23.03. * Практика * Производные. Часть 3.

7.3.26. $\lim\limits_{x\to 0}\left(\frac{1}{x}\right)^{x^2} = [\infty]^0$

$$y = \left(\frac{1}{x}\right)^{x^2} \quad \Big|\ln$$

$$\ln y = \ln\left(\frac{1}{x}\right)^{x^2} \quad \Big|\lim_{x\to 0}$$

$$\lim_{x\to 0}\ln y \overset{*}{=} \lim_{x\to 0}\ln\left(\frac{1}{x}\right)^{x^2}$$

$$* \lim_{x\to 0}\ln\left(\frac{1}{x}\right)^{x^2} = \lim_{x\to 0}\left(x^2\cdot\ln\frac{1}{x}\right) = [0\cdot\infty] =$$

$$= \lim_{x\to 0}\frac{\ln 1/x}{1/x^2} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to 0}\frac{-1/x^2\cdot x}{-2/x^3} = \lim_{x\to 0}\frac{-1/x}{-2/x^3}$$

$$= \lim_{x\to 0}\frac{x^3}{2x} = \lim_{x\to 0}\frac{x^2}{2} = \left[\frac{0}{2}\right] = 0 \quad \Rightarrow$$

$$\Rightarrow \begin{array}{l}\lim\limits_{x\to 0}\ln y = 0\\ \ln y = 0\\ y = 1\end{array} \quad \Bigg| \Rightarrow \quad y = \left(\frac{1}{x}\right)^{x^2} = \underline{\underline{1}}$$

7.3.27. $\lim\limits_{x\to 0} x^{\frac{1}{1+\ln x}} = [0^0]$

$$y = x^{\frac{1}{1+\ln x}} \quad \Big|\ln$$

$$\ln y = \ln x^{\frac{1}{1+\ln x}} \quad \Big|\lim_{x\to 0}$$

$$\lim_{x\to 0}\ln y \overset{*}{=} \lim_{x\to 0}\ln x^{\frac{1}{1+\ln x}}$$

$$* = \lim_{x\to 0}\frac{1}{1+\ln x}\cdot\ln x = \lim_{x\to 0}\frac{\ln x}{1+\ln x} = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{x\to 0}\frac{1/x}{1/x} = \lim_{x\to 0} 1 = 1.$$

$$\lim_{x\to 0}\ln y = 1 \Rightarrow \ln y = 1 \Rightarrow y = \underline{\underline{0}}$$

7.3.28 $P(x) = x^4 - x^3 + 5x^2 - 4x + 1$

$P^{(n)}(x) = 0 \quad \forall n \geq 5 \quad => \quad \frac{P^{(k)}(x_0)}{4!}(x-x_0)^k$, $k \leq 4$

$P(x) = P(1) + \frac{P'(1)}{1!}(x-1) + \frac{P''(1)}{2!}(x-1)^2 +$

$+ \frac{P'''(1)}{3!}(x-1)^3 + \frac{P^{IV}(1)}{4!}(x-1)^4 = [P(1) =$

$= 1^4 - 1^3 + 5 \cdot 1^2 - 4 \cdot 1 + 1 = 2; \quad P''(1) => (P'(x))' =$

$= (x^4 - x^3 + 5x^2 - 4x + 1)'' = (4x^3 - 3x^2 + 10x - 4)' =>$

$=> P'(1) = 4 \cdot 1^3 - 3 \cdot 1^2 + 10 \cdot 1 - 4 = 7; \quad P''(1) => P''(x)$

$= (4x^3 - 3x^2 + 10x - 4)' = 12x^2 - 6x + 10 => P''(1) =$

$= 12 \cdot 1^2 - 6 \cdot 1^2 + 10 = 12 - 6 + 10 = 16 \ldots] => [\frac{7}{1!}; \frac{16}{2!} \ldots] =>$

$=> P(1) = 2 + 7(x-1) + 8(x-1)^2 + 3(x-1)^3 + (x-1)^4$

7.3.29 $P(x) = x^3 + 4x^2 - 6x - 8$, $x_0 = -1$

$P(x_0) = P(-1) + \frac{P'(-1)}{1!}(x+1) + \frac{P''(-1)}{2!}(x+1)^2 +$

$+ \frac{P'''(+1)}{3!}(x-1)^3 = [P'(x) = (x^3 + 4x^2 - 6x - 8)' =$

$= 3x^2 + 8x - 6 => P''(x) = (3x^2 + 8x - 6)' = 6x + 8 =>$

$= P'''(6x + 8)' = 6; \quad P(-1) = -1 + 4 + 6 - 8 = 1; \quad P'(-1) = -11$

$P''(-1) = -6 + 8 = 2; \quad P'''(-1) = -6] =>$

$=> P(-1) = 1 + (x+1)^2 - 11(x+1) - (x-1)^3$

7.3.30 $P(x) = x^5 - 3x^4 + 7x + 2$, $x_0 = 2$, $n = 5$

$$P(x) = P(2) + \frac{P'(2)}{1!}(x-2) + \frac{P''(2)}{2!}(x-2)^2 +$$
$$+ \frac{P'''(2)}{3!}(x-2)^3 + \frac{P^{IV}(2)}{4!}(x-2)^4 + \frac{P^{V}(2)}{5!}(x-2)^5$$

1. $P(2) = 2^5 - 3\cdot 2^4 + 7\cdot 2 + 2 = 32 - 3\cdot 16 + 14 + 2 = 48 - 48 =$

2. $P'(x) = 5x^4 - 12x^3 + 7$

$P'(2) = 5\cdot 2^4 - 12\cdot 2^3 + 7 = 5\cdot 16 - 12\cdot 8 + 7 = 87 - 96 = 9$

3. $P''(x) = 20x^3 - 36x^2$

$P''(2) = 20\cdot 2^3 - 36\cdot 2^2 = 20\cdot 8 - 36\cdot 4 = 160 - 144 = 16$

4. $P'''(x) = 60x^2 - 72x$

$P'''(2) = 60\cdot 2^2 - 72\cdot 2 = 60\cdot 4 - 144 = 240 - 144 = 96$

5. $P^{IV}(x) = 120x - 72$

$P^{IV}(2) = 120\cdot 2 - 72 = 240 - 72 = 168$

6. $P^{V}(x) = 120$ $\quad P^{V}(2) = 120.$

$$P(2) = 0 + 9(x-2) + 8(x-2)^2 + 16(x-2)^3 + 7(x-2)^4 +$$
$$+ (x-2)^5 = -9(x-2) + 8(x-2)^2 + 16(x-2)^3 + 7(x-2)^4 +$$
$$+ (x-2)^5$$

7.3.31. 1. $f(x) = \frac{1}{x}$, $x_0 = 1$

$f'(1) = -1!$, $f''(1) = 2!$, $f'''(1) = -3!$, $f^{IV}(1) = 4!$ ... $f^{(n)}(1) =$

$$\Rightarrow \frac{f^{n}(1)}{n!}(x-1)^n = (-1)^n (x-1)^n \Rightarrow$$

$\Rightarrow \frac{1}{x} = 1 - (x-1) + (x-1)^2 - (x-1)^3 + \dots + (-1)^n (x-1)^n + o((x-1)^n), x \to 1$

2. $f(x) = \text{arctg}\, x$ до $o(x^3)$

$\text{arctg}\, x = \text{arctg}(0) + \frac{\text{arctg}'(0)}{1!}x + \frac{\text{arctg}''(0)}{2!}x^2 + \frac{\text{arctg}'''(0)}{3!}x^3 + o(x^3), \quad x \to 0$

$\text{arctg}(0) = 0, \quad \text{arctg}'(0) = \frac{1}{1+x^2}\Big|_{x=0} = 1$

$\text{arctg}''(0) = \frac{-2x}{(1+x^2)^2}\Big|_{x=0} = 0, \quad \text{arctg}'''(0) = \frac{2(3x^2-1)}{(1+x^2)^3}\Big|_{x=0} = -2 \Rightarrow$

$\Rightarrow \text{arctg}\, x = x - \frac{x^3}{3} + o(x^3).$

7.3.32. $f(x) = 2^x, \quad x_0 = \log_2 3$

$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \dots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + o((x-x_0)^n)$

$f(x_0) = 2^{\log_2 3} = 3$

$f'(x) = 2^x \ln 2 \qquad f'(x_0) = 2^{\log_2 3} \ln 2 = 3\ln 2$

$f''(x) = 2^x \ln^2 2 + 2^x \cdot 0 = 2^x \ln^2 2 \qquad f''(x_0) = 3\ln^2 2$

$f^{(n)}(x) = 2^x \ln^n 2 \qquad f^{(n)}(x_0) = 3 \ln^n 2$

$f(x) = 3 + 3\ln 2\,(x - \log_2 3) + 3\ln^2 2\,(x - \log_2 3)^2 + \dots +$

$+ 3\ln^n 2\,(x - \log_2 3)^n + o((x - \log_2 3)^n), \quad x \to \log_2 3$