Интегралы часть №4 (1) Шульман Эмиль

8.4.12 $\int\frac{dx}{x+\sqrt[3]{x^2}}=[x=t^3\Rightarrow dx=3t^2dt]=\int\frac{3t^2dt}{t^3+t^2}=3\int\frac{t^2dt}{t^2(t+1)}=3\int\frac{dt}{t+1}=3\ln|t+1|+C=$
$=3\ln|\sqrt[3]{x}+1|+C$

8.4.13 $\int\frac{\sqrt{x}}{1+\sqrt[4]{x^3}}dx=\left[\begin{matrix}n=2\\q=4\end{matrix}\right|\Rightarrow НОК(2;4)=4\Rightarrow x=t^4\Rightarrow dx=4t^3dt]=\int\frac{\sqrt{t^4}}{1+\sqrt[4]{t^{12}}}\cdot 4t^3dt=$
$=4\int\frac{t^2\cdot t^3dt}{1+t^3}=4\int\frac{t^5dt}{t^3+1}=4\int\frac{t^2(t^3+1)\cdot t}{t^3+1}dt=4\left(\int t^2dt-\int\frac{t^2dt}{t^3+1}\right)=4\left(\int t^2dt-\right.$
$\left.-\frac{1}{3}\int\frac{d(t^3+1)}{t^3+1}\right)=4\left(\frac{t^3}{3}-\frac{1}{3}\ln|t^3+1|\right)+C=\frac{4}{3}(x^{\frac{1}{4}})^3-\frac{4}{3}\ln|(x^{\frac{1}{4}})^3+1|+C=$
$=\frac{4}{3}x^{\frac{3}{4}}-\frac{4}{3}\ln|x^{\frac{3}{4}}+1|+C$

8.4.15 $\int\frac{\sqrt{x}\,dx}{x-\sqrt[3]{x^2}}=\left[\begin{matrix}n=2\\q=3\end{matrix}\right|\Rightarrow НОК(2,3)=6\Rightarrow x=t^6\Rightarrow dx=6t^5dt]=\int\frac{t^3\cdot 6t^5dt}{t^6-t^4}=$
$\int\frac{6t^8dt}{t^4(t^2-1)}=6\int\frac{t^4dt}{t^2-1}=6\int\frac{t^4-1+1}{t^2-1}dt=6\int\frac{(t^2-1)(t^2+1)}{t^2-1}dt=6\int\frac{dt}{t^2-1}=6\int t^2dt+6\int dt+$
$+6\int\frac{dt}{t^2-1}=6\frac{t^3}{3}+6t+\frac{6}{2}\ln\left|\frac{t-1}{t+1}\right|+C=2t^3+6t+3\ln\left|\frac{t-1}{t+1}\right|+C=2\sqrt{x}+6\sqrt[6]{x}+3\ln\left|\frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1}\right|+C$

8.4.16 $\int\frac{\sqrt{x}\,dx}{1+\sqrt{x}}=[x=t^2\Rightarrow dx=2tdt]=\int\frac{2t\cdot tdt}{1+t}=2\int\frac{t^2-1+1}{t+1}dt=2\int(t-1)dt+2\int\frac{dt}{t+1}=$
$=2\int tdt-2\int dt+2\int\frac{dt}{t+1}=t^2-2t+2\cdot\ln|t+1|+C=x-2\sqrt{x}+2\ln|\sqrt{x}+1|+C$

8.4.18 $\int\frac{\sqrt{x+2}}{x}dx=[x+2=t^2\Rightarrow x=t^2-2;\,dx=2tdt]=\int\frac{t}{t^2-2}\,tdt=\int\frac{t^2-2+2}{t^2-2}dt=\int dt+$
$+2\int\frac{dt}{t^2-2}=t+\frac{2}{2\sqrt{2}}\ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right|+C=t+\frac{1}{\sqrt{2}}\ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right|+C=\sqrt{x+2}+\frac{1}{\sqrt{2}}\ln\left|\frac{\sqrt{x+2}-\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}\right|+C$

8.4.20 $\int\frac{dx}{(x+1)^{3/2}+(x+1)^{1/2}}=[x+1=t^2;\,dx=2tdt]=2\int\frac{tdt}{t^3+t}=2\int\frac{tdt}{t(t^2+1)}=2\int\frac{dt}{t^2+1}=$
$2\,arctg\,t+C=2\,arctg\sqrt{x+1}+C$

8.4.21 $\int\frac{\sqrt{1+x}+1}{\sqrt{1+x}-1}dx=\int\frac{\sqrt{1+x}-1+2}{\sqrt{1+x}-1}dx=\int dx+2\int\frac{2dx}{\sqrt{1+x}-1}=[1+x=t\Rightarrow$
$\Rightarrow dx=2tdt]=\int dx+4\int\frac{tdt}{t-1}=\int dx+4\int\frac{t-1+1}{t-1}dt=\int dx+4\int dt+4\int\frac{dt}{t-1}=$
$=x+4t+4\ln|t-1|+C=x+4\sqrt{1+x}+4\ln|\sqrt{1+x}-1|+C$

8.4.22. $\int\frac{x-1}{\sqrt{2x-1}}dx=[2x-1=t^2\Rightarrow x=\frac{t^2+1}{2}\Rightarrow dx=\frac{1}{2}\cdot tdt=tdt]=\int\frac{\frac{t^2+1}{2}-1}{t}\cdot tdt=$
$\int\frac{1}{2}(t^2-1)dt=\frac{1}{2}\int t^2dt-\frac{1}{2}\int dt=\frac{1}{2}\cdot\frac{t^3}{3}-\frac{1}{2}t+C=\frac{1}{6}t^3-\frac{1}{2}t+C=\frac{1}{6}\sqrt{(2x-1)^3}-\frac{1}{2}\sqrt{2x+1}+C=$
$=\frac{1}{2}\sqrt{2x-1}\left(\frac{1}{3}(2x-1)-1\right)+C=\sqrt{2x-1}\left(\frac{1}{3}x-\frac{2}{3}\right)+C=\frac{1}{3}\sqrt{2x-1}(x-2)+C$

8.4.23 $\int\frac{dx}{\sqrt{1-2x}-\sqrt[4]{1-2x}}=[1-2x=t^4\Rightarrow x=\frac{1-t^4}{2}\Rightarrow dx=2t^3dt]=\int\frac{2t^3dt}{t^2-t}=\int\frac{2t^2dt}{t-1}=$
$=2\int\frac{t^2-1+1}{t-1}dt==2\int\left(t+1+\frac{1}{t-1}\right)dt=\frac{2t^2}{2}+2t+2\ln|t-1|+C=\sqrt{1+2x}+2\sqrt[4]{1-2x}+2\ln$
$|\sqrt[4]{1-2x}-1|+C$

8.4.24 $\int \frac{1}{(2-x)^2} \cdot \sqrt{\frac{2-x}{2+x}}\, dx = \Big[\frac{2-x}{2+x} = t^2 \Rightarrow x = 2 \cdot \frac{1-t^2}{1+t^2} \Rightarrow dx =$

$= 2 \cdot \frac{-2t(1+t^2) - 2t(1-t^2)\,dt}{(1+t^2)^2} = -4t \cdot \frac{1+t^2+1-t^2}{(1+t^2)^2}\,dt = -\frac{8t\,dt}{(t^2+1)^2} \Big] =$

$= \int \frac{1}{\left(2-2\left(\frac{1-t^2}{1+t^2}\right)\right)^2} \cdot t \left(\frac{-8t\,dt}{(t^2+1)^2} \right) = -\int \frac{2t^2\,dt}{\left(1-\frac{1-t^2}{1+t^2}\right)^2 (t^2+1)^2} = -2\int \frac{t^2\,dt}{\left(\frac{2t^2}{1+t^2}\right)^2 (t^2+1)^2} =$

$= -2\int \frac{1}{4} t^2\,dt = -\frac{1}{2}\int t^2\,dt = -\frac{1}{2} \cdot \frac{t^3}{3} + C = -\frac{1}{6}t^3 + C = C - \frac{1}{6}\sqrt{\left(\frac{2-x}{2+x}\right)^3}$

8.4.34 $\int \sqrt{x}(1+\sqrt{x})^3\,dx = \Big[m = \frac{1}{2};\ n = \frac{1}{2};\ p = 3 \in Z \Rightarrow x = t^2 \Rightarrow dx = 2t\,dt \Big] =$

$= \int t(1+t)^3 \cdot 2t\,dt = 2\int t^2(1+t)^3\,dt = 2\int t^2(t^3+3t^2+3t+1)\,dt = 2\int (t^5+3t^4+3t^3+t^2)\,dt =$

$= 2\left(\frac{t^6}{6} + \frac{3t^5}{5} + \frac{3t^4}{4} + \frac{t^3}{3} \right) + C = \frac{1}{3}x^3 + \frac{6}{5}x^2\sqrt{x} + \frac{3}{2}x^2 + \frac{2}{3}x\sqrt{x} + C$

8.4.35 $\int \sqrt[3]{x^3-4} \cdot x^2\,dx = \int x^2(x^3-4)^{\frac{1}{3}}\,dx = \Big[m = 2;\ n = 3\ \ p = \frac{1}{3} \notin Z \Rightarrow 2)\ \frac{m+1}{n} = \frac{3}{3} =$

$= 1 \in Z \Rightarrow x^3-4 = t^3 \Rightarrow x = \sqrt[3]{t^3+4} \Rightarrow dx = \frac{t^2\sqrt[3]{t^3+4}}{t^3+4}\,dt \Big] = \int \sqrt[3]{(t^3+4)^2}\,(t^3+4-4)^{\frac{1}{3}} \cdot$

$\cdot \frac{t^2\sqrt[3]{t^3+4}}{t^3+4}\,dt = \int t^3\,dt = \frac{t^4}{4} + C = \frac{1}{4}(x^3-4)\sqrt[3]{x^3-4} + C$

8.4.36 $\int \frac{dx}{\sqrt{1-2x-x^2}} = \int \frac{dx}{\sqrt{(1-x)^2}} = \int \frac{dx}{1-x} = -\ln(1-x) + C$

8.4.40 $\int \frac{\sqrt{1-x^2}}{x}\,dx = \int x^{-1}(1-x^2)^{\frac{1}{2}}\,dx = \Big[m = -1,\ n = 2,\ p = \frac{1}{2} \notin Z \Rightarrow 2) =$

$\frac{m+1}{n} = 0 \in Z \Rightarrow 1-x^2 = t^2 \Rightarrow x = \sqrt{1-t^2} \Rightarrow dx = \frac{-t\,dt}{\sqrt{1-t^2}} \Big] = \int \frac{t}{\sqrt{1-t^2}} \cdot \frac{-t\,dt}{\sqrt{1-t^2}} =$

$\int \frac{-t^2\,dt}{1-t^2} = -\int \frac{t^2-1+1}{t^2-1}\,dt = -\int dt - \int \frac{dt}{t^2-1} = -t - \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + C =$

$= -\sqrt{1-x^2} - \frac{1}{2}\ln\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right| + C$

8.4.42 $\int x\sqrt[5]{x-2}\,dx = \Big[m = 1,\ n = 1\ \ p = \frac{1}{5} \notin Z \Rightarrow 2)\ \frac{m+1}{n} = \frac{1+1}{1} = 2 \in Z \Rightarrow$

$\Rightarrow x-2 = t^5;\ x = t^5-2,\ dx = 5t^4\,dt \Big] = \int (t^5-2)t \cdot 5t^4\,dt = 5\int (t^5-2)t^5\,dt =$

$= 5\int (t^{10}-2t^5)\,dt = 5\int t^{10}\,dt - 10\int t^5\,dt = \frac{5t^{11}}{11} - \frac{10t^6}{6} + C = 5t^6\left(\frac{t^5}{11} - \frac{1}{3}\right) + C =$

$= (5x-10)\sqrt{x-2}\left(\frac{3x+5}{33}\right) + C$