Дифференциал. Часть 2. Лагинова Софья

7.2.16 $y = 2^{\cos x}$

$dy = (2^{\cos x})' dx = (-2^{\cos x} \ln 2 \sin x)\, dx$

7.2.17. $y = \ln^3 \sin x$

$dy = (\ln^3 \sin x)' dx = \left(\frac{3\ln^2 \sin x \cdot \cos x}{\sin x}\right) dx = 3\ln^2 \sin x \cdot \operatorname{ctg} x\, dx$

7.2.18 $f(x) = \sqrt[3]{x^5 - 1}$

$df(x) = (\sqrt[3]{x^5 - 1})' dx = \left(\frac{1}{3}(x^5 - 1)^{-2/3} \cdot 5x^4\right) dx = \frac{5x^4}{3\sqrt[3]{(x^5-1)^2}}\, dx$

7.2.19 $S(t) = \frac{\sqrt{t}}{t-1}$

$dS(t) = \left(\frac{\sqrt{t}}{t-1}\right)' dt = \left(\frac{\frac{(t-1)}{2\sqrt{t}} - \sqrt{t}}{(t-1)^2}\right) dt = \frac{(t-1) - 2t}{\frac{2\sqrt{t}}{(t-1)^2}}\, dt =$

$= \frac{t - 1 - 2t}{2\sqrt{t}(t-1)^2}\, dt = -\frac{1+t}{2\sqrt{t}(t-1)^2}\, dt$

7.2.20. $y = 4x^2 + 1$, $x_0 = 1$, $\Delta x = 0{,}02$

$\Delta y = (4 \cdot (x + \Delta x)^2 + 1 - (4x^2 + 1)) =$ ~~$4(1 + 0{,}02)^2 + 1 =$~~

~~$4 \cdot 1^2 + 1 = 4(1{,}02)^2 + 1 = 5 =$~~ $4(x^2 + 2x \cdot \Delta x + (\Delta x)^2) + 1 -$

$- 4x^2 - 1 = 4x^2 + 8x \cdot \Delta x + (\Delta x)^2 - 4x^2 = 8x \cdot \Delta x + (\Delta x)^2$

$dy(x_0 = 1, \Delta x = 0{,}02) = 8 \cdot (1 \cdot 0{,}02) + (0{,}02)^2 = 0{,}16 + 0{,}0004 =$

$= 0{,}1604.$

7.2.21 $y = |x|$, $x_0 = 10$, $\Delta x = -0{,}1$

$\Delta y = (|\Delta x + x| - |x|)$

$dy(x_0 = 10,\ \Delta x = -0{,}1) = |-0{,}1 + 10| - |10| = 9{,}9 - 10 = -0{,}1$

7.2.22 $\sin 29^\circ$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\,\Delta x$

$f(x) = \sin(x)$

$f(x_0 + \Delta x) = \sin(29^\circ) = \sin(30^\circ - 1) \Rightarrow [x_0 = 30,\ \Delta x = 1] \Rightarrow$

$\Rightarrow f(30^\circ) + f'(x_0) \cdot 1 = \sin 30^\circ + \frac{1}{} \cos 30^\circ = \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$

$\sin 29^\circ \approx \frac{1-\sqrt{3}}{2}$

7.2.23 $\operatorname{arctg}(1{,}05)$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0) \cdot \Delta x$

$f(x) = \operatorname{arctg} x$

$f(x_0 + \Delta x) = \operatorname{arctg}(1{,}05) = \operatorname{arctg}(1 + 0{,}05) \Rightarrow [x_0 = 1,\ \Delta x = 0{,}05] \Rightarrow$

$\Rightarrow f(1) + f'(x_0) \cdot 0{,}05 = \operatorname{arctg} 1 + \frac{1}{1+1^2} \cdot 0{,}05 = \frac{\pi}{4} + \frac{1}{2} \cdot \frac{5}{100} =$

$= \frac{\pi}{4} + \frac{5}{200} = \frac{\pi}{4} + \frac{1}{40} = \frac{1}{4}(\pi + 0{,}1)$

$\operatorname{arctg}(1{,}05) \approx 0{,}25(\pi + 0{,}1)$

7.2.24 $(0{,}99)^4$

(2) $f(x_0 + \Delta x) \approx f(x_0) + f'(x_0) \cdot \Delta x$

$x_0 = 1$, $\Delta x = -0{,}01$

$f(x_0 + \Delta x) = 1^4 + (1^4)'(-0{,}01) = 1 - 0{,}04 = 0{,}96$

7.2.25. $y = \frac{x-1}{x+1}$, dy, d^2y - ?

1. $y' = \left(\frac{x-1}{x+1}\right)' = \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2}$

$dy = y'dx = \frac{2dx}{(x+1)^2}$

2. $y'' = \left(\frac{2}{(x+1)^2}\right)' = -\frac{2((x+1)^2)'}{(x+1)^4} = -\frac{4}{(x+1)^3}$

$d^2y = -\frac{4}{(x+1)^3}dx^2$

7.2.26. $y = x(\ln x - 1)$, dy, d^2y - ?

$y' = (x(\ln x - 1))' = x'(\ln x - 1) + x(\ln x - 1)' = \ln x - 1 + 1 = \ln x$

$dy = (\ln x)dx$

$y'' = (\ln x)' = \frac{1}{x}$

$d^2y = \frac{dx^2}{x}$

7.2.27 $y = x^n$, dy, d^2y, d^3y - ?

$y' = (x^n)' = nx^{n-1}$ $\quad dy = nx^{n-1}dx$

$d^2y = (y')'dx^2 = n(n-1)x^{n-2}dx^2$

$d^3y = (y'')'dx^3 = n(n-1)(n-2)x^{n-3}dx^3$