Follow Your Ideology: Measuring Media Ideology on Social Networks

January 2017

Or How We Determined That Fox News is Conservative

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Average American voter	50.1
Average Republican legislator	16.1
Average Democratic legislator	84.4

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- Our contribution: new measure to quantify ideology based on structure of social media networks (Twitter)

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Ideology: primary dimension of political conflict, *common* to elites and voters (Jessee 2009, Tausanovitch and Warshaw 2013)

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– Following Bonica (2013), we estimate θ_i and ϕ_j using correspondence analysis. Mathematically close to log-linear ideal point model (Lowe, 2008).

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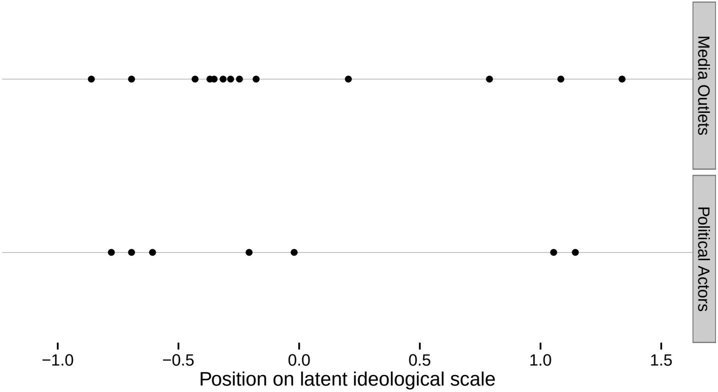

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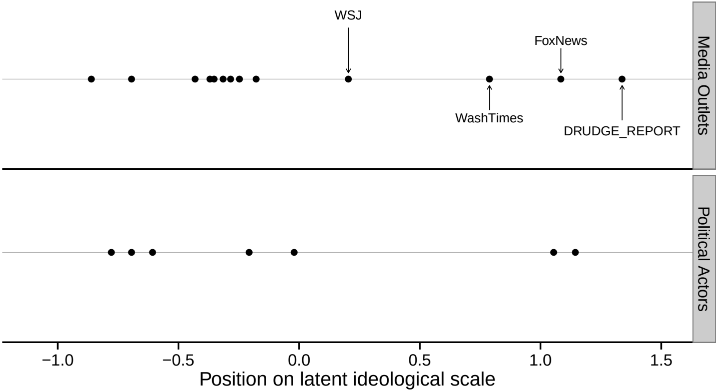

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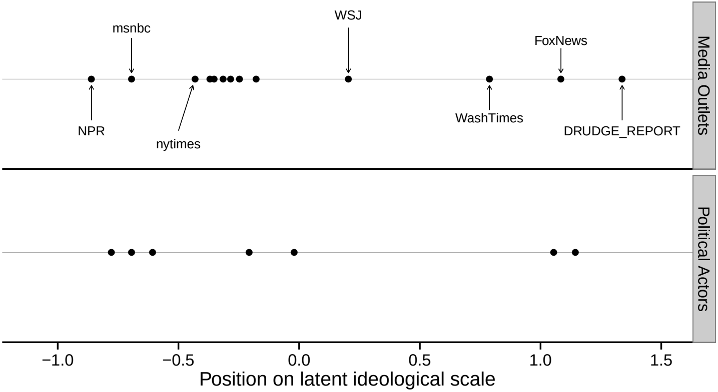

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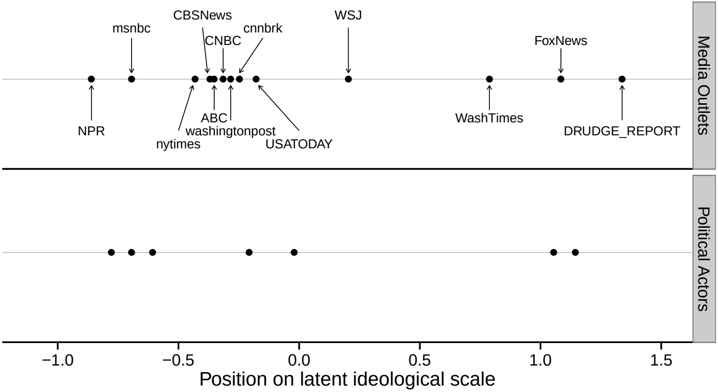

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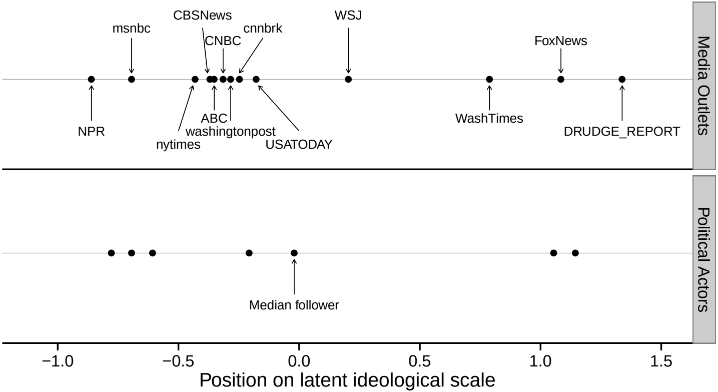

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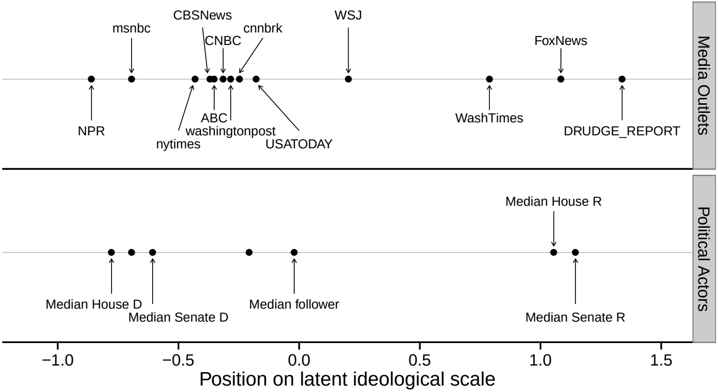

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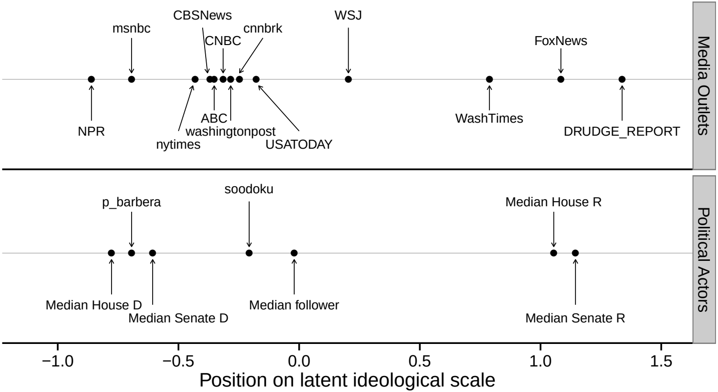

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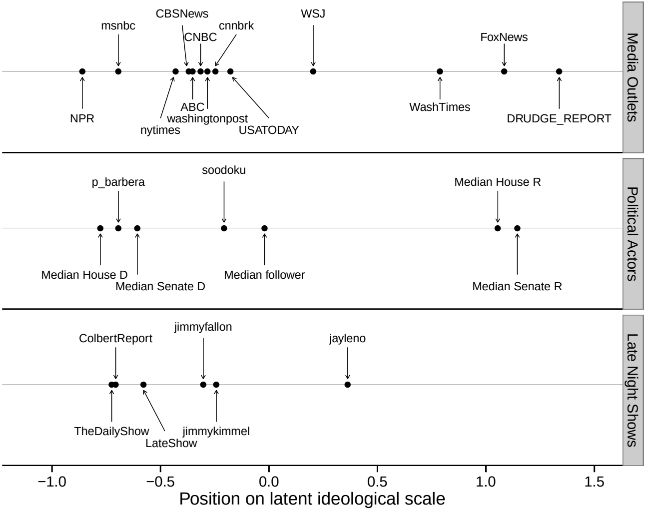

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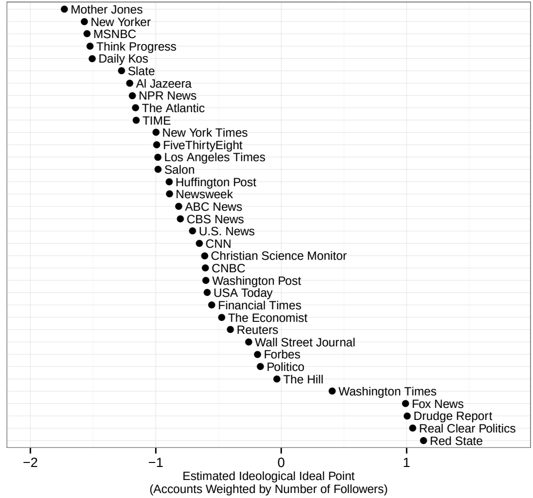
Weighted estimates of media ideology (all outlets)



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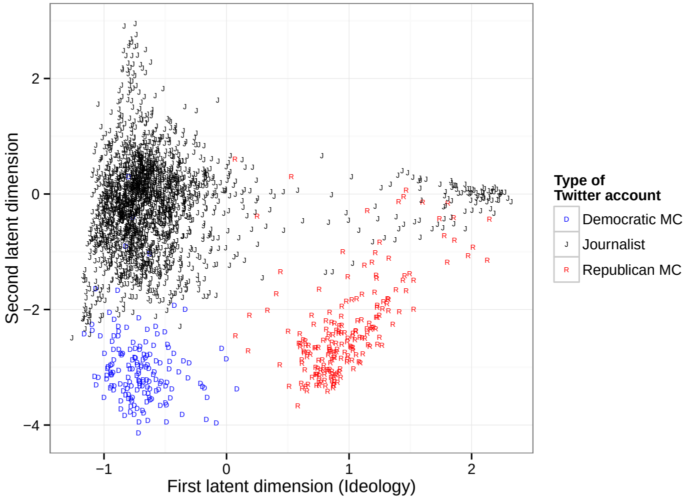
Two-dimensional ideology estimates for outlets and politicians



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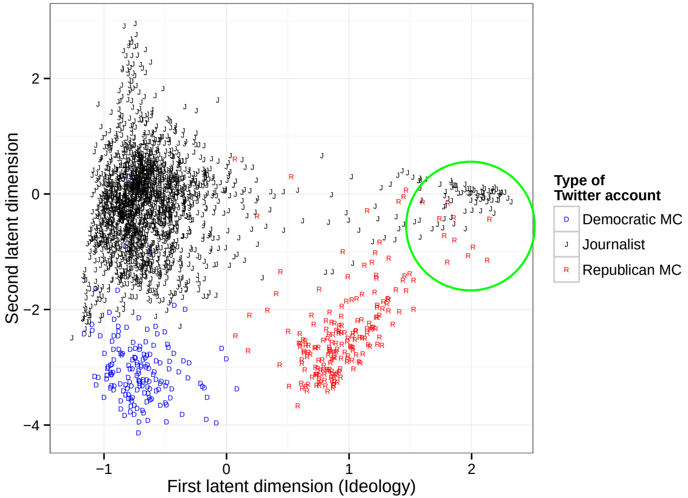

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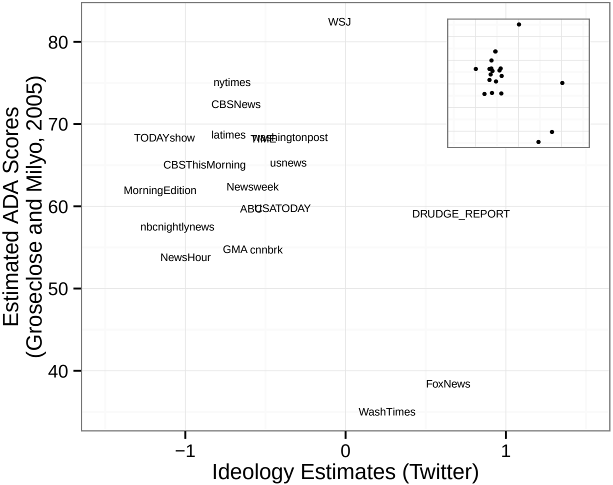

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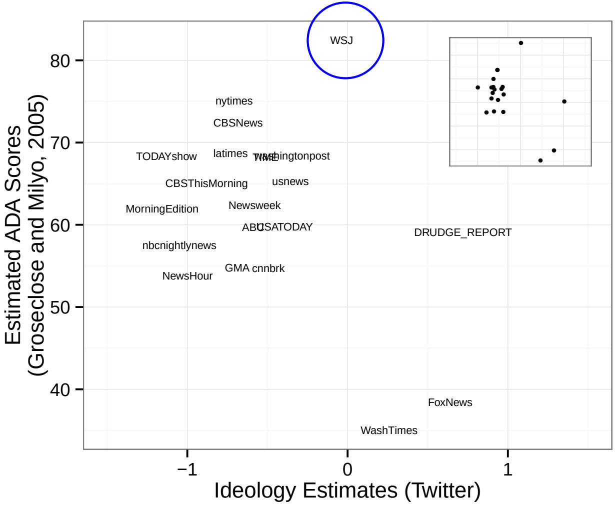

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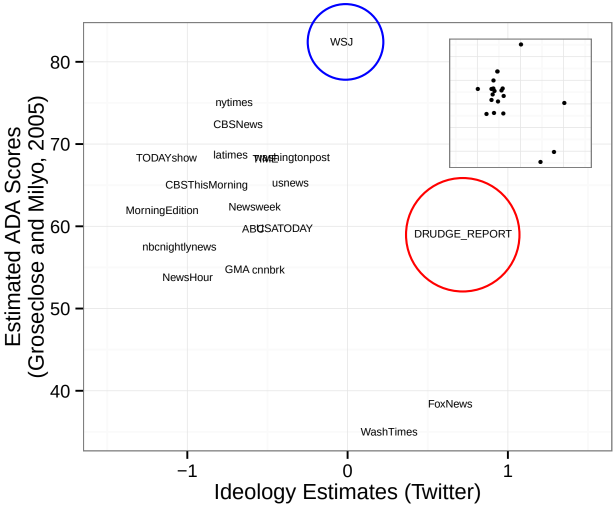

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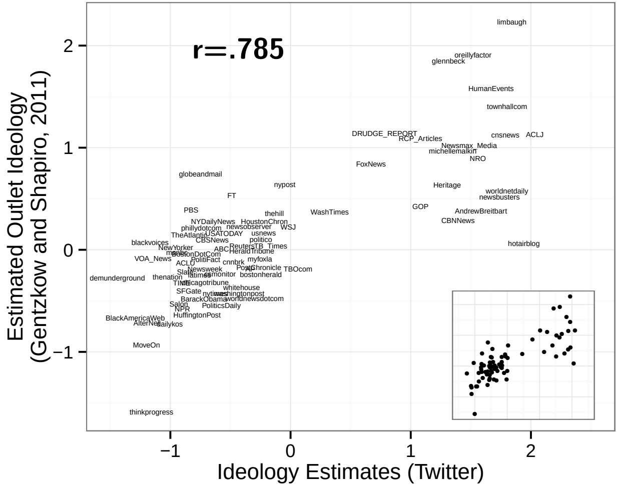

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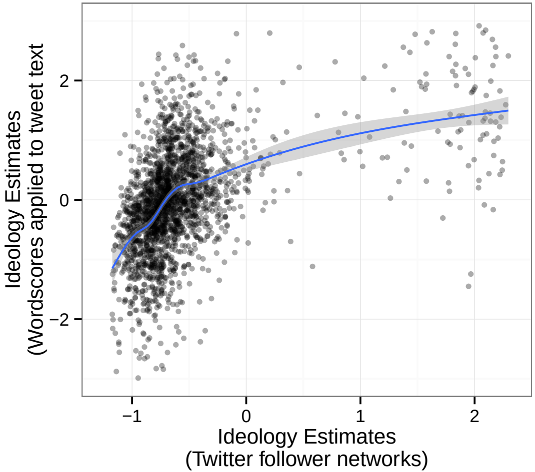
Comparing text-based and network-based methods: ρ=0.47



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  - Estimated ideology for n=306 journalists

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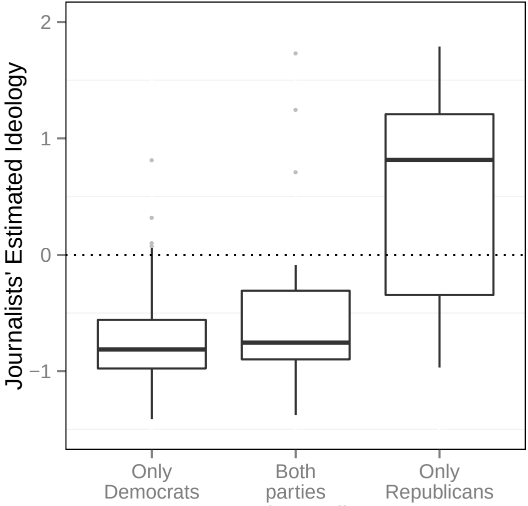
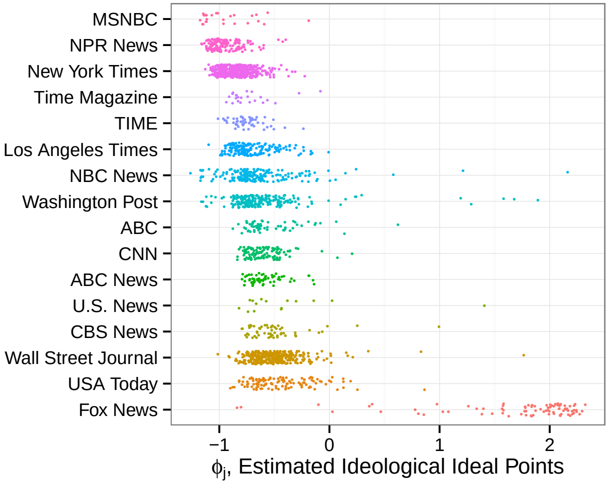
Journalists' ideology estimates and their campaign contributions

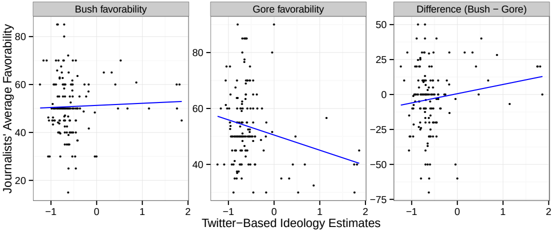


Intra-media ideological heterogeneity

Journalists are clustered within outlets, but high heterogeneity



Ideological Self-Selection



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  - Journalists' private beliefs and content slant are correlated
  - Significant heterogeneity in journalists' ideology within outlets

Correspondence analysis (Greenacre, 1993)

- $\mathbf{Y}$ matrix of nonnegative numbers ($n \times m$)
- Correspondence matrix: $\mathbf{P} = \mathbf{Y} / \sum_{ij} y_{ij}$
- Row and column masses, $\mathbf{r}$ and $\mathbf{c}$, where $r_i = \sum_j p_{ij}$ and $c_j = \sum_j p_{ij}$ (row and column sums)
- Diagonal matrices $\mathbf{D}_r = \mathrm{diag}(\mathbf{r})$ and $\mathbf{D}_c = \mathrm{diag}(\mathbf{c})$
- Basic computational algorithm:
  1. Standardized residuals: $\mathbf{S} = \mathbf{D}_r^{1/2}(\mathbf{P} - \mathbf{r}\mathbf{c}^T)\mathbf{D}_c^{1/2}$
  2. SVD: $\mathbf{S} = \mathbf{U}\mathbf{D}_\alpha\mathbf{V}^\mathbf{T}$ where $\mathbf{U}^\mathbf{T}\mathbf{U} = \mathbf{V}^\mathbf{T}\mathbf{V} = \mathbf{I}$
  3. Compute coordinates:

     Rows: ¯ $= \mathbf{D}_r^{1/2}\mathbf{U}$
     Columns: ` $= \mathbf{D}_c^{1/2}\mathbf{V}$
  4. Supplementary points (e.g. column $\mathbf{h}$ of length n):
     - 4.1 Standardize: $\tilde{\mathbf{h}} = \mathbf{h} / \sum_i h_i$
     - 4.2 Project: $\mathbf{g} = \tilde{\mathbf{h}}^T$¯