$= \frac{1}{2} \ln |x^2+9| - \frac{2}{3} \operatorname{arctg} \frac{x}{3} - \ln|x-1| + \ln|x-2| \ldots$

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8.5.12. $\int \sin^2 x \cos^2 x \, dx = \left[\sin^2 x = \frac{1-\cos 2x}{2} ; \cos^2 x = \frac{1+\cos 2x}{2} \right] =$

$= \int \frac{(1-\cos 2x)\cdot(1+\cos 2x)}{4} dx = \int \frac{1-\cos^2 2x \, dx}{4} = \frac{1}{4} \int dx -$

$-\frac{1}{4}\int \cos^2 2x\,dx = [\cos^2 2x = \frac{1+\cos 4x}{2}] = \frac{1}{4}\int dx -$

$-\frac{1}{4}\cdot\frac{1}{2}\int(1+\cos 4x)\,dx = \frac{1}{4}\int dx - \frac{1}{8}\int dx - \frac{1}{8}\int \cos 4x\,dx$

$= [\,4x = t \to dx = \frac{1}{4}dt\,] = \frac{1}{4}\int dx - \frac{1}{8}\int dx - \frac{1}{32}\int \cos t\,dt$

$= \frac{x}{4} - \frac{x}{8} - \frac{\sin t}{32} + C = \frac{x}{4} - \frac{x}{8} - \frac{\sin 4x}{32} + C =$

$= \frac{8x - 4x - \sin 4x}{32} + C = \frac{4x - \sin 4x}{32} + C$

8.5.14 $\int \cos 2x \cdot \sin 4x\,dx = [\int 2\cos 2x \sin 2x \cos 2x\,dx =$

$= 2\int \cos 2x \sin 2x \cos 2x\,dx = 2\int(1-2\sin^2 x)(1-2\sin^2 x)$

$\cdot 2\sin x\cos x\,dx = 4\int(1-2\sin^2 x - 2\sin^2 x + 4\sin^4 x)$

$\cdot 2\sin x\cos x\,dx = 4(\int \sin x\cos x\,dx - \int 2\sin^3 x\cos x\,dx -$

$-\int 2\sin^3 x\cos x\,dx + \int 4\sin^5 x\cos x\,dx) = 4(\int \frac{1}{2}\sin 2x\,dx -$

$-4\int \sin^3 x\cos x\,dx + 4\int \sin^5 x\cos x\,dx)] = [\sin\alpha\cdot\cos\beta =$

$= \frac{1}{2}(\sin(\alpha-\beta) + \sin(\alpha+\beta))] = \int \frac{1}{2}(\sin 2x + \sin 6x)\,dx$

$= \frac{1}{2}\int \sin 2x\,dx + \frac{1}{2}\int \sin 6x\,dx = -\frac{1}{2}\cdot\frac{1}{2}\cos 2x -$

$-\frac{1}{12}\cos 6x + C = C - \frac{1}{4}\cos 2x - \frac{1}{12}\cos 6x$

$\int dx -$

$\int \cos 4x \, dx$

$\int \cos t \, dt$

$C =$

8.5.15. $\int \sin\frac{x}{2} \sin\frac{3x}{2} \, dx = \left[\sin\alpha \sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)\right] = \int \frac{1}{2}\left(\cos\left(\frac{x}{2} - \frac{3}{2}x\right) - \cos\left(\frac{x}{2} + \frac{3}{2}x\right)\right) dx =$

$= \frac{1}{2}\int (\cos x - \cos 2x) \, dx = \frac{1}{2}\int \cos x \, dx - \frac{1}{2}\int \cos 2x \, dx =$

$= \frac{1}{2}\sin x - \frac{1}{4}\sin 2x + C$

8.5.17 $\int \operatorname{ctg}^3 x \, dx = \int \operatorname{ctg} x \cdot \left(\frac{1}{\sin^2 x} - 1\right) dx =$

$= \int \left(\frac{\cos x}{\sin x} - \operatorname{ctg} x\right) dx = \int \frac{\cos x}{\sin^3 x} \, dx - \int \operatorname{ctg} x \, dx =$

$= [\, t = \sin x \Rightarrow dt = \cos x \, dx \,] = \int \frac{dt}{t^3} - \int \operatorname{ctg} x \, dx =$

$= \frac{t^{-2}}{-2} - \ln|\sin x| + C = -\frac{1}{2\sin^2 x} - \ln|\sin x| + C$

8.5.18. $\int \operatorname{tg}^2 x \, dx = \int \left(\frac{dx}{\cos^2 x} - 1\right) = \int \frac{dx}{\cos^2 x} - \int dx =$

$= \operatorname{tg} x - x + C$