Интегралы часть №3 (1) Шульман Э9-41б

8.3.19 $\int \frac{5dx}{x+\sqrt{2}} = 5\cdot \ln|x+\sqrt{2}|+C$

8.3.20 $\int \frac{4dx}{(x-\frac{1}{2})^3} = [A=4, \alpha=\frac{1}{2}, k=3] = \frac{4}{1-3}\cdot\frac{1}{(x-\frac{1}{2})^{3-1}}+C =$

$= -\frac{4}{2}\cdot\frac{1}{(x-\frac{1}{2})^2}+C = \frac{-2}{(x-\frac{1}{2})^2}+C$

8.3.21 $\int \frac{7dx}{(x+3)^6} = [A=7; \alpha=-3; k=6] = \frac{7}{1-6}\cdot\frac{1}{(x+3)^{6-1}}+C = \frac{-7}{5}\cdot\frac{1}{(x+3)^5}+C$

8.3.22 $\int \frac{dx}{(3x+2)^4} = [t=3x \Rightarrow dt=3dx \Rightarrow dx=\frac{dt}{3}] = \frac{1}{3}\int\frac{1\cdot dt}{(t+2)^4} =$

$= [A=1; \alpha=-2; k=4] = \frac{1}{3}\cdot\frac{1}{1-4}\cdot\frac{1}{(x+2)^{4-1}}+C = -\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{(x+2)^3}+C = -\frac{1}{9(3x+2)^3}+C$

8.3.23 $\int \frac{dx}{(x^2-4x+8)} = [x^2-4x+8=0 \Rightarrow D=16-4\cdot 8<0, A=0, B=0 \Rightarrow] =$

$\int\frac{dx}{(x-2)^2+4} = [t=x-2 \Rightarrow dt=dx] = \int\frac{dt}{t^2+4} = \int\frac{dt}{t^2+2^2} = \frac{1}{2}\operatorname{arctg}\frac{t}{2}+C =$

$= \frac{1}{2}\operatorname{arctg}\frac{x-2}{2}+C$

8.3.24 $\int\frac{dx}{x^2+x+1} = \int\frac{dx}{(x+0.5)^2+0.75} = [t=x+0.5 \Rightarrow dx=dt] =$

$= \int\frac{dx}{t^2+0.75} = \int\frac{dx}{t^2+(\frac{\sqrt{3}}{2})^2} = \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}+C$

8.3.26 $\int\frac{5x+2}{x^2+2x+16}dx = [2^2+4\cdot 10<0; A=5, B=2, p=2, q=10] =$

$\Rightarrow Ax+B = \frac{5}{2}(2x+2)+2-\frac{5\cdot 2}{2} = \frac{1}{5}(2x+2)-3 \Rightarrow \frac{5}{2}\int\frac{2x+2}{x^2+2x+10}dx -$

$- \frac{15}{2}\int\frac{dx}{x^2+2x+10} = [t=x^2+2x+10 \Rightarrow dt=(2x+2)dx; m=x+1, dm=dx] =$

$= \frac{5}{2}\left(\int\frac{dt}{t} - 3\int\frac{dm}{m^2+3^2}\right) = \frac{5}{2}\ln|t| - \frac{15}{2}\cdot\frac{1}{3}\operatorname{arctg}\frac{m}{3}+C =$

$= \frac{5}{2}\ln|x^2+2x+10| - \frac{5}{2}\operatorname{arctg}\frac{x+1}{3}+C$

8.3.29 $\int\frac{x-1}{(x^2+2x+3)^2}dx = [D=2^2-4\cdot 3=4-12<0, A=1, B=-1, p=2; q=3]$

$Ax+B \Rightarrow \frac{1}{2}(2x-1)-1+\frac{2\cdot 1}{2} = (x-1)-0 \quad \frac{1}{2}\int\frac{(2x-2)dx}{(x^2+2x+3)^2} = [t=x^2+2x+3 \Rightarrow$

$\Rightarrow dt(2x+2)dx] = \frac{1}{2}\int\frac{dt}{t^2} = \frac{1}{2}\int t^{-2}dt = \frac{1}{2}\cdot\frac{t^{-2+1}}{-2+1}+C = -\frac{1}{2}t^{-1}+C = \frac{-1}{2t}+C =$

$= \frac{-1}{x^2+2x+3}+C$

8.3.33 $\int\frac{2x-3}{(x-1)(x+2)}dx = \Big[\frac{2x-3}{(x-1)(x+2)} = \frac{A}{x-1}+\frac{B}{x+2} \Rightarrow A(x+2)+$ 2

$+B(x-1) = 2x+2A+Bx-B = x(A+B)+(2A-B) \Rightarrow \{A+B=2; 2A-B=-3; 3A=-1 \Rightarrow$

$\Rightarrow A=-\frac{1}{3} \Rightarrow -\frac{1}{3}+B=2; B=2+\frac{1}{3}=\frac{7}{3}\Big] = \int\left(-\frac{1}{3(x-1)}+\frac{7}{3(x+2)}\right)dx = -\frac{1}{3}$

$\int\frac{dx}{x-1}+\frac{7}{3}\int\frac{dx}{x+2} = \frac{7}{3}\ln|x+2|-\frac{1}{3}\ln|x-1|+C$

8.3.34 $\int\frac{x-4}{(x-2)(x-3)}dx = \Big[\frac{x-4}{(x-2)(x-3)} = \frac{A}{x-2}+\frac{B}{x-3} \Rightarrow x-4 = A(x-3)+B(x-2)$

$= Ax-3A+Bx-2B = x(A+B)-3A-2B \Rightarrow \begin{cases}A+B=1\\3A-2B=-4\end{cases} \Rightarrow \begin{cases}2A+2B=2\\-3A-2B=-4\end{cases} \Rightarrow$

$\Rightarrow A=-2 \Rightarrow -2+B=1 \Rightarrow B=3\Big] = \int\left(-\frac{2}{x-2}+\frac{3}{x-3}\right)dx = -2\int\frac{dx}{x-2}+3\int\frac{dx}{x-3} =$

$= 3\ln|x-3|-2\ln|x-2|+C$

8.3.35 $\int\frac{x\,dx}{x^2-4x-5} = \Big[x^2-4x-5=0; D=16+4\cdot5 \Rightarrow x_1=\frac{4+6}{2}=5, x_2=-1\Big] =$

$=\int\frac{x\,dx}{(x-5)(x+1)} = \Big[\frac{x}{(x-5)(x+1)} = \frac{A}{x-5}+\frac{B}{x+1} \Rightarrow x = A(x+1)+B(x-5) =$

$= Ax+A+Bx-5B = x(A+B)+A-5B \Rightarrow \begin{cases}A+B=1\\A-5B=0\end{cases} \Rightarrow 6B=1 \Rightarrow B=\frac{1}{6} \Rightarrow A=$

$=1-\frac{1}{6}=\frac{5}{6}\Big] = \int\left(\frac{5}{6(x-5)}+\frac{1}{6(x+1)}\right)dx = \frac{5}{6}\int\frac{dx}{x-5}+\frac{1}{6}\int\frac{dx}{x+1} =$

$=\frac{5}{6}\ln|x-5|+\frac{1}{6}\ln|x+1|+C$

8.3.36 $\int\frac{2x^2-11}{x^2+x-6}dx = \int\frac{2(x^2+x-6)-2x+1}{x^2+x-6} = 2\int dx-\int\frac{2x-1}{x^2+x-6}dx =$

$=\Big[x^2+x-6=0 \Rightarrow D=1+4\cdot6=25 \Rightarrow x_1=\frac{-1+5}{2}=2; x_2=\frac{-1-5}{2}=-3\Big] =$

$=2\int dx-\int\frac{2x-1}{(x-2)(x+3)}dx = \Big[\frac{2x-1}{(x-2)(x+3)} = \frac{A}{x-2}+\frac{B}{x+3} \Rightarrow 2x-1 = A(x+3)+B(x-2)$

$= Ax+3A+Bx-2B = x(A+3A-2B \Rightarrow \begin{cases}A+B=2\\3A-2B=-1\end{cases} \cdot 2 = \begin{cases}2A+2B=4\\3A-2B=-1\end{cases} \Rightarrow$

$5A=3 \Rightarrow A=\frac{3}{5} \Rightarrow B=\frac{7}{5}\Big] = 2\int dx-\int\left(\frac{3}{5(x-2)}+\frac{7}{5(x+3)}\right)dx = 2\int dx -$

$-\frac{3}{5}\int\frac{dx}{x-2}+\frac{7}{5}\int\frac{dx}{x+3} = 2x-\frac{3}{5}\ln|x-2|+\frac{7}{5}\ln|x+3|+C$

8.3.37 $\int\frac{-3x^2+x+19}{(x-4)(x-2)(x+1)}dx = \Big[\frac{A}{x-4}+\frac{B}{x-2}+\frac{C}{x+1} \Rightarrow -3x^2+x+19 = A(x-2)(x+1)$

$+B(x-4)(x+1)+C(x-4)(x-2) = Ax^2-2Ax+Ax-2A+Bx^2-4Bx+Bx-4B+$

$+Cx^2-4Cx-2Cx+8C = x^2(A+B+C)+x(-A-3B-6C)+(-2A-4B+8C) \Rightarrow$

$= \begin{cases}A+B+C=-3\\-A-3B-6C=1\\-2A-4B+8C=19\end{cases} \Bigg| \Rightarrow \begin{matrix}A=-\frac{5}{2}\\B=-\frac{3}{2}\\C=1\end{matrix}\Bigg| = \int\left(\frac{-5}{2(x-4)}-\frac{3}{2(x-2)}+\frac{1}{x+1}\right)dx$

3

$= -\frac{5}{2}\int\frac{dx}{x-4} - \frac{3}{2}\int\frac{dx}{x-2} + \int\frac{dx}{x+1} = -\frac{5}{2}\ln|x-4| - \frac{3}{2}\ln|x-2| +$

$+\ln|x+1| + C$

8.3.38 $\int\frac{x-1}{(x+1)(x^2-4)}dx = \int\frac{x-1}{(x+1)(x-2)(x+2)}dx = \Big[\frac{x-1}{(x+1)(x-2)(x+2)} =$

$= \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+2} \Rightarrow x-1 = A(x^2-4) + B(x+1)(x+2) + C(x+1)(x-2) =$

$= 2x^2 - 4A + Bx^2 + Bx + 2Bx + 2B + Cx^2 + Cx - 2Cx - 2C \Rightarrow x^2(A+B+C) +$

$+ x(3B - C) - 4A + 2B - 2C \Rightarrow \begin{cases} A+B+C=0 \\ 3B - C = 1 \\ -4A + 2B - 2C = -1 \end{cases} \Rightarrow C = 3B - 1 \Rightarrow$

$\Rightarrow \begin{cases} A+B+3B=1 \\ -4A + 2B - 2(3B-1) = -1 \end{cases} = \begin{cases} A = 1-4B \\ -4(1-4B + 2B - 2(3B-1) = -1 \end{cases} \Rightarrow$

$\Rightarrow \left| \begin{array}{l} -4+16B+2B-6B+2=-1 \\ 12B = 1 \Rightarrow B = \frac{1}{12}; \quad C = -\frac{3}{3} \\ A = 1-4\cdot\frac{1}{2} = \frac{2}{3} \end{array} \right] = \int\left(\frac{2}{3(x+1)} + \frac{1}{12(x-2)} - \frac{3}{4(x+2)}\right)dx =$

$= \frac{2}{3}\ln|x+1| + \frac{1}{12}\ln|x-2| - \frac{3}{4}\ln|x+2| + C$

8.3.39 $\int\frac{x^2+2}{(x^2-1)(x+1)^2}dx = \Big[\frac{x^2+2}{(x^2-1)(x+1)^2} = \frac{x^2+2}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{(x+1)^3} + \frac{C}{(x+1)^2} + \frac{D}{x+1} =$

$\Rightarrow x^2+2 = A(x+1)^3 + B(x-1) + C(x^2-1) + D(x-1)(x+1)^2 = Ax^3 + 3Ax^2 + 3Ax +$

$+ A + Bx - B + Cx^2 - C + Dx^3 + Dx^2 - Dx - D = x^3(A+D) + x^2(3\cdot A + C + D) + x(3A+B-D) +$

$+ A - B - C - D \Rightarrow \begin{cases} A+D=0 \\ 3A+C+D=1 \\ 3A+B-D=0 \\ A-B-C-D=2 \end{cases} \Rightarrow \begin{cases} D=-A \\ 3A+C-A=1 \\ 3A+B+A=0 \\ A-B-C+A=2 \end{cases} \Rightarrow \begin{cases} C=1-2A \\ A+B=0 \\ A-B-1+2A = A=2 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} B=-4A \\ A+4A-1+2A+A=2 \end{cases} \Rightarrow \begin{array}{l} 8A=3=3 \quad A=\frac{3}{8} \quad D=-\frac{3}{8} \\ B=-3/2 \ ; \ C=1/4 \end{array}\Big] =$

$= \int\left(\frac{3}{8(x-1)} - \frac{3}{2(x+1)^3} + \frac{1}{4(x+1)^2} - \frac{3}{8(x+1)}\right)dx = \frac{3}{8}\int\frac{dx}{x-1} - \frac{3}{2}\int\frac{dx}{(x+1)^3} + \frac{1}{4}\int\frac{dx}{(x+1)^2} - \frac{3}{8}\int\frac{dx}{x+1} =$

$= \frac{3}{8}\ln|x-1| + \frac{3}{4}\frac{1}{(x+1)^2} - \frac{1}{4}\frac{1}{x+1} - \frac{3}{8}\ln|x+1| + C$

8.3.40 $\int\frac{2x+3}{(x-2)^3}dx = \Big[\frac{2x+3}{(x-2)^3} = \frac{A}{(x-2)^3} + \frac{B}{(x-2)^2} + \frac{C}{x-2} = 2x+3 = A + B(x-2) +$

$+ C(x-2)^2 = A - Bx - 2B + Cx^2 - 4Cx + 4C = x^2(C) + x(B-4C) + A - 2B + 4C =$

$= \begin{cases} C=0 \\ B-4C = B-4\cdot 0 = 2 \\ A - 2\cdot 2 + 4\cdot 0 = 3 \end{cases} \Rightarrow \begin{cases} C=0 \\ B=2 \\ A=7 \end{cases}\Big] = \int\left(\frac{7}{(x-2)^3} + \frac{2}{(x-2)^2}\right)dx = 7\int\frac{dx}{(x-2)^3} +$

4

$$+2\int\frac{dx}{(x-2)^2}=7\cdot\left(-\frac{1}{2(x-2)^2}\right)+2\left(-\frac{1}{x-2}\right)+C=-\frac{7}{2(x-2)^2}-\frac{2}{x-2}+C=\frac{4x-1}{2(x-2)^2}+C$$

8.3.49 $\int\frac{x^2dx}{x^3+5x^2+8x+4}=\left[x^3+5x^2+8x+4=0\ (x^2+3x+2)(x+2)=0;\ x^2+3x+2=0\right.$
$\left.D=3^2-4\cdot2=1\quad x_1=\frac{-3+1}{2}=-1\quad x_2=\frac{-3+1}{2}=-2\right]$

$$\Rightarrow\int\frac{x^2dx}{(x+2)^2(x+1)}=\left[\frac{A}{(x+1)}+\frac{B}{(x+2)^3}-\frac{C}{(x+2)}\Rightarrow x^2=A(x+2)^2+B(x+1)+C(x+2)(x+1)\right.$$

$$=Ax^2+4Ax+4A+Bx+B+Cx^2+36x+2C=x^2(A+C)+x(4A+B+3C)+4A+B+$$

$$2C;\ \begin{cases}A+C=1\\4A+B+3C=0\\4A+B+2C=0\end{cases}\Rightarrow\left.\begin{cases}A=1-6\\C=0\\B=-4\end{cases}\right]=\int\left(\frac{1}{x+1}-\frac{4}{(x+2)}\right)dx=\int\frac{dx}{x-1}-4\int\frac{dx}{(x+2)^2}=$$

$$=\ln|x-1|+\frac{4}{x+2}+C$$

8.3.53 $\int\frac{\cos x\,dx}{(\sin x-1)(\sin x+2)}=\left[t=\sin x\Rightarrow dt=\cos x\,dx\right]=\int\frac{dt}{(t-1)(t+2)}=$

$$=\left[\frac{1}{(t-1)(t+2)}=\frac{A}{t-1}+\frac{B}{t+2}\Rightarrow 1=A(t+2)+B(t-1)=At+2A+Bt-B=\right.$$

$$\left.=t(A+B)+2A-B\Rightarrow\begin{cases}A+B=0\\2A-B=1\end{cases}\Rightarrow 3A=1\Rightarrow A=\frac{1}{3};\ \frac{1}{3}+B=0\Rightarrow B=-\frac{1}{3}\right]=$$

$$=\int\left(\frac{1}{3}\frac{1}{t-1}-\frac{1}{3}\frac{1}{t+2}\right)dt=\frac{1}{3}\int\frac{dt}{t-1}-\frac{1}{3}\int\frac{dt}{t-2}=\frac{1}{3}\ln|t-1|-\frac{1}{3}\ln|t+2|+$$

$$+C=\frac{1}{3}\ln|\sin x-1|-\frac{1}{3}\ln|\sin x+2|+C$$

8.3.54 $\int\frac{\sin^4x\,dx}{\cos x}=\int\sin^3x\,tg\,x\,dx=\left[\begin{matrix}u'=\frac{1}{\cos^2x} & u=tg\,x\\ v'=\sin^3x & \end{matrix}\Rightarrow v=\int\sin^3x\,dx=\right.$

$$=\int(1-\cos^2x)\sin x=\left[t=\cos x\to dt=-\sin x\,dx\right]=\int(t^2-1)dt=\int t^2dt-\int dt=$$

$$\left.=\frac{t^3}{3}-t=\frac{1}{3}\cos^3x-\cos x\right]=tg\,x\left(\frac{\cos^3x}{3}-\cos x\right)-\int\frac{1}{\cos^2x}\cdot\left(\frac{\cos^3x}{3}-\cos x\right)dx=$$

$$=\frac{\sin\cos^4x}{3}-\sin x\cos^2x-\int\left(\frac{1}{3}\cos x\right)dx=\frac{1}{3}\sin x\cos^4x-\sin\cos^3x-$$

$$-\frac{1}{3}\int\cos x\,dx+\int\frac{dx}{\cos x}=\frac{1}{3}\sin x\cos^0x(\cos^2x-1)-\frac{1}{3}\sin x+\ln\left|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|+$$

$$+C=-\frac{1}{3}\sin^0x\cos^1x-\frac{1}{3}\sin x+\ln\left|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|+C$$