Интегралы часть N 1. 1 Шульман ЭМи16

8.1.29 $\int \frac{dx}{x^2\sqrt{x}} = \int x^{-2}\cdot x^{-\frac{1}{2}}dx = \int x^{-\frac{5}{2}}dx =$

$= \left[\int x^{\alpha}dx = \frac{x^{\alpha+1}}{\alpha+1} + C\right] = \frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + C = \frac{-2}{3x\sqrt{x}} + C$

8.1.30 $\int \frac{dx}{x^2+3} = \left[\int \frac{dx}{x^2+a^2} = \frac{1}{a}\,\mathrm{arctg}\,\frac{x}{a} + C,\ a=\sqrt{3}\right] =$

$= \int \frac{dx}{x^2+(\sqrt{3})^2} = \frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{x}{\sqrt{3}} + C$

8.1.31 $\int \frac{1}{5^x}dx = \int \left(\frac{1}{5}\right)^x dx = \left[\int a^x dx = \frac{a^x}{\ln a} + C,\ a=\frac{1}{5}\right] =$

$= \frac{\left(\frac{1}{5}\right)^x}{\ln\frac{1}{5}} + C = \frac{\frac{1}{5^x}}{\ln 0{,}2} + C = \frac{1}{5^x \ln 0{,}2} + C = \frac{-1}{5^x\ln 5} + C$

8.1.32 $\int \frac{dx}{\sqrt{4-x^2}} = \left[\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C\ \ a=2\right] = \int \frac{dx}{\sqrt{2^2-x^2}} =$

$= \arcsin\frac{x}{2} + C$

8.1.33 $\int \frac{dx}{\sqrt{x^2-1}} = \left[\int \frac{dx}{\sqrt{x^2+a}} = \ln\left|x+\sqrt{x^2+a}\right| + C,\ a=-1\right] =$

$= \ln\left|x+\sqrt{x^2-1}\right| + C$

8.1.34 $\int \frac{dx}{x^2-25} = \left[\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C\ \ a=5\right] = \int \frac{dx}{x^2-5^2} =$

$= \frac{1}{10}\ln\left|\frac{x-5}{x+5}\right| + C$

8.1.35 $\int \left(x+\frac{2}{x}\right)^2 dx = \int \left(x^2 + \frac{4x}{x} + \frac{4}{x^2}\right)dx = \int x^2dx + \int 4dx + \int \frac{4}{x^2}dx =$

$= \int x^2dx + 4\int dx + 4\int \frac{dx}{x^2} = \int x^2dx + 4\int dx + 4\int x^{-2}dx = \frac{x^3}{3} + 4\cdot\frac{x^2}{2} + 4\cdot\frac{x^2}{2} +$

$+ 4\cdot\frac{x^{-1}}{-1} + C = \frac{x^3}{3} + 2x^2 - \frac{4}{x} + C$

8.1.36 $\int \frac{dx}{4x^2+1} = \frac{1}{4}\int \frac{dx}{x^2+4} = \frac{1}{4}\int \frac{dx}{x^2+2} = \frac{1}{4}\cdot\frac{1}{2}\,\mathrm{arctg}\,\frac{x}{2} + C = \frac{1}{8}\,\mathrm{arctg}\,\frac{x}{2} + C$

8.1.37 $\int \left(7^x - \frac{8}{x} + 4\cos x\right)dx = \int 7^x dx - 8\int \frac{dx}{x} + 4\int \cos x\,dx =$

$= \frac{7^x}{\ln 7} - 8\ln|x| + 4\sin x + C$

8.1.38 $\int\left(\frac{\sqrt{3}}{\cos^2 x} - \sqrt[3]{x} - \frac{2}{x^4}\right)dx = \sqrt{3}\int\frac{dx}{\cos^2 x} - \int x^{1/3}dx - 2\int x^{-4}dx =$

$\sqrt{3}\,\mathrm{tg}\,x - \frac{x^{4/3}}{\frac{4}{3}} + \frac{2x^{-3}}{3} + C = \sqrt{3}\,\mathrm{tg}\,x - \frac{3x^{4/3}}{4} + \frac{2}{3x^3} + C$

8.1.39 $\int\frac{\sqrt{x} - 3\sqrt[5]{x^2} + 1}{\sqrt[4]{x}}dx = \int\frac{\sqrt{x}}{\sqrt[4]{x}}dx - \int\frac{3\sqrt[5]{x^2}}{\sqrt[4]{x}}dx + \int dx =$

$= \int x^{1/2}\cdot x^{-\frac{1}{4}}dx - 3\int x^{\frac{2}{5}}\cdot x^{-\frac{1}{4}}dx + \int dx = \int x^{\frac{1}{4}}dx - 3\int x^{\frac{3}{20}}dx + \int dx =$

$= \frac{x^{\frac{5}{4}}}{\frac{5}{4}} - \frac{3x^{\frac{23}{20}}}{\frac{23}{20}} + x + C = \frac{4x^{5/4}}{5} - \frac{60x\sqrt[20]{x^3}}{23} + x + C$

8.1.40 $\int(0.7x^{-0.1} + 0.2(0.5)^x)dx = 0.7\int x^{-0.1}dx + 0.2\int(0.5)^x dx =$

$= 0.7\cdot\frac{x^{0.9}}{0.9} + 0.2\cdot\frac{(0.5)^x}{\ln 0.5} + C = \frac{7x^{0.9}}{9} - \frac{1}{2^x\cdot 5\ln 2} + C$

8.1.41 $\int(5\,\mathrm{sh}\,x - 7\,\mathrm{ch}\,x + 1)dx = 5\int\mathrm{sh}\,x\,dx - 7\int\mathrm{ch}\,x\,dx + \int dx =$

$= 5\,\mathrm{ch}\,x - 7\,\mathrm{sh}\,x + x + C$

8.1.42 $\int(x^2 - 1)(\sqrt{x} + 4)dx = \int(x^2\sqrt{x} - \sqrt{x} + 4x^2 - 4)dx =$

$= \int x^2 x^{\frac{1}{2}}dx - \int x^{1/2}dx + 4\int x^2 dx - 4\int dx = \int x^{\frac{5}{2}}dx - \int x^{\frac{1}{2}}dx + 4\int x^2 dx -$

$- 4\int dx = \frac{x^{\frac{7}{2}}}{\frac{7}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{4x^3}{3} - 4x + C = \frac{2x^{\frac{7}{2}}}{7} - \frac{2x^{3/2}}{3} + \frac{4x^3}{3} - 4x + C$

8.1.43 $\int\frac{7 - \sqrt{x^2 + \pi}}{\sqrt{x^2 + \pi}}dx = \int\frac{7\,dx}{\sqrt{x^2 + \pi}} - \int dx = 7\int\frac{dx}{\sqrt{x^2 + \pi}} - \int dx =$

$= 7\ln|x + \sqrt{x^2 + \pi}| - x + C$

8.1.44 $\int\left(\frac{\sqrt{x} - 5}{x}\right)^3 dx = \int\frac{(\sqrt{x} - 5)^3}{x^3}dx = \int\frac{x^{\frac{3}{2}} - 3x\cdot 5 + 3\sqrt{x}\cdot 25 - 125}{x^3}dx =$

$\int x^{\frac{3}{2}}\cdot x^{-3}dx - \int\frac{15x\,dx}{x^3} + 75\int\frac{x^{\frac{1}{2}}}{x^3}dx - 125\int\frac{dx}{x^3} = \int x^{-\frac{3}{2}}dx - 15\int x^{-2}dx + 75\int x^{-\frac{5}{2}}dx -$

$- 125\int x^{-3}dx = \frac{125}{2x^2} - \frac{50}{x\sqrt{x}} + \frac{15}{x} - \frac{2}{\sqrt{x}} + C$

8.1.45 $\int\sin 7x\,dx = \left[t = 7x \Rightarrow dt = (7x)'_x dx = 7dx \Rightarrow dx = \frac{1}{7}dt\right] =$

$= \int\sin t\,\frac{1}{7}dt = \frac{1}{7}\int\sin t\,dt = -\frac{1}{7}\cos t + C = -\frac{1}{7}\cos 7x + C$

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8.1.45 $\int \sin 7x\,dx = [t = 7x \Rightarrow dt = (7x)'\,dx = 7dx \Rightarrow$
$\Rightarrow dx = \frac{1}{7}dt] = \int \sin t\,\frac{1}{7}dt = \frac{1}{7}\int \sin t\,dt = -\frac{1}{7}\cos t + C =$
$= -\frac{1}{7}\cos 7x + C$

8.1.46 $\int \sqrt[5]{2x-8}\,dx = [t = 2x-8 \Rightarrow dt = (2x-8)'\,dx = 2dx \Rightarrow$
$\Rightarrow dx = \frac{1}{2}dt] = \int \sqrt[5]{t}\,\frac{1}{2}dt = \frac{1}{2}\int t^{1/5}dt = \frac{1}{2}\cdot\frac{t^{\frac{6}{5}}}{\frac{6}{5}} + C = \frac{1}{2}\cdot\frac{5t^{\frac{6}{5}}}{6} + C =$
$= \frac{5(2x-8)^{6/5}}{12} + C$

8.1.47 $\int (1-4x)^{2001}dx = [t = 1-4x \Rightarrow dt = -4dx,\ dx = \frac{-dt}{4}] =$
$= -\int t^{2001}\cdot\frac{1}{4}dt = -\frac{1}{4}\int t^{2001}dt = -\frac{1}{4}\cdot\frac{t^{2002}}{2002} + C = \frac{-(1-4x)^{2002}}{8008} + C$

8.1.48. $\int \frac{dx}{9x+7} = \frac{1}{9}\int \frac{dt}{t} = [t = 9x+7;\ dt = 9dx;\ dx = \frac{1}{9}dt] =$
$= \frac{1}{9}\cdot \ln|t| + C = \frac{1}{9}\ln|9x+7| + C$

8.1.49 $\int \frac{dx}{(6x+11)^4} = [t = 6x+11;\ dt = 6dx;\ dx = \frac{1}{6}dt] =$
$= \frac{1}{6}\int \frac{dt}{t^4} = \frac{1}{6}\int t^{-4}dt = -\frac{1}{6}\cdot\frac{t^{-3}}{3} + C = -\frac{1}{18t^3} + C = \frac{-1}{18(6x+11)^3}$

8.1.50 $\int \frac{dx}{25x^2+1} = \frac{1}{25}\int \frac{dx}{x^2+\frac{1}{25}} = \frac{1}{25}\int \frac{dx}{x^2+(0.2)^2} = \frac{1}{25}\cdot\frac{1}{0.2}\,\mathrm{arctg}\,\frac{x}{0.2} + C =$
$= \frac{1}{5}\,\mathrm{arctg}\,5x + C$

8.1.51 $\int 3^{2-11x}dx = [t = 2-11x;\ dt = -11dx;\ dx = -\frac{1}{11}dt] =$
$= -\frac{1}{11}\int 3^t dt = -\frac{1}{11}\cdot\frac{3^t}{\ln 3} + C = \frac{-3^{2-11x}}{11\ln 3} + C$

8.1.52 $\int \frac{dx}{\sqrt{4x^2-1}} = \frac{1}{16}\int \frac{dx}{\sqrt{x^2-\frac{1}{4}}} = \frac{1}{16}\ln\left|x+\sqrt{x^2-0.25}\right| + C$

8.1.53 $\int \sin^2 3x\,dx = [3x = t;\ dt = 3dx;\ dx = \frac{1}{3}dt] =$
$= \frac{1}{3}\int \sin^2 t\,dt = \frac{1}{3}\int \frac{1-\cos 2t}{2}dt = \frac{1}{3}\cdot\frac{1}{2}\int dt - \frac{1}{3}\cdot\frac{1}{2}\int \cos 2t\,dt =$
$= \frac{1}{6}\int dt - \frac{1}{6}\int \cos 2t\,dt = \frac{1}{6}\int dt - \frac{1}{6}\cdot\frac{1}{2}\int \cos 2t\,d2t = \frac{1}{6}t - \frac{1}{12}\sin 2t + C =$
$= \frac{x}{2} - \frac{1}{12}\sin 6x + C$

$8.1.54 \int \cos^2 8x\,dx = [t=8x \quad dt = 8dx \quad dx = \frac{1}{8}dt] =$ 4

$= \frac{1}{8}\int \cos^2 t\,dt = \frac{1}{8}\int \frac{1+\cos 2t}{2}\,dt = \frac{1}{8}\cdot\frac{1}{2}\int dt + \frac{1}{8}\cdot\frac{1}{2}\int \cos 2t\,dt =$

$= \frac{1}{16}\int dt + \frac{1}{16}\cdot\frac{1}{2}\int \cos 2t\,d2t = \frac{1}{16}t + \frac{1}{32}\sin 2t + C = \frac{x}{2} + \frac{\sin 16x}{32} + C$

$8.1.55 \int tg^2 x\,dx = [\int \frac{1+\cos 2x}{1-\cos 2x}\,dx] = [\cancel{tg^2 x} + 1 = \frac{1}{\cos^2 x} \Rightarrow tg^2 x = \frac{1}{\cos^2 x} - 1] = \int (\frac{1}{\cos^2 x} - 1)\,dx = \int \frac{dx}{\cos^2 x} - \int dx = tg\,x - x + C$

$8.1.56 \int \frac{4x-1}{x-5}\,dx = \int [\frac{dx}{x-5}] + \int \frac{4x}{x-5}\,dx = \ln(x-5) + 4\int \frac{x}{x-5}\,dx =$

$\ln(x-5) + 4\int (1+\frac{5}{x-5})\,dx = \ln(x-5) + 4(x + 5\ln(x-5)) + C = 4x + 21\ln(x-5) + C$

$8.1.60 \int \frac{\sin 2x}{\cos x}\,dx = \int \frac{2\sin x\cos x}{\cos x}\,dx = \int 2\sin x\,dx = 2\int \sin x\,dx =$

$= -2\cos x + C = C - 2\cos x$