интегралы часть №2 ① Шульмохн

8.2.33 $\int \cos(6x+1)dx = [t=6x+1 \Rightarrow dt = d(6x+1) = 6dx \Rightarrow dx = dt/6] = \int \cos t \cdot \frac{1}{6}dt = \frac{1}{6}\int \cos dt = \frac{1}{6}\sin t + C = \frac{1}{6}\sin(6x+1)+C$

8.2.34 $\int \frac{dx}{\sqrt[3]{(5x-2)^4}} = [t=5x-2 \Rightarrow dt = 5dx \Rightarrow dx = \frac{1}{5}dt] = \frac{1}{5}\int \frac{dt}{\sqrt[3]{t^4}} = \frac{1}{5}\int t^{-\frac{4}{3}}dt = \frac{1}{5}\cdot\frac{t^{-\frac{4}{3}+1}}{-4/3+1} + C = \frac{1}{5}\cdot\frac{t^{-1/3}}{-1/3} + C = \frac{-3}{5\sqrt[3]{t}} + C$

8.2.35 $\int \frac{\sqrt{tg\,x}\,dx}{\cos^2 x} = [t = \frac{1}{\cos^2 x} \Rightarrow dt = d(\frac{1}{\cos^2 x});\ t = tg\,x \Rightarrow dt = d(tg\,x) = \frac{1}{\cos^2}dx] = \int \sqrt{t}\,dt = \int t^{\frac{1}{2}}dt = \frac{t^{1/2+1}}{1/2+1} + C = \frac{t^{3/2}}{3/2} + C = \frac{2\sqrt{t^3}}{3} + C = \frac{2tg\,x\sqrt{tg\,x}}{3} + C$

8.2.36 $\int \frac{e^x dx}{e^{2x}+9} = [t = e^{2x}+9 \Rightarrow dt = d(e^{2x}+9) = 2e^{2x}dx \Rightarrow] = \int \frac{dt}{2e^x t} = [t = e^x \Rightarrow dt = e^x dx] = \int \frac{dt}{t^2+9} = \int \frac{dt}{t^2+3^2} = \frac{1}{3}arctg\frac{t}{3} + C = \frac{1}{3}arctg\frac{e^x}{3} + C$

8.2.37 $\int \frac{x^5 dx}{\sqrt{x^6+7}} = [t = x^6+7 \Rightarrow dt = 6x^5dx \Rightarrow x^5dx = \frac{dt}{6}] = \frac{1}{6}\int \frac{dt}{\sqrt{t}} = \frac{1}{6}\cdot 2\sqrt{t} + C = \frac{\sqrt{x^6+7}}{3} + C$

8.2.39 $\int \frac{(2x+3)dx}{(x^2+3x-1)^4} = [t = x^2+3x-1 \Rightarrow dt = (2x+3)dx] = \int \frac{dt}{t^4} = \int t^{-4}dt = \frac{t^{-4+1}}{-4+1} + C = \frac{(x^2+3x-1)^{-3}}{-3} + C = \frac{-1}{3(x^2+3x-1)^3} + C$

8.2.40 $\int \cos^{11}2x \cdot \sin 2x\,dx = [t = \cos 2x \Rightarrow dt = -2\sin x\,dx \Rightarrow \sin 2x\,dx = \frac{dt}{-2}] = \int t^{11}dt/-2 = -\frac{1}{2}\int t^{11}dt = \frac{-1}{2}\cdot\frac{t^{11+1}}{11+1} + C = \frac{-t^{12}}{2\cdot 12} + C = \frac{-\cos^{12}2x}{24} + C$

8.2.41 $\int \frac{7^{\sqrt{x}}dx}{\sqrt{x}} = [t = \sqrt{x} \Rightarrow dt = \frac{1}{2\sqrt{x}}dx = \frac{1}{2\sqrt{x}}dx \Rightarrow \frac{dx}{\sqrt{x}} = 2dt] = \int 7^t \cdot 2dt = 2\int 7^t dt = \frac{2\cdot 7^t}{\ln 7} + C = \frac{2\cdot 7^{\sqrt{x}}}{\ln 7} + C$

8.2.42 $\int \frac{e^{1/x}}{x^2}dx = [t = \frac{1}{x} \Rightarrow dt = \frac{-1}{x^2}dx \Rightarrow \frac{dx}{x^2} = -dt] = \int e^t\cdot(-dt) = -\int e^t dt = -e^t + C = -e^{1/x} + C$

8.2.43 $\int \frac{\ln 5x\,dx}{x} = [t = \ln 5x \Rightarrow dt = \frac{5dx}{x} \Rightarrow \frac{dx}{x} = \frac{dt}{5}] = \int \frac{t\,dt}{5} = \frac{1}{5}\int t\,dt = \frac{1\cdot t^2}{5\cdot 2} + C = \frac{\ln^2 5x}{10} + C$

8.2.44 $\int ctg\,x\,dx = \int \frac{\cos x}{\sin x}dx = [t = \sin x \Rightarrow dt = \cos x\,dx] = \int \frac{dt}{t} = \ln|x| + C = \ln|\sin x| + C$

8.2.48 $\int \frac{x\,dx}{x^4+1} = [t^2 = x^4+1 \Rightarrow dt = 4x^3dx;\ \overset{II}{;}\ t = x^2 \Rightarrow dt = 2x\,dx \Rightarrow x\,dx = \frac{dt}{2}] = \frac{1}{2}\int \frac{dt}{t^2+1} = \frac{1}{2}\, arctg\, x^2 + C$

8.2.49 $\int e^{-x^3} x^2 dx = [t = -x^3 \Rightarrow dt = -3x^2dx \Rightarrow x^2dx = \frac{dt}{-3}] = \frac{1}{3}\int e^t\cdot dt = -\frac{1}{3}e^t + C = -\frac{1}{3}e^{-x^3} + C$

8.2.50 $\int \frac{x^2dx}{\sqrt{x^6-4}} = [t = x^3 \Rightarrow dt = 3x^2dx \Rightarrow x^2dx = \frac{dt}{3}] = \frac{1}{3}\int \frac{dt}{\sqrt{t^2-4}} = \frac{1}{3}\cdot \arcsin\frac{t}{2} + C\ \frac{1}{3}\arcsin\frac{x^3}{2} + C$

8.2.55 $\int \frac{3\sqrt{x} - 2\cos\frac{1}{x^2}}{x^3}dx = \int \frac{3\sqrt{x}}{x^3}dx - \int \frac{2\cos\frac{1}{x^2}}{x^3}dx = 3\int x^{1/2}\cdot x^{-3}dx - 2\int \frac{\cos\frac{1}{x^2}}{x^3}dx = 3\int x^{-2\frac{1}{2}}dx - 2\int x^{-3}\cos x^{-2}dx =$
$= 3\int x^{-\frac{5}{2}}dx + \int -2x^{-3}\cos x^{-2}dx = [t = x^{-2} \Rightarrow dt = \frac{-2}{x^3}dx] =$
$= 3\cdot\frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + \sin x^{-2} + C = \frac{-2}{x\sqrt{x}} + \sin\frac{1}{x^2} + C$

8.2.56 $\int \frac{7x+2}{\sqrt{x^2+10}}dx\ \int \frac{7x}{\sqrt{x^2+10}}dx + 2\int \frac{dx}{\sqrt{x^2+10}} = [t = x^2+10 \Rightarrow dt = 2x\,dx]$
$3.5\int \frac{2x\,dx}{\sqrt{x^2+10}} + 2\int \frac{dx}{\sqrt{x^2+10}} = 3.5\int \frac{dt}{\sqrt{t}} + 2\int \frac{dx}{\sqrt{x^2+10}} = 3.5\cdot 2\sqrt{t} + 2\cdot\ln|x + \sqrt{x^2+10}| + C =$
$= 7\sqrt{x^2+10} + 2\ln|x + \sqrt{x^2+10}| + C$

8.2.57 $\int \frac{dx}{e^x + e^{-x}} = \int \frac{sh\,x}{2}dx = \frac{1}{2}\int sh\,x\,dx = \frac{1}{2}ch\,x + C$

8.2.60 $\int \frac{1-6x}{(x+1)(x-1)}dx = \int \frac{(1-6x)dx}{x^2-1} = \int \frac{dx}{x^2-1} + \int \frac{-6x\,dx}{x^2-1} = [t = x^2-1 \Rightarrow dt = 2x\,dx]$
$= \int \frac{dx}{x^2-1} - 3\int \frac{dt}{t} = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| - 3\ln|x^2-1| + C$

8.2.61 $\int (\cos^2x - \sin^2x)\sqrt[3]{1+\sin^2x}\,dx = \int \cos 2x\sqrt[3]{1+\sin 2x}\,dx =$
$= [t = 1+\sin 2x \Rightarrow dt = 2\cos 2x\,dx \Rightarrow \cos 2x\,dx = \frac{dt}{2}] = \frac{1}{2}\int \sqrt[3]{t}\,dt =$
$\frac{1}{2}\int t^{\frac{1}{3}}dt = \frac{1}{2}\cdot\frac{t^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C = \frac{1}{2}\cdot\frac{t^{4/3}}{4/3} + C = 3\cdot\frac{1}{2}\cdot\frac{t\sqrt[3]{t}}{4} + C = \frac{3(1+\sin 2x)\sqrt[3]{1+\sin 2x}}{8}$

8.2.69 $\int x\ln x\,dx = [u = \ln x \Rightarrow u' = \frac{1}{x}\ \ v' = x \Rightarrow \int x\,dx = \frac{x^2}{2}] =$
$= \ln x\cdot\frac{x^2}{2} - \int \frac{x^2}{2}\cdot\frac{1}{x}dx = \frac{x^2\ln x}{2} - \int \frac{x}{2}dx = \frac{x^2\ln x}{2} - \int \frac{x}{2}dx = \frac{x^2\ln x}{2} - \frac{1}{2}\int x\,dx$
$\frac{x^2\ln x}{2} - \frac{1}{2}\cdot\frac{x^2}{2} + C = \frac{x^2\ln x}{2} - \frac{x^2}{4} + C = \frac{2x^2\ln x - x^2}{4} + C = \frac{x^2(2\ln x - 1)}{4} + C$

8.2.70 $\int(2x+3)\cos x\,dx=[u=2x+3\Rightarrow u'=2;\ v'=\cos x\Rightarrow v=\sin x]=$ ③

$=(2x+3)\sin x-\int\sin x\cdot 2dx=(2x+3)\sin x-2\int\sin x\,dx=(2x+3)\sin x-2\cos x+C$

8.2.71 $\int x\cdot \operatorname{sh}5x\,dx=[u=\operatorname{sh}5x\Rightarrow u'=5\operatorname{ch}5x;\ v'=x\Rightarrow v=\frac{x^2}{2}]=$

$=\operatorname{sh}5x\cdot\frac{x^2}{2}-\int\frac{x^2}{2}\cdot 5\operatorname{ch}5x\,dx=\frac{x^2\operatorname{sh}5x}{2}-\frac{5}{2}\int x^2\operatorname{ch}5x\,dx=$

$=[u=x\Rightarrow u'=1;\ v'=\operatorname{sh}5x\Rightarrow v=\frac{1}{5}\operatorname{ch}5x]=\frac{x\operatorname{ch}5x}{5}-\frac{1}{5}\int\operatorname{ch}5x\,dx=$

$=\frac{x\operatorname{ch}5x}{5}+\frac{1}{5}\cdot\frac{1}{5}\operatorname{sh}5x+C=\frac{5x\operatorname{ch}-\operatorname{sh}5x}{25}+C$

8.2.72 $\int\frac{x\cos x}{\sin^3x}dx=[u=x\cos x\Rightarrow u'=\cos x-x\sin x,\ v=\frac{1}{\sin^3x}\Rightarrow$

$=v=\int\frac{1}{\sin^3x}dx=[t=\sin x\Rightarrow dt=\cos x\,dx\Rightarrow dx=\frac{dt}{\cos x}\Rightarrow\sin^3x=$

$=\frac{3\sin x-\sin 3x}{4}]=\int\frac{4dx}{3\sin x-\sin 3x}=[u=x\Rightarrow u'=1;\ v'=\operatorname{ctg}x/\sin^2x\Rightarrow$

$\Rightarrow u'=1\ v=\int\frac{\operatorname{ctg}x\,dx}{\sin^2x}=[t=\operatorname{ctg}x\Rightarrow dt=\frac{dx}{\sin^2x}\Rightarrow -dt=\frac{dx}{\sin^2x}]=-\frac{1}{2}t^2+C=$

$=-\frac{1}{2}\operatorname{ctg}^2x+C]=-\frac{1}{2}x\operatorname{ctg}^2x-\int-\frac{1}{2}\operatorname{ctg}^2x\,dx=[x=\operatorname{arcctg}t;\ dx=-dt/(1+t^2)]=$

$\int\operatorname{ctg}^2x\,dx=-\int\frac{t^2}{1+t^2}dt=-\int\frac{1+t^2-1}{1+t^2}dt=-\int dt+\int\frac{dt}{t^2+1}=-t+\operatorname{arcctg}$

$=\operatorname{arctg}(\operatorname{ctg}x)-\operatorname{ctg}x]=-\frac{1}{2}x\operatorname{ctg}^2x+\frac{1}{2}\operatorname{arctg}(\operatorname{ctg}x)-\frac{1}{2}\operatorname{ctg}x+C=$

$=-\frac{1}{2}\operatorname{ctg}x(x\operatorname{ctg}x+1)+\frac{1}{2}\operatorname{arctg}(\operatorname{ctg}x)+C$

8.2.73 $\int x^2\ln x\,dx=[u=\ln x\Rightarrow u'=\frac{1}{x};\ v'=x^2\Rightarrow v=\frac{x^3}{3}]=$

$=\frac{x^3\ln x}{3}-\int\frac{1}{x}\cdot\frac{x^3}{3}dx=\frac{x^3\ln x}{3}-\frac{1}{3}\int x^2dx=\frac{x^3\ln x}{3}-\frac{x^3}{9}+C=$

$=\frac{3x^3\ln x-x^3}{9}+C=\frac{x^3(3\ln x-1)}{9}+C$

8.2.75 $\int x^3e^xdx=[v'=e^x\Rightarrow v=e^x;\ u=x^3\Rightarrow u'=3x^2]=$

$=x^3e^x-\int 3x^2e^xdx=x^3e^x-3\int x^2e^xdx=[u=x^2\Rightarrow u'=2x;v=e^x]$

$=x^3e^x-3(x^2e^x+3(2\cdot(xe^x-\int e^xdx))=[u=x\Rightarrow u'=1;\ v=e^x]=$

$=x^3e^x-3x^2e^x+3(2\cdot(xe^x-\int e^xdx))=x^3e^x-3x^2e^x+6xe^x-6e^x+C=$

$=e^x(x^3-3x^2+6x-6)$

8.2.76 $\int\frac{\arccos x\,dx}{\sqrt{1+x}}=\int\arccos x(1+x)^{-1/2}dx=[u=\arccos x\Rightarrow$

$=u'=\frac{-1}{\sqrt{1+x^2}};\ v'=(1+x)^{-1/2}\Rightarrow v=\frac{(1+x)^{1/2}}{1/2}=\sqrt{1+x}\cdot 2]=2\arccos x\sqrt{1+x}+2\int$

$\int\frac{\sqrt{1+x}}{\sqrt{1-x^2}}dx=[1+x=t\Rightarrow x=t-1\Rightarrow dx=dt;\ (t-1)^2=t^2-2t+1]=$

$= \int \frac{\sqrt{t}}{\sqrt{2t-t^2}}\, dt \; \{ \int \sqrt{\frac{t}{2t-t^2}}\, dt = \int \frac{dt}{\sqrt{2-t}}] = 2(x\arccos x\sqrt{1+x} - 2\sqrt{1-x}) + C$

$8.2.78 \int \frac{x^2dx}{(x^2-1)^2} = \int x \cdot \frac{xdx}{(x^2-1)^2} = [u = x \Rightarrow u' = 1; v' = \frac{x}{(x^2-1)^2} \Rightarrow v =$

$= \int \frac{xdx}{(x^2-1)^2} = [t = x^2-1 \Rightarrow dt = 2x\,dx \Rightarrow xdx = \frac{dt}{2}] = \frac{1}{2}\int \frac{dt}{t^2} =$

$= \frac{1}{2}\int t^{-2}dt = \frac{1}{2}\cdot\frac{t^{-1}}{-1} = \frac{-1}{2t} + C \Rightarrow v = \frac{-1}{2(x^2-1)}] = \frac{-x}{2(x^2-1)} + \int \frac{dx}{2(x^2-1)} =$

$= \frac{-x}{2(x^2-1)} + \frac{1}{2}\int \frac{dx}{x^2-1} = \frac{1}{2}\left(\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| - \frac{x}{x^2-1}\right) + C$

$8.2.81 \int e^{\sqrt{x}}dx = [x = t^2 \Rightarrow t = \sqrt{x} \Rightarrow dx = 2t\,dt] = 2\int e^t t\,dt =$

$= [u = t \Rightarrow u' = 1,\ v' = e^t \Rightarrow v = e^t] = 2(te^t - \int e^t dt) = 2te^t - 2e^t + C =$

$= 2e^t(t-1) + C = 2e^{\sqrt{x}}(\sqrt{x}-1) + C$

$8.2.83 \int x^3 e^{x^2}dx = [t = x^2 \Rightarrow dt = 2xdx \Rightarrow xdx = \frac{dt}{2}] =$

$= \frac{1}{2}\int t\cdot e^t dt = [u = t \Rightarrow u' = 1;\ v' = e^t = v = e^t] = \frac{1}{2}(te^t - \int e^t dt) = \frac{te^t}{2} - \frac{1}{2}e^t + C =$

$= \frac{e^{x^2}}{2}(x^2-1) + C$

$8.2.84 \int \ln(x+\sqrt{x^2+1})dx = [v' = 1;\ v = x;\ u = \ln(x+\sqrt{x^2+1}) \Rightarrow$

$u' = \frac{1}{\sqrt{x^2+1}}] = x\ln(x+\sqrt{x^2+1}) - \int \frac{xdx}{\sqrt{x^2+1}} = [t = x^2+1 \Rightarrow dt = 2xdx \Rightarrow xdx =$

$\frac{dt}{2}] = x\ln(x+\sqrt{x^2+1}) - \int \frac{dt}{2\sqrt{t}} = x\ln(x+\sqrt{x^2+1}) - \frac{1}{2}\int \frac{dt}{\sqrt{t}} = x\ln(x+\sqrt{x^2+1}) -$

$+ C = x\ln(x+\sqrt{x^2+1}) - \sqrt{x^2+1} + C$

$8.2.87 \int x\sin\sqrt{x}\,dx = [t^2 = x \Rightarrow t = \sqrt{x} \Rightarrow dx = 2t\,dt] = \int t^2\sin t\,dt$

$= [u = t^2 \Rightarrow u' = 2t;\ v' = \sin t \Rightarrow v = \cos t] = t^2\cos t - 2\int t\cos t\,dx$

$= [u = t \Rightarrow u' = 1;\ v' = \cos t;\ v = \sin t] = t^2\cos t - 2t\sin t + 2\int \sin t\,dx =$

$= x\cos\sqrt{x} - 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$

$8.2.89 \int \frac{\cos\sqrt{x}}{\sqrt{x}}dx\,[x = t^2 \Rightarrow dx = 2t\,dt] = [t = \sqrt{x}] = 2\int \frac{\cos t\cdot t\,dt}{t} = 2\int \cos t\,dt$

$= 2\sin t + C = 2\sin\sqrt{x} + C$

8.2.91 $\int \frac{dx}{\sin x} = \Big[\frac{1}{\sin x} = \operatorname{cosec} t;\ v' = 1 \Rightarrow v = x;\ u = \sin^{-1} x \Rightarrow$

$\Rightarrow u' = \frac{-\cos x}{\sin^2 x} \Big] = \frac{x}{\sin x} + \int \frac{x \cos x}{\sin^2 x} dx = \frac{x}{\sin x} + \int \frac{x\, ctg}{\sin x} dx =$

$= \Big[u = \sin^{-1} x = u' = -\frac{2\cos x}{\sin^2 x} \Big] = v' = x\, ctg\, x \Rightarrow \ln|tg \frac{x}{2}| + C$

8.2.93 $\int \frac{e^{arctg x} + 8x}{1+x^2} dx = \int \frac{e^{arctg x}}{1+x^2} dx + \int \frac{8x\,dx}{1+x^2} = \Big[arctg\, x = t \Rightarrow$

$\Rightarrow dt = \frac{dx}{1+x^2} \Big] = \int e^t dt + \int \frac{8x\,dx}{1+x^2} = \Big[m = 1+x^2 \Rightarrow dm = 2x\,dx \Big] =$

$= \int e^t dt + \int \frac{4 \cdot dm}{m} = \int e^t dt + 4 \int \frac{dm}{m} = e^t + 4 \ln|m| + C =$

$= e^{arctg x} + 4 \ln|1+x^2| + C$