University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

Linear Programming and Optimazation

MATB61 Winter 2020

Answers to the odd number questions may be found at the end of the textbook.

Selected answers to the assignment # 1

Section 1.1

2. x: number of machines of model A, y: number of machines of model B.

$$\begin{aligned} \min\, & 15000x + 20000y \\ 30x + 50y &\geq 320 \\ x + 2y &\leq 12 \\ x, y &\geq 0 \end{aligned}$$

4. x_1, x_2, x_3: number of acres of corn, soybeans, oats respectively.

$$\begin{aligned} \max\, & 40x_1 + 30x_2 + 20x_3 \\ x_1 + x_2 + x_3 &\leq 12 \\ 6x_1 + 6x_2 + 2x_3 &\leq 48 \\ 36x_1 + 24x_2 + 18x_3 &\leq 360 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

6. x, y: number of barrels using four-field/five-field collector

$$\begin{aligned} \min\, & 0.14x + 0.18y \\ x + y &\leq 2500000 \\ .18x - 0.12y &\leq 0 \\ x, y &\geq 0 \end{aligned}$$

The second inequality comes from the constraint

$$1.5x + 1.8y \leq 0.84(2x + 2y)$$

8. x_1, x_2, x_3: amount invested in utilities stock, electronics stock and bonds (in tens of thousands of dollars).

$$\begin{aligned} \max\, & 0.09x_1 + 0.04x_2 + 0.05x_3 \\ x_1 + x_2 + x_3 &\leq 20 \\ 0.5x_1 + 0.5x_2 - 0.5x_3 &\leq 0 \\ x_1 &\leq 4 \\ x_3 &\geq 7 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

10. x_{ij} ith component of jth mixture, where components 1, 2, 3, 4 are alkylate, catalytic cracked, straight run, isopentane and mixtures 1, 2 are high and low octane.

$$\max 6.5\sum_{i=1}^{4} x_{i1}+7.5\sum_{i=1}^{4} x_{i2}-7.2(x_{11}+x_{12})-4.35(x_{21}+x_{22})-3.8(x_{31}+x_{32})-4.3(x_{41}+x_{42})$$

$$\begin{aligned}
108x_{11}+94x_{21}+87x_{31}+108x_{41} &= 100\sum_{i=1}^{4} x_{i1}\\
99x_{12}+87x_{22}+80x_{32}+100x_{41} &= 90\sum_{i=1}^{4} x_{i2}\\
5x_{11}+6.5x_{21}+4x_{31}+18x_{41} &= 7\sum_{i=1}^{4} x_{i1}\\
5x_{12}+6.5x_{22}+4x_{32}+18x_{41} &= 7\sum_{i=1}^{4} x_{i2}\\
x_{11}+x_{12} &\le 700\\
x_{21}+x_{22} &\le 600\\
x_{31}+x_{32} &\le 900\\
x_{41}+x_{42} &\le 500\\
\sum_{i=1}^{4} x_{i1} &= 1300\\
\sum_{i=1}^{4} x_{i2} &= 800
\end{aligned}$$

$x_{ij} \ge 0$ for $i = 1, 2, 3, 4; j = 1, 2$

In addition:

1. x = number of days per week mine X is operated

   y = number of days per week mine Y is operated

   minimize

   $180x + 160y$

   subject to

   $6x + y \ge 12$

   $3x + y \ge 8$

   $4x + 6y \ge 24$

   $x \le 5$

   $y \le 5$

   $x, y \ge 0$

2. a) Set $x_1 = x_1' - 1$ and $x_2 = 2 - x_2'$.

   Max $z = 4x_1' - 2x_2' + x_3$

   Subject to $x_1' + 3x_2' + x_3 \le 6$

   $5x_1' + 3x_3 \le 10$

   $-5x_1' - 3x_3 \le -10$

   $x_1' - x_2' + x_3 \le 0$

   $x_1', x_2', x_3 \ge 0.$

b) Maximize $x + 2y + 4z$
Subject to $3x + 2y - 8z \leq 0$
$-5x - 4x + 6z \leq 0$
$x, y, z \geq 0.$

3. a) Set $x_3 = x_3^+ - x_3^-$.

$$\text{Max } z = -x_1 - 2x_2 - x_3^+ + x_3^-$$
$$\text{Subject to } x_1 - x_2 + x_3^+ - x_3^- - s_1 = 2$$
$$x_1 + x_2 + s_2 = 3$$
$$x_1 + x_2 - s_3 = -3$$
$$x_1, x_2, x_3^+, x_3^- \geq 0, s_i \geq 0, i = 1, 2, 3.$$

b) Set $x = x^+ - x^-$, $y = -y'$ and $z = z' + 1$.

$$\text{Max } z = -x^+ + x^- - 12y' - 3z'(-3)$$
$$\text{Subject to } 5x^+ - 5x^- + y' - 2z' = 12$$
$$-2x^+ + 2x^- + y' + 10z' + s_1 = 20$$
$$z' + s_2 = 3$$
$$x^+, x^-, y', z', s_1, s_2 \geq 0.$$