5. **Практика** **Простейшие интегралы**

8.1.2 $\int x^{10}dx = \left[\int x^{\alpha}dx = \frac{x^{\alpha+1}}{\alpha+1} + C\right] = \frac{x^{10+1}}{10+1} = \frac{x^{11}}{11} + C$

8.1.3 $\int \sqrt[4]{x}\,dx = \int x^{1/4}dx = x^{5/4}/5/4 + C = \frac{4}{5}x^{5/4}$

8.1.4 $\int dx/x^7 = x^{-6}/-6 + C = -1/6x^6 + C$

8.1.5 $\int \frac{dx}{x^2+9} = \left[\frac{dx}{x^2+\alpha^2}\ \operatorname{arctg}\frac{x}{a} + C,\ a \neq 0,\ a=3\right] = \frac{1}{3}\operatorname{arctg}\frac{x}{3} + C$

8.1.6 $\int \frac{dx}{x^2-\frac{1}{2}} = \left[\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C,\ a \neq 0,\ a=\sqrt{\frac{1}{2}}\right.$

$= \frac{1}{2\sqrt{1/2}}\cdot\ln\left|\frac{x-\sqrt{1/2}}{x+\sqrt{1/2}}\right| + C = \frac{\sqrt{2}}{2}\ln\left|\frac{(\sqrt{2}x-1)\sqrt{2}}{\sqrt{2}(\sqrt{2}x+1)}\right| = \frac{\sqrt{2}}{2}\ln\left|\frac{\sqrt{2}x-1}{\sqrt{2}x+1}\right|$

8.1.7. $\int \frac{dx}{\sqrt{x^2+3}} = \ln\left|x+\sqrt{x^2+3}\right| + C$

8.1.9 $\int \frac{x^4+x^2-6x}{x^3}dx = \int\left(\frac{x^4}{x^3}+\frac{x^2}{x^3}-\frac{6x}{x^3}\right)dx = \int x\,dx +$

$+ \int\frac{dx}{x} - 6\int\frac{dx}{x^2} = \frac{x^2}{2} + \ln|x| + \frac{6}{x} + C$

8.1.10. $\int\left(\frac{5}{x}-\frac{10}{\sqrt[4]{x^3}}-\frac{3}{x^2+7}\right)dx = 5\int\frac{dx}{x}-10\int\frac{dx}{\sqrt[4]{x^3}}-3\int\frac{dx}{x^2+7} =$

$= 5\ln|x| - 10\frac{x^{1/4}}{1/4} - \frac{3}{\sqrt{7}}\operatorname{arctg}\frac{x}{\sqrt{7}} + C = 5\ln|x| + 40\sqrt[4]{x} - \frac{3}{\sqrt{7}}\operatorname{arctg}\frac{x}{\sqrt{7}} + C$

8.1.11. $\int\sqrt{x}\,(x^2+1)dx = \int(x^2\sqrt{x}+\sqrt{x})dx = \int x^2\cdot x^{1/2}dx + \int x^{1/2}dx =$

$= \int x^{2.5}dx + \int x^{0.5}dx = \frac{x^{3.5}}{3.5} + \frac{x^{1.5}}{1.5} + C = \frac{2x^{7/2}}{7} + \frac{2x^{3/2}}{3} + C$

8.1.12. $\int\frac{3+\sqrt{4-x^2}}{\sqrt{4-x^2}}dx = \int\frac{3}{\sqrt{4-x^2}}dx + \int\frac{\sqrt{4-x^2}}{\sqrt{4-x^2}}dx = 3\int\frac{dx}{\sqrt{4-x^2}} + \int dx =$

$= 3\arcsin\frac{x}{2} + x + C$

8.1.13. $\int\frac{(x^3+2)^2}{\sqrt{x}}dx = \int\frac{x^6+4x^3+4}{\sqrt{x}}dx = \int\frac{x^6}{\sqrt{x}}dx + 4\int\frac{x^3}{\sqrt{x}}dx + 4\int\frac{dx}{\sqrt{x}} =$

$= \int x^6\cdot x^{-1/2}dx + 4\int x^3\cdot x^{-1/2}dx + 4\int dx = \int x^{5\frac{1}{2}}dx + 4\int x^{2\frac{1}{2}}dx + 4\int dx\,x^{-1/2}$

$= \frac{x^{6\frac{1}{2}}}{6\frac{1}{2}} + \frac{4x^{3\frac{1}{2}}}{3\frac{1}{2}} + 4\cdot 2\sqrt{x} + C = \frac{2x^{\frac{13}{2}}}{13} + \frac{8x^{\frac{7}{2}}}{7} + 8\sqrt{x} + C$

8.1.14 $\int\left(4\sin x + 8x^3 - \frac{11}{\cos^2 x}\right)dx = 4\int\sin x\,dx + 8\int x^3dx - 11\int\frac{dx}{\cos^2 x} =$

$= -4\cos x + \frac{8x^4}{4} - 11\operatorname{tg}x + C = -4\cos x + 2x^4 - 11\operatorname{tg}x + C$

8.1.16. $\int\cos 2x\,dx = \frac{1}{2}\cdot 2\int\cos 2x\,dx = \frac{1}{2}\int\cos(2x)\,2dx =$

$= \frac{1}{2}\int\cos 2x\,d2x = \frac{1}{2}\sin 2x + C$

8.1.17. $\int(9x+2)^{17}dx = [t = 9x+2 \Rightarrow dt = d(9x+2) \Rightarrow (9x+2)'_x = 9dx] =$

$= \int t^{17}\cdot\frac{dt}{9} = \frac{1}{9}\int t^{17}dt = \frac{t^{18}}{9\cdot 18}\cancel{dt} + C = \frac{(9x+2)^{18}}{162} + C$

8.1.18 I $\int \frac{dx}{8x-1} = [t = 8x-1 \Rightarrow dt = d(8x-1) = (8x-1)'_x dx = 8dx \Rightarrow dx = \frac{1}{8}dt]$

$= \frac{1}{8}\int \frac{dt}{t} = \frac{1}{8} \ln|t| + C = \frac{\ln|8x-1|}{8} + C$

II

$= \frac{1}{8} \cdot 8 \int \frac{dx}{8x-1} = \frac{1}{8}\int \frac{8dx}{8x-1} = \frac{1}{8}\int \frac{d(8x-1)}{8x-1} = \frac{\ln|8x-1|}{8} + C$

8.1.19 $\int 4^{3-5x} dx = [t = 3-5x \Rightarrow dt = d(3-5x) = (3-5x)'_x dx =$

$= -\frac{1}{5}dt] = -\frac{1}{5}\int 4^t dt = \frac{-4^t}{5\ln 4} + C = \frac{-4^{3-5x}}{5\ln 4} + C$

8.1.20 $\int \sqrt{3x+4}\, dx = [t = 3x+4 \Rightarrow dt = d(3x+4) = \frac{1}{3}dt = dx] =$

$= \frac{1}{3}\int \sqrt{t}\, dt = \frac{1}{3} \cdot \frac{t^{1\frac{1}{2}}}{1\frac{1}{2}} + C = \frac{2t^{3/2}}{9} + C = \frac{2}{9}\sqrt[2]{(3x+4)^3} + C$

8.1.25. $\int \frac{dx}{3x^2-25} = \left[\int \frac{dx}{3x^2-a^2}\right] = \int \frac{dx}{3(x^2-25/3)} = \frac{1}{3}\int \frac{dx}{x^2-\frac{25}{3}} =$

$= \frac{1}{3}\int \frac{dx}{x^2-\left(\frac{5}{\sqrt{3}}\right)^2} = \frac{1}{3} \cdot \frac{1}{2 \cdot \frac{5}{\sqrt{3}}} \ln\left|\frac{x-5/\sqrt{3}}{x+5/\sqrt{3}}\right| + C = \frac{\sqrt{3}}{30} \ln\left|\frac{\cancel{5}\sqrt{3}x-5}{\cancel{5}\sqrt{3}x+5}\right| + C$

8.1.23 $\int \cos^2 x\, dx = [\cos^2 \alpha = (1+\cos 2\alpha)/2] = \int \frac{1+\cos 2x}{2} dx =$

$= \frac{1}{2}\int dx + \frac{1}{2}\int \cos 2x\, dx = \frac{1}{2}\int dx + \frac{1}{2} \cdot \frac{1}{2}\int \cos 2x\, d2x = \frac{x}{2} + \frac{1}{4}\sin 2x + C$

8.1.24. $\int \frac{x-2}{x+3} dx = \int \frac{x+3-3-2}{x+3} dx = \int \frac{(x+3)-5}{(x+3)} dx =$

$= \int \frac{x+3}{x+3} dx - 5\int \frac{dx}{x+3} = \int dx - 5\int \frac{dx}{x+3} = \int dx - 5\int \frac{d(x+3)}{x+3} =$

$= x - 5\ln|x+3| + C$

8.1.25. $$\int \frac{x^2 dx}{x^2-9} = \int \frac{x^2-9+9}{x^2-9} dx = \int \frac{x^2-9}{x^2-9}dx + \int \frac{9}{x^2-9} dx =$$

$$= \int dx + 9\int \frac{dx}{x^2-9} = x + 9 \cdot \frac{1}{2 \cdot 3} \ln \left| \frac{x-3}{x+3} \right| + C =$$

$$= x + \frac{3}{2} \ln \left| \frac{x-3}{x+3} \right| + C$$

8.1.26. $$\int \frac{5+\sin^3 x}{\sin^2 x} dx = 5\int \frac{dx}{\sin^2 x} + \int \frac{\sin^3 x}{\sin^2 x} dx =$$

$$= 5\int \frac{dx}{\sin^2 x} + \int \sin x \, dx = -5 \operatorname{ctg} x - \cos x \, dx + C$$

8.2.2. $$\int \sqrt{4x-5}\, dx = [t = 4x-5, \Rightarrow dt = d(4x-5) = (4x-5)'_x dx =$$

$$= 4dx \Rightarrow dx = \frac{1}{4} dt] = \int \sqrt{t} \cdot \frac{1}{4} dt = \frac{1}{4}\int \sqrt{t}\, dt = \frac{1}{4} \int t^{\frac{1}{2}} dt =$$

$$= \frac{1}{4} \cdot \frac{t^{\frac{3}{2}}}{3/2} = \frac{\sqrt{t^3}}{6} + C = \frac{t}{6}\sqrt{t} + C = \frac{4x-5}{6}\sqrt{4x-5} + C$$

8.2.3 $$\int \frac{dx}{(3x+2)^4} = [t = 3x+2,\ dt = d(3x+2) = 3dx \Rightarrow dx = \frac{1}{3} dt] =$$

$$= \frac{1}{3}\int \frac{dt}{t^4} = \frac{1}{3} \int t^{-4} dt = \frac{1}{3} \cdot \frac{t^{-3}}{-3} + C = -\frac{1}{9(3x+2)^3} + C$$

8.2.4. $$\int \sin^3 x \cos x \, dx = [t = \sin x \Rightarrow dt = d(\sin x) = \cos x \, dx] = \int t^3 dt =$$

$$= \frac{t^4}{4} + C = \frac{1}{4}\sin^4 x + C = [\int \sin^3 x \, d(\sin x) = \frac{(\sin x)^{3+1}}{3+1} + C = \frac{1}{4}\sin^4 x + C]$$

8.2.5. $$\int e^{x^3} x^2 dx = [t = x^3 \Rightarrow dt = d(x^3) = (x^3)'_x dx \Rightarrow 3x^2 dx \Rightarrow$$

$$\Rightarrow x^2 dx = \frac{1}{3} dt] = \int e^t \frac{1}{3} dt = e^t/3 + C = \frac{1}{3} e^{x^3} + C$$

8.2.6. $$\int \frac{\ln^5 x \, dx}{x} = [t = \ln x \Rightarrow dt = d(\ln x) = (\ln x)'_x dx = \frac{1}{x} dx = \frac{dx}{x}] =$$

$$= \int t^5 dt = \frac{t^6}{6} + C = \frac{\ln^6 x}{6} + C$$

8.2.7. $\int \frac{\sin x\,dx}{\cos x+1} = \left[t = \cos x+1 \Rightarrow dt = d(\cos x+1) = (\cos x+1)'_x dx = -\sin x\,dx \Rightarrow \sin x\,dx = -dt \right]$

$= \int \frac{-dt}{\cos x+1} = -\int \frac{dt}{t} = -\ln|t| + C = -\ln|\cos x+1| + C$

8.2.8. $\int \frac{x^2 dx}{x^3+1} = \left[t = x^3+1 \Rightarrow dt = d(x^3+1) = (x^3+1)'_x dx = 3x^2 dx \Rightarrow x^2 dx = \frac{dt}{3} \right]$

$= \frac{1}{3}\int \frac{dt}{t} = \frac{1}{3}\ln|t| + C = \frac{1}{3}\ln|x^3+1| + C$

8.2.9. $\int \frac{\operatorname{arctg} x\,dx}{x^2+1} = \left[t = \operatorname{arctg} x \Rightarrow dt = (\operatorname{arctg} x)'_x dx = \frac{1}{1+x^2}dx = \frac{1}{x^2+1}dx = \frac{dx}{x^2+1} \right] = \int t\,dt =$

$= \frac{t^2}{2} + C = \frac{\operatorname{arctg}^2 x}{2} + C$