University of Toronto at Scarborough
Department of Computer & Mathematical Sciences

Term Test

MATB61 Linear Programming and Optimization

Examiner: X. Jiang — Duration: 110 minutes

1. [12 points] Set up a linear programming model. (**Please don't solve it!**)

A Supermarket requires different numbers of full-time employees on different days of the week. Union rules state each employee must work 5 consecutive days and then receive two days off. Find the minimum number of employees needed.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Staff needed	16	20	17	21	23	25	18

2. [18 points] Suppose that we apply the Simplex method to a given LP problem and obtain the following tableau:

	x_1	x_2	x_3	x_4	x_5	x_6	
u_1	0	a	0	1	1	0	e
u_2	1	f	0	2	0	-4	5
u_3	0	-3	d	-4	0	1	1
	0	c -1	0	5	0	b	12

Specify the ranges of values for the parameters *a, b, c, d, e, f* ($\geq, >, \leq, <, =$) that make each of the following statements true. Assume that the original problem was a maximization problem.

a) The tableau describes an infeasible basic solution.
b) The tableau describes an optimal basic feasible solution.
c) The tableau describes a basic feasible solution, but it is not the optimal solution.
d) The tableau describes a basic feasible solution, but the problem is unbounded and the simplex algorithm cannot proceed any further.
e) The tableau describes an optimal basic feasible solution which is not unique.
f) The current basic solution is feasible. At the next iteration, x_2 is the only candidate for entering the basis.
g) The current basic solution is feasible. At the next iteration, a degenerate BFS will occur in the next tableau.

3. [22 points]

1) **[10 points]** Convert the following LP problem into standard form:

$$\begin{aligned} \text{Minimize} \quad & 2x_1 - x_2 + x_3 \\ \text{Subject to} \quad & |x_1 - 2x_2| \leq 2 \\ & 3x_1 - 5x_2 - x_3 = -1 \\ & x_1 \geq 0,\ 2 \leq x_2 \leq x_3. \end{aligned}$$

2) **[8 points]** Assume a LP problem: Max $z = c^T x$, subject to $Ax \leq b$, $x \geq 0$ has an unique optimal solution. Prove that the optimal solution may not be reached at x_0 if x_0 is not an extreme point of the feasible region.

3) **[4 points]** Multiple choices:

a) Given an LP problem

Maximize $p = 4x + 2y + 7z$

subject to $2x - y + 4z \leq 18$

$4x + 2y + 5z \leq 10$ $x \geq 0, y \geq 0, z \geq 0$

Which of the following is(are) basic feasible solution(s) of the LP problem?

i) [5/2, 0, 1], ii) [9, 0, 0], iii) [0, 0, 9/2], iv) [0, 5, 0]

b) Which of the following statements is true?

i) The set of solutions to an inequality is a hyperplane.
ii) An extreme point is a basic feasible solution.
iii) [1, 0] is a convex combination of [-1, 0], [0, 5] and [4, 1].
iv) any point in S is an interior point of any two distinct points in S.

4. [18 points] Solve the following LP problems graphically.

a) Maximize $z = -x + y$

Subject to $x - 3y \leq 4$

$x + 2y \geq 4$

$x, y \geq 0.$

b) Minimize $z = 4x + 2y$

Subject to $2x - 3y \leq 12$

$4x - y \geq 1$

$2x + y \geq 8$

$x, y \geq 0.$

5. [15 points] Solve the following LP Problem by the Big M method:

Maximize $z = 4x_1 + 5x_2 - 3x_3$

Subject to $x_1 + 2x_2 + x_3 = 10$

$x_1 - x_2 \geq 6$

$x_1 + 3x_2 + x_3 \leq 14$

$x_1, x_2, x_3 \geq 0.$

6. [15 points] Use the two phase method to solve the following LP problem.

Maximize $z = 5x_1 + 3x_2 + x_3$

Subject to:

$2x_1 + x_2 + x_3 \leq 4$

$x_1 + 2x_2 - x_3 \geq 3$

$x_2 + 2x_3 \leq 2$

$x_i \geq 0$, i = 1, 2, 3