Практика. Интегрирование. Часть 3.

8.3.14. $\int \frac{x+2}{x^2-6x+5}\,dx = \Big[x^2-6x+5=0 \Rightarrow D = 9-1\cdot5=4 \Rightarrow$

$\Rightarrow x_1 = 5,\quad x_2 = 1 \Rightarrow x^2-6x+5 = (x-5)(x-1) \Big] =$

$= \int \frac{x+2}{(x-5)(x-1)}\,dx = \Big[\frac{x+2}{(x-5)(x-1)} = \frac{A}{x-5} + \frac{B}{x-1} \Rightarrow$

$\Rightarrow x+2 = A(x-1)+B(x-5) = Ax - A + Bx - 5B =$

$= (A+B)x + (-A-5B) \Rightarrow \begin{cases} A+B=1 \\ -A-5B=2 \end{cases} \Rightarrow -4B = 3 \Rightarrow B=$

$\Rightarrow A - \frac{3}{4} = 1 \Rightarrow A = \frac{7}{4} \Big] = \int\left(\frac{7}{4(x-5)} - \frac{3}{4(x-1)}\right)dx =$

$= \frac{7}{4}\int\frac{dx}{x-5} - \frac{3}{4}\int\frac{dx}{x-1} = \frac{7}{4}\ln|x-5| -$

$- \frac{3}{4}\ln|x-1| + C$

8.3.15 $\int \frac{dx}{x^4+x^2} = \Big[x^4+x^2=0 \Rightarrow x^2(x^2+1)=0 \Rightarrow x^2=0$

ии $x^2+1=0 \Rightarrow x=0 \Big] = \int \frac{dx}{x^2(x^2+1)} = \Big[\frac{1}{x^2(x^2+1)} = \frac{A}{x^2} +$

$+ \frac{B}{x^2+1} \Rightarrow \quad 1 = A(x^2+1) + Bx^2 = Ax^2 + A + Bx^2 = x^2(A+B) + A \Rightarrow$

$\Rightarrow \begin{cases} A+B=0 \\ A=1 \end{cases} \Rightarrow B=-1 \Big] = \int \left(\frac{1}{x^2} - \frac{1}{x^2+1} \right) dx =$

$= \int \frac{dx}{x^2} - \int \frac{dx}{x^2+1} = \frac{1}{x} - \operatorname{arctg} x + C$

8.3.16 $\int \frac{x^5+x^4-8}{x^3-4x}\,dx = \int \frac{x^5+x^4-8}{x(x^2-4)}\,dx = \int \frac{x^5+x^4-8}{x(x-2)(x+2)}\,dx$

$\Rightarrow$

$$\begin{array}{r|l} x^5+x^4+0x^3+0x^2+0x-8 & x^3+0x^2-4x \\ \cline{2-2} -\;x^5+0x^4-4x^3 \qquad\qquad & x^2+x+4 \\ \hline x^4+4x^3+0x^2 \qquad\quad & \\ -\;x^4+0x^3-4x^2 \qquad\quad & \\ \hline 4x^3+4x^2+0x \qquad & \\ -\;4x^3+0x^2-16x \qquad & \\ \hline 4x^2+16x-8 \qquad & \end{array}$$

$\int \frac{x^5+x^4-8}{x^3-4x}\,dx = \int \left(\frac{(x^3-4x)(x^2+x+4)}{x^3-4x} + 4\frac{x^2+4x-2}{x^3-4x} \right) dx =$

$= \int (x^2+x+4)\,dx + 4\int \frac{x^2+4x-2}{x^3-4x}\,dx = \Big[^{1)} \frac{x^2+4x-2}{x(x-2)(x+2)} =$

$= \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2} \Rightarrow x^2+4x-2 = Ax^2 - 4A + Bx^2 + 2Bx +$
$+ Cx^2 - 2Cx = x^2(A+B+C) + x(2B-2C) - 4A \Rightarrow$

$\Rightarrow \begin{cases} A+B+C=1 \\ 2B-2C=4 \\ -4A=-2 \end{cases} \Rightarrow \begin{cases} A=\frac{1}{2} \\ B=C+2 \\ C+2+C=\frac{1}{2} \end{cases} \Rightarrow \begin{array}{l} A=\frac{1}{2} \\ C=-\frac{3}{4} \\ B=\frac{5}{4} \end{array} \Big] =$

$= \int (x^2+x+4)\,dx + 4\int \left(\frac{1}{2x} + \frac{5}{4(x-2)} - \frac{3}{4(x+2)}\right) dx =$

$= \int x^2 dx + \int x\,dx + 4\int dx + 2\int \frac{dx}{x} + 5\int \frac{dx}{x-2} - 3\int \frac{dx}{x+2}$

$= \frac{x^3}{3} + \frac{x^2}{2} + 4x + 2\ln|x| + 5\ln|x-2| - 3\ln|x+2| + C$

8.3.17. $\int \frac{dx}{x^3-8} = \int \frac{dx}{(x-2)(x^2+2x+4)} = \Big[\frac{A}{x-2} +$

$+ \frac{Bx+C}{x^2+2x+4} \Rightarrow A(x^2+2x+4) + (Bx+C)(x-2) = 1 \Rightarrow$

$\Rightarrow x^2(A+B) + x(2A-2B+C) + (4A-2C) \Rightarrow$

$\Rightarrow \begin{cases} A+B=0 \\ 2A-2B+C=0 \\ 4A-2C=1 \end{cases} \Rightarrow \begin{cases} A=-B \\ -2B-2B+C=0 \\ C=0 \end{cases} \Rightarrow \begin{cases} A=\frac{1}{4} \\ C=0 \\ B=-\frac{1}{4} \end{cases} \Big] =$

$= \frac{1}{4}\left(\int \frac{d(x-2)}{x-2} - \int \frac{x\,dx}{x^2+2x+4}\right) = \frac{1}{4}\left(\int \frac{d(x-2)}{x-2} -\right.$

$\left. - \frac{1}{2}\int \frac{(2x+2)\,dx}{x^2+2x+4} + \int \frac{dx}{x^2+2x+4}\right) = \Big[2)\ t = x^2+2x+4 \Rightarrow$
$\Rightarrow dt = (2x+2)\,dx;$

$3)\ y = x+1 \Rightarrow dy = dx$
$a = \sqrt{3}$ $\Big] = \frac{1}{4}\left(\int \frac{d(x-2)}{x-2} - \frac{1}{2}\int \frac{dt}{t} + \int \frac{dy}{y^2+3}\right) =$

$= \frac{1}{4}\ln|x-2| - \frac{1}{8}\ln|t| + \frac{1}{4}\cdot\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{y}{\sqrt{3}} + C =$

$= \frac{1}{4}\ln|x-2| - \frac{1}{8}\ln|x^2+2x+4| + \frac{\sqrt{3}}{12}\operatorname{arctg}\frac{x+1}{\sqrt{3}} + C$

8.3.18. $\int \frac{7x^3-10x^2+50x-7}{(x^2+9)(x^2+x-2)}\,dx = \Big[x^2+9=0$ к.н

$x^2+x-2=0 \Rightarrow D=9 \Rightarrow x_1 = 1;\ x_2 = -2 \Rightarrow (x-1)(x+2)$

$$= \int \frac{7x^3 - 10x^2 + 50x - 77}{(x^2+9)(x-1)(x+2)} = \left[\frac{Ax+B}{x^2+9} + \frac{C}{x-1} + \frac{D}{x+2} \Rightarrow \right.$$

$$\Rightarrow 7x^3 - 10x^2 + 50x + 77 = Ax^3 + Bx^2 + Ax^2 + Bx - 2Ax -$$

$$-2B + Cx^3 + 2Cx^2 + 9Cx + 18C + Dx^3 - Dx^2 + 9Dx - 9D =$$

$$= x^3(A+C+D) + x^2(A+B+2C-D) +$$

$$+ x(-2A + B + 9C + 9D) + (-2B + 18C - 9D) \Rightarrow$$

$$\Rightarrow \begin{cases} A + C + D = 7 \\ -2A + B + 9C + 9D = 50 \\ A + B + 2C - D = -10 \\ -2B + 18C - 9D = 77 \end{cases} \Rightarrow \begin{cases} A = 7 - C - D \\ B = -17 - C + 2D \\ -2(7 - C - D) + 2D - C - 17 + 9C + 9D = 50 \\ -2(2D - C - 17) + 18C - 9D = -77 \end{cases}$$

$$\Rightarrow \begin{cases} 10C + 13D = 81 \\ 20C - 13D = -111 \end{cases} \Rightarrow \begin{matrix} 30C = -30 \\ C = -1 \end{matrix}, \; D = 7; \; B = -2; \; A = 1; \left.\right] =$$

$$= \int \frac{(x-2)\,dx}{x^2+9} + \int \frac{-dx}{x+1} + \int \frac{7\,dx}{x+2} = \int \frac{x\,dx}{x^2+9} -$$

$$- 2\int \frac{dx}{x^2+9} - \int \frac{d(x-1)}{x-1} + 7\int \frac{d(x+2)}{x+2} = \frac{1}{2}\int \frac{2x\,dx}{x^2+4} -$$

$$- 2\int \frac{dx}{x^2+9} - \int \frac{d(x-1)}{x-1} + 7\int \frac{d(x+2)}{x+2} = \left[2x\,dx = d(x^2+9) \right] =$$

$$= \frac{1}{2} \ln|x^2+9| - \frac{2}{3} \operatorname{arctg} \frac{x}{3} - \ln|x-1| + 7\ln|x+2| + C$$