Домашняя работа. Обратная матрица. Логинова София 3112.

1.4.37

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0{,}5 & 0 \end{pmatrix}$$

1. $\det A = 0+0+0-0+1-0 = 1 \neq 0 \Rightarrow \exists A^{-1}$

2. $A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 0 & 2 \\ 0{,}5 & 0 \end{vmatrix} = 0-1 = -1$

$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = -(0-0) = 0$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 0 & 0 \\ 0 & 0{,}5 \end{vmatrix} = 0-0 = 0$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 0 & 0 \\ 0{,}5 & 0 \end{vmatrix} = -(0-0) = 0$

$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0-0 = 0$

$A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} -1 & 0 \\ 0 & 0{,}5 \end{vmatrix} = -(-0{,}5-0) = 0{,}5$

$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 0 & 0 \\ 0 & 2 \end{vmatrix} = (0-0) = 0$

$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} -1 & 0 \\ 0 & 2 \end{vmatrix} = -(-2-0) = 2$

$A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0-0 = 0$

3. $\tilde{A} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0{,}5 \\ 0 & 2 & 0 \end{pmatrix}^T = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0{,}5 & 0 \end{pmatrix}$

4. $A^{-1} = \frac{1}{\det A} \cdot \tilde{A} = \frac{1}{1} \cdot \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0{,}5 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0{,}5 & 0 \end{pmatrix}$

1.4.35

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 8 & 3 & -6 \\ -4 & -1 & 3 \end{pmatrix}$$

1. $\det A = 9 + 8 + \underline{24} - 12 - 6 - \underline{24} =$
$= 0 + 17 - 18 = -1 \neq 0 \Rightarrow \exists A^{-1}$

2. $A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 3 & -6 \\ -1 & 3 \end{vmatrix} = 9 - 6 = 3$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 8 & -6 \\ -4 & 3 \end{vmatrix} = -(24-24) = 0$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 8 & 3 \\ -4 & -1 \end{vmatrix} = (-8+12) = 4$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix} = -(3-1) = -2$

$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & -1 \\ -4 & 3 \end{vmatrix} = (3-4) = -1$

$A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 1 \\ -4 & -1 \end{vmatrix} = -(-1+4) = -3$

$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 1 & -1 \\ 3 & -6 \end{vmatrix} = (-6+3) = -3$

$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & -1 \\ 8 & -6 \end{vmatrix} = -(-6+8) = -2$

$A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 1 & 1 \\ 8 & 3 \end{vmatrix} = 3 - 8 = -5$

3. $\tilde{A} = \begin{pmatrix} 3 & 0 & 4 \\ -2 & -1 & -3 \\ -3 & -2 & -5 \end{pmatrix}^T = \begin{pmatrix} 3 & -2 & -3 \\ 0 & -1 & -2 \\ 4 & -3 & -5 \end{pmatrix}$

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4. $A^{-1} = \frac{1}{\det A} \cdot \tilde{A} = \frac{1}{-1} \cdot \begin{pmatrix} 3 & -2 & -3 \\ 0 & -1 & -2 \\ 4 & -3 & -5 \end{pmatrix} = \begin{pmatrix} -3 & 2 & 3 \\ 0 & 1 & 2 \\ -4 & 3 & 5 \end{pmatrix}$

$12 - 6 - \underline{24} =$

$0 \Rightarrow \exists A^{-1}$

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & 2 \\ 4 & 1 & 4 \end{pmatrix} \qquad \det A = -\cancel{4} + \cancel{4} + 8 + 8 - 2 - \cancel{8} = 6 \neq 0 \Rightarrow \exists A^{-1}$$

2. $A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix} = -4 - 2 = -6$

$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & 2 \\ 4 & 4 \end{vmatrix} = -(8-8) = 0$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} = 2 + 4 = 6$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = -(4-2) = -2$

$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & 2 \\ 4 & 4 \end{vmatrix} = 4 - 8 = -4$

$A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} = -(1-4) = 3$

$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} = 2 + 2 = 4$

$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = -(2-4) = 2$

$A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1 - 2 = -3$

3. $\tilde{A} = \begin{pmatrix} -6 & 0 & 6 \\ -2 & -4 & 3 \\ 4 & 2 & -3 \end{pmatrix}^T = \begin{pmatrix} -6 & -2 & 4 \\ 0 & -4 & 2 \\ 6 & 3 & -3 \end{pmatrix}$

4. $A^{-1} = \frac{1}{\det A} \cdot \tilde{A} = \frac{1}{6} \begin{pmatrix} -6 & -2 & 4 \\ 0 & -4 & 2 \\ 6 & 3 & -3 \end{pmatrix} = \begin{pmatrix} -1 & -1/3 & 2/3 \\ 0 & -2/3 & 1/3 \\ 1 & 1/2 & -1/2 \end{pmatrix}$

1. 4. 40

$$A = \begin{pmatrix} 3 & 4 & 2 \\ 2 & -4 & -3 \\ 1 & 5 & 1 \end{pmatrix}$$

1. $\det A = -12 + 20 - 12 + 8 + 45 - 8 = 41$

$\Rightarrow \exists A^{-1}$

2. $A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -4 & -3 \\ 5 & 1 \end{vmatrix} = -4 + 15 = 11$

$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -(2+3) = -5$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & -4 \\ 1 & 5 \end{vmatrix} = (10+4) = 14$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 4 & 2 \\ 5 & 1 \end{vmatrix} = -(4-10) = 6$

$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3 - 2 = 1$

$A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} = -(15-4) = -11$

$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 4 & 2 \\ -4 & -3 \end{vmatrix} = -12 + 8 = -4$

$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix} = -(-9-4) = 13$

$A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 3 & 4 \\ 2 & -4 \end{vmatrix} = -12 - 8 = -20$

3. $\tilde{A} = \begin{pmatrix} 11 & -5 & 14 \\ 6 & 1 & -11 \\ -4 & 13 & -20 \end{pmatrix}^T = \begin{pmatrix} 11 & 6 & -4 \\ -5 & 1 & 13 \\ 14 & -11 & -20 \end{pmatrix}$

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4. $A^{-1} = \frac{1}{\det A} \cdot \tilde{A} = \frac{1}{41} \cdot \begin{pmatrix} 11 & 6 & -4 \\ -5 & 1 & 13 \\ 14 & -11 & -20 \end{pmatrix} = \begin{pmatrix} 11/41 & 6/41 & -4/41 \\ -5/41 & 1/41 & 13/41 \\ 14/41 & -11/41 & -20/41 \end{pmatrix}$

1.4.41

$A = \begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{pmatrix}$

1. $\det A = 0 + 24 - 3 + 6 - 27 - 6 = 0 = 0 \Rightarrow \nexists A^{-1}$

Т.к определитель равен нулю обратной матрицы не существует.

1.4.42.

$A = \begin{pmatrix} 5 & 8 & -1 \\ 2 & -3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$

1. $\det A = -45 - 4 + 16 - 3 - 20 - 48 = -104 \neq 0 \Rightarrow \exists A^{-1}$

2. $A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -3 & 2 \\ 2 & 3 \end{vmatrix} = -9 - 4 = -13$

$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} = -(6-2) = -4$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} = 4 + 3 = 7$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 8 & -1 \\ 2 & 3 \end{vmatrix} = -(24+2) = -26$

$A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix} = 15 + 1 = 16$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 5 & 8 \\ 1 & 2 \end{vmatrix} = -(10-8) = -2$

$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 8 & -1 \\ -3 & 2 \end{vmatrix} = 16 + 3 = 13$

$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 5 & -1 \\ 2 & 2 \end{vmatrix} = -(10+2) = -12$

$A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 5 & 8 \\ 2 & -3 \end{vmatrix} = -15 - 16 = -31$

3. $\tilde{A} = \begin{pmatrix} -13 & -4 & 7 \\ -26 & 16 & -2 \\ 13 & -12 & -31 \end{pmatrix}^T = \begin{pmatrix} -13 & -26 & 13 \\ -4 & 16 & -12 \\ 7 & -2 & -31 \end{pmatrix}$

4. $A^{-1} = \frac{1}{\det A} \cdot \tilde{A} = \frac{1}{-104} \cdot \begin{pmatrix} -13 & -26 & 13 \\ -4 & 16 & -12 \\ 7 & -2 & -31 \end{pmatrix} = \begin{pmatrix} 1/8 & 1/4 & -1/8 \\ 1/26 & -2/13 & 3/26 \\ -7/104 & 1/52 & 31/104 \end{pmatrix}$

1.4.43.

$$A = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix} \qquad 1.\ \det A = 4+1+1-2-1-2 = 1 \neq 0 \Rightarrow \exists A^{-1}$$

$$2.\ \Gamma = \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ II+I \\ III+I \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right) \begin{array}{l} I+II+III \\ \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right) \qquad 3.\ A^{-1} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

1.4.44.

$$A = \begin{pmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{pmatrix} \qquad 1.\ \det A = 54+45+28-27-40-63 =$$

$$= 5+1-9 = -3 \neq 0 \Rightarrow \exists A^{-1}$$

$$2.\ \Gamma = \left(\begin{array}{ccc|ccc} 2 & 7 & 3 & 1 & 0 & 0 \\ 3 & 9 & 4 & 0 & 1 & 0 \\ 1 & 5 & 3 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ II-I \\ III-I \end{array} \sim \left(\begin{array}{ccc|ccc} 2 & 7 & 3 & 1 & 0 & 0 \\ 1 & 2 & 1 & -1 & 1 & 0 \\ -1 & -2 & 0 & -1 & 0 & 1 \end{array}\right) \begin{array}{l} I-2\cdot II \\ \\ III+II \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 0 & 3 & 1 & 3 & -2 & 0 \\ 1 & 2 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) \begin{array}{l} I-III \\ II-III \\ \\ \end{array} \sim \left(\begin{array}{ccc|ccc} 0 & 3 & 0 & 5 & -3 & -1 \\ 1 & 2 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) \begin{array}{l} I:3 \\ \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 0 & 1 & 0 & 5/3 & -1 & -1/3 \\ 1 & 2 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) \updownarrow \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 5/3 & -1 & -1/3 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) \begin{array}{l} I-2II \\ \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -7/3 & 2 & -1/3 \\ 0 & 1 & 0 & 5/3 & -1 & -1/3 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) \qquad 3.\ A^{-1} = \begin{pmatrix} -7/3 & 2 & -1/3 \\ 5/3 & -1 & -1/3 \\ -2 & 1 & 1 \end{pmatrix}$$

1.4.45.

$$A = \begin{pmatrix} 1 & 2 & -2 & 4 \\ 2 & 6 & 1 & 0 \\ 3 & 0 & 1 & 2 \\ -1 & 4 & 5 & -4 \end{pmatrix}$$

1. $$\det A = \begin{vmatrix} 1 & 2 & -2 & 4 \\ 2 & 6 & 1 & 0 \\ 3 & 0 & 1 & 2 \\ -1 & 4 & 5 & -4 \end{vmatrix} \begin{matrix} \\ \mathrm{II} - 2\mathrm{I} \\ \mathrm{III} - 3\mathrm{I} \\ \mathrm{IV} + \mathrm{I} \end{matrix} =$$

$$= \begin{vmatrix} 1 & 2 & -2 & 4 \\ 0 & 2 & 5 & -8 \\ 0 & -6 & 7 & -10 \\ 0 & 6 & 3 & 0 \end{vmatrix} \begin{matrix} \\ \\ \mathrm{III} + 3\mathrm{II} \\ \mathrm{IV} - 3\mathrm{II} \end{matrix} = \begin{vmatrix} 1 & 2 & -2 & 4 \\ 0 & 2 & 5 & -8 \\ 0 & 0 & 22 & -34 \\ 0 & 0 & -12 & 24 \end{vmatrix} \begin{matrix} \\ \\ \\ \mathrm{IV} + \mathrm{III} \cdot \frac{6}{11} \end{matrix} =$$

$$= \begin{vmatrix} 1 & 2 & -2 & 4 \\ 0 & 2 & 5 & -8 \\ 0 & 0 & 22 & -34 \\ 0 & 0 & 0 & 60/11 \end{vmatrix} = 60/11 \cdot (-1)^{4+4} \cdot 2 \cdot 1 \cdot 22 = \frac{60 \cdot 2 \cdot 22}{11} = 240$$

$$240 \neq 0 \Rightarrow \exists A^{-1}$$

2. $$\Gamma = \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 2 & 6 & 1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 1 & 2 & 0 & 0 & 1 & 0 \\ -1 & 4 & 5 & -4 & 0 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \\ \mathrm{II} - 2\mathrm{I} \\ \mathrm{III} - 3\mathrm{I} \\ \mathrm{IV} + \mathrm{I} \end{matrix} =$$

$$= \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 5 & -8 & -2 & 1 & 0 & 0 \\ 0 & -6 & 7 & -10 & -3 & 0 & 1 & 0 \\ 0 & 6 & 3 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \\ \mathrm{II} : 2 \\ \\ \\ \end{matrix} = \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2.5 & -4 & -1 & 0{,}5 & 0 & 0 \\ 0 & -6 & 7 & -10 & -3 & 0 & 1 & 0 \\ 0 & 6 & 3 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \mathrm{I} - 2\mathrm{II} \\ \\ \mathrm{III} + 6\mathrm{II} \\ \mathrm{IV} - 6\mathrm{II} \end{matrix}$$

$$= \left(\begin{array}{cccc|cccc} 1 & 0 & -7 & 12 & 3 & -1 & 0 & 0 \\ 0 & 1 & 2.5 & -4 & -1 & 0{,}5 & 0 & 0 \\ 0 & 0 & 22 & -34 & -9 & 3 & 1 & 0 \\ 0 & 0 & -12 & 24 & 7 & -3 & 0 & 1 \end{array}\right) \begin{matrix} \\ \\ \mathrm{III} : 22 \\ \\ \end{matrix} \simeq$$

$$= \left(\begin{array}{cccc|cccc} 1 & 0 & -7 & 12 & 3 & -1 & 0 & 0 \\ 0 & 1 & 2.5 & -4 & -1 & 0{,}5 & 0 & 0 \\ 0 & 0 & 1 & -17/11 & -9/22 & 3/22 & 1/22 & 0 \\ 0 & 0 & -12 & 24 & 7 & -3 & 0 & 1 \end{array}\right) \begin{matrix} \mathrm{I} + 7 \cdot \mathrm{III} \\ \mathrm{II} - 2{,}5\,\mathrm{III} \\ \\ \mathrm{IV} + 12 \cdot \mathrm{III} \end{matrix} =$$

$$= \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 13/11 & 3/22 & -1/22 & 7/22 & 0 \\ 0 & 1 & 0 & -3/22 & 1/44 & 3/44 & -5/44 & 0 \\ 0 & 0 & 1 & -17/11 & -9/22 & 3/22 & 1/22 & 0 \\ 0 & 0 & 0 & 60/11 & 23/11 & -15/11 & 6/11 & 1 \end{array}\right) \quad \overline{IV} : 60/11 \quad =$$

$$= \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 13/11 & 3/22 & -1/22 & 7/22 & 0 \\ 0 & 1 & 0 & -3/22 & 1/44 & 3/44 & -5/44 & 0 \\ 0 & 0 & 1 & -17/11 & -9/22 & 3/22 & 1/22 & 0 \\ 0 & 0 & 0 & 1 & 23/60 & -0{,}25 & 0{,}1 & 11/60 \end{array}\right) \quad \begin{array}{l} \overline{I} - \frac{13}{11} \cdot \overline{IV} \\ \overline{II} + 3/22 \cdot \overline{IV} = \\ \overline{III} + 17/11 \cdot \overline{IV} \end{array}$$

$$= \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -19/60 & 0{,}25 & 0{,}2 & -13/60 \\ 0 & 1 & 0 & 0 & 0{,}075 & 0{,}125 & -0{,}1 & 0{,}025 \\ 0 & 0 & 1 & 0 & 11/60 & -0{,}25 & 0{,}2 & 17/60 \\ 0 & 0 & 0 & 1 & 23/60 & -0{,}25 & 0{,}1 & 11/60 \end{array}\right)$$

3. $$A^{-1} = \begin{pmatrix} -19/60 & 0{,}25 & 0{,}2 & -13/60 \\ 0{,}075 & 0{,}125 & -0{,}1 & 0{,}025 \\ 11/60 & -0{,}25 & 0{,}2 & 17/60 \\ 23/60 & -0{,}25 & 0{,}1 & 11/60 \end{pmatrix}$$