IB Diploma Extended Essay
Physics

Deducing the power output required to operate a human powered helicopter through a scale experiment

3,927 Words

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Abstract

The following research is conducted with the aim to understand the physical requirements expected of a "Human engine" to successfully power a Human Powered Helicopter, with respect physiological limitations. Hence the research question **"What power output is required to successfully operate a Human Powered Helicopter?".**

The research reveals the variable of power output to be 553 Watts for a 80 kg individual when operating in ground effect, a phenomenon which reduces the power requirement of the aircraft near the ground, and 887.9 Watts as a lower limit out of ground effect.

The variable of power output required is found using a scale model setup with a RC helicopter with the same airfoils as the "Gamera II" human powered helicopter; The relationship between lift generated and frequency of the airfoil was found and combined with Blade Element Theory to find a variable called coefficient of lift for the miniature airfoil (0.074 ± 0.017[1]). This subsequently lead to the power output required to fly the full-size human powered helicopter.

The deduction that Human Powered Helicopters are inoperable for long durations both in and out of ground effect is made in the conclusion.This deduction is made using human power output data and limitations indicated in "The Human Power Plant" by A. Evans, Human Powered Helicopters are only operable for short times 1~1.5 minutes when they are optimized for efficiency.

Abstract Word Count: 229 Words

[1] Dimensionless

Introduction /Aim

The Human Powered Helicopter (abbreviated HPH, see fig. 1.) is a modern *human powered vehicle* currently under development. The first of these helicopters left the ground in 1989 and soon other HPH's quickly set a flight time of 7.1 s in the same year (AeroVelo Inc., "Background" ; Naito 1). Previously, developing a functional HPH was seen as an aeronautical challenge due to lack of information on the physics involved and poor design (Naito 1). This was further exacerbated by the limited power output of the "human engine" because "[Human] power output diminishes as the duration of exercise increases" (Wilkie 1), making human power a vital aspect to consider when designing a HPH.

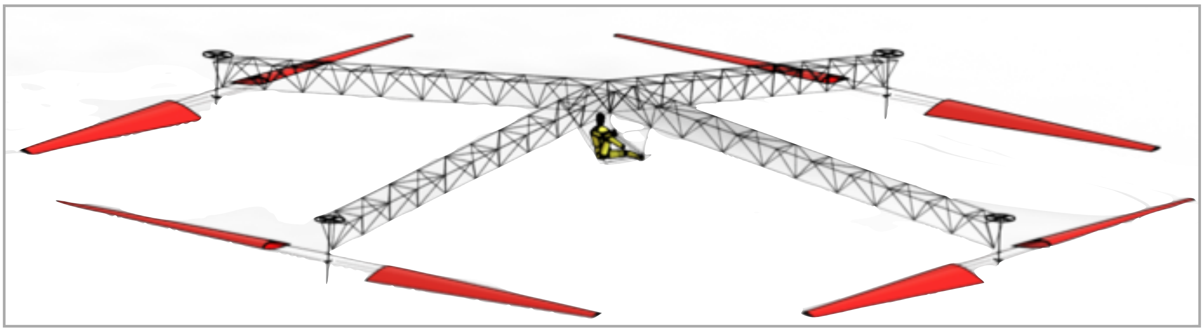


Fig. 1. A visualization of the "Gamera II" Human Powered Helicopter, which is powered using a bike; Berry et. al.; "Design Optimization of Gamera II: A Human Powered Helicopter."; page 1; University of Maryland; Web; 30 Oct; 2013.

The modern HPH community is in a dilemma because even drastic measures such as using innovative materials and reducing mass reflect little improvement on flight time despite the increase in efficiency. Modern HPH's such as the Gamera II[1] (fig.1.), are only capable of sustaining 60 seconds of flight time and reaching 3 meters of vertical height, as shown by the recent achievement of the AHS Sikorsky Prize[2] (AHS International).

[2] The AHS Sikorsky prize was (broken 2013) a 33 year old unbroken record which required a HPH to "fly for at least 60 seconds, reach an altitude of at least 3 meters (9.8 feet) and remain hovering over a 10 by 10 meter (32.8 by 32.8 foot) area" (AHS International)

The consequence of this increased efficiency is that, firstly, the flight time of a HPH is less dependent on the parameters of the vehicle itself, such as mass or lift generated, and more dependent on the “human engine’s” physiological capabilities, Secondly, the problem also puts the view of HPH as a renewable form of transport in jeopardy.

This increasing importance of the issue relating to human performance has lead to the research question **“What is the power output required to successfully operate a Human Powered Helicopter?”** The aim of the investigation is to find the power output required to fly a HPH in an ascending state and use existing academic research to evaluate the possibility of the HPH as a form of transportation.

Significance

The significance of the research question is two-fold, firstly, the question allows us to evaluate the HPH, a "green" vehicle, as an alternative form of transport (or supplement to) the oil dependent cars existing today, hence evaluating the HPH's capabilities as viable form of transport could lead to mass production. The HPH could be avant-grade, a no emission flying vehicle which could revolutionize the transport industry.

Secondly, the question allows provide us an insight into the limits of human performance. data from current HPH flights gives the power output for an ascending state to be above 550 Watts[3] (AeroVelo Inc., "Draft FAI World Record Claim."; AeroVelo Inc., "Technical Information"), this value is higher than that of common man (Evans 9).Furthermore, additional research, i.e "Wing in ground effect - a new approach for the Human Powered Helicopter" by A. Krenik and J. Götz also indicates flying the HPH to 3 meters is a task on human limits, the duo states: "...[flying a HPH] is hard enough to accomplish, but concerning the mandatory pull up to three meters height, the very scarce human power resources become evident." (12).

Important notes

The term "helicopter blade" commonly refers to a helicopter's **rotary wings or airfoil** in colloquial english (Oxford Dictionaries).The latter term is more accurate and shall be used throughout the investigation.

[3] This is the estimate power output to hover for a 80 kg pilot pedaling for 1 minute on a HPH (AeroVelo Inc., "Technical Information").Hence, any power output greater than this would make the HPH climb.

Procedure and Background Information

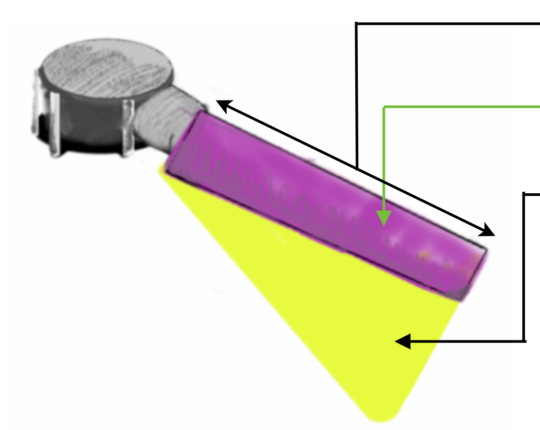
The power output of a HPH is found using of a scale model. Using scale model rotary wings ensures “kinematic similarity”, a term implying all forces on both the scale model and full size model <u>are scale factor of one another</u> (Cambridge-MIT Institute).

The investigation uses scale model rotary wings used in the “Gamera II” HPH. The Gamera II HPH was particularly selected as it’s one of the most efficient HPH’s created and has a official world record of 65.1s in flight time (University of Maryland). The key physics from Blade Element Theory and the Actuator disk theory, two physical models to find forces on airfoils, are used in to find the value of power.

Previous research on the topic of power output has been conducted by numerous researchers, including HPH creators themselves. AeroVelo, the creator of a HPH called “Atlas”, has given an estimate power output of 550 Watts to hover for an 80kg pilot (AeroVelo Inc., “Technical Information”). Furthermore, the analysis of flight data taken from the Atlas shows a similar value of 533 Watts minimum for a climbing state (AeroVelo.Inc, "Draft FAI World Record Claim.").

Introduction to terminology and variables

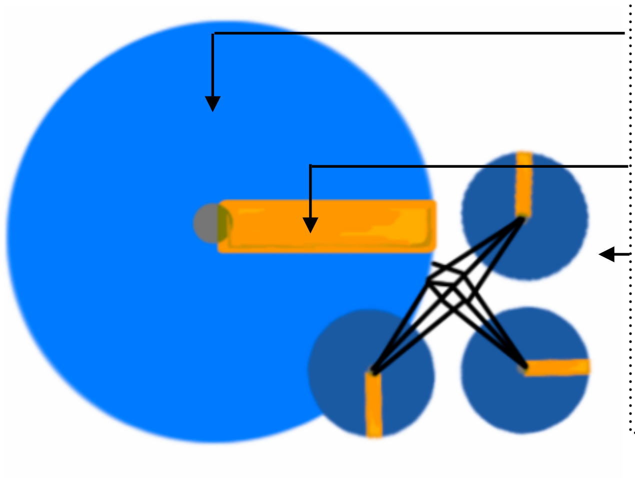
The following investigation relies on seven metrics of a HPH. These are described below:



Radius of airfoil (r / units: m) - The radius of an airfoil blade.

Surface area of airfoil (S / units: m^2) - The surface area of one face of the airfoil.

Frequency of rotation (f / units: Hz) - The frequency of the rotary wing.



Actuator disk area (A / units: m^2) - The area covered by an airfoil over period of 1 rotation.

Number of rotary wings per rotor (N) - in this diagram, $n = 1$.

Number of rotors on HPH (M) - For example, $M = 4$ in this diagram.

Weight of HPH (W / units: N)

Lift (L / units: N) - The lift generated by a HPH.

Other metrics used in the investigation are:

Coefficient of lift (c_l / No units) - a dimensionless coefficient, which contains complex variables such as angle of attack and Reynolds number of air (NASA, and Tom Benson, "The Lift Coefficient")

Power (P / units: W) - A general term used throughout the investigation. A more specific variant, called *ideal power* (P_{id}), stands for the power requirement to hover (Aerostudents 1)

Air density (ρ / units: kg m^{-3}) - The density of air, which is 1.275 kg m^{-3} (Helmenstine).

Velocity (v / m s^{-1}) - General term, velocity experienced by any moving object.

Theoretical analysis of HPH flight

The forces governing all helicopters in flight are similar. The rotation of the rotary wings forces air downwards (Weisstein), and creates lift (L) which counteracts the helicopter's weight (W). According to Newton's 3rd law, a hovering state occurs if the lift is equal to the weight of the helicopter ($L = W$) as shown in fig.1.

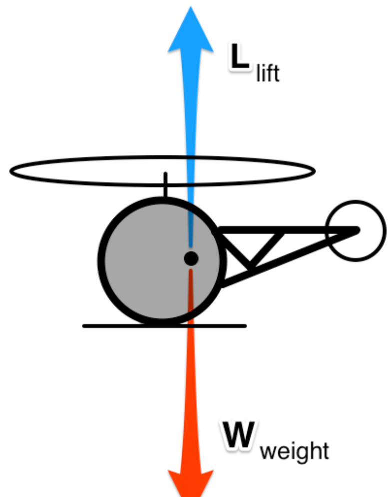


Fig. 1. The force diagram of Helicopter in flight.

For the situation in figure 1, we can find the ideal ideal power[4] (P_{id}) using the equation

$P = Fv$. Since $F = W$, in fig. 1. we get equation 1.0:

$$P_{id} = Wv \qquad (1.0)$$

$$v_i = \sqrt{\frac{W}{2M\rho A}} \qquad (2.0)$$

$$P_{id} = W\sqrt{\frac{W}{2M\rho\pi r^2}}$$

ρ air density
A area of the actuator disk, where $A = \pi r^2$.
r radius of the helicopter blade.
W weight of the HPH.
M number of rotors on the HPH.

v is the velocity of helicopter in direction of lift, this is called *induced velocity* v_i in aerodynamics. The equation for v_i (2.0) from the "Helicopter Aerodynamics - Actuator Disk Theory." (Aerostudents 1) is combined with equation 1.0 .
This the gives equation 3.0 for ideal power.
If the HPH has many rotors, "*M*" or number of helicopter rotors is factored into 2.0 and other equations

$$P_{id} = \sqrt{\frac{W^3}{2M\rho\pi r^2}} \qquad (3.0)$$

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[4] The power required for a rotorcraft, such as a helicopter, to hover (Aerostudents 1)

Gamera II Human Powered Helicopter information

To calculate the ideal power of the we must know certain parameters of the Gamera II HPH itself, such as airfoil radius, weight etc. These are provided as table 1 below:

Table 1: Gamera II Human Powered Helicopter information	
Mass (kg) unoccupied	32.3
Mass (kg) occupied	112.3 (assuming 80 kg human engine)
Radius of airfoil (r) (m)	6.5
Taper ratio a measure of taper of the blade	3:1
width (m) calculated, see Appendix 1	1.5
Number of blades per rotor (N)	2
Number of rotors (M)	4
Surface area (S) calculated, see Appendix 1	6.5
Source: Berry et al.; "Design Optimization of Gamera II: A Human Powered Helicopter.";(n.d.); 1-19; *The Alfred Gessow Rotorcraft Center, Aerospace Engineering;* University of Maryland; Web; 30 Oct; 2013	

The width of the rotary wing was not provided, however, it was possible to calculate it using rotor solidity (σ_e), see Appendix 1. Furthermore, no error was given for these values.

We insert the values for weight $W = 1100N$ [5] and radius $r = 6.5m$ into equation 3.0 to get ideal power :

$$P_{id} = \sqrt{\frac{W^3}{2M\rho\pi r^2}} = \sqrt{\frac{(1100)^3}{2(4)(1.275)(6.5)^2\pi}}$$ Substitute in values

$$P_{id} = \sqrt{\frac{1.331\times10^9}{430.95\pi}}$$ Further simplify

$$P_{id} = 990 \ W$$ (2 s.f) due to 6.5

Here, we can see the value for hovering , 990 Watts is substantially higher than 550 Watts, the experimental value from AeroVelo, even though the value is ideal and no energy has been lost (AeroVelo Inc., “Technical Information”). This unexpected case of ideal power being higher

[5] $112.3 \ kg \times 9.81N \ kg^{-1} = 1100 \ N$ (3 s.f)

than experimental power is indicative of an aerodynamic force called ground effect. Ground effect lowers the operating power requirement for a HPH:

> "Ground effect is a well-known phenomenon in which rotorcraft or airplanes experience an increase in performance when operating near the ground....[this results in] a decrease in power requirements for the same thrust."(Berry et al. 5).

The effect of this force and how to prevent it from affecting the evaluation of a HPH as a mode of transport is discussed later in this investigation[6].

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[6] See p.18 "Removing Ground Effect"

Using Blade Element Theory to find ascending HPH power requirement

Because the previous equation 3.0 only applies to HPH in a hovering state, it is inappropriate for finding the power of a HPH in a state of ascent, where $L \neq W$. To find the power required for different rates of ascent, we must express lift as a function of frequency of the rotary blades $L(f)$, because $L \propto f$ for any HPH.

We find the relationship using the Blade Element Theory (BET), **a mathematical model to deduce forces on an airfoil over the course of a single rotation** (Aerostudents 1). In BET, the rotary wing is first divided into infinitesimal annuli[7] along the radius (Seep.1 fig.2.). The force of lift is calculated over each annulus, which is followed by integration of force across the radius to give the formula for the lift (Aerostudents 2).

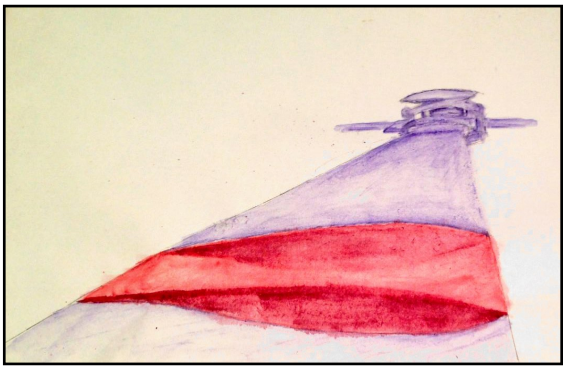

Fig. 2. A visualization of the infinitesimal annuli used in Blade Element Theory.

To apply BET, we must consider the movement of each annulus first, because the HPH's wings rotate, they experience an increasing velocity away from center of rotation (fig.3. next page)

[7] small sections.

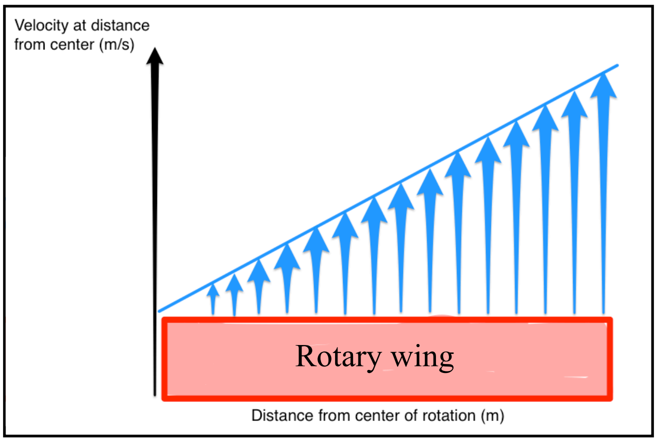

Fig.3. Velocity experienced by each segment of a rotary wing is different.

The velocity of each annuli is given in equation 3.1

$$v = \omega r \equiv 2\pi fr \tag{3.1}$$

3.1 is then combined with equation 3.2, the equation for lift, given by NASA (NASA, "The lift equation").

$$L = \tfrac{1}{2} c_l \rho S v^2 \tag{3.2}$$

Equation 3.2 is used to find the lift force on a airfoil moving at velocity *v* (NASA, "The lift equation").

- S, Surface area of the airfoil.
- c_l, Coefficient of lift.
- ρ, Air density.

We can combine equation 3.1 and 3.2 to get equation 3.3.

$$L = \tfrac{1}{2} c_l \rho S (2\pi fr)^2$$

Expand , Simplify

$$L = 2 c_l \rho S \pi^2 f^2 r^2 \tag{3.3}$$

[3.3] is the mathematical expression of lift experienced by a annulus hence we can now apply BET by integrating 3.3 with respect to radius of the airfoil. The process leads equation 4.0 which gives the lift in terms of *f (frequency)*.

BET

$$L = \int 2c_l \rho S \pi^2 f^2 r^2 \;\delta r$$ Integrate 3.3 with respect to *r*

$$L = 2c_l \rho S \pi^2 f^2 \int r^2 \;\delta r$$ Remove constants from integral

$$L = \frac{2}{3} c_l \rho S \pi^2 f^2 r^3 \qquad (4.0)$$

Note that for N rotary wings, and M motors on a HPH, equation 4.0 will be $L = \frac{2}{3} MNc_l \rho S \pi^2 f^2 r^3$

Finally, we use $P = Fv$, where $v = v_i$ [8], $F = L$ and equation 4.0 to get equation 5.0, power when ascending.

$$P = Lv_i$$

$$P = \frac{2}{3} MNc_l \rho S \pi^2 f^2 r^3 \sqrt{\frac{W}{2M\rho\pi r^2}} \rightarrow P = \frac{MNSc_l r^2 f^2 \sqrt{2\rho\pi^3 W/M}}{3} \qquad (5.0)$$

- c_l, Coefficient of lift.
- ρ, Air density.
- S, Surface area of the airfoil.
- M, number of motors
- f, frequency of rotation of rotary wing.
- r, Radius of rotary wing
- *W,* Weight of HPH
- N, number of blades

[8] The induced velocity, refer to equation 2.0 on page 7 of this investigation.

Experiment Aim

$$P = \frac{MNSc_l r^2 f^2 \sqrt{2\rho\pi^3 W / M}}{3} \qquad (5.0)$$

The aim of the investigation is to find the power output required to successfully operate a HPH and if the HPH is a viable form of human transport. Looking at equation 5.0, used to find the power output, we know all parameters of the Gamera II HPH aside from its coefficient of lift (c_l). Hence, the aim of the experiment is to find an accurate value for this constant. This will allow us to deduce power output for the Human Engine to fly the HPH.

Experiment setup and method

The coefficient lift was found using a model helicopter by exploring the relationship between the lift generated by helicopter (L) and frequency of rotation of rotary wings (f). Looking back we know that $L \propto f^2$, so we expect the relationship to be quadratic[9]. Fig. 4. shows the experiment setup.

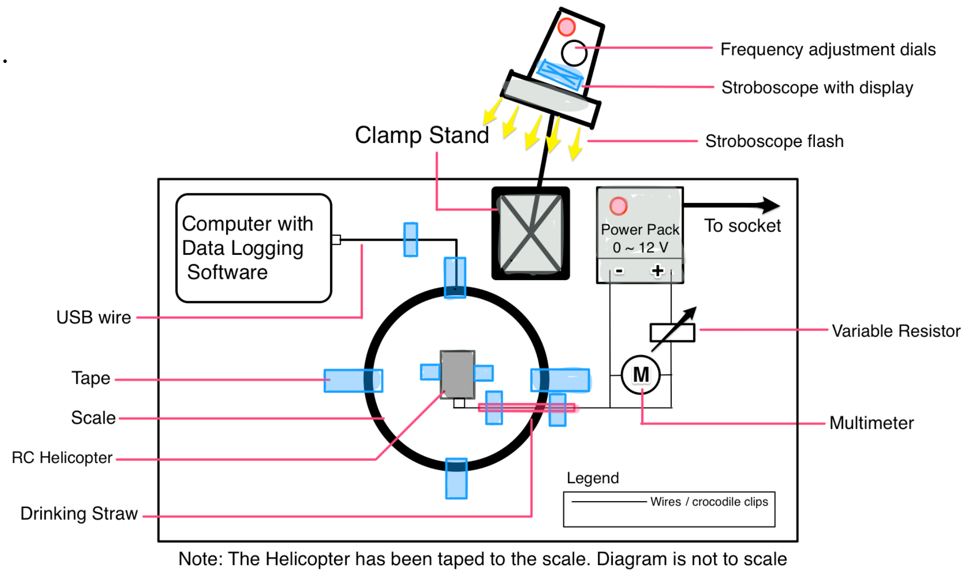

Fig. 4. The experiment setup used to find c_l

The experiment setup above uses numerous equipment to record and manipulate the frequency and lift of the helicopter. For practical reasons, the model helicopter is dismantled and connected to a mains DC electricity supply as this gives it an infinite supply of electricity to operate on without having to recharge batteries.

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[9] Equation 4.0

The rotary wing frequency (independent variable) was controlled through the voltage supplied. A higher voltage means more rotations per minute (RPM) of the airfoils. This method allowed me to overcome the difficulties of precisely controlling the helicopter, but required great understanding of the helicopter’s circuitry itself. For details, see appendix 2.

The frequency of the helicopter’s wing was recorded using a stroboscope [10] using the “stroboscopic effect”. The effect occurs when the **frequency of stroboscope flashes equals the frequency of rotary wings**, this creates the illusion of a stationary airfoil as shown in fig. 5. This illusion allows for optimal and precise measurement of the cyclic, rotating, motion of the blades and is cheaper compared to other methods.

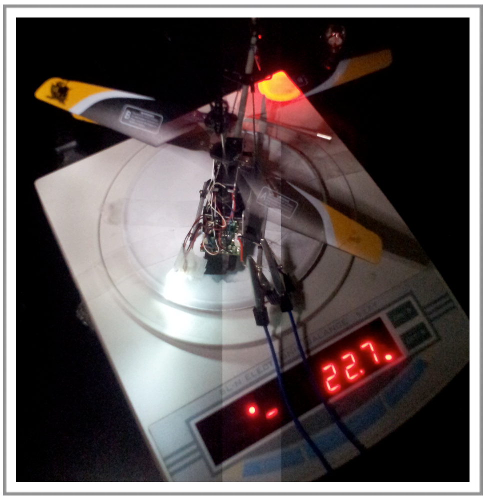

Fig. 5. The picture of the stroboscopic effect in action from trial experiment, the rotary wings are actually spinning and inducing a force on the scale, but the picture appears still. The frequency of stroboscope flash equals the frequency of blade rotation.

The average lift generated (dependent) was measured using weight scale (see fig. 5.) when the helicopter operated, a mass was induced on the weight scale which was digitally recorded using “Logger Pro 3.6.0”[11] an computer as a data-acquisition interface.

The variables are shown in table 2 (next page):

[10] A Stroboscope is a lamp capable of modulating it’s frequency on demand and emits flashes of light.

[11] Logger Pro 3.6.0 Copyright © 2007 Vernier Software & Technology

Table 2: Variable information

Independent	Dependent	Controlled
Average frequency of rotary wing (Rotations min^{-1})	Mass induced on weight scale (grams)	Inclination of experiment surface
Method used to record (in)dependent variable	Position of stroboscope on RC helicopter	Rotary wings of RC helicopter
Distance between rotary wings and weight scale	The darkness of the experiment room	Current provided to the RC helicopter

Previous trials

Previous trials were conducted as a proof-of-concept and to find major limitations or unforeseen errors in the to be corrected for in the future. Learning form the trials, amendments were made to the method as follows:

- Fixed all stationary equipment to minimize zero-offset error.
- Made sure the scale and lab desk were at level, this prevented the unforeseen problem of lift force dividing into components.
- Used automated data-acquisition procedures, giving better precision.
- Gave helicopter 5 minute rest time to prevent wear on motors
- Removed ground effect (see below).

Preparation

The preparation aimed to remove the biggest limitations learnt in the trials, this was planned for as I bought a helicopter with an easily accessible chassis and extra rotary wings.

Modifying power source

The experiment needed to provide the scale helicopter with a constant source of DC electricity. This was a practical decision because it removed any time restrictions imposed[12].This is done by removing the outer casing, cutting the battery terminals and connecting them to the DC source. See Appendix 2.

Removing ground effect

The influence of ground effect on HPH is highly detrimental to the evaluation of the vehicle as a *human powered* form of transport. Assistance provided by the ground effect in this investigation only serves to expand the values of time a human engine can sustain HPH flight and in turn give skewed results for coefficient of lift other values.

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[12] The small battery capacity of 300 mAh (milliamp hours) and 3.9 V in most RC helicopters only allow for 5~8 minutes flying time followed by 30~40 minute recharging time.

In the trials conducted, it was found that the assisting lift provided by the ground effect increases as the frequency of the helicopter increases, hence ground effect was hard to mathematically remove, it is not a constant, systematic error. However, in the main experiment, all ground effect was removed by simply positioning the rotary wings upside-down. When the helicopter operated, all the air is forced upwards instead of downwards towards ground. This removed ground effect.

Uncertainties and limitations

Using a scale model approach to find the value for c_l and human power output saves having to build a real size HPH. However, this method also puts many limitations on the model and its results.

The underlying assumption of "kinematic similarity"[13] only occurs when there is geometric similarity between the scale model and the real life HPH, while this is true for the rotary wings and the lift induced, it is not true for the body of the scale model. This results in a completely different flow of air around the body of the RC aircraft compared to the HPH, and thereby alters result of ground effect around the scale model compared to the HPH. This is the reason behind why I have chosen to remove all ground effect from the experiment. It would have been a better investigation to involve ground effect, however this is just not possible with the restrictions imposed.

Another limitation of using a scale model is that all uncertainties derived will not be applicable to the real-life HPH as they will be too negligible. Regardless I shall list the uncertainties of the equipment used in the scale model below:

<u>Equipment Uncertainties</u>

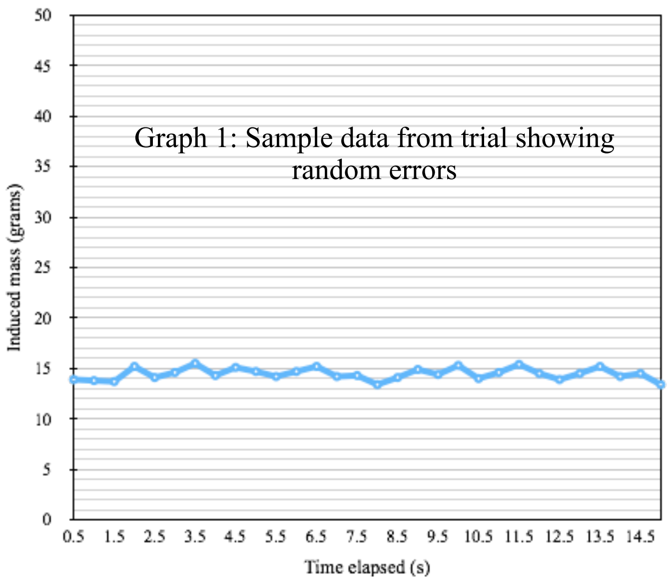
- Digital stroboscope/ tachometer has uncertainty of ± 1 FPM (Flash per minute) or ± 1/60 Hz, hence the rotary wing's frequency's uncertainty is similar[14].
- Digital weight scale uncertainty ± 0.1 grams, hence ± 0.001 N.
- Multimeter used to measure voltage and amperage has uncertainty of ± 0.01 Amps / Volts.

[13] A term implying <u>all forces</u> on both scale model and full size model <u>are scale factor of one another</u> (Cambridge-MIT Institute), first mentioned page 7.

[14] We use the stroboscope to measure the frequency of the blades.

Average (statistical mean) uncertainty

The trial experiment revealed (see graph. 1) the mass induced by the helicopter during operation fluctuates and is not constant. This fluctuation can be accounted as a random error in the experiment because it differs every trial. To counteract this error we take every data set from and its **mean** instead. The error of this mean, called *standard error,* is given below :



$$\pm\delta m = \frac{\sigma_m}{\sqrt{n}}$$ (Palmer 3).

where σ_m is the standard deviation of the total data set and n is the number of data points in total.This is done for every trial.

Method

Open DC power supply, helicopter, weight scale and computer with data logging software. Calibrate the scales to remove zero-offset error. Ensure the circuit is closed and helicopter isn't open. Start the data logger (sampling rate 2 Hz) to record mass on weight scale and open the circuit, starting the helicopter rotor. To adjust the frequency of the helicopter, adjust the variable resistor accordingly; Increase the resistance to decrease rotations, decrease the resistance to increase the rotations.

Once set at the required helicopter frequency, open the stroboscope and adjust the frequency of the flashes for the stroboscopic effect to take place. **Always** start from the highest possible value and slowly decrease the dial of the stroboscope, as this always gives the right rotary wing frequency.When the illusion that the blades aren't moving is created, the desired frequency has been reached.

Further adjust the variable resistor and the fine tune the stroboscope to reach desired exact helicopter frequency. Let the helicopter run for a durations of 20 seconds (40 data points), then close the power pack then. Allow helicopter motors to rest for 5 minutes to prevent wear to motor.Write down helicopter frequency and use the data logger to find average mass induced on weight scale.

Repeat the following procedure four times for each value of helicopter frequency (4 trials). Further repeat 4 trials for a total of 10 different frequencies in graduations of 300 RPM, range 300~3000 RPM.

Results

The copious amount raw data sampled by Logger Pro is shown in Appendix 3[15]. The following table gives the average induced mass per trial in a tabular format (Table 3). Color keys are provided to help readability

Table 3: Raw data table		Trials			
		T1	T2	T3	T4
Keys to reading table below	300 RPM (5 Hz)	300	300	300	300
	±0.0	1.4	1.4	1.4	1.4
	600 RPM (10 Hz)	600	600	600	600
Helicopter frequency (RPM) ± 1 RPM	±0.0	1.6	1.6	1.6	1.6
	900 RPM (15 Hz)	900	900	900	900
	±0.0	2.1	2.1	2.1	2.1
	1200 RPM (20 Hz)	1200	1200	1200	1200
Average mass induced (gram)	±0.0	2.7	2.6	2.7	2.7
	1500 RPM (25 Hz)	1500	1500	1500	1500
	±0.1	3.2	3.1	3.1	3.2
Standard error for average (mean) mass $\pm\delta m$ (1 d.p)	1800 RPM (30 Hz)	1800	1800	1800	1800
	±0.1	4.3	4.2	4.2	4.2
	2100 RPM (35 Hz)	2100	2100	2100	2100
	±0.1	4.9	4.9	4.9	4.9
	2400 RPM (40 Hz)	2400	2400	2400	2400
	±0.2	6.3	6.2	6.2	6.2
	2700 RPM (45 Hz)	2700	2700	2700	2700
	±0.2	7.3	7.4	7.3	7.5
	3000 RPM (50 Hz)	3000	3000	3000	3000
	±0.2	7.2	7.4	7.3	7.3

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Analysis of obtained data

- The data points from trial 1~ 4 are averaged to give one value of average induced mass per frequency. i.e for 300 RPM in Table 3, row 3:

$$(1.4+1.4+1.4+1.4)/4 = 1.4 \ (1 \ d.p)$$

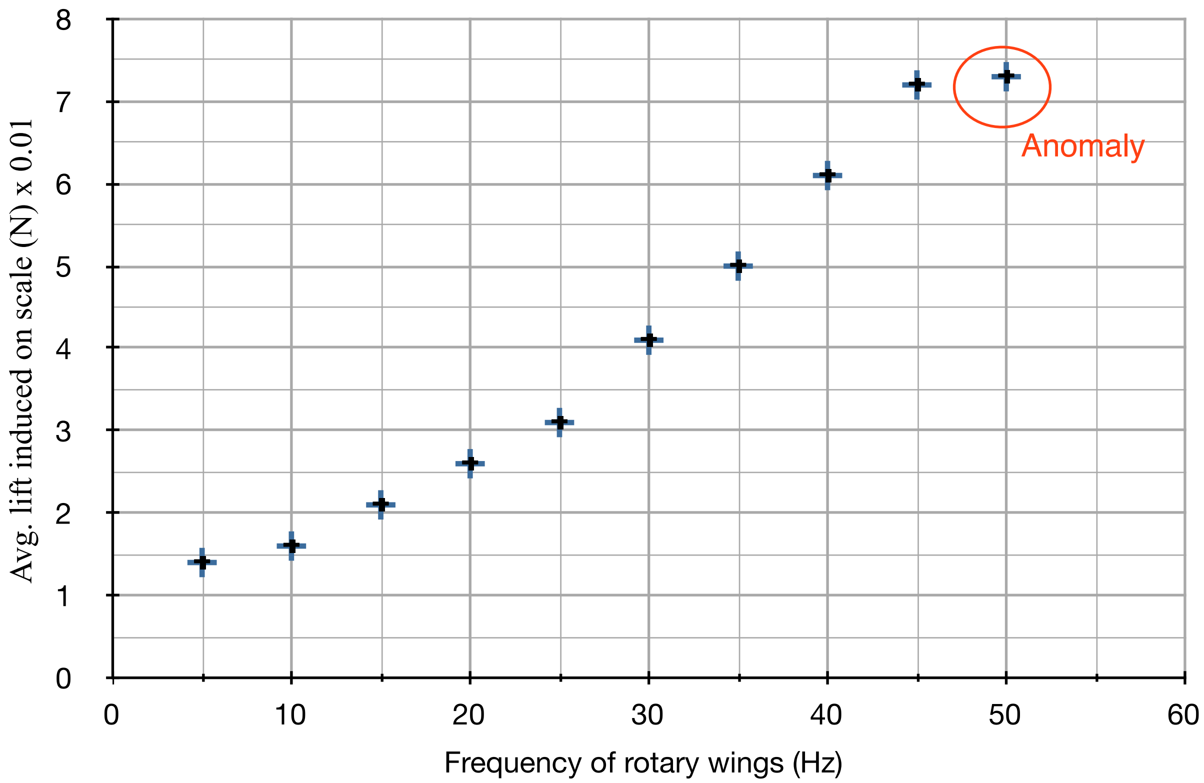
- The rotary blade frequency is converted from units RPM to Hz (divide by 60) and the dependent variable is converted from grams to Newtons (divide 1000 then multiply by 9.81).This is conversion also applied to uncertainties.

300 RPM / 60 = 60 Hz *(1.4 g * 9.81)/1000 = 0.014 N (2 s.f)*

± 1 RPM /60 = ±0.02 Hz (1 s.f)

- Finally these data points are plotted in graph 2.

Graph 2: Average lift induced vs. the frequency of the rotary wings.



The errors bars in graph 2 are too negligible to be seen.

The increasing standard error in table 3 suggests the propagation of random error increases with greater frequency, this is further supported by looking at graph 2. In graph 2, we see the data conforms to the quadratic[16] relationship between f and L until it reaches 45 Hz. Here, the halt of the progressive increase in lift and increasing standard error suggest the last data point lies on the brink of the rotary wing's lift properties. At such high rotations, I observed the rotary wings bending under force, reducing lift produced.

Overall, this shows it would be practical to categorize the last datum as anomaly when finding c_l.

Graph 2 also shows a systematic zero-offset error of 0.013 N (1.3 grams) as the best interpolated line of data (remove anomalies) gives $L = 2.7 \times 10^{-5} f^2 + 1 \times 10^{-4} f + 1.3 \times 10^{-2}$. This is corrected for.

[16] See derived equation 4.0

Finding true value of c_l and the power output of a HPH

$$L = \frac{2}{3} M N c_l \rho S \pi^2 f^2 r^3 \quad (4.0)$$

The value of c_l is found by plugging in the average data points from table 3 back into the equation 4.0 with other parameters of the model helicopter, shown in table 4

Table 4: Model helicopter information	
Rotary wing surface area (S)(m^2)	0.018 ± 0.00045 (2.5%)
Radius of airfoil (r) (m)	0.11 ± 0.0005
Number of blades per rotor (N)	2
Number of rotors (M)	1

The value of c_l obtained using the information in Table 4 and 3 result in table 5[17]:

Table 5: Value for coefficient of lift		
Frequency of rotary wing (Hz) ± 1/60	Value for coefficient of lift (2 s.f)	**Average of coefficient** 0.074 ± 0.017 (no units) The overall uncertainty in range for raw values
5	0.10	
10	0.074	
15	0.077	
20	0.074	
25	0.072	
30	0.072	
35	0.070	
40	0.068	
45	0.066	

[17] Anomaly not included and removed systematic error 0.012 N beforehand.

Finding state of ascent power requirement

The average coefficient of lift (c_l) is substituted into the equation for HPH power [5.0],

$$P = \frac{MNSc_l r^2 f^2 \sqrt{2\rho\pi^3 W / M}}{3}$$

with parameters of the Gamera II HPH from table 1 (This assumes the weight of the human pilot is 80 kg). The insertion of these variables and simplification leads to the following:

$$P = \frac{2 \times 4 \times 6.5^3 \times 0.074 \times f^2 \sqrt{2 \times 1.275 \times \pi^3 \times 1100 / 4}}{3}$$
$$P = \frac{162.58 \times f^2 147.46}{3}$$
$$P = 7991.35 f^2$$

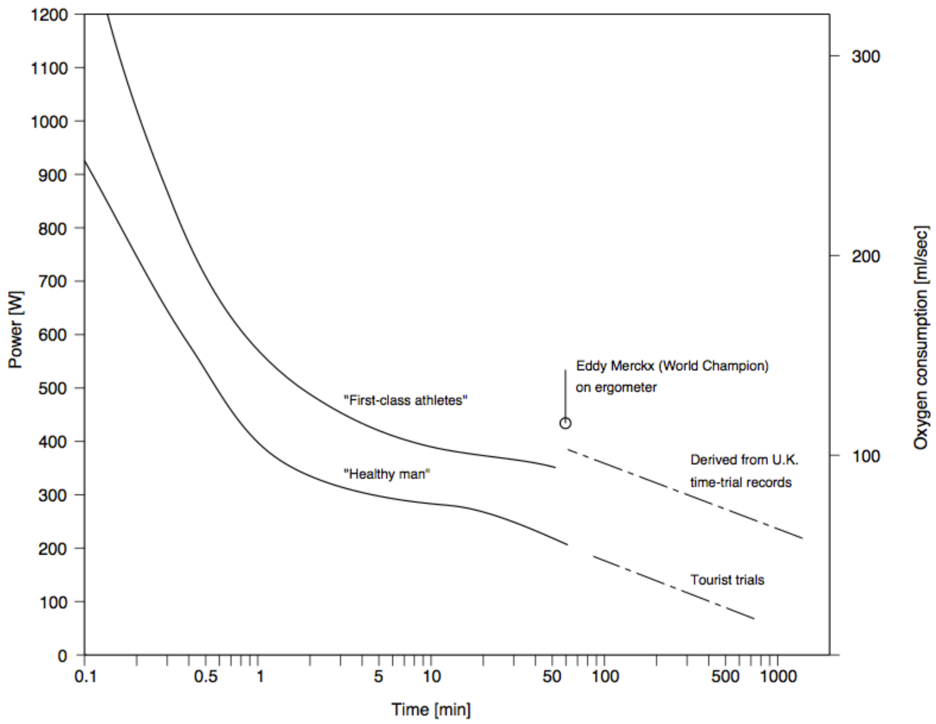
The creators of the Gamera II state the operating frequency to be 1/3 Hz (20 RPM) in the journal "Control and Stability Characteristics of Gamera II: A Human Powered Helicopter" (Staruk et. al 9).

Inserting this figure into the equation for power gives us **887.9 ± 0.017 W**[18] for a 80 kg human engine to power the Gamera II without any assistance from ground effect.

[18] The only error/uncertainty bearing figure in the equation is c_l, this small error reflects on the previous statement of the error of a scale model as not applicable to a real model.

Human Power capabilities

Since the task of powering a HPH is breaks down glycogen[19] and high energy phosphates while producing lactic acid in the body, it is a form of exercise which can only be sustained for short period of time (Evans 1). Evans, in his work, “The human power plant” states the time a man can sustain 800 Watts to be 10 seconds (Evans 1), this is further supported by graph 3 (next page):



Graph 3. The time till fatigue for different levels of power output; Evans, A. "The Human Power Plant."; Royal Aeronautical Society Human Powered Aircraft Group Conference (1989); page 9; Royal Aeronautical Society. Web.

Graph 3 shows the limited capacity of human power under exercise conditions (adapted from Evans).Here, previous research reveals the time a human can sustain power output till fatigue is considerably low for high power output, which lasts in the magnitude of seconds to a whole minute. In contrast, the polar opposite occurs for smaller power outputs, for example 100W can be sustained for a magnitude of hours in a feat of endurance. Finally, fitness itself is also a considerable factor, the ability of the individual can increase sustained time by more than 50% for short duration exercise and has even more pronounced effects in long duration exercise.

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[19] Anaerobic Glycogenolysis (Evans 1)

Conclusion and Evaluation

The disparity between the two value of powers required to reach a hovering state, 990 Watts theoretical and 887.9 Watts experimental for a 80 kg human engine is large, but can be explained considering the limitations of the scale experiment conducted. The lack of application of errors to real size Human Power Helicopter in a scale experiment hides possible large uncertainties in the final value[20], making it appear incorrect.This however doesn't mean the result is just wrong, the experimental estimate of 887.9 watts represents a feasible lower estimate of power required to fly a HPH out of ground effect that is significant in its own right.

The measurements are valid considering the aircraft is operating out of ground effect but are significantly lower than the 533 watts from real HPH flight data in ground effect given by AeroVelo, this allows a glimpse into the power savings underground effect. Sadly, current research on the impact of ground effect on the overall efficiency of a HPH is under-explored and a big unresolved topic suggested for future human powered aircraft research.

Combining our knowledge of the power requirements both to fly an HPH both in and out of ground effect with the physical capabilities of normal human being it becomes possible to answer the question **"What power output is required successfully operate a Human Powered Helicopter?"** to be 550+ Watts for a 80kg (6.9 W/kg) person in ground effect for short period of time (1~1.5 minutes[21]), under the assumptions the individual is highly trained and the helicopter is optimized for efficiency and assistance from ground effect . For all other situations such as out of ground effect, the task is simply beyond the human engine's limitations. As a direct consequence, the human powered helicopter is also unlikely to become a form of transport and take to the skies.

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[20] For example the uncertainty of ± 120 Watts would mean the 887.9 value lied in the range of answer, however, we can't be sure of this using a scale model.

[21] Found using graph 3

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Appendix 1 - Calculating width rotary blade using rotor solidity.

Data for rotary wing

Assume width is *c*. Berry et. al gives the taper ratio as 3:1, length 6.5 m, rotor solidity (σ_e) as 0.049 (Berry et al 13).

The journal article "Helicopter Aerodynamics - Actuator disk theory" Rotor Solidity is defined as **area of a single airfoil to the actuator disk area**, and states the equation as $\sigma_e = \frac{Rc}{\pi R^2}$ (Aerostudents 3). where R is the radius of the airfoil and c is the "camber" otherwise the width of the airfoil.

Because the taper ratio is 3:1, the shape of the blade is that of a trapezoid, hence the surface area is $S = \frac{\left(\frac{c}{3} + c\right)R}{2} = \frac{2cR}{3}$. The insertion of this equation in $\sigma_e = \frac{Rc}{\pi R^2}$ leads to the $c = 1.5m$ (see following equation simplification on left).

Now that we know the value for *c* we can further find the surface area S

$$0.049 = \frac{\frac{2cR}{3}}{\pi R^2}$$

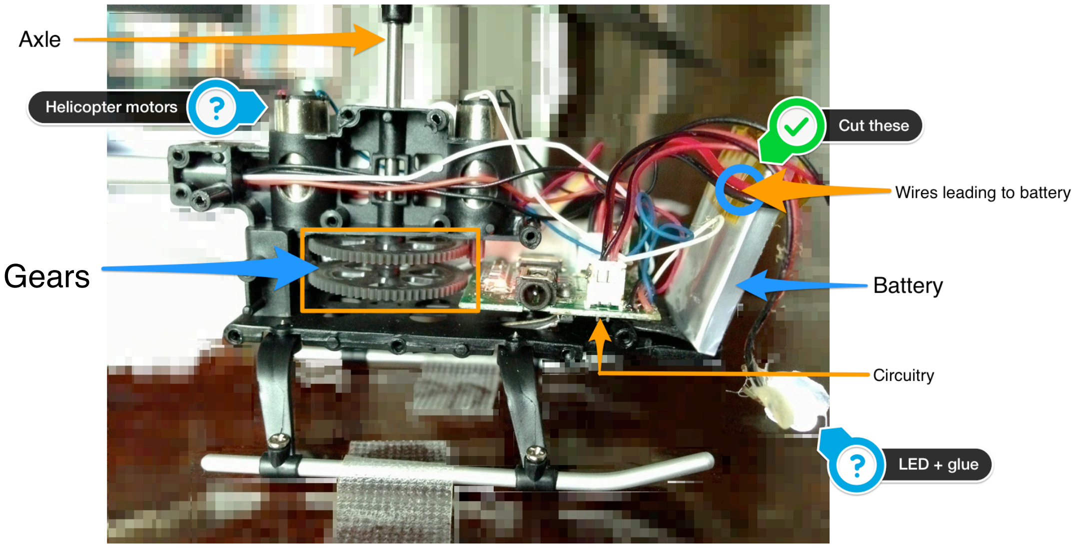
$$S == \frac{2cR}{3} = \frac{2 \times 1.5 \times 6.5}{3} = \frac{2 \times 1.5 \times 6.5}{3} = 6.5m^2$$

$$0.049 = \frac{13c}{126.75\pi}$$

$$6.21075\pi = 13c$$
$$c = 1.5m$$

Appendix 2 - Helicopter internals

The pictures shows the helicopter, in an opened state. Graphical instructions have been attached in order to dictate the process of replacing power source step-by-step.



After cutting and removing the battery, simply connect terminals of the helicopter to a DC power source (such as a power pack).

Appendix 3 - Raw Data from Logger Pro Software

Time (s) ± 0.1 s	Trial 1	Trial 2	Trial 3	Trial 4		Trial 1	Trial 2	Trial 3	Trial 4
aim frequency (RPM)	300	300	300	300		600	600	600	600
0.5	1.3	1.3	1.4	1.4		1.8	1.6	1.5	1.8
1	1.4	1.3	1.4	1.4		1.7	1.7	1.7	1.7
1.5	1.4	1.3	1.1	1.4		1.5	1.7	1.6	1.2
2	1.5	1.5	1.4	1.1		1.5	1.6	1.7	1.9
2.5	1.3	1.4	1.4	1.2		1.8	1.7	1.7	1.8
3	1.7	1.3	1.6	1.3		1.5	1.6	1.6	1.6
3.5	1.5	1.4	1.5	1.3		1.7	1.7	1.5	1.5
4	1.3	1.3	1.2	1.4		1.6	1.6	1.5	1.7
4.5	1.4	1.3	1.4	1.3		1.6	1.5	1.6	1.5
5	1.3	1.3	1.6	1.5		1.6	1.6	1.8	1.5
5.5	1.3	1.3	1.4	1.3		1.5	1.7	1.6	1.6
6	1.3	1.2	1.5	1.5		1.6	1.7	1.6	1.7
6.5	1.3	1.4	1.3	1.5		1.7	1.6	1.7	1.6
7	1.5	1.4	1.5	1.3		1.6	1.5	1.6	1.7
7.5	1.4	1.3	1.5	1.4		1.6	1.8	1.7	1.7
8	1.3	1.5	1.3	1.5		1.8	1.5	1.3	1.7
8.5	1.4	1.3	1.3	1.4		1.6	1.6	1.6	1.6
9	1.5	1.2	1.5	1.5		1.7	1.7	1.6	1.4
9.5	1.4	1.4	1.3	1.3		1.6	1.7	1.5	1.6
10	1.4	1.4	1.4	1.2		1.6	1.5	1.6	1.6
10.5	1.4	1.3	1.5	1.4		1.7	1.5	1.7	1.8
11	1.2	1.2	1.3	1.3		1.6	1.5	1.6	1.5
11.5	1.4	1.4	1.4	1.5		1.6	1.5	1.5	1.6
12	1.3	1.4	1.4	1.4		1.6	1.6	1.7	1.5
12.5	1.3	1.5	1.5	1.5		1.7	1.5	1.7	1.6
13	1.4	1.4	1.4	1.3		1.6	1.5	1.6	1.5
13.5	1.5	1.4	1.3	1.2		1.5	1.5	1.6	1.6
14	1.3	1.3	1.4	1.6		1.6	1.5	1.7	1.7
14.5	1.3	1.4	1.2	1.4		1.6	1.4	1.6	1.6
15	1.5	1.3	1.3	1.4		1.5	1.7	1.5	1.5
15.5	1.4	1.5	1.5	1.2		1.5	1.7	1.6	1.6
16	1.3	1.4	1.3	1.4		1.4	1.6	1.5	1.7
16.5	1.4	1.5	1.3	1.3		1.6	1.6	1.7	1.3
17	1.5	1.2	1.3	1.3		1.7	1.6	1.6	1.7
17.5	1.5	1.4	1.4	1.4		1.7	1.7	1.5	1.6
18	1.4	1.2	1.3	1.3		1.7	1.7	1.7	1.5
18.5	1.4	1.2	1.3	1.5		1.7	1.6	1.7	1.6
19	1.3	1.6	1.5	1.3		1.4	1.7	1.7	1.8
19.5	1.4	1.4	1.4	1.3		1.7	1.5	1.4	1.5
20	1.4	1.3	1.4	1.5		1.7	1.6	1.7	1.6

Time (s) ± 0.1 s	Trial 1	Trial 2	Trial 3	Trial 4		Trial 1	Trial 2	Trial 3	Trial 4
aim frequency (RPM)	900	900	900	900		1200	1200	1200	1200
0.5	1.9	2.3	2.2	1.9		3.1	2.6	2.7	2
1	2.4	2	2.2	1.8		2.6	2.1	3	2.5
1.5	2.1	2.1	1.9	1.9		2.6	2.5	2.4	3.3
2	2.3	2.2	2	2.3		2.5	2.6	2.7	2.1
2.5	2	2.2	1.9	1.8		2.9	2.7	2.4	2.6
3	2.3	2.2	1.8	2.1		3.2	2.5	2.5	2.5
3.5	1.9	2.2	1.9	2.3		2.4	2.3	1.9	2.6
4	2.2	2	2.5	2.2		1.7	3	2.4	2.7
4.5	1.9	2.2	2.1	2.1		2.4	2.3	2.8	3
5	2	2.3	1.9	1.9		2.9	3	2.6	2.6
5.5	2	2.2	2.1	1.9		2.9	2.6	2.8	3
6	1.9	2	2.2	2.3		2.8	2.8	2.3	3.1
6.5	2.5	2.3	2	2.2		2.4	2.3	2.8	3.1
7	2	1.9	2	1.9		2.3	2.6	3.0	2.7
7.5	2.3	2	1.8	2		3	3.2	2.2	2.4
8	1.9	2	2.1	2.2		2.5	3.3	2.5	2.6
8.5	2	1.8	1.9	2.1		2.4	2.8	2.8	2.2
9	2.8	2.1	2	1.9		2.3	2.1	1.7	2.9
9.5	1.9	2.1	2	1.8		2.2	2.4	2.5	2.7
10	2.1	1.9	2	1.8		2.9	2.8	2.5	2.4
10.5	1.9	2.4	2.2	2.4		3.4	1.9	2.8	3.2
11	2	2.3	2	2		2.8	2.3	2.8	2.3
11.5	2	2	1.9	1.9		2.9	2.4	2.5	2.9
12	2.2	2.1	1.5	2		3.1	2.2	2.7	2.1
12.5	2.1	2.1	1.9	2.4		2.4	2.7	2.4	2.8
13	2.5	2.1	2.2	2.1		2.5	2.7	2.6	2.3
13.5	1.8	2.1	1.8	1.9		2.7	2.6	2.4	3.1
14	2.3	1.8	1.7	1.7		2.6	2.5	2.7	2.6
14.5	2.2	1.9	1.9	2.5		2.7	2.4	2.4	2.7
15	2.1	1.7	1.7	2		3.2	3.0	2.7	2.6
15.5	1.8	2.2	1.9	2.2		2.7	2.3	2.9	2.8
16	2.1	2.3	2.5	2.1		2.3	2.4	2	2.6
16.5	2.1	2	2.2	1.9		2.7	2.4	2.6	2.8
17	1.9	1.8	2.4	1.9		2.5	2.4	2.9	2.7
17.5	1.9	1.8	2	1.8		2.5	3	2.9	2.2
18	2.0	1.8	1.7	2.1		3.0	2.6	2.6	2.7
18.5	2.3	2.2	1.7	2.1		2.5	2.2	2.7	2.3
19	1.9	2.2	1.9	1.8		2.6	2.9	2.5	2.5
19.5	2.1	1.7	1.8	2		2.5	2.6	2.6	2.4
20	2.2	2.0	2.0	2.2		2.7	2.4	2.9	2.3

Time (s) ± 0.1 s	Trial 1	Trial 2	Trial 3	Trial 4		Trial 1	Trial 2	Trial 3	Trial 4
aim frequency (RPM)	1500	1500	1500	1500		1800	1800	1800	1800
0.5	4.3	4	3.2	3		3.7	3.7	3.8	4.5
1	3.1	3.4	3.0	3.8		4.4	4.6	4	4.1
1.5	3.0	3	2.9	3.2		4.6	4.4	4.3	4.1
2	3.0	3.8	3.0	3		4.5	4.4	4.5	4.2
2.5	3.0	3.3	3.1	3.3		3.7	4.4	4.9	4.4
3	3.3	3.8	3.8	3.1		4.3	4.1	4.4	4.2
3.5	3.3	3.8	3.2	3.4		4.0	3.9	3.6	4.3
4	3.3	3.3	3.2	3.3		4.2	4.5	4	3.5
4.5	3.2	3.0	3.1	3.6		3.5	3.6	4.6	3.4
5	3.2	3.0	3.9	3.3		4.1	4.4	3.7	4.4
5.5	3.0	3.1	2.9	3.1		4.1	4.3	4.0	4.5
6	3.8	3.2	3.2	2.9		3.9	4.6	4.1	4.4
6.5	2.9	3.4	3.6	3.4		4.5	3.6	4.1	4.1
7	3.4	3.1	2.7	3.6		3.6	4.5	4.7	4.3
7.5	2.9	3	3	3.5		4.5	4.0	4.4	4.3
8	3.1	3.8	3.4	3.2		3.8	4.0	3.4	4.5
8.5	2.9	3.8	3.3	4		4.6	4.1	4.6	4.1
9	3.6	3.7	3.3	3.3		4.0	4.1	3.6	4.5
9.5	3	3.7	3.8	3.3		4.8	4.6	4.6	4.2
10	3.5	3.6	2.6	2.8		3.9	3.8	5.0	4.1
10.5	2.9	3.1	3.4	3.1		4.0	4.4	3.8	4.9
11	3.3	3.1	3.4	3.3		4.0	3.6	5.0	4.2
11.5	3.3	3.6	3.3	3.4		4.1	4.1	5.0	4.1
12	3.3	3.3	3.1	4.1		3.5	4.4	3.6	3.7
12.5	3	3.1	3.4	3.4		4.1	4.1	4.2	4.3
13	3	3.2	3.7	3.6		4.8	3.6	4.3	4.5
13.5	3	2.8	3.6	2.9		4.4	3.9	3.7	4.3
14	3.4	3.7	2.3	3.8		4.0	4.1	3.5	4
14.5	2.6	3.6	3	3.2		3.6	4.1	3.8	4.4
15	3	3	3	2.9		4.9	4.5	4	3.4
15.5	3.4	3	3.1	3.4		4.4	3.9	4.1	3.4
16	2.9	3	3.1	3.9		4.6	4.5	4.8	4.6
16.5	3.7	3.6	3.3	3.3		4.6	3.7	4.2	3.9
17	3	3.7	3	3.2		4.4	3.6	3.5	4.6
17.5	3.2	3.4	3.6	3.2		4.7	3.6	3.7	4
18	3.1	3.5	3.3	3.6		3.9	4	4.1	4.1
18.5	3.5	3.3	2.9	3.9		4.8	4	4	3.7
19	3.7	3.8	2.9	3.2		3.9	4.5	4.1	3.9
19.5	3.4	3.2	3.3	3.5		4.1	4.7	3.8	4.1
20	3.8	3.6	3.5	3.5		4.1	4.2	4.3	3.7

Time (s) ± 0.1 s	Trial 1	Trial 2	Trial 3	Trial 4		Trial 1	Trial 2	Trial 3	Trial 4
aim frequency (RPM)	2100	2100	2100	2100		2400	2400	2400	2400
0.5	4.8	5.2	5.4	5.1		7.5	6.3	7.3	5.6
1	4.4	4.8	4.2	5.5		5.5	6.6	5.4	5.9
1.5	5.1	5.7	4.5	4.7		6.7	6.3	5.5	7.4
2	4.8	6	4.6	5.3		5.5	6.4	5.8	6.3
2.5	5.9	5.2	5.5	5.3		6	6.5	6.6	6.8
3	5.2	4.9	5.5	5.8		6.8	7.3	5.7	6.6
3.5	5.5	5.2	4.1	4.8		6.2	6.9	5.8	5.5
4	4.6	5.1	4.5	4.6		7.3	5.5	6.5	6.5
4.5	5.0	4.2	4.3	4.9		6.0	7.1	5.5	6.6
5	4.9	5.7	6.1	5.1		5.3	5.9	7	6.7
5.5	5.7	4.5	4.6	4.8		6.5	5.8	6.4	7.0
6	6.0	4.5	5.5	4.5		6.2	5.9	6.1	5.7
6.5	5.3	4.8	5.3	4.7		5.0	6.2	6	5.5
7	5.3	5.5	4.3	5.3		6.1	5.8	5.7	6.1
7.5	4.9	5.1	4.6	5		6.1	6.0	6.1	5.4
8	4.4	5.1	6	5.7		5.8	7.3	7.9	6.7
8.5	4.6	4.5	5.3	4.7		7.0	6.8	6.6	6.9
9	4.3	5.2	4.1	4.4		5.7	5.5	5.3	6.4
9.5	5.4	4.8	5.5	4.6		7.6	6.2	5.9	6.7
10	5.8	5.7	4.4	5.2		6.9	7.2	5.9	6
10.5	4.7	5.5	5.5	4.8		6.6	6.3	6.1	5.8
11	5.5	5.1	4.6	5.1		5.3	5.4	5.7	5.7
11.5	4.7	5.6	4.6	4.4		6.7	6.5	6.6	5.5
12	5.7	4.4	5.1	4.8		7.2	6	5.8	6.2
12.5	4.5	5.2	5.9	5.3		5.8	6.3	5.9	5.1
13	4.2	5.1	5.1	4.6		6.8	6	7.0	6.7
13.5	4.2	5.6	4.1	5.5		6.9	6.2	5.5	5.8
14	4.2	4.9	5.6	4.6		6.2	6.1	6.4	6.3
14.5	4.6	4.5	5.4	4.5		6.4	5.7	6	5.4
15	4.7	6.1	4.9	5.4		6.3	5.9	5.6	6.5
15.5	5.6	5.1	4.8	5.5		5.6	7	7.2	7.3
16	5.2	5.9	5.7	4.4		5.3	5.3	6.4	6.5
16.5	4.6	4.8	5.6	5		5.9	5.7	6.2	5.6
17	4.7	5.9	5.4	5.2		6.7	5.4	5.9	6.7
17.5	4.5	5.2	4.5	5.4		6.9	7.5	7.1	6.8
18	5.2	5	4.6	5.3		6.2	6.5	6.2	5.8
18.5	6	4	4.8	4.6		6.1	6	6.6	7
19	6.4	4.4	5.3	5.5		5.8	6.3	6	5.9
19.5	5.1	4.4	5	4.8		6.1	5.9	5.5	5.9
20	5.1	4.9	5.4	5.4		5.8	6.6	5.6	5

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Time (s) ± 0.1 s	Trial 1	Trial 2	Trial 3	Trial 4		Trial 1	Trial 2	Trial 3	Trial 4
aim frequency (RPM)	2700	2700	2700	2700		3000	3000	3000	3000
0.5	7.7	7	7.7	6.4		7.2	7.9	6.8	7.9
1	8.1	7.6	7.3	7.7		7.5	7.6	6.9	7.3
1.5	6.9	7.7	7.5	6.8		8	8.6	8.2	8
2	7.3	8	7.6	8.7		7.7	6.5	7.8	7
2.5	7	6.8	8.1	7.1		7.8	7	6.7	7.2
3	8.2	8.3	7.5	7.6		6.6	6.5	7.7	7.4
3.5	7.4	8.2	6.9	7.8		7.1	7.9	7.6	8.7
4	7.9	7.6	7	6.8		7.2	7.2	7.4	7.8
4.5	6.9	7.7	6.4	7.1		7.4	7.7	7.5	7.7
5	7.3	7.7	7.5	7.1		7	6.1	8.1	7.6
5.5	7	6.9	6.5	7.8		7	7.7	7.3	7.5
6	6.8	8.1	6.9	7.8		7.6	8.7	8.7	7
6.5	7	7.3	6.9	6.7		6.8	7.6	7.2	6.1
7	6.3	7.6	7.2	8.3		6.3	7.4	7.3	7.4
7.5	6.2	7.1	7.5	8		5.9	7.8	7.6	8.4
8	7.7	7.5	6	7.6		7.2	7.3	7.5	6.9
8.5	7.6	8.2	7.7	6.7		6.3	7.5	7.4	8.3
9	7.6	7.4	7.2	7.2		8.6	7.6	7.1	6.9
9.5	6.7	7.6	7.9	7.7		7.6	7.8	6.7	8.2
10	7.6	8.3	6.9	6.5		7.9	7.4	7.3	7.7
10.5	7.1	7.4	7.7	7.9		7	8	7.8	7.1
11	6.7	8.1	7.9	7.8		7.5	6.4	7.6	8.1
11.5	7.9	7.2	7.2	8.4		6.5	7.5	6.7	8.1
12	7.4	8.7	7.3	8.1		8.9	7.7	8.3	7.4
12.5	7.4	8.4	6.5	7.1		7.8	7	6.8	6.9
13	7	8.4	8.4	7.7		7.9	8.3	7	6.7
13.5	7.2	7.8	7.1	6.3		7.3	7.8	7.5	7.4
14	8.1	7.9	7.9	6.8		7.7	6.9	7.5	7.6
14.5	7	8.1	6.9	7.9		7.9	8.4	7.8	6.2
15	8.5	7.3	7.2	7.2		6.3	8.1	7.3	7
15.5	7.4	7.5	8.5	8.3		6.9	6.9	6.8	7.4
16	7.7	7.3	7.4	6.7		7.5	7.7	7.9	7.5
16.5	6.8	6.9	6.7	7.4		7.8	7.6	6.8	9
17	7.5	7.6	6.7	7.9		8	7.4	6.5	6.9
17.5	7.6	8.2	7.3	8.1		7.9	7.6	7.2	7.3
18	6.8	7.8	6.5	7.1		7.9	7.6	7.9	8
18.5	6.9	8.4	6.7	8.5		7.2	7.1	7.9	6
19	6.3	7.6	5.5	7.9		6.3	7.6	7.8	7.1
19.5	7.2	6.7	7.1	7.8		8.2	6.6	6.7	8.3
20	7.6	6.9	7.3	6.9		7	8.6	6.4	8.7