University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

Linear Programming and Optimazation

MATB61 Winter 2020

Assignment # 3

You are expected to work on this assignment prior to your tutorial in the week of Jan. 27th. You may ask questions about this assignment in that tutorial. In your tutorial in the week of Feb. 3rd, you will be asked to write a quiz based on this assignment & assignment 2 and/or related material from the lectures in week 2 & 3, and textbook readings.

Midterm Test
Wednesday, February 26th, 5 – 7pm
IC120, IC130

Note: The Maple code for finding all basic solutions is on the next page. You may use it to check your calculation of basic solutions if you may access Maple software.

The following problems are not to be handed-in.

Textbook: Elementary Linear Programming With Applications
B. Kolman & R. Beck, 2nd edition

Read: Chapter 1 section 4, 5 and Lecture notes

Problems:

1. Section 1.4 #1 - #8 #15 #16
2. Please find all basic solutions and finally find extreme points in Section 1.4 # 6 and #8 .
3. Section 1.5 #1 - #9 #12

In addition:
Let S be a convex set and P is an extreme point of the set. Prove that S\{P} is a convex set.

Note: There are answers at the back of the textbook for the odd number questions.

Maple code: (Find basic solutions)

- with(linalg): with(LinearAlgebra):
- with(combinat,choose):
- basic1:=proc(f,A,b) local m,n,A1,d,i; global Ci,A2,k;
- m,n :=Dimensions(A);
- A1:=IdentityMatrix(m);
- A2:=<<A>|<A1>>;
- d:=choose(n+m,m);
- k:=(((n+m)!)/(((m)!)*((n)!)));
- for i from 1 to k do;
- C(i):=LinearSolve(A2[1..m,d[i]],b);
- print(C(i),d[i]);
- od;
- end proc;

For example: Maximize f(x, y) = 3x+2y

Subject to

.

$$-x+y \le +2$$
$$x+y \le +3$$
$$\frac{2}{5}x-y \le -\frac{1}{5}$$
$$x, y \ge 0$$

- G:=Matrix([[-1,1],[1,1],[2/5,-1]]);
- b:=<2, 3,-1/5>;
- basic1(3,G,b);

$$\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, [1,2,3] \qquad \begin{bmatrix} -3 \\ -1 \\ 7 \end{bmatrix}, [1,2,4] \qquad \begin{bmatrix} \frac{1}{2} \\ \frac{5}{2} \\ \frac{21}{10} \end{bmatrix}, [1,2,5] \qquad \begin{bmatrix} -\frac{1}{2} \\ \frac{3}{2} \\ \frac{7}{2} \end{bmatrix}, [1,3,4] \qquad \begin{bmatrix} 3 \\ 5 \\ -\frac{7}{5} \end{bmatrix}, [1,3,5]$$

[2, 1, 3, 0,0] b [1/2,5/2, 0, 0, 21/10] b b

f(2, 1) = 8 f(1/2, 5/2) = 13/2

$$\begin{bmatrix} -2 \\ 5 \\ \frac{3}{5} \end{bmatrix}, [1,4,5] \qquad \begin{bmatrix} \frac{1}{5} \\ \frac{9}{5} \\ \frac{14}{5} \end{bmatrix}, [2,3,4] \qquad \begin{bmatrix} 3 \\ -1 \\ \frac{14}{5} \end{bmatrix}, [2,3,5] \qquad \begin{bmatrix} 2 \\ 1 \\ \frac{9}{5} \end{bmatrix}, [2,4,5] \qquad \begin{bmatrix} 2 \\ 3 \\ -\frac{1}{5} \end{bmatrix}, [3,4,5]$$

b [0, 1/5, 9/5, 14/5, 0] b [0, 2, 0, 1, 9/5] b

f(0, 1/5) = 2/5 f(0, 2) = 4

The 1st column vector gives the value of the nonzero variable in the basic solution and the 2nd row vector indicates the position of the nonzero variable.