Производные часть №6 (1) Шуломан Эмиль

11.5.41. $z=\cos(ax+e^y)$

$\frac{\partial z}{\partial x}=(\cos(ax+e^y))'_x=-\sin(ax+e^y)\cdot a=-a\sin(ax+e^y)$

$\frac{\partial^2 z}{\partial x\partial y}=(-a\sin(ax+e^y))'_y=-a\cos(ax+e^y)\cdot e^y=-ae^y\cos(ax+e^y)$

$\frac{\partial^3 z}{\partial x\partial y^2}=(-ae^y\cos(ax+e^y))'_y=-ae^y\cos(ax+e^y)+ae^y\sin(ax+e^y)\cdot e^y=$

$=ae^y(e^y\sin(ax+e^y)-\cos(ax+e^y))$

11.5.42 $z=\frac{x^4-8xy^3}{x-2y}\Rightarrow z=\frac{x(x^3-(2y)^3)}{x-2y}=\frac{x(x-2y)(x^2+2xy+4y^2)}{x-2y}=$

$=x^3+2x^2y+4xy^2$

$\frac{\partial z}{\partial x}=3x^2+4xy+4y^2$

$\frac{\partial^2 z}{\partial x^2}=6x+4y$

$\frac{\partial^3 z}{\partial x^2\partial y}=4$

11.5.43 $u=x\ln(xy)\Rightarrow x\ln x+x\ln y$

$\frac{\partial u}{\partial x}=\ln x+\ln y+1$; $\frac{\partial^2 u}{\partial x^2}=\frac{1}{x}$; $\frac{\partial^3 u}{\partial x^2\partial y}=0$

11.5.44 $u=x^3\sin y+y^3\sin x$

$\frac{\partial^3 u}{\partial x\partial y\partial z}=0$

11.5.45 $u=e^{xyz}$

$\frac{\partial u}{\partial x}=yze^{xyz}$

$\frac{\partial^2 u}{\partial x\partial y}=ze^{xyz}+xze^{xyz}\cdot yz=ze^{xyz}(xyz+1)$

$\frac{\partial^3 u}{\partial x\partial y\partial z}=e^{xyz}(xyz+1)+xyze^{xyz}(xyz+1)+xyze^{xyz}=$

$= e^{xyz}(xyz + 1 + xyz(xyz+1) + xyz) = e^{xyz}(x^2y^2z^2 + 3xyz + 1)$ 2

11.5.47 $u = (x-x_0)^p (y-y_0)^q$

$\frac{\partial u}{\partial x} = (y-y_0)^q p (x-x_0)^{p-1}$

$\frac{\partial^p u}{\partial x^p} = p!(x-x_0)(y-y_0)^q$

$\frac{\partial^{p+1} u}{\partial x^p \partial y} = p!(x-x_0) q (y-y_0)^{q-1}$

$\frac{\partial^{p+q} u}{\partial x^p \partial y^q} = p!\,q!(x-x_0)(y-y_0)$

11.5.48 $u = \frac{x+y}{x-y}$

$m=1;\ \frac{\partial u}{\partial x} = \frac{-2y}{(x-y)^2} = (-1)^m \cdot \frac{2y \cdot 1}{(x-y)^{m+1}}$

$m=2;\ \frac{\partial^2 u}{\partial x^2} = \frac{4y}{(x-y)^3} = (-1)^m \cdot \frac{2y \cdot 1 \cdot 2}{(x-y)^{m+1}}$

$m=3;\ \frac{\partial^3 u}{\partial x^3} = \frac{-12y}{(x-y)^4} = (-1)^m \frac{2y \cdot 1 \cdot 2 \cdot 3}{(x-y)^{m+1}}$

$\frac{\partial^m u}{\partial x^m} = (-1)^m \frac{m! \cdot 2y}{(x-y)^{m+1}}$

$n=1;\ \frac{\partial^{m+1}}{\partial x^m \partial y} = (-1)^m 2m! \cdot \frac{(x-y)^{m+1} - y(m+1)(x-y)^m}{(x-y)^{2m+2}} =$

$= (-1)^{m+1} 2m! \cdot \frac{1}{(x-y)^{m+2}} (y(m+2) - x)$

$n=2\quad \frac{\partial^{m+2} u}{\partial x^m \partial y^2} = (-1)^{m-1} \cdot 2m! \frac{(m+2)(x-y)^{m+2}}{(x-y)^{2m+4}} - \frac{(y(m+2)-x)(m+2)(x-y)^{m+1}}{(x-y)^{2m+4}} =$

$= (-1)^{m+2} \cdot 2m! \cdot \frac{(m+2)}{(x-y)^{m+3}} (y(m+3) - 2x);$

$\frac{\partial^{m+n} u}{\partial x^m \partial y^n} = (-1)^{m+n} \cdot \frac{2(m+n)!}{(m+n)(x-y)^{m+n+1}} (y(m+n+1) - nx)$

11.5.49 $u = (x^2+y^2)e^{x+y}$

$\frac{\partial u}{\partial x} = e^{x+y}(x^2+y^2) + e^{x+y} \cdot 2x = e^{x+y}(x^2+2x+y^2)$ $m=1$

$\frac{\partial^2 u}{\partial x^2} = e^{x+y}(x^2+2x+y^2) + e^{x+y}(2x+2) = e^{x+y}(x^2+4x+2+y^2);\ m=2$

$\frac{\partial^3 u}{\partial x^3} = e^{x+y}(x^2+4x+2+y^2) + e^{x+y}(2x+4) = e^{x+y}(x^2+6x+6+y^2);\ m=3$

③

$$\frac{\partial^m n}{\partial x^m} = e^{x+y}(x^2+2mx+(m-1)m+y^2)$$

$$\frac{\partial^{m+1} n}{\partial x^m \partial y} = e^{x+y}(x^2+2mx+m(m-1)+y^2)+e^{x+y}\cdot 2y = e^{x+y}(x^2+2mx+m(m-1)+y^2+2y)\ n=1$$

$$\frac{\partial^{m+2} u}{\partial x^m \partial y^2} = e^{x+y}(x^2+2mx+m(m-1)+y^2+2y)+e^{x+y}(2y+2) = e^{x+y}(x^2+2mx+m(m-1)$$

$$+y^2+4y+2);\ n=1$$

$$\frac{\partial^{m+3} u}{\partial x^m \partial y^3} = e^{x+y}(x^2+2mx+m(m-1)+y^2+y^2+4y+2)+e^{x+y}(2y+4) = e^{x+y}(x^2+$$

$$2mx+m(m-1)+y^2+6y+6)\ n=3$$

$$\frac{\partial^{m+n} u}{\partial x^m \partial y^n} = e^{x+y}(x^2+2mx+m(m-1)+y^2+2ny+n(n-1))$$

11.5.51 $d^{10}u - ?$; $u = \ln(x+y)$

$$d^{10}u = \frac{\partial^{10}u}{\partial x^{10}}dx^{10} + \frac{\partial^{10}u}{\partial x^9 \partial y}dx^9 dy + \dots + \frac{\partial^{10}u}{\partial x \partial y^9}dx dy^9 + \frac{\partial^{10}}{\partial y^{10}}dy^{10}$$

$$\frac{du}{d} = \frac{1}{x+y} \quad \frac{\partial^2 u}{\partial x^2} = \frac{-1}{(x+y)^2} \quad \frac{\partial^3 u}{\partial x^3} = \frac{2}{(x+y)^3} \Rightarrow \frac{\partial^{10}u}{\partial x^{10}} = \frac{-9!}{(x+y)^{10}}$$

$$\frac{\partial^{10}u}{\partial x^9 \partial y} = \left(\frac{8!}{(x+y)^9}\right)'_y = \frac{-9!}{(x+y)^{10}}$$

$$\frac{\partial^{10}\varphi}{\partial x^8 \partial y^2} = \left(\frac{-7!}{(x+8)^8}\right)'_y = \frac{8!}{(x+y)^9} \Rightarrow \frac{\partial^{10}}{\partial x^n \partial y^m} = \frac{(-1)^n n!}{(x+y)^{n+1}}$$

$$d^{10}u = \frac{-9!}{(x+y)^{10}}dx^{10} - \frac{9!}{(x+y)^{10}}dx^9 dy - \frac{9!}{(x+y)^{10}}dx^8 dy^2 - \frac{9!}{(x+y)^{10}}dx^2 dy^8 -$$

$$\frac{9!}{(x+y)^{10}}dx dy^9 - \frac{9!}{(x+y)^{10}}dy^{10} = \frac{-9!}{(x+y)^{10}}(dx^{10}+dx^9 dy + dx dy^9 + dy^{10})$$