq P(Y|C)$ for some values of A,C,Y

From the table above we get the following three aggregation tables:

<table>
<tr><th>C</th><th>Y</th><th>P(C,Y)</th><th>P(C)</th></tr>
<tr><td>YES</td><td>YES</td><td>P(C= YES, Y = YES) = 0.144 + 0.064 = 0.208</td><td rowspan="2">P(C = YES) = 0.144 + 0.042 + 0.064 + 0.032 = 0.282</td></tr>
<tr><td>YES</td><td>NO</td><td>P(C= YES, Y = NO) = 0.042 + 0.032 = 0.074</td></tr>
<tr><td>NO</td><td>YES</td><td>P(C= NO, Y = YES) = 0.036 + 0.016 = 0.052</td><td rowspan="2">P(C = NO) = 0.036 + 0.378 + 0.016 + 0.288 = 0.718</td></tr>
<tr><td>NO</td><td>NO</td><td>P(C= NO, Y = NO) = 0.378 + 0.288 = 0.666</td></tr>
</table>

P(Y = YES) = 0.144 + 0.036 + 0.064 + 0.016 = 0.26

P(Y = NO) = 0.042 + 0.378 + 0.032 + 0.288 = 0.74

A	Y	P(A,Y)
male	YES	P(A = male, Y = YES) = 0.144 + 0.036 = 0.18
male	NO	P(A = male, Y = NO) = 0.042 + 0.378 = 0.42
female	YES	P(A = female, Y = YES) = 0.064 + 0.016 = 0.08
female	NO	P(A = female, Y = NO) = 0.032 + 0.288 = 0.32

A	C	P(A,C)
male	YES	P(A = male, C = YES) = 0.144 + 0.042 = 0.186
male	NO	P(A = male, C = NO) =0.036 + 0.378 = 0.414
female	YES	P(A = female, C = YES) = 0.064 + 0.032 = 0.096
female	NO	P(A = female, C = NO) = 0.016 + 0.288 = 0.304

Using data above, we get

A	C	Y	P(C|A,Y)	P(C|Y)	P(Y|A,C)	P(Y|C)
male	YES	YES	$\frac{P(C=YES,A=male,Y=YES))}{P(A=male,Y=YES)} = \frac{0.144}{0.18} = 0.8$	$\frac{P(C=YES,Y=YES))}{P(Y=YES)} = \frac{0.208}{0.26} = 0.8$	$\frac{P(C=YES,A=male,Y=YES))}{P(A=male,C=YES)} = \frac{0.144}{0.186} = 0.774$	$\frac{P(Y=YES,C=YES))}{P(C=YES)} = \frac{0.208}{0.282} = 0.738$
male	YES	NO	$\frac{P(C=YES,A=male,Y=NO))}{P(A=male,Y=NO)} = \frac{0.042}{0.42} = 0.1$	$\frac{P(C=YES,Y=NO))}{P(Y=NO)} = \frac{0.074}{0.74} = 0.1$	$\frac{P(C=YES,A=male,Y=NO))}{P(A=male,C=YES)} = \frac{0.042}{0.186} = 0.226$	$\frac{P(Y=NO,C=YES))}{P(C=YES)} = \frac{0.074}{0.282} = 0.262$
male	NO	YES	$\frac{P(C=NO,A=male,Y=YES))}{P(A=male,Y=YES)} = \frac{0.036}{0.18} = 0.2$	$\frac{P(C=NO,Y=YES))}{P(Y=YES)} = \frac{0.052}{0.26} = 0.2$	$\frac{P(C=NO,A=male,Y=YES))}{P(A=male,C=NO)} = \frac{0.036}{0.414} = 0.087$	$\frac{P(Y=YES,C=NO))}{P(C=NO)} = \frac{0.052}{0.718} = 0.072$
male	NO	NO	$\frac{P(C=NO,A=male,Y=NO))}{P(A=male,Y=NO)} = \frac{0.378}{0.42} = 0.9$	$\frac{P(C=NO,Y=NO))}{P(Y=NO)} = \frac{0.666}{0.74} = 0.9$	$\frac{P(C=NO,A=male,Y=NO))}{P(A=male,C=NO)} = \frac{0.378}{0.414} = 0.913$	$\frac{P(Y=NO,C=NO))}{P(C=NO)} = \frac{0.666}{0.718} = 0.928$
female	YES	YES	$\frac{P(C=YES,A=female,Y=YES))}{P(A=female,Y=YES)} = \frac{0.064}{0.08} = 0.8$	$\frac{P(C=YES,Y=YES))}{P(Y=YES)} = \frac{0.208}{0.26} = 0.8$	$\frac{P(C=YES,A=female,Y=YES))}{P(A=female,C=YES)} = \frac{0.064}{0.096} = 0.667$	$\frac{P(Y=YES,C=YES))}{P(C=YES)} = \frac{0.208}{0.282} = 0.738$
female	YES	NO	$\frac{P(C=YES,A=female,Y=NO))}{P(A=female,Y=NO)} = \frac{0.032}{0.32} = 0.1$	$\frac{P(C=YES,Y=NO))}{P(Y=NO)} = \frac{0.074}{0.74} = 0.1$	$\frac{P(C=YES,A=female,Y=NO))}{P(A=female,C=YES)} = \frac{0.032}{0.096} = 0.333$	$\frac{P(Y=NO,C=YES))}{P(C=YES)} = \frac{0.074}{0.282} = 0.262$
female	NO	YES	$\frac{P(C=NO,A=female,Y=YES))}{P(A=female,Y=YES)} = \frac{0.016}{0.08} = 0.2$	$\frac{P(C=NO,Y=YES))}{P(Y=YES)} = \frac{0.052}{0.26} = 0.2$	$\frac{P(C=NO,A=female,Y=YES))}{P(A=female,C=NO)} = \frac{0.016}{0.304} = 0.053$	$\frac{P(Y=YES,C=NO))}{P(C=NO)} = \frac{0.052}{0.718} = 0.072$
female	NO	NO	$\frac{P(C=NO,A=female,Y=NO))}{P(A=female,Y=NO)} = \frac{0.288}{0.32} = 0.9$	$\frac{P(C=NO,Y=NO))}{P(Y=NO)} = \frac{0.666}{0.74} = 0.9$	$\frac{P(C=NO,A=female,Y=NO))}{P(A=female,C=NO)} = \frac{0.288}{0.304} = 0.947$	$\frac{P(Y=NO,C=NO))}{P(C=NO)} = \frac{0.666}{0.718} = 0.928$

This shows that separation holds but not sufficiency.