Интеграл часть II/5 (1)

8.5.19 $\int \frac{dx}{\cos x} = \left[t = tg\frac{x}{2};\ dx = \frac{2dt}{1+t^2};\ \cos x = \frac{1-t^2}{1+t^2}\right] =$

$= \int \frac{1+t^2}{1-t^2} \cdot \frac{2dt}{1+t^2} = 2\int \frac{dt}{1-t^2} = -2\int \frac{dt}{t^2-1} = -2\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + C = \ln\left|\frac{tg\frac{x}{2}+1}{tg\frac{x}{2}-1}\right| + C$

8.5.20 $\int \frac{dx}{1-\sin x} = \left[t = tg\frac{x}{2};\ \ldots d(1+t^2) = t\,dx;\ 1-\sin x = \frac{1+t^2-2t}{1+t^2} = \ldots\right]$

$= \left[\frac{(t-1)^2}{1+t^2}\right] = \int \frac{1+t^2}{(t-1)^2}\cdot\frac{2dt}{1+t^2} = 2\int \frac{dt}{(t-1)^2} = \left[t-1=n \Rightarrow dt = dn\right] = 2\int \frac{dn}{n^2} =$

$= 2\int n^{-2}dn = 2\cdot\frac{n^{-1}}{-1} + C = \frac{-2}{n} + C = \frac{-2}{t-1} + C = \frac{-2}{tg\frac{x}{2}-1} + C$

8.5.23 $\int \frac{dx}{3\sin x + 4\cos x + 5} = \left[dx = \frac{2dt}{1+t^2};\ \sin x = \frac{2t}{1+t^2};\ \cos x = \frac{1-t^2}{1+t^2}\right] =$

$= \int \frac{2dt}{1+t^2}\left(\frac{6t - 1 + t^2 + 5 + 5t^2}{1+t^2}\right) = 2\int \frac{dt}{6t^2+6t+4} = \int \frac{dt}{3t^2+3t+2} = \int \frac{dt}{1{,}5(t+\frac{1}{2})^2 + \frac{3}{4}\cdot 2} =$

$= \left[\sqrt{3}\,t + \frac{1}{\sqrt{3}} = n \Rightarrow dn = \sqrt{3}\,dt = \ldots dt = \frac{dn}{\sqrt{3}}\right] = \frac{1}{\sqrt{3}}\int \frac{dn}{n^2 + (\frac{\sqrt{15}}{3}x)^2} =$

$= \frac{1}{\sqrt{3}}\cdot\frac{3}{\sqrt{15}}\,arctg\frac{3n}{\sqrt{15}} + C = \frac{3}{\sqrt{45}\cdot\sqrt{3}}\,arctg\frac{3\sqrt{15}\,t+3}{\sqrt{15}} + C = \frac{3}{\sqrt{51}}\,arctg\frac{3t+1}{\sqrt{5}} + C =$

$= \frac{3}{\sqrt{51}}\,arctg\frac{3tg\frac{x}{2}+3}{\sqrt{51}} + C$

8.5.26 $\int \frac{dx}{4\sin^2 x + 5\cos^2 x} = \left[tg x = t;\ dx = \frac{dt}{1+t^2};\ \sin x = \frac{t}{\sqrt{1+t^2}};\ \cos x = \frac{1}{\sqrt{1+t^2}}\right] =$

$= \int \frac{dt}{1+t^2}\cdot\left(\frac{4t^2+5}{1+t^2}\right) = \int \frac{dt}{4t^2+5} = \left[n = 2t \Rightarrow dn = 2\,dt = \ldots dt = \frac{dn}{2}\right] =$

$= \frac{1}{2}\int \frac{dn}{n^2+5^2} = \frac{1}{2}\cdot\frac{1}{5}\,arctg\frac{n}{5} + C = \frac{1}{10}\,arctg\frac{2t}{3} + C = \frac{1}{10}\,arctg\frac{2tg x}{3} + C$

8.5.27 $\int \frac{dx}{1+3\cos^2 x} = \left[tg x = t;\ dx = \frac{dt}{1+t^2};\ \cos x = \frac{1}{\sqrt{1+t^2}}\right] = \int \frac{dt}{1+t^2}\cdot\left(\frac{1+t^2}{1+3t^2}\right) =$

$= \int \frac{dt}{t^2+4} = \int \frac{dt}{t^2+2^2} = \frac{1}{2}\,arctg\frac{t}{2} + C = \frac{1}{2}\,arctg\frac{tg x}{2} + C$

8.5.29 $\int \sin^5 x\,dx = \left[t = \cos x \Rightarrow dx = -\frac{dt}{\sqrt{1-t^2}};\ \sin x = \sqrt{1-t^2}\right] =$

$= -\int \frac{dt}{\sqrt{1-t^2}}\cdot(\sqrt{1-t^2})^5 = -\int (\sqrt{1-t^2})^4\,dt = -\int (1-t^2)^2\,dt = -\left(\int dt - \int 2t^2\,dt + \int t^4\,dt\right) =$

$= -t + \frac{2t^3}{3} - \frac{t^5}{5} + C = C - \cos x + \frac{2\cos^3 x}{3} - \frac{\cos^5 x}{5}$

8.5.30 $\int \sin^4 x \cos^5 x\,dx = \left[t = \sin x \Rightarrow dx = \frac{dt}{\sqrt{1-t^2}};\ \cos x = \sqrt{1-t^2}\right] =$

$= \left[t^4\cdot\frac{dt}{\sqrt{1-t^2}}\cdot(\sqrt{1-t^2})^5 = \int t^4\cdot(1-t^2)^2\,dt = \int t^4(1-2t^2+t^4)\,dt =\right.$

$= \int t^4 dt - 2\int t^6 dt + \int t^8 dt = \frac{t^5}{5} - \frac{2t^7}{7} + \frac{t^9}{9} + C = \frac{\sin^5 x}{5} - \frac{2\sin^7 x}{7} + \frac{\sin^9 x}{9} + C$ (2)

8.5.34 $\int \frac{\sin^3 x dx}{\cos^4 x} = \int \frac{\cos x \sin^2 x dx}{\cos^4 x} = 2\int \frac{\sin^2 x dx}{\cos^6 x} = \left[t = \cos x \Rightarrow dx = \frac{dt}{-\sin x}; \sin x = \sqrt{1-t^2}\right] = -2\int \frac{\sqrt{1-t^2}}{t^6} \frac{dt}{\sqrt{1-t^2}} = -2\int \frac{dt}{t^6} = -2\int t^{-6} dt = -2\frac{t^{-5}}{-5} + C =$

$= \frac{2}{5} t^{-5} + C = \frac{2}{5\cos^5 x} + C$

8.5.36 $\int \sin x \cdot \sin 3x dx = \left[\frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2} = \sin\alpha \cdot \sin\beta\right] = \int \frac{\cos 2x - \cos 4x}{2} dx = \frac{1}{2}\int \cos 2x dx - \frac{1}{2}\int \cos 4x dx =$

$= \frac{1}{4}\int \cos 2x d(2x) - \frac{1}{8}\int \cos 4x d(4x) = \frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x + C$

8.5.37 $\int \sin\frac{x}{11} \cdot \cos\frac{x}{33} dx = \frac{1}{2}\int \left(\sin\left(\frac{x}{11} + \frac{4x}{11}\right) + \sin\left(\frac{x}{11} - \frac{4x}{11}\right)\right) dx =$

$= \frac{1}{2}\int \sin\left(-\frac{3x}{11}\right) dx + \frac{1}{2}\int \sin\frac{5x}{11} dx = -\frac{11}{6}\int \sin\frac{3x}{11} d\left(\frac{3x}{11}\right) + \frac{1}{2}\cdot\frac{11}{5}\int \sin\frac{5x}{11} d\left(\frac{5x}{11}\right) =$

$= C + \frac{11}{6}\cos\frac{3x}{11} - \frac{11}{10}\cos\frac{5x}{11}$

8.5.38 $\int \cos x \cos 3x dx = \frac{1}{2}\int (\cos(3x-x) + \cos(3x+x)) dx = \frac{1}{2}\int \cos 2x dx +$

$+ \frac{1}{2}\int \cos 4x dx = \frac{1}{4}\int \cos 2x d(2x) + \frac{1}{8}\int \cos 4x d(4x) = \frac{\sin 2x}{4} + \frac{\sin 4x}{8} + C$

8.5.41 $\int \operatorname{tg}^4 \frac{x}{2} dx = \int \operatorname{tg}^2 \frac{x}{2} \operatorname{tg}^2 \frac{x}{2} dx = \left[1 + \operatorname{tg}^2 \frac{x}{2} = \sec^2 \frac{x}{2}\right] =$

$= \int \operatorname{tg}^2 \frac{x}{2} \left(\sec^2 \frac{x}{2} - 1\right) dx = \int \operatorname{tg}^2 \frac{x}{2} \sec^2 \frac{x}{2} dx - \int \operatorname{tg}^2 \frac{x}{2} dx = \int \frac{1}{\cos^2 \frac{x}{2}} \cdot \frac{1}{\cos^2 \frac{x}{2}} dx -$

$- \int \frac{1}{\cos^2 \frac{x}{2}} dx - \int \frac{1}{\cos^2 \frac{x}{2}} dx + \int dx = \left[\cos^2 x = \frac{1+\cos 2x}{2}\right] = 4\int \frac{dx}{(1+\cos x)^2} -$

$= 4\int \frac{dx}{1 + 2\cos x + \cos^2 x} = 8\int \frac{dx}{2 + 4\cos x + \cos 2x + 1} = \frac{\operatorname{tg} 3x}{3} + \operatorname{tg} x + x + C$