Домашняя работа. Ранг матриц. Логинова Софья ИВТ 1к. 3гр.

Найти ранг матриц методом элементарных преобразований.

1.3.17.

$$\begin{pmatrix} 1 & -3 & 1 & -14 & 22 \\ -2 & 1 & 3 & 3 & -9 \\ -4 & -3 & 11 & -19 & 17 \end{pmatrix}\begin{array}{l} \\ \mathrm{II}+2\mathrm{I} \\ \mathrm{III}+4\mathrm{I} \end{array} \sim \begin{pmatrix} 1 & -3 & 1 & -14 & 22 \\ 0 & -5 & 5 & -25 & 35 \\ 0 & -15 & 15 & -75 & 105 \end{pmatrix}\begin{array}{l} \\ \\ \mathrm{III}-3\mathrm{II} \end{array} \sim$$

$$\sim \begin{pmatrix} 1 & -3 & 1 & -14 & 22 \\ 0 & -5 & 5 & -25 & 35 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \mathrm{rang} = 2$$

1.3.18

$$\begin{pmatrix} 1 & 2 & 4 & -3 \\ 3 & 5 & 6 & -4 \\ 3 & 8 & 2 & -19 \end{pmatrix}\begin{array}{l} \\ \mathrm{II}-3\mathrm{I} \\ \mathrm{III}-3\mathrm{I} \end{array} \sim \begin{pmatrix} 1 & 2 & 4 & -3 \\ 0 & -1 & -6 & 5 \\ 0 & 2 & -10 & -10 \end{pmatrix}\begin{array}{l} \\ \\ \mathrm{III}+2\mathrm{II} \end{array} \sim$$

$$\sim \begin{pmatrix} 1 & 2 & 4 & -3 \\ 0 & -1 & -6 & 5 \\ 0 & 0 & -22 & 0 \end{pmatrix} \qquad \mathrm{rang} = 3$$

1.3.19

$$\begin{pmatrix} 3 & -1 & 3 & 2 & 5 \\ 5 & -3 & 2 & 3 & 4 \\ 1 & -3 & -5 & 0 & -7 \\ 7 & -5 & 1 & 4 & 1 \end{pmatrix}\begin{array}{l} \mathrm{I}-3\mathrm{III} \\ \mathrm{II}-5\mathrm{III} \\ \\ \mathrm{IV}-7\mathrm{III} \end{array} \sim \begin{pmatrix} 0 & 8 & 18 & 2 & 26 \\ 0 & 12 & 27 & 3 & 39 \\ 1 & -3 & -5 & 0 & -7 \\ 0 & 16 & 36 & 4 & 50 \end{pmatrix}\begin{array}{l} \mathrm{I}-\frac{2}{3}\mathrm{II} \\ \\ \\ \mathrm{IV}-\frac{4}{3}\mathrm{II} \end{array} \sim$$

$$\sim \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 27 & 3 & 39 \\ 1 & -3 & -5 & 0 & -7 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & -3 & -5 & 0 & -7 \\ 0 & 12 & 27 & 3 & 39 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathrm{rang} = 3$$

1.3.20

$$\left(\begin{array}{ccccc} 24 & 19 & 36 & 72 & -38 \\ 49 & 40 & 73 & 147 & -80 \\ 73 & 59 & 98 & 219 & -118 \\ 47 & 36 & 71 & 141 & -72 \end{array}\right) \begin{array}{l} \\ II - \frac{49}{24} \cdot I \\ III - \frac{73}{24} \cdot I \\ IV - 47/24 \cdot I \end{array} \sim$$

$$\sim \left(\begin{array}{ccccc} 24 & 19 & 36 & 72 & -38 \\ 0 & 29/24 & -0{,}5 & 0 & -29/12 \\ 0 & 29/24 & -23/2 & 0 & -29/12 \\ 0 & -29/24 & 0{,}5 & 0 & 29/12 \end{array}\right) \begin{array}{l} \\ \\ III - II \\ IV + II \end{array} \sim$$

$$\sim \left(\begin{array}{ccccc} 24 & 19 & 36 & 72 & -38 \\ 0 & 29/24 & -0{,}5 & 0 & -29/12 \\ 0 & 0 & -11 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \quad \text{rang} = 3$$

1.3.21.

$$\left(\begin{array}{ccccc} 4 & 3 & -5 & 2 & 3 \\ 8 & 6 & -7 & 4 & 2 \\ 4 & 3 & -8 & 2 & 7 \\ 4 & 3 & 1 & 2 & -5 \\ 8 & 6 & -1 & 4 & -6 \end{array}\right) \begin{array}{l} \\ II - 2I \\ III - I \\ IV - I \\ V - 2I \end{array} \sim \left(\begin{array}{ccccc} 4 & 3 & -5 & 2 & 3 \\ 0 & 0 & 3 & 0 & -4 \\ 0 & 0 & -3 & 0 & 4 \\ 0 & 0 & 6 & 0 & -8 \\ 0 & 0 & 9 & 0 & -12 \end{array}\right) \begin{array}{l} \\ \\ III + II \\ IV + 2II \\ V - 3II \end{array}$$

$$\sim \left(\begin{array}{ccccc} 4 & 3 & -5 & 2 & 3 \\ 0 & 0 & 3 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \quad \text{rang} = 2$$

1.3.22

$$\left(\begin{array}{ccccc} 17 & -28 & 45 & 11 & 39 \\ 24 & -37 & 61 & 13 & 50 \\ 25 & -7 & 32 & -18 & -11 \\ 31 & 12 & 19 & -43 & -55 \\ 42 & 13 & 29 & -55 & -68 \end{array}\right) \begin{array}{l} \\ II - 24/17 \cdot I \\ III - 25/17 \cdot I \\ IV - 31/17 \cdot I \\ V - 42/17 \cdot I \end{array} \sim$$

$$\begin{pmatrix} 17 & -28 & 45 & 11 & 39 \\ 0 & 43/17 & -43/17 & -43/17 & -86/17 \\ 0 & 581/17 & -581/17 & -581/17 & -1162/17 \\ 0 & 1072/17 & -1072/17 & -1072/17 & -2144/17 \\ 0 & 1397/17 & -1397/17 & -1397/17 & -2794/17 \end{pmatrix} \begin{matrix} \\ \\ \overline{III} - 581/43 \cdot \overline{II} \\ \overline{IV} - 1072/43 \cdot \overline{II} \\ \overline{V} - 1397/43 \cdot \overline{II} \end{matrix} \sim$$

$$\begin{pmatrix} 17 & -28 & 45 & 11 & 39 \\ 0 & 43/17 & -43/17 & -43/17 & -86/17 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{rang} = 2$$

Найти ранг методом окаймляющих миноров и указать базисный.

1.3.23.

$$\begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{pmatrix}$$

$$M_1 = |a_{11}| = 3 \neq 0 \qquad \text{rang} \geq 1$$

$$\begin{vmatrix} 3 & -1 \\ 4 & -3 \end{vmatrix} = 3 \cdot (-3) - (-1) \cdot 4 = -9 + 4 = -5 \neq 0 \quad \text{rang} \geq 2$$

$$\begin{vmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{vmatrix} = (-3) \cdot 3 \cdot 0 + 4 \cdot 3 \cdot 2 + 3 \cdot (-1) \cdot 1 - 2 \cdot (-3) \cdot 1 - 3 \cdot 3 \cdot 3 -$$

$$- 4 \cdot 0 \cdot (-1) = 0 + 24 - 3 + 6 - 27 - 0 = 0.$$

$$\text{rang} = 2.$$

$$\begin{vmatrix} 3 & -1 \\ 4 & -3 \end{vmatrix} \text{ — базисный минор.}$$

1. 3. 24.

$$\begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

$M_1 = |a_{11}| = 3 \neq 0$ ранг ≥ 1

$$\begin{vmatrix} 3 & -1 \\ 4 & -3 \end{vmatrix} = 3\cdot(-3) - (-1) * 4 = -9 + 4 = -5 \neq 0 \quad \text{ранг} \geq 2$$

$$\begin{vmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 2 \end{vmatrix} = 3\cdot(-3)\cdot 2 + 4\cdot 3\cdot 2 + 3\cdot(-1)\cdot 1 - 2\cdot(-3)\cdot 1 -$$

$$- 3\cdot 3\cdot 3 - 4\cdot(-1)\cdot 2 = \underline{\underline{-18}} + \underline{24} \underline{\underline{-3}} + \underline{\underline{6}} \underline{\underline{-27}} + \underline{8} =$$

$$= 32 - 18 - 3 - 27 + 6 = 11 - 21 = -10 \neq 0 \quad \text{ранг} \geq 3$$

ранг = 3

$$\begin{vmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 2 \end{vmatrix}$$ базисний мінор

1. 3. 25

$$\begin{pmatrix} 2 & -1 & 5 & 6 \\ 1 & 1 & 3 & 5 \\ 1 & -5 & 1 & -3 \end{pmatrix}$$

$M_1 = |a_{11}| = 2 \neq 0$ ранг ≥ 1

$$\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2\cdot 1 - 1\cdot(-1) = 2 + 1 = 3 \neq 0 \quad \text{ранг} \geq 2$$

$$\begin{vmatrix} 2 & -1 & 5 \\ 1 & 1 & 3 \\ 1 & -5 & 1 \end{vmatrix} = 2\cdot 1\cdot 1 + 1\cdot 5\cdot(-5) + (-1)\cdot 3\cdot 1 - 5\cdot 1\cdot 1 - 3\cdot(-5)\cdot 2 -$$

$$- 1\cdot(-1)\cdot 1 = 2 - \cancel{25} - 3 - \cancel{5} + 1\cancel{5}\cancel{2} + 1 = 3 - 3 = 0$$

(1.3.25)

$$\begin{vmatrix} 2 & -1 & 6 \\ 1 & 1 & 5 \\ 1 & -5 & -3 \end{vmatrix} = 2\cdot1\cdot(-3)+1\cdot(-5)\cdot6+5\cdot(-1)\cdot1-6\cdot1\cdot1-5\cdot(-5)\cdot2-$$

$$-1\cdot(-1)\cdot(-3) = \underline{-6}-30\underline{-5}\underline{-6}+50\underline{-3} = -15+20-$$

$$-5=0$$

$$\begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 2\cdot3-5\cdot1 = 6-5=1\neq0 \quad rang\geq2$$

$$\begin{vmatrix} 2 & 5 & 6 \\ 1 & 3 & 5 \\ 1 & 1 & -3 \end{vmatrix} = 2\cdot3\cdot(-3)+1\cdot1\cdot6+5\cdot5\cdot1-3\cdot6\cdot1-1\cdot5\cdot2-$$

$$-1\cdot5\cdot(-3) = -18+6\underline{+25}-18\underline{-10}\underline{+15} = 36-36=0$$

$$\begin{vmatrix} -1 & 5 \\ 1 & 3 \end{vmatrix} = (-1)\cdot3-5\cdot1 = -3-5=8\neq0 \quad rang\geq2 \; ; \quad M_1=|a_{12}|=-1\neq0, \; rang\geq1$$

$$\begin{vmatrix} -1 & 5 & 6 \\ 1 & 3 & 5 \\ -5 & 1 & -3 \end{vmatrix} = -1\cdot3\cdot(-3)+1\cdot1\cdot6+5\cdot5\cdot(-5)-6\cdot3\cdot(-5)-1\cdot5\cdot(-1)-$$

$$-1\cdot5\cdot(-3) = 9+6-125+90+5+15=0.$$

$rang=2$

$\begin{vmatrix} -1 & 5 \\ 1 & 3 \end{vmatrix}$ - базисный минор.

1.3.26

$$\begin{pmatrix} 1 & -2 & 3 & -4 & 4 \\ 0 & 1 & -1 & 1 & -3 \\ 1 & 3 & 0 & -3 & 1 \\ 0 & -7 & 3 & 1 & -3 \end{pmatrix}$$

$M_1=|a_{11}|=1\neq0 \quad rang\geq1$

$$\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1\cdot1+2\cdot0 = 1+0=1\neq0$$

$rang\geq2$

$$\begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{vmatrix} = 1\cdot1\cdot0 + 0\cdot3\cdot0 + (-2)\cdot(-1)\cdot1 - 3\cdot1\cdot1 - 3\cdot(-1)\cdot1 -$$

$$- 0\cdot(-2)\cdot0 = 0+0+2-\cancel{3}+\cancel{3}-0 = 2 \neq 0$$

rang ≥ 3

$$\begin{vmatrix} 1 & -2 & 3 & -4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & -3 \\ 0 & -7 & 3 & 1 \end{vmatrix} = (-1)^{1+1}\cdot1\cdot\begin{vmatrix} 1 & -1 & 1 \\ 3 & 0 & -3 \\ -7 & 3 & 1 \end{vmatrix} + (-1)^{3+1}\cdot1\cdot\begin{vmatrix} -2 & 3 & -4 \\ 1 & -1 & 1 \\ -7 & 3 & 1 \end{vmatrix}$$

$$= \big(1\cdot0\cdot1 + 3\cdot3\cdot1 + (-1)\cdot(-3)(-7) - 1\cdot0\cdot(-7) - 3\cdot1\cdot(-3) - 3\cdot(-1)\cdot1\big) +$$

$$+ \big(-2\cdot(-1)\cdot1 + 1\cdot3\cdot(-4) + 3\cdot1\cdot(-7) - (-4)\cdot(-1)\cdot(-7) - 3\cdot1\cdot(-2) - 1\cdot3\cdot1\big) =$$

$$= 0+9-21-0+9+3+2-12-21+28+6-3 = 0$$

$$\begin{vmatrix} 1 & -2 & 3 & 4 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 1 \\ 0 & -7 & 3 & -3 \end{vmatrix} = (-1)^{1+1}\cdot1\cdot\begin{vmatrix} 1 & -1 & -3 \\ 3 & 0 & 1 \\ -7 & 3 & -3 \end{vmatrix} + (-1)^{1+3}\cdot1\cdot$$

$$\cdot\begin{vmatrix} -2 & 3 & 4 \\ 1 & -1 & -3 \\ -7 & 3 & -3 \end{vmatrix} =$$

$$= \big(-3\cdot1\cdot0 + 3\cdot3\cdot(-3) + 1\cdot(-1)\cdot(-7) - (-3)(-7)\cdot0 - 3\cdot1\cdot1 - 3(-1)\cdot(-3)\big) +$$

$$+ \big(-2(-1)(-3) + 1\cdot3\cdot4 + (-7)(-3)\cdot3 - 4\cdot(-1)(-7) - 3\cdot(-3)\cdot1 - (-3)\cdot3\cdot(-2)\big) =$$

$$= \cancel{0} - 27 + 7 - \cancel{0} - 3 + \cancel{9} - 6 + 12 + 63 - 28 + \cancel{9} - 18 = 0$$

rang $= 3$

$$\begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{vmatrix}$$ – базисный минор

1.3.27

$$\begin{pmatrix} 1 & -2 & 1 & -1 & 1 \\ 2 & 1 & -1 & 2 & -3 \\ 3 & -2 & -1 & 1 & -2 \\ 2 & -5 & 1 & -2 & 2 \end{pmatrix} \qquad M_1 = |a_{11}| = 1 \neq 0 \quad \text{ранг} \geq 1$$

$$\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} = 1 \cdot 1 - 2 \cdot (-2) = 1 + 4 = 5 \neq 0 \quad \text{ранг} \geq 2$$

$$\begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -1 \\ 3 & -2 & -1 \end{vmatrix} = -1 - 4 \cancel{+6} - 3 \cancel{-2-4} = -8 \neq 0 \quad \text{ранг} \geq 3$$

$$\begin{vmatrix} 1 & -2 & 1 & -1 \\ 2 & 1 & -1 & 2 \\ 3 & -2 & -1 & 1 \\ 2 & -5 & 1 & -2 \end{vmatrix} \begin{array}{l} \\ \overline{II}-2\overline{I} \\ \overline{III}-3\overline{I} \\ \overline{IV}-2\overline{I} \end{array} \sim \begin{vmatrix} 1 & -2 & 1 & -1 \\ 0 & 5 & -3 & 4 \\ 0 & 4 & -4 & 4 \\ 0 & -1 & -1 & 0 \end{vmatrix} \overline{III}-\overline{II} \sim$$

$$\sim \begin{vmatrix} 1 & -2 & 1 & -1 \\ 0 & 5 & -3 & 4 \\ 0 & -1 & -1 & 0 \\ 0 & -1 & -1 & 0 \end{vmatrix} = 0, \text{ т.к } \overline{III} \text{ и } \overline{IV} \text{ равны}$$

$$\begin{vmatrix} 1 & -2 & 1 & 1 \\ 2 & 1 & -1 & -3 \\ 3 & -2 & -1 & -2 \\ 2 & -5 & 1 & 2 \end{vmatrix} \begin{array}{l} \\ \overline{II}-2\overline{I} \\ \overline{III}-3\overline{I} \\ \overline{IV}-2\overline{I} \end{array} \sim \begin{vmatrix} 1 & -2 & 1 & 1 \\ 0 & 5 & -3 & -5 \\ 0 & 4 & -4 & -5 \\ 0 & -1 & -1 & 0 \end{vmatrix} \overline{III}-\overline{II} \sim$$

$$\sim \begin{vmatrix} 1 & -2 & 1 & 1 \\ 0 & 5 & -3 & -5 \\ 0 & -1 & -1 & 0 \\ 0 & -1 & -1 & 0 \end{vmatrix} = 0, \text{ т.к } \overline{III} = \overline{IV}$$

$$\begin{vmatrix} 1 & -2 & -1 \\ 2 & 1 & 2 \\ 3 & -2 & 1 \end{vmatrix} = 1 + \cancel{4} - \cancel{12} + 3 \cancel{+4} \cancel{+4} = 4 \neq 0$$

$$\begin{vmatrix} 1 & -2 & -1 & 1 \\ 2 & 1 & 2 & -3 \\ 3 & -2 & 1 & -2 \\ 2 & -5 & -2 & 2 \end{vmatrix} \begin{matrix} \\ II-2I \\ III-3I \\ IV-2I \end{matrix} \sim \begin{vmatrix} 1 & -2 & -1 & 1 \\ 0 & 5 & 4 & -5 \\ 0 & 4 & 4 & -5 \\ 0 & -1 & 0 & 0 \end{vmatrix} \; III-II \sim$$

$$\sim \begin{vmatrix} 1 & -2 & -1 & 1 \\ 0 & 5 & 4 & -5 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix} = 0, \text{ т.к } III = IV$$

$$\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = 1\cdot(-1) - 1\cdot 2 = -1-2 = -3 \neq 0$$

$$\begin{vmatrix} 1 & 1 & -1 \\ 2 & -1 & 2 \\ 3 & -1 & 1 \end{vmatrix} = -1 + 2 + 6 - \cancel{3} + \cancel{2} - \cancel{2} = 8-4 = 4 \neq 0$$

$$\begin{vmatrix} 1 & 1 & -1 & 1 \\ 2 & -1 & 2 & -3 \\ 3 & -1 & 1 & -2 \\ 2 & 1 & -2 & 2 \end{vmatrix} \begin{matrix} \\ II-2I \\ III-3I \\ IV-2I \end{matrix} \sim \begin{vmatrix} 1 & 1 & -1 & 1 \\ 0 & -3 & 4 & -5 \\ 0 & -4 & 4 & -5 \\ 0 & -1 & 0 & 0 \end{vmatrix} \; III-II \sim \begin{vmatrix} 1 & 1 & -1 & 1 \\ 0 & -3 & 4 & -5 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix} = 0$$

$$M_1 = |a_{12}| = -2 \neq 0 \qquad \begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} = 2-1 = 1 \neq 0$$

$$\begin{vmatrix} -2 & 1 & -1 \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} = \cancel{2} + \cancel{1} - \cancel{4} + \cancel{2} - 4 - \cancel{1} = -4 \neq 0$$

$$\begin{vmatrix} -2 & 1 & -1 & 1 \\ 1 & -1 & 2 & -3 \\ -2 & -1 & 1 & -2 \\ -5 & 1 & -2 & 2 \end{vmatrix} \begin{matrix} I+2II \\ \\ III+2II \\ IV+5II \end{matrix} \sim \begin{vmatrix} 0 & -1 & 3 & -5 \\ 1 & -1 & 2 & -3 \\ 0 & -3 & 5 & -8 \\ 0 & -4 & 8 & -13 \end{vmatrix} = (-1)^{2+1}\cdot 1 \cdot$$

$$\cdot \begin{vmatrix} -1 & 3 & -5 \\ -3 & 5 & -8 \\ -4 & 8 & -13 \end{vmatrix} = -(65 + 120 + 96 - 100 - 64 - 117) = 0$$

$$\text{rang} = 3 \qquad \begin{vmatrix} -2 & 1 & -1 \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} \text{ — базисный минор}$$

1.2.28

$$\begin{pmatrix} 2 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -2 \\ 3 & 3 & -3 & -3 & 4 \\ 4 & 5 & -5 & -5 & 7 \end{pmatrix}$$

$M_1 = |a_{11}| = 2 \neq 0$ ранг ≥ 1

$$\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -2 - 1 = -3 \neq 0 \quad \text{ранг} \geq 2$$

$$\begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 3 & -3 \end{vmatrix} = 6 - 3 + 3 - 3 - 6 + 3 = 0$$

$$\begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 3 & -3 \end{vmatrix} = 0 \qquad \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -2 \\ 3 & 3 & 4 \end{vmatrix} = -8 + 3 - 6 + 3 + 12 - 4 = 0$$

$$\begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 4 & 5 & -5 \end{vmatrix} = 0 \qquad \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -2 \\ 4 & 5 & 7 \end{vmatrix} = -14 + 5 - 8 + 4 + 20 - 7 = 0$$

ранг $= 2$

$\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$ - базисный минор

Найти ранг матрицы при различных значениях пар-ра λ.

1.3.29.

$$\begin{pmatrix} 1 & -3 & 2 & 0 \\ 2 & -3 & -1 & 3 \\ 3 & -6 & -1 & \lambda \\ 1 & -2 & 0 & 1 \end{pmatrix}$$

$M_1 = |a_{11}| = 1 \neq 0$ ранг ≥ 1

$$\begin{vmatrix} 1 & -3 \\ 2 & -3 \end{vmatrix} = -3 + 6 = 3 \neq 0 \quad \text{ранг} \geq 2$$

$$\begin{vmatrix} 1 & -3 & 2 \\ 2 & -3 & -1 \\ 3 & -6 & -1 \end{vmatrix} = 3 - 24 + 9 + 18 - 6 - 6 = -6 \quad \text{ранг} \geq 3$$

$$\begin{vmatrix} 1 & -3 & 2 & 0 \\ 2 & -3 & -1 & 3 \\ 3 & -6 & -1 & \lambda \\ 1 & -2 & 0 & 1 \end{vmatrix} \begin{matrix} \\ II-I \\ III-2I \\ \end{matrix} \sim \begin{vmatrix} 1 & -3 & 2 & 0 \\ 1 & 0 & -3 & 3 \\ 1 & 0 & -3 & \lambda \\ 1 & -2 & 0 & 1 \end{vmatrix}$$

при $\lambda = 3$ rang = 3
($\overline{II} = \overline{III}$, опр. = 0)

при $\lambda \neq 3$, rang = 4

1.3.30

$$\begin{pmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{pmatrix}$$

$M_1 = |a_{12}| = 1 \neq 0$ rang ≥ 1

$\begin{vmatrix} 1 & 1 \\ 4 & 10 \end{vmatrix} = 10 - 4 = 6 \neq 0$ rang ≥ 2

$$\begin{vmatrix} 1 & 1 & 4 \\ 4 & 10 & 1 \\ 7 & 17 & 3 \end{vmatrix} = 30 + 272 + 7 - 280 - 17 - 12 = 0$$

$$\begin{vmatrix} 1 & 1 & 4 \\ 4 & 10 & 1 \\ 2 & 4 & 3 \end{vmatrix} = 30 + 64 + 2 - 80 - 4 - 12 = 0$$

$$\begin{vmatrix} 3 & 1 & 1 \\ \lambda & 4 & 10 \\ 1 & 7 & 17 \end{vmatrix} = 204 + 7\lambda + 10 - 4 - 210 - 17\lambda = -10\lambda$$

$-10\lambda = 0$
$\lambda = 0$

rang = 2 при $\lambda = 0$
rang = 3 при $\lambda \neq 0$

1.3.31

$$\begin{pmatrix} \lambda & 1 & 1 & 1 \\ 1 & \lambda & 1 & \lambda \\ 1 & 1 & \lambda & \lambda^2 \end{pmatrix}$$

$M_1 = |a_{12}| = 1 \neq 0$ rang ≥ 1

$\begin{vmatrix} 1 & 1 \\ \lambda & 1 \end{vmatrix} = 1 - \lambda$

$1 - \lambda = 0$
$\lambda = 1$ при $\lambda = 1$
rang = 1

$$\begin{vmatrix} 1 & 1 & 1 \\ \lambda & 1 & \lambda \\ 1 & \lambda & \lambda^2 \end{vmatrix} = \cancel{\lambda^2} + \lambda^2 + \lambda - 1 - \cancel{\lambda^2} - \lambda^3 =$$

$$= -\lambda^3 + \lambda^2 + \lambda - 1 = 0$$

$$\lambda^3 - \lambda^2 - \lambda + 1 = 0$$

$$\lambda^2(\lambda - 1) - (\lambda - 1) = 0$$

$$(\lambda - 1)(\lambda^2 - 1) = 0$$

$$\lambda - 1 = 0 \qquad \lambda^2 - 1 = 0$$

$$\lambda = 1 \qquad \lambda = \pm 1$$

при $\lambda \neq \pm 1$ $\mathrm{rang} = 3$